Security Analysis of Several Group Signature Schemes

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February, 2003

Abstract. At Eurocrypt'91, Chaum and van Heyst introduced the concept of group signature. In such a scheme, each group member is allowed to sign messages on behalf of a group anonymously. However, in case of later disputes, a designated group manager can open a group signature and identify the signer. In recent years, researchers have proposed a number of new group signature schemes and improvements with different levels of security. In this paper, we present a security analysis of three group signature schemes proposed in [22, 24, 26]. By using similar methods, we successfully identify several *universally forging attacks* on these three schemes. Using our attacks, anyone (not necessarily a group member) can forge valid group signature on any message such that the forged signature cannot be opened by the group manager. Furthermore, we point out that similar attacks can also be applied to several other schemes [8, 16, 19, 21, 23]. Therefore, these eight group signature schemes all are *insecure*.

Keywords: digital signature, group signature, forgery, cryptanalysis.

1 Introduction

A group signature scheme, first introduced by Chaum and van Heyst in [5], allows each group member to sign messages on behalf of a group anonymously. However, in case of later disputes, a designated group manager can open a group signature and then identify the true signer. A secure group signature scheme must satisfy the following six properties [1–3, 5]:

1. **Unforgeability:** Only group members are able to sign messages on behalf of the group.
2. **Anonymity:** Given a valid signature of some message, identifying the actual signer is computationally hard for everyone but the group manager.
3. **Unlinkability:** Deciding whether two different valid signatures were computed by the same group member is computationally hard.
4. **Exculpability:** Neither a group member nor the group manager can sign on behalf of other group members.
5. **Traceability:** The group manager is always able to open a valid signature and identify the actual signer.
6. **Coalition-resistance:** A colluding subset of group members (even if comprised of the entire group) cannot generate a valid signature that the group manager cannot link to one of the colluding group members.

In general, group signature schemes can be classified into two different types: The schemes based on *signatures of knowledge* [3] and the schemes designed by *straightforward constructions*. The schemes in $[3, 4, 1, 27]$ belong to the first type, while the schemes proposed by $[7, 8, 24, 21, 22, 16, 26]$ belong to the second type. Some of the first type schemes are provably secure, but all those schemes are not much efficient. For example, as one of the most efficient schemes belonging this type, the scheme in [4] sill needs about 13,000 RSA modular multiplications in generation and verification a group signature (see Section 5.6 of [4]). The second type schemes are very efficient since generation and verification of a signature only need to compute several standard signatures. However, no existing scheme of the second type is proved to have provable security.

In 1998, Lee and Chang presented an efficient group signature scheme based on the discrete logarithm [8]. Their scheme is obviously linkable since two same pieces of information are included in all group signatures generated by the same group member. To provide unlinkability, Tseng and Jan proposed an improved group signature scheme in [21]. However, Sun pointed out that this improved scheme is still linkable [20]. After that, Tseng and Jan proposed another improvement to provide unlinkability [22].

At the same time, base on the Shamir's idea of identity(ID)-based cryptosystems [18], Tseng and Jan proposed an ID-based group signature scheme in [24]. Later, Popescu presented a modification to the Tseng-Jan scheme in [23], and Xian and You proposed a group signature scheme with strong separability such that the group manager can be split into a membership manager and a revocation manager [26].

In this paper, we present a security analysis of three group signature schemes proposed in $[22, 24, 26]$. By using similar methods, we successfully identify several *universally forging attacks* on these three schemes. Using our attacks, anybody can easily forge valid group signature on an arbitrary message. In addition, we point out that similar attacks apply to the schemes in $[8, 16, 19, 21, 23]$. Therefore, the group signature schemes in $[8, 16, 19, 21–24, 26]$ all are *universally forgeable*, i.e., anyone (not necessarily a group member) is able to generate a valid group signature on any message, which cannot be opened by the group manager. In our description, we not only describe how to attack these schemes, but also explain why and how we find our attacks.

The rest of this paper is organized as follows. We review and analyze the Tseng-Jan scheme I [22], the Tseng-Jan scheme II [24] and the Xia-You scheme [26] in Sections 2, 3 and 4 , respectively. Finally, the concluding remarks are given in Section 5. In the appendix A, we show that our method can be used to attack Kim et al.'s convertible group signature scheme [7].

2 Tseng-Jan Group Signature Scheme I

2.1 Review of Tseng-Jan Scheme I

Tseng-Jan group signature scheme I [22] is based on discrete logarithm problem. We review this scheme in this subsection.

Setup. Let p and q be two large primes such that $q|(p-1)$, and g a generator with order q in $\mathbb{Z}_p$. Each group member U_i selects his secret key $x_i \in_R \mathbb{Z}_q^*$, and computes his public key $y_i := g^{x_i} \bmod p$. Similarly, the group manager (GM) selects his secret key $x \in_R \mathbb{Z}_q^*$, and computes his public key $y := g^x \bmod p$. Furthermore, GM selects a one-way hash function $h(\cdot)$. To join the group, a group member U_i sends his public key y_i to GM. Then, GM randomly chooses a random number $k_i \in_R [1, q]$, computes and sends back the following pair (r_i, s_i) to U_i privately:

$$r_i := g^{-k_i} \cdot y_i^{k_i} \bmod p, \quad s_i := k_i - r_i x \bmod q. \tag{1}$$

U_i can check the validity of his certificate (x_i, r_i, s_i) by

$$g^{s_i} y^{r_i} r_i \equiv (g^{s_i} y^{r_i})^{x_i} \bmod p. \tag{2}$$

Signing. To sign a message M, U_i first selects four random numbers $a, b, d, t \in_R \mathbb{Z}_q^*$, then calculates a signature (R, S, A, B, C, D, E) as follows:

$$\begin{array}{l} A: = r_i^a \bmod p, \\ B: = as_i - b \cdot h(A||C||D||E) \bmod q, \\ C: = r_i a - d \bmod q, \\ D: = g^b \bmod p, \\ E: = y^d \bmod p, \\ \alpha_i: = g^B y^C E D^{h(A||C||D||E)} \bmod p, \\ R: = \alpha_i^t \bmod p, \\ S: = t^{-1}(h(M||R) - Rx_i) \bmod q. \end{array} \tag{3}$$

Verification. On receiving a signature (R, S, A, B, C, D, E) on a message M, a verifier first computes α_i as above and check the validity of the signature by

$$\alpha_i^{h(M||R)} \equiv (\alpha_i \cdot A)^R \cdot R^S \bmod p. \tag{4}$$

Note that the above equality holds since we have the following equations:

$$g^{s_i} y^{r_i} = g^{k_i} \bmod p, \quad \alpha_i = g^{ak_i} \bmod p, \quad \text{and} \quad \alpha_i A = \alpha_i^{x_i} \bmod p. \tag{5}$$

Open. To identify the signer of a valid group signature (R, S, A, B, C, D, E) on a message M, GM first computes the corresponding α_i and then find the signer by searching which pair (r_i, s_i, k_i) satisfies $\alpha_i \equiv (g^C \cdot E^{x^{-1}})^{r_i^{-1} \cdot k_i} \bmod p$, where x^{-1} and $r_i^{-1} \cdot k_i$ all are computed in $\mathbb{Z}_q$.

2.2 Security Analysis of Tseng-Jan Scheme I

Forging Signatures. Now we want to forge a group signature on an arbitrary message M even though we do not know any certificate, i.e., we need to find a tuple (R, S, A, B, C, D, E) that satisfies the following two verification equations:

$$\begin{cases} \alpha_i = g^B y^C E D^{h(A||C||D||E)} \bmod p, \\ \alpha_i^{h(M||R)} = (\alpha_i \cdot A)^R \cdot R^S \bmod p. \end{cases} \tag{6}$$

Note that in the generation of a signature, A, D, E and R all are some powers to the bases g and y. At the same time, C is embedded in the hash value $h(A||C||D||E)$.

Therefore, we can define A, D, E, R as some known powers of g and y, and choose a value for C. Then, we try to solve B and S from equation (6). For this sake, we choose nine numbers $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, C \in \mathbb{Z}_q$ to define A, D, E and R as follows (all in $\mathbb{Z}_p$)

$$A := g^{a_1}y^{b_1}, \quad D := g^{a_2}y^{b_2}, \quad E := g^{a_3}y^{b_3}, \quad R := g^{a_4}y^{b_4}.$$

Then, we evaluate the two hash values $h := h(A||C||D||E)$, $h' := h(M||R)$, and replace the corresponding variables in equation (6) with the above expressions. Therefore, we get the following two equations for unknown variables of B and S:

$$\begin{cases} (B + a_3 + a_2h)h' = (B + a_3 + a_2h)R + a_1R + a_4S \bmod q, \\ (C + b_3 + b_2h)h' \ = (C + b_3 + b_2h)R + b_1R + b_4S \bmod q. \end{cases} \tag{7}$$

Therefore, if $b_4 \neq 0$ and $R \neq h' \bmod q$ (i.e., $R \neq h(M||R) \bmod q$.), we get the following solutions for S and B:

$$\begin{cases} S = b_4^{-1}[(C + b_3 + b_2h)(h' - R) - b_1R] \bmod q, \\ B = (a_1R + a_4S)(h' - R)^{-1} - (a_3 + a_2h) \bmod q. \end{cases} \tag{8}$$

For summary, in the Tseng-Jan group signature scheme I [22], an attacker can forge a group signature on any message M as follows:

1. Select nine random numbers $a_1, a_2, a_3, a_4, b_1, b_2, b_3, b_4, C \in_R \mathbb{Z}_q$ such that $b_4 \neq 0$.
2. Define $A := g^{a_1}y^{b_1}$, $D := g^{a_2}y^{b_2}$, $E := g^{a_3}y^{b_3}$, and $R := g^{a_4}y^{b_4}$ (all in $\mathbb{Z}_p$).
3. Evaluate $h := h(A||C||D||E)$ and $h' := h(m||R)$.
4. Determine if $R = h' \bmod q$. If yes, go to step (1); otherwise, continue.
5. Compute S and B according to equation (8).
6. Output (R, S, A, B, C, D, E) as a group signature on the message M.

The correctness of the above attack can be verified directly. When one such forged group signature is given, of course, GM cannot find the signer. At the same time, note that in the above attack $R = h' \bmod q$ occurs only with a negligible probability since $h(\cdot)$ is a one-way hash function. Therefore, in general, our attack will succeed just by one try. Furthermore, for simplicity, some of those nine random numbers can be set as zeroes. For example, if we set $a_1 = b_2 = b_3 = a_4 = 0$, A, D, E and R can be computed simply: $A := y^{b_1} \bmod p$, $D := g^{a_2} \bmod p$, $E := g^{a_3} \bmod p$, $R := y^{b_4} \bmod p$. In such case, S and B can be computed by $S = b_4^{-1}(Ch' - CR - b_1R) \bmod q$ and $B = -(a_3 + a_2h) \bmod q$.

Forging Certificates. The authors of [8, 21] noted that for any group member U_i, (r_i, s_i) is a Nyberg-Rueppel signature [12] on message $y_i^{k_i}$. However, this *does not* imply that only GM can generate a valid certificate. Now, we demonstrate how to forge a certificate $(\bar{x}_i, \bar{r}_i, \bar{s}_i)$ that satisfies the equation (2). For this sake, we choose $a_0, b_0 \in \mathbb{Z}_q^*$, and define $\bar{r}_i := g^{a_0}y^{b_0} \bmod p$. Then, from equation (2), we have the following equation for unknown $\bar{x}_i$ and $\bar{s}_i$:

$$g^{\bar{s}_i}y^{\bar{r}_i}g^{a_0}y^{b_0} = (g^{\bar{s}_i}y^{\bar{r}_i})^{\bar{x}_i} \bmod p.$$

From the above equation, we get the following two equations for $\bar{x}_i$ and $\bar{s}_i$:

$$\bar{s}_i + a_0 = \bar{s}_i \cdot \bar{x}_i \bmod q, \quad \text{and} \quad \bar{r}_i + b_0 = \bar{r}_i \cdot \bar{x}_i \bmod q.$$

Therefore, we obtain the solutions for $\bar{x}_i$ and $\bar{s}_i$: $\bar{x}_i = 1 + b_0\bar{r}_i^{-1} \bmod q$ and $\bar{s}_i = a_0 b_0^{-1}\bar{r}_i \bmod q$. The forged certificate $(\bar{x}_i, \bar{r}_i, \bar{s}_i)$ satisfies equation (2) since $g^{\bar{s}_i} y^{\bar{r}_i} \bar{r}_i = g^{a_0 b_0^{-1}\bar{r}_i} y^{\bar{r}_i} g^{a_0} y^{b_0} = g^{a_0 b_0^{-1}\bar{r}_i(1+b_0\bar{r}_i^{-1})} y^{\bar{r}_i(1+b_0\bar{r}_i^{-1})} = (g^{\bar{s}_i} y^{\bar{r}_i})^{\bar{x}_i} \bmod p$.

Now, an attacker can use the forged certificate $(\bar{x}_i, \bar{r}_i, \bar{s}_i)$ to generate valid group signature on any message M as a group member dose. Firstly, the attacker chooses $a, b, d, t \in_R \mathbb{Z}_q^*$ and computes $A := \bar{r}_i^a \bmod p, B := a\bar{s}_i - b \cdot h(A||C||D||E) \bmod q, C := \bar{r}_i a - d \bmod q, D := g^b \bmod p$ and $E := y^d \bmod p$. Then, he computes $\bar{\alpha}_i := g^B y^C E D^{h(A||C||D||E)} = (\bar{\beta}_i)^a \bmod p$, where $\bar{\beta}_i := g^{\bar{s}_i} y^{\bar{r}_i} \bmod p$. Finally, he gets $R := \bar{\alpha}_i^t \bmod p$ and $S := t^{-1}[h(M||R) - R\bar{x}_i] \bmod q$. By using the facts that $\bar{\alpha}_i = (\bar{\beta}_i)^a \bmod p$ and $\bar{\alpha}_i A = (\bar{\beta}_i)^{a\bar{x}_i} \bmod p$, it is not difficult to verify that the resulting tuple (R, S, A, B, C, D, E) satisfies the verification equation (4), i.e., the forged group signature for the message M is valid.

Remark 1. The schemes proposed in [8, 21, 19] all are subject to similar attacks due to their similar structures. Especially, the above forged certificate can be directly used to generate valid group signatures in those schemes since all those schemes use the same certificate as in Tseng-Jan scheme I [22].

3 Tseng-Jan Group Signature Scheme II

3.1 Review of Tseng-Jan Scheme II

Tseng-Jan group signature scheme II [24] involves four parties: a trusted authority (TA), the group manager (GM), the group members, and the verifiers. TA acts as a third party to setup the system parameters. GM selects the group public/secret key pair. He (jointly with TA) issues certificates to new users who wants to join the group. Then, group members can anonymously sign on behalf of the group by using their membership certificates and verifiers check the validity of a group signature by using the group public key. In case of disputes, GM opens the contentious group signatures to reveal the identity of the actual signer.

System Initialization. In order to set up the system, TA sets a modulus $n = p_1 p_2$ where p_1 and p_2 are two large prime numbers (about 120 decimal digits) such that $p_1 = 3 \bmod 8$, $p_2 = 7 \bmod 8$, and $(p_1 - 1)/2$ and $(p_2 - 1)/2$ are smooth, odd and co-prime. Furthermore, $(p_1 - 1)/2$ and $(p_2 - 1)/2$ should contain several prime factors of about 20 decimal digits but no large prime factors. In this case, it is easy for TA to find the discrete logarithms for p_1 and p_2 [10, 11, 14, 15]. TA also defines e, d, v, t satisfying $ed = 1 \bmod \phi(n)$ and $vt = 1 \bmod \phi(n)$. Then, he selects an element g of large order in $\mathbb{Z}_n^*$, and computes $F := g^v \bmod n$. GM selects a secret key x and computes the corresponding public key $y := F^x \bmod n$. GM also chooses a hash function $h(\cdot)$. The public parameters are $(n, e, g, F, y, h(\cdot))$, and the secret parameters are (p_1, p_2, d, v, t, x).

When a user U_i (with identity information D_i) wants to join the group, TA and GM computes and sends the following s_i and x_i to U_i, respectively.

$$s_i := et \cdot \log_g ID_i \bmod \phi(n), \quad \text{and} \quad x_i := {ID_i}^x \bmod n. \tag{9}$$

where

$$ID_i := \begin{cases} D_i, & \text{if Jacobi symbol } (D_i|n) = 1; \\ 2D_i, & \text{if Jacobi symbol } (D_i|n) = -1. \end{cases} \quad (10)$$

The equation (10) guarantees the existence of the discrete logarithm of ID_i to the base g [11]. The membership certificate of the user U_i is (s_i, x_i).

Signing and Verification. To sign a message M, U_i first chooses two random integers r_1 and $r_2 \in \mathbb{Z}_n$. Then, U_i computes his group signature (A, B, C, D) on the message M as follows:

$$\begin{aligned} A &:= y^{r_1} \bmod n \\ B &:= y^{r_2 e} \bmod n \\ C &:= s_i + r_1 \cdot h(M||A||B) + r_2 e \\ D &:= x_i \cdot y^{r_2 \cdot h(M||A||B||C)} \bmod n. \end{aligned} \quad (11)$$

Note that comparing with the scheme [23], the D in equation (11) is computed in a different way. Upon receiving a signature tuple (A, B, C, D) on message M, a verifier can verify the validity of this signature by checking whether

$$D^e A^{h(M||A||B)} B \equiv y^C B^{h(M||A||B||C)} \bmod n. \quad (12)$$

Open. GM with the secret key x can identify the signer of a signature by finding the ID_i that satisfies the following equation:

$$(ID_i)^{xe} \equiv D^e \cdot B^{-h(M||A||B||C)} \bmod n. \quad (13)$$

3.2 Security Analysis of Tseng-Jan Scheme II

In [24], Tseng and Jan provide detailed security analysis to show that their scheme is secure against forgeries. However, we can identify two attacks that allow anybody to forge valid group signature on any message.

Forging Signatures. Similar to what we did in Section 2.2, we want to forge a group signature for an arbitrary message M even without any membership certificate. Note that the verification equation (12) is about some powers of A, B, D and y. So we first define A, B, D as some known powers to the base y, and then try to solve C from equation (12). Therefore, we choose three random number r_1, r_2, r_4 and define A, B, D as follows (A and B have the same forms as in equation (11)):

$$A := y^{r_1} \bmod n; \quad B := y^{r_2 e} \bmod n; \quad D := y^{r_4} \bmod n.$$

Then, from the verification equation (12), we get the condition for the value C:

$$r_4 e + r_1 \cdot h(M||A||B) + r_2 e = C + r_2 e \cdot h(M||A||B||C) \bmod \phi(n). \quad (14)$$

We have selected r_1, r_2 and r_4, so A, B, D and then hash value $h(M||A||B)$ all are fixed. Therefore, finding a solution for unknown value C from equation (14) seems difficult because we do not know the modulus $\phi(n)$ and the value of C is embedded in the hash value $h(M||A||B||C)$. However, we note that solving equation (14) seems really difficult only if r_1, r_2 and r_4 are truly selected as *random* numbers. But, we are attackers. So we have the freedom to choose some special values for r_1, r_2 and r_4. In other words, to get a solution for the value C, we can let those numbers satisfy some specific relationships. Up to this point, it is not difficult to find the following solution for equation (14):

$$C := r_1 \cdot h(M||A||B) + r_2 e \in \mathbb{Z}^+; \quad r_4 := r_2 \cdot h(M||A||B||C) \in \mathbb{Z}^+.$$

Now, we summary our attack on the Tseng-Jan scheme II [24] as follows:

1. Firstly, select two random numbers r_1, r_2.
2. Then define $A := y^{r_1} \bmod n$, and $B := y^{r_2 e} \bmod n$.
3. Compute $C := r_1 \cdot h(M||A||B) + r_2 e \in \mathbb{Z}^+$.
4. Define $r_4 := r_2 \cdot h(M||A||B||C) \in \mathbb{Z}^+$, and then compute $D := y^{r_4} \bmod n$.
5. Output (A, B, C, D) as a group signature on the message M.

It is easy to check that the above attack is correct. At the same time, when such a forged signature is given, the group manager cannot find any group member to take responsible for it.

In fact, if we choose a new random number r_3, the values of C and D in the above attack can be randomized by defining C and r_4 as follows

$$C := r_1 \cdot h(M||A||B) + r_2 e + r_3 e \in \mathbb{Z}^+; \quad r_4 := r_2 \cdot h(M||A||B||C) + r_3 \in \mathbb{Z}^+.$$

Furthermore, we have another idea to solve equation (14): First define A, B and C, then calculate hash values of $h(M||A||B)$ and $h(M||A||B||C)$, and finally solve r_4 for D. However, it seems difficult to find the value of r_4 from equation (14) since we do not know the values of modulus $\phi(n)$ and $e^{-1} \bmod \phi(n)$. But we notice that we can find a value for r_4 if e can be eliminated from equation (14). Here is the trick. We use $r_1 e$ to replace r_1 (i.e., $A := y^{r_1 e} \bmod p$) and define $C := r_3 e$ (in $\mathbb{Z}$) for some random number r_3, then r_4 can be attained:

$$r_4 := r_3 + r_2 \cdot h(M||A||B||C) - r_1 \cdot h(M||A||B) - r_2 \in \mathbb{Z}.$$

Forging Certificates. From equation (9), we have $g^{s_i} = ID_i^{et}$. Therefore, $y^{s_i} = F^{xs_i} = g^{vxs_i} = (ID_i)^{etvx} = (ID_i)^{ex} = {x_i}^e \bmod n$. That is, any valid membership certificate (s_i, x_i) satisfies the following condition:

$${x_i}^e \equiv y^{s_i} \bmod n.$$

However, the above equation does not guarantee that only TA and GM together can generate a valid certificate. In fact, if k is a non-negative integer, there are two ways to generate valid certificates: (1) A group member U_i can generate a new certificate $(ks_i, x_i^k \bmod n)$; (2) Anybody (not necessarily a group member) can use $(\bar{s}_i = ke, \bar{x}_i = y^k \bmod n)$ as a valid certificate. Given a valid group signature generated by using such forged certificates, of course, GM cannot identify the signer.

Remark 2. In [16], Popescu proposed a group signature scheme which is a modification of the scheme in [23]. We find that similar attacks apply to the schemes in [23, 16], i.e., these two schemes are also universally forgeable.

4 Xia-You Group Signature Scheme

4.1 Review of Xia-You Scheme

Setup of Trusted Authority (TA). TA generates two prime numbers p_1 and p_2 satisfying the same conditions listed in the Setup of Tseng-Jan scheme II and sets $m := p_1 p_2$. In this case, it is easy for TA to find the discrete logarithms modulo p_1 and p_2. An integer g is chosen such that $g < \min\{p_1, p_2\}$. Finally, TA publishes (m, g) but keeps the prime factors p_1 and p_2 as his secret.

Generating Private Keys. Since a signer U_i's identity information D_i (which is smaller than m) is not guaranteed to have a discrete logarithm modulo the composite number m, TA computes ID_i by equation (10) (respect to modulus m). Now TA computes the private key x_i for U_i as the discrete logarithm of ID_i to the base g:

$$ID_i = g^{x_i} \bmod m. \tag{15}$$

Finally, TA sends x_i to U_i in a secure way and U_i can check the validity of x_i by verifying equation (15). The reader can refer to [10, 11] for details.

Setup of Group Manager (GM). GM chooses two large primes p_3 and p_4 such that $p_3 - 1$ and $p_4 - 1$ are not smooth, and sets $n = p_3 p_4$ such that $n > m$. Let e be an integer satisfying $\gcd(e, \phi(n)) = 1$, and computes d such that $ed = 1 \bmod \phi(n)$. Then, GM chooses two integers $x \in \mathbb{Z}_m, h \in \mathbb{Z}_m^*$, and then computes $y := h^x \bmod m$ as the group public key. Let $H(\cdot)$ be a collision-resistant hash function that maps $\{0,1\}^*$ to $\mathbb{Z}_m$. The group public key is (n, e, h, y, H) and GM's secret key is (x, d, p_3, p_4).

Generating Membership Keys. When a signer U_i wants to join the group, GM computes the membership key z_i of U_i as follows

$$z_i = ID_i^d \bmod n. \tag{16}$$

Then, z_i is sent to U_i in a secure way and U_i checks the validity of z_i by verifying $ID_i = z_i^e \bmod n$.

Signing. To sign a message M, U_i first chooses five random numbers $\alpha, \beta, \theta, \omega \in \mathbb{Z}_m$ and $\delta \in \mathbb{Z}_n$, and then computes the signature (A, B, C, D, E, F, G) as follows:

$$\begin{aligned}
A &:= y^{\alpha} \cdot z_i \bmod n, \\
B &:= y^{\omega} \cdot ID_i, \\
C &:= h^{\omega} \bmod m, \\
D &:= H(y||g||h||A||B||\hat{B}||C||v||t_1||t_2||t_3||M), \\
E &:= \delta - D(\alpha e - \omega), \\
F &:= \beta - D\omega, \\
G &:= \theta - Dx_i,
\end{aligned} \tag{17}$$

where

$$\hat{B} := B \bmod m, \quad v := (A^e / B) \bmod n;$$
$$t_1 := y^{\delta} \bmod n, \quad t_2 := y^{\beta} \cdot g^{\theta} \bmod m, \quad t_3 := h^{\beta} \bmod m.$$

Verification. A verifier accepts a signature (A, B, C, D, E, F, G) on a message M if and only if

$$D \equiv H(y||g||h||A||B||\hat{B}||C||v||t_1{}'||t_2{}'||t_3{}'||M), \tag{18}$$

where $\hat{B}$ and v are computed as in signing equation, i.e., $\hat{B} = B \bmod m, v = (A^e/B) \bmod n$, but t_1', t_2' and t_3' are given by the following equations

$$t_1' := v^D y^E \bmod n, \quad t_2' := \hat{B}^D y^F g^G \bmod m, \quad t_3' := C^D h^F \bmod m. \tag{19}$$

Open. Given a valid group signature (A, B, C, D, E, F, G) on a message M, the group manager can identify the signer by finding the ID_i such that

$$ID_i = B \cdot C^{-x} \bmod m.$$

4.2 Security Analysis of Xia-You Scheme

Xia and You claimed that their scheme [26] satisfies all the security properties listed in Section 1. However, in this subsection we will present two attacks to show that Xia-You scheme [26] is insecure.

Forging Signatures. Using similar method used in the previous sections, it is still possible to forge a group signature on an arbitrary given message M even without any membership certificate (ID_i, x_i, z_i) for Xia-You scheme. Note that to satisfy the verification equations (18) and (19), we can first choose A, B, C and t_1, t_2, t_3, then we get D by evaluating the corresponding hash value, and finally try to solve the values of E, F and G from equation (19). If we observe equations (17)-(19) carefully, we will know that a good strategy is to choose A, B, t_1 and t_2 as some known representations of bases y and g, but C and t_3 as powers of h. Therefore, we can choose ten random numbers $a_1, a_2, a_3, a_4, a_5, b_1, b_2, b_3, b_4, b_5$ to define A, B, C and t_1, t_2, t_3 as follows:

$$\begin{array}{lcl} A := y^{a_1} \cdot g^{a_2} \bmod n, & & t_1 := y^{b_1} \cdot g^{b_5} \bmod n, \\ B := y^{a_3} \cdot g^{a_4}, & \text{and} & t_2 := y^{b_2} \cdot g^{b_3} \bmod m, \\ C := h^{a_5} \bmod m. & & t_3 := h^{b_4} \bmod m. \end{array}$$

Then, we compute $\hat{B} := B \bmod m$, $v := (A^e/B) \bmod n = y^{a_1 e - a_3} \cdot g^{a_2 e - a_4} \bmod n$ and evaluate the hash value $D = H(y||g||h||A||B||\hat{B}||C||v||t_1||t_2||t_3||M)$. At last, to get the values of E, F and G, we replace the occurrences of $t_1', t_2', t_3', \hat{B}$ and v in equations (19) by t_1, t_2, t_3, B and $y^{a_1 e - a_3} \cdot g^{a_2 e - a_4} \bmod n$, respectively, and then we have

$$\begin{array}{l} b_1 = (a_1 e - a_3)D + E \bmod \phi(n), \\ b_5 = (a_2 e - a_4)D \bmod \phi(n), \\ b_2 = a_3 D + F \bmod \phi(m), \\ b_3 = a_4 D + G \bmod \phi(m), \\ b_4 = a_5 D + F \bmod \phi(m). \end{array}$$

In general, we cannot find a solution for (E, F, G) from the above equation system. However, we can set the ten numbers, i.e., $a_1, \cdots, b_5$, satisfying specific relationships such that the above equation system has one solution. First, please note that we should set $b_5 = 0$. Because D is determined by those ten numbers, we cannot require $b_5 = (a_2 e - a_4)D \bmod \phi(n)$ again. $b_5 = 0$ also implies that $a_2 e - a_4 = 0$, i.e., $a_4 = a_2 e$ (in $\mathbb{Z}$). Secondly, we notice that F has to satisfy the third and the fifth equations at the same time, so we should set these two equations as the same one. This means that

$a_5 = a_3$ and $b_4 = b_2$. Therefore, under the conditions of $b_5 = 0$, $a_4 = a_2 e$, $a_5 = a_3$ and $b_4 = b_2$, we get the following solution for (E, F, G) even though we do not know the values of $\phi(m)$ and $\phi(n)$:

$$E := b_1 + (a_3 - a_1 e)D \in \mathbb{Z}, \quad F := b_2 - a_3 D \in \mathbb{Z}, \quad G := b_3 - a_2 e D \in \mathbb{Z}.$$

In summary, to forge a valid group signature on a message M, an attacker can work as follows:

1. First of all, select six random numbers $a_1, a_2, a_3, b_1, b_2, b_3$.
2. Then, define $A := y^{a_1} \cdot g^{a_2} \bmod n$, $B := y^{a_3} \cdot g^{a_2 e}, C := h^{a_3} \bmod m, t_1 := y^{b_1} \bmod n, t_2 := y^{b_2} \cdot g^{b_3} \bmod m, t_3 := h^{b_2} \bmod m$.
3. Compute $\hat{B} := B \bmod m$ and $v := (A^e/B) \bmod n = y^{a_1 e - a_3} \bmod n$, and then $D := H(y||g||h||A||B||\hat{B}||C||v||t_1||t_2||t_3||M)$.
4. Compute $E := b_1 + (a_3 - a_1 e)D \in \mathbb{Z}, F := b_2 - a_3 D \in \mathbb{Z}$, and $G := b_3 - a_2 e D \in \mathbb{Z}$.
5. Output (A, B, C, D, E, F, G) as a group signature on the message M.

Again, it is not difficult to verify that the above attack is successful.

Forging Certificates. Similarly, we can get the following conditions for a valid membership certificate $(\overline{ID}_i, \bar{x}_i, \bar{z}_i)$:

$$\bar{z}_i^e = \overline{ID}_i \bmod n, \quad \text{and} \quad \overline{ID}_i = g^{\bar{x}_i} \bmod m.$$

These two conditions are the exact equations (15) and (16). So, it seems that valid membership certificates can only be generated jointly by TA and GM. However, for any non-negative integer k, it is not difficult to see that (1) A group member U_i with membership certificate (ID_i, x_i, z_i) can generate a valid membership certificates $(ID_i^k, kx_i, z_i^k \bmod n)$, and (b) anyone (not necessarily a group member) can use $(\overline{ID} := g^{ke}, \bar{x} := ke, \bar{z} := g^k \bmod n)$ as a valid certificate to generate group signature on any message. Given a valid signature generated by using such forged membership certificate, of course, GM cannot identify the signer.

5 Concluding Remarks

In this paper, by using similar methods, we successfully identified several universally forging attacks on three group signature schemes proposed in [22, 24, 26]. Using our attacks, anybody (not necessarily a group member) can forge valid group signature on any message. At the same time, we point out that similar attacks apply to the schemes in [8, 16, 19, 21, 23]. Therefore, the group signature schemes in [8, 16, 19, 21–24, 26] all are universally forgeable.

In addition, using our method, we can unify some existing attacks on Kim et al.'s convertible group signature scheme [7] in a family. Those existing attacks are pointed out by [9, 17, 25] independently and accidentally. Furthermore, we find a new problem in Kim et al.'s scheme, that is, a valid group signature signed by one group member is also a possible valid group signature of other group members for the same message. Therefore, their group signature scheme is information-theoretically *anonymous* even for the group manager, and hence all valid group signatures are completely *untraceable* and *unlinkable*. Our attacks on the scheme [7] are given in Appdendix A.

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A Kim-Park-Won Convertible Group Signature Scheme

A.1 Review of Kim-Park-Won Scheme

To set up a system, the GM first chooses three primes p', q', f such that $p := 2fp' + 1$ and $q := 2fq' + 1$ are also primes. Then, the GM sets $n = p\,q$ and selects an element $g \in \mathbb{Z}_n^*$ of order f, i.e., $g^f = 1 \bmod n$. Furthermore, the GM chooses $\gamma \in \mathbb{Z}_{\phi(n)}^*$ and computes d such that $\gamma d = 1 \bmod \phi(n)$. Let ID_G be the identity information of the group, $h(\cdot)$ a secure hash function. Finally, the GM makes $(n, \gamma, f, g, h(\cdot), ID_G)$ as public information, (d, p', q') as his private key.

To joint the group, a user U_i with identity information ID_i chooses a random secret number $s_i \in (0, f)$, then computes $y_i := g^{s_i}$ and sends (ID_i, y_i) to the GM. Then, the GM computes and sends following x_i to U_i securely:

$$x_i := (ID_G \cdot y_i)^{-d} \bmod n. \tag{20}$$

At the same time, to identify signers in case of disputes, the GM stores (ID_i, y_i, x_i) into a complete list for all registered group members.

To generate a group signature (e, z_1, z_2) on message M, user U_i first chooses two random numbers $r_1 \in_R [0, f), r_2 \in_R [0, n)$ and then computes:

$$\begin{aligned} V &:= g^{r_1} r_2^{\gamma} \bmod n \\ e &:= h(V||M) \\ z_1 &:= r_1 + s_i e \bmod f \\ z_2 &:= r_2 {x_i}^{e} \bmod n. \end{aligned} \tag{21}$$

To verify a group signature (e, z_1, z_2), a verifier checks whether

$$e \equiv h(\bar{V}||M), \quad \text{where } \bar{V} := (ID_G)^e g^{z_1} {z_2}^{\gamma} \bmod n. \tag{22}$$

To open a valid group signature (e, z_1, z_2) for message M, the GM first calculates $\bar{V} := (ID_G)^e g^{z_1} {z_2}^{\gamma} \bmod n$, and then searches his list of all (ID_j, y_j, x_j) to find the signer U_i if U_i's (x_i, y_i) satisfies the following equality

$$g^{z_1} \equiv \bar{V} \cdot z_2^{-\gamma} \cdot x_i^{e\gamma} \cdot y_i^e \bmod n. \tag{23}$$

A.2 Security Analysis of Kim-Park-Won Scheme

Forging Signatures. Now, we first try to use the similar method used in previous sections to forge a valid group signature on any given message M under the assumption that we do not know any valid membership certificate. Note that the verification equation is to evaluate a hash value, and we have assumed that $h(\cdot)$ is a secure hash function. Therefore, if we first choose value for e, it seems difficult to find a tuple (V, z_1, z_2) such that both relations in verification equation (22) are satisfied. So we go in the other direction, i.e., we first choose a value for V and calculate $e := h(V||M)$, then we try to find a pair (z_1, z_2) satisfying the following equality:

$$V \equiv (ID_G)^e g^{z_1} {z_2}^{\gamma} \bmod n.$$

Note that the above equation is about several powers of ID_G, g and z_2, so we choose four numbers, a_1, a_2, b_1, b_2, and then define V and z_2 as follows

$$V := (ID_G)^{a_1} g^{b_1} \bmod n, \quad z_2 := (ID_G)^{a_2} g^{b_2} \bmod n.$$

Replacing all occurrences of V and z_2 in equation (22) with the above two expressions, respectively, we get the following equation:

$$(ID_G)^{a_1} g^{b_1} \equiv (ID_G)^{e + a_2\gamma} g^{z_1 + b_2\gamma} \bmod n.$$

Then, we have

$$\begin{cases} a_1 = e + a_2\gamma \bmod ord(ID_G) \\ b_1 = z_1 + b_2\gamma \bmod f \end{cases}, \quad \text{or} \quad \begin{cases} a_1 = e + a_2\gamma \bmod \phi(n) \\ b_1 = z_1 + b_2\gamma \bmod f \end{cases}. \tag{24}$$

Where $ord(ID_G)$ denotes the multiplicative order of element $ID_G \in \mathbb{Z}_n^*$, and $e := h(V||M) = h(ID_G^{a_1} g^{b_1} \bmod n||M)$.

In the above two equation systems, given a_1, b_1 (and then V, e), finding solutions for b_2 and z_1 are very easy since modulus f is known. However, finding a solution for a_2 seems difficult since we do not know any value of $ord(ID_G), \phi(n)$, $\gamma^{-1} \bmod \phi(n)$ or $\gamma^{-1} \bmod ord(ID_G)$. But, in the following three special settings, we can find some solutions.

(1) $ID_G^{2f} = 1 \bmod n$, i.e., $ord(ID_G) = 2, f$, or $2f$. In this case, an attacker can forge valid group signature by setting $a_2 := (a_1 - e)\gamma^{-1} \bmod ord(ID_G)$. This is the attack pointed out in [17]. However, if the suggested parameters are used, i.e., $|p'| = |q'| \approx 234$ and $|f| \approx 160$ [7], we note that this case occurs only with a negligible probability $(4f^2 - 1)/n < 1/2^{466}$.

(2) Since the GM knows the value of $\phi(n)$, he can generate a valid group signature by setting $a_2 := (a_1 - e)\gamma^{-1} \bmod \phi(n)$. In fact, this is a trivial result. Because in general group signature schemes, including Kim-Park-Won scheme, GM always can create nonexistent membership certificate and generate group signature.

(3) The value of $ID_G^d \bmod n$ is known. In this case, if we define $z_2 := (ID_G^d)^{\bar{a}_2} g^{b_2} \bmod n$, then the equation for $\bar{a}_2$ will become:

$$a_1 = e + \bar{a}_2 \cdot d\gamma \bmod \phi(n).$$

Since $d\gamma = 1 \bmod \phi(n)$, one trivial solution is attained $\bar{a}_2 := a_1 - e \in \mathbb{Z}^+$ if $a_1 - e > 0$. If we assume $h(\cdot) \leq l$ and choose a_1 such that $a_1 \geq 2^l$, we will always have $a_1 - e > 0$. However, how to get the value of $ID_G^d \bmod n$? The methods are given in the next part.

Forging Certificates. A valid membership certificate is defined by equation (20), which is a RSA signature of GM on the message $(ID_G \cdot y_i)^{-1}$. However, this does not imply that valid membership certificates can only be generated by the GM. It is easy to know that the following equation defines a valid membership certificate $(\bar{s}_i, \bar{x}_i)$ too, since it is a variant of equation (20):

$$ID_G \cdot g^{\bar{s}_i} \cdot \bar{x}_i^{\gamma} = 1 \bmod n. \quad (25)$$

Let U_i and U_j, with certificates (s_i, x_i) and (s_j, x_j) respectively, be two colluding group members, then they have several ways to forge a valid membership certificate $(\bar{s}, \bar{x})$.

(a) For any integer $k > 1$, define $\bar{s} := ks_i - (k-1)s_j \bmod f$ and $\bar{x} := x_i^k \cdot x_j^{-(k-1)} \bmod n$. This method works since $\bar{x} := x_i^k \cdot x_j^{-(k-1)} = (ID_G \cdot g^{ks_i-(k-1)s_j})^{-d} = (ID_G \cdot g^{\bar{s}})^{-d} \bmod n$.

(b) If they choose an integer $\delta > 0$ and define $s_j := s_i + \delta \bmod f$, they can get the value of $g^{\delta d}$ by $g^{\delta d} := x_i \cdot x_j^{-1} \bmod n$. Then, for any integer $k > 1$, define $\bar{s} := s_i + k\delta \bmod f$ and $\bar{x} := x_i \cdot (g^{\delta d})^{-k} \bmod n$. $(\bar{s}, \bar{x})$ is a valid certificate because $\bar{x} = x_i \cdot (g^{\delta d})^{-k} = (ID_G \cdot g^{s_i})^{-d}(g^{\delta k})^{-d} = (ID_G \cdot g^{s_i+k\delta})^{-d} = (ID_G \cdot g^{\bar{s}})^{-d} \bmod n$. Specifically, if $\delta = 1$, then we get $g^d = x_i \cdot x_j^{-1} \bmod n$ and $(ID_G)^{-d} = x_i \cdot (g^d)^{s_i} \bmod n$; if $\delta = s_i$, i.e., $s_j = 2s_i \bmod f$, we get $(ID_G)^{-d} = (x_i)^2 \cdot x_j^{-1} \bmod n$. Therefore, $ID_G^d \bmod n$ is available.

(c) If they set $s_i := ab$ and $s_j := ab + b$ for two known positive integers a and b, g^{bd} can be attained by computing $x_i \cdot x_j^{-1} \bmod n$, and then ID_G^{-d} can be attained by computing $x_i \cdot (g^{bd})^a \bmod n$. When g^{bd} and ID_G^{-d} are known, they can generate a valid certificate $(\bar{s}, \bar{x})$ by defining $\bar{s} := bk \bmod f$ and $\bar{x} := ID_G^{-d} \cdot (g^{bd})^{-k} \bmod n$, for any integer $k > 1$. This attack was first found by Lim and Lee [9].

In the above three cases, two colluding group members are needed. However, if the system allows a user own two certificates at the same time or an old group member can get a new certificate when he joins the same system for the second time, a group member alone can mount above attacks successfully.

Signer Identification. For a valid group signature (e, z_1, z_2) on message M, if replacing the occurrence of $\bar{V}$ in equation (23) by $ID_G^e g^{z_1} z_2^{\gamma} \bmod n$, we have

$$g^{z_1} \equiv ID_G^e \cdot g^{z_1} \cdot z_2^{\gamma} \cdot z_2^{-\gamma} \cdot x_i^{e\gamma} \cdot g^{s_i e} \bmod n,$$

i.e., $1 = (ID_G \cdot g^{s_i} \cdot x_i^{\gamma})^e \bmod n$. However, according to equation (25), we know $1 \equiv ID_G \cdot g^{s_i} \cdot x_i^{\gamma}$ for every certificate (s_i, x_i). This shows that given a valid group signature, equation (23) is an equality for all certificates (s_i, x_i). In other words, equation (23) cannot be used to identify the signer because all certificates (s_i, x_i) satisfy it. Wang et. al first pointed out this problem [25], but they have no explanations for it. Now we point out the reason: If (e, z_1, z_2) is U_i's valid group signature on message M, it is also U_j's valid group signature on message M. More specifically, we denote $\delta := s_j - s_i \bmod f$ and assume that U_i chooses two random numbers r_1 and r_2 to generate his signature (e, z_1, z_2) as in equation (21). Then, it is easy to check that (e, z_1, z_2) is also a valid signature of U_j for the same message if U_j chooses $\bar{r}_1 := r_1 - \delta e \bmod f$ and $\bar{r}_2 := r_2 g^{\delta de} \bmod n$ as his own two random numbers and then generates his signature. Therefore, for the same message, the signature spaces of any two group member are the same. So, it is impossible (in information theoretic sense) to trace the signer even for the GM. Therefore, Kim-Park-Won scheme [7] is totally anonymous and unlinkable even for the GM.