Fuzzy Identity Based Encryption
Preliminary Version

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Abstract

We introduce a new type of Identity Based Encryption (IBE) scheme that we call Fuzzy Identity Based Encryption. A Fuzzy IBE scheme allows for a private key for an identity **id** to decrypt a ciphertext encrypted with another identity **id'** if and only if the identities **id** and **id'** are close to each other as measured by some metric (*e.g.* Hamming distance). A Fuzzy IBE scheme can be applied to enable encryption using biometric measurements as identities. The error-tolerance of a Fuzzy IBE scheme is precisely what allows for the use of biometric identities, which inherently contain some amount of noise during each measurement.

In this paper we present a construction of a Fuzzy IBE scheme that uses groups with efficiently computable bilinear maps. Additionally, our construction does not use Random Oracles. We prove the security of our scheme under the Selective-ID security model.

1   Introduction

There has been recent interest about the challenge of generating cryptographic keys from biometric inputs. The primary difficulty in generating a strong key from a biometric input is that the measured value of the a biometric can change slightly upon each sampling. This effect can be explained by differences in sampling devices, environmental noise, or small changes in the human trait itself. This inherent non-determinism makes it difficult to extract a cryptographic key from a biometric input.

Recent work has produced techniques to derive cryptographic keys from biometric inputs for symmetric key applications. For example, Monrose et al. [12, 11, 10] develop techniques to extract secrets from keyboard typing dynamics and later voice prints by using a form of error-tolerant secret sharing. Other work by Davida *et al.* [4] and Juels and Wattenberg [7] use error-correcting codes to compensate for the noise in the biometric input.

These techniques are useful for symmetric key cryptography applications such as password authentication and symmetric key encryption. However, there does not seem to be a clear way to move these techniques into the realm of public key cryptography. In particular the work above seems does not fit into the paradigm of Identity Based Encryption.

We propose a new type of Identity Based Encryption that we call *Fuzzy Identity Based Encryption*. In a Fuzzy Identity Based encryption scheme a user with secret key for the identity **id** is able to decrypt a ciphertext encrypted with the public key **id'** if and only if **id** and **id'** are within a certain distance of each other as judged by some metric. For the remainder of this paper we view identities as n bit vectors and use the Hamming distance between them as the metric of distance. However, a Fuzzy Identity Based Encryption scheme could be built using some other metric.

Motivating Example   The existence of a practical Fuzzy Identity Based Encryption scheme would allow for the encryption of data using a biometric input as the public key. This could be useful in the following scenario. Consider a patient that is rushed into an emergency medical visit. A collection of medical data will be created from the visit including possibly the results of tests that will not be ready until days later. The patient would like to be able to conveniently access this data later by retrieving it from a public storage server. However, since the privacy of medical information is considered to be very important the data would need to be encrypted in such a way that only the patient could access it.

Given the urgency of the initial visit it is unrealistic to require that the patient has on his person a device that can store a cryptographic key or to require that he remember a unique identifier (as might be required if we were to use Identity Based Encryption in the traditional manner). However, the medical staff would be able to measure an inherent biometric identity from the patient. Using a Fuzzy IBE scheme the medical staff could use this biometric identity to encrypt information for the patient so that he could later retrieve at his convenience.

Advantages of biometric-based IBE over traditional IBE In many situations, using biometric-based IBE would have a number of important advantages over "traditional" IBE. We argue that the use of biometric identities fits the framework of Identity Based Encryption very well and is a very valuable application of it.

First, the process of obtaining a secret key from an authority is very natural and straightforward. In traditional Identity Based Encryption schemes a user with a certain identity, for example "Bob Smith", will need to go to an authority to obtain the private key corresponding to the identity. In this process the user will need to "prove" to the authority that he is indeed entitled to this identity. This will typically involve presenting supplementary documents or credentials. The type of authentication that is necessary is not always clear and robustness of this process is questionable (the supplementary documents themselves could be subject to forgery) Typically, there will exist a tradeoff between a system that is expensive in this step and one that is less reliable.

In contrast if a biometric is used as an identity then the verification process for an identity is very clear. The user must demonstrate ownership of the biometric under the supervision of a well trained operator. If the operator is able to detect imitation attacks, for example playing the recording of a voice, then the security of such this phase is only limited by the quality of the biometric technique itself. We emphasize that the biometric measurement for an individual need not be kept secret (indeed it won't be if it is to be used as a public key). We must only guarantee that an attacker cannot fool the human attendee that delegates private keys into believing that an attacker owns a biometric identity that he does not.

Secondly, using a biometric as an identity has the advantage that identities are unique if the underlying biometric is of a good quality. Some types of traditional identities such as the name "Bob Smith" will clearly not be unique and this could lead to problems, although the advent of Hierarchial Identity Based Encryption does alleviate this problem.

Finally, the fact that a biometric identity is an inherent trait and therefore is always with a person is useful. In several situations such as our medical example the user might not have the foresight to carry an cryptographic devices or even obtain a unique traditional identity.

Our Contributions We formalize the notion of Fuzzy Identity Based Encryption and provide a construction for a Fuzzy Identity Based Encryption scheme. Our construction uses groups for which an efficient bilinear map exists, but for which the Computational Diffie-Hellman problem is assumed to be hard. We achieve our result by applying the techniques of Shamir Secret Sharing [15] where the a polynomial can be reconstructed in the exponent of a group.

We prove our scheme secure under an adapted version of the Selective-ID security model first proposed by Canetti et al. [3]. Additionally, our construction does not use Random Oracles. We reduce the security of our scheme to an assumption that is similar to the Bilinear Decisional Diffie-Hellman assumption.

1.1   Related Work

Shamir [16] first proposed the concept of Identity Based Encryption. Boneh and Franklin [2] presented the first Identity Based Encryption scheme that was both practical and secure. Their solution made use of groups for which there was an efficiently computable bilinear map, but for which the Computational Diffie-Hellman problem is believed to be hard.

Canetti et al. [3] proposed the first construction for IBE that was provably secure outside the Random Oracle model. To prove Security they described a slightly weaker model of Security known as the Selective-ID model, in which the adversary declares which identity he will attack before the global public parameters are generated. Since their construction views identities as bit strings a Bilinear map computation is required for each bit in the identity.

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Boneh and Boyen [1] describe two IBE scheme that are also proven to be secure in the Selective-ID model. Additionally, their constructions only require respectively two and one pairing computations per decryption and are thus significantly more efficient.

It is interesting that our work can be viewed as a new method for a standard IBE scheme that is secure without Random Oracles in the Selective-ID model. Our Fuzzy IBE scheme becomes a standard one when the error-tolerance parameter, d, is set to 0. However, since the decryption phase requires a pairing computation for every bit of the identity it would serve as a much less efficient standard IBE scheme than that of Boneh and Boyen [1].

Other work in applying biometrics to cryptography has focused on the derivation of a secret from a biometric. This secret can be then used for operations such as encryption or UNIX style password authentication.

Monrose et al. [12, 11, 10] examined the use of keystroke dynamics and later voice prints as biometrics for this purpose. The authors use a type of secret sharing in which the biometric would decided which keyshares from a stored set to use in reconstructing the secret. Some stored shares are bogus to prevent someone with the wrong biometric from unlocking the secret. Their scheme provides error-tolerance by storing two valid secrets for points in the biometric measurement that are not reliable for a particular user.

Davida et al. [4] proposed a scheme where they store the check bits for an error correcting code of a biometric input along with the hash of the input. The check bits are used to compute the original input from one that is reasonably close. Juels and Wattenberg [7] improve upon this work using a novel technique in which the biometric input is treated as a corrupted codeword.

Juels and Sudan [8] presented a scheme that uses set overlap as the measurement between biometric templates instead of Hamming distance. Dodis, Rezyin, and Smith [5] describe a general primitive for extracting uniform randomness from biometric inputs. Additionally, they give constructions for the metrics of Hamming distance, set overlap, and edit distance.

The primary distinguishing feature of our work from the other related work on biometrics above is that we view the biometric input as public identity as opposed to a secret that the human poses. Our only physical requirement is that the biometric cannot be imitated such that a trained human operator would be fooled.

Finally, the use of polynomial interpolation within the exponents was discussed with Philippe Golle and Jessica Staddon [6] during work on the problem of conjunctive keyword search on encrypted data.

1.2 Organization

The rest of the paper is organized as follows. In Section 2 we formally define a Fuzzy Identity Based Encryption scheme including the Selective-ID security model for one. In Section 3 we describe the security assumption our scheme reduces to. In Section 4 we describe our construction of a Fuzzy Identity Based Encryption scheme. In Section 5 we prove the security of our scheme. Finally, we conclude in Section 6.

2 Definitions

In this section we define two Selective-ID models of security for Fuzzy Identity Based Encryption. The Basic Fuzzy Selective-ID game is very similar to the standard Selective-ID model with the exception that the adversary is only allowed to query for secret keys for identities that are greater than d Hamming Distance away from the targeted identity.

In the second game which we call Multiple Fuzzy Selective-ID the adversary chooses multiple number of identities that are simultaneously targets. This model of security captures how an attack against a biometric identity will occur in a practice. The attacker will want to target a certain biometric identity, but the identities that the ciphertexts are encrypted with will deviate slightly from the original identity. These deviations in practice will be from environmental noise and therefore will be independent of the public parameters. This game captures this case since the attacker can model the random distribution of identities in the multiple identities that he chooses to attack.

We describe the two security games below and show that the security of the Multiple Fuzzy Selective-ID game reduces to the Basic Fuzzy Selective-ID game in Section A of the Appendix.

Basic Fuzzy Selective-ID

Init The adversary declares the identity α that he wishes to be challenged upon.

Setup The challenger runs the setup phase of the algorithm and tells the adversary the public parameters.

Phase 1 The adversary is allowed issue queries private keys for an identity γ where the Hamming distance between α and γ is greater than d.

Challenge The adversary submits two equal length messages m_0, m_1. The challenger flips a random coin b and encrypts m_b. The ciphertext is passed to the adversary.

Phase 2 Phase 1 is repeated.

Guess The adversary outputs a guess b' of b.

The advantage of an adversary $\mathcal{A}$ in this game is $\Pr[b' = b] - \frac{1}{2}$.

Multiple Fuzzy Selective-ID

Init The adversary declares l identities $\alpha_1, \ldots alpha_l$ that he wishes to be challenged upon where l is bounded by a polynomial of the security parameter κ.

Setup The challenger runs the setup phase of the algorithm and tells the adversary the public parameters.

Phase 1 The adversary is allowed issue queries private keys for an identity γ where the Hamming distance between α_i and γ is greater than d for all i.

Challenge The adversary submits $2l$ messages $m_{1,0}, \ldots, m_{l,0}$ and $m_{1,1}, \ldots, m_{l,1}$ where $m_{i,0}, m_{i,1}$ are of equal length for all i. The challenger flips a random coin b and encrypts $m_{1,b}, \ldots, m_{l,b}$ with the respective identities $\alpha_1, \ldots \alpha_l$. The ciphertexts are passed to the adversary.

Phase 2 Phase 1 is repeated.

Guess The adversary outputs a guess b' of b.

The advantage of an adversary $\mathcal{A}$ in this game is $\Pr[b' = b] - \frac{1}{2}$.

3 Complexity Assumption

Let $\mathcal{G}_1$ be a group of prime order p with an admissible bilinear map into $\mathcal{G}_2$ and g be a generator of $\mathcal{G}_1$ and h be a generator of $\mathcal{G}_2$ where $e(g, g) = h$. Our assumption follows.

3.1 Modified Decisional Bilinear Diffie-Hellman (BLDDH) Assumption

The simulator chooses $a, b, c, z \in \mathbf{Z}_p$ at random. The goal of the adversary is to be able to distinguish the tuple

$$A = g^a, B = g^b, C = g^c, Z = h^{\frac{ab}{c}}$$

from

$$A = g^a, B = g^b, C = g^c, Z = h^z$$

with more than a negligible advantage.

4 Our Construction

4.1 Description

Let $\mathcal{G}_1$ be bilinear group of prime order p, g be a generator of $\mathcal{G}_1$, and h be a generator of $\mathcal{G}_2$. We let the security parameter, k, determine the size of the group. Additionally let $e : \mathcal{G}_1 \times \mathcal{G}_1 \to \mathcal{G}_2$ denote the bilinear map where $e(g,g) = h$. The security parameter κ will determine the size of the groups.

We construct a Fuzzy Identity Based Encryption scheme with identities of length n and error-tolerance d in the following manner.

Setup(n,d) To generate the public parameters first select $x_0, (x_{1,0}, x_{1,1}) \ldots (x_{n,0}, x_{n,1}) \in \mathbf{Z}_p$ so that all x values are distinct.

Next, choose $(t_{1,0}, t_{1,1}) \ldots (t_{n,0}, t_{n,1})$ uniformly at random from $\mathbf{Z}_p$. Finally, choose y_0 uniformly at random in $\mathbf{Z}_p$. The published public parameters are:

$$(T_{1,0} = g^{t_{1,0}}, T_{1,1} = g^{t_{1,1}}), \ldots, (T_{n,0} = g^{t_{n,0}}, T_{n,1} = g^{t_{n,1}}), x_0, (x_{1,0}, x_{1,1}), \ldots, (x_{n,0}, x_{n,1}), Y = h^{y_0}.$$

The master key is:

$$(t_{1,0}, t_{1,1}), \ldots, (t_{n,0}, t_{n,1}), y_0.$$

Key Generation To generate a private key for identity **id** the following steps are taken. A $(n-1) - d$ degree polynomial q is randomly chosen such that $q(x_0) = y_0$. Let $\mathbf{id}(j)$ denote the jth bit of **id**. The private key is

$$D_1 = g^{\frac{q\left(x_{1,\mathbf{id}(1)}\right)}{t_{1,\mathbf{id}(1)}}}, \ldots, D_n = g^{\frac{q\left(x_{n,\mathbf{id}(n)}\right)}{t_{n,\mathbf{id}(n)}}}.$$

Encryption Encryption with the public key $\mathbf{id}'$ and message $m \in \mathcal{G}_2$ proceeds in the following way.

First, a random value $r \in \mathbf{Z}_p$ is chosen. The ciphertext is then published as

$$(\mathbf{id}', E_0 = mY^r = mh^{ry_0}, E_1 = (T_{1,\mathbf{id}'(1)})^r = g^{rt_{1,\mathbf{id}'(1)}}, \ldots, E_n = (T_{1,\mathbf{id}'(1)})^r = g^{rt_{n,\mathbf{id}'(\mathbf{n})}}).$$

Decryption A private key for identity **id** can decrypt a ciphertext encrypted with $\mathbf{id}'$ when the Hamming distance between **id** and $\mathbf{id}'$ is less than or equal to d.

The intuitive idea behind decryption is to compute the bilinear map between E_i and D_i when $\mathbf{id}(i) = \mathbf{id}'(i)$. The t values will cancel out and we will have (at least) $n - d$ points of the polynomial $rq(x)$ in the exponent. Since the x values are publicly known, the polynomial can be interpolated in the exponent to evaluate it at x_0 to get $rq(x_0) = ry_0$ (in the exponent). Once h^{ry_0} is calculated it can be divided out from the ciphertext to get the message. A detailed description follows.

For the remainder of this section we leave out the second subscript where it is obvious and define it to be relative to the identity **id** (e.g. x_1 indicates $x_{1,\mathbf{id}(1)}$).

We first define the Lagrangian multiplier $\triangle_{i,S,\mathbf{id}}$ for index i relative to a set S and identity **id**.

$$\triangle_{i,S,\mathbf{id}}(x) = \prod_{j \in S, j \neq i} \frac{x - x_j}{x_i - x_j}$$

To decrypt we first define S as a set of the first $n - d$ indices i for which $\mathbf{id}(i) = \mathbf{id}'(i)$.

In the first phase of the decryption the variable u is calculated as

$$u = \prod_{i \in S} e(D_i, E_i)^{\triangle_{i,S,\mathbf{id}}(x_0)} = \prod_{i \in S} e(g^{\frac{q(x_i)}{t_i}}, g^{rt_i})^{\triangle_{i,S,\mathbf{id}}(x_0)} = \prod_{i \in S} h^{rq(x_i)\triangle_{i,S,\mathbf{id}}(x_0)} = h^{rq(x_0)}$$

In this step the $(n-1) - d$ degree polynomial $rq(x)$ is interpolated (in the exponent) to be evaluated at the point x_0. The interpolation is possible since the $(n-1) - d$ degree polynomial $rq(x)$ is known at $n - d$ points.

Finally, the message m is recovered as

$$\frac{E_0}{u} = \frac{mh^{ry'}}{h^{ry'}} = m.$$

4.2 Computational Cost

Encryption of a message will consist of $n + 1$ exponentiations. The cost of decryption will be dominated by the $n - d$ bilinear map computations.

4.3 Flexible Error-Tolerance

In the scheme described above the error-tolerance d is set as a global system parameter. However, in some situations it is desirable for the error-tolerance to be set by the party encrypting the message. This could be useful if the encrypting party had a less accurate biometric reader and wanted to raise the error-tolerance to compensate (this would of course come at the expense of a greater risk in security).

In the construction described above the dealer generated $2n$ public T-values where the subscripts corresponded to the values of an identity. For example, $T_{2,0}$ corresponds to the second bit of an identity being 0 and the encrypter will raise $T_{2,0}$ to r if and only if the public key **id**′ has a 0 for its second bit.

We can achieve a more flexible solution to error-tolerance. In the setup phase we add n more T-values $T_{n+1}, \ldots T_{2n}$ where these values have no correspondence to the identity and n more x-values $x_{n+1} \ldots x_{2n}$ If the encrypter wishes to encrypt with a chosen error-tolerance d' he will produce $E_1, \ldots E_n$ in an identity dependent manner as before. Additionally, he will generate $E_{n+1}, \ldots, E_{n+d'}$ as $(T_{n+1})^r, \ldots, (T_{n+d'})^r$ in an identity independent manner. That is the encrypter will raise d' extra points to the r power.

For the private key the dealer will now choose a random $n - 1$ degree polynomial so that n points are needed to interpolate. In an analogous fashion the dealer will give $D_1, \ldots, D_n$ key parts that are related to the secret key identity and $D_{n+1}, \ldots, D_{2n}$ that are not. A holder of a secret key can decrypt the ciphertext by computing the bilinear map between D_i and E_i at least $n - d'$ indices, i, where the identities agree in addition to computing the bilinear map at the d' indices that are independent of the identity.

4.4 Extending to the Set Intersection Metric

We believe that we can incorporate the recent IBE scheme of Boneh and Boyen [1] into the techniques presented above to produce a scheme in which the size of the master public key is constant (independent of the length of the identities). Using this method we believe we can design an efficient method for creating a Fuzzy IBE scheme that uses identities that are sets of attributes and uses set overlap as the distance metric between identities.

5 Proof of Security

We prove that the security of our scheme in the Selective-ID model reduces to the hardness of our Modified Bilinear Decisional Diffie-Hellman assumption.

Suppose there exists a computationally efficient adversary $\mathcal{A}$ that can attack our scheme in the Selective-ID model with advantage ϵ. We build a simulator $\mathcal{B}$ that can play the Modified BLDDH game with advantage $\frac{\epsilon}{2}$.

The steps in the simulation are as follows.

- The challenger flips a fair binary coin μ outside of $\mathcal{B}$'s view. If $\mu = 0$ the challenger gives $(A, B, C, D) = (g^a, g^b, g^c, h^{\frac{ab}{c}})$, otherwise if $\mu = 1$ $(A, B, C, D) = (g^a, g^b, g^c, h^z)$ for random a, b, c, z.

- Next, $\mathcal{B}$ will run $\mathcal{A}$ and receive the challenge identity α, an n-bit vector.

- $\mathcal{B}$ then generates the public parameters as follows. First, the groups $\mathcal{G}_1, \mathcal{G}_2$ of prime order p are chosen based on the security parameter and the generators $g \in \mathcal{G}_1$ and $h = e(g, g)$ are chosen. Then the simulator assigns $Y = A = g^a$. Next, for all integers i between 1 and n the simulator chooses $w_i, \beta_i \in \mathbf{Z}_p$ uniformly at random in $\mathbf{Z}_p$. The simulator then assigns $T_{i,\alpha_i} = C^{\beta_i} = g^{c\beta_i}$ and $T_{i,\overline{\alpha_i}} = g^{w_i}$. Finally, all x values are assigned to be distinct members in $\mathbf{Z}_p$.

Since $a, w_1, \ldots w_n, \beta_1 \ldots \beta_n$ are chosen at random the distribution of the public parameters in the simulation is identical to that of the actual construction.

- In the next step $\mathcal{A}$ makes requests for private keys where the identity of the private keys are greater than d Hamming Distance from α.

  Suppose $\mathcal{A}$ asks for such a private key γ. We first define three sets of indices $\Gamma, \Gamma', S \subseteq \{1 \ldots n\}$ in the following manner.

$$\Gamma = \{i : \gamma_i = \alpha_i\}$$

$$\Gamma' \text{ be any set such that } \Gamma' \cap \Gamma = \emptyset \text{ and } |\Gamma' \cup \Gamma| = (n-1) - d$$

$$S = \Gamma \cup \Gamma' \cup \{0\}$$

  Next, we define the decryption key components D_i for $i \in \Gamma \cup \Gamma'$ as follows.

$$\text{If } i \in \Gamma : \ D_i = g^{s_i} \text{ where } s_i \text{ is chosen randomly in } \mathbf{Z}_p.$$

$$\text{If } i \in \Gamma' : \ D_i = g^{\frac{\lambda_i}{w_i}} \text{ where } \lambda_i \text{ is chosen randomly in } \mathbf{Z}_p.$$

  The intuition behind these assignments is that we are implicitly choosing a random $(n-1)d$ degree polynomial $q(x)$ by choosing its value the $(n-1) - d$ points randomly in addition to having $q(x_0) = y_0$. For $i \in \Gamma$ we have $q(x_i) = c\beta_i s_i$ and for $i \in \Gamma'$ we have $q(x_i) = \lambda_i$. The simulator can calculate the other D_i values where $i \notin \Gamma \cup \Gamma'$ since the simulator knows the discrete log of $T_{i, \overline{\alpha_i}}$. Again, we omit the second subscript for clarity and define it relative to the private key identity γ. The simulator makes the assignments as follows.

$$\text{If } i \notin \Gamma \cup \Gamma' : \ D_i = \left(\prod_{j \in \Gamma} C^{\frac{\beta_j s_j \triangle_{j,S}(x_i)}{w_i}}\right)\left(\prod_{j \in \Gamma'} g^{\frac{\lambda_j \triangle_{j,S}(x_i)}{w_i}}\right) Y^{\frac{\triangle_{0,S}(x_i)}{w_i}}$$

  Using interpolation the simulator is able to calculate $D_i = g^{\frac{q(x_i)}{t_i}}$ for $i \notin \Gamma \cup \Gamma'$ where q(x) was implicitly defined by the random assignment of the other $n - d - 1$ $D_i \in \Gamma \cup \Gamma'$ and Y. Therefore, the simulator is able to construct a private key for the identity γ. Furthermore, the distribution of a the private key for a γ is identical to that of the original scheme.

- The adversary, $\mathcal{A}$, will submit two challenge messages m_1, m_0 to the simulator. The simulator flips a fair binary coin ν and returns an encryption of m_ν. The encryption is performed as follows. We let $E_0 = Z m_\nu$ and $E_i = B^{\beta_i} = g^{b\beta_i}$.

  If $\mu = 0$ then $Z = g^{\frac{ab}{c}}$. If we let $r' = \frac{b}{c}$ then we have $E_0 = Z m_\nu = g^{\frac{ab}{c}} = g^{ar'} = Y^{r'}$ and $E_i = B^{\beta_i} = g^{b\beta_i} = g^{\frac{b}{c}c\beta_i} = g^{r'c\beta_i} = (T_i)^{r'}$. Therefore, the ciphertext is an encryption of the message m_ν under the public key α.

  Otherwise, if $\mu = 1$ then $Z = g^z$. We then have $E_0 = m_\nu g^z$ since z is random E_0 will be a random element of $\mathcal{G}_2$ from the adversaries view and the message contains no information about m_ν.

- $\mathcal{A}$ will then ask for more private keys. The simulator will respond to these requests exactly as before.

- $\mathcal{A}$ will submit a guess ν' of ν. If $\nu = \nu'$ the simulator will output $\mu' = 0$ to indicate that it was given a Modified BLDDH-tuple otherwise it will output $\mu' = 1$ to indicate it was given a random 4-tuple.

As shown in the construction the simulator's generation of public parameters and private keys is identical to that of the actual scheme.

In the case where $\mu = 1$ the adversary gains no information about ν. Therefore, we have $\Pr[\nu \neq \nu'|\mu = 1] = \frac{1}{2}$. Since the simulator guesses $\mu' = 1$ when $\nu \neq \nu'$ we have $\Pr[\mu' = \mu|\mu = 1] = \frac{1}{2}$.

If $\mu = 0$ then the adversary sees an encryption of m_ν. The adversary's advantage in this situation is ϵ by definition. Therefore, we have $\Pr[\nu = \nu'|\mu = 0] = \frac{1}{2} + \epsilon$. Since the simulator guesses $\mu' = 0$ when $\nu = \nu'$ we have $\Pr[\mu' = \mu|\mu = 0] = \frac{1}{2} + \epsilon$.

The overall advantage of the simulator in the Modified BLDDH game is $\frac{1}{2}\Pr[\mu' = \mu|\mu = 0] + \frac{1}{2}\Pr[\mu' = \mu|\mu = 1] = \frac{1}{2}(\frac{1}{2} + \epsilon) + \frac{1}{2}\frac{1}{2} = \frac{1}{2} + \frac{1}{2}\epsilon$.

5.1 Chosen-Ciphertext Security

Our security definitions and proofs have been in the chosen-plaintext model. Our scheme can be extended to the chosen-ciphertext model by applying the technique of using simulation-sound NIZK proofs to achieve chosen-ciphertext security [13, 14, 9] as described by Canetti et al. [3].

5.2 Security in Standard IBE Model

While our main security proof is in the slightly weaker Selective-ID model we conjecture that our scheme is secure in the standard IBE model of security. In this model the adversary commits to the identity that he will attack after seeing the public parameters and receiving chosen private keys. We assert that our scheme can be reduced to a much stronger (non-standard) assumption in which the adversary makes interactive queries to an oracle.

6 Conclusion

We introduced the concept of Fuzzy Identity Based Encryption, which allows for error-tolerance between the identity of a private key and the public key used to encrypt a ciphertext. Fuzzy Identity Based Encryption has a direct application to Identity Based Encryption using a biometric as a public key.

We presented a construction of a Fuzzy IBE scheme that uses Hamming distance as the distance metric between identities. Finally, we proved our scheme under the Selective-ID model by reducing it to an assumption that can be viewed as a modified version of the Bilinear Decisional Diffie-Hellman assumption.

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A   Security Game Reduction

Suppose we have an adversary $\mathcal{A}$ that is successful with advantage ϵ in the Multiple Fuzzy Selective-ID game for a certain scheme. Furthermore, suppose $\mathcal{A}$ picks at most l target identities in the first phase. We can then build a simulator $\mathcal{B}$ that is successful with advantage at least $\frac{\epsilon}{2l}$ in the Basic Fuzzy Selective-ID game.

We first define hybrid experiments for the Multiple Fuzzy Selective-ID game.

For $0 \leq i \leq l$ we define H_i as a game in which the adversary $\mathcal{A}$ is given an encryption of a random message for the first i ciphertexts (encrypted under the respective first i identities) and the rest of the ciphertexts are determined by the coinflip b as in the defined game. $\mathcal{A}$'s advantage in H_0 is ϵ by our assumption and $\mathcal{A}$'s advantage in H_l is 0 since the ciphertexts will be completely independent of b. Therefore, by the triangle inequality there exists a $0 \leq j < l$ such that $\mathcal{A}$ has a greater advantage in H_j than H_{j+1} by at least $\frac{\epsilon}{l}$.

$\mathcal{B}$ plays the Basic Fuzzy Selective-ID game in the following way. $\mathcal{B}$ will first run $\mathcal{A}$ and get $l' \leq l$ target identities $\alpha_1 \ldots \alpha_{l'}$. $\mathcal{B}$ will then submit α_{j+1} as the target identity for its game. $\mathcal{B}$ then receives the public parameters for its scheme and passes these on to $\mathcal{A}$.

$\mathcal{B}$ will then respond to private key queries that $\mathcal{A}$ can make by making the same private key queries itself and passing these back to $\mathcal{A}$. The only private key queries that $\mathcal{B}$ is not allowed to make are within d Hamming distance of α_{j+1}. These are also not allowed by $\mathcal{A}$. Therefore, $\mathcal{B}$ can respond to all legal queries made by $\mathcal{A}$.

In the challenge phase $\mathcal{A}$ will submit $m_{1,0}, \ldots, m_{l',0}$ and $m_{1,1}, \ldots, m_{l',1}$. $\mathcal{B}$ will select a random message R and flip a coin b. $\mathcal{B}$ then submits $m_0 = R, m_1 = m_{j+1,b}$ as the ciphertexts it wishes to be challenged on in its game. The challenger will flip a coin β outside $\mathcal{B}$'s view. $\mathcal{B}$ will then receive back ciphertext C which is an encryption of m_β with the key α_{j+1} from the challenger.

It then assigns ciphertexts $c_1, \ldots, c_{l'}$ int the following way. It creates $c_1, \ldots, c_j$ as the message R encrypted under the respective identities $\alpha_1, \ldots \alpha_j$. It assigns $c_{j+1} = C$. Finally, it creates $c_{j+2}, \ldots, c_{l'}$ by encrypting $m_{j+2,b}, \ldots m_{l',b}$ under $\alpha_{j+2}, \ldots, \alpha_{l'}$ respectively. (If $j \geq l'$ then only the first step needs to be taken.) The ciphertexts are then passed to $\mathcal{A}$.

Subsequent private key queries are then satisfied as before.

$\mathcal{A}$ then outputs its guess b'. If $b = b'$ then $\mathcal{B}$ outputs its guess $\beta' = 1$, otherwise it outputs $\beta' = 0$.

If $\beta = 1$ then C is an encryption of $m_{j+1,b}$ and $\mathcal{A}$ in playing the hybrid game H_j. Otherwise, if $\beta = 0$ then C is an encryption of R and $\mathcal{A}$ is playing the hybrid game H_{j+1}. It follows that $\Pr[b' = b | \beta = 1] - \Pr[b' = b | \beta = 0] \geq \frac{\epsilon}{l}$. Since $B' = 1$ iff $b = b'$ we then have $\Pr[\beta' = 1 | \beta = 1] - \Pr[\beta' = 1 | \beta = 0] \geq \frac{\epsilon}{l}$. Therefore, $\mathcal{B}$ has an advantage of at least $\frac{\epsilon}{2l}$ in the Basic Fuzzy Selective-ID game.