

# The Exact Security of an Identity Based Signature and its Applications

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**Abstract.** This paper first positively answers the previously open question of whether it was possible to obtain an optimal security reduction for an identity based signature (IBS) under a plausible computational assumption. We revisit the Sakai-Ogishi-Kasahara IBS that was recently proven secure by Bellare, Namprempe and Neven through a general framework applying to a large family of schemes. We show that their modified SOK-IBS scheme can be viewed as a one-level instantiation of Gentry and Silverberg's alternative hierarchical IBS the exact security of which was never considered before. As an application, we propose a modification of Boyen's "Swiss Army Knife" identity based signature encryption (IBSE) that presents better security reductions and satisfies the same strong security requirements with a similar efficiency.

**Keywords.** ID-based cryptography, signatures, signcryption, security proofs, exact security

## 1 Introduction

Identity based cryptography has become a very fashionable topic in the last couple of years. The motivation of this concept, introduced by Shamir in 1984 ([51]), was to simplify key management and avoid the use of digital certificates. The trick was to let a public key be publicly and uniquely derivable from a human-memorizable binary sequence corresponding to an information non-ambiguously identifying its owner (e-mail address, IP address combined to a user name, social security number,...) while the associated private keys can only be computed by a trusted Private Key Generator (PKG) thanks to a master secret. This paradigm allows bypassing the trust problems that arise in traditional certificate-based public key infrastructures (PKIs). Indeed, since a public key 'is' its owner's identity, it becomes useless to bind them by a digital certificate. Although a PKG's public key still has to be certified, the need of digital certificates is really reduced as reasonably many users may depend on the same PKG.

Since the concept's appearance in 1984, several practical identity based signature schemes (IBS) have been devised in the late 80's ([26], [28]) and also after 2001 ([17],[31],[50],[46]). On the other hand, finding a practical identity based encryption scheme (IBE) remained an open challenge until 2001 when Boneh and Franklin ([13]) proposed to use bilinear maps over algebraic curves to elegantly solve the challenge. After that, these fashionable bilinear maps provided plenty of other applications (that are not listed here but their references can be found in [4]) including various particular kinds of signatures: blind, ring, undeniable, proxy,etc.

Along the evolution of public key cryptography from 1976, there has been a graduate evolution tending to a necessity to provide security proofs for asymmetric cryptosystems in the sense that the existence of an attacker against them would imply a probabilistic polynomial time algorithm to solve a hard number theoretic problem. In 1993, motivated by the perspective to achieve provable security for efficient protocols, Bellare and Rogaway introduced the random oracle model ([8]) that was previously implicitly suggested in [26] and in which hash functions are used as black box by attackers for whom they are also indistinguishable from perfectly random functions. Although it is well known that security in the random oracle model does not imply security in the real world

as shown by several papers ([16],[5]) exhibiting pathological cases of provably secure schemes for which no secure implementation exists, it still seems to be a good principle to give security proofs 'at least' in the random oracle model when proposing a new asymmetric cryptosystem.

In the area of provable security, the last couple of years saw the rise of a new trend consisting of providing tight security reductions for asymmetric cryptosystems ([9],[47],etc.): the security of a cryptographic protocol is said to be tightly related to a hard number theoretic problem if an attacker against the scheme implies an efficient algorithm solving the problem with roughly the same advantage. This led several authors to provide search for new security proofs for systems that were already well known to be secure in the random oracle model or for some of their variants ([21],[22],[42]) or to devise new schemes that, although apparently less efficient than existing ones at first sight, provide much better security guarantees for the same security parameters and are then eventually more efficient for a similar desired level of security ([29],[34],etc.).

Although concerned with the provable security of identity based signatures, the research community did not really focus on providing really strong security arguments for the various IBS proposed in the literature up to now. Indeed, Paterson's IBS still has no formal security proof while Hess and Cha-Cheon gave proofs under the Diffie-Hellman assumption for their respective scheme but these proofs were both obtained through Pointcheval and Stern's forking lemma ([48],[49]) which does not yield tight security reductions as already argued in several previous papers ([29],[34]). Libert and Quisquater ([37]) recently proposed a scheme that may be viewed as an identity based transformation of the Goh-Jarecki Diffie-Hellman based signature ([29]). Unfortunately, their scheme is computationally expensive and its security relies on the Bilinear Diffie-Hellman assumption instead of the weaker Computational Diffie-Hellman one.

At Eurocrypt 2004, Bellare, Namprempre and Neven ([6]) defined a framework to provide security proofs for a large family of IBS by considering the security against passive, active and concurrent attacks of underlying 'convertible' identifications schemes (i.e. that can be converted into identity based identification schemes (IBI)). Unfortunately, as mentioned in [44], their framework does not end up with explicitly tight security bounds for the resulting family of IBS that includes the schemes originally described [31],[17],[52],[28],... and a variant of the one in [50]. We think the latter is of particular interest since it is possible to prove its security without considering the underlying identification scheme: in fact, explicitly tight reductions can be obtained without using the BNN framework in a black-box fashion. That is one of the concerns of the present paper.

An improved security reduction was recently achieved by Kurosawa and Heng ([35]) for the Cha-Cheon scheme by also considering the underlying identity based identification scheme. They exhibited a polynomial time reduction from the Diffie-Hellman problem to a chosen-message attacker that avoids the use of the forking technique: an attacker with a given advantage  $\epsilon$  is used to build an algorithm to solve the Diffie-Hellman problem with probability  $c \cdot \epsilon^2/q_E$  where  $c$  is a constant  $q_E$  denotes a bound on the number of identities corrupted by the adversary.

In the present work, we show that Bellare et al.'s modified Sakai-Ogishi-Kasahara identity based signature ([50]) has an even tighter security proof under the Diffie-Hellman assumption: from an attacker with advantage  $\epsilon$ , we build a polynomial time algorithm for the Diffie-Hellman problem with an advantage  $c \cdot \epsilon/q_E$  and we stress that a heuristic due to Katz and Wang allows eliminating the  $q_E$  degradation in the security bound even without loss of efficiency. Finally, we also show that a fully optimal reduction from a potentially stronger but reasonable assumption also exists.

A second contribution of this paper is to revisit Boyen's "Swiss Army Knife" identity based signature-encryption (IBSE) protocol by depicting a scheme achieving the same functionalities and satisfying the same strong security requirements while having much better security reductions than Boyen's IBSE (indeed, the only drawback of the latter construction is to have a poor exact security). Our IBSE follows a construction almost similar to Boyen's one. For applications requiring authentication and privacy without necessitating all of IBSE's features, we describe a slightly more efficient 'monolithic' identity based signcryption (IBSC) based on SOK-IBS that is also provably

secure in a strong security model.

Before starting with showing our proof for the modified SOK-IBS, we first recall the properties of bilinear maps that turned out to be (almost) unavoidable tools in the design of identity based cryptosystems. The next section gives formal definitions of presumed hard computational problems from which our reductions are made and then recalls an extended security model for identity based signatures. The modified SOK-IBS is described in 3.1 and its security analysis is given in paragraph 3.2. Finally, its application to ID-based signature/encryption is presented in section 4.

## 2 Preliminaries

### 2.1 Bilinear maps and Diffie-Hellman problems

Let  $k$  be a security parameter and  $q$  be a  $k$ -bit prime number. Let us consider groups  $\mathbb{G}_1$  and  $\mathbb{G}_2$  of the same prime order  $q$ . For our purposes, we need a bilinear map  $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \rightarrow \mathbb{G}_2$  satisfying the following properties:

1. Bilinearity:  $\forall P, Q \in \mathbb{G}_1, \forall a, b \in \mathbb{Z}_q^*$ , we have  $\hat{e}(aP, bQ) = \hat{e}(P, Q)^{ab}$ .
2. Non-degeneracy: for any  $P \in \mathbb{G}_1$ ,  $\hat{e}(P, Q) = 1$  for all  $Q \in \mathbb{G}_1$  iff  $P = \mathcal{O}$ .
3. Computability: there is an efficient algorithm to compute  $\hat{e}(P, Q) \forall P, Q \in \mathbb{G}_1$ .

As shown in [13], such non-degenerate admissible maps over cyclic groups can be obtained from the Weil or the Tate pairing over supersingular elliptic curves or abelian varieties.

We now recall the definitions of the Computational Diffie-Hellman problems and of one of its variants.

**Definition 1.** *Let us consider cyclic group  $\mathbb{G}_1$  of prime order  $q$ ,*

- *The **Computational Diffie-Hellman problem (CDH)** in  $\mathbb{G}_1$  is, given  $\langle P, aP, bP \rangle$  for unknown  $a, b \in \mathbb{Z}_q$ , to compute  $abP \in \mathbb{G}_1$ .*
- *The **one more CDH problem (1m-CDH)** is, given  $\langle P, aP \rangle \in \mathbb{G}_1$  for an unknown  $a \in \mathbb{Z}_q$ , and access to a target oracle  $\mathcal{T}_{\mathbb{G}_1}$  returning randomly chosen elements  $Y_i \in \mathbb{G}_1$  (for  $i = 1, \dots, q_t$ ,  $q_t$  being the exact number of queries to this oracle) as well as a multiplication oracle  $\mathcal{H}_{\mathbb{G}_1, a}(\cdot)$  answering  $aW \in \mathbb{G}_1$  when queried on an input  $W \in \mathbb{G}_1$ , to produce a list  $((Z_1, j_1), \dots, (Z_{q_t}, j_{q_t}))$  of  $q_t$  pairs such that  $Z_i = aY_{j_i} \in \mathbb{G}_1$  for all  $i = 1, \dots, q_t$ ,  $1 \leq j_i \leq q_t$  and  $q_h < q_t$  where  $q_h$  denotes the number of queries made to the multiplication oracle.*

The one more CDH problem was introduced in [10] where it was initially called "Chosen-Target CDH problem" and used to prove the security of a blind signature scheme obtained from the BLS signature ([14]) with a technique similar to the one used by Bellare et al. ([7]) to prove the security of Chaum's RSA based blind signature ([18]). Its commonly assumed intractability was more recently used in [35] and [6] to prove the security of identification schemes built on top of identity based signatures proposed in [17] and [31] and of a variant of the scheme described in [50]. In the upcoming sections, we will sometimes refer to this assumption as the "one more CDH assumption".

### 2.2 Security notions for identity based signatures

We consider the notion of strong existential unforgeability already considered in [1] and [11] that is slightly stronger than the usual notion of existential unforgeability under chosen-message attacks introduced in [30].

**Definition 2.** *An identity based signature scheme is said to be **strongly existentially unforgeable** under chosen-message attacks if no probabilistic polynomial time (PPT) adversary has a non-negligible advantage in this game:*

1. The challenger runs the setup algorithm to generate the system's parameters and sends them to the adversary.
2. The adversary  $\mathcal{F}$  performs a series of queries:
  - Key extraction queries:  $\mathcal{F}$  produces an identity  $ID$  and receives the private key  $d_{ID}$  corresponding to  $ID$ .
  - Signature queries:  $\mathcal{F}$  produces a message  $M$  and an identity  $ID$  and receives a signature on  $M$  that was generated by the signature oracle using the private key corresponding to the identity  $ID$ .
3. After a polynomial number of queries,  $\mathcal{F}$  produces a tuple  $(ID^*, M^*, \sigma^*)$  made of an identity  $ID^*$ , whose corresponding private key was never asked during stage 2, and a message-signature pair  $(M^*, \sigma^*)$  such that  $\sigma^*$  was not returned by the signature oracle on the input  $(M^*, ID^*)$  during stage 2 for the identity  $ID^*$ .

The forger  $\mathcal{F}$  wins the game if the signature verification algorithm outputs 1 when it is run on the tuple  $(ID^*, M^*, \sigma^*)$ . The forger's advantage is defined to be its probability of producing a forgery taken over the coin-flippings of the challenger and  $\mathcal{F}$ .

The above security notion was not considered in previous papers tackling with identity based signatures but it is interesting to notice that the schemes proposed in [17] and [31] are also provably secure in this strengthened model.

### 3 An identity based signature with tight security reductions

The present section revisits the modified Sakai-Ogishi-Kasahara signature ([6]) by considering it as a one level instantiation of a (randomized) version of Gentry and Silverberg's alternative hierarchical IBS ([27]). This scheme is the same as the one obtained by applying Bellare et al.'s extended Fiat-Shamir heuristic ([6]) to the SOK identity based identification scheme (that is only secure against passive attacks as shown in [6]).

#### 3.1 The scheme

The signature that was commonly called SOK-IBS in ([6]) (for Sakai-Ogishi-Kasahara Identity Based Signature) is made of four algorithms that are depicted on figure 1.

From an efficiency point of view, the signature issuing algorithm has the same complexity as Cha and Cheon's one ([17]) while the verification algorithm is slightly more expensive: it requires 3 pairing computations against only two for the verification algorithms of the schemes described in [17] and [31]. As will be shown in the next subsection, the advantage of the present scheme over the previously cited ones is to yield much better security guarantees in the random oracle model for similar security parameters.

Interestingly, although derived from an identity based identification scheme (IBI) that is only secure against passive attacks, the modified SOK-IBS has tighter reductions than other IBS for which the underlying IBI is secure against stronger attacks. In fact, it can also be regarded as an identity based extension of a randomized version of Boneh et al.'s short signature ([14]). That is the reason why so tight security reductions can be obtained.

#### 3.2 The exact security of SOK-IBS

This security analysis first presents a security reduction from the CDH problem to a chosen-message attacker against SOK-IBS that is more efficient than any other known security reduction (including those given in [35],[6]) for existing identity based signatures ([17],[31],etc.). In a second step, we give an even more efficient reduction from the one more CDH problem.

**Setup:** given a security parameter  $k$ , the PKG chooses groups  $\mathbb{G}_1$  and  $\mathbb{G}_2$  of prime order  $q > 2^k$ , a generator  $P$  of  $\mathbb{G}_1$ , a randomly chosen master key  $s \in \mathbb{Z}_q^*$  and the associated public key  $P_{pub} = sP$ . It also picks cryptographic hash functions of same domain and range  $H_1, H_2 : \{0, 1\}^* \rightarrow \mathbb{G}_1^*$ . The system's public parameters are

$$\text{params} = (\mathbb{G}_1, \mathbb{G}_2, \hat{e}, P, P_{pub}, H_1, H_2).$$

**Keygen:** given an user's identity  $ID$ , the PKG computes  $Q_{ID} = H_1(ID) \in \mathbb{G}_1$  and the associated private key  $d_{ID} = sQ_{ID} \in \mathbb{G}_1$  that is transmitted to the user.

**Sign:** in order to sign a message  $M$ ,

1. picks  $r \xleftarrow{R} \mathbb{Z}_q$  and computes  $U = rP \in \mathbb{G}_1$  and then  $H = H_2(ID, M, U) \in \mathbb{G}_1$ .
2. Compute  $V = d_{ID} + rH \in \mathbb{G}_1$ .

The signature on  $M$  is the pair  $\sigma = \langle U, V \rangle \in \mathbb{G}_1 \times \mathbb{G}_1$ .

**Verify:** to verify a signature  $\sigma = \langle U, V \rangle \in \mathbb{G}_1 \times \mathbb{G}_1$  on a message  $M$  for an identity  $ID$ , the verifier first takes  $Q_{ID} = H_1(ID) \in \mathbb{G}_1$  and  $H = H_2(ID, M, U) \in \mathbb{G}_1$ . He then accepts the signature if  $\hat{e}(P, V) = \hat{e}(P_{pub}, Q_{ID})\hat{e}(U, H)$  and rejects it otherwise.

**Fig. 1.** The SOK-IBS scheme

**Theorem 1.** *In the random oracle model, if a PPT forger  $\mathcal{F}$  has an advantage  $\epsilon$  in forging a signature in an attack modelled by the game of definition 2 when running in a time  $t$  and asking  $q_{H_i}$  queries to random oracles  $H_i$  ( $i=1,2$ ),  $q_E$  queries to the key extraction oracle and  $q_S$  queries to the signature oracle, then the CDH problem can be solved with an advantage*

$$\epsilon' > \frac{\epsilon - (q_S(q_{H_2} + q_S) + 1)/2^k}{e(q_E + 1)}$$

within a time  $t' < t + (q_{H_1} + q_{H_2} + q_E + 2q_S)t_m + (q_S + 1)t_{mm}$  where  $e$  denotes the base of the natural logarithm,  $t_m$  is the time to compute a scalar multiplication in  $\mathbb{G}_1$  and  $t_{mm}$  is the time to perform a multi-exponentiation in  $\mathbb{G}_1$ .

**Proof.** We start by describing how a forger  $\mathcal{F}$  can be used by a probabilistic polynomial time algorithm  $\mathcal{B}$  to solve the CDH problem. Let  $(X = xP, Y = yP) \in \mathbb{G}_1 \times \mathbb{G}_1$  be a random instance of the CDH problem taken as input by  $\mathcal{B}$ . The latter initializes  $\mathcal{F}$  with  $P_{pub} = X$  as a system's overall public key. The forger  $\mathcal{F}$  then starts performing queries such as those described in definition 2. These queries are answered by  $\mathcal{B}$  as follows (without loss of generality, we assume that, for any key extraction query or signature query involving an identity, a  $H_1$  oracle query was previously issued for the same identity):

- queries on oracle  $H_1$ : when an identity  $ID$  is submitted to the  $H_1$  oracle, as in Coron's proof technique ([21]),  $\mathcal{B}$  flips a coin  $T \in \{0, 1\}$  that yields 0 with probability  $\delta$  and 1 with probability  $1 - \delta$ .  $\mathcal{B}$  then picks  $u \xleftarrow{R} \mathbb{Z}_q^*$ . If  $T = 0$  then the hash value  $H_1(ID)$  is defined as being  $uP \in \mathbb{G}_1$ . If  $T = 1$ , then  $\mathcal{B}$  returns  $uY \in \mathbb{G}_1$ . In both cases,  $\mathcal{B}$  inserts a tuple  $(ID, u, T)$  in a list  $L_1$  to keep track of the way it answered the query.
- Key extraction queries: when  $\mathcal{F}$  requests the private key associated to an identity  $ID$ ,  $\mathcal{B}$  recovers the corresponding  $(ID, u, T)$  from  $L_1$  (recall that such a tuple must exist because of the aforementioned assumption). If  $T = 1$ , then  $\mathcal{B}$  outputs "failure" and halts because it is unable to coherently answer the query. Otherwise, it means that  $H_1(ID)$  was previously defined to be  $uP \in \mathbb{G}_1$  and  $uP_{pub} = uX \in \mathbb{G}_1$  is then returned to  $\mathcal{F}$  as a private key associated to  $ID$ .

- queries on oracle  $H_2$ : when a tuple  $(ID, M, U)$  is submitted to the  $H_2$  oracle,  $\mathcal{B}$  first scans a list  $L_2$  to check whether  $H_2$  was already defined for that input. If it was, the previously defined value is returned. Otherwise,  $\mathcal{B}$  picks a random  $v \xleftarrow{R} \mathbb{Z}_q^*$ , stores the tuple  $(ID, M, U, v)$  in the list  $L_2$  and returns  $vP \in \mathbb{G}_1$  as a hash value to  $\mathcal{F}$ .
- Signature queries: when  $\mathcal{F}$  queries the signature oracle on a message  $M$  for an identity  $ID$ ,  $\mathcal{F}$  first recovers the previously defined value  $Q_{ID} = H_1(ID) \in \mathbb{G}_1$  from  $L_1$ . It then chooses  $t, \nu \xleftarrow{R} \mathbb{Z}_q^*$  before setting  $V = tP_{pub} = tX \in \mathbb{G}_1$ ,  $U = \nu P_{pub} = \nu X \in \mathbb{G}_1$  and defining the hash value  $H_2(ID, M, U)$  as  $\nu^{-1}(tP - Q_{ID}) \in \mathbb{G}_1$  ( $\mathcal{B}$  halts and outputs "failure" if  $H_2$  turns out to be already defined for the input  $(ID, M, U)$ ). The pair  $(U, V)$  is returned to  $\mathcal{F}$  and appears as a valid signature from the latter's point of view.

Eventually, the forger  $\mathcal{F}$  produces a message  $M^*$ , an identity  $ID^*$  and a fake signature  $\langle U^*, V^* \rangle$  for the pair  $(M^*, ID^*)$  and  $\mathcal{B}$  then recovers the triple  $(ID^*, u^*, T^*)$  from  $L_1$ . If  $T^* = 0$ , then  $\mathcal{B}$  outputs "failure" and stops. Otherwise, it goes on and the list  $L_2$  must contain an entry  $(ID^*, M^*, U^*, v^*)$  with overwhelming probability (otherwise,  $\mathcal{B}$  stops and outputs "failure"). Hence, since  $H^* = H_2(ID^*, M^*, U^*)$  was defined to be  $v^*P \in \mathbb{G}_1$ , if  $\mathcal{F}$  succeeded in the game with the view it was provided with,  $\mathcal{B}$  knows that

$$\hat{e}(P, V^*) = \hat{e}(X, Q_{ID^*})\hat{e}(U^*, H^*)$$

with  $H^* = v^*P \in \mathbb{G}_1$  and  $Q_{ID^*} = u^*Y \in \mathbb{G}_1$  for some known elements  $u^*, v^* \in \mathbb{Z}_q^*$ . Then, it also knows that

$$\hat{e}(P, V^* - v^*U^*) = \hat{e}(X, u^*Y)$$

and that  $u^{*-1}(V^* - v^*U^*) \in \mathbb{G}_1$  is the solution to the CDH instance  $(X, Y) \in \mathbb{G}_1 \times \mathbb{G}_1$ .

When assessing  $\mathcal{B}$ 's probability of failure, one readily checks that its probability to fail in handling a signing query because of a conflict on  $H_2$  is at most  $q_S(q_{H_2} + q_S)/2^k$  (as  $L_2$  never contains more than  $q_{H_2} + q_S$  entries) while the probability for  $\mathcal{F}$  to output a valid forgery  $\langle U^*, V^* \rangle$  on  $M^*$  without asking the corresponding  $H_2(ID^*, M^*, U^*)$  query is at most  $1/2^k$ . Finally, by an analysis similar to Coron's one ([21]), the probability  $\delta^{q_E}(1 - \delta)$  for  $\mathcal{B}$  not to fail in a key extraction query or because  $\mathcal{F}$  produces its forgery on a 'bad' identity  $ID^*$  is greater than  $1 - 1/e(q_E + 1)$  when the optimal probability  $\delta_{opt} = q_E/(q_E + 1)$  is used when handling key extraction queries. Eventually, it comes that  $\mathcal{B}$ 's advantage is at most

$$\frac{\epsilon - (q_S(q_{H_2} + q_S) + 1)/2^k}{e(q_E + 1)}.$$

□

**Efficiency of the reduction.** We note that the obtained reduction is tighter than for any previously known ID-based signature scheme: at this stage, our bound on  $\epsilon'$  is already much better than Kurosawa and Heng's one ([35]) that was  $O(\epsilon^2/eq_E)$ . As an example, for  $k = 160$ , if we allow  $q_{H_1}, q_{H_2} < 2^{60}$  and  $q_E, q_S < 2^{30}$ , we have  $q_S(q_{H_2} + q_S)/2^k < 2 \times 2^{90}/2^{160} = 2^{-69}$ . If we assume that the advantage of an attacker in solving CDH is at most  $\epsilon' < 2^{-60}$ , we obtain that  $(\epsilon - 2^{-69})/2^{32} \leq \epsilon' < 2^{-60}$  and the probability for an attacker to break SOK-IBS is bounded by  $\epsilon \leq 2^{-28} + 2^{-69} < 2 \times 2^{-28} = 2^{-27}$ <sup>1</sup>.

**An IBS as secure as the CDH problem.** We point out that one can obtain an identity based signature as secure as the Diffie-Hellman problem by the use of a technique suggested by Katz and

<sup>1</sup> We have to mention that the upper bound on  $\mathcal{B}$ 's probability to fail in simulating the signing oracle could be improved by using another proof technique due to Coron ([22]) but, in the present situation, this would degrade the bound  $t'$  on  $\mathcal{B}$ 's running time.

Wang in [34] (and recalled in appendix A) to improve the exact security of the Boneh-Franklin IBE through a perfect simulation of the key extraction oracle. The resulting reduction from the CDH problem is then essentially optimal since the solver’s advantage for the CDH problem is then  $\epsilon' > (\epsilon - (q_S(q_{H_2} + q_S) + 1)/2^k)/2$ . In order to avoid the need of computing twice the pairing  $\hat{e}(P_{pub}, Q_{ID})$  (once for  $Q_{ID} = H_1(ID||0)$  and once for  $Q_{ID} = H_1(ID||1)$ , see [34] for details) at a signature verification, the signer simply appends a single bit to the signature to indicate which of  $Q_{ID||0} = H_1(ID||0)$  or  $Q_{ID||1} = H_1(ID||1)$  is the effective public key. We end up with a verification algorithm that is slightly less efficient than in the Cha-Cheon IBS (only an extra pairing computation is needed) while the signature issuing algorithm as well as the length of signatures remain the same. Furthermore, for a given desired security level this modification of the SOK-IBS is more efficient since it allows using a much shorter security parameter  $k$ .

**Achieving an even tighter reduction.** The theorem below shows that an even tighter reduction exists from the potentially stronger one more CDH assumption. The solver’s advantage against the one more CDH problem is then twice as large as in the reduction from the CDH problem. The theorem’s proof is straightforward and is omitted here.

**Theorem 2.** *In the random oracle model, if a PPT adversary  $\mathcal{F}$  has an advantage  $\epsilon$  against the strong unforgeability of SOK-IBS in a chosen-message attack when running in a time  $t$ , asking  $q_{H_i}$  queries to random oracles  $H_i$  ( $i=1,2$ ),  $q_E$  key extraction queries and  $q_S$  queries to the signature oracle, then there is an algorithm  $\mathcal{B}$  to solve the one more CDH problem with an advantage  $\epsilon' > \epsilon - (q_S(q_{H_2} + q_S) + 1)/2^k$  in a time  $t' < t + (q_{H_2} + 2q_S)t_m + (q_S + 1)t_{mm}$  where  $t_m$  is the cost of a scalar multiplication in  $\mathbb{G}_1$  and  $t_{mm}$  is the time to perform a multi-exponentiation in  $\mathbb{G}_1$ .*

We now obtain an excellent exact security: with  $k = 160$  and the previously given values of  $q_{H_i}$ , for  $i = 1, 2$  and  $q_S$ , assuming that the one more CDH problem cannot be solved with a probability  $\epsilon'$  greater than  $2^{-60}$ , then no attacker can break the scheme with a better advantage than  $\epsilon < 2^{-60} + 2^{-69} < 2^{-59}$ . Finding a so tight security reduction for an identity based signature appeared as an open problem before this work.

## 4 Identity based signcryption schemes with tighter security reductions

Since 2002, a couple of identity based protocols jointly performing signature and encryption have been studied ([15],[19],[39],[36],[43]). In 2003 ([15]), Boyen proposed a nice two-layer design of signature/encryption (IBSE) that provably satisfies strong security notions among which chosen-ciphertext security, signature non-repudiation, ciphertext authentication and ciphertext anonymity. This scheme also provides detachable signatures that cannot be linked to the original ciphertext (this property was called ciphertext unlinkability in [15]). The only drawback of Boyen’s IBSE is not to have tight security reductions, especially in the proof of unforgeability that relies on Pointcheval and Stern’s forking lemma. In fact, none of the previously known identity based signcryption schemes (IBSC) can be proved to be unforgeable with a tight reduction. Improvements for the schemes in [39] and [36] are possible by using intermediate reductions from the underlying signature’s unforgeability (that are in turn derived from the security of an underlying identity based identification protocol by similar arguments to those given in [35]) but the new bounds are still not as tight as those obtained for the scheme described in this section.

SOK-IBS provides a very natural construction based on a randomness re-use to perform encryption. This construction results in an identity based signature/encryption that provably satisfies the same security requirements as Boyen’s scheme with a similar efficiency and much better reductions (especially for the non-repudiation aspect). In the security analysis, because of space limitation, we restricted ourselves to only detail proofs of CCA-security (even against ”insider” attacks as for

the schemes in [15] and [19]) and signature strong unforgeability but our scheme can also be shown to be also anonymous (i.e. a ciphertext conveys no information on its intended recipient nor about who its originator is) and to allow detachable signatures that are unlinkable to their associated ciphertexts.

By some parameter's adjustment, this construction can be turned into a somewhat more efficient monolithic identity based signcryption (IBSC) that offers better security guarantees than other previous IBSC proposals ([39],[36]) and that is also provably anonymous (unlike the solution proposed in [38]). It is useful for applications that do not require to extract signed message from ciphertexts in such a way that extracted signatures cannot be linked to the original ciphertext they are embedded in. A strengthened security model for such monolithic IBSC schemes is presented in an extended version of the paper.

Before presenting our solution, let us first recall definitions of underlying hard problems on which the security of our scheme is shown to rely.

#### 4.1 Underlying hard problems

**Definition 3.** Given groups  $\mathbb{G}_1$  and  $\mathbb{G}_2$  of prime order  $q$ , a bilinear map  $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \rightarrow \mathbb{G}_2$  and a generator  $P$  of  $\mathbb{G}_1$ ,

- The **Bilinear Diffie-Hellman problem (BDH)** is, given  $\langle P, aP, bP, cP \rangle$  for unknown  $a, b, c \in \mathbb{Z}_q$ , to compute  $\hat{e}(P, P)^{abc} \in \mathbb{G}_2$ .
- The **Decisional Bilinear Diffie-Hellman problem (DBDH)** is, given  $P, aP, bP, cP \in \mathbb{G}_1$  and  $h \in \mathbb{G}_2$  to decide whether  $h = \hat{e}(P, P)^{abc}$  or not. According to the terminology of [45], tuples of the form  $\langle P, aP, bP, cP, \hat{e}(P, P)^{abc} \rangle \in \mathbb{G}_1^4 \times \mathbb{G}_2$  are called *Bilinear Diffie-Hellman tuples*.
- The **Gap Bilinear Diffie-Hellman problem (GBDH)** is, given  $\langle P, aP, bP, cP \rangle$ , for unknown  $a, b, c \in \mathbb{Z}_q^*$ , to compute  $\hat{e}(P, P)^{abc}$  with the help of a DBDH oracle that is able to decide within a unit time whether a tuple  $\langle P, a'P, b'P, c'P, h' \rangle \in \mathbb{G}_1^4 \times \mathbb{G}_2$  is such that  $h' = \hat{e}(P, P)^{a'b'c'}$  or not.

The DBDH problem was recently used in [12] to prove the security of an identity based encryption scheme in a standard computational model. Its was previously considered in [37] where the GBDH problem (whose denomination emanates from a terminology due to Okamoto and Pointcheval ([45])) was considered for the first time.

#### 4.2 The SOK-IBSE scheme

We call this new scheme SOK-IBSE (for SOK-like Identity Based Signature-Encryption). The protocol is given by the six algorithms depicted on figure 2. As done in [15], we assume the receiver of a ciphertext has no a priori knowledge on the sender's identity that is thus encrypted together with the plaintext.

A random salt  $\tau$  of length  $\delta$  is also encrypted together with the message and the sender's identity at the encryption operation. This random string, that is hashed with the sender and receiver's identities when computing the  $X$  component of the ciphertext from the  $U$  component of the signature, aims at providing the ciphertext unlinkability property by preventing someone observing a signed message  $(M, \langle U, V \rangle)$  for a signer's identity  $ID_A$  to know how this message-signature pair should be encrypted into a ciphertext intended to some receiver  $ID_B$ . That is why it is also encrypted into one of the ciphertext's components to allow the receiver to detach the signature.

We re-use Boyen's construction ([15]) but, in order to achieve better security reductions, we also hash the ciphertext's  $X$  component and the recipient's public key  $Q_{ID_B}$  together with the pairing's result at the encryption operation.



**Setup:** is identical to its counterpart in SOK-IBS except that five hash functions are needed:  $H_1 : \{0, 1\}^* \rightarrow \mathbb{G}_1$ ,  $H_2 : \{0, 1\}^{n_0+n_1+\ell} \rightarrow \mathbb{G}_1$ ,  $H_3 : \mathbb{G}_1^2 \times \mathbb{G}_2 \rightarrow \{0, 1\}^\ell$ ,  $H_4 : \mathbb{G}_1 \rightarrow \{0, 1\}^\lambda$  and  $H_5 : \{0, 1\}^{2n_1+\delta} \rightarrow \mathbb{Z}_q^*$  where  $\ell$  is the bitlength of  $\mathbb{G}_1$ 's elements. The space of plaintexts is  $\mathcal{M} := \{0, 1\}^{n_0}$ , and the ciphertext space is  $\mathcal{C} := \mathbb{G}_1 \times \{0, 1\}^\ell \times \{0, 1\}^{n_0+n_1+\delta}$  where  $n_1$  denotes the maximum length of identifiers and  $\delta$  is another security parameter explained below. A symmetric cipher  $(\mathcal{E}, \mathcal{D})$  of keylength  $\lambda$  is also chosen.

**Keygen:** is the same as in SOK-IBS.

**Sign:** given a message  $M$  and the sender's private key  $d_{ID_A}$ ,

1. pick  $r \in \mathbb{Z}_q^*$ , compute  $U = rP$  and  $H = H_2(ID_A, M, U) \in \mathbb{G}_1$ ,
2. set  $V = d_{ID_A} + rH \in \mathbb{G}_1$ ,
3. return  $\langle M, r, U, V, ID_A \rangle$ .  
to Encrypt

**Encrypt:** given  $\langle M, r, U, V, ID_A \rangle$  and the recipient's identity  $Q_{ID_B} = H_1(ID_B) \in \mathbb{G}_1$ ,

1. compute  $x = H_5(ID_A, ID_B, \tau) \in \mathbb{Z}_q$  for  $\tau \xleftarrow{R} \{0, 1\}^\delta$  and  $X = xU \in \mathbb{G}_1$ ,
2. set  $W = V \oplus H_3(X, Q_{ID_B}, g^{xr})$ , with  $g = \hat{e}(P_{pub}, Q_{ID_B})$ ,
3. compute  $Z = \mathcal{E}_\kappa(M || ID_A || \tau)$ , with  $\kappa = H_4(V) \in \{0, 1\}^\lambda$ , and the final ciphertext is  $\langle X, W, Z \rangle$ .

**Decrypt:** given a ciphertext  $\langle X, W, Z \rangle$  and the private key  $d_{ID_B}$ ,

1. compute  $V = W \oplus H_3(X, Q_{ID_B}, g')$  with  $g' = \hat{e}(X, d_{ID_B})$  and then  $(M || ID_A || \tau) = \mathcal{D}_\kappa(Z)$  with  $\kappa = H_4(V) \in \{0, 1\}^\lambda$ ,
2. compute  $Q_{ID_A} = H_1(ID_A) \in \mathbb{G}_1$ ,  $U = x^{-1}X \in \mathbb{G}_1$  with  $x = H_5(ID_A, ID_B, \tau) \in \mathbb{Z}_q^*$ ,
3. return  $\langle M, ID_A, U, V \rangle$  to Verify

**Verify:** given  $\langle M, ID_A, U, V \rangle$ ,

Return 1 if  $\hat{e}(P, V) = \hat{e}(P_{pub}, Q_{ID_A})\hat{e}(U, H)$ , where  $H = H_2(ID_A, M, U) \in \mathbb{Z}_q^*$  and  $Q_{ID_A} = H_1(ID_A) \in \mathbb{G}_1$  and 0 otherwise.

**Fig. 2.** The SOK-IBSE Signature/Encryption scheme

**Efficiency discussions.** From an efficiency point of view, for a given security parameter  $k$ , SOK-IBSE has exactly the same cost as Boyen's scheme while ciphertexts produced by the latter are  $\delta$  bits shorter. A naive composition of SOK-IBS with the Boneh-Franklin IBE or with any of its variants (e.g. the one in [2]) could not have final ciphertexts shorter than  $4\ell + n_0 + n_1$  (indeed, the whole signature would need to be encrypted with the message and the sender's identity for ciphertext unlinkability purposes) while those of SOK-IBSE are no longer than  $3\ell + n_0 + n_1$  in the worst case (we can assume  $\delta \leq \ell$  as explained below).

**Other comparisons with Boyen's IBSE.** The size  $\delta$  of the random seed, that is exactly the difference between the length of ciphertexts in Boyen's scheme and their size in SOK-IBSE, is flexible: it is an unlinkability parameter depending on the specific application: if detached signatures are required to be computationally unlinkable to their ciphertexts, it is recommended to take  $\delta = 160$ . If senders only need to be able to deny having created any ciphertext from a third party's view and to discourage anyone to try and link their detached signatures to any ciphertext, a short seed of 20 bits is sufficient. Finally, if a user does not need his/her signatures to be unlinkable to the ciphertext they are embedded in, one can set  $\delta = 0$  and  $x = 1$  to obtain a monolithic single-layer identity based signcryption (IBSC). It is then recommended to hash the recipient's identity  $ID_B$  together with the sender's one in the evaluation of  $H \in \mathbb{G}_1$  upon the ciphertext's construction in

order for the notion of ciphertext strong unforgeability to be satisfied in the monolithic security model (that is described in an extended version of this paper while the resulting efficient IBSC is described in appendix D).

We must concede that this method to achieve ciphertext unlinkability is not practical for multi-recipient signature/encryption purposes (unless one is not concerned with the leakage of the information that all components of the multi-recipient ciphertext hide the same signed message) but, in such a scenario, one can easily use Boyen’s method (that consists of setting  $x = H_5(\hat{e}(d_{IDA}, Q_{IDB})) = H_5(\hat{e}(Q_{IDA}, d_{IDB}))$  instead of using a hidden random seed  $\tau$ ) without being concerned with the irreflexivity assumption (i.e. the assumption that a ciphertext’s sender and receiver are always distinct entities) that causes a further degradation when dropped in [15] (indeed the use of a decisional BDH oracle helps in achieving a perfect simulation).

Interestingly, the use of a random seed to handle unlinkability concerns allows the sender and the receiver not to depend on the same PKG since the hash value of the sender’s identity is never paired with the receiver’s private key (and vice versa). As a result, if all users have confidence in the public keys of all PKGs (this could be achieved by the use of PKG certificates), a sender can be imagined to append a tag indicating which PKG he/she depends on to his/her identity when completing the encryption operation. A tradeoff between identity and certificateful public key cryptography is then obtained: digital certificates are again necessary but their use is much more moderate than in traditional PKIs (since only PKGs public keys need to be certified) and, for large scale applications requiring both privacy and authentication, this is much more flexible than a hierarchical sequential composition ID-based signature-encryption. What is more, this new certificate-and-identity based signature/encryption solution preserves the anonymity of ciphertexts unlike hierarchical schemes (recall that, in Gentry and Silverberg’s ones, a ciphertext’s length reveals the depth of its intended recipient in the hierarchy).

The security proofs presented in the next section only consider the case of a single authority but they can be easily adapted to the multi-authority setting.

### 4.3 Security analysis of SOK-IBSE

We refer to appendix B for a recall of the security models of chosen-ciphertext security and signature non-repudiation defined by Boyen ([15]) for IBSE schemes. The following theorems, for which the proofs can be found in appendix, claim that SOK-IBSE satisfies these two notions with tighter security bounds than the original IBSE. It can also be shown to satisfy the other notions of ciphertext authentication and ciphertext anonymity that are also formally described in [15].

**Theorem 3.** *In the random oracle model, if a chosen-ciphertext adversary  $\mathcal{A}$  has an advantage  $\epsilon$  against SOK-IBSE when running in a time  $t$ , making  $q_{H_i}$  queries to the random oracles  $H_i$  ( $i = 1, \dots, 4$ ),  $q_{SE}$  signature/encryption queries and  $q_{DV}$  decryption/verification queries, then there exists a PPT algorithm  $\mathcal{B}$  solving the GBDH problem with an advantage*

$$\epsilon' \geq \frac{1}{e^{(q_E + 1)}} \left( \epsilon - q_{SE} \frac{q_{SE} + q_{H_2}}{2^k} - q_{DV} \left( \frac{q_{H_4}}{2^\ell} + \frac{1}{2^k} \right) \right)$$

*within a time  $t' \leq t + (q_{H_1} + q_E + 3q_{SE})t_m + q_{DV}t_{exp} + 2q_{DV}t_p + q_{H_3}O(1)$  where  $e$  denotes the base of the natural logarithm,  $t_p$  is the time to perform a pairing evaluation,  $t_m$  is the cost of a multiplication in  $\mathbb{G}_1$  and  $t_{exp}$  is the time to perform an exponentiation in  $\mathbb{G}_2$ .*

**Proof.** given in appendix. □

We observe that the reduction is much tighter than for any previously known identity based authenticated encryption scheme: for common security parameters  $k = \ell = 160$ , if we allow  $q_{H_i} \leq 2^{60}$  for  $i = 1, \dots, 4$  and  $q_{SE}, q_{DV} \leq 2^{30}$ , we have  $\epsilon' \geq 2^{-32}\epsilon - 2^{-100}$ . Unfortunately, unlike what happens

with SOK-IBS, the reduction is not optimal since we still have a loss of  $2^{30}$  in the security bound. This loss can be removed by applying the Katz-Wang trick again but this would then require two pairing computations at encryption. An alternative solution to keep the scheme's efficiency would be to make its security rely on a stronger and non-standard assumption that can be called "one more Gap Bilinear Diffie-Hellman" and that consists of a relaxed version of the GBDH one obtained by providing the simulator with a powering oracle raising elements of  $\mathbb{G}_2$  to a fixed hidden power beside its decisional oracle. The simulator would then have to solve  $t$  challenge while asking strictly less than  $t$  queries to its powering oracle.

The next theorem claims the strong existential unforgeability of SOK-IBSE.

**Theorem 4.** *In the random oracle model, if an adversary  $\mathcal{F}$  has an advantage  $\epsilon$  against the strong existential unforgeability (ESUF-IBSC-CMCA) of SOK-IBSE when running in a time  $t$ , making  $q_{H_i}$  queries to the random oracles  $H_i$  ( $i = 1, \dots, 4$ ),  $q_{SE}$  signature/encryption queries and  $q_{DV}$  decryption/verification queries, then there exists an algorithm  $\mathcal{B}$  to solve the GBDH problem in a time with an advantage*

$$\epsilon' \geq \frac{1}{e^{(q_E + 1)}} \left( \epsilon - q_{SE} \frac{q_{SE} + q_{H_2}}{2^k} - q_{DV} \left( \frac{q_{H_4}}{2^\ell} + \frac{1}{2^k} \right) - 2^{-\ell} - 2^{-k} - 2^{n_0 + n_1 + \delta - 2\lambda} \right)$$

within a time  $t' \leq t + (q_{H_1} + q_E + 3q_{SE})t_m + q_{DV}t_{exp} + 2(q_{DV} + 1)t_p + q_{H_3}O(1)$  where  $e$  is the base of the natural logarithm,  $t_p$  is the time to perform a pairing evaluation,  $t_m$  denotes the cost of a multiplication in  $\mathbb{G}_1$  and  $t_{exp}$  is the time to perform an exponentiation in  $\mathbb{G}_2$ .

**Proof.** given in appendix □

## 5 Conclusions

In this paper, we showed that optimal security reductions are also achievable for identity based signatures. We showed that the variant of the Sakai-Ogishi-Kasahara IBS proposed by Bellare et al. ([6]) has an improved but still loose reduction. Our proof does not make use of the forking lemma ([48],[49]) and bypasses the BNN framework ([6]) to directly achieve a reduction from scratch.

Interestingly, Bellare et al.'s modified SOK-IBS, that can be viewed as derived from the Gentry-Silverberg second hierarchical scheme, has an efficiency comparable to other existing IBS proposals such as those described in [17] or [31] and much stronger security guarantees for similar security parameters since it can be made essentially as secure as the Diffie-Hellman problem at a negligible cost. We also showed that Bellare et al.'s first modified SOK-IBS is also as secure as the one more CDH problem. We think that, beside their interest for identity based cryptography, these results can also find application to key evolving signatures that were shown to be somewhat related to IBS schemes ([23],[53]).

As an application of our results, we showed that SOK-IBS could be turned into an identity based signature encryption (IBSE) similar to Boyen's one ([15]) but providing much better security guarantees or into a strongly secure monolithic identity based signcryption scheme (IBSC) depicted in appendix D. The obtained reductions are not optimal for SOK-IBSE: recall that we still have a loss of  $q_E$  in our bound (unless using the Katz-Wang trick on the sender as well as the receiver's side but, in this case, this would degrade the scheme's efficiency: the sender would have to compute an additional pairing at encryption while the receiver would might have to perform 5 pairing evaluation), where  $q_E$  is a bound on the number of corrupted identities. Nevertheless, we think it is a real breakthrough in terms of provable security for identity based cryptosystems. Finally, one of our scheme's advantage over the first IBSE is that it can be scaled up into a multi-authority signature encryption protocol.

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## Appendix

### A. The Katz-Wang trick to avoid security degradation in the Boneh-Franklin IBE

This trick, suggested by Dan Boneh and discussed in [34], derives from a technique to achieve tight security reductions for digital signature schemes without using random salts. The idea is that a message is always signed in the same way: the signer initially sets up a random seed  $SK$ . When a message  $m$  has to be signed, it is first hashed together with a message-dependent bit  $b_m = H'(SK||m)$  into a hash value  $H(b_m||m)$  that is then signed using RSA or using the BLS ([14]) signature scheme (the bit  $b_m$  is appended to the signature to speed up the signature verification). In the security proof, when the simulator is asked for the hash value of a pair  $(b_i||m_i)$ , where  $b_i \in \{0, 1\}$  and  $m \in \mathcal{M}$ , if  $b_i = b_{m_i}$ , the simulator returns a value that will subsequently allow him to answer a signature query on  $m_i$ . On the other hand, if  $b_i = \bar{b}_m$ , the simulator answers a value that would allow him to solve the underlying number theoretic problem he takes as input if the adversary eventually produces its forgery  $(m^*, \langle b^*, m^* \rangle)$  on the message  $m_i$  and if  $b^* \neq b_{m^*}$ .

This proof technique can be used for RSA (with Full Domain Hash or PSS) as well as for other signature schemes such as the BLS short signature. In order to apply it to improve the security reductions of IBE thanks to a perfect simulation of the key extraction oracle, we consider that a PKG initially chooses a secret random seed  $SK$  and that the private key  $d_{ID}$  associated to an identity  $ID$  is given by  $d_{ID} = sH(b_{ID}||ID)$  where the identity dependent bit  $b_{ID} = H'(SK, ID) \in \{0, 1\}$  is obtained through a random oracle  $H'$ . By doing so, someone wanting to use an identity  $ID$  for encryption or to verify a signature never knows which of  $Q_{0||ID} = H_1(0||ID) \in \mathbb{G}_1$  or  $Q_{1||ID} = H_1(1||ID) \in \mathbb{G}_1$  must be used as an effective public key. That is the reason why two pairings must then be computed at encryption in the Boneh-Franklin IBE and four pairings might need to be computed in a signature verification operation with the SOK-IBS.

In the security proofs, when the simulator is asked for a hash value  $H_1(b_{ID_i}||ID_i)$ , he returns  $\mu P \in \mathbb{G}_1$  in such a way that he can subsequently return a valid private key  $\mu P_{pub}$ , where  $\mu \stackrel{R}{\leftarrow} \mathbb{Z}_q^*$ , for the identity  $ID_i$ . When the adversary asks the hash value  $H_1(\bar{b}_{ID_j}, ID_j)$ , the simulator then returns  $\mu(yP)$  where  $yP \in \mathbb{G}_1$  is a part of a CDH or a BDH instance. In the case of SOK-IBS, when the adversary produces a forgery  $\langle U^*, V^* \rangle$  on a message  $M^*$  and an identity  $ID^*$ , there is a probability of 1/2 that the signature verification algorithm accepts the signature for the public key  $Q_{\bar{b}_{ID^*}||ID^*}$  and that this forgery then allows the simulator to solve his CDH instance.

### B. Formal security model for identity based signature/encryption (IBSE)

In [15], Boyen gives formal definitions for the security of two-layer identity based signature/encryption schemes. For the chosen-ciphertext security of his scheme, he considers "insider attacks" (i.e. attacks led by attackers that learn the private key of the challenge ciphertext's sender at some moment of the game). We recall this definition as well as the one of "encrypted signature non-repudiation". These definitions can be skipped by the reader who is familiar with the model of [15].

**Definition 4.** *An identity based signature/encryption scheme (IBSE) is said to be **adaptively chosen-ciphertext secure** (IND-IBSE-CCA) if no PPT adversary has a non-negligible advantage in the following game.*

1. *The challenger runs the Setup algorithm on input of a security parameter  $k$  and sends the domain-wide parameters to the cca-adversary  $\mathcal{A}$ .*
2.  *$\mathcal{A}$  starts to adaptively query the following oracles:*
  - *Key extraction oracle: given an identity  $ID$ , it returns the extracted private key associated to it.*

- *Signature/encryption oracle*: given a sender and a receiver's identities  $ID_S, ID_R$  and a plaintext  $M$ , it returns an encryption under the receiver's identity  $ID_R$  of the message  $M$  signed in the name of the sender  $ID_S$ .
  - *Decryption/verification oracle*: given a receiver's identity  $ID_R$  and a ciphertext  $\sigma$ , it generates the private key  $d_{ID_R} = \text{Keygen}(ID_R)$  and returns either a triple  $(M, s, ID_S)$  made of a valid message-signature pair  $(M, s)$  for the sender's identity  $ID_S$  or the  $\perp$  symbol if  $\sigma$  does not decrypt into a valid message-signature pair.
3. After the **find** stage,  $\mathcal{A}$  produces two equal-length plaintexts  $M_0, M_1 \in \mathcal{M}$  and two identities  $ID_S^*$  and  $ID_R^*$  on which it wishes to be challenged. It may not have corrupted the private key corresponding to  $ID_R^*$  in the **find** stage.
  4. The challenger picks a bit  $b \leftarrow_R \{0, 1\}$  and computes  $C = \text{Sign/Encrypt}(M_b, d_{ID_S^*}, ID_R^*)$  which is sent to  $\mathcal{A}$ .
  5. In the **guess** stage,  $\mathcal{A}$  asks new queries as in the **find** stage. This time, it may not issue a key extraction request on  $ID_R^*$  and it cannot submit  $C$  to the decryption/verification oracle for the target identity  $ID_R^*$ .
  6. Finally,  $\mathcal{A}$  outputs a bit  $b'$  and wins if  $b' = b$ .

$\mathcal{A}$ 's advantage is defined as  $\text{Adv}(\mathcal{A}) := |2 \times \Pr[b' = b] - 1|$ .

In the following definition, the forger is allowed to corrupt the receiver's identity  $ID_R$ . The motivation for this is to really achieve the non-repudiation property: a dishonest recipient cannot send a ciphertext to himself on behalf of Alice and to try and convince a third party that Alice was the author of the signed-message embedded in it.

**Definition 5.** An identity based signature/encryption scheme (IBSE) is said to be **strongly existentially signature-unforgeable** against adaptive chosen messages and ciphertexts attacks (ESUF-IBSE-CMCA) if no PPT adversary can succeed in the following game with a non-negligible advantage:

1. the challenger runs the Setup algorithm on input  $k$  and gives the system-wide public key to the adversary  $\mathcal{F}$ .
2.  $\mathcal{F}$  starts querying the oracles as in the previous definition.
3. Finally,  $\mathcal{F}$  outputs a pair  $(\sigma^*, ID_R^*)$  and wins the game if  $\text{Decrypt/verify}(\sigma^*, d_{ID_R^*}^*)$  (i.e. the result of the decryption/verification oracle on the ciphertext  $\sigma^*$  under the private key associated to  $ID_R^*$ ) is a valid message-signature  $(M^*, s^*)$  pair for an uncorrupted sender's identity  $ID_S^*$  such that no signature/encryption query involved  $M^*, ID_S^*$  and some receiver  $ID_R'$  (possibly different from  $ID_R^*$ ) and resulted in a ciphertext  $\sigma'$  whose decryption under the private key  $d_{ID_R'}$  is the alleged forgery  $(M^*, s^*, ID_S^*)$ .

The adversary's advantage is its probability of victory.

This notion is called "strong unforgeability" because the forger is allowed to produce a ciphertext decrypting into a triple  $(M^*, s^*, ID_S^*)$  such that a signature/encryption query involved  $M^*, ID_S^*$  and  $ID_R' \neq ID_R^*$  but resulted into a ciphertext  $\sigma'$  such that the oracle call  $\text{Decrypt/verify}(\sigma', d_{ID_R'})$  produces a triple  $(M^*, s, ID_S) \neq (M^*, s^*, ID_S^*)$ .

## C. Security proofs for SOK-IBSE

### C.1 Proof of theorem 3

Let  $(P, aP, bP, cP)$  be a random instance of the GBDH problem taken as input by a simulator  $\mathcal{B}$  and let  $\mathcal{O}_P^{DBDH}(\cdot)$  be an oracle deciding within a short and constant time whether a given tuple  $(a'P, b'P, c'P, h') \in \mathbb{G}_1^3 \times \mathbb{G}_2$  is a correct BDH one or not. We show how  $\mathcal{B}$  uses a chosen-ciphertext attacker  $\mathcal{A}$  against SOK-IBSE to solve this problem. To achieve this, it first pre-computes  $\alpha = \hat{e}(cP, bP)$  and  $\beta = \hat{e}(cP, P)$  and initializes  $\mathcal{A}$  under the system-wide public key  $P_{pub} = cP \in \mathbb{G}_1$ . In order to coherently answer  $\mathcal{A}$ 's requests,  $\mathcal{B}$  maintains lists  $L_i$  to keep track of answers given to queries on random oracles  $H_i$  (for  $i = 1, \dots, 4$ ). Without loss of generality, we may assume that any key extraction query, signature/encryption or decryption/verification query involving an identity is preceded by a  $H_1$  query on the same identity. We also assume that  $\mathcal{A}$  never submits to the decryption/verification oracle a ciphertext-identity pair obtained from the signature/encryption oracle. Indeed, since the decryption/verification algorithm is deterministic,  $\mathcal{B}$  would be able to answer such a request by simply keeping in memory the plaintexts and the random coins used when simulating any signature/encryption query.

In the simulation, the  $H_3$  oracle needs to be simulated in a somewhat special fashion. Namely, an auxiliary list  $L'_3$  is also used and the decisional oracle  $\mathcal{O}_P^{DBDH}(\cdot)$  is called upon each  $H_3$  query. In more details,

- $H_1$  queries are dealt with as in the proof of theorem 1. Namely, when an identity  $ID$  is submitted to the  $H_1$  oracle,  $\mathcal{B}$  flips a coin  $T$  taking the value 0 with probability  $\delta$  and the value 1 with probability  $1 - \delta$ .  $\mathcal{B}$  then chooses a random  $u \xleftarrow{R} \mathbb{Z}_q^*$  before returning  $uP \in \mathbb{G}_1$  if  $T = 0$  and  $u(bP) \in \mathbb{G}_1$  otherwise. The entry  $(ID, u, T)$  is then inserted into the list  $L_1$ .
- $H_2$  queries on input  $(ID_{A,i}, M_i, U_i)$ :  $\mathcal{B}$  returns a randomly sampled element  $h_{2,i} \xleftarrow{R} \mathbb{G}_1$ . The whole tuple  $(ID_{A,i}, M_i, U_i, h_{2,i})$  is then inserted into  $L_2$ .  $\mathcal{B}$  must of course answer the previously stored  $h_{2,i}$  if an entry of this form was already present in  $L_2$ .
- $H_3$  queries on an input  $(X_i, Q_i, g_i) \in \mathbb{G}_1^2 \times \mathbb{G}_2$ : if the list  $L_3$  already contains an entry  $(X_i, Q_i, g_i, h_{3,i}, \cdot)$ , then  $\mathcal{B}$  returns  $h_{3,i}$ . Otherwise,  $\mathcal{B}$  checks whether  $L'_3$  contains an entry of the form  $(X_i, Q_i, \cdot, h_{3,i})$  such that, when queried on  $(P, P_{pub}, X_i, Q_i, g_i)$ , the oracle  $\mathcal{O}_P^{DBDH}(\cdot)$  returns 1. If it does,  $\mathcal{B}$  returns  $h_{3,i}$  and stores the pair  $(X_i, Q_i, g_i, h_{3,i}, 1)$  in  $L_3$ . If no entry of the above form is found in  $L'_3$ , a random  $h_{3,i} \xleftarrow{R} \{0, 1\}^\ell$  is returned to  $\mathcal{A}$  and the information  $(X_i, Q_i, g_i, h_{3,i}, t_i)$ , where  $t_i = \mathcal{O}_P^{DBDH}(P, P_{pub}, X_i, Q_i, g_i)$ , is stored in  $L_3$ .
- $H_4$  and  $H_5$  queries are handled in the usual fashion by returning a randomly sampled element in the appropriate range  $(\{0, 1\}^\lambda$  or  $\mathbb{Z}_q)$  and updating the corresponding list.

The other kinds of queries are tackled with as follows:

- Key extraction query: when  $\mathcal{A}$  asks for the private key associated to an identity  $ID$ ,  $\mathcal{B}$  looks at the corresponding entry  $(ID, u, T)$  that must exist in  $L_1$ . If  $T = 1$ , it aborts and outputs "failure". Otherwise, it returns  $uP_{pub} = u(cP) \in \mathbb{G}_1$ .
- Signature/encryption query: for a plaintext  $M$  and a pair of identities  $(ID_A, ID_B)$  submitted to the signature/encryption oracle,  $\mathcal{B}$  first recovers  $Q_{ID_A} = H_1(ID_A)$  and  $Q_{ID_B} = H_1(ID_B)$  from  $L_1$ . It then picks  $t, \nu \xleftarrow{R} \mathbb{Z}_q$ , computes  $U = \nu P_{pub}$ ,  $V = t P_{pub}$  before setting the hash value  $H_2(ID_A, M, U)$  to  $\nu^{-1}(tP - Q_{ID_A})$ . It aborts and outputs "failure" if  $H_2$  is already defined at the point  $(ID_A, M, U)$ . Otherwise, it runs the  $H_5$  simulation algorithm to obtain  $x = H_5(ID_A, ID_B, \tau) \in \mathbb{Z}_q$ , for a random  $\tau \xleftarrow{R} \{0, 1\}^\delta$ , and then calculates  $X = xU \in \mathbb{G}_1$ . At this point, depending on whether  $L_3$  contains an entry  $(X, Q_{ID_B}, g, h_3, 1)$ , two cases can be distinguished: if it does,  $\mathcal{B}$  sets  $W = V \oplus h_3$  for the corresponding  $h_3$  and returns the ciphertext  $\langle X, W, \mathcal{E}_{H_4(V)}(M || ID_A || \tau) \rangle$  where the value of  $H_4$  is obtained through simulation. If no entry of



the aforementioned form exists in  $L_3$ ,  $\mathcal{B}$  picks a random string  $h_3 \leftarrow \{0, 1\}^\ell$  and stores the tuple  $(X, Q_{ID_B}, \cdot, h_3)$  in  $L'_3$  before defining  $W = V \oplus h_3 \in \{0, 1\}^\ell$ ,  $\kappa = H_4(V) \in \{0, 1\}^\lambda$  (obtained by simulation) and  $Z = \mathcal{E}_\kappa(M || ID_A || \tau)$ . The ciphertext  $\langle X, W, Z \rangle$  is finally returned to  $\mathcal{A}$ .

- Decryption/verification query: for a ciphertext  $\sigma = \langle X, W, Z \rangle$  and a recipient's identity  $ID_B$  submitted to the decryption/verification oracle,  $\mathcal{B}$  starts by recovering the corresponding  $Q_{ID_B}$  from the list  $L_1$ . With overwhelming probability, a unique entry  $(X, Q_{ID_B}, g, h_3, 1)$  must exist in  $L_3$ , for some  $h_3 \in \{0, 1\}^\ell$ , if  $\sigma$  was correctly formed.  $\mathcal{B}$  then looks into  $L_4$  for the unique entry  $(V, h_4)$ , for some string  $V \in \{0, 1\}^\ell$  representing a point on the curve, such that  $h_3 = V \oplus W \in \{0, 1\}^\ell$  ( $h_3$  being the output of  $H_3(X, Q_{ID_B}, g)$  where  $\mathcal{O}_P^{DBDH}(P, P_{pub}, X, Q_{ID_B}, g) = 1$ ). The only possible pair of such entries in  $L_3$  and  $L_4$  is further examined as follows: if  $(M || ID_A || \tau) = \mathcal{D}_{h_4}(Z)$  is such that the hash value  $h_2 = H_2(M, ID_A, U) \in \mathbb{G}_1$  satisfies  $\hat{e}(P, V) = \hat{e}(P_{pub}, Q_{ID_A})\hat{e}(U, h_2)$ , where  $U = x^{-1}X \in \mathbb{G}_1$  for an  $x = H_5(ID_A, ID_B, \tau)$  obtained by  $H_5$  simulation (note that this condition can be checked by only computing two pairings and one exponentiation since  $\hat{e}(P_{pub}, Q_{ID_A})$  can be obtained by raising  $\alpha$  or  $\beta$  to a known power),  $\mathcal{B}$  returns the message-identity pair  $(M || ID_A)$  together with the appended signature  $(U, V)$ . The  $\perp$  symbol is returned if the appropriate  $H_3$  query was not asked or if no entry of  $L_4$  satisfies the aforementioned conditions (i.e.  $V \oplus W$  matches the right output of  $H_3$  and  $\hat{e}(P, V) = \hat{e}(P_{pub}, Q_{ID_A})\hat{e}(U, h_2)$  with  $h_2 = H_2(\mathcal{D}_{h_4}(Z), ID_B, U)$ ).

Once  $\mathcal{A}$  decides that the find stage is over, it outputs a pair  $(M_0, M_1)$  of plaintexts that will be ignored by the simulator  $\mathcal{B}$  and an uncorrupted recipient's identity  $ID_B$ . If  $T = 0$  for the corresponding entry  $(ID_B, u, T)$  of  $L_1$ , then  $\mathcal{B}$  fails. Otherwise, it sends the challenge  $\sigma = \langle aP, W, Z \rangle$ , where  $W \stackrel{R}{\leftarrow} \{0, 1\}^\ell$  and  $Z \stackrel{R}{\leftarrow} \{0, 1\}^{n_0+n_1+\delta}$  are random strings, to  $\mathcal{A}$ . The latter will be unable to realize that  $\sigma$  is not an actual ciphertext unless it asks the hash value  $H_3(aP, u(bP), \hat{e}(P, P)^{(abc)u})$  during the simulation. This would provide  $\mathcal{B}$  with the solution to the GBDH problem since it knows  $u$  and the  $\mathcal{O}_P^{DBDH}$  oracle can decide whether a candidate  $(P, aP, u(bP), cP, g)$  is a valid BDH tuple or not.

During the guess stage,  $\mathcal{A}$  keeps on issuing new queries that are handled as in the find stage but, this time, for any query  $(aP, u(bP), g)$  with  $g \in \mathbb{G}_2$  made to the  $H_3$  oracle,  $\mathcal{B}$  submits  $(P, aP, u(bP), cP, g)$  to the  $\mathcal{O}_P^{DBDH}(\cdot)$  oracle. If the latter returns 1,  $\mathcal{B}$  then stops and outputs  $g^{1/u} \in \mathbb{G}_2$  as a result. Since  $\mathcal{A}$  is assumed to have a non-negligible advantage  $\epsilon$  in the IND-IDSC-CCA game, one can easily show (as done in many papers in the literature, see [13] for example) that it must ask the hash value  $H_3(aP, u(bP), \hat{e}(P, P)^{(abc)u})$  with a probability at least  $\epsilon$  during the game provided the simulation is perfect.

The simulation can be readily checked to only be non perfect if a ciphertext is wrongly rejected by the simulator  $\mathcal{B}$  or if the latter reaches a state of "failure" during the game. The probability for the second undesirable event to occur because of a 'bad' key extraction query or because of a 'bad' chosen target identity  $ID_B$  is bounded by  $1/e(q_E + 1)$ . On the other hand,  $\mathcal{B}$ 's probability to fail in answering a signature/encryption query because of a hash collision is not greater than  $q_{SE}(q_{SE} + q_{H_2})/2^k$ .

Finally, the probability to incorrectly reject a ciphertext at some moment of the simulation is bounded by  $q_{DV}(q_{H_4}/2^\ell + 1/2^k)$ . Indeed, for a given decryption/verification query on a pair  $(\sigma = \langle X, W, Z \rangle, ID_B)$ , the  $\perp$  symbol is returned if no hash query  $H_3(X, Q_{ID_B}, g)$  was made for the only valid BDH tuple  $(P, P_{pub}, X, Q_{ID_B}, g)$ . In this case, if this query is subsequently asked by  $\mathcal{B}$ , there is a probability of  $q_{H_4}/2^\ell$  that the answer hits  $W \oplus V_i$  ( $0 \leq i \leq q_{H_4}$ ) for some entry  $(V_i, h_{4,i})$  of  $L_4$ . On the other hand, if the appropriate  $H_3(X, Q_{ID_B}, g) = W \oplus V$  was asked for some  $(V, h_4)$  in  $L_4$  but the corresponding  $H_2(ID_A, M, U)$ , with  $(M || ID_A || \tau) = \mathcal{D}_{h_4}(Z)$  and  $U = x^{-1}X$  for  $x = H_5(ID_A, ID_B, \tau)$ , was not asked by  $\mathcal{B}$ , the probability for this query to be subsequently answered in the only way that renders  $\sigma$  valid from  $\mathcal{A}$ 's view is at most  $1/2^k$  (indeed, there is a single value of  $h_2 \in \mathbb{G}_1$  such that  $\hat{e}(P, V)\hat{e}(P_{pub}, Q_{ID_A})^{-1} = \hat{e}(U, h_2)$ ).

The bound on  $\mathcal{B}$ 's running time easily comes by noting that each  $H_1$  or key extraction query requires one scalar multiplication in  $\mathbb{G}_1$  while answering a signature/encryption query implies to perform two multiplications and one computation of the type  $aP + bQ \in \mathbb{G}_1$ . Finally, decryption/verification queries all require two pairing evaluations as well as an exponentiation in  $\mathbb{G}_2$  and every  $H_3$  query implies a query to the  $\mathcal{O}_P^{DBDH}$  oracle that takes a constant time.

□

## C.2 Proof of theorem 4

Let  $(P, aP, bP, cP)$  be a random instance of the GBDH problem given as input to  $\mathcal{B}$  and let  $\mathcal{O}_P^{DBDH}(\cdot)$  be an oracle solving DBDH instances in short and constant time. The simulator  $\mathcal{B}$  runs the forger  $\mathcal{F}$  on the system-wide key  $P_{pub} = cP \in \mathbb{G}_1$  and answers all oracles requests exactly as in the proof of IND-IBSE-CCA security: the  $\mathcal{O}_P^{DBDH}(\cdot)$  oracle is again necessary to handle decryption/verification and  $H_3$  queries (this is why we have a reduction from the GBDH problem instead of the CDH or the one more CDH ones).

At the end of the game, the forger  $\mathcal{F}$  halts and comes up with a ciphertext  $\sigma^* = \langle X, W, Z \rangle$  and a possibly corrupted receiver's identity  $ID_B^*$ . If  $ID_B^*$  is an identity for which  $\mathcal{B}$  defined  $H_1(ID_B^*)$  as a known power of  $P$ , then the private key  $d_{ID_B^*}$  is computable for  $\mathcal{B}$  that can then open  $\sigma^*$  into a valid message-signature pair  $(M^*, s^*)$ , with  $s^* = \langle U, V \rangle$ , for some uncorrupted sender's identity  $ID_A^*$ . If  $H_1(ID_B^*)$  was rather defined as a power of  $bP$ ,  $\mathcal{B}$  can extract the embedded message-signature pair  $(M^*, s^*)$  as well as the sender's identity  $ID_A^*$  by submitting  $(\sigma^*, ID_B^*)$  to its own decryption/verification oracle. As we will see, it can successfully extract the clear triple  $(M^*, s^*, ID_A^*)$  with overwhelming probability (i.e. if the appropriate  $H_2$ ,  $H_3$  and  $H_4$  queries were made by  $\mathcal{F}$  regarding the intended forgery). If the hash value  $Q_{ID_A^*}$  was defined to be  $u^*(bP) \in \mathbb{G}_1$  for a known  $u^* \in \mathbb{Z}_q^*$  in  $L_1$ , then  $\mathcal{B}$  can extract the solution  $abP$  to the CDH problem  $(P, aP, bP)$  as in the proofs of strong unforgeability of SOK-IBS (and then obtain the solution to the easier initial GBDH problem). If the hash value  $Q_{ID_A^*}$  was rather defined to be  $uP \in \mathbb{G}_1$ , then  $\mathcal{B}$  ends and fails because the produced forgery is useless.

Let us now assess  $\mathcal{F}$ 's probability to successfully output an encrypted existential forgery for an uncorrupted identity without having asked the appropriate queries to  $H_2$ ,  $H_3$  and  $H_4$ . First, its probability to create a pair  $\langle U^*, V^* \rangle$  such that  $\hat{e}(P, V^*) = \hat{e}(P_{pub}, Q_{ID_A^*})\hat{e}(U^*, H^*)$  with  $H^* = H_2(ID_A^*, M^*, U^*)$  is at most  $1/2^k$  if it never asks the latter hash value. On the other hand, given a valid fake signature  $\langle U^*, V^* \rangle$  on  $M^*$  for an identity  $ID_A^*$ , its probability to properly encrypt it into a valid ciphertext  $\langle X^*, W^*, Z^* \rangle$  intended to a recipient  $ID_B^*$  without querying  $H_3$  on the input  $(X^*, Q_{ID_B^*}, g)$  such that  $(P, P_{pub}, X^*, Q_{ID_B^*}, g)$  is a valid BDH tuple is not greater than  $1/2^\ell$ . Finally, if the appropriate  $H_2$  and  $H_3$  queries are made but no entry of  $L_4$  allows the decryption/verification simulator to complete the extraction,  $\mathcal{F}$  cannot turn a given fake signature  $\langle U^*, V^* \rangle$  for a pair  $(M^*, ID_A^*)$  into a correctly encrypted message-signature pair  $\langle X^*, W^*, Z^* \rangle$  with a probability greater than  $2^{n_0+n_1-2\lambda}$ .

□

## D. The monolithic SOK-IBSC scheme

The protocol consists of the four algorithms on the figure below. As done in [15] and [38], we assume the receiver of a ciphertext has no a priori knowledge about the sender's identity that is recovered during the de-signcryption operation.

**Setup:** given a security parameter  $k$ , the PKG chooses groups  $\mathbb{G}_1$  and  $\mathbb{G}_2$  of prime order  $q > 2^k$ , a generator  $P$  of  $\mathbb{G}_1$ , a randomly chosen master key  $s \in \mathbb{Z}_q^*$  and the associated public key  $P_{pub} = sP$ . It also chooses a symmetric encryption scheme  $(\mathcal{E}, \mathcal{D})$  and cryptographic hash functions  $H_1, H_2 : \{0, 1\}^{n_0+2n_1+\ell} \rightarrow \mathbb{G}_1^*$ ,  $H_3 : \mathbb{G}_1^2 \times \mathbb{G}_2 \rightarrow \{0, 1\}^\ell$  and  $H_4 : \mathbb{G}_1 \rightarrow \{0, 1\}^\lambda$  where  $\ell$  denotes the length of the representation of  $\mathbb{G}_1$ 's elements,  $\lambda$  is the length of symmetric keys for  $(\mathcal{E}, \mathcal{D})$  while  $n_0$  and  $n_1$  are respectively the size of plaintexts and the bitlength of identifiers. The system-wide parameters are now

$$\mathbf{params} = (\mathbb{G}_1, \mathbb{G}_2, \hat{e}, P, P_{pub}, \ell, n_0, n_1, \lambda, \mathcal{E}, \mathcal{D}, H_1, H_2, H_3, H_4).$$

**Keygen:** as in SOK-IBS, given an identity  $ID$ , the PKG computes  $Q_{ID} = H_1(ID) \in \mathbb{G}_1$  and the associated private key  $d_{ID} = sQ_{ID} \in \mathbb{G}_1$  that is securely sent to the user.

**Signcrypt:** given a message, the parameters  $\mathbf{params}$ , the sender's private key  $d_{ID_A}$  and the recipient's public key  $Q_{ID_B} = H_1(ID_B)$ , this algorithm

1. picks  $r \xleftarrow{R} \mathbb{Z}_q$  and computes  $U = rP \in \mathbb{G}_1$  and then  $H = H_2(M, ID_A, ID_B, U) \in \mathbb{G}_1$ ,
2. sets  $V = d_{ID_A} + rH \in \mathbb{G}_1$
3. hides the signature  $W = V \oplus H_3(U, Q_{ID_B}, g^r) \in \{0, 1\}^\ell$  where  $g = \hat{e}(P_{pub}, Q_{ID_B})$ ,
4. encrypts the message and the sender's identity with a hash value of  $V$  together with the other components of the ciphertext as a symmetric key:  $Z = \mathcal{E}_\kappa(M || ID_A) \in \{0, 1\}^{n_0+n_1}$  with  $\kappa = H_4(V) \in \{0, 1\}^\lambda$ .

The ciphertext is  $\sigma = \langle U, W, Z \rangle \in \mathbb{G}_1 \times \{0, 1\}^{\ell+n_0+n_1}$

**De-Signcrypt:** upon receiving a ciphertext  $\sigma = \langle U, W, Z \rangle$  and given the recipient's private key  $d_{ID_B}$ , this algorithm

1. first checks that  $U$  is a point on the curve on which  $\mathbb{G}_1$  is defined and rejects  $\sigma$  if it is not,
2. computes  $V = W \oplus H_3(U, Q_{ID_B}, g')$  where  $g' = \hat{e}(U, d_{ID_B})$  and rejects  $\sigma$  if  $V$  is not a point on the curve,
3. computes  $(M || ID_A) = \mathcal{D}_\kappa(Z)$  where  $\kappa = H_4(V) \in \{0, 1\}^\lambda$  and then  $Q_{ID_A} = H_1(ID_A) \in \mathbb{G}_1$ .
4. The message-signature pair  $(M, \langle U, V \rangle)$  is accepted for the sender's identity  $ID_A$  if and only if  $\hat{e}(P, V) = \hat{e}(P_{pub}, Q_{ID_A}) \hat{e}(U, H)$  where  $H = H_2(M, ID_A, ID_B, U) \in \mathbb{G}_1$ .

**Fig. 3.** The SOK-IBSC signcrypt scheme