Signcryption in Hierarchical Identity Based Cryptosystem

Sherman S.M. Chow[1⋆], Tsz Hon Yuen[2], Lucas C.K. Hui[1], and S.M. Yiu[1]

[1] Department of Computer Science
University of Hong Kong
Pokfulam, Hong Kong
{smchow, hui, smyiu}@cs.hku.hk
[2] Department of Information Engineering
Chinese University of Hong Kong
Shatin, Hong Kong
thyuen4@ie.cuhk.edu.hk

Abstract. In many situations we want to enjoy confidentiality, authenticity and non-repudiation of message simultaneously. One approach to achieve this objective is to "sign-then-encrypt" the message, or we can employ special cryptographic scheme like signcryption. Two open problems about identity-based (ID-based) signcryption were proposed in [15]. The first one is to devise an efficient forward-secure signcryption scheme with public verifiability and public ciphertext authenticity, which is promptly closed by [10]. Another one which still remains open is to devise a hierarchical ID-based signcryption scheme that allows the user to receive signcrypted messages from sender who is under another sub-tree of the hierarchy. This paper aims at solving this problem by proposing two concrete constructions of hierarchical ID-based signcryption.

Key words: Data security, hierarchical identity-based signcryption, bilinear pairings

1 Introduction

In traditional public key infrastructure, certificates leak data and are not easily located. Strict online requirement removes offline capability, and validating policy is time-consuming and difficult to administer. Moreover, traditional PKI may not provide a good solution in many scenarios. For example, in tetherless computing architecture (TCA) [21] where two mobile hosts wanting to communicate might be disconnected from each other and also from the Internet. As exchange of public keys is impossible in this disconnected situation, identity-based (ID-based) cryptosystem fits in very well since the public key can be derived from the identity of another party [20].

In many situations we want to enjoy confidentiality, authenticity and non-repudiation of message simultaneously. A traditional approach to achieve this objective is to "sign-then-encrypt" the message, or we can employ special cryptographic scheme like signcryption. A recent direction is to merge the concept of ID-based cryptography and signcryption. Two open problems about ID-based signcryption were proposed in [15]. The first one is to devise an efficient forward-secure signcryption scheme with public verifiability and public ciphertext authenticity, which is promptly closed by [10]. Another one which still remains open is to devise a hierarchical ID-based signcryption scheme that allows the user to receive signcrypted messages from sender who is under another sub-tree of the hierarchy. This paper aims at solving this problem.

As shown by [7], the integrity checking necessary for security against adaptive adversaries can be obtained from the signature. We employ the same idea here to proposed two concrete constructions of hierarchical ID-based signcryption.

⋆ corresponding author

1.1 Applications

ID-based cryptography is suitable for the use of commercial organizations. In their settings, the inherent key-escrow of property is indeed beneficial, where the big boss has the power to monitor his/her employees' Internet communications if necessary. Hierarchical structure is common in nowadays' organizations, single trusted authority for generation of private key and authentication of users may be impractical, all these motivated the need of hierarchical ID-based cryptosystem.

Moreover, hierarchical ID-based cryptosystem is also useful in other scenarios, such as in TCA, a computing architecture with the concept of "regions", which can be viewed as a branch of the hierarchy[14, 20].

1.2 Related Work

Malone-Lee gave the first ID-based signcryption scheme [17]. This scheme is not semantically secure as the signcrypted text produced is a concatenation of a signature by a variant of Hess's ID-based signature [13] and a ciphertext by a simplified version of Boneh and Franklin's ID-based encryption [4]. In short, the signature of the message is visible in the signcrypted message.

On the other hand, Nalla and Reddy's ID-based signcryption scheme [19] cannot provide public verifiability as well as public ciphertext authenticity since the verification can only be done with the knowledge of recipient's private key. Libert and Quisquater proposed three ID-based signcryption schemes [15]. None of them can satisfy the requirements for public verifiability and forward security at the same time.

Boyen's multipurpose ID-based signcryption scheme [5] is the first scheme that provides public verifiability and forward security and is also provably secure. However, this scheme aims at providing ciphertext unlinkability and anonymity. So, a third party cannot verify the origin of the ciphertext, thus the scheme does not satisfy the requirement of public ciphertext authenticity. We remark that Boyen's scheme is very useful in applications that require unlinkability and anonymity.

The public verifiability of the signcrypted message usually can only be checked with some ephemeral data computed by the intended recipient of the signcrypted message. The notion of verifiable pairing was introduced in [8] to ensure the non-repudiation property of the ID-based signcryption by disallowing the intended recipient to manipulate the ephemeral data.

In 2004, [18] claimed that they were the first one closing the open problem proposed by [15]; however, the open problem was indeed closed by [10] in 2003. Recently, a simple but secure ID-based signcryption scheme was proposed in [7] and an ID-based signcryption scheme with exact security was proposed in [16]. The first blind ID-based signcryption scheme was proposed in [22]. This scheme offers the option to choose between authenticated encryption and ciphertext unlinkability. The generic group and pairing model was also introduced in this paper. Notice that none of the previously mentioned schemes works with hierarchical ID-based cryptosystem.

2 Preliminaries

Before presenting our results, we give the definition of a hierarchical ID-based signcryption scheme by extending the framework in previous work (e.g. [10, 22]). We also review the definitions of groups equipped with a bilinear pairing and the related complexity assumptions.

2.1 Framework of Hierarchical ID-based Signcryption Schemes

An ID-based signcryption (IDSC) scheme consists of six algorithms: `Setup`, `Extract`, `Sign`, `Encrypt`, `Decrypt` and `Verify`. `Setup` and `Extract` are executed by the private key generators (PKGs henceforth). Based on the security level parameter, `Setup` is executed to generate the master secret and common public parameters. `Extract` is used to generate the private key for any given identity. The algorithm `Sign` is used to produce the signature of a signer on a message, it also outputs some ephemeral data for the use of `Encrypt`; `Encrypt` takes the message, the signature, the ephemeral data produced by `Sign` and the recipient's identity to produce a signcrypted text. `Decrypt` takes the input of secret key and decrypt the signcrypted text to give the message and the corresponding signature, finally `Verify` is used by any party to verify the signature of a message.

In the hierarchical ID-based signcryption (HIDSC henceforth), PKGs are arranged in a tree structure, the identities of users (and PKGs) can be represented as vectors. A vector of dimension ℓ represents an identity at depth ℓ. Each identity ID of depth ℓ is represented as an ID-tuple $ID|\ell = \{ID_1, \cdots, ID_\ell\}$. The algorithms of HIDSC have similar functions to those of IDSC except that the `Extract` algorithm in HIDSC will generate the private key for a given identity which is either a normal user or a lower level PKG. The private key for identity ID of depth ℓ is denoted as $S_{ID|\ell}$ (or S_{ID} if the depth of ID does not related to the discussion). The functions of `Setup`, `Extract`, `Sign`, `Encrypt`, `Decrypt` and `Verify` in HIDSC are described as follows.

- `Setup`: Based on the input of an unary string 1^k where k is a security parameter, it outputs the common public parameters *params*, which include descriptions of a finite message space, a finite signature space and a finite signcrypted text space. It also outputs the master secret s, which is kept secret by the root private key generator (PKG).
- `Extract`: Based on the input of an arbitrary identity ID of depth j, it makes use of the secret key $S_{ID|j-1}$ (if $j = 1$, the input of the algorithm is s, which is the master secret of the root PKGs, instead of $S_{ID|j-1}$) to output the private key $S_{ID|j}$ for ID.
- `Sign`: Based on the input (M, S_{ID}), it outputs a signature σ and some ephemeral data r.
- `Encrypt`: Based on the input $(M, S_A, ID_B, \sigma, r)$, it outputs a signcrypted message C.
- `Decrypt`: Based on the input (C, S_B, ID_B), it outputs the message M, the corresponding signature σ and the purported signer ID_A.
- `Verify`: Based on the input (σ, M, ID), it outputs $\top$ for "true" or $\perp$ for "false", depending on whether σ is a valid signature of message M signed by ID or not.

These algorithms must satisfy the standard consistency constraint of hierarchical ID-based signcryption, i.e. if $\{\sigma, r\} = \texttt{Sign}(M, S_A)$, $C = \texttt{Encrypt}(S_A, ID_B, M, \sigma, r)$ and $\{M', ID_{A'}, \sigma'\} = \texttt{Decrypt}(C, S_B)$, we must have $M = M'$, $ID_A = ID_{A'}$ and $\top = \texttt{Verify}(\sigma', M, ID_A)$.

2.2 Bilinear Pairing

Let $(\mathbb{G}, \cdot)$ and $(\mathbb{G}_1, \cdot)$ be two cyclic groups of prime order q and g be a generator of $\mathbb{G}$. The bilinear pairing is given as $\hat{e} : \mathbb{G} \times \mathbb{G} \to \mathbb{G}_1$, which satisfies the following properties:

1. *Bilinearity*: For all $u, v \in \mathbb{G}$ and $a, b \in \mathbb{Z}$, $\hat{e}(u^a, v^b) = \hat{e}(u, v)^{ab}$.
2. *Non-degeneracy*: $\hat{e}(g, g) \neq 1$.
3. *Computability*: There exists an efficient algorithm to compute $\hat{e}(u, v)$ $\forall u, v \in \mathbb{G}$.

2.3 Diffie-Hellman Problems

Definition 1. *The computational Diffie-Hellman problem (CDHP) in* $\mathbb{G}$ *is defined as follows: Given a 3-tuple* $(g, g^a, g^b) \in \mathbb{G}^3$*, compute* $g^{ab} \in \mathbb{G}$*. We say that the* (t, ϵ)*-CDH assumption holds in* $\mathbb{G}$ *if no* t*-time algorithm has advantage at least* ϵ *in solving the CDHP in* $\mathbb{G}$*.*

Definition 2. *The bilinear Diffie-Hellman problem (BDHP) in* $\mathbb{G}$ *is defined as follows: Given a 4-tuple* $(g, g^a, g^b, g^c) \in \mathbb{G}^4$ *and a pairing function* $\hat{e}(\cdot, \cdot)$*, compute* $\hat{e}(g, g)^{abc} \in \mathbb{G}_1$*. We say that the* (t, ϵ)*-BDH assumption holds in* $\mathbb{G}$ *if no* t*-time algorithm has advantage at least* ϵ *in solving the BDHP in* $\mathbb{G}$*.*

Definition 3. *The decisional bilinear Diffie-Hellman problem (DBDHP) in* $\mathbb{G}$ *is defined as follows: Given a 5-tuple* $(g, g^a, g^b, g^c, T) \in \mathbb{G}^4 \times \mathbb{G}_1$ *and a pairing function* $\hat{e}(\cdot, \cdot)$*, decides whether* $T = \hat{e}(g, g)^{abc}$*. We say that the* (t, ϵ)*-DBDH assumption holds in* $\mathbb{G}$ *if no* t*-time algorithm has advantage at least* ϵ *in solving the DBDHP in* $\mathbb{G}$*.*

3 Security model

We present our security model for indistinguishability, existential unforgeability and ciphertext authenticity for HIDSC.

3.1 Indistinguishability

Indistinguishability for HIDSC against adaptive chosen ciphertext attack (IND-CCA2) is defined as in the following IND-CCA2 game.

1. The simulator selects the public parameter and sends the parameter to the adversary.
2. There are three oracles except the random oracles (hash oracles).
   - **Key extraction oracle** $\mathcal{KEO}$: Upon the input of an identity, the key extraction oracle outputs the private key corresponding to this identity.
   - **Signcryption oracle** $\mathcal{SO}$: Upon the input of the message M, the sender ID_A, the recipient ID_B, the signcryption oracle produces a valid signcryption C.
   - **Unsigncryption oracle** $\mathcal{UO}$: Upon the input of the ciphertext C, the sender ID_A and the recipient ID_B, the unsigncryption oracle outputs the decryption result and the verification outcome.

   The adversary is allowed to perform a polynomial number of oracle queries adaptively, but oracle query to $\mathcal{KEO}$ with input ID_B is not allowed.
3. The adversary generates M_0, M_1, ID_A, ID_B, and sends them to the simulator. The simulator randomly chooses $b \in_R \{0, 1\}$ and delivers the challenge ciphertext C to the adversary where $\{\sigma, r\} = \texttt{Sign}(M, S_A)$ and $C = \texttt{Encrypt}(S_A, ID_B, M_b, \sigma, r)$. M_0 and M_1 should be of equal length, and no oracle query have been made and will be made to $\mathcal{SO}$ with input (M_0, ID_A, ID_B) and (M_1, ID_A, ID_B) throughout the game.
4. The adversary can again perform a polynomial number of oracle queries adaptively, but oracle query to $\mathcal{UO}$ for the challenge ciphertext (defined later) from the simulator is not allowed.
5. The adversary tries to compute b.

The adversary wins the game if he can guess b correctly. The *advantage* of the adversary is the probability, over half, that he can compute b accurately.

Definition 4. *(Indistinguishability) A hierarchical ID-based signcryption scheme is* IND-CCA2 *secure if no PPT adversary has a non-negligible advantage in the IND-CCA2 game.*

Our security notion above is a strong one. It incorporates previous security notions including *insider-security* in [1] and *indistinguishability* in [17].

Notice that if we set the adversary to send the recipient identity ID_B to the simulator before step 1 (say, in an initialization stage) in the game, the security is reduced to the indistinguishability against *selective identity*, adaptive chosen ciphertext attack (IND-sID-CCA2).

3.2 Existential unforgeability

Existential unforgeability against adaptive chosen message attack (EU-CMA2) for HIDSC is defined as in the following EU-CMA2 game. The adversary is allowed to query the random oracles, $\mathcal{KEO}$, $\mathcal{SO}$ and $\mathcal{UO}$ (which are defined above) with the restriction that oracle query to $\mathcal{KEO}$ with input ID_A is not allowed.

The game is defined as follows:

1. The simulator selects the public parameter and sends it to the adversary.
2. The adversary is allowed to perform a polynomial number of oracle queries adaptively.
3. The adversary delivers a recipient identity ID_B and a ciphertext C.

The adversary wins the game if he can produce a valid (C, ID_B) such that C can be decrypted, under the private key of ID_B, to a message M, a sender identity ID_A and a signature σ which passes the verification test and no $\mathcal{SO}$ request that resulted in a ciphertext C, whose decryption under the private key of ID_B is the claimed forgery (σ, M, ID_A).

Definition 5. *(Existential Unforgeability) A hierarchical ID-based signcryption scheme is* EU-CMA2 *secure if no PPT adversary has a non-negligible probability in winning the EU-CMA2 game.*

The adversary is allowed to get the private key of the recipient in the adversary's answer. This gives us an *insider-security* in [1].

Notice that if we set the adversary to send the sender identity ID_A to the simulator in Step 1 in the game, the security is reduced to the existential unforgeability against *selective identity*, adaptive chosen ciphertext attack (EU-sID-CMA2).

3.3 Ciphertext Authenticity

Ciphertext authenticity against adaptive chosen message attack (AUTH-CMA2) for HIDSC is defined as in the following AUTH-CMA2 game. The adversary is allowed to query the random oracles, $\mathcal{KEO}$, $\mathcal{SO}$ and $\mathcal{UO}$, which are defined above. The game is defined as follows:

1. The simulator selects the public parameter and sends the parameter to the adversary.
2. The adversary is allowed to perform a polynomial number of oracle queries adaptively.
3. The adversary delivers a recipient identity ID_B and a ciphertext C.

The adversary wins the game if he can produce a valid (C, ID_B) such that C can be decrypted, under the private key of ID_B, to a message M, sender identity ID_A and a signature σ which passes the verification test.

Oracle query to $\mathcal{KEO}$ with input ID_A and ID_B is not allowed. The adversary's answer (C, ID_B) should not be computed by $\mathcal{SO}$ before.

Definition 6. *(Ciphertext Authenticity) A hierarchical ID-based signcryption scheme is* AUTH-CMA2 *secure if no PPT adversary has a non-negligible probability in winning the AUTH-CMA2 game.*

Outsider-security is considered in this model since the adversary is not allowed to get the private key of the recipient in the adversary's answer. This model represents the attack where a signature is re-encrypted by using a public key with unknown secret key.

4 Scheme 1

4.1 Construction

Let ℓ be the number of levels of the hierarchy to be supported. Let H_1, H_2 and H_3 be three cryptographic hash functions where $H_1 : \{0,1\}^* \rightarrow \mathbb{G}$ and $H_2 : \{0,1\}^* \rightarrow \mathbb{G}$, and $H_3 : \mathbb{G}_1 \rightarrow \{0,1\}^{k_0+k_1+n}$ where k_0 is the number of bits required to represent an element of $\mathbb{G}$, k_1 is the maximum number of bits required to represent an identity (of depth ℓ) and n is the maximum number of bits of a message to be signcrypted. Our first construction of a hierarchical ID-based signcryption scheme is given below. The construction is based on the idea in [12].

`Setup:` On the input of a security parameter $k \in \mathbb{N}$, the root PKG uses the BDH parameter generator [4] to generate $\mathbb{G}$, $\mathbb{G}_1$, q and $\hat{e}(\cdot,\cdot)$, where q is the order of groups $\mathbb{G}$ and $\mathbb{G}_1$. Then the root PKG executes the following steps.

1. Select an arbitrary generator P_0 from $\mathbb{G}$.
2. Pick a random s_0 from $\mathbb{Z}_p$, which is the system's master secret key.
3. Compute $Q_0 = {P_0}^{s_0}$.
4. The public system parameters are

$$\texttt{params} =< \mathbb{G}, \mathbb{G}_1, \hat{e}(\cdot,\cdot), q, P_0, Q_0, H_1(\cdot), H_2(\cdot), H_3(\cdot) > .$$

`KeyGen:` For an entity with $ID|k-1 = \{ID_1, ID_2, \cdots, ID_{k-1}\}$ of depth $k-1$ (for root PKG, its depth is defined as 0 and its identity is defined as empty string ϵ), it uses its secret key $S_{ID|k-1}$ (or the master secret s_0 of the root PKGs, if $k = 1$) to generate the secret key for a user $ID|k$ (where the first $k-1$ elements of $ID|k$ are those in $ID|k-1$) as follows.

1. Compute $P_{ID|k} = H_1(ID_1, ID_2, \cdots, ID_{k-1}, ID_k)$.
2. Pick random s_{k-1} from $\mathbb{Z}_p$ (this step is not necessary for the root PKG as s_0 is already defined).
3. Set the private key of the user to be $S_{ID|k} = S_{ID|k-1} \cdot {P_{ID|k}}^{s_{k-1}} = \prod_{i=1}^{k} {P_{ID|i}}^{s_{i-1}}$, where $S_{ID|0}$ is defined as the identity element in $\mathbb{G}$.
4. Send the values of $Q_i = {P_0}^{s_i}$ for $1 \leq i \leq k-1$ as "verification points" to the user.

`Sign:` For a user $A|k = \{A_1, A_2, \cdots, A_k\}$ with secret key $S_{A|k} = \prod_{i=1}^{k} {P_{A|i}}^{s_{i-1}}$ and the points $Q_i = {P_0}^{s_i}$ for $1 \leq i \leq k-1$ to sign on a message M, he/she follows the steps below.

1. Pick a random number r from $\mathbb{Z}_p^*$.
2. Compute $P_M = H_2(M)$.
3. Compute $\sigma = S_{A|k} \cdot {P_M}^r$.

4. Return $\{\sigma, Q_1, Q_2, \cdots, Q_{k-1}, Q_M = {P_0}^r\}$ as the signature and return r as the ephemeral data for `Encrypt`.

`Encrypt:` To signcrypt the message M to user $B|l$, the steps below are used.

1. Compute $P_{B|j} = H_1(B_1, B_2, \cdots, B_j)$ for $1 \leq j \leq l$.
2. Pads the identity A with a chain of zero bits if it is not of depth ℓ.
3. Return ciphertext $C =$

$$\{{P_{B|2}}^r, \cdots, {P_{B|l}}^r, (M||\sigma||A) \oplus H_3(\hat{g}^r), Q_1, Q_2, \cdots, Q_M\}$$

where $\hat{g} = \hat{e}(Q_0, P_{B|1}) \in \mathbb{G}_1$ and $\oplus$ represents the bitwise XOR.

`Decrypt:` For user $B|l$ with secret key $S_{B|l} = \prod_{i=1}^{l} {P_{B|i}}^{s'_{i-1}}$ and the points $Q'_i = {P_0}^{s'_i}$ for $1 \leq i \leq l-1$ to decrypt the signcrypted message C, the steps below are used.

1. Let $C = \{U_2, \cdots, U_l, V, Q_1, Q_2, \cdots, Q_M\}$
2. Compute $V \oplus H_3(\hat{e}(Q_M, S_{B|l}) / \prod_{i=2}^{l} \hat{e}(Q'_{i-1}, U_i)) = M||\sigma||A$.
3. Return $\{M, \sigma, A, Q_1, Q_2, \cdots, Q_M\}$.

`Verify:` For A's signature $\{\sigma, Q_1, Q_2, \cdots, Q_{k-1}, Q_M\}$, everyone can do the following to verify its validity.

1. Compute $P_M = H_2(M)$.
2. Compute $P_{A|i} = H_1(A_1, A_2, \cdots, A_i)$ for $1 \leq i \leq k$.
3. Return $\top$ if $\hat{e}(P_0, \sigma) / \prod_{i=2}^{k} \hat{e}(Q_{i-1}, P_{A|i}) = \hat{e}(Q_0, P_{A|1}) \hat{e}(Q_M, P_M)$.

4.2 Efficiency Analysis

The signcrypted message is shortened by one $\mathbb{G}_1$ element, as compared with using the schemes HIDE and HIDS in [12] together. Moreover, chosen ciphertext secure HIDE requires the transformation in Section 3.2 of [12], while our scheme does not require such transformation as the integrity checking of the ciphertext is obtained from the signature.

4.3 Security analysis

Theorem 1. *Suppose that the (t, ϵ)-BDH assumption holds in $\mathbb{G}$, then the above scheme is $(t', q_S, q_H, q_E, q_R, \epsilon)$-adaptive chosen ciphertext (IND-CCA2) secure for arbitrary q_S, q_H, q_E, q_R, and any $t' < t - o(t)$.*

Proof. Dealer $\mathcal{D}$ gives (g, g^a, g^b, g^c) to Simulator $\mathcal{S}$ and wants $\mathcal{S}$ to compute $\hat{e}(g, g)^{abc}$. Set $P_0 = g, Q_0 = g^a$. $\mathcal{S}$ sends the system parameter to $\mathcal{A}$. $\mathcal{S}$ randomly picks μ with $1 \leq \mu \leq q_H$.

Phase 1: Query on H_1 for input $(A_1, \cdots, A_k)$:

- If $k = 1$, the μ-th query to H_1 with $k = 1$ is back patched to g^b. The corresponding identity is denoted as $ID^*{}_b$. Adds the entry $< ID^*{}_b, g^b >$ to tape L_1 and returns g^b.
- Otherwise, randomly picks $\lambda \in \mathbb{Z}_p$; add $< A_1, \cdots, A_k, \lambda >$ to L_1 and returns g^λ.

When there is a query on H_2 for input M, randomly picks $\lambda \in \mathbb{Z}_p$; adds $< M, \lambda >$ to L_2 and returns $(g^a)^\lambda$. Query on H_3 is handled by producing a random element from the codomain, and adding both query and answer to tape L_3.

Key Extraction Oracle ($\mathcal{KEO}$): For input identity $A = \{A_1, \cdots, A_k\} \in {\mathbb{Z}_p}^k$ where $k \leq \ell$.

- If $A_1 = ID^*{}_b$, then aborts the simulation.
- Otherwise, look up at the tape $L_K =< ID_1, \cdots, ID_u, \alpha_1, \cdots, \alpha_{u-1} >$ which stores the previously extracted keys. Let y be the maximal value such that $\{ID_1, \cdots, ID_y\} = \{A_1, \cdots, A_y\}$ for some tuple $< ID_1, \cdots, ID_u, \alpha_1, \cdots, \alpha_{u-1} > \in L_K$. Then:
  - For $1 \leq i \leq y$, get α_i from the list and set $Q_i = g^{\alpha_i}$. Get $P_i = H_1(A_1, \cdots, A_i)$ from L_1 and also get λ from $(A_1, \lambda) \in L_1$.
  - For $y < i \leq k$, query the value of P_i from H_1. Randomly generate $\alpha_i \in \mathbb{Z}_p$.
  - Put $< I_1, \cdots, I_k, \alpha_1, \cdots, \alpha_{k-1} >$ in L_K. Set the private key as $S_{A|k} = \prod_{i=1}^{k} {P_i}^{s_{i-1}} = (g^a)^\lambda \cdot {P_2}^{\alpha_1} \cdots {P_k}^{\alpha_{k-1}}$. Returns $S_{A|k}$ and $Q_i = g^{\alpha_i}$ for $1 \leq i \leq k-1$.

  Note that the private key satisfies the required form.

Signcryption Oracle ($\mathcal{SO}$): For input message M, sender $A|k = \{A_1, \cdots, A_k\}$, and recipient $B|l = \{B_1, \cdots, B_l\}$.

- If $A_1 = ID^*{}_b$, query P_M from H_2 and obtain λ_M from $< M, \lambda_M > \in L_2$. Query $P_{A|i}$ from H_1 and obtain λ_i from $< A_1, \cdots, A_i, \lambda_i > \in L_1$, for $1 \leq i \leq k$. Randomly generate $\alpha_i \in \mathbb{Z}_p$ for $1 \leq i \leq k$. Compute $\sigma = (g^a)^{(\alpha_k \lambda_M)} \prod_{i=2}^{k} g^{\lambda_i \alpha_{i-1}}, Q_i = g^{\alpha_i}$ for $1 \leq i \leq k-1$, $Q_M = (g^{\alpha_k})(g^b)^{-1/\lambda_M}$. Query $P_{B|i}$ from H_1 and obtain get λ_{Bi} from $< B_1, \cdots, B_i, \lambda_{Bi} > \in L_1$, for $1 \leq i \leq l$. Compute $U_i = (g^{\alpha_k})(g^b)^{-\frac{\lambda_{Bi}}{\lambda_M}}$ for $2 \leq i \leq l$, $V = (M||\sigma||A|k) \oplus H_3(\hat{e}(g^a, (g^{\alpha_k})(g^b)^{-\frac{\lambda_{B1}}{\lambda_M}}))$. Return the ciphertext $C = \{U_2, \cdots, U_l, V, Q_1, \cdots, Q_{k-1}, Q_M\}$. $\mathcal{S}$ puts $< A|k, B|l, M, C >$ in L_S. It is easy to see that the signature will pass the verification test:

$$
\begin{aligned}
&\hat{e}(P_0, \sigma) / \prod_{i=2}^{k} \hat{e}(Q_{i-1}, P_{A|i}) \\
&= \hat{e}(g, (g^{(a\alpha_k \lambda_M)}) \prod_{i=2}^{k} g^{\lambda_i \alpha_{i-1}}) / \prod_{i=2}^{k} \hat{e}(g^{\alpha_{i-1}}, g^{\lambda_i}) \\
&= \hat{e}(g, g^{(a\alpha_k \lambda_M)}) \hat{e}(g, \prod_{i=2}^{k} g^{\lambda_i \alpha_{i-1}}) / \prod_{i=2}^{k} \hat{e}(g, g^{\lambda_i \alpha_{i-1}}) \\
&= \hat{e}(g^{\alpha_k}, g^{(a\lambda_M)}) \prod_{i=2}^{k} \hat{e}(g, g^{\lambda_i \alpha_{i-1}}) / \prod_{i=2}^{k} \hat{e}(g, g^{\lambda_i \alpha_{i-1}}) \\
&= \hat{e}(g^a, g^b) \hat{e}(g^{\alpha_k}, g^{(a\lambda_M)}) \hat{e}(g^{a\lambda_M}, (g^b)^{-1/\lambda_M}) \\
&= \hat{e}(g^a, g^b) \hat{e}((g^{\alpha_k})(g^b)^{-1/\lambda_M}, g^{a\lambda_M}) \\
&= \hat{e}(Q_0, P_{A|1}) \hat{e}(Q_M, P_M).
\end{aligned}
$$

- Otherwise, $\mathcal{S}$ retrieves the private key of $A|k$ using the same way as $\mathcal{KEO}$ and then uses it to run signcryption and gets ciphertext C. $\mathcal{S}$ puts $< A|k, B|l, M, C >$ in L_S.

Un-signcryption Oracle ($\mathcal{UO}$): For input sender $A|k = \{A_1, \cdots, A_k\}$, recipient $B|l = \{B_1, \cdots, B_l\}$ and ciphertext $C = \{U_2, \cdots, U_l, V, Q_1, \cdots, Q_M\}$.

- For the case $B_1 = ID^*{}_b$, $\mathcal{S}$ finds if $< A|k, B|l, M, C >$ is in L_S. If so, returns M. Otherwise, $\mathcal{S}$ searches for all combinations $< M, \sigma >$ such that $< M, h_2 > \in L_2$, $< g', h_3 > \in L_3$, for some h_2, h_3, under the constraints that $\hat{e}(g, \sigma) = \hat{e}(g^a, P_{A|1})\hat{e}(Q_M, h_2)\prod_{i=2}^{k}\hat{e}(Q_{i-1}, P_{A|i})$ and $h_3 \oplus V = M||\sigma||A|k$. $\mathcal{S}$ simply picks one of the valid message M from the above and return it as answer. If no such tuple is found, the oracle signals that the ciphertext is invalid.
- For other cases, $\mathcal{S}$ retrieves the private key of $B|l$ using the same way as $\mathcal{KEO}$ and then uses it to decrypt and verify.

Witness Extraction: As in the IND-CCA2 game, at some point $\mathcal{A}$ chooses plaintext M_0, M_1, and sender $A|k$ on which he wishes to be challenged. $\mathcal{S}$ retrieves the private key of $A|k$ and $Q_1, \cdots, Q_{k-1}, Q_M$ using the same way as $\mathcal{KEO}$. $\mathcal{S}$ queries $P_{B|i}$ from H_1 and obtain get λ_{Bi} from $< B_1, \cdots, B_i, \lambda_{Bi} > \in L_1$, for $2 \leq i \leq l$. $\mathcal{S}$ randomly picks $V \in \{0,1\}^{k_0+k_1+n}$ and responds with challenge ciphertext $C = \{(g^c)^{\lambda_{B2}}, \cdots, (g^c)^{\lambda_{Bl}}, V, Q_1, \cdots, Q_{k-1}, g^c\}$. All further queries by $\mathcal{A}$ are processed adaptively as in the oracles above, with no private key extraction of $B|l$. Finally, $\mathcal{A}$ returns its final guess b'. $\mathcal{S}$ ignores the answer from $\mathcal{A}$, randomly picks an entry $< g', h_3 >$ in L_3, and returns g' as the solution to the BDH problem.

If the recipient identity is $B|l = \{B_1, \cdots, B_l\}$ with $B_1 = ID^*{}_b$, to recognize the challenge ciphertext is incorrect, $\mathcal{A}$ needs to query random oracle H_3 with $g' = \hat{e}(Q_0, P_{B|1})^c = \hat{e}(g, g)^{abc}$. It will leave an entry $< g', h_3 >$ on L_3, from which $\mathcal{S}$ can extract $g' = \hat{e}(g, g)^{abc}$. □

Theorem 2. *Suppose that the (t, ϵ)-CDH assumption holds in $\mathbb{G}$, then the above scheme is $(t', q_S, q_H, q_E, q_R, \epsilon)$-adaptive chosen message (EU-CMA2) secure for arbitrary q_S, q_H, q_E, q_R, and any $t' < t - o(t)$, $\epsilon' > \frac{\epsilon}{e^2 q_S q_E}$.*

Proof. Dealer $\mathcal{D}$ gives (g, g^a, g^b) to Simulator $\mathcal{S}$ and wants $\mathcal{S}$ to compute g^{ab}. Set $g_1 = g^a, g_2 = g^b$. The initialization, setup and the simulation of oracles are similar to those in the proof of Theorem 1. The difference is that probabilistic simulations are used in the simulation of two hash oracles: the one for hashing the identity ($H_1(\cdot)$) and the one for hashing the message ($H_2(\cdot)$).

Queries on oracle H_1 for identity $(A_1, \cdots, A_k)$: If $k = 1$, $\mathcal{S}$ embeds part of the challenge g^b in the answer of many H_1 queries [11]. $\mathcal{S}$ picks $\lambda \in_R \mathbb{F}_q^*$ and repeats the process until λ is not in the list L_1. $\mathcal{S}$ then flips a coin $W_1 \in \{0, 1\}$ that yields 0 with probability ζ_1 and 1 with probability $1 - \zeta_1$. (ζ_1 will be determined in the probability analysis shortly afterward.) If $W_1 = 0$, then the hash value $H_1(A_1)$ is defined as g^λ; else returns $H_1(A_1) = (g^b)^\lambda$ if $W_1 = 1$. In either case, $\mathcal{S}$ stores $< A_1, \lambda, W_1 >$ in the list L_1.

On the other hand, if $k > 1$, $\mathcal{S}$ performs the simulation as that in the proof of Theorem 1.

Queries on oracle H_2 for message M : In this case, $\mathcal{S}$ embeds the remaining part of the challenge g^a in the answer of many H_2 queries. $\mathcal{S}$ picks $\beta \in_R \mathbb{F}_q^*$ and repeats the process until β is not in the list L_2. $\mathcal{S}$ then flips a coin $W_2 \in \{0, 1\}$ that yields 0 with probability ζ_2 and 1 with probability $1 - \zeta_2$. (ζ_2 will be determined later.) If $W_2 = 0$, then the hash value $H_2(M)$ is defined as $(g^a)^\beta$; else returns $H_2(M) = g^\beta$ if $W_2 = 1$. In either case, $\mathcal{S}$ stores $< M, \beta, W_2 >$ in the list L_2.

Witness Extraction: After such probabilistic behaviour is introduced to the simulation, $\mathcal{S}$ will fail for the $\mathcal{KEO}$ query of $(A_1, \cdots, A_k)$ if $W_1 = 1$ is found in the corresponding entry of A_1 in L_1. The $\mathcal{SO}$ query for the signcryption of message M done by $(A_1, \cdots, A_k)$ will fail too when $W_1 = 1$ and $W_2 = 1$ are found in the corresponding entry of A_1 in L_1 and M in L_2 respectively.

At the end of the game, $\mathcal{A}$ returns a forgery $C = \{U_2, \cdots, U_l, V, Q_1, Q_2, \cdots, Q_{k-1}, Q_M\}$ which is the signcryption of message M done by $(A_1, \cdots, A_k)$. $\mathcal{S}$ cannot solve the CDH problem if the forgery is not related to the problem instance at all, i.e. when $W_1 = 0$ is found in the corresponding entry of A_1 in the list L_1 and $W_1 = 1$ and $W_2 = 0$ are found in the corresponding entry of A_1 in L_1 and M in L_2 respectively.

For successful cases, $\mathcal{S}$ gets the forged signature $\{\sigma, Q_1, Q_2, \cdots, Q_{k-1}, Q_M\}$ by the decryption of the signcrypted text. Suppose that λ_i is the corresponding entry of $P|i$ in the list L_1 and β is the corresponding entry of P_M in the list L_2, since $\sigma = \prod_{i=1}^{k} ({P_i}^{s_{i-1}}) \cdot {P_M}^r = g^{ab} \cdot \prod_{i=2}^{k} ({P_i}^{s_{i-1}}) \cdot {P_M}^r$, $\mathcal{C}$ can compute the solution of the CDH problem by $\sigma / \prod_{i=2}^{k} ({Q_{i-1}}^{\lambda_i}) \cdot Q_{k-1}, {Q_M}^{\beta}$.

Probability Analysis: The probability that $\mathcal{S}$ answers to all private key extraction queries is ${\zeta_1}^{q_E}$. $\mathcal{S}$ can answer all signcrypt queries for users $H_1(A_1, \cdots, A_k)$ where $< A_1, \lambda, 0 >$ is in the list L_1, so the worst case for $\mathcal{S}$ to answer all signcrypt queries correctly happens when all signcrypt requests are for users $H_1(A_1, \cdots, A_k)$ where $< A_1, \lambda, 1 >$ is in the list L_1. For these class of users, $\mathcal{S}$ can still signcrypt given the message is M where $< M, \beta, 0 >$ can be found in the list L_2, so the probability for $\mathcal{S}$ to successfully answer all signcrypt requests is ${\zeta_2}^{q_S}$.

Finally, the probability that $\mathcal{A}$ makes a forged signature for user $H_1(A_1, \cdots, A_k)$ where $< A_1, \lambda, 1 >$ is in the list L_1 is $1 - \zeta_1$ and the probability that $\mathcal{A}$ makes a forged signature on message M where $(M, \beta, 1)$ is in the list L_2 is $1 - \zeta_2$. Hence the probability for $\mathcal{S}$ to solve CDH problem successfully is $f_{q_E}(\zeta_1) f_{q_S}(\zeta_2)$ where $f_x(\zeta) = \zeta^x(1 - \zeta)$. Simple differentiation shows that $f_x(\zeta)$ is maximized when $\zeta = 1 - (x+1)^{-1}$, and the corresponding probability is $\frac{1}{x}(1 - \frac{1}{x+1})^{x+1}$. So the maximum probability for $\mathcal{S}$ to solve CDH problem successfully is

$$\frac{1}{q_S q_E}(1 - \frac{1}{q_S + 1})^{q_S+1}(1 - \frac{1}{q_E + 1})^{q_E+1}$$

For large q_S and q_E, this probability is approximately equal to $1/e^2 q_S q_E$.

□

Theorem 3. *Suppose that the (t, ϵ)-CDH assumption holds in $\mathbb{G}$, then the above scheme is $(t', q_S, q_H, q_E, q_R, \epsilon)$-adaptive chosen message (AUTH-CMA2) secure for arbitrary q_S, q_H, q_E, q_R, and any $t' < t - o(t)$.*

Proof. Dealer $\mathcal{D}$ gives (g, g^a, g^b, g^c) to Simulator $\mathcal{S}$ and wants $\mathcal{S}$ to compute $\hat{e}(g, g)^{abc}$. Set $P_0 = g, Q_0 = g^c$. $\mathcal{S}$ sends the system parameter to $\mathcal{A}$. $\mathcal{S}$ randomly picks μ_a, μ_b with $1 \leq \mu_a, \mu_b \leq q_H$.

Phase 1: Query on H_1 for input $(A_1, \cdots, A_k)$:

- If $k = 1$, the μ_a distinct query to H_1 with $k = 1$ is back patched to g^a. The corresponding identity is denoted as ${ID^*}_a$. Adds the entry $< {ID^*}_a, g^a >$ to tape L_1 and returns g^a.
- If $k = 1$, the μ_b distinct query to H_1 with $k = 1$ is back patched to g^b. The corresponding identity is denoted as ${ID^*}_b$. Adds the entry $< {ID^*}_b, g^b >$ to tape L_1 and returns g^b.
- Otherwise, randomly picks $\lambda \in \mathbb{Z}_p$; add $< A_1, \cdots, A_k, \lambda >$ to L_1 and returns g^λ.

When there is a query on H_2 for input $(A_1, \cdots, A_k, M)$, randomly picks $\lambda \in \mathbb{Z}_p$; add $< M, \lambda >$ to L_2 and returns $(g^c)^\lambda$. Query on H_3 is handled by producing a random element from the codomain, and adding both query and answer as a single tuple to tape L_3.

Key Extraction Oracle ($\mathcal{KEO}$): For input identity $A|k\{A_1, \cdots, A_k\} \in {\mathbb{Z}_p}^k$ where $k \leq \ell$, if $A_1 = {ID^*}_a$ or $A_1 = {ID^*}_b$, aborts the simulation. Otherwise proceeds as in $\mathcal{KEO}$ of the proof of Theorem 1.

Signcryption Oracle ($\mathcal{SO}$): For input message M, sender $A|k = \{A_1, \cdots, A_k\}$, and recipient $B|l = \{B_1, \cdots, B_l\}$.

- If $A_1 \neq {ID^*}_a$ and $A_1 \neq {ID^*}_b$, $\mathcal{S}$ retrieves the private key of $A|k$ using the same way as $\mathcal{KEO}$, then uses it to run signcryption and gets ciphertext C. $\mathcal{S}$ puts $< A|k, B|l, M, C, r >$ in L_S.
- If $A_1 = {ID^*}_a$ or $A_1 = {ID^*}_b$, proceeds as in $\mathcal{SO}$ of the proof of Theorem 1. The only change is that $\mathcal{S}$ puts $< A|k, B|l, M, C, {\lambda_M}^{-1} >$ in L_S.

Un-signcryption Oracle ($\mathcal{UO}$): For input sender $A|k = \{A_1, \cdots, A_k\}$, recipient $B|l = \{B_1, \cdots, B_l\}$ and ciphertext $C = \{U_2, \cdots, U_l, V, Q_1, \cdots, Q_{k-1}, Q_M\}$.

- If $B_1 = {ID^*}_a$ or $B_1 = {ID^*}_b$, proceeds as in $\mathcal{UO}$ of the proof of Theorem 1.
- Otherwise, $\mathcal{S}$ retrieves the private key of $A|k$ using the same way as $\mathcal{KEO}$ and then uses it to decrypt and verify.

Witness Extraction: As in the AUTH-CMA2 game, finally $\mathcal{A}$ returns a recipient identity $B|l$ and a ciphertext C. $\mathcal{S}$ does the followings:

- With probability $q_S/(q_S + q_R)$ choose a random element from L_S and a random element $< g', h_3 >$ from L_3. If the element chosen from L_3 has the form of $< A|k, B|l, M, C, r >$, compute $\hat{e}(\sigma, g^b)/\prod_{i=2}^{k} \hat{e}({Q_{i-1}}^{\lambda_{Ai}}, g^b)\hat{e}(g^{ar\lambda_M}, g^b)$, where $< A_1, \cdots, A_i, \lambda_{Ai} >$ and $< M, \lambda_M >$ is in L_1 and L_2 respectively. If the chosen element has the form of $< B|l, A|k, M, C, r >$, then compute as in the above with $B|l$ substituting $A|k$ if possible.
- With probability $q_R/(q_S + q_R)$ choose a random element from L_S and a random element $< g', h_3 >$ from L_3. If the element chosen from L_3 has the form of $< A|k, B|l, M, C, r >$, compute ${g'}^{-\lambda_M}$, where $< M, \lambda_M > \in L_2$. On the other hand, if the chosen element has the form of $< B|l, A|k, M, C, r >$, compute ${g'}^{-\lambda_M}$, where $< M, \lambda_M > \in L_2$.

Now we analyze the simulation. Suppose that the $\mathcal{SO}$ responds to query $(A|k, B|l, M)$ and caused an entry $< A|k, B|l, M, C, r >$ being added to L_S, if σ is a valid signature, then $\hat{e}(g, g)^{abc} = \hat{e}(\sigma, g^b)/\prod_{i=2}^{k} \hat{e}({Q_{i-1}}^{\lambda_{Ai}}, g^b)\hat{e}(g^{ar\lambda_M}, g^b)$. On the other hand, if the $\mathcal{UO}$ can decrypt with $H_3(g')$ where $g' = \hat{e}(Q_0, P_{B|1})^r = \hat{e}(Q_M, g^{bc}) = \hat{e}(g^{-a/\lambda_M}, g^b c)$, then we can obtain $\hat{e}(g, g)^{abc} = {g'}^{-\lambda_M}$. □

5 Scheme 2

5.1 Construction

Let H be a cryptographic hash function where $H : \{0, 1\}^* \rightarrow \mathbb{Z}_p$. We use $H(\cdot)$ to hash the string representing the identity into an element in ${\mathbb{Z}_p}^k$, the same hash function will be used in the signing algorithm too. Similar to [3], H is not necessarily a full domain hash function.

Notice that the identity string is hashed to $\mathbb{Z}_p$ instead $\mathbb{G}$ in scheme 1, so we use I_i to denote $H(ID_i)$ for $1 \leq i \leq \ell$, where ℓ is the number of levels of the hierarchy to be supported. Our second construction of HIDSC, based on the ideas in [9] and [3], is given below.

`Setup:` On the input of a security parameter $k \in \mathbb{N}$, the root PKG uses the BDH parameter [4] to generate $\mathbb{G}$, $\mathbb{G}_1$, q and $\hat{e}(\cdot,\cdot)$, where q is the order of groups $\mathbb{G}$ and $\mathbb{G}_1$. Then the root PKG executes the following steps.

1. Select α from $\mathbb{Z}_p^*$, $h_1, h_2, \cdots, h_\ell$ from $\mathbb{G}$ and two generators g, g_2 from $\mathbb{G}^*$,
2. The public parameters are: $\{g, g_1 = g^\alpha, g_2, h_1, h_2, \cdots, h_\ell, \hat{e}(g_1, g_2)\}$.
3. The master secret key is $d_{ID|0} = {g_2}^\alpha$.

`KeyGen:` For a user $ID|k-1 = \{ID_1, ID_2, \cdots, ID_{k-1}\}$ of depth $k-1$, he/she uses his/her secret key $d_{ID|k-1}$ to generate the secret key for a user $ID|k$ (where the first $k-1$ elements of $ID|k$ are those in $ID|k-1$) as follows.

1. Pick random r_k from $\mathbb{Z}_p$.
2. $d_{ID|k} = \{d_0 {F_k(I_k)}^{r_k}, d_1, \cdots, d_{k-1}, g^{r_k}\}$, where $F_k(x)$ is defined as ${g_1}^x h_k$.

`Sign:` For a user $ID|k$ with secret key $\{{g_2}^\alpha \prod_{j=1}^{k} F_j(I_j)^{r_j}, g^{r_1}, \cdots, g^{r_k}\}$ to sign on a message M, he/she follows the steps below.

1. Pick a random number s from $\mathbb{Z}_p^*$.
2. Compute $h = H(M, \hat{e}(g_1, g_2)^s)$.
3. Repeat Steps 1-3 in case the unlikely event $s + h = 0$ occurs.
4. For $j = \{1, 2, \cdots, k\}$, compute $y_j = {d_j}^{s+h}$.
5. Compute $z = {d_0}^{s+h}$.
6. Return $\{s, y_1, y_2, \cdots, y_k, z\}$ as the signature.

`Encrypt:` To signcrypt a message $M \in \mathbb{G}_1$ to user $ID|l = \{ID_1, ID_2, \cdots, ID_l\}$, the ciphertext to be generated is

$$\{F_1(I_1)^s, F_2(I_2)^s, \cdots, F_l(I_l)^s, \hat{e}(g_1, g_2)^s \cdot M, g^s, y_1, y_2, \cdots, y_k, z\}.$$

`Decrypt:` For a user $ID'|l$ with secret key $\{d'_0 = {g_2}^\alpha \prod_{j=1}^{l} F_j(I'_j)^{r'_j}, d'_1 = g^{r'_1}, \cdots, d'_l = g^{r'_l}\}$ to decrypt the signcrypted text $\{u_1, \cdots, u_l, v, w, y_1, y_2, \cdots, y_k, z\}$, he/she follows the steps below.

1. Compute $\sigma = \hat{e}(g_1, g_2)^s$ by $\hat{e}(w, d'_0) / \prod_{j=1}^{l} \hat{e}(u_j, d'_j)$.
2. Obtain the message M by $v \cdot \sigma^{-1}$

`Verify:` For $ID|k = \{ID_1, ID_2, \cdots, ID_k\}$'s signature $\{\sigma, y_1, y_2, \cdots, y_k, z\}$, everyone can do the following to verify its validity.

1. Compute $h = H(M, \sigma)$.
2. Return $\top$ if $\hat{e}(g, z) = \sigma \cdot \hat{e}(g_1, {g_2}^h \prod_{j=1}^{k} {y_j}^{I_j}) \prod_{j=1}^{k} \hat{e}(y_j, h_j)$, $\perp$ otherwise.

5.2 Efficiency Analysis

The signcrypted message is shortened by one $\mathbb{G}_1$ element, as compared with using the scheme in [9] and [3] together. Moreover, chosen ciphertext secure HIDE requires the transformation in Section 4 of [6], while our scheme does not require such transformation as the integrity checking of the ciphertext is obtained from the signature.

5.3 Security Analysis

Theorem 4. *Suppose that the (t, ϵ)-Decision BDH assumption holds in $\mathbb{G}$, then the above scheme is $(t', q_S, q_H, q_E, q_R, \epsilon)$-selective identity, adaptive chosen ciphertext (IND-sID-CCA2) secure for arbitrary q_S, q_H, q_E, q_R, and any $t' < t - o(t)$.*

Proof. Dealer $\mathcal{D}$ gives (g, g^a, g^b, g^c, T) to Simulator $\mathcal{S}$ and wants $\mathcal{S}$ to output 1 if $T = \hat{e}(g, g)^{abc}$ or output 0 otherwise. Set $g_1 = g^a, g_2 = g^b, g_3 = g^c$.

Initialization: Adversary $\mathcal{A}$ sends an identity $ID^* = \{ID_1{}^*, \cdots, ID_k{}^*\} \in Z_p^k$ of depth $k \leq \ell$ that it intends to attack to $\mathcal{S}$.

Setup: $\mathcal{S}$ randomly picks $\alpha_1, \cdots, \alpha_\ell \in \mathbb{Z}_p$ and defines $h_j = {g_1}^{-I_j^*} g^{\alpha_j} \in \mathbb{G}$ for $j = 1, \cdots, \ell$. $\mathcal{S}$ sends the system parameter $(g, g_1, g_2, h_1, \cdots, h_\ell, \hat{e}(g_1, g_2))$ to $\mathcal{A}$.

Phase 1: Query on H for input (M, σ):

- If $(M, \sigma, h) \in L$ for some h, return h.
- Otherwise, randomly picks $h \in \mathbb{Z}_p$; add (M, σ, h) to L and returns h.

Key Extraction Oracle ($\mathcal{KEO}$): For input identity $ID = \{ID_1, \cdots, ID_u\} \in {\mathbb{Z}_p}^u$ where $u \leq \ell$.

- If $ID = ID^*$ or ID is a prefix of ID^* , then aborts the simulation.
- Otherwise, let j be the smallest index such that $I_j \neq I_j^*$. $\mathcal{S}$ firstly derives a private key for identity $\{I_1, \cdots, I_j\}$, from which it then construct a private key for ID. $\mathcal{S}$ randomly picks $r_1, \cdots, r_j \in \mathbb{Z}_p$ and sets:

$$d_0 = {g_2}^{\frac{-\alpha_j}{I_j - I_j^*}} \prod_{v=1}^{j} F_v(I_v)^{r_v}, d_1 = g^{r_1}, \cdots, d_{j-1} = g^{r_j - 1}, d_j = {g_2}^{\frac{-1}{I_j - I_j^*}} g^{r_j}$$

We now show that $(d_0, d_1, \cdots, d_j)$ is a valid random private key for $(I_1, I_2, \cdots, I_j)$. Let $\tilde{r}_j = r_j - b/(I_j - I_j^*)$, then we have:

$${g_2}^{\frac{-\alpha_j}{I_j - I_j^*}} F_j(I_j)^{r_j} = {g_2}^{\frac{-\alpha_j}{I_j - I_j^*}} ({g_1}^{I_j - I_j^*} g^{\alpha_j})^{r_j} = {g_2}^{a} ({g_1}^{I_j - I_j^*} g^{\alpha_j})^{r_j - \frac{b}{I_j - I_j^*}} = {g_2}^{a} F_j(I_j)^{\tilde{r}_j}$$

So the private key satisfies the required form.

Signcryption Oracle ($\mathcal{SO}$): For input message M, sender $ID_{A|k} = \{ID_{A1}, \cdots, ID_{Ak}\}$, and recipient $ID_{B|l} = \{ID_{B1}, \cdots, ID_{Bl}\}$.

- If $ID_{A|k}$ equals ID^* or a prefix of ID^*, then $\mathcal{S}$ randomly chooses $h \in \mathbb{Z}_p$, and computes $\sigma = \hat{e}(g_1, g_2)^{-h}$. Then $\mathcal{S}$ randomly picks $r_1, \cdots, r_k \in \mathbb{Z}_p$, computes $y_v = {g_2}^{r_v}$ for $1 \leq v \leq k$ and $z = \prod_{v=1}^{k} {g_2}^{r_v \alpha_v}$. Then $\mathcal{S}$ adds the tuple (M, σ, h) to L to force the random oracle $H(M, \sigma) = h$. Finally, $\mathcal{S}$ returns the ciphertext $C =$
  $\{F_1(I_{B1})^{-h}, F_2(I_{B2})^{-h}, \cdots, F_l(I_{Bl})^{-h}, \sigma \cdot M, g^{-h}, y_1, y_2, \cdots, y_k, z\}$.
  $\mathcal{S}$ puts $< ID_{A|k}, ID_{B|l}, M, C, -h, h >$ in L_S.
- Otherwise, $\mathcal{S}$ retrieves the private key of $ID_{A|k}$ using the same way as $\mathcal{KEO}$ and then uses it to run signcryption and gets ciphertext C. $\mathcal{S}$ puts $< ID_{A|k}, ID_{B|l}, M, C, s, h >$ in L_S.

Un-signcryption Oracle ($\mathcal{UO}$): For input sender $ID_{A|k} = \{ID_{A1}, \cdots, ID_{Ak}\}$, recipient $ID_{B|l} = \{ID_{B1}, \cdots, ID_{Bl}\}$ and ciphertext $C = \{u_1, \cdots, u_l, v, w, y_1, \cdots, y_k, z\}$.

- For the case $ID_{B|l} = ID^*$, $\mathcal{S}$ finds if $(ID_{A|k}, ID_{B|l}, M, C, s, h)$ is in L_S. If so, returns M. Otherwise, $\mathcal{S}$ searches for a valid M in all entries $< M, \sigma, h >\in L$, under the constraints that $\sigma \cdot M = v$, $\sigma = \hat{e}(g, z)/(\hat{e}(g_1, {g_2}^h \prod_{j=1}^{k} {y_j}^{ID_{A|j}}) \prod_{j=1}^{k} \hat{e}(y_j, h_j))$ and $\hat{e}(w, F_j(I_{Bj})) = \hat{e}(g, u_j)$ for $1 \leq j \leq l$. $\mathcal{S}$ simply picks a message in one of the valid M in the above and return it as the answer. If no such tuple is found, the oracle signals that the ciphertext is invalid.
- For other cases, $\mathcal{S}$ retrieves the private key of $ID_{A|k}$ using the same way as $\mathcal{KEO}$ and then uses it to decrypt and verify.

Witness Extraction: As in the IND-sID-CCA2 game, at some point $\mathcal{A}$ chooses plaintext M_0, M_1, and sender $ID_{A|k}$ on which he wishes to be challenged. $\mathcal{S}$ picks a random bit $b \in \{0, 1\}$ and responds with challenge ciphertext $C = \{{g_3}^{\alpha_1}, \cdots, {g_3}^{\alpha_l}, T \cdot M_b, g_3, y_1, \cdots, y_k, z\}$, where $(y_1, \cdots, y_k, z)$ is a valid signature from $ID_{A|k}$. All further queries by $\mathcal{A}$ are processed adaptively as in the oracles above. Finally, $\mathcal{A}$ returns its final guess b'. If $b = b'$, then $\mathcal{S}$ outputs 1 meaning $T = \hat{e}(g, g)^{abc}$. Otherwise it outputs 0 meaning $T \neq \hat{e}(g, g)^{abc}$.

If the recipient identity is ID^*, then the value of $\hat{e}(g_1, g_2)^s$ is equal to $\hat{e}(g^a, g^b)^c = \hat{e}(g, g)^{abc}$. If $\mathcal{A}$ has the advantage ϵ to guess b correctly, then $\mathcal{S}$ has the advantage ϵ to solve the DBDHP. □

Theorem 5. *Suppose that the (t, ϵ)-CDH assumption holds in $\mathbb{G}$, then the above scheme is $(t', q_S, q_H, q_E, q_R, \epsilon')$-selective identity, adaptive chosen message (EU-sID-CMA) secure for arbitrary q_S, q_H, q_E, q_R, and any $t' < t - o(t)$, $\epsilon' > \epsilon \cdot (1 - \frac{q_S(q_H + q_S)}{q})$.*

Proof. (Sketch) Dealer $\mathcal{D}$ gives (g, g^a, g^b) to Simulator $\mathcal{S}$ and wants $\mathcal{S}$ to compute g^{ab}. Set $g_1 = g^a, g_2 = g^b$. The initialization, setup and the simulation of oracles are the same as the proof of Theorem 4. At the end of the game, $\mathcal{A}$ returns a forgery $C = \{u_1, \cdots, u_l, v, w, y_1, \cdots, y_k, z\}$ using h from H query. By forking lemma, we rewind $\mathcal{A}$ to the time when the H query was issued and get $C' = \{u'_1, \cdots, u'_l, v', w', y'_1, \cdots, y'_k, z'\}$ using h' from H query. We can get $d_j = (y_j / y'_j)^{(h-h')^{-1}}$ for $1 \leq j \leq k$. Then we can calculate $d_0 = (z/z')^{(h-h')^{-1}}$. Finally we can get ${g_2}^{\alpha} = d_0 / \prod_{j=1}^{k} {d_j}^{\alpha_j}$ which is the solution to the CDH problem. □

Let us consider the possibility for $\mathcal{SO}$ to fail. The only possibility for introducing an error is in defining $H(M, \sigma)$ which is already defined. Since σ takes its value uniformly at random in $\mathbb{G}_1$, the chance for the occurrence of one of these events is at most $(q_H + q_S)/q$ for each query. Therefore over the whole simulation, the chance of an error is at most $q_S(q_H + q_S)/q$. Hence $\mathcal{S}$ succeeds with probability at least $\epsilon \cdot (1 - \frac{q_S(q_H + q_S)}{q})$. □

Theorem 6. *Suppose that the (t, ϵ)-CDH assumption holds in $\mathbb{G}$, then the above scheme is $(t', q_S, q_H, q_E, q_R, \epsilon)$-selective identity, adaptive chosen message (AUTH-sID-CMA2) secure for arbitrary q_S, q_H, q_E, q_R, and any $t' < t - o(t)$.*

Proof. (Sketch) By the construction of the game of AUTH-CMA2, we can see that if an adversary wants to win the game, he either forges a signature from a signer or forges an encryption using a valid signature.

For the first case, by theorem 5, if an adversary can forge a signature in the above scheme, then he can solve the CDH problem.

For the second case, let the adversary gets a signature $\{s, y_1, y_2, \cdots, y_k, z\}$, where $y_j = {d_j}^{s+h}$, for $j = \{1, 2, \cdots, k\}$ and $z = {d_0}^{s+h}$, and gets $\hat{e}(g_1, g_2)^s, g^s$ from the corresponding signcryption with recipient identity not $ID_{B|l} = \{ID_{B1}, \cdots, ID_{Bl}\}$. Then the adversary needs to forge an encryption by computing $\{F_1(I_{B1})^s, F_2(I_{B2})^s, \cdots, F_l(I_{Bl})^s\}$ from the knowledge of $\{F_1(I_{B1}), F_2(I_{B2}), \cdots, F_l(I_{Bl}), \hat{e}(g_1, g_2), g\}$. This is the same as the CDH problem.

Therefore, if an adversary wants to win the game, he has to solve the CDH problem. □

6 Conclusion

Two concrete constructions of hierarchical identity based signcryption are proposed, which closed the open problem proposed by [15]. Our schemes are provably secure under the random oracle model [2]. Moreover, our schemes do not require transformation which is necessary for the case of hierarchical identity based encryption as the integrity checking of the ciphertext is obtained from the signature. We believe that hierarchical identity based signcryption schemes are useful in nowadays commercial organization and also in new network architecture such as tetherless computing architecture. Future research directions include further improvement on the efficiency of hierarchical identity based signcryption schemes and achieving other security requirements such as public ciphertext authenticity ([10, 15]) or ciphertext anonymity ([5]).

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