Zero-Knowledge Blind Identification For Smart Cards Using Bilinear Pairings

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Abstract

Identification protocols based on the Computational Discrete Logarithm (CDH) problem generally assume the intractability of the underlying Decisional Diffie Hellman Problem (DDHP). Due to this, the security of all such schemes in a pairing based scenario is doubtful. In this paper, we propose a two-round zero-knowledge identification protocol using bilinear pairings. Our proposed protocol has two contrasting features to traditional identification schemes: (1) The scheme requires the verifier to toss his coins before the prover. (2) The coin tosses of the verifier are secret while the coin tosses of the prover are not. As a consequence, we obtain a *blind* identification scheme with complete zero knowledge. Traditionally in an identification scheme, a passive adversary watching the communication gains information intended only for the verifier. For instance, from watching the transcript in the Fiat-Shamir zero knowledge identification scheme, an adversary also learns the outcome of the protocol (i.e. whether the identification succeeds or not). The blinding property of our scheme eliminates this disadvantage while still ensuring zero knowledge.

Finally, as a natural extension of our scheme, we present the concept of 'all or none' group identification protocol that can be used to authenticate together an arbitrary number of users in a batch such that if the identification fails, it is impossible for the users to know which one cheated. We also prove the security of our scheme and give some interesting applications including anonymous seller credit card payments. The cryptographic primitives can be efficiently encapsulated in smart cards designed for Elliptic Curve Cryptography (ECC). The private key must be included in a tamperproof device inside the smart card.

1 Introduction

The user of zero-knowledge proofs for identification has interesting implications. For instance, a verifier cannot later impersonate the prover using the transcript of the proof. Zero-knowledge identification does not solve all the problems since it is still susceptible to a man-in-the middle attack, where a passive adversary simply relays the messages between the prover and the verifier. In a typical scenario, this adversary has the same information as the verifier. We present a slightly stronger variation of zero-knowledge identification where a passive adversary does not learn anything new from observing the transcript of the protocol. This paper has two motivations; firstly, to provide a secure 'pairing' variant of identification schemes based on the discrete logarithm problem and secondly, to provide an efficient smart card based zero-knowledge identification scheme which does not leak any information to a passive adversary. In other words, the scheme allows users to correctly identify a prover to a verifier such that (a) if both the prover and verifier are honest, the protocol leaks only the same one bit of information (that the prover's statement is indeed true) in arbitrarily many runs of the protocol and (b) the protocol leaks this one bit of information only to the honest verifier. A passive adversary watching the entire communication still gains no knowledge about the correctness of prover's statement after arbitrarily many runs. To do this, we require the verifier to toss secret coins while requiring the prover to toss public coins. Additionally, we require that the verifier toss his coins before the prover. We also give some extension of the scheme including two-way authentication, key agreement, single-key signcryption (which allows a user Alice, for example to send a signed and encrypted message to another user Bob without involving Bob's keys at all!), hidden signatures (which allow Alice to send a message along with a hidden signature to Bob) and online credit card/cheque payments.

The cryptographic primitives can be implemented efficiently in smart cards; one-way identification requires only two elliptic curve point multiplications while two-way identification requires three multiplications and two pairing computations for each user. Unlike most pairing based schemes which involve identity based cryptography, our model is based on a standard certificate based infrastructure. Due to this we lose certain benefits offered only by identity based mechanisms (like implicit key authentication). However, at the same time we are unaffected by the key escrow problem inherent in all identity based systems.

2 Background

In this section, we will review some existing identification mechanisms. Assume that Alice and Bob are two users and Alice wishes to identify herself to Bob. We only consider one-way identification and ignore the case of Bob identifying himself to Alice.

2.1 Public Key Encryption

Alice has public encryption function E_a and a secret decryption function D_a.

1. Alice begins by claiming to know D_a.
2. Bob generates a random challenge message m and encrypts it using E_a to get ciphertext $c = E_a(m)$. Bob sends c to Alice over an insecure channel.
3. Alice computes $m' = D_a(c)$ and sends back m' to Bob over the same insecure channel.
4. Bob accepts if $m = m'$.

This protocol is correct and sound but has an inherent disadvantage. It may be a security requirement of Alice that only the sender of c (whether it is Bob or not) may know her identity. An adversary watching the communication can also check that $E_a(m') = c$ and consequently obtain information about the outcome of the identification that only Bob was supposed to have. The Fiat-Shamir identification protocol [1] also suffers from this drawback. Additionally in this protocol, an adversary is able to obtain decryption of an arbitrary ciphertext intended for Alice. A modified version of the protocol overcomes this problem by using a hash function $\mathcal{H} : \{0,1\}^* \mapsto \{0,1\}^*$. In step 3, instead of sending m, Alice sends $m'' = \mathcal{H}(m')$ to Bob who accepts if $m'' = \mathcal{H}(m)$. However, this protocol still does not satisfy our strong notion of zero-knowledge because Alice's reply is deterministic. If an adversary replays the same message c to Alice later on and receives the same m'', he can be reasonably confident that he is talking to Alice without knowing m. To avoid this attack, Alice must insert some randomness into her reply each time.

2.2 Diffie-Hellman Based Approaches

Let $\mathbb{Z}_p$ be a multiplicative group where computing discrete logarithms is hard. Let g be a generator of $\mathbb{Z}_p$. Alice has a private key $x \in \mathbb{Z}_{p-1}$ and public key $y = g^x \in \mathbb{Z}_p$. Let $\mathcal{H} : \mathbb{Z}_p \mapsto \{0,1\}^*$ be a cryptographic hash function.

1. Alice begins by claiming to know x, the discrete logarithm of y to base g.
2. Bob generates a random challenge message $u \in \mathbb{Z}_{p-1}$ and sends $v = g^u$ to Alice over an insecure channel.
3. Alice generates a random $r \leftarrow \mathbb{Z}_p$ and computes $w_1 = \mathcal{H}(w+r)$. She sends back the tuple $\langle w_1, r \rangle$ back to Bob.
4. Bob accepts if $w_1 = \mathcal{H}(y^u + r)$.

This protocol is correct and sound and an adversary watching the communication cannot tell the outcome of the protocol without knowing u. Bob cannot obtain v^x for any chosen v of his choice even if Alice has no guarantee that Bob

did indeed generate v properly. The protocol is unconditionally secure if $\mathcal{H}$ is a random oracle. Observe that Alice must toss coins to insert randomness r into her reply to avoid the chosen text attack. The identification protocol presented in this paper is based on a similar idea. Like this one, the protocol is two round and begins by the verifier tossing secret coins. The prover then tosses public coins. Our scheme also satisfies the security requirement of Alice that a passive adversary does not learn the outcome of the protocol. Other Identification schemes based on the discrete logarithm problem have been proposed earlier, for example in [2, 3]. All these schemes, however, assume that the underlying DDH problem is computationally hard. Due to this, some of their security properties are lost when used in a pairing based scenario where the DDH is easy. The purpose of this paper was to present an identification protocol for smart cards using bilinear pairings satisfying this requirement and not relying on hash functions or random oracles.

3 Bilinear Pairings

Pairing based cryptography is based on the existence of efficiently computable non-degenerate bilinear maps which can be abstractly described as follows: Let $\mathbb{G}_1$ be a cyclic additive group of prime order q and $\mathbb{G}_2$ be a cyclic multiplicative group of the same order. Assume that computing the discrete logarithm in both $\mathbb{G}_1$ and $\mathbb{G}_2$ is hard. A bilinear pairing is a map $e : \mathbb{G}_1 \times \mathbb{G}_1 \mapsto \mathbb{G}_2$ that satisfies the following properties [4, 5]:

1. *Bilinearity*: $e(aP, bQ) = e(P, Q)^{ab}$ $\forall P, Q \in \mathbb{G}_1$ and $a, b \in \mathbb{Z}_q$
2. *Non-degeneracy*: $P \neq 0 \Rightarrow e(P, P) \neq 1$
3. *Computability*: e is efficiently computable

The above properties also imply:

$$e(P + Q, R) = e(P, R) \cdot e(Q, R) \ \forall P, Q, R \in \mathbb{G}_1.$$

$$e(P, Q + R) = e(P, Q) \cdot e(P, R) \ \forall P, Q, R \in \mathbb{G}_1$$

Typically, the map e will be derived from either the Weil or Tate pairing on an elliptic curve over a finite field. Despite the complex mathematics involved in constructing such maps, cryptographic protocols based on pairings can be described entirely without ever referring to the actual implementation. We refer the reader to [4, 5, 6] for more details. Pairings and other parameters should be selected in proactive for efficiency and security. For appropriately selected parameters, the following problems are computationally intractable [7]:

(a) *Discrete Logarithm Problem (DLP)*: Given $P, aP \in \mathbb{G}_1$, compute $a \in \mathbb{Z}_q$

(b) *Diffie Hellman Problem (DHP)*: Given $P, aP, bP \in \mathbb{G}_1$, compute $abP \in \mathbb{G}_1$

(d) *Decisional Linear Diffie-Hellman Problem (DLDHP)*: Let $x, y \leftarrow \mathbb{Z}_q$. For any given $P, Q, R, xP, yQ, S \in \mathbb{G}_1$, decide if $S = (x + y)R$. The security of our protocol is based on a variant of the DLDHP.

While the following problem is always easy:

(e) *Decisional Diffie Hellman Problem (DDHP)*: Given $P, aP, bP, Q \in \mathbb{G}_1$, decide if $Q = abP$

4 Setup PKI

In the rest of the discussion, we will using a PKI which will be setup as follows: A central authority is responsible for generating the security parameters. A trusted CA is responsible for certifying the public keys. To participate in the protocol each user must have a certified public key (the process of certification is outside the scope of our protocol). The setup proceeds as follows:

1. Let $e : \mathbb{G}_1 \times \mathbb{G}_1 \mapsto \mathbb{G}_2$ be a bilinear mapping and $P \in \mathbb{G}_1$ be a generator of $\mathbb{G}_1$. The parameters $\langle e, q, \mathbb{G}_1, P \rangle$ are generated by the trusted authority and made public in an authentic way.

2. Each participant $\mathcal{ID}_i$ generates $x_i \leftarrow \mathbb{Z}_q$ as the private key. The corresponding public key is $Y_i = x_i P \in \mathbb{G}_1$. Each user also obtains a certificate from the CA linking the identity $\mathcal{ID}_i$ and the public key Y_i. In other words, the CA fixes the pairs $\langle Y_i, \mathcal{ID}_i \rangle$.

5 Blind Identification

Assume that user $\mathcal{ID}$ having secret key $x \in \mathbb{Z}_q$ and public key $Y = xP \in \mathbb{G}_1$ wants to prove to server $\mathcal{S}$, the knowledge of x. Additionally, $\mathcal{ID}$ wants to ensure that no one except the verifier $\mathcal{S}$ gets convinced of this fact from watching the communication (in other words, the proof needs to be verifier dependent and a dishonest verifier does not get convinced about the statement). We will assume the infrastructure of section 4. The identification is done as follows:

1. $\mathcal{ID}$ starts by claiming to know $x \in \mathbb{Z}_q$, the discrete logarithm of $Y \in \mathbb{G}_1$ to base P.

2. The verifier $\mathcal{S}$ generates $r \leftarrow \mathbb{Z}_q$ and computes a challenge $R = rY$. It makes R public. Typically the challenge should have a very short lifetime.

3. $\mathcal{ID}$ generates $Q \leftarrow \mathbb{G}_1$ and computes $Z = \frac{1}{x}R + xQ$. It sends Z, Q as its proof to $\mathcal{S}$.

4. $\mathcal{S}$ accepts if $e(Z, P) = e(P, P)^r \cdot e(Q, Y)$.

6 Security Proof (Sketch)

We claim that this test will pass with a high probability if and only if $\mathcal{ID}$ knows x *and* $\mathcal{S}$ knows r. If the former is false, the verification fails with a high probability while if the latter is false, no knowledge about x is given out in the process and the outcome of the proof is undecidable.

1. Correctness: The properties of bilinear maps ensure that the verification is always successful if no one cheats.

2. Soundness: We will analyze the soundness assuming that one of the parties is dishonest. The trivial case when both parties are dishonest is ignored. The soundness property holds because:

   (a) Dishonest Prover: Given P, xP, rxP, Computing a pair Z', Q' such that $e(Z', P) = e(rP, P) \cdot e(Q', xP)$ is infeasible without knowledge of r or x due to the hardness of the DHP in $\mathbb{G}_1$ as shown in theorem 4.4 of [8] (cf. aggregate extraction). Additionally, an honest verifier keeps r secret. Thus, the proof is sound from a verifier's view.

   (b) Dishonest Verifier: A dishonest verifier will generate R non-randomly. In other words, a dishonest verifier will not know r. We note that given P, xP, rxP, Q, Z, deciding if $Z = rP + xQ$ is infeasible without knowledge of r or x due to the hardness of the DLDHP in $\mathbb{G}_1$ as shown in [7] (cf. linear encryption) where a lower bound on the hardness of the problem in relation to the DHP is mentioned.

3. Honest Verifier Zero-Knowledge: $\mathcal{S}$ can simulate the proof without interaction by generating an accepting transcript $\{Z', r, Q', R, Y, P\}$ as follows: Generate $r, \alpha \leftarrow \mathbb{Z}_q$. Compute $R = rY$, $Q' = \alpha P$, $Z' = rP + \alpha Y$. It is easy to see that the simulated and real distributions are indistinguishable. Also observe that an unlimited number of accepting transcripts can be generated using the same coin tosses r.

4. Computational Zero-Knowledge: It is easy to see that the notion of dishonest verifier in our context is irrelevant since the notion of an 'accepting' proof is meaningless if the outcome is undecidable. Thus, in simulating a dishonest verifier we could very well generate a random message and 'tell' the verifier that this is an accepting transcript. We give a slightly different and intuitive definition of computational zero-knowledge (without using a black-box). Suppose that a dishonest verifier could gain some trivial information about x from interaction with an honest prover. The protocol is computationally zero-knowledge if this distribution (learned from interaction with the honest prover) is indistinguishable from the distribution generated from an honest-verifier simulation. In other words, the dishonest verifier could very well have simulated the interaction. It is easy to see that if the coin tosses of the prover are truly random, the values (Z, Q) sent in step 2 of each round of the proof (irrespective of the

verifier) are uniformly distributed. Another way of saying this is that the information gained by a dishonest verifier is less than or equal to that gained by an honest verifier. We observe that our system is equivalent to a non-interactive zero knowledge proof since the zero-knowledge property is independent of the coin tosses of the verifier.

7 Blind Group Identification

This scheme enables a group of users to identify themselves to a server such that: (a) The identification test passes if none of the users cheat, (b) if any users cheat, the test will fail with a high probability, (c) it is not possible for the server or the users to know which person cheated. An important application for this type of scheme is in the following type of group systems: Assume that two users Alice and Bob want to identity themselves jointly to a server (for example, because they don't trust each other to individually login to the server without the other's approval). Alice wants to ensure that the identification succeeds if and only if the other user is really Bob. Bob has a similar requirement.

Assume that $\{\mathcal{ID}_1, \mathcal{ID}_2, \ldots \mathcal{ID}_n\}$ are the set of users who want to jointly identify themselves. It is necessary that each user $\mathcal{ID}_i$ must have a certified public key Y_i as described earlier. The goal of the protocol is that all users will simultaneously identify themselves to a server $\mathcal{S}$.

In other words, each user $\mathcal{ID}_i$ will prove possession of the discrete logarithm x_i of Y_i (to base P) such that $\mathcal{S}$ cannot be convinced about any of the individual statements separately. That is, the proof is valid only on all the statements together: "$\mathcal{ID}_i$ knows x_i" $\forall i : 1 \leq i \leq n$ but not on any of the individual statements like "$\mathcal{ID}_1$ knows x_1" or "$\mathcal{ID}_2$ knows x_2" independently of the others. Such a proof is called an *additive zero knowledge* proof [9]. We will assume the infrastructure of section 4. The identification is done as follows:

1. The n provers $\mathcal{ID}_1, \mathcal{ID}_2 \ldots \mathcal{ID}_n$ start by claiming to $\mathcal{S}$ that they know the discrete logarithms $x_1, x_2, \ldots x_n \in \mathbb{Z}_q$ of $Y_1, Y_2, \ldots Y_n \in \mathbb{G}_1$ (to base P) respectively.

2. The verifier $\mathcal{S}$ generates $r_1, r_2, \ldots r_n \leftarrow \mathbb{Z}_q$ and computes $R_i = r_i Y_i$. It makes the list of challenges $R_i, \mathcal{ID}_i$ public. The challenge values have a short lifetime (up to a few minutes).

3. Each $\mathcal{ID}_i$ generates randomly $Q_i \leftarrow \mathbb{G}_1$ and computes $Z_i = \frac{1}{x_i} R_i + x_i Q_i$

4. All users then collaborate to jointly compute the value $Z = \sum_{j=1}^{j=n} Z_j$. This computation is hidden from $\mathcal{S}$ so that individual values Z_j are effectively hidden the it's view. The combined proof $\{Z, Q_1, Q_2 \ldots Q_n\}$ is sent to $\mathcal{S}$.

5. $\mathcal{S}$ computes $r = \sum_{j=1}^{j=n} r_j$ and accepts if $e(Z, P) = e(P, P)^r \cdot \prod_{j=1}^{j=n} e(Q_j, Y_j)$.

8 Security Proof (Sketch)

We claim that this test will pass if and only if each $\mathcal{ID}_i$ knows x_i. To summarize the goals of the protocol, the individual users can jointly authenticate themselves to the server such that:

(a) If all users are honest, the server always accepts.

(b) If any of the users are dishonest, the server rejects with a high probability.

(c) The protocol is zero knowledge. It is not possible for anyone (including the server) to know which user cheated.

(d) Collusions are possible between users but not with the server (the server is trusted).

The protocol is secure based on the following observations:

1. Correctness: The properties of bilinear maps ensure that the verification is always successful if none of $\mathcal{ID}_i$ cheat.

2. Soundness: We discuss the soundness using a similar reasoning as above. The soundness property holds because:

   (a) Dishonest Prover: Computing individual proofs Z_i, Q_i without knowledge of x_i is infeasible due to similar reasoning as above for single user blind identification. Consequently the verifier will reject.

   (b) Dishonest Verifier: A dishonest verifier does not gain any information about the correctness of the statements after interaction with the provers. The above reasoning for single user identification can be extended here.

3. Zero Knowledge: For simplicity, we will ignore the dishonest verifier case where the coin tosses of $\mathcal{S}$ are non-randomly generated since the reasoning of the single user scenario above suffices. We will prove the zero-knowledge property of the protocol from the following perspectives:

   (a) Honest Verifier Zero Knowledge: $\mathcal{S}$ can generate a valid accepting transcript on its own corresponding to $\{r_1, R_1, Q_1\}$ $\forall i : 1 \leq i \leq n$ as follows: $\mathcal{S}$ generates $\alpha_i \leftarrow \mathbb{Z}_q$ $\forall i$ and computes $Q_i = \alpha_i P$, $R_i = r_i Y_i$. Then $Z = \sum_{j=1}^{j=n} r_i P + \alpha_i Y_i$.

   (b) Honest Prover Secrecy: Assume that all the provers are honest and thus, $\mathcal{S}$ accepts. In this case computing individual proofs Z_i just from $Z, Q_1, Q_2 \ldots Q_n$ such that $e(Z_i, P) = e(P, P)^{r_i} \cdot e(Q_i, Y_i)$ $\forall i$ is infeasible without knowledge of each x_i due to the hardness of the DHP in $\mathbb{G}_1$ as shown in theorem 4.4 of [8] (cf. aggregate extraction).

(c) Dishonest Prover Secrecy: Assume that some of the provers are dishonest. In this case, deciding if any given $Z_i = \frac{1}{x_i} R_i + x_i Q_i$ is infeasible without knowledge of x_i or r_i due to the Linear Diffie Hellman Assumption [7]. That is, given $P, x_i P, r_i x_i P, Q, Z_i \in \mathbb{G}_1$, deciding if $Z_i = r_i P + x_i Q$ is infeasible without knowing at least one of $\{x_i, r_i\}$. Therefore if $\mathcal{S}$ rejects, none of the provers know which pairs of (Z_i, Q_i) correspond to invalid proofs (if the individual coin tosses r_i of $\mathcal{S}$ are kept secret and $\mathcal{S}$ is honest, no information is leaked to the provers). Similarly if the individual values Z_i are kept secret (from $\mathcal{S}$), the identity of the dishonest provers is still concealed from $\mathcal{S}$.

Finally, if the joint computation of Z is carried out in a way that any one individual prover or a small coalition of provers can know Z_i's for only a small fraction of users, the identities of dishonest provers can still be effectively hidden, even if $\mathcal{S}$ can be coerced to reveal all the coin tosses r_i.

9 Other Extensions

In this section we will provide several extensions of our scheme. We refer to the definitions of sections 4 and 5. The private keys x_i can either be generated by users or a trusted authority. The public key are assumed to be certified in the former case. Note that the identification is a two round protocol, with the verifier sending the challenge R in the first step and the prover sending the response Z, Q in the second step. The private key for each smart card is encapsulated in a tamperproof chip. Signing access to this key is given via a secure Key Encapsulation Mechanism (KEM). The corresponding public key is also present along with a certificate. Smart cards may be purchased from a (reputed) third party and must be registered with the relevant organization (like a bank) before they can be used. To register a smart card, a bank simply provides a certificate.

9.1 Authenticated Encryption (Signcryption)

Assume that user $\mathcal{ID}$ is identifying itself to the server $\mathcal{S}$ using the blind identification scheme of section 5. $\mathcal{ID}$ can encrypt a random message $M \in \mathbb{G}_1$ intended for $\mathcal{S}$ using the challenge R as follows: Let $M = xQ$. Using this relation, $\mathcal{ID}$ computes $Q = \frac{1}{x} M$ and $Z = \frac{1}{x} R + xQ$ as before. The authenticated ciphertext is (Z, Q). An honest verifier $\mathcal{S}$ can compute the message M as follows: First it checks that the identification condition is true (i.e. it checks that (Z, Q) is indeed an accepting configuration). Then it computes $M = Z - rP$. Authentication is provided due to the zero-knowledge property; the verifier is assured that the sender is indeed $\mathcal{ID}$. It may appear that non-repudiation as such is not provided because $\mathcal{S}$ cannot later prove in a court that the message was sent by $\mathcal{ID}$ since $\mathcal{S}$ could very well have generated an accepting transcript without interaction with the prover by simulating the entire protocol.

However, note that in most cases when the transcript is simulated, the resulting message M will be meaningless. If $\mathcal{S}$ presents the tuple $\langle M, Q\rangle$ in a court, a judge can verify that that $e(M,P) = e(Q,Y)$. If the message is meaningful and the equality is valid, the judge is convinced that Q was indeed computed by $\mathcal{ID}$ and not by $\mathcal{S}$. Hence Q also serves as a signature on M. The signature and encryption scheme is secure against Chosen Plaintext Attacks (IND-CPA) under the Diffie-Hellman assumption in $\mathbb{G}_1$ [4]. However, it is insecure against Chosen Ciphertext Attacks (IND-CCA) because (a) existential forgery of signatures is always possible and (b) given two ciphertext $\{(Z,Q), (Z',Q')\}$ and one of the plaintexts $\{xQ, xQ'\}$, it is always possible to associate it with its corresponding ciphertext.

Despite having the disadvantage of being IND-CCA insecure, our scheme offers an interesting feature; if in addition to $\langle M, Q\rangle$, the verifier $\mathcal{S}$ also produces $\langle Z, r\rangle$ in the court and Z has been signed by $\mathcal{ID}$ (using some scheme, which are irrelevant to us), then $\mathcal{S}$ can also claim that the message was *directly* received from $\mathcal{ID}$. To validate this, the judge checks if $e(Z,P) = e(P,P)^r \cdot e(Q,Y)$. This tells the judge that the message was signed by $\mathcal{ID}$ and also *intended* for the holder of r (which turns out to be $\mathcal{S}$ in this case) since even $\mathcal{ID}$ does not have the ability to compute r. Observe that this encryption/non-repudiation is provided from using only $\mathcal{ID}$'s certified public key(s) in contrast to most other schemes that require a certified key for each party. This feature is a new type of non-repudiation (or commitment) that can be used in electronic payment systems as described later.

9.2 Signatures

To construct an IND-CCA secure signature scheme from the protocol of section 5, we simply remove the verifier from the protocol and set its challenge $R = 0 \in \mathbb{G}_1$. The prover's response is extracted from the message using a hash function. That is, $Q = h(M)$ where M is a message and $h : \{0,1\}^* \mapsto \mathbb{G}_1$ is a cryptographic hash function. The signature of user $\mathcal{ID}$ is $S = xQ$. This is exactly the short signature scheme of Boneh et al. [4]. To verify that (M, S) is a valid message-signature pair, check that $e(S,P) = e(h(M), Y)$.

9.3 Hidden Signatures

In the scenario where user $\mathcal{ID}$ identifies to the server $\mathcal{S}$ (section 5), $\mathcal{ID}$ can also send plaintext messages along with hidden signatures such that only $\mathcal{S}$ can extract the signature. Of course, once extracted, the signatures provide the same non-repudiation as ordinary signatures. Like the previous scheme, the message to be signed is M and $Q = h(M)$. However, in this case, the verifier is not ignored. The hidden signature of $\mathcal{ID}$ on M is $Z = \frac{1}{x}R + xQ$. On receiving Z, an honest verifier can extract the original signature $S = Z - rP$. The verification condition is $e(S,P) = e(h(M), Y)$ like before.

9.4 Authenticated Key Agreement

Using the protocol of section 5, authenticated key agreement between any two parties is possible. User $\mathcal{ID}_a$ having public key x_aP and private key x_a wants to establish a shared key with user $\mathcal{ID}_b$ having public key x_bP and private key x_b. The protocol is essentially an extension of the two round identification protocol. We provide two variants, the first for illustrative purposes.

9.4.1 Three-Round Key Agreement

This protocol requires three rounds and is based on the traditional model for two-way authentication. In addition to two-way authentication, this protocol can also be used for key agreement.

1. $\mathcal{ID}_a$ generates $r_a \leftarrow \mathbb{Z}_q$ and computes $R_a = r_aY_b = r_ax_bP$. $\mathcal{ID}_a$ initiates the protocol by sending R_a to $\mathcal{ID}_b$
2. $\mathcal{ID}_b$ generates $Q \leftarrow \mathbb{G}_1$ and $r_b \leftarrow \mathbb{Z}_q$. $\mathcal{ID}_b$ computes $Z_b = \frac{1}{x_b}R_a + x_bQ$ and $R_b = r_bY_a = r_bx_aP$. It sends $\langle Z_b, Q, R_b\rangle$ to $\mathcal{ID}_a$.
3. $\mathcal{ID}_a$ accepts $\mathcal{ID}_b$'s authentication if $e(Z_b, P) = e(P, P)^{r_a} \cdot e(Q, Y_b)$. If so, it also computes $Z_a = \frac{r_a}{x_a}R_b = r_ar_bP$ and sends Z_a to $\mathcal{ID}_b$.
4. $\mathcal{ID}_b$ accepts $\mathcal{ID}_a$'s authentication if $e(r_bR_a, P) = e(Z_a, Y_b)$. That is, $\mathcal{ID}_b$ checks that $\langle Z_a, Y_b, r_bR_a\rangle$ is indeed a valid DDH tuple. After this step, both parties are authenticated to each other. The shared key can be either of $\{r_aP, r_bP, x_bQ\}$.

In the first three steps, $\mathcal{ID}_b$ identifies itself to $\mathcal{ID}_a$. In the fourth step, $\mathcal{ID}_a$ identifies itself to $\mathcal{ID}_b$ by proving the knowledge of r_a, the discrete logarithm of R_a to base Y_b that was sent in step 1 (since a correct value Z_a simultaneously proves knowledge of x_a and r_a). We note that the protocol is still zero-knowledge because a passive adversary is unable to decide if the authentication was successful after watching the communication. We note that it is possible to combine the first and last steps together as demonstrated in the next variant.

9.4.2 Two-Round Key Agreement

Using the protocol of section 5, a two round authenticated key agreement is also possible. As before, user $\mathcal{ID}_a$ having public key x_aP and private key x_a wants to establish a shared key with user $\mathcal{ID}_b$ having public key x_bP and private key x_b. Additionally, we need a cryptographic hash function $\mathcal{H} : \mathbb{G}_1 \mapsto \mathbb{G}_1$. The protocol is essentially a proof of 'knowledge of knowledge' and is unconditionally secure under the following scenario: $\mathcal{ID}_b$ proves knowledge of $\mathcal{ID}_a$'s knowledge. That is, $\mathcal{ID}_a$ initiates by saying "I know x_a and r" and $\mathcal{ID}_b$ replies by saying "I know x_b if you know x_a and r".

1. $\mathcal{ID}_a$ generates $r \leftarrow \mathbb{Z}_q$. It computes $R = rY_b$ and $Z_a = rP + x_a\mathcal{H}(R)$ and initiates the protocol by sending (R, Z_a) to $\mathcal{ID}_b$

2. On receiving (R, Z_a), $\mathcal{ID}_b$ computes $S = \frac{1}{x_b}R$ and $Q_a = \mathcal{H}(R)$. It checks that $e(Z_a, P) = e(S, P) \cdot e(Q_a, Y_a)$ and if this test passes $\mathcal{ID}_b$ accepts $\mathcal{ID}_a$'s authentication. If $\mathcal{ID}_b$ wants to continue with the process, it generates $Q_b \leftarrow \mathbb{G}_1$, computes $Z_b = S + x_b Q_b$ and sends (Z_b, Q_b) to $\mathcal{ID}_a$. It also keeps S as the shared key.

3. $\mathcal{ID}_a$ accepts $\mathcal{ID}_b$'s authentication if $e(Z_b, P) = e(P, P)^r \cdot e(Q_b, Y_b)$. If so, it keeps rP as the shared key.

We claim that in the second step, $\mathcal{ID}_b$ will accept if and only if $\mathcal{ID}_a$ knows r_a and x_a. To see this, first note that (Z_a, Q_a) is a zero knowledge identification proof of $\mathcal{ID}_a$. Due to this, there is no guarantee that the proof was generated by $\mathcal{ID}_a$ (since it could also have been efficiently simulated according to section 5). However, observe that if this protocol is simulated, the resulting Q_a will almost certainly be random. A simulator cannot choose a predetermined value of Q_a since there appears to be no way to output an accepting configuration for a specific Q_a without knowledge of x_a. The use of the hash function additionally ensures that the simulator did not have control even over the random coin tosses r. Hence, for this particular instance, we can safely assume that the simulation must have been carried out by $\mathcal{ID}_a$. The second and third steps of the protocol involve the identification of $\mathcal{ID}_b$ to $\mathcal{ID}_a$ using the protocol of section 5 keeping r as the random coin tosses of verifier $\mathcal{ID}_a$. We feel that this brief analysis is sufficient to understand the security of the protocol.

9.5 Online Credit Card Payments

In this section, we will present a simple online payment system with some interesting security features. The protocol requires only one certified key. The seller of a product need not provide a certified key to the buyer, effectively remaining anonymous. The seller must produce some identification to the credit card processor or the bank to ensure that the payment is successful. If the buyer notices a disputed transaction on his credit cart statement, he can ask the bank to reveal the identity of the party who received the money. If the transaction is not disputed, the seller can remain completely anonymous. Moreover, we provide the additional advantage of 'single-use' transactions, that is after having successfully processed a payment, the seller cannot later reuse the same information to process another identical payment. We will assume the identification scheme of section 5 where the buyer is $\mathcal{ID}$ and the seller is $\mathcal{S}$. The protocol also involves a third party $\mathcal{B}$ which could be a bank or a credit card processor. The certified public key of $\mathcal{ID}$ is $Y = xP$. This key could itself serve as a credit card number. We also assume two cryptographic hash functions $\mathcal{H}_1 : \{0,1\}^* \mapsto \mathbb{G}_1$ and $\mathcal{H}_2 : \mathbb{G}_1 \mapsto \mathbb{G}_1$.

1. The buyer $\mathcal{ID}$ begins by visiting the website of $\mathcal{S}$ and initiating a purchase transaction. The details of the transaction are encapsulated in a request $\mathcal{REQ}$. The tuple $\langle \mathcal{ID}, \mathcal{REQ}, Y \rangle$ is sent to $\mathcal{S}$

2. $\mathcal{S}$ generates random $r \leftarrow \mathbb{Z}_q$ and computes $R = rY = rxP$. It also creates a contract $\mathcal{CON}$ containing the payment amount, transaction date, time and other details (though it will possibly not mention the identity of the seller or the commodity for sale to protect privacy). It sends $\langle \mathcal{CON}, R \rangle$ to $\mathcal{ID}$. It is understood that transactions are accepted as valid by the bank only for a short specified deadline (say five minutes) from the time mentioned in the contract.

3. $\mathcal{ID}$ checks that the contract is correct and computes $Q = \mathcal{H}_1(\mathcal{CON})$. To initiate the payment, $\mathcal{ID}$ computes $Z_1 = \frac{1}{x}R + xQ$ and $Z_2 = x\mathcal{H}_2(Z_1)$. It sends $\langle Z_1, Z_2 \rangle$ back to $\mathcal{S}$ and saves $\langle \mathcal{CON}, R \rangle$ in its database until it receives its next credit card statement from the bank.

4. $\mathcal{S}$ computes $Q = \mathcal{H}_1(\mathcal{CON})$ verifies that $e(Z_1, P) = e(P, P)^r \cdot e(Q, Y)$ and $e(Z_2, P) = e(\mathcal{H}_2(Z_1), Y)$. If both checks pass, $\mathcal{S}$ forwards the tuple $\langle Z_1, Z_2, r, Y, \mathcal{ID}, \mathcal{S}, \mathcal{CON} \rangle$ as a payment request to the bank $\mathcal{B}$.

5. On receiving a payment request, $\mathcal{B}$ does the same verification as $\mathcal{S}$; that is, it computes $Q = \mathcal{H}_1(\mathcal{CON})$ and verifies $e(Z_1, P) = e(P, P)^r \cdot e(Q, Y)$ and $e(Z_2, P) = e(\mathcal{H}_2(Z_1), Y)$. It also ensures that the $\langle r, \mathcal{ID} \rangle$ pair has not been previously used by checking its database. Finally, the bank checks the date and time specified in $\mathcal{CON}$ and ensures that it is within the specified expiry period (five minutes) of the current time. If all checks pass, $\mathcal{B}$ accepts this transaction, deducts the amount specified in Q from $\mathcal{ID}$'s account, credits that amounts to $\mathcal{S}$'s account, saves the tuple $\langle Z_1, Z_2, \mathcal{ID}, \mathcal{S}, \mathcal{CON}, r \rangle$ in its database and returns `success` to $\mathcal{S}$.

6. The bank's reply is forwarded to the buyer along with a receipt of a successful transaction.

7. If the bank receives another transaction with the same $\langle r, \mathcal{ID} \rangle$ pair in the future, it outputs `failure`. For security reasons, it also saves the corresponding $\langle \mathcal{S}, \mathcal{S}', r, \mathcal{ID} \rangle$ in a *blacklist* where $\mathcal{S}'$ is the identity of the other seller corresponding to the same pair. This blacklist can be used for further investigation if necessary.

8. Sometime in the future, the bank sends $\mathcal{CON}$ in a credit card statement to $\mathcal{ID}$. If some transaction is disputed, $\mathcal{ID}$ reports the corresponding $\langle \mathcal{CON}, R \rangle$ back to the bank, along with some evidence (eg. a transaction receipt with a `failure` response). The bank can easily trace the disputed seller $\mathcal{S}$ using its database after validating that $R = rY$.

9.6 Identity Based Cryptography

Using the primitives for identification of section 5, any smart card user $\mathcal{S}$ can setup an identity based encryption scheme with a group of smart cards dynamically. The interesting feature of our scheme is that the private keys can be distributed over an insecure channel. The infrastructure is roughly as follows:

messages for a user $\mathcal{ID}$ can be encrypted using the public key $\mathcal{ID}$. The private key for decryption is given out by $\mathcal{S}$ over an insecure public channel but masked using the *signcryption* procedure of 9.1. Only the real user is able to extract the secret key. Before this is done, however, user $\mathcal{ID}$ must first produce some personal authenticating information (like a passport photocopy) that can truly establish the identity and is a one-time requirement. This request for a private key can be sent over an unencrypted channel as long as it can be authenticated; that is, it must be ensured that the person requesting the key for $\mathcal{ID}$ is indeed $\mathcal{ID}$. For our purpose, we assume that a signature is used. The private key of the user $\mathcal{S}$ who acts as the KGC for this setup is $x \in \mathbb{Z}_q$ and the corresponding public key is $Y = xP \in \mathbb{G}_1$. Let $\mathcal{H}_1 : \{0,1\}^* \mapsto \mathbb{G}_1$ be a cryptographic hash function. The public key of $\mathcal{ID}_i$ is implicitly understood to be $Q_i = \mathcal{H}_1(\mathcal{ID}_i)$

1. All users must have a prior certified public key to authenticate its requests to $\mathcal{S}$. Each user $\mathcal{ID}_i$ generates $r_i \leftarrow \mathbb{Z}_q$ and computes $R_i = r_i Y = r_i xP$. $\mathcal{ID}_i$ then signs R_i using its certified private key and sends R_i to $\mathcal{S}$ over an insecure channel.

2. The KGC $\mathcal{S}$ verifies the signatures and thus authenticates the request of users. For each valid request R_i of $\mathcal{ID}_i$, the KGC computes $Q_i = \mathcal{H}_1(\mathcal{ID}_i)$ and $Z_i = \frac{1}{x}R_i + xQ_i$. It makes each Z_i public via an insecure channel.

3. If $\mathcal{ID}_i$ knows corresponding r_i, he/she can compute the private key xQ_i after authenticating it using the method described in section 9.1. The encryption/decryption can be done exactly as described in [5] after this step. The zero-knowledge property ensures that only the right user can compute the private key from Z_i.

We briefly describe the encryption scheme here (details can be obtained from [5]). Let $\mathcal{H}_2 : \mathbb{G}_2 \mapsto \{0,1\}^k$ be a cryptographic hash function. A random k bit message M for $\mathcal{ID}_i$ is encrypted as follows: generate $\alpha \leftarrow \mathbb{Z}_q$, compute $Q_i = \mathcal{H}_1(\mathcal{ID}_i)$, $C_1 = M \oplus \mathcal{H}_2(e(\alpha Q_i, Y))$ and $C_2 = \alpha P$. The ciphertext (C_1, C_2) is sent to $\mathcal{ID}_i$ who decrypts $M = C_1 \oplus \mathcal{H}_2(e(C_2, xQ_i))$.

10 Summary

In this paper, we proposed the notion of *zero knowledge blind identification*. Informally, in such a protocol, an honest prover reveals only one (intended) bit of information to an honest verifier and reveals less than that information to a dishonest verifier. In effect, using our scheme, any user can correctly identify to a random server and a passive adversary cannot learn anything about the outcome of the identification. Hence we coin the term blind identification. This is an additional feature provided by our scheme in contrast to previously proposed schemes. In an intuitive sense, ours is a model of a non-interactive zero knowledge proof since the security of the protocol is independent of the honesty of the verifier. The constructions presented in this paper arise from the work on identity based encryption [5], group signatures [7], aggregate signatures [8],

chained signatures [10, 11] and additive zero knowledge proofs [9]. The security of our protocol relies on the hardness of deciding if $Z = \frac{1}{x}R + xQ$ for given P, xP, Q, Z and a chosen R. We feel that this problem should be further studied.

In section 9, we show how these simple identification primitives can be used for constructing complex mechanisms like key agreement, digital signatures, encryption and signcryption. As a simple application of our smart card scheme, we propose a model for online credit card and cheque transactions. The protocol can be used in conjunction with the Secure Electronic Transaction (SET) specification or in a completely different infrastructure. As some other applications, we mention subliminal identification, designated verifier proofs and multiuser authentication. For optimal security, the primitives for signing are best implemented in a tamperproof chip supporting elliptic curve point addition and doubling operations. As observed, all the verification primitives require one or two pairing computations and deal with public keys only. Consequently, they are not restricted to a secure tamperproof device and can be implemented on faster processors. We refer the reader to [4] for details on constructing the hash functions used in this paper.

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