

Is it possible to have CBE from CL-PKE?

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Abstract. Recently, Al-Riyami and Paterson proposed a generic conversion from CL-PKE (Certificateless Public Key Encryption) to CBE (Certificate Based Encryption) and claimed that the derived CBE scheme is secure and even more efficient than the original scheme of Gentry. In this paper, we show that their conversion is wrong due to the flaw of the security proof. Our result supports the impossibility to relate both notions in any directions. In addition, it leads the new concrete CBE scheme by Al-Riyami and Paterson to be invalidated.

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1 Introduction

The notions of Certificate Based Encryption (CBE) and Certificateless Public Key Encryption (CL-PKE) are proposed as alternative approaches to overcome several drawbacks of conventional PKIs and Identity Based Encryption (IBE). CBE proposed by Gentry [4] provides an implicit certification mechanism for a conventional PKI and allows a periodical update of certificate status. As conventional PKIs, each user in CBE generates his own public/private key pair and request a long-lived certificate from the CA. But, CA uses identity based cryptography to generate the long-lived certificate as well as short-lived certificates (i.e., certificate status). A short-lived certificate can be pushed only to the owner of the public/private key pair and acts as a partial decryption key. So CBE provides implicit certification, while it is not subjected to the private key escrow problem inherent in IBE.

On the other hand, CL-PKE is designed to overcome the key escrow limitation of IBE. As IBE, each user has a unique identifier and the (partial) private key associated with that identifier is computed by a Key Generation Center

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(KGC), who knows some special master secret, and distributed to the user with that identifier. However, unlike a traditional IBE scheme, the user also publishes a public key, based on a secret value which the user alone knows. This user secret value is also contained in the user's private key. So the KGC does not know the user's private key that implies the escrow freeness. Note that the user's public key need not to be certified as in conventional PKIs.

Although CBE and CL-PKE were developed independently, both of them were motivated to provide alternative scheme with merits of PKI and IBE at the same time. So a natural question to establish the connection of two concepts arose. After it was briefly recognized in [1], Yum and Lee gave a generic construction for CL-PKE from IBE [6] and explored the relationships between IBE, CBE and CL-PKE [7]. However their construction is lack of consideration about the full security model in [1] by placing a certain extra limitations on the adversaries [2]. Recently, Al-Riyami and Paterson in [2] presented a generic conversion of a secure CBE scheme from a secure CL-PKE scheme, also explained why it is unlikely to be forthcoming in the opposite direction.

In this note, we point out that the claim of Al-Riyami and Paterson about relationship between CBE and CL-PKE is wrong. More precisely, their conversion from CL-PKE to CBE has a critical flaw in the security proof, which finally brings the evidence that each concept has its own unique advantage even with many aspects in common.

This paper is organized as follows. Section 2 and 3 briefly reviews the definition and security model for CL-PKE and CBE, respectively. Much of the detail such as a concrete scheme will be omitted, stating only what is needed for our discussion. Section 4 points out some problems for the generic construction of PKC 2005 and Section 5 draws conclusions.

2 Certificateless Public Key Encryption

In this section, we review the definition and security model for CL-PKE from [1].

Definition 1. A certificateless public key encryption scheme is specified by seven algorithms (CL.Setup, CL.PartialPrivateKey, CL.SetSecret, CL.SetPrivate, CL.SetPublic, CL.Enc, CL.Dec) such that:

- CL.Setup is a probabilistic algorithm that takes security parameter k as input and returns the system parameters params and master-key.
- CL.PartialPrivateKey is a deterministic algorithm that takes params , master-key and an identifier for entity A , $\text{ID}_A \in \{0, 1\}^*$ as inputs and returns a partial private key D_A .
- CL.SetSecret is a probabilistic algorithm that takes params as input and returns a secret value x_A .
- CL.SetPrivate is a deterministic algorithm that takes params , D_A , and x_A as input and returns a private key S_A .

- **CL.SetPublic** is a deterministic algorithm that takes **params**, and x_A as input and returns a public key P_A .
- **CL.Enc** is a deterministic algorithm that takes **params**, a message M , P_A , and ID_A as inputs and returns either a ciphertext C or the null symbol \perp indicating an encryption failure.
- **CL.Dec** is a deterministic algorithm that takes **params**, C , and S_A as inputs and returns a message M or a message \perp indicating a decryption failure.

2.1 Security Model for CL-PKE

Here we list the actions that an IND-CCA adversary \mathcal{A} against a CL-PKE scheme may carry out and discuss how each action should be handled by the challenger \mathcal{C} for that adversary.

1. **Extract Partial Private Key of Entity A :** \mathcal{C} responds by running algorithm **CL.PartialPrivateKey** to generate D_A for entity A .
2. **Extract Private Key for Entity A :** If A 's public key has not been replaced then \mathcal{C} can respond by running algorithm **CL.SetPrivate** to generate the private key S_A for entity A . It is assumed that the adversary does not make such queries for entities whose public keys have been changed.
3. **Request Public Key of Entity A :** \mathcal{C} responds by running algorithm **CL.SetPublic** to generate the public key P_A for entity A . If necessary, first runs algorithm **CL.SetSecret**.
4. **Replace Public Key of Entity A :** \mathcal{A} can repeatedly replace the public key P_A for any entity A with any value P'_A of its choice. The current value of an entity's public key is used by \mathcal{C} in any computations or responses to \mathcal{A} 's requests.
5. **Decryption Query for Ciphertext C and Entity A :** If \mathcal{A} has not replaced the public key of entity A , then \mathcal{C} responds by running algorithm **CL.SetPrivate** to obtain the private key S_A , then running **CL.Dec** on ciphertext C and private key S_A and returning the output to \mathcal{A} . It is assumed that \mathcal{C} should properly decrypt ciphertexts, even for those entities whose public keys have been replaced.

The IND-CCA security model distinguishes two types of adversary. A Type I adversary is able to change public keys of entities at will, but does not have access to the **master-key**. A type II adversary is equipped with **master-key** but is not allowed to replace public keys. This adversary models security against an eavesdropping KGC. The security game proceeds in three phases; in the middle challenge phase, the adversary selects a challenge identifier ID_{ch} and corresponding public key P_{ch} , and is given a challenge ciphertext C^* . We provide a detailed description of the two adversary types and the security game next.

CL-PKE Type I IND-CCA Adversary: Adversary \mathcal{A}_I does not have access to **master-key**. However, \mathcal{A}_I may request public keys and replace public keys with values of its choice, extract partial private and private keys and make decryption

queries, all for identities of its choice. \mathcal{A}_I cannot extract the private key for ID_{ch} at any point, nor request the private key for any identifier if the corresponding public key has already been replaced. \mathcal{A}_I cannot both replace the public key for the challenge identifier ID_{ch} in some phase. Furthermore, in Phase 2, \mathcal{A}_I cannot make a decryption query on the challenge ciphertext C^* for the combination (ID_{ch}, P_{ch}) that was used to encrypt M_b .

CL-PKE Type II IND-CCA Adversary: Adversary \mathcal{A}_{II} does have access to master-key, but may not replace public keys of entities. Adversary \mathcal{A}_{II} can compute partial private keys for itself, given master-key. It can also request public keys, make private key extraction queries and decryption queries, all for identities of its choice. The restrictions on this type of adversary are that it cannot replace public keys at any point, nor extract the private key for ID_{ch} at any point. Additionally, in Phase 2, \mathcal{A}_{II} cannot make a decryption query on the challenge ciphertext C^* for the combination (ID_{ch}, P_{ch}) that was used to encrypt M_b .

Definition 2. A CL-PKE scheme is said to be IND-CCA secure if no polynomially bounded adversary \mathcal{A} of Type I or Type II has a non-negligible advantage in the following game:

- **Setup:** Challenger \mathcal{C} takes a security parameter k as input and runs the CL.Setup algorithm. It gives \mathcal{A} the resulting system parameters **params**. If \mathcal{A} is of Type I, then \mathcal{C} keeps master-key to itself, otherwise, it gives master-key to \mathcal{A} .
- **Phase 1:** \mathcal{A} issues a sequence of request described above. These queries may be asked adaptively, but are subject to the rules on adversary behavior defined above.
- **Challenge Phase:** Once \mathcal{A} decides that Phase 1 is over it outputs the challenge identifier ID_{ch} and two equal length plaintexts M_0, M_1 . \mathcal{C} now picks a random bit $b \in \{0, 1\}$ and computes C^* , then encryption of M_b under the current public key P_{ch} for ID_{ch} . Then C^* is delivered to \mathcal{A} .
- **Phase 2:** Now \mathcal{A} issues a second sequence of requests as in Phase 1, again subject to the rules on adversary behavior above.
- **Guess:** Finally, \mathcal{A} outputs a guess $b' \in \{0, 1\}$. The adversary wins the game if $b = b'$. We define \mathcal{A} 's advantage in this game to be $\text{Adv}(\mathcal{A}) := 2|\Pr[b = b'] - \frac{1}{2}|$.

3 Certificate Based Encryption

In this section, we briefly review the definition and security model for CBE from [2].

Definition 3. A certificate based encryption scheme is defined by six algorithms (CBE.Setup, CBE.SetKeyPair, CBE.Certify, CBE.Consolidate, CBE.Enc, CBE.Dec) such that:

- **CBE.Setup** is a probabilistic algorithm that takes a security parameter k as input and returns sk_{CA} and public parameters params that include the description of a string space Λ .
- **CBE.SetKeyPair** is a probabilistic algorithm that takes params as input and returns a public key pk and a private key sk .
- **CBE.Certify** is a deterministic certification algorithm that takes an input $\langle \text{sk}_{\text{CA}}, \text{params}, \tau, \lambda \in \Lambda, \text{pk} \rangle$ and returns Cert'_{τ} , which is sent to the client. Here τ is a string identifying a time period, while λ contains other information needed to certify the client such as the client's identifying information, and pk is a public key.
- **CBE.Consolidate** is a deterministic certificate consolidation algorithm that takes $\langle \text{params}, \tau, \lambda, \text{Cert}'_{\tau} \rangle$ as input and optionally $\text{Cert}_{\tau-1}$. It returns Cert_{τ} , the certificate used by a client in time period τ .
- **CBE.Enc** is a probabilistic algorithm that takes $\langle \tau, \lambda, \text{params}, \text{pk}, M \rangle$ as input, where M is a message. It returns a ciphertext C for the message M .
- **CBE.Dec** is a deterministic algorithm that takes $\langle \text{params}, \text{Cert}_{\tau}, \text{sk}, C \rangle$ as input in time period τ . It returns either a message M or the special symbol \perp indicating a decryption failure.

3.1 Security Model for CBE

Here we also list the actions that an IND-CCA adversary \mathcal{A} against a CBE scheme may carry out and discuss how each action should be handled by the challenger \mathcal{C} for that adversary.

- **Extract Certificate of Entity A :** \mathcal{C} responds by running **CBE.Certify** on input $\langle \text{sk}_{\text{CA}}, \text{params}, \tau, \lambda, \text{pk} \rangle$ to generate Cert'_{τ} for entity A .
- **Decryption Query for Ciphertext C and Entity A :** \mathcal{C} responds by running **CBE.Certify** and **CBE.Consolidate** on input $\langle \text{sk}_{\text{CA}}, \text{params}, \tau, \lambda, \text{pk} \rangle$ to obtain Cert_{τ} , then running **CBE.Dec** on ciphertext C , private key sk , and Cert_{τ} and returning the output to \mathcal{A} .

In [4, 2], security for CBE is defined using two different games and the adversary chooses which game to play. In **GAME 1**, the adversary models an uncertified entity and in **GAME 2**, the adversary models the certifier in possession of the master-key sk_{CA} attacking a fixed entity's public key. Let \mathcal{A}_1 be a Game 1 IND-CCA adversary and let \mathcal{A}_2 be a Game 2 IND-CCA adversary.

CBE Game 1 IND-CCA Adversary: \mathcal{A}_1 does not have access to sk_{CA} . However, \mathcal{A}_1 may request public keys, extract certificate and make decryption queries, all for public keys of its choice. \mathcal{A}_1 must provide a private key sk along with the corresponding public key pk in all of its queries. This enables the challenger to handle decryption queries.

CBE Game 2 IND-CCA Adversary: \mathcal{A}_2 does not have access sk_{CA} , but does not get to choose a challenge public key to attack. Instead, it is given a specific public key from \mathcal{C} at the start of the game. So \mathcal{A}_2 can compute Cert'_r for any public key pk , given sk_{CA} . In [2], it is restricted to work with the fixed value of params , while \mathcal{A}_2 in [4] is allowed to work with multiple values of params . But this restriction is sufficiently reasonable because CA does not change its public parameters frequently. Furthermore, it is required to connect the notions of CBE and CL-PKE [2].

Definition 4. A CBE scheme is said to be IND-CCA secure if no polynomially bounded adversary \mathcal{A} of Type I or Type II has a non-negligible advantage in the following game:

- **Setup:** Challenger \mathcal{C} takes a security parameter k as input and runs the CBE.Setup algorithm. It gives \mathcal{A} the resulting system parameters params . If \mathcal{A} is of GAME 1, then \mathcal{C} keeps sk_{CA} to itself, otherwise, it gives sk_{CA} and $\langle \text{pk}_{\text{ch}}, \text{sk}_{\text{ch}} \rangle$ obtained by running CBE.SetKeyPair to \mathcal{A} .
- **Phase 1:** \mathcal{A} issues a sequence of requests described above. These queries may be asked adaptively, but are subject of the rules on adversary behavior defined above.
- **Challenge Phase:** Once \mathcal{A} decides that Phase 1 is over it outputs the challenge time period τ_{ch} , certifying information λ_{ch} and two equal length plaintexts M_0, M_1 . If \mathcal{A} is of GAME 1, then \mathcal{C} additionally gives $\langle \text{pk}_{\text{ch}}, \text{sk}_{\text{ch}} \rangle$ to \mathcal{A} . \mathcal{C} now picks a random bit $b \in \{0, 1\}$ and computes C^* , then encryption of M_b under the current public key pk_{ch} . Then C^* is delivered to \mathcal{A} .
- **Phase 2:** Now \mathcal{A} issues a second sequence of requests as in Phase 1, again subject of the rules on adversary behavior above.
- **Guess:** Finally, \mathcal{A} outputs a guess $b' \in \{0, 1\}$. The adversary wins the game if $b = b'$. we define \mathcal{A} 's advantage in this game to be $\text{Adv}(\mathcal{A}) := 2|\Pr[b = b'] - \frac{1}{2}|$.

4 CBE from CL-PKE at PKC 2005

At PKC 2005, Al-Riyami and Paterson provided a conversion method constructing a CBE scheme Π^{CBE} using the algorithms of a generic CL-PKE scheme Π^{CL} as components [2]. They claimed that Π^{CBE} is secure based on the security of Π^{CL} . The main aspect of their method is to define the identifier of the user used in Π^{CL} to include a certain public key of Π^{CBE} and short-lived certificates in the Π^{CBE} are obtained from the partial private keys in the Π^{CL} .

Let Π^{CL} be a CL-PKE scheme with algorithms (CL.Setup, CL.PartialPrivateKey, CL.SetSecret, CL.SetPrivate, CL.SetPublic, CL.Enc, CL.Dec). Then a CBE scheme Π^{CBE} can be defined as follows:

- **CBE.Setup:** On input a security parameter k , first run CL.Setup(k) to obtain master-key and $\text{params}^{\text{CL}}$. Then set $\text{sk}_{\text{CA}} = \text{master-key}$ and Λ be any subset of $\{0, 1\}^*$. Define $\text{params}^{\text{CBE}}$ by extending $\text{params}^{\text{CL}}$ to include a description of Λ .

- CBE.Setup: On input $\text{params}^{\text{CBE}}$ of an entity A , extract $\text{params}^{\text{CL}}$ then run $\text{CL.SetSecret}(\text{params}^{\text{CL}}) = x_A$ and $\text{CL.SetPublic}(\text{params}^{\text{CL}}, x_A) = P_A$. The output $\langle \text{pk}, \text{sk} \rangle = \langle P_A, x_A \rangle$.
- CBE.Certify: On input $\langle \text{sk}_{CA}, \text{params}^{\text{CBE}}, \tau, \lambda, \text{pk} \rangle$, extract $\text{params}^{\text{CL}}$. Set $\text{ID}'_A = \text{params}^{\text{CBE}} \parallel \tau \parallel \lambda \parallel \text{pk}$ and run $\text{CL.PartialPrivateKey}(\text{params}^{\text{CL}}, \text{sk}_{CA}, \text{ID}'_A) = D_A$. The output $\text{Cert}'_\tau = D_A$.
- CBE.Consolidate: On input $\langle \text{params}^{\text{CBE}}, \tau, \lambda, \text{Cert}'_\tau \rangle$, outputs Cert'_τ .
- CBE.Enc: On input $\langle \tau, \lambda, \text{params}^{\text{CBE}}, \text{pk}, M \rangle$, extract $\text{params}^{\text{CL}}$ and set $\text{ID}'_A = \text{params}^{\text{CBE}} \parallel \tau \parallel \lambda \parallel \text{pk}$. The output $C = \text{CL.Enc}(\text{params}^{\text{CL}}, M, \text{pk}, \text{ID}'_A)$.
- CBE.Dec: On input $\langle \text{params}^{\text{CBE}}, \text{Cert}'_\tau, \text{sk}, C \rangle$ in time period τ , extract $\text{params}^{\text{CL}}$ and set $D_A = \text{Cert}'_\tau$, $x_A = \text{sk}$ then run $\text{CL.SetPrivate}(\text{params}^{\text{CL}}, D_A, x_A) = S_A$. The output is $\text{CL.Dec}(\text{params}^{\text{CL}}, C, S_A)$.

The security proof of Π^{CBE} provided in [2] is only restricted to GAME 1. For the security proof of the GAME 2, the authors in [2] only claimed that similar ideas of the proof for GAME 1 may be applied. However we provide a serious observation so that it does not seem to be clear the security of their conversion can be proven also in GAME 2. First, we briefly introduce the idea used in the proof of GAME 1. Let \mathcal{A}_1 be a GAME 1 IND-CCA adversary against Π^{CBE} with advantage ϵ . Using \mathcal{A}_1 as a black-box, a Type I IND-CCA adversary \mathcal{B}_I against Π^{CL} can be constructed as follows:

\mathcal{B}_I simulates CBE.Setup of Π^{CBE} by setting Λ to be an arbitrary subset of $\{0, 1\}^*$ and $\text{params}^{\text{CBE}}$ to be an extension of $\text{params}^{\text{CL}}$ which includes a description of Λ . \mathcal{B}_I then gives $\text{params}^{\text{CBE}}$ to \mathcal{A}_1 . To handle a sequence of queries issued by \mathcal{A}_1 , \mathcal{B}_I sets an identifier $\text{ID}'_A = \text{params}^{\text{CBE}} \parallel \tau \parallel \lambda \parallel \text{pk}$ of the entity A using the information τ, λ and pk included in the query of \mathcal{A}_1 and replaces the public key with the value pk . In GAME 1, it is always possible because \mathcal{B}_I is permitted to replace public keys with values of its choice. Then \mathcal{B}_I makes some queries to \mathcal{C} for the identifier ID'_A and relays \mathcal{C} 's response to \mathcal{A}_1 .

But this approach for \mathcal{A}_1 cannot be applied to the GAME 2 adversary directly. Let \mathcal{A}_2 be a GAME 2 IND-CCA adversary against Π^{CBE} with advantage ϵ . To construct a Type II IND-CCA adversary \mathcal{B}_{II} against Π^{CL} using \mathcal{A}_2 , \mathcal{B}_{II} should simulate CBE.Setup of Π^{CBE} as \mathcal{B}_I and then give $\text{params}^{\text{CBE}}$ and sk_{CA} to \mathcal{A}_2 . Furthermore, a specific key pair $\langle \text{pk}_{\text{ch}}, \text{sk}_{\text{ch}} \rangle$ should be given to \mathcal{A}_2 before launching it. To perform this, \mathcal{B}_{II} requests a public key for any ID and returns it to \mathcal{A}_2 as the challenge public key pk_{ch} , then sets $\text{ID}'_A = \text{params}^{\text{CBE}} \parallel \tau \parallel \lambda \parallel \text{pk}_{\text{ch}}$. Note that \mathcal{B}_{II} working in GAME 2 is not allowed to replace a public key for ID'_A with pk_{ch} . This implies the public key for ID'_A cannot be set as desired. So even \mathcal{B}_{II} makes decryption queries on input $\langle C, \text{ID}'_A \rangle$ to \mathcal{C} , the response can never be expected to be the correct answer for $\langle \tau, \lambda, \text{pk}_{\text{ch}}, \text{sk}_{\text{ch}}, C \rangle$ requested by \mathcal{A}_2 . Thus, we can conclude that a secure CBE scheme is not possibly obtained from a secure CL-PKE, at least using the conversion that Al-Riyami and Paterson constructed. This is somehow surprising result compared to how much it seems likely to be related.

5 Conclusion

We show that the generic construction of a CBE scheme from a secure CL-PKE scheme proposed by Al-Riyami and Paterson does not satisfy the security model for CBE. Based on the observation for the opposite conversion by [2], we clarify that it is still an open problem to relate these two concepts.

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