HB^{++}: a Lightweight Authentication Protocol Secure against Some Attacks

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Abstract

At Crypto'05, Juels and Weis introduce HB^+, an enhancement of the Hopper and Blum (HB) authentication protocol. This protocol HB^+ is proven secure against active attacks, though preserving HB's advantages: mainly, requiring so few resources to run that it can be implemented on an RFID tag. However, in a wider adversarial model, Gilbert, Robshaw and Sibert exhibit a very effective attack against HB^+.

We here show how a modification of the HB^+ protocol thwarts Gilbert et al's attack. The resulting protocol, HB^{++}, remains a good candidate for RFID tags authentication.

Keywords. HB^+ protocol, active attacks, RFID.

1 Introduction

The problems of security and privacy for Radio Frequency Identification (RFID) have recently attracted many technical research.

RFID systems are made of three components: some tags, a reader, and a database which contains information on the tagged objects. Tags (transponders) follow the ISO and EPC [8] standards and communicate with the reader (transceiver) over the air. One main constraint here is that these tags have to be quite inexpensive (the order of magnitude is US cents) and thus they can embed only scarce resources, of which only some part is dedicated to security. Typically, computations are hardwired and some thousands of logic gates are kept for cryptography. This means that tags seem, at first glance, difficult targets for the implementation of classical cryptographic schemes, even if Feldhofer, Dominikus and Wolkerstorfer [9] have described an implementation of the AES algorithm which looks promising. Anyway, the introduction of new cryptographic schemes, requiring less resources, is today tempting.

In the typical setting, each tag comes with a unique identifier and an adversary should not be able to counterfeit tag responses. Many authentication protocols for RFID tags have been proposed so far (see e.g. in 2003 [18, 26], [12, 13, 16, 24] in 2004, [1, 3, 7, 22] in 2005, see also Juels [17] for a general survey and [2] for fresh references). Notably, at Crypto'05, HB^+, a lightweight cryptographic authentication scheme very well suited for low-cost hardware implementation, was introduced by Juels and Weis [19]. It provides a symmetric-key protocol allowing tags to identify themselves on the reader (the reader does not need to know a priori which tags and secrets are involved for the protocol to work). HB^+ is presented as an improvement of the HB protocol, which had been introduced in [14]. The security of the HB protocol does not rely on classical symmetric key cryptography solutions, but rather on the hardness of the computational Learning Parity with Noise (LPN) problem [4, 5, 15]. While the HB protocol is made to be secure against passive attacks only, the aim of HB^+ is to be resistant to active attacks. A proof of security is provided but at the same time, Gilbert, Robshaw and Sibert [10] describe a man-in-the-middle attack on HB^+ not covered by the corresponding security model.

The principal contribution of our work is to improve the HB^+ protocol in order to avoid the attacks of [10] and [25], while keeping its design principles and, thus, its advantages. We call HB^{++} our new protocol. In fact, HB^{++} can be seen as running HB^+ twice under independent secrets but with correlated challenges. Moreover, the secrets are renewed at each authentication. Two functions are shared by all the tags and readers; one is introduced to link together challenges of the protocol, the other is needed to determine secrets used for an authentication. At the end, the HB^{++} protocol seems to us a good susbtitute for HB^+ for RFID tags authentication.

The paper is organised as follows. In Sect. 2, we recall the HB^+ protocol. In Sect. 3, we summarize Gilbert et al's attack of the HB^+ protocol [10]. In Sect. 4, we introduce our protocol HB^{++}, we show that it is at least as secure as the HB^+ protocol and even resists some man-in-the-middle attacks. Section 5 concludes.

2 The HB^+ protocol

A brief description of one round of the HB^+ protocol is given by Fig. 1 where $a.x$ stands for the scalar product of the binary vectors a and x, and $\oplus$ is the exclusive or.

The two k-bit vectors x and y are secret keys shared by the tag and the

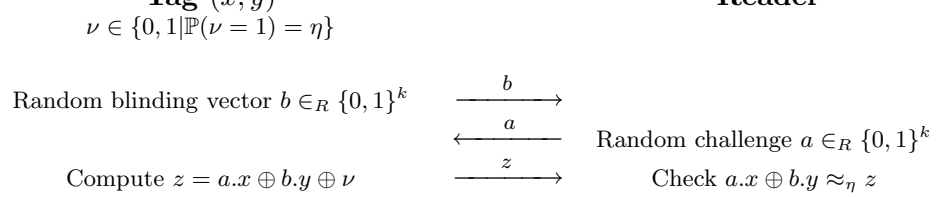

Figure 1: One round of HB^+

reader. Note that an extra noise is added to the response $a.x \oplus b.y$ by the tag, this error bit ν equals 1 with probability η.

The HB^+ round described by Fig. 1 is repeated r times and the tag is successfully authenticated if the check fails about ηr times (this is what is denoted by $\approx_\eta$ in Fig. 1 and in the following).

Remark 1 *The principal difference between the* HB^+ *and* HB *protocols is the introduction of* y *and* b *in the* HB^+ *protocol in order to avoid active attacks.*

In [19], the authors define a security model, and then show how to reduce an attack on HB to an attack on HB^+.

The security of the HB protocol is based on the Learning Parity with Noise (LPN) problem. Juels and Weis extend this result in their security model to HB^+ and explain how an attack on HB^+ can be used to solve an instance of the LPN problem (see Sect. 4.1 for an extension to our ideas).

Unfortunately, they do not take into account the extra information given by the result (positive or negative) of the protocol and this is exploited during the attack [10] (see Sect. 3).

3 A man-in-the-middle attack against HB^+

In [10], an attack is described against the HB^+ protocol. It is a linear-time man-in-the-middle attack where an adversary located between the reader and the tag is able to modify the challenge at every round. The adversary chooses a vector δ in $\{0,1\}^k$ and when a challenge a is sent by the reader, he intercepts the challenge and makes a switch to $a + \delta$ (see Fig. 2). Hence, at the end of the round, the reader will receive $\tilde{z} = (a + \delta).x \oplus b.y \oplus \nu$ from the tag.

This is repeated along all the rounds in order to deduce information from the success or failure of the authentication. Indeed, if the authentication

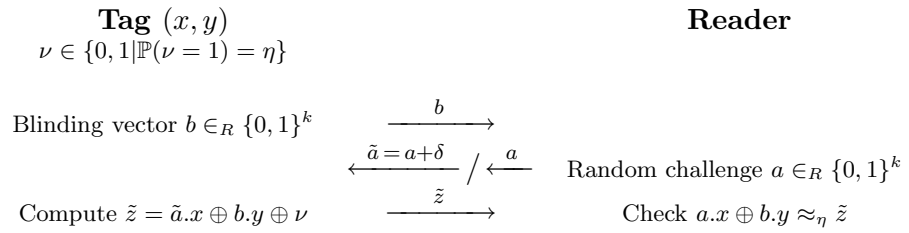

Figure 2: An effective attack against HB$^+$

succeeds (resp. fails), we have $\delta.x = 0$ (resp. $\delta.x = 1$) with a high probability. So one can recover x "bit after bit" by varying δ progressively.

Remark 2 *This attack holds to recover y too, as an adversary can send $b + \delta$ instead of b to the reader.*

4 Proposed solution

4.1 First attempt for HB^{++}

The protocol HB^{++} needs two new secrets x', y', and f a permutation of the set $\{0,1\}^k$ as described in Sect. 4.2. This protocol simply consists in computing corresponding responses to given challenges (a, b), $(f(a), f(b))$ and for the tag to send these responses together with independent errors ν and ν', i.e. $z = a.x \oplus b.y \oplus \nu$ and $z' = f(a).x' \oplus f(b).y' \oplus \nu'$ (see Fig. 3).

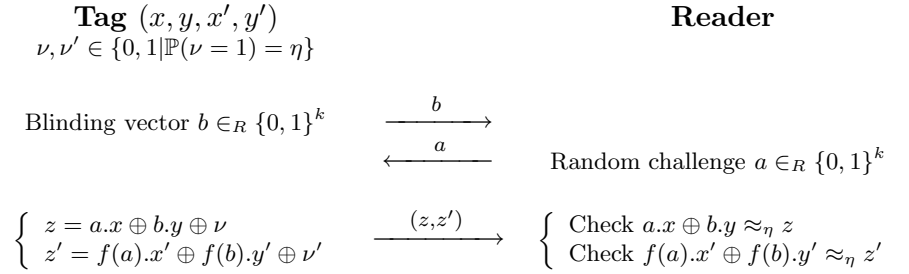

Figure 3: First attempt

The model of active security standing for the HB$^+$ protocol in [19] can be translated to this first construction.

Proposition 1 *An adversary who has the capability of breaking a random*

sequence of challenges-responses of this first attempt of HB^{++} *can successfully attack* HB^{+}*.*

Proof. Indeed, if an adversary $\mathcal{A}$ obtains a sequence of challenges-responses $\mathcal{S} = \{a_i, b_i, a_i.x \oplus b_i.y \oplus \nu_i\}_{i \in I}$ from successive rounds of the HB^{+} protocol between a tag $\mathcal{T}_{x,y}$ and a reader $\mathcal{R}$, then, by randomly picking x', y' and a variable ν' such that $\mathbb{P}(\nu' = 1) = \eta$, he can simulate a sequence of challenges-responses

$$\{a_i, b_i, a_i.x \oplus b_i.y \oplus \nu_i, f(a_i).x' \oplus f(b_i).y' \oplus \nu'_i\}_{i \in I}$$

of successive rounds of this first attempt of the HB^{++} protocol between $\mathcal{R}$ and a tag $\mathcal{T}_{x,y,x',y'}$. Thus his ability to cryptanalyse this protocol allows $\mathcal{A}$ to recover the value of x, y, x' and y' given a sufficiently large number of challenges-responses, and so to gain the knowledge of the secrets of the original tag $\mathcal{T}_{x,y}$. □

If $\mathcal{A}$ needs to use an active attack for this last point, the only constraint is to obtain the sequence $\mathcal{S}$ of challenges-responses by applying the same modification on a and b during the rounds of HB^{+} as if he was trying the attack on the new protocol.

The reduction to the LPN problem, which ensures the security of HB and HB^{+} against a passive attack, is always true for the new protocol.

Let wt_H stand for the hamming weight.

Definition 1 (LPN problem) *Let A be a random $q \times k$ binary matrix, let X be a random k-bit vector, let η be a constant noise parameter, and let $\vec{\nu}$ be a random q-bit vector such that* $\mathrm{wt}_H(\vec{\nu}) \leq \eta q$*.*

Given A, η, and $\vec{z} = AX \oplus \vec{\nu}$, find a k-bit vector X' such that $\mathrm{wt}_H(AX' \oplus \vec{z}) \leq \eta q$*.*

Proposition 2 *If a "passive" adversary has the capacity of breaking this first attempt of the* HB^{++} *protocol with 4 secrets of size k, he can also solve a random instance of the LPN problem of size $2k$.*

Proof. The adversary $\mathcal{A}$ can recover the secrets given a sufficiently large sequence.

Let A a random $q \times 2k$ binary matrix, X a random $2k$-bit vector, $\vec{\nu}$ a random q-bit vector such that $\mathrm{wt}_H(\vec{\nu}) \leq \eta q$ and $\vec{z} = AX \oplus \vec{\nu}$. $\mathcal{A}$ can construct the k-bit vectors x, y, a_i, b_i for $i \in \{1, \ldots, q\}$ such that:

$$X = \begin{pmatrix} x \\ y \end{pmatrix}$$

and

$$A = \begin{pmatrix} a_1 & b_1 \\ \vdots & \vdots \\ a_i & b_i \\ \vdots & \vdots \\ a_q & b_q \end{pmatrix}$$

The adversary $\mathcal{A}$ can interpret $\vec{z} = (a_i.x \oplus b_i.y \oplus \nu_i)_{i=1\ldots q}$ as responses of the HB$^+$ protocol. As in the HB^{++} protocol the errors ν_i are independant of the errors ν'_i, by taking random vectors $x', y', \vec{\nu'}$ and by computing $\vec{z'} = (f(a_i).x' \oplus f(b_i).y' \oplus \nu'_i)_{i=1\ldots q}$, then $(\vec{z}, \vec{z'})$ can be viewed as responses of the new protocol which allows $\mathcal{A}$ to recover $X = \begin{pmatrix} x \\ y \end{pmatrix}$. □

4.2 Protection against Gilbert et al.'s attack

We primarily choose f in order to thwart the attack presented in [10] but f has also to be taken with a low complexity and must not desequilibrate the distribution of scalar products.

As f is taken as a bijection, the last point is always true, the distribution of values does not change:

$$\forall x \in \{0,1\}^k,\ \mathbb{P}(c \in \{c | f(c).x = 0\}) = \mathbb{P}(c \in \{c | c.x = 0\}).$$

Henceforth, we focus on the first point. In order to avoid the attack [10], f is chosen such that Δ_f is small with:

$$\Delta_f = \max_{\delta \neq 0, \gamma} |\{a \in \{0,1\}^k | f(a+\delta) + f(a) = \gamma\}|.$$

In fact, this comes to force f to respect only a small number of linear relations, such that ultimately no linear relation holds for all the rounds.

Definition 2 *Let $f : \mathbb{F}_2^k \to \mathbb{F}_2^k$ be a vectorial boolean function, for $u \neq 0, v \in \mathbb{F}_2^k$, let*

$$\delta_f(u,v) = \big|\{a \in \{0,1\}^k | f(a+u) + f(a) = v\}\big|.$$

Remark 3 *This $\delta_f(u,v)$ has been introduced in [6] to measure the resistance of an S-box against differential cryptanalysis. We have*

$$\Delta_f = \max_{u \neq 0, v} \delta_f(u,v).$$

And for instance, the lower the value Δ_f will be, the more resistant against differential cryptanalysis the function f will be.

We will see in the sequel how Δ_f can be used to measure the resistance against the attack of Gilbert et al.

One can try to extend the attack [10] by corrupting (a, b) with $G(a, b) = (g_1(a, b), g_2(a, b))$, where $G \neq Id$, and sending $g_2(a, b)$ to the reader and $g_1(a, b)$ to the tag such that the reader will check if

$$\begin{cases} g_1(a,b).x \oplus b.y \oplus \nu & \approx_\eta \quad a.x \oplus g_2(a,b).y \\ f(g_1(a,b)).x' \oplus f(b).y' \oplus \nu' & \approx_\eta \quad f(a).x' \oplus f(g_2(a,b)).y' \end{cases}$$

i.e. if

$$\begin{cases} (g_1(a,b) + a, b + g_2(a,b)).(x,y) \oplus \nu & \approx_\eta \quad 0 \\ (f(g_1(a,b)) + f(a), f(b) + f(g_2(a,b))).(x',y') \oplus \nu' & \approx_\eta \quad 0 \end{cases} \tag{1}$$

Fortunately, an adversary does not know the result of this comparison but only the result of the authentication which depends on the results of all the r rounds of the protocol. So, if one wants to obtain some information on the secrets via this method, (1) has to be independent of a and b. We suppose also that an adversary has no knowledge of x, y, x' and y' and so they have to be considered as random vectors. In consequence, to achieve an attack, $\delta_1^{(x,y)}, \delta_2^{(x,y)}, \lambda_1^{(x',y')}$ and $\lambda_2^{(x',y')}$ have to be chosen such that the following equalities stand for all the r rounds:

$$\begin{cases} g_1(a,b) &= a + \delta_1^{(x,y)} \\ g_2(a,b) &= b + \delta_2^{(x,y)} \\ f(g_1(a,b)) &= f(a) + \lambda_1^{(x',y')} \\ f(g_2(a,b)) &= f(b) + \lambda_2^{(x',y')} \end{cases}$$

If $\{(a_i, b_i)\}_{i=1..r}$ is the set of all the values used during the r rounds, those equalities induce two linear relations involving f: $\forall i \in \{1, \ldots, r\}$,

$$f(a_i + \delta_1^{(x,y)}) + f(a_i) = \lambda_1^{(x',y')},$$
$$f(b_i + \delta_2^{(x,y)}) + f(b_i) = \lambda_2^{(x',y')}.$$

As $\Delta_f = \max_{\delta \neq 0, \gamma} \left| \{a \in \{0,1\}^k | f(a+\delta) + f(a) = \gamma\} \right|$ is small, these relations are verified during all the rounds only with a small probability. So it is possible to deduce something on the secrets from the success or failure of the authentication only with a small probability $\mathbb{P}$, which verifies:

$$\mathbb{P} \leq \left(\frac{\Delta_f}{2^k} \right)^r.$$

Consequently, the smaller Δ_f is, the smaller $\mathbb{P}$ is, and we have thus the following criterion for candidate functions f:

Criterion 1 *The security of the* HB^{++} *protocol against generalizations of the active attack described in [10] is ensured whenever the function f satisfies the following property: Δ_f is small enough such that $\left(\frac{\Delta_f}{2^k}\right)^r$ is negligible.*

An example of construction of function f is given for realistic parameters in Appendix A.1.

4.3 Another man-in-the-middle attack due to Wagner [25]

This first construction remains sensitive to an attack due to Wagner where the idea is to modify both the challenges sent by the reader and the responses received from the tag along one authentication. For understanding concern, we describe the method on a particular case.

Assume that part of the output of function f depends only on few bits of its input: for instance, say that the first five bits of $f(a)$ can be computed from the first five bits of a. An adversary can then try to find the value of, say, the first three bits of x and x' as follows:

1. he makes a guess for these six bits;
2. for each challenge a, he tries to choose δ such that:
   (a) all its bits are 0, except the first three ones that can be 0 or 1,
   (b) the same holds for $\delta' = f(a \oplus \delta) \oplus f(a)$, i.e.
   $$\delta = (*, *, *, 0, \ldots, 0), \quad \delta' = (*, *, *, 0, \ldots, 0);$$
3. he replaces the reader's challenge a by $a \oplus \delta$ and the tag responses z, z' by $z \oplus \delta.x$ and $z' \oplus \delta'.x'$, respectively;
4. at the end of the protocol, if the adversary has succeeded in constructing such triplets (a, δ, δ') along the rounds, he can exploit the result of the authentication. On one hand, if the authentication fails, he chooses another value for the first three bits of x and x'; on the other hand, if the authentication succeeds, this increases his confidence in his choice.

Indeed, we see in Fig. 4 that if the adversary has made a good guess the responses he sends to the reader are $\tilde{z} \oplus \delta.x = z$ and $\tilde{z'} \oplus \delta'.x' = z'$.

Actually, the existence of (a, δ, δ')'s verifying conditions 2a and 2b is likely to occur according to our initial hypothesis on f. In this example, if the same holds for the other output bits, after these 6 bits are recovered,

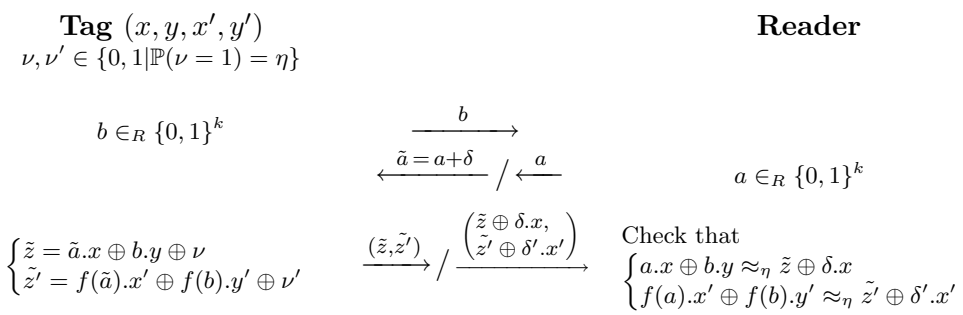

Figure 4: Wagner's attack

the adversary can iteratively extend his knowledge of x,x' by one bit at a time using a similar process, until he have learned all of x,x'. This attack requires a number of iterations that is linear in the key size.

Note that Wagner's attack seems closer than Gilbert et al's one to a general man-in-the-middle attack. As the adversary modifies the messages in both ways, he changes challenge a at his will, and the responses $\tilde{z}$, $\tilde{z'}$ are partly random, since the adversary tries all possible values for some bits of x, x'.

To counter this attack, we add in the computations of $f(a).x'$ and $f(b).y'$ a rotation which depends on the current round; i.e. we let

$$z' = rot(f(a), \rho).x' \oplus rot(f(b), \rho).y' \oplus \nu'$$

where ρ stands for the index of the current round. This way, an adversary has to take into consideration in turn all portions of the secrets during one authentication, and so the attack is not practicable anymore for large k.

We address the problem of the security of the protocol across different authentications in the next section.

4.4 Description of HB^{++}

As already mentionned in Sect. 4.1, we conserve proofs of security for the model of adversary of [19]. Along one authentication, we design a protocol which resists to known man-in-the-middle attacks [10, 25]. Furthermore, among several authentications, we now execute this protocol under renewed secrets.

The HB^{++} protocol can now be fully described.

Each tag comes with a unique secret Z. At the beginning of each authentication, two challenges are exchanged between the reader and the tag.

These challenges are derived under Z with a universal hash function h to obtain x, x', y and y'. These keys x, x', y and y' are then used to perform the authentication via r successive rounds.

Figure 5 illustrates the round ρ of the protocol.

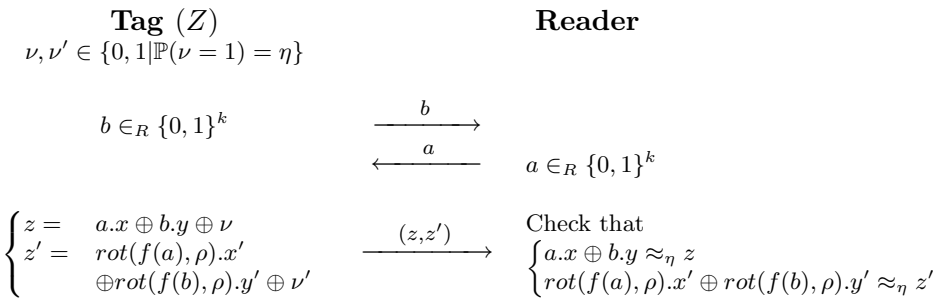

Figure 5: One round of the HB^{++} protocol

An example of construction of function h is given for realistic parameters in Appendix A.2.

5 Conclusion

The main contribution of this paper is to present HB^{++}, a new identification protocol which can be used as a replacement of HB^{+} for low-cost pervasive computing devices. At the price of making more computations than in HB^{+}, it allows to achieve security in a stronger adversarial model than HB^{+} as it is resistant to the attacks [10, 25] and at least as secure as HB^{+} in its adversarial model. This point was left as "an essential line for future work" in [19]. In fact, with HB^{++}, we switch from the "detection security model" to a more classical one (i.e. a "prevention-based" model).

The way we improve HB^{+}, i.e. forcing challenges to a specific form, is, to the best of our knowledge, new.

Its security reduction, against any man-in-the-middle attack, to a hard problem is left as an open question. Our point is that for attacks considered by Juels and Weis, security proofs continue to hold. For man-in-the-middle attacks – at this time – we only rely on know-how techniques as, for instance, this is the case for the design of block ciphers. Note that here, the adversary is severly constrained in his actions as he has only access to the result of the authentication at the end of the entire protocol.

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References

[1] G. Ateniese, J. Camenisch, and B. de Medeiros. Untraceable RFID tags via insubvertible encryption. In *Conference on Computer and Communications Security – CCS'05*. ACM Press, 2005.

[2] G. Avoine. `http://lasecwww.epfl.ch/~gavoine/rfid/`.

[3] G. Avoine and P. Oechslin. A scalable and provably secure hash based RFID protocol. In *International Workshop on Pervasive Computing and Communication Security – PerSec 2005*, pages 110–114. IEEE Computer Society Press, 2005.

[4] A. Blum, M. L. Furst, M. J. Kearns, and R. J. Lipton. Cryptographic primitives based on hard learning problems. In *Advances in Cryptology – CRYPTO'93*, Lecture Notes in Computer Science, pages 278–291. Springer-Verlag, 1993.

[5] A. Blum, A. Kalai, and H. Wasserman. Noise-tolerant learning, the parity problem, and the statistical query model. In *STOC 2000*, pages 435–440, 2000.

[6] F. Chabaud and S. Vaudenay. Links between differential and linear cryptanalysis. In A. D. Santis, editor, *Advances in Cryptology – EUROCRYPT'94*, volume 950 of *Lecture Notes in Computer Science*, pages 356–365. Springer-Verlag, 1994.

[7] S. Dominikus, E. Oswald, and M. Feldhofer. Symmetric authentication for RFID systems in practice. Handout of the Ecrypt Workshop on RFID and Lightweight Crypto, July 2005.

[8] EPC. Electronic product code global inc. `http://www.epcglobalinc.org/`.

[9] M. Feldhofer, S. Dominikus, and J. Wolkerstorfer. Strong authentication for RFID systems using the AES algorithm. In M. Joye and J.-J. Quisquater, editors, *Workshop on Cryptographic Hardware and Embedded Systems – CHES 2004*, volume 3156 of *Lecture Notes in Computer*

Science, pages 357–370, Boston, Massachusetts, USA, August 2004. IACR, Springer-Verlag.

[10] H. Gilbert, M. Robshaw, and H. Sibert. An active attack against HB^+ - a provably secure lightweight authentication protocol. Cryptology ePrint Archive, Report 2005/237, 2005. `http://eprint.iacr.org/`.

[11] R. Gold. Maximal recursive sequences with 3-valued crosscorrelation functions. *IEEE Trans. on Inform. Theory*, 14:154–156, 1968.

[12] P. Golle, M. Jakobsson, A. Juels, and P. Syverson. Universal re-encryption for mixnets. In T. Okamoto, editor, *The Cryptographers' Track at the RSA Conference – CT-RSA*, volume 2964 of *Lecture Notes in Computer Science*, pages 163–178. Springer-Verlag, 2004.

[13] D. Henrici and P. Müller. Hash-based enhancement of location privacy for radio-frequency identification devices using varying identifiers. In R. Sandhu and R. Thomas, editors, *International Workshop on Pervasive Computing and Communication Security – PerSec 2004*, pages 149–153. IEEE Computer Society, 2004.

[14] N. J. Hopper and M. Blum. Secure human identification protocols. In C. Boyd, editor, *Advances in Cryptology - ASIACRYPT 2001*, volume 2248 of *Lecture Notes in Computer Science*, pages 52–66. Springer-Verlag, 2001.

[15] J. Håstad. Some optimal inapproximability results. In *STOC 1997*, pages 1–10, 1997.

[16] A. Juels. "yoking-proofs" for RFID tags. In R. Sandhu and R. Thomas, editors, *International Workshop on Pervasive Computing and Communication Security – PerSec 2004*, pages 138–143. IEEE Computer Society, 2004.

[17] A. Juels. RFID security and privacy: A research survey. To appear in the *IEEE Journal on Selected Areas in Communication*, 2006.

[18] A. Juels, R. Rivest, and M. Szydlo. The blocker tag: Selective blocking of RFID tags for consumer privacy. In V. Atluri, editor, *Conference on Computer and Communications Security – ACM CCS*, pages 103–111. ACM Press, 2003.

[19] A. Juels and S. Weis. Authenticating pervasive devices with human protocols. In V. Shoup, editor, *Advances in Cryptology – CRYPTO'05*, volume 3126 of *Lecture Notes in Computer Science*, pages 293–308. Springer-Verlag, 2005.

[20] J.-P. Kaps, K. Yüksel, and B. Sunar. Energy scalable universal hashing. *IEEE Trans. on Computers*, 54:1484–1495, 2005.

[21] J. Katz and J. S. Shin. Parallel and concurrent security of the HB and HB$^+$ protocols. Cryptology ePrint Archive, Report 2005/461, 2005. `http://eprint.iacr.org/`.

[22] D. Molnar, A. Soppera, and D. Wagner. A scalable, delegatable pseudonym protocol enabling ownership transfer of RFID tags. In B. Preneel and S. Tavares, editors, *Selected Areas in Cryptography – SAC 2005*, Lecture Notes in Computer Science. Springer-Verlag, 2005.

[23] K. Nyberg and L. R. Knudsen. Provable security against differential cryptanalysis. In E. F. Brickell, editor, *Advances in Cryptology – CRYPTO'92*, volume 740 of *Lecture Notes in Computer Science*, pages 566–574. Springer-Verlag, 1992.

[24] J. Saito, J.-C. Ryou, and K. Sakurai. Enhancing privacy of universal re-encryption scheme for RFID tags. In L. Jang, M. Guo, G. Gao, and N. Jha, editors, *Embedded and Ubiquitous Computing – EUC 2004*, volume 3207 of *Lecture Notes in Computer Science*, pages 879–890. Springer-Verlag, 2004.

[25] D. Wagner. Private communication, December 2005.

[26] S. Weis, S. Sarma, R. Rivest, and D. Engels. Security and privacy aspects of low-cost radio frequency identification systems. In D. Hutter, G. Müller, W. Stephan, and M. Ullmann, editors, *International Conference on Security in Pervasive Computing – SPC 2003*, volume 2802 of *Lecture Notes in Computer Science*, pages 454–469. Springer-Verlag, 2003.

A An example of practical settings

A.1 Construction of f

Proposition 3 ([23]) *Let $f : \mathbb{F}_2^k \to \mathbb{F}_2^k$, then $\Delta_f \geq 2$. In case of equality, f is said to be* Almost Perfect Nonlinear (APN).

Proposition 4 ([11]) *Let $s = 2^j + 1$, known as a Gold exponent, with $\gcd(k, j) = 1$. If k is odd, the power function F defined as $F : x \mapsto x^s$ over $\mathbb{F}_{2^k}$ is a permutation and APN.*

Let $(\alpha_1, \ldots, \alpha_k)$ be a basis of $\mathbb{F}_{2^k}$ over $\mathbb{F}_2$, and $\varphi : (x_i)_{i=1..k} \in \mathbb{F}_2^k \mapsto \sum_i x_i \alpha_i$ the associated isomorphism. Let $s = 2^j + 1$ be a Gold exponent, F the corresponding power function over $\mathbb{F}_{2^k}$ and $f = \varphi^{-1} \circ F \circ \varphi : \mathbb{F}_2^k \to \mathbb{F}_2^k$. Hence f is a permutation and APN, that is $\Delta_f = 2$. Moreover, it is easy to see that f is a quadratic function. But, even if it is quadratic, f has a large complexity in terms of elementary operations, for a large k.

A way to reduce the complexity is to use a composition of functions defined over subspaces of $\mathbb{F}_2^k$. In particular, in the following case, the value Δ_f is easy to compute.

Proposition 5 *Let $k = k_1 + k_2$, $f : \mathbb{F}_2^k \to \mathbb{F}_2^k$ defined for $a = (a_1, a_2) \in \mathbb{F}_2^{k_1} \times \mathbb{F}_2^{k_2}$ by $f(a) = (f_1(a_1), f_2(a_2))$ with $f_i : \mathbb{F}_2^{k_i} \to \mathbb{F}_2^{k_i}$. We have*

$$\Delta_f = \max(\Delta_{f_1}\Delta_{f_2}, \Delta_{f_1} 2^{k_2}, \Delta_{f_2} 2^{k_1}).$$

For example, we can use this construction with a "good" function $g : \mathbb{F}_2^{k_1} \to \mathbb{F}_2^{k_1}$ with low complexity (e.g. a Gold power function over a small field) and define $f : \mathbb{F}_2^k \to \mathbb{F}_2^k$ as $f(a_1, \ldots, a_j) = (g(a_1), \ldots, g(a_j))$ where $k = jk_1$. For well-chosen parameters, this function f satisfies all the conditions to design the HB^{++} protocol: f is a permutation, has a low complexity and $\Delta_f = \Delta_g \times 2^{(j-1)k_1}$ is small compared to 2^k.

Let $k = 80$, the best known algorithm to solve the relying LPN problem has a computational runtime and needs a number of challenges greater than 2^{35} [19] (with $\eta = 1/4$).

Let $k_1 = 5$, $j = 16$ and $(\alpha_1, \ldots, \alpha_{k_1})$ be a basis of $\mathbb{F}_{2^{k_1}}$ over $\mathbb{F}_2$, and $\varphi : \mathbb{F}_2^{k_1} \to \mathbb{F}_{2^{k_1}}, (x_i)_{i=1..k_1} \mapsto \sum_i x_i \alpha_i$ the associated isomorphism.

We construct $f : \mathbb{F}_2^k \to \mathbb{F}_2^k$ thanks to the power function

$$\begin{array}{rccc} g: & \mathbb{F}_{2^{k_1}} & \to & \mathbb{F}_{2^{k_1}} \\ & x & \mapsto & x^3 \end{array}$$

by

$$f(a_1, \ldots, a_j) = (\tilde{g}(a_1), \ldots, \tilde{g}(a_j)),$$

for $a = (a_1, \ldots, a_j) \in (\mathbb{F}_2^{k_1})^j$ and $\tilde{g}(x) = \varphi^{-1} \circ g \circ \varphi(x)$.

As explained above, g is a permutation and $\Delta_g = 2$ ($s = 3$ is a Gold exponent, so g is an APN function). Hence, f is a permutation and

$$\Delta_f = 2^{(j-1)k_1+1} = 2^{k-4}.$$

Thus, the probability for an attack, like the one described in [10], to succeed is lower than $(2^{-4})^r$. For $r \geq 20$, the probability of success is smaller than 2^{-80}.

One remaining constraint has to be checked: f must have a low complexity.

We set the representation of the field $\mathbb{F}_{2^{k_1}}$ as $\mathbb{F}_{2^{k_1}} = \mathbb{F}_2[X]/(P)$ where $P = X^5 + X^2 + 1$ is an irreducible polynomial over $\mathbb{F}_2$. For α a root of P in $\mathbb{F}_{2^{k_1}}$, let $(\alpha_1, \ldots, \alpha_{k_1}) = (1, \alpha, \alpha^2, \alpha^3, \alpha^4)$ be the canonical basis of $\mathbb{F}_{2^{k_1}}$. For this basis, a description of $\tilde{g} : \mathbb{F}_2^5 \to \mathbb{F}_2^5$ is given below.

$$\begin{aligned}
\tilde{g} : (x_0, x_1, x_2, x_3, x_4) \longmapsto (&x_0 \oplus x_1x_3 \oplus x_1x_2 \oplus x_2x_3 \oplus x_0x_4,\\
&x_0x_1 \oplus x_2 \oplus x_0x_3 \oplus x_3 \oplus x_3x_4 \oplus x_4,\\
&x_0x_2 \oplus x_0x_1 \oplus x_1x_2 \oplus x_2x_4 \oplus x_0x_4 \oplus x_3x_4 \oplus x_4,\\
&x_1 \oplus x_2 \oplus x_2x_4 \oplus x_2x_3 \oplus x_3 \oplus x_0x_4 \oplus x_4,\\
&x_0x_4 \oplus x_1x_2 \oplus x_0x_2 \oplus x_2x_3 \oplus x_1x_3 \oplus x_3 \oplus x_3x_4 \oplus x_1x_4 \oplus x_2x_4)
\end{aligned}$$

The computation of $\tilde{g}$ requires the evaluation of 10 AND and 29 XOR.

Finally, suppose that each authentication requires 40 rounds. At each round, we rotate $f(a)$ and $f(b)$ by 2 bits. At the end of an authentication, with these rotations, the output of function f, $f(a)$ (resp. $f(b)$), is thus related to all 2-bit blocks of x' (resp. y') during the computation of the scalar product.

A.2 Construction of h

We follow [20], choose $h = \mathrm{WH}^{\mathrm{T}}$-16 and, from now, we adopt the notation of this article (here we let $w = 16$).

We have WH^{T}-16 : $\{0,1\}^{n \times w} \to \{0,1\}^{t \times w}$ and here choose $t = 20$ and $n = 10$.

The key, Z, has $n + 2(t-1)$ w-bit words, i.e. 768 bits. Let $Z = Z_1, \ldots, Z_{n+2(t-1)}$ where each Z_i is a w-bit word.

The output of WH$^{\mathrm{T}}$-16 is made of t w-bit words. And for each of these words, n words Z_i of the key are involved in the computation:

$$\begin{aligned} \text{WH}^{\mathrm{T}}\text{-16}(M) \quad = \quad & \big(\text{WH-16}(M; Z_1, \ldots, Z_n), \text{WH-16}(M; Z_3, \ldots, Z_{n+2}), \ldots, \\ & \text{WH-16}(M; Z_{2t-1}, \ldots, Z_{n+2t-2})\big), \end{aligned}$$

where the function WH-16 needs 460 gates to be implemented and consumes only 2.95 μW at 500 kHz.

The following result is proven in [20].

Theorem 1 *The function* WH$^{\mathrm{T}}$-16 *is universal on equal-length strings with collision probability of* 2^{-wt}.

To compute new secrets x, x', y and y', challenges exchanged at the beginning of each authentication are 80 bits long and are concatenated together to form the input of $h =$ WH$^{\mathrm{T}}$-16.