A Collision Attack on a Double-Block-Length Hash Proposal [*]

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Abstract. At FSE 2006 Shoichi Hirose proposed a construction for double-block-length hash functions [3]. This construction only requires a block cipher where the key length is greater than the block length. In this article we present a collision attack on the proposal with DESX as underlying block cipher.
Keywords: hash functions, double-block-length hash functions, cryptanalysis, collision

1 The Double-Block-Length Proposal

Shoichi Hirose proposed a double-block-length hash function defined as following [3]:

$$g_i = e_k(g_{i-1}) \oplus g_{i-1} \tag{1}$$
$$h_i = e_k(g_{i-1} \oplus c) \oplus g_{i-1} \oplus c, \tag{2}$$

where c is an arbitrary constant ($c \neq 0$), and e_k ($k = h_{i-1}||m_i$) any block cipher. The construction is shown in Figure 1.

In their paper they give two examples how AES-192 or AES-256 can be used as underlying block cipher e_k but they do not explicitly forbid to use another cipher. The only requirement for their scheme is that the key length has to be greater than the block length of the cipher.

2 The collision attack

Assume that DESX [4] is used as underlying block cipher with the following settings (see also Figure 2):

$$y = DES_k(x \oplus k_1) \oplus k_2, \tag{3}$$

where $k_1 = h_{i-1}$, k are the first 56 bits of the input message block and k_2 are the remaining 64 bits of the input message block, i.e. $m = k||k_2$ and $|m| = 120$, $|k| = 56$, and $|k_2| = 64$. For DESX we have $|g_i| = |h_i| = |c| = 64$.
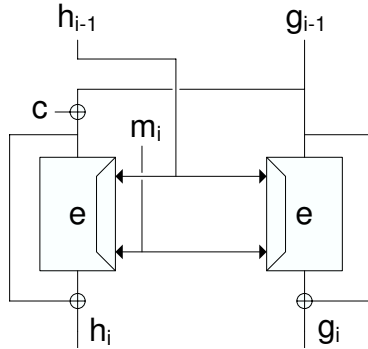
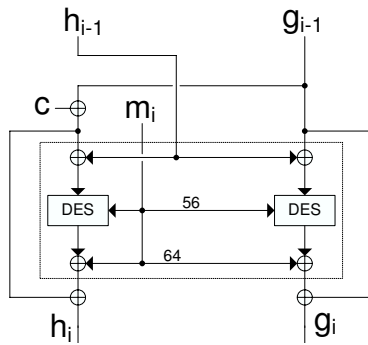**Fig. 1.** The proposed scheme for any block cipher e_k

Fig. 2. The proposed scheme with DESX

For this setting we can easily construct a two-block collision for the DBL hash proposal. We stick to the convention of [1] to denote a difference by $h' = h \oplus h^*$.

For the first iteration the input message block has an arbitrary difference in the right-most 64 bits (k_2):

$$m_1 = u||v \tag{4}$$
$$m_1^* = u||v^* \tag{5}$$
$$m_1' = 0||v', \tag{6}$$

where u, v are arbitrary 56-bit values and v' is an arbitrary difference. To construct the two-block collision we only have to choose the second message block with the same difference as for the first message block where the first 56 bits can be any value z:

$$m_2 = z||v \tag{7}$$
$$m_2^* = z||v^* \tag{8}$$
$$m_2' = 0||v' . \tag{9}$$

If we do so we have a collision after two iterations. After this iteration we can start with the same attack again. So the only restriction we have is that we need two message blocks that have the same difference in the right-most 64 bits. The DES keys in both iterations can be different.

The same attack can be applied if a block cipher following the Even-Mansour construction [2] is used as underlying block cipher.

3   3-Block Collisions

3.1   3-Block Collision—Configuration 1

Based on (3) we can use the following settings to produce 3-block collisions (see Figure 3): $k = h_{i-1}$, k_1 are the first 64 bits of the input message block and k_2 are the remaining 64 bits of the input message block or vice versa, i.e. $m = k_1||k_2$ or $m = k_2||k_1$, respectively. Since $|h_{i-1}| = 64$ and $|k| = 56$ we have to truncate h_{i-1} to 56 bits. Which bits are truncated does not have any impact on the attack.
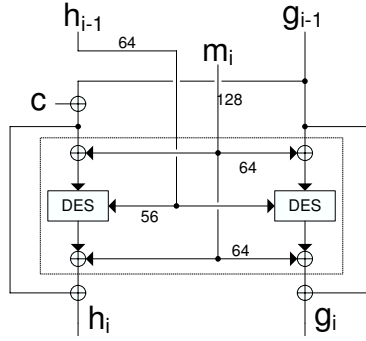

Fig. 3. The proposed scheme with DESX for a 3-block collision—configuration 1

Table 1 shows the differences for each iteration. The differences a' and u'

Table 1. Differences for a 3-block collision—configuration 1

iteration	h'_{i-1}	g'_{i-1}	k'_1	k'_2	h'_i	g'_i
i	0	0	0	a'	a'	a'
$i+1$	a'	a'	u'	v'	0	c'
$i+2$	0	c'	c'	c'	0	0

can be chosen arbitrarily whereas the difference v' has to be chosen such that $v' = a' \oplus z'$, where z' is the output difference of the left DES instance in iteration $i + 1$. It is clear that we cannot predict z' but once we have chosen the input

messages and the differences a' and u' we can easily compute it. The same holds for the difference c'. We cannot predict it but we can compute it after the messages and other differences have been chosen.

3.2   3-Block Collision—Configuration 2

We use the following settings (see Figure 4): $k_2 = h_{i-1}$, k_1 are the first 64 bits of the input message block and k are the remaining 56 bits of the input message block or vice versa, i.e. $m = k_1||k$ or $m = k||k_1$, respectively.
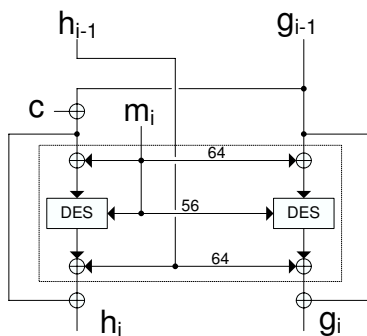

Fig. 4. The proposed scheme with DESX for a 3-block collision—configuration 2

Table 2 shows the differences for each iteration. The differences u', v' can be chosen arbitrarily. The differences a' and b' can be computed once we have chosen the input messages and the differences u', v' for the first iteration.

Table 2. Differences for a 3-block collision—configuration 2

iteration	h'_{i-1}	g'_{i-1}	k'	k'_1	h'_i	g'_i
i	0	0	u'	v'	a'	b'
$i+1$	a'	b'	0	b'	$a' \oplus b'$	$a' \oplus b'$
$i+2$	$a' \oplus b'$	$a' \oplus b'$	0	$a' \oplus b'$	0	0

4   Conclusion and Further Work

We have shown that for the proposal of Shoichi Hirose the underlying block cipher is important for the security against collision attacks. For DESX as underlying block cipher we can easily create collisions. This is work in progress.

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