

# A Survey of Certificateless Encryption Schemes and Security Models

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**Abstract.** In this paper we survey the different security models that have been proposed for certificateless encryption, comment on their practical significance, and propose a consistent, new nomenclature for these models. We also survey all known certificateless encryption schemes and point out that there are no known certificateless encryption schemes that achieve the highest levels of security without using the random oracle methodology. Lastly, we discuss the difficulties in proving the security of a certificateless scheme in the standard model, and propose possible ways of finding a solution to this problem.

## 1 Introduction

In 1978, Rivest, Shamir and Adleman [22] proposed the first public-key encryption scheme. This scheme was a concrete realisation of a seemingly paradoxical conjecture of Diffie and Hellman [15]: that it was possible for an entity (the sender) to securely send another entity (the receiver) a message without these two entities having a pre-existing shared secret key, without any online contact between them, and even without the receiver knowing that they were about to receive a message. This functionality is achieved by generating a pair of keys instead of just one: a public key that is widely distributed for encryption, and a related private key that is kept secret and used for decryption.

History, though, has shown that public key encryption has significant practical problems. In particular, the sender has to be sure that the public key that they have is the correct public key for the receiver. Hence, we require public key infrastructures — a series of trusted third parties that can be relied upon to check a receiver's identity and vouch for the connection between that identity and a particular public key. Public key management is the most costly, cumbersome and inefficient part of any framework that makes use of public key cryptography.

One way of avoiding the tiresome need for a public key infrastructure is to use an identity-based encryption scheme [24]. The best that a public key infrastructure can do is to confirm a link between a digital identity and a public key, where a digital identity is some bitstring that uniquely identifies the user in some context. An identity-based encryption scheme removes the need for a public key infrastructure by setting an entity's public key to be equal to their

digital identity. Of course, in such a situation, an entity cannot be expected to compute their own private key; hence, there must exist a trusted third party who initially sets up the system and uses their secret knowledge of the system to compute private keys for other entities.

Identity-based encryption seems to remove the need for a public key infrastructure, replacing it with the need for a key generation centre that computes a user's private key for them. This is more efficient, but has a significant disadvantage too. The fact that the trusted third party computes the private decryption keys for a particular entity means that that trusted third party can decrypt all messages sent to that entity. Thus, an honest-but-curious key generation centre can read the messages of every user in the system.

In 2003, Al-Riyami and Paterson proposed a new type of encryption scheme that avoids the drawbacks of both traditional public-key encryption and identity-based encryption [2]. They termed this new type of encryption *certificateless public-key encryption* (CL-PKE) because their encryption scheme did not require a public key infrastructure. Roughly speaking, their idea was to combine the functionality of a public key scheme with that of an identity based scheme. Hence, to encrypt a message, a sender requires both the receiver's identity and a public key value produced by the receiver. Similarly, to decrypt a ciphertext, a receiver requires the partial private key corresponding to their identity (which is given to them by a key generation centre) and the private key corresponding to the public key they distributed.

This paper surveys and extends the known results about certificateless encryption schemes. In particular, we will propose a new nomenclature that can be used to clarify the contradictory definitions that are currently prominent in the area, and we will examine some of the conjectured facts about the difficulty of proving the security of certificateless encryption schemes. The paper is arranged as follows:

- Section Two contains a discussion of the various types of security models proposed for use with certificateless encryption. A new nomenclature for these models is proposed.
- Section Three surveys known CL-PKE constructions and examines their security in light of the simplified security notions given in Section Two.
- Section Four discuss the difficulties that can arise when attempting to prove the security of a CL-PKE scheme in the standard model and discusses the various techniques that might be used to provide such a proof.
- Section Five concludes the paper.

## 2 Security Models For CL-PKE

This section will examine the various security models proposed for a certificateless public-key encryption scheme. This has been an area of some controversy. As we shall see, the original security definitions proposed by Al-Riyami and Paterson [1, 2] are very strong, and have been criticised for not realistically reflecting an attacker's capabilities. Furthermore, we suggest that Al-Riyami and Paterson's

security definitions are not consistent in their strength, i.e. that a certificateless scheme is held to a higher standard of security with regard to Type I attackers, than the standard to which it is held with regard to Type II attackers. We also survey the most common relaxations Al-Riyami and Paterson’s security notions and propose a new nomenclature.

## 2.1 CL-PKE Schemes

The definition of a certificateless public-key encryption scheme was introduced by Al-Riyami and Paterson [1, 2]. A certificateless public-key encryption scheme is defined by seven probabilistic, polynomial-time algorithms:

- **Setup**: This algorithm takes as input a security parameter  $1^k$  and returns the master private key  $msk$  and the master public key  $mpk$ . This algorithm is run by a KGC in order to initially set up a certificateless system.
- **Extract-Partial-Private-Key**: This algorithm takes as input the master public key  $mpk$ , the master private key  $msk$ , and identifier  $ID \in \{0, 1\}^*$ . It outputs a partial private key  $d_{ID}$ . This algorithm is run by a KGC once for each user, and the corresponding partial private key is distributed to that user in a suitably secure manner.
- **Set-Secret-Value**: This algorithm takes as input the master public key  $mpk$  and an entity’s identifier  $ID$  as input, and outputs a secret value  $x_{ID} \in \mathcal{S}$  for that identity. This algorithm is run once by the user. Note that the secret value space  $\mathcal{S}$  is somehow defined by the master public key  $mpk$  and an entity’s identity  $ID$ .
- **Set-Private-Key**: This algorithm takes as input the master public key  $mpk$ , an entity’s partial private key  $d_{ID}$  and an entity’s secret value  $x_{ID} \in \mathcal{S}$ . It outputs the full private key  $sk_{ID}$  for that user. This algorithm is run once by the user.
- **Set-Public-Key**: This algorithm takes as input the master public key  $mpk$  and an entity’s secret value  $x_{ID} \in \mathcal{S}$ . It outputs a public key  $pk_{ID} \in \mathcal{PK}$  for that user. This algorithm is run once by the user and the resulting public key is widely and freely distributed. The public-key space  $\mathcal{PK}$  for a particular user is defined by the master public key  $mpk$  and the user’s identity  $ID$ .
- **Encrypt**: This algorithm takes as input the master public key  $mpk$ , a user’s identity  $ID$ , a user’s public key  $pk_{ID} \in \mathcal{PK}$  and a message  $m \in \mathcal{M}$ . It outputs either a ciphertext  $C \in \mathcal{C}$  or the error symbol  $\perp$ . Note that the message space  $\mathcal{M}$  and the ciphertext space  $\mathcal{C}$  are somehow defined by a combination of the master public key  $mpk$ , the user’s public key  $pk_{ID}$  and the user’s identity  $ID$ .
- **Decrypt**: This algorithm takes as input the master public key  $mpk$ , a user’s private key  $sk_{ID}$  and a ciphertext  $C \in \mathcal{C}$ . It returns either a message  $m \in \mathcal{M}$  or the error symbol  $\perp$ .

A certificateless public-key encryption scheme allows anybody to encrypt a message for a particular receiver using publicly available information (in exactly

the same way as a traditional public-key encryption scheme or an identity-based encryption scheme). However, unlike a traditional public-key encryption scheme, no certificates are needed. This is because an attacker who publishes a false public key  $pk_{ID}$  for an identity will still not be able to decrypt messages encrypted under that public key, because the key generation centre will not release the partial public key  $d_{ID}$  to the attacker and so the attacker will not be able to compute the full private key.

This functionality is also given by identity-based cryptography, but identity-based encryption schemes have the disadvantage that the key generation centre can always decrypt messages. A certificateless public-key encryption scheme does not have this disadvantage. An honest-but-curious KGC can always compute an identity's partial private key  $d_{ID}$ , but since they will not know the secret value  $x_{ID}$  associated with that entity's public key  $pk_{ID}$ , they will not be able to form the full private key either. Of course, a malicious KGC that masquerades as a user by publishing a false public key can still break the security of the system; however, it is unclear how to prevent this threat and we will not consider it any further.

### An equivalent method for constructing public keys

We may replace the **Set-Secret-Value** and **Set-Public-Key** algorithms with a single **Set-Public-Key** algorithm that works as follows:

- **Set-User-Keys**: This algorithm takes as input the master public key  $mpk$  and an entity's identifier  $ID$  as input, and outputs a secret value  $x_{ID}$  and a public key  $pk_{ID}$  for that identity. This algorithm is run once by the user.

This is functionally equivalent formulation to the formulation given by Al-Riyami and Paterson. It is easy to see that a new **Set-User-Keys** algorithm can be obtained by running the **Set-Secret-Value** algorithm and then the **Set-Public-Key** algorithm of a scheme presented in the original way.

A little bit of thought is required to show that a certificateless scheme presented in this new formulation can also be presented in the old formulation. Suppose that the new **Set-User-Keys** algorithm takes as input  $p(k)$  random bits. We define the **Set-Secret-Value** algorithm to be the algorithm that outputs a set of  $p(k)$  random bits,  $x'_{ID}$ . The old **Set-Public-Key** algorithm takes these random bits  $x'_{ID}$ , runs **Set-User-Keys** to determine a public/private key pair, and outputs the public key. Similarly, the **Set-Private-Key** algorithm runs **Set-User-Keys** using the random bits  $x'_{ID}$  to determine the 'proper' secret value  $x_{ID}$  and then uses this to create a private key in the normal way.

### The Baek, Safavi-Naini and Susilo formulation

The only significant departure from the Al-Riyami and Paterson definition of a certificateless encryption scheme was proposed by Baek, Safavi-Naini and Susilo [4]. In their model, a public key can only be computed after a partial pri-

vate key has been obtained<sup>1</sup>. In other words, they make a slight change to the **Set-Public-Key** algorithm:

- **Set-Public-Key**: This algorithm takes as input the master public key  $mpk$ , an entity’s partial private key  $d_{ID}$  and secret value  $x_{ID}$ . It outputs a public key  $pk_{ID}$  for that user. This algorithm is run once by the user and the resulting public key is widely distributed.

This slight change (along with the corresponding changes to the security models) allow Baek, Safavi-Naini and Susilo to prove the security of a certificateless encryption scheme based on the CDH problem alone (i.e. without requiring elliptic curve pairings for either the scheme or the security proof). This is the only known scheme that does not require a pairing. The only slight drawback of this formulation is that it does not allow messages to be encrypted “into the future”. Under the Al-Riyami and Paterson formulation, an entity may publish a public key  $pk_{ID}$  without necessarily knowing the partial private key, and therefore they may receive messages that they cannot decrypt the KGC releases the partial private key to them. However, the Baek, Safavi-Naini and Susilo formulation requires that the entity has obtained the partial private key before they can release their full public key; hence, an entity that releases a public key must necessarily have enough information to compute the full private key.

Since the Al-Riyami and Paterson formulation is more prevalent, we will continue to use it in this paper; however, we expect all of our results to apply to the Baek, Safavi-Naini and Susilo formulation too.

## 2.2 The General Security Model

The security of a certificateless encryption scheme is expressed by two (very similar) games. In this section we will describe a basic framework. In both cases, an attacker  $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$  is trying to break the IND-CCA2 security, the formal model describing confidentiality. The game runs as follows:

1. The challenger generates a master key pair  $(mpk, msk) = \text{Setup}(1^k)$ .
2. The attacker executes  $\mathcal{A}_1$  on  $mpk$  and (possibly) some extra information  $aux$ . During its execution  $\mathcal{A}_1$  may have access to certain oracles (described subsequently).  $\mathcal{A}_1$  terminates by outputting an identity  $ID^*$ , two messages of equal length  $(m_0, m_1)$ , and some state information  $state$ .
3. The challenger randomly chooses a bit  $b \in \{0, 1\}$  and computes the challenge ciphertext  $C^* = \text{Encrypt}(mpk, ID^*, pk_{ID^*}, m_b)$  using the value of  $pk_{ID^*}$  currently associated with the identity  $ID^*$ . If the public key  $pk_{ID^*}$  does not exist, then the challenger computes a public key  $pk_{ID^*}$  for  $ID^*$  by running the **Set-Secret-Value** and **Set-Public-Key** algorithms.

<sup>1</sup> Technically, the Baek *et al.* model proposed that the KGC return a partial public key (used to help compute the full public key) and a partial private key (used to help compute the full private key). However, the security model they provided is equivalent to the one given here.

4. The attacker executes  $\mathcal{A}_2$  on the input  $(C^*, state)$ . During its execution  $\mathcal{A}_2$  may have access to certain oracles (described subsequently).  $\mathcal{A}_2$  terminates by outputting a guess  $b'$  for  $b$ .

The attacker wins the game if  $b = b'$  and its advantage is defined to be:

$$|\Pr[b = b'] - 1/2| \quad (1)$$

It now remains to define the oracles that the attacker may have access to:

- **Request Public Key:** The attacker supplies an identity  $ID$  and the challenger responds with the public key  $pk_{ID}$  for  $ID$ . If the identity  $ID$  has no associated public key, then the challenger generates a public key for  $ID$  by running **Set-Public-Key** and **Set-Secret-Value** (if necessary).
- **Replace Public Key:** This oracle models the attacker's ability to convince a legitimate user to use an invalid public key. This can happen because public keys are no longer verified by a trusted third party, and a user may be given a false public key by an attacker and believe it to be correct. The attacker supplies an identity  $ID$  and a valid public key value  $pk_{ID} \in \mathcal{PK}$ , and the challenger replaces the current public key value (if it exists) with the value  $pk_{ID}$ .
- **Extract Partial Private Key:** The attacker supplies an identity  $ID$  and the challenger responds with the partial private key  $d_{ID}$ . If the identity has no partial private key, then the challenger generates a partial private key by running **Extract-Partial-Private-Key** on  $ID$  using  $msk$ .
- **Extract Private Key:** The attacker supplies an identity  $ID$  and the challenger responds with the private key  $sk_{ID}$ . If the identity has no associated private key, then the challenger generates a private key using **Set-Private-Key** (after running **Set-Secret-Value** and **Extract-Partial-Private-Key** if necessary).

An attacker may also have access to one or more different types of decryption oracle:

- **Strong Decrypt:** The attacker supplies an identity  $ID$  and a ciphertext  $C$ , and the challenger responds with the decryption of  $C$  under the private key  $sk_{ID}$ . Note that if the attacker has replaced the public key for  $ID$ , then this oracle should return the correct decryption of  $C$  using the private key that is associated with the current public key  $pk_{ID}$  (or  $\perp$  if no such private key exists).
- **Weak Decrypt:** The attacker supplies an identity  $ID$ , a secret value  $x_{ID} \in \mathcal{S}$ , and a ciphertext  $C$ . The challenger computes that full private key  $sk_{ID}$  for the identity from the (correct) partial private key  $d_{ID}$  and the supplied secret value  $x_{ID}$ ; then returns the decryption of  $C$  under this private key  $sk_{ID}$ . If either process fails, then the oracle returns  $\perp$ . Note that this functionality can be achieved by a strong decryption oracle.
- **Decrypt:** The attacker supplies an identity  $ID$  and a ciphertext  $C$ , and the challenger responds with the decryption of  $C$  under the original private

key  $sk_{ID}$  for  $ID$ . Note that this functionality can be achieved by a strong decryption oracle.

A certificateless scheme is proven secure by showing that any attacker attempting to break the scheme only has a negligible chance of success.

**Definition 1 (Negligible Function).** *A function  $f : \mathbb{N} \rightarrow \mathbb{R}$  is said to be negligible if, for all polynomial  $p$ , there exists an integer  $N(p)$  such that  $|f(x)| \leq 1/p(x)$  for all  $x \geq N(p)$ .*

### 2.3 Type I Attackers

The Type I security model is designed to protect against a third party attacker (i.e. anyone except the legitimate receiver or the KGC) who is trying to gain some information about a message from its encryption. There has been some debate about how to precisely formulate this notion and we survey the main attempts in this section. We also comment on their correctness, and provide a new and consistent nomenclature for the different notions.

#### Strong Type I Security

The original definition proposed by Al-Riyami and Paterson is as follows.

**Definition 2.** *A certificateless encryption scheme is Strong Type I secure if every probabilistic, polynomial-time attacker  $\mathcal{A}^I = (\mathcal{A}_1^I, \mathcal{A}_2^I)$  has negligible advantage in winning the IND-CCA2 game subject to the following oracle constraints:*

- $\mathcal{A}^I$  cannot extract the private key for the challenge identity  $ID^*$  at any time,
- $\mathcal{A}^I$  cannot extract the private key of any identity for which it has replaced the public key,
- $\mathcal{A}^I$  cannot extract the partial private key of  $ID^*$  if  $\mathcal{A}^I$  replaced the public key  $pk_{ID^*}$  before the challenge was issued,
- $\mathcal{A}_2^I$  cannot query the strong decrypt oracle on the challenge ciphertext  $C^*$  for the identity  $ID^*$  unless the public key  $pk_{ID^*}$  used to create the challenge ciphertext has been replaced,
- $\mathcal{A}^I$  cannot query the weak decrypt or decrypt oracles (although this functionality can be given by the strong decrypt oracle).

*In this model, the attacker is given no extra information, i.e. aux is the empty bit-string.*

This model gives as much power as possible to the attacker. Its only restrictions are those necessary to prevent attacks that would trivially allow the attacker to win, and to prevent the attacker from asking for the private key of a user with a replaced public key. This latter restriction was only made because it was considered too difficult to achieve a notion of security that doesn't have this restriction.

It should be noted that the model expects the challenger to be able to correctly respond to decryption queries made on identities for which the attacker

has replaced the public key. This is a very strong notion of security and it is unclear whether it represents a realistic attack scenario. In general, decryption oracles are provided to an attacker to model the fact that the attacker may be able to gain some information from a legitimate receiver about the decryptions of some ciphertexts (for example, by bribing the legitimate receiver to give up the message or by deducing whether a ciphertext is a valid encryption of a particular message by observing the legitimate receiver's behaviour after receiving the ciphertext). This situation cannot happen if we replace a public key: when we replace a public key, we are duping a *sender* into encrypting a message using a false public key that the receiver has not published. Under no circumstances will the receiver then attempt to decrypt that ciphertext using the private key corresponding to that replaced public key. Hence, providing a decryption oracle that will accurately decrypt ciphertexts encrypted under the replaced public key gives the attacker more power than it would have in practice.

This represents an interesting philosophical question in the construction of security models: do we give the attacker as much power as is possible (perhaps subject to the restriction that we must still be able to construct secure certificateless encryption schemes)? Or should the model only try to reflect a realistic attacker's abilities? The former approach leads to strong security models, and potentially more complex schemes. The latter approach may lead to more efficient schemes, but a scheme's security can only be guaranteed if an attacker's abilities have been correctly modelled.

### Weak Type Ia Security

Several authors have judged Al-Riyami and Paterson's Type I security model to be too strong and proposed weaker versions. We will consider each of the major alternatives in turn. The strongest of these definitions, which we will term Weak Type Ia security, has been put forward by Bentahar *et al.* [9].

**Definition 3.** *A certificateless encryption scheme is Weak Type Ia secure if every probabilistic, polynomial-time attacker  $\mathcal{A}^I$  has negligible advantage in winning the IND-CCA2 game subject to the following oracle constraints:*

- $\mathcal{A}^I$  cannot extract the private key for the challenge identity  $ID^*$  at any time,
- $\mathcal{A}^I$  cannot extract the private key of any identity for which it has replaced the public key,
- $\mathcal{A}^I$  cannot extract the partial private key of  $ID^*$  if  $\mathcal{A}^I$  replaced the public key  $pk_{ID^*}$  before the challenge was issued,
- $\mathcal{A}^I$  cannot query the strong decrypt oracle,
- $\mathcal{A}_2^I$  cannot query the decrypt oracle on the challenge ciphertext  $C^*$  for the identity  $ID^*$ .

*In this model, the attacker is given no extra information, i.e.  $aux$  is the empty bit-string.*

It should be noted that the original notion of Weak Type Ia security [9] did not give the attacker the ability to request decryptions using the original



private key value *after* the public key had been replaced. We make this small change to make the model more realistic. It does not affect the results presented by Bentahar *et al.*

The Weak Type Ia model seems to most realistically reflect the potential abilities of an attacker. The attacker can replace public keys with arbitrary values of its choice, thus allowing for senders to be duped, but the attacker can still ask a legitimate receiver to decrypt any ciphertext with his original private key value (using the decrypt oracle). Furthermore, the attacker may be able to dupe a legitimate receiver into changing his public key and secret value to that of the attacker’s choice (using a combination of the replace public key oracle and the weak decrypt oracle), and so obtain decryptions using private keys formed from arbitrary secret values.

### Weak Type Ib Security

A weakening of this model gives Weak Type Ib security [28]:

**Definition 4.** *A certificateless encryption scheme is Weak Type Ib secure if every probabilistic, polynomial-time attacker  $\mathcal{A}^I$  has negligible advantage in winning the IND-CCA2 game subject to the following oracle constraints:*

- $\mathcal{A}^I$  cannot extract the private key for the challenge identity  $ID^*$  at any time,
- $\mathcal{A}^I$  cannot extract the private key of any identity for which it has replaced the public key,
- $\mathcal{A}^I$  cannot extract the partial private key of  $ID^*$  if  $\mathcal{A}^I$  replaced the public key  $pk_{ID^*}$  before the challenge was issued,
- $\mathcal{A}^I$  cannot query the strong decrypt or weak decrypt oracles,
- $\mathcal{A}_2^I$  cannot query the decrypt oracle on the challenge ciphertext  $C^*$  and the identity  $ID^*$ .

*In this model, the attacker is given no extra information, i.e.  $aux$  is the empty bit-string.*

In this model, the attacker can replace public keys (i.e. dupe senders) and can ask for decryptions of ciphertexts using the original private key values (using the decrypt oracle), but cannot dupe a recipient into publishing a new public key value chosen by the attacker. This reflects security in a situation where users generate their public key values correctly (i.e. by using the **Set-Secret-Value** and **Set-Public-Key** algorithms) and never change their public key values once they are set.

### Weak Type Ic Security

Lastly, mostly for comparison with Type II attackers, we present a final weak notion of security. This model of security was briefly considered in an early version of a paper by Baek and Wang [5]. This notion of security can be achieved by a public-key encryption scheme alone.

**Table 1.** A Summary of the Oracle Access Provided to a Type I Attacker

	Request Public Key	Replace Public Key	Extract Partial Private Key	Strong Decrypt	Weak Decrypt	Decrypt
Strong Type I	✓	✓	✓	✓		
Weak Type Ia	✓	✓	✓		✓	✓
Weak Type Ib	✓	✓	✓			✓
Weak Type Ic	✓		✓			✓

**Definition 5.** A certificateless encryption scheme is Weak Type Ic secure if every probabilistic, polynomial-time attacker  $\mathcal{A}^I$  has negligible advantage in winning the IND-CCA2 game subject to the following oracle constraints:

- $\mathcal{A}^I$  cannot replace any public keys at any time,
- $\mathcal{A}^I$  cannot extract the private key for the challenge identity  $ID^*$  at any time,
- $\mathcal{A}^I$  cannot query the strong decrypt or weak decrypt oracles,
- $\mathcal{A}_2^I$  cannot decrypt the challenge ciphertext  $C^*$  for the identity  $ID^*$ .

In this model, the attacker is given no extra information, i.e.  $aux$  is the empty bit-string.

The different types of oracle access that these models give to an attacker is summarised in Table 1. It is simple to deduce the following relationships between the different notions of Type I security:

$$\text{Strong Type I} \Rightarrow \text{Weak Type Ia} \Rightarrow \text{Weak Type Ib} \Rightarrow \text{Weak Type Ic}$$

where  $A \Rightarrow B$  if any scheme that is  $A$  secure must necessarily be  $B$  secure.

### The partial private key of the challenge identity

There are some further variations on these security models. Several schemes are proven secure in a weakened model in which a Type I attacker is not allowed to query the partial private key extraction oracle on the challenge identity  $ID^*$ . We denote these models with an asterisk; for example, the Weak Type Ib\* model is exactly the same as the Weak Type Ib security model except that the attacker is not allowed to query the partial private key extraction oracle on the challenge identity.

This security model also has a natural interpretation. It assumes that the attacker is unable to get hold of an identity’s partial private keys except in specialised cases (for example, for the attacker’s own identity or where an entity has been completely corrupted). This can be achieved by a system in which the KGC delivers the partial private key through some confidential channel. This is not an unreasonable assumption, given that the security models assume that an entity receives its partial private key through an integrity protected channel. Of course, this means that the KGC must be especially vigilant in verifying the identities of users before issuing partial private keys.

### Cheng and Comley Oracles

Cheng and Comley [10] have also proposed a variation on the above security models. In the Cheng and Comley variation, the attacker is not given access to an Extract Private Key oracle, but is instead given access to a Extract Secret Value oracle:

- **Extract Secret Value:** The attacker supplies an identity  $ID$  and the challenger responds with the secret value  $x_{ID}$  associated with that entity. If the identity has no associated secret value, then the challenger generates one by running `Set-Secret-Value`.

The attacker may not query both the Extract Partial Private Key oracle and the Extract Secret Value oracle on the challenge identity. Furthermore, the attacker may not query the Extract Secret Value oracle on any identity for which it has replaced the public key. We denote a security model in which the attacker has access to an Extract Secret Value oracle using a dagger; for example, the Weak Type Ib<sup>†</sup> model is exactly the same as the Weak Type Ib security model except that the attacker has access to an Extract Secret Value oracle instead of an Extract Private Key oracle.

This change gives rise to slightly more powerful security models. The attacker can simulate an Extract Private Key oracle by making queries to both the Extract Partial Private Key and Extract Secret Value oracles, and then assembling the full private key itself. However, the attacker now has the ability to find out the secret value associated with a public key (and, in particular, the public key associated with the challenge identity). This is not an ability that the attacker is guaranteed to have in the normal security models.

The main problem with Extract Secret Value oracles is that they do not appear to reflect reality. In a normal mode of use, a secret value will be used to generate a public/private key pair for an entity and then deleted. It does not seem likely that an attacker will be able to extract the secret value at any point. Hence, we do not believe security models involving Extract Secret Value oracles to be correct for certificateless encryption schemes used in a normal way. These oracles should be included in any model that attempts to model the situation where certificateless encryption is used to encrypt messages ‘into the future’. In such a situation, we have to protect against a curious receiver (who knows his own secret value) who wishes to read a message before being issued his partial private key.

## 2.4 Type II Attackers

### Weak Type II Security

The second security definition states that an honest-but-curious key generation centre should not be able to break the confidentiality of the scheme. Hence, we allow the attacker to have access to master private key by setting  $aux = msk$ . This means that we do not have to give the attacker explicit access to an extract

partial private key oracle, as they are able to compute this value for themselves. The most important point about Type II security is that the KGC can trivially break the scheme if it is allowed to replace the public key for the challenge identity *before* the challenge is issued. Al-Riyami and Paterson chose to prevent this from occurring by forbidding the KGC from replacing any public keys at all, proposing the following model:

**Definition 6.** *A certificateless encryption scheme is Weak Type II secure if every probabilistic, polynomial-time attacker  $\mathcal{A}^{II} = (\mathcal{A}_1^{II}, \mathcal{A}_2^{II})$ , which is given the auxiliary information  $aux = msk$ , has negligible advantage in winning the IND-CCA2 game subject to the following oracle constraints:*

- $\mathcal{A}^{II}$  cannot extract the private key for the challenge identity  $ID^*$  at any time,
- $\mathcal{A}^{II}$  cannot query the extract partial private key oracle at any time,
- $\mathcal{A}^{II}$  cannot replace public keys at any point in time,
- $\mathcal{A}^{II}$  cannot query the strong decrypt or weak decrypt oracles at any time,
- $\mathcal{A}_2^{II}$  cannot query the decrypt oracle on the challenge ciphertext  $C^*$  and the identity  $ID^*$ .

This roughly corresponds to the weakest notion of security proposed for Type I attackers, and it is easy to see that any scheme that is Weak Type II secure is necessarily Weak Type Ic secure. Furthermore, this notion of security can be achieved by a public key encryption scheme alone.

### Strong Type II Security

By denying the KGC the ability to replace public keys or query , we might be denying it the ability to perform certain attacks that might occur in practice, and we are certainly not providing it with the huge level of power provided to a Strong Type I attacker. Hence, we should consider whether the KGC gains any advantages if we allow it to replace public keys (subject to the restriction that it cannot replace the public key of the challenge identity until after the challenge has been issued) or allow it access to more powerful decryption oracles.

Clearly, if we do not give the attacker access to a strong decryption oracle, then the ability to replace public keys is of no use to the attacker. This is because the challenger never gives a response based on a replaced public key value; hence, the attacker gains no advantage by replacing a public key. Similarly, the weak decryption oracle is of no use to an attacker because the attacker can always compute the full private key of a user given their identity  $ID$  and their secret value  $x_{ID}$ . Hence, all of the Weak Type II security models that we might propose (based on the Weak Type I security models) are equivalent.

However, if we follow the principle that we should give the attacker as much power as possible, then there is some merit in considering a Strong Type II security model. This gives an equivalent security level for the scheme against Type II attackers as is demanded for Type I attackers. It is unreasonable to require a scheme to meet the Strong Type I security level, without also requiring to meet the Strong Type II security level.

**Definition 7.** A certificateless encryption scheme is Strong Type II secure if every probabilistic, polynomial-time attacker  $\mathcal{A}^{II} = (\mathcal{A}_1^{II}, \mathcal{A}_2^{II})$ , which is given the auxiliary information  $aux = msk$ , has negligible advantage in winning the IND-CCA2 game subject to the following oracle constraints:

- $\mathcal{A}^{II}$  cannot extract the private key for the challenge identity  $ID^*$  at any time,
- $\mathcal{A}^{II}$  cannot extract the private key of any identity for which it has replaced the public key,
- $\mathcal{A}^{II}$  cannot query the extract partial private key oracle at any time,
- $\mathcal{A}_1^{II}$  cannot output a challenge identity  $ID^*$  for which it has replaced the public key,
- $\mathcal{A}_2^{II}$  cannot query the strong decrypt oracle on the challenge ciphertext  $C^*$  for the identity  $ID^*$  unless the public key  $pk_{ID^*}$  used to create the challenge ciphertext has been replaced,
- $\mathcal{A}^{II}$  cannot query the weak decrypt or decrypt oracles (although this functionality can be given by the strong decrypt oracle).

The different types of oracle access that these models give to an attacker is summarised in Table 2.

**Table 2.** A Summary of the Oracle Access Provided to a Type II Attacker

	Request Public Key	Replace Public Key	Strong Decrypt	Decrypt
Strong Type II	✓	✓	✓	
Weak Type II	✓			✓

### 3 Surveying Certificateless Encryption Schemes

There have been several attempts at producing secure certificateless encryption schemes, using various models of security, and we will briefly discuss these constructions in this section. The constructions are summarised in Table 3. We differentiate between concrete constructions, which give a full description of a specific certificateless scheme, and generic constructions, which explain how to construct a certificateless scheme from other primitives.

**Al-Riyami 1/Yum-Lee 1** The Al-Riyami 1/Yum-Lee 1 construction [1, 28] is a generic construction of a certificateless encryption scheme from an IND-CCA2 secure public-key encryption scheme and an IND-CCA2 secure identity-based encryption scheme. It is a sequential construction, in which the full ciphertext is formed by first encrypting the message with the public-key encryption scheme, and then encrypting this ciphertext with the identity-based encryption scheme. Its security proof uses a weakened version of the Weak Type Ib model in which the attacker cannot query the partial key extraction oracle on the challenge identity  $ID^*$ . Libert and Quisquater [20] show that if we allow partial key extraction

**Table 3.** A Survey of Certificateless Encryption Schemes

Scheme	Reference	Style	Simplifying Model	Type I Model	Type II Model	Broken?
Al-Riyami 1 / Yum-Lee 1	[1, 28]	Generic		Weak Type Ib*	Weak Type II	[19, 20]
Al-Riyami 2	[1]	Generic		No proof given	No proof given	Sec. 3
Al-Riyami 3	[1]	Generic		No proof given	No proof given	[20]
Al-Riyami-Paterson 1	[1, 2]	Concrete	ROM	Strong Type I	Weak Type II	
Al-Riyami-Paterson 2	[1, 3]	Concrete	ROM	Strong Type I	Weak Type II	[20, 30]
Cheng-Comley	[10]	Concrete	ROM	Weak Type Ib*†	Weak Type II	
Baek-Safavi-Naini-Susilo	[4]	Concrete	ROM	Strong Type I*	Weak Type II	
Bentahar-Farshim-Malone-Lee-Smart	[9]	Generic	ROM	Weak Type Ia	Weak Type II	
Libert-Quisquater 1	[20]	Generic	ROM	Strong Type Ia	Weak Type II	
Libert-Quisquater 2	[20]	Generic	ROM	Strong Type I	Weak Type II	
Libert-Quisquater 3	[20]	Generic	ROM	Strong Type I	Weak Type II	
Libert-Quisquater 4	[20]	Concrete	ROM	Strong Type I	Weak Type II	
Shi-Li	[26]	Concrete	ROM	Strong Type I	Weak Type II	
Yum-Lee 2	[29]	Generic	ROM	Weak Type Ia*	Weak Type II	[19]

\* = The attacker is not allowed to query the partial private key extraction oracle on the challenge identity

† = The attacker has access to a secret value extraction oracle instead of a private key extraction oracle

oracle queries on the identity  $ID^*$ , then the scheme can be broken. Galindo, Morillo and Ràfols [19] demonstrated that similar techniques can be used to show that the scheme is not Weak Type II secure.

**Al-Riyami 2** The second Al-Riyami scheme is the “opposite” of the Al-Riyami 1/Yum–Lee 1 scheme, i.e. one composes an identity-based encryption scheme and a public-key encryption scheme by first applying the identity-based scheme and then applying the public-key encryption scheme. Al-Riyami does not offer a security proof for this scheme. Libert and Quisquater [20] note that this scheme is not Strong Type I secure; however, the scheme has greater security problems. The scheme is actually insecure in the Weak Type Ib model. An attacker can break the scheme by initially replacing the public key with one for which the corresponding private key is known. After receiving the challenge ciphertext, the attacker decrypts the public-key portion of the ciphertext using the (known) private key. This leaves the portion of the ciphertext encrypted using the identity-based encryption scheme. The attacker re-encrypts this ciphertext using the original public key value and submits this new ciphertext to a decryption oracle, which returns the challenge message.

**Al-Riyami 3** The third Al-Riyami construction uses an identity-based encryption scheme and a public-key encryption scheme in parallel. The identity-based encryption scheme and the public-key encryption scheme are used to encrypt shares of the message which (if both shares are recovered) can be used to reconstruct the message. Al-Riyami does not offer a security proof for this scheme, and the scheme has now been broken by Libert and Quisquater [20].

**Al-Riyami–Paterson 1–2** These schemes are concrete and require an elliptic curve pairing. The second scheme was independently broken by Libert and Quisquater [20], and Zhang and Feng [30].

**Baek–Safavi-Naini–Susilo** The Baek–Safavi-Naini–Susilo [4] construction is a concrete construction that uses the slightly altered Baek–Safavi-Naini–Susilo formulation of a CL-PKE scheme. It is proven secure in a slightly weakened version of the Strong Type I security model, in which attacker cannot query the partial key extraction oracle on the challenge identity. However, unlike the Yum–Lee 1 construction, it is not clear whether this scheme can be broken if an attacker is able to make partial key extraction queries on the challenge identity. Therefore, we consider its full security to be an open problem. It is the only known scheme which does not require an elliptic curve pairing to be implemented.

**Bentahar *et al.*** Bentahar *et al.* [9] have two major contributions to the theory of certificateless public-key encryption. The first is to propose a general model for a certificateless KEM–DEM encryption scheme, similar to that proposed for public-key encryption schemes by Cramer and Shoup [11]. This allows for greater flexibility and efficiency when constructing a CL-PKE. Unfortunately, the nature of their construction is such that only Weak Type Ia security can be achieved, although this can be achieved in the standard model. This should

be compared with the generic Fujisaki–Okamoto style construction given by Libert–Quisquater, which provides Strong Type I security, but only in the random oracle model. The second contribution of Bentahar *et al.* is to provide a generic construction for a CL-PKE KEM from a weakly secure public-key encryption scheme and a weakly secure identity-based encryption scheme. This generic construction uses a form of parallel composition of the public-key and identity-based encryption schemes, and is proven secure in the random oracle model.

**Cheng–Comley** Cheng–Comley [10] present a concrete CL-PKE construction based on the use of pairings. The scheme is proven secure in a variant of the Weak Type Ib security model. In particular, Cheng–Comley do not allow the partial public key oracle to be queried on the challenge identity. Furthermore, the attacker does not get access to an oracle which returns an entity’s full private key, but instead have access to an oracle which will return the secret value for a (non-replaced) public key. The Cheng–Comley scheme proves full security by applying the Fujisaki–Okamoto transform [18] to a weaker CL-PKE scheme. It is unclear whether this transformation is valid or not for certificateless encryption schemes (in the given model). Libert and Quisquater [20] have proposed a Fujisaki–Okamoto-style transform that converts a weakly secure CL-PKE into a strongly secure CL-PKE, under which the Cheng and Comley scheme appears secure.

**Libert–Quisquater 1–3** Libert–Quisquater [20] provide four CL-PKE constructions. The first three of these constructions are generic constructions for a CL-PKE from a public-key encryption scheme and an identity-based encryption scheme. Libert–Quisquater note that, even though the generic sequential and parallel composition constructions (Al-Riyami 1/Yum–Lee 1, Al-Riyami 2 and Al-Riyami 3) are insecure against active attackers, these schemes are secure against passive attackers. Libert–Quisquater obtain three strongly secure CL-PKE schemes by developing a certificateless analogue of the Fujisaki–Okamoto transform [17] and applying this transformation to the three weakly secure certificateless schemes.

**Libert–Quisquater 4** The final scheme that Libert–Quisquater [20] propose is a concrete construction based on the Sakai–Kasahara [23] identity-based encryption scheme and an ElGamal-style public-key encryption scheme [16]. As such, it is similar to the Shi–Li CL-PKE scheme, although the Libert–Quisquater scheme is more efficient.

**Shi–Li** The Shi–Li scheme [26] is adapted from an earlier scheme by the same authors [25] and based on the Sakai–Kasahara [23] identity-based encryption scheme and an ElGamal-style public-key encryption scheme [16]. As such it is similar to, although slightly less efficient than, the Libert–Quisquater 4 CL-PKE scheme. The current version of the paper also claims to achieve full security by applying the Fujisaki–Okamoto transform to a weaker scheme. Libert and Quisquater [20] have shown this to be insufficient to give full security; however,



by applying the improved Fujisaki–Okamoto–style transform proposed by Libert and Quisquater, full security can still be achieved.

**Yum–Lee 2** The second generic construction provided by Yum and Lee [29] is similar to the first in that it is the sequential composition of an encryption scheme controlled by the user and an encryption scheme controlled by the KGC. In the Al-Riyami 1/Yum-Lee 1 construction, the user’s encryption scheme is a public key encryption scheme and the KGC’s encryption scheme is an identity-based encryption scheme. In this construction, both encryption schemes are identity-based. The authors claim to prove the security of the scheme in a weakened version of the Weak Type Ia security model, in which the attacker may not query the partial key extraction oracle on the challenge identity, and in the Weak Type II security model. Galindo, Morillo and Ràfols [19] show that it does not achieve security in the Weak Type II model. Similar techniques can be used to show that it does not achieve Type I security in any model in which the attacker is allowed to query the partial key extraction oracle on the challenge identity.

## 4 On the Difficulty of Achieving Full Security in the Standard Model

Several authors [1–4, 20] have presented schemes that are secure in the Strong Type I and Weak Type II models by using the the random oracle methodology. Other authors have not been so successful in proving the full security of their schemes [9, 10, 26, 28, 29] and have chosen to use weaker security models. Indeed, it has been suggested by several members of the cryptographic community that full security proofs in the standard model cannot be achieved because “a Strong Type I decryption simulator is a Weak Type II attacker”. In this section, we will investigate this claim, showing that certain proof techniques may not be used to prove the full security of a certificateless public-key encryption scheme in the standard model. After this we discuss possible solutions to the problem of proving full security.

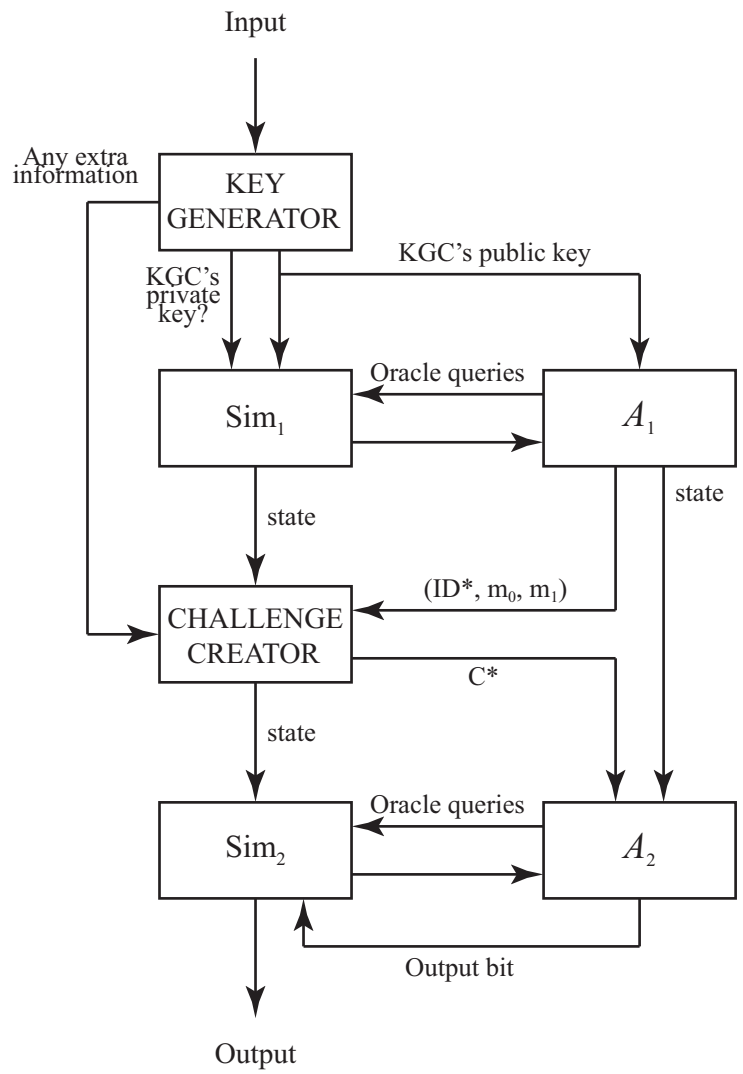
An observational or black-box security proof is one in which an algorithm, called a *solver*, uses the attacker as a subroutine in solving a mathematical problem. The attacker is run as an independent subroutine, with no interference from the programme that is running it, i.e. as a black box. The solver answers all of the attacker’s queries using a simulator, which must answer these queries well enough that the attacker cannot distinguish the simulator’s responses from authentic ones. The solver computes its output based on observation of the queries that the attacker makes. This is a common format for many security proofs.

Consider the following specific type of black-box proof for the security of a certificateless scheme against a Strong Type I attacker. It should be noted that several assumptions are made as to the structure of the solver in the following proof. We shall discuss the correctness of these assumptions and the relevance

of the result after the proof has been presented. We show the structure of the solver in Figure 1. The structure of the solver breaks down as follows:

1. The solver initially receives an instance of a problem that it needs to solve; this could be, for example, an instance of the CDH problem or a RSA problem. The solver executes a key generation algorithm which, based on the problem instance, generates a master public key  $mpk$  and possibly, depending on the nature of the solver, a master private key  $msk$ . It should be noted that the solver’s key generation algorithm is likely to be vastly different to the “proper” key generation algorithm  $\mathcal{G}$ ! However, it is important that this key pair be indistinguishable from a proper key pair produced by the  $\text{Setup}$  algorithm. The key generation algorithm may also produce some extra information, based on the problem instance, which will be used to compute the challenge ciphertext.
2. The solver executes the attacker  $\mathcal{A}_1^I$  on the input  $mpk$ . We note that the attacker is completely independent of the solver, i.e. no part of the solver has access to the program code of the attacker nor the attacker’s random coins except for the subroutines that executes  $\mathcal{A}_1^I$  and  $\mathcal{A}_2^I$ . Any oracle query made by the  $\mathcal{A}_1^I$  should be answered by a simulator  $Sim_1$ . This simulator takes the master public key  $mpk$  and, if it has been generated, the master private key  $msk$  as input. The simulator must answer oracle queries in such a way that no attacker can distinguish (with non-negligible advantage) between oracle access to the simulator and oracle access to the appropriate certificateless algorithms.
3. The attacker  $\mathcal{A}_1^I$  terminates by outputting an identity  $ID^*$ , two equal-length messages  $m_0$  and  $m_1$ , and some state information  $state(\mathcal{A}_1^I)$ . At this point the simulator  $Sim_1$  also terminates and outputs some state information  $state(Sim_1)$ .
4. A challenge ciphertext  $C^*$  is then created by a ciphertext creation algorithm. This algorithm takes as input the triple  $(ID^*, m_0, m_1)$  output by the attacker  $\mathcal{A}_1^I$ , the state information  $state(\mathcal{A}_1^I)$  output by the simulator  $Sim_1$  and any extra information provided by the key generation algorithm. It outputs both the challenge ciphertext  $C^*$  and some state information  $state$ .
5. The solver then executes  $\mathcal{A}_2^I$  on the inputs  $C^*$  and  $state(\mathcal{A}_1^I)$ . Again, this algorithm may make oracle queries. These queries are answered by a simulator  $Sim_2$  which takes the state information  $state$  provided by the ciphertext creator as input. As before, we require that the simulator answers oracle queries in such a way that no attacker can distinguish (with non-negligible advantage) between oracle access to the simulator and oracle access to the appropriate correct algorithms.
6. The attacker  $\mathcal{A}_2^I$  terminates by outputting a guess  $b'$  for the bit  $b$ . The simulator  $Sim_2$  takes this as a final input and outputs its guess for the solution to the problem instance.

**Theorem 1.** *If there exists a black-box proof in the above form for the security of a certificateless encryption scheme against Strong Type I attackers, then that certificateless encryption scheme is insecure against Weak Type II attackers.*



**Fig. 1.** The Structure of a Solver for a Black Box Proof of Security for a Strong Type I Attacker

*Proof* Suppose a black-box proof of this form exists for the security of a scheme against Strong Type I attackers, and consider a particular Strong Type I attacker  $\mathcal{A}^I = (\mathcal{A}_1^I, \mathcal{A}_2^I)$ , in which  $\mathcal{A}_1^I$  works as follows:

1. Receive the public key  $mpk$  from the challenger.
2. Choose an identity  $ID$ , then randomly generate a secret value  $x_{ID}$  for that identity using **Set-Secret-Value** and a public key  $pk_{ID}$  for that identity using **Set-Public-Key**.
3. Replace the public key of the identity  $ID$  with the value  $pk_{ID}$ .
4. Choose two distinct messages  $m_0$  and  $m_1$  from the message space of the encryption scheme.
5. Select a bit  $b \in \{0, 1\}$  uniformly at random.
6. Compute the encryption  $C$  of message  $m_b$  using the identity  $ID$  and the public keys  $pk_{ID}$  and  $mpk$ .
7. Submit  $C$  to the decryption oracle.
8. Receive the decryption of  $C$  from the decryption oracle.
9. Output  $(ID, m_0, m_1)$  and no state information.

The second part of the attacker,  $\mathcal{A}_2^I$ , is not important to our discussion. Since the simulator must answer oracle queries in a way that cannot be distinguished from the real oracle by any attacker, the simulator must respond to  $\mathcal{A}_1^I$ 's oracle query with the correct message  $m_b$  (as  $\mathcal{A}_1^I$  "knows" what the simulator should return) with overwhelming probability. We use this fact to construct a Type II attacker.

Consider a Weak Type II attacker  $\mathcal{A}^{II} = (\mathcal{A}_1^{II}, \mathcal{A}_2^{II})$ , in which  $\mathcal{A}_1^{II}$  runs as follows:

1. Receive the public/private key pair  $(mpk, msk)$  from the challenger.
2. Choose an identity  $ID$ .
3. Choose two distinct messages  $m_0$  and  $m_1$  from the message space of the encryption scheme.
4. Output the triple  $(ID, m_0, m_1)$  and the state information  $(mpk, msk, ID, m_0, m_1)$ .

After receiving this output, the challenger will then generate a random secret value  $x_{ID}$  for the identity  $ID$  using **Set-Secret-Value** and a public key  $pk_{ID}$  for that identity using **Set-Public-Key**. It will then choose a bit  $b \in \{0, 1\}$  uniformly at random and compute the encryption  $C^*$  of  $m_b$  the identity  $ID$  and the public keys  $pk_{ID}$  and  $mpk$ .  $\mathcal{A}_2^{II}$  runs as follows:

1. Receive the challenge ciphertext  $C^*$  and the state information  $(mpk, msk, ID, m_0, m_1)$  from the challenger.
2. Request the public key  $pk_{ID}$  for the identity  $ID$ .
3. Execute  $Sim_1$  (initialised using the master public key  $mpk$  and, if required, the master private key  $msk$ ) to decrypt the ciphertext  $C^*$  using the identity  $ID$  and the public key  $pk_{ID}$ . The simulator will return a message  $m$ .
4. If  $m = m_0$  output 0, otherwise output 1.

Since the key pair generated by the solver’s key generation algorithm is indistinguishable from a key pair produced by the `Setup` algorithm, the Type II attacker will undertake the same steps as the Type I attacker up to the point in which it has received the response from the simulator  $Sim_1$ . Hence, the simulator will output the correct message  $m = m_b$  with overwhelming probability. Thus, the Type II attacker will break the certificateless scheme with overwhelming probability.  $\square$

So we must ask the question: is it possible to prove that a certificateless scheme is secure in the standard model? Many people have chosen to interpret the relationship between Strong Type I simulators and Weak Type II attackers as meaning that it is impossible to construct a scheme that is provably secure against both Strong Type I and Strong Type II attackers. However, the existence of schemes [1–4, 20] which are provably secure against both Strong Type I and Weak Type II attackers in the random oracle model suggests that schemes can exist which are fully secure. We suggest that any attempt to prove the security of a certificateless scheme in the standard model must make use of one of the following avenues:

- **The Weak Type Ia model should be accepted.** A simple, although intellectually unsatisfying, solution would be to accept that the definition of a Type I attacker is too strong, and that the appropriate security definition should be the Weak Type Ia model. This would make the powers of the attackers in the Type I and Type II model consistent, and it doesn’t seem to deprive the attacker of any avenue of attack that they are likely to have in practice.
- **The keys produced by the solver’s key generation algorithm should not be indistinguishable from the Setup algorithms output.** In our demonstration that a Type I simulator can be adapted to give a Type II attacker we insisted that the key pair produced by the solver’s key generation algorithm must be indistinguishable from random. This allowed us to use the simulator  $Sim_1$  with the keys generated by the `Setup` algorithm, even though the algorithm is designed to use the keys generated by the solver’s key generation algorithm. However, in order for an observational proof to work, we only require that the Type I attacker be unable to distinguish the master public key from random. It might be possible to construct a security proof based on a solver whose key generation algorithm outputs public/private key pairs of a special form, but no attacker can distinguish public keys of this form from valid public keys output by the `Setup` algorithm.
- **The simulator  $Sim_1$  should take extra information as input.** If a Type I simulator is to become a Type II attacker, then all the inputs to the simulator must be available in the Type II model (or, at least, simulated in such a way that the attacker cannot distinguish between the real input produced by the Type I solver’s key generation algorithm and the input given to the simulator by the Type II attacker that is running it). However, if this input is unavailable, then the Type II attacker will not be able to execute  $Sim_1$  correctly. This could mean that a standard model proof could be constructed

based on an assumption that explicitly provides extra information, such as the DDH assumption or the q-BDHI assumption.

- **The attacker should gain no advantage when given incorrect responses.** Often security proofs revolve around showing that responses given to an algorithm are indistinguishable from the responses given by a legitimate oracle. In many cases, though, the queries that might help an attacker decide whether it is interacting with a legitimate oracle or a simulator do not help the attacker break the scheme. Heuristically we can often see that even if the attacker does “realise” that it is interacting with an incorrect oracle (i.e. alter its behaviour), their success probability is not significantly raised or lowered, and so they will still only be able to break the scheme if and only if they can solve the corresponding hard problem. However, no theoretical framework exists to examine the effects of improper oracle responses on an attacker’s behaviour.
- **Game hopping techniques should be used.** In a game hopping proof, an attacker  $\mathcal{A}$  is not used as a subroutine in a solver, but instead run as normal in the correct attack environment [8, 27]. This environment is slowly altered until it is shown that the attacker can have no significant advantage in winning. The proof shows that each time we change the environment, the attacker’s advantage only increases a small amount. Therefore, if the attacker has no significant advantage in the final environment, then it could have had no significant advantage in the first (normal attack) environment. There is no reason why a game-hopping proof of security for a certificateless scheme against Type I attackers should imply the existence of a successful Type II attacker.
- **The attacker’s black-box subroutine should be opened.** In every current security proof for an asymmetric encryption or encryption-like scheme, the attacker is always considered to be a black box with its own source of random coins. However, when we execute the attacker subroutine within a solver, there is no reason why we should treat it as a black box. There is no reason, beyond the complexity involved in developing proofs, to stop a solver from using an attacker in a non-black-box way. This could involve choosing the attacker’s coins in a way that is more advantageous; changing or resetting an attacker’s state during execution; or rewinding and re-running an attacker if it makes unsuitable queries or to force it to find multiple, related solutions to a problem (in a manner similar to the forking lemma for signatures [21]). Such an approach has already been discussed in the field of zero-knowledge proofs [6].
- **Assumptions based on extractors should be used.** Observational security proofs involve running a solver to solve a given instance of a (hard) problem. This assumes that the instance of the underlying problem can be presented as input to the algorithm. However, a small number of security proofs have been reduced to problems of a different nature, which we call *extractor problems*. An extractor problem asks whether it is possible, for every attacker  $\mathcal{A}$  that outputs problem instances, to find an extractor  $\mathcal{A}^*$  that can solve these problem instances *given knowledge of the way they were gener-*

*ated.* For example, the DHK assumption [7, 12–14]. This is a much stronger security assumption, but one that may allow the construction of security proofs in the Strong Type I and Strong Type II models.

- **Completely new proof techniques should be developed.** Currently, there are only two techniques that are commonly used for proving the security of any public key encryption algorithm: observational proofs and game-hopping proofs. This small number of techniques means that proofs are formulaic and the results often predictable. If the field of provable security is to move forward, then new proof techniques must be developed.

## 5 Conclusion

We draw several conclusions from this survey. First, we conclude that Al-Riyami and Paterson’s security models are inconsistent in their strength. A scheme proven secure against Al-Riyami and Paterson’s definitions is held to a higher standard against Type I attackers than Type II attackers. We provided the equivalent strong definition for Type II attackers.

We have also noted that the strong decryption oracles to which an attacker is given access in the Al-Riyami and Paterson model do not reflect reality. However, wherever possible it would be advantageous to prove a scheme’s security in these model in order to give a ‘margin of error’ for the models. In cases where a scheme cannot be proven secure in the Strong Type I and Strong Type II models, it should be proven secure in the Weak Type Ia and Weak Type II models. These models most closely reflect possible real-world attacks against certificateless encryption schemes.

The problems associated with constructing a certificateless encryption scheme that is provably secure in the Strong Type I and Weak Type II models without using the random oracle methodology were also considered. It was shown that the most obvious method for proving secure would not work, but that there are other methods that could conceivably be used to construct solutions. Of these, perhaps the most promising included the use of underlying assumptions that gave the Type I decryption simulator extra information (such as the DDH and  $q$ -BDHI problems) or the use of game-hopping proofs.

Lastly, it should be noted that certificateless encryption is not yet a viable cryptographic solution. There remain too many un-answered questions about how certificateless encryption would be used in practice. Almost all of these concern the infrastructure that needs to surround a certificateless encryption scheme. Most, notable, certificateless encryption schemes cannot be considered viable unless the problems of public-key distribution and key revocation are solved. Any solution to the problem of public-key distribution has to prevent a user being overwhelmed with ‘false’ public keys, and the problem of select a real public key from a false one.

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