Collisions for two branches of FORK-256

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Abstract. This note presents analysis of the compression function of a recently proposed hash function, FORK-256. We exhibit some unexpected differentials existing for the step transformation and show their possible uses in collision-finding attacks on different simplified variants of FORK-256. Finally, as a concrete application of those observations we present a method of finding chosen IV collisions for a variant of FORK-256 reduced to two branches : either 1 and 2 or 3 and 4.

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1 Introduction

Most of dedicated hash functions published in the last 15 year follow more or less closely design ideas used by R. Rivest in his functions MD4 [10, 11] and MD5 [12]. Using terminology from [13], their step transformations are all based on source-heavy Unbalanced Feistel Networks (UFN) and employ bitwise Boolean functions. Apart from MD4 and MD5 other examples include RIPEMD [9], HAVAL [18], SHA-1 [7] and also SHA-256 [8]. A very nice feature of all these designs is that they all are very fast in software implementations on modern 32-bit processors and use only a small set of basic instructions executed in constant-time like additions, rotations and Boolean functions.

However, traditional wisdom says that monoculture is dangerous. This proved to be true also in the world of hash functions. Ground-breaking attacks on MD4, MD5 by X. Wang *et al.* [16, 14] were later refined and applied to attack SHA-0 [17] and SHA-1 [15] as well as some other hash functions.

Since source-heavy UFNs with Boolean functions seem to be susceptible to attacks similar to Wang's because only one register is changed after each step and the attacker can manipulate it to a certain extent, one could try designing a hash function using the other flavour of UFNs, namely target-heavy UFNs where changes in one register influence many others. This is the case with designed in 1995 hash function Tiger [1] (tailored for 64-bit platforms) and a recently proposed FORK-256 [3] which will be the focus of this paper.

1.1 Notation

Throughout the paper we will use the notation presented below. Unless stated otherwise, all words are 32-bit and can be seen as elements of $\mathbb{Z}_{2^{32}}$ or $\mathbb{Z}_2^{32}$.

$X + Y$ integer addition / addition modulo 2^{32} (depending on the context),
$X - Y$ integer subtraction / modular subtraction of two words X, Y,
$X \oplus Y$ bitwise XOR of two words X, Y,
$ROL^a(X)$ rotation of bits of the word X by a positions left.

1.2 A brief description of FORK-256

FORK-256 is a dedicated hash function recently proposed by Hong *et al.* [3, 4]. It is based on the classical Merkle-Damgård iterative construction with the compression function that maps 256 bits of state and 512 bits of message to 256 bits of a new state. For the complete description we refer interested readers to [3], here we only present an outline necessary to understand main ideas of the rest of this paper.

The compression function consists of four parallel branches $BRANCH_j$, $j = 1, 2, 3, 4$, each one of them using a different permutation of 16 message words M_i, $i = 0, \ldots, 15$ and the same set of chaining variables $CV = (A, B, C, D, E, F, G, H)$. The compression function updates the set of chaining variables according to the formula

$$CV_{i+1} = CV_i + \{[BRANCH_1(CV_i, M) + BRANCH_2(CV_i, M)] \oplus [BRANCH_3(CV_i, M) + BRANCH_4(CV_i, M)]\} \quad ,$$

where modular and XOR additions are performed word-wise. This construction can be seen as a further extension of the design principle of two parallel lines used in RIPEMD [9].

Each branch function $BRANCH_j$, $j = 1, 2, 3, 4$ consists of eight steps. In each step $k = 0, \ldots, 7$ branch function updates its own copy of eight chaining variables according to the following formulae

$$\begin{aligned}
A_{j,k+1} &:= H_{j,k} + ROL^{21}(g(E_{j,k} + M_{\sigma_j(2k+1)}) \oplus ROL^{17}(E_{j,k} + M_{\sigma_j(2k+1)} + \delta_{\pi_j(2k+1)}),\\
B_{j,k+1} &:= A_{j,k} + M_{\sigma_j 2k} + \delta_{\pi_j(2k)},\\
C_{j,k+1} &:= B_{j,k} + f(A_{j,k} + M_{\sigma_j(2k)}) \oplus g(A_{j,k} + M_{\sigma_j(2k)} + \delta_{\pi_j(2k)}),\\
D_{j,k+1} &:= C_{j,k} + ROL^5(f(A_{j,k} + M_{\sigma_j(2k)})) \oplus ROL^9(g(A_{j,k} + M_{\sigma_j(2k)} + \delta_{\pi_j(2k)})),\\
E_{j,k+1} &:= D_{j,k} + ROL^{17}(f(A_{j,k} + M_{\sigma_j(2k)})) \oplus ROL^{21}(g(A_{j,k} + M_{\sigma_j(2k)} + \delta_{\pi_j(2k)})),\\
F_{j,k+1} &:= E_{j,k} + M_{\sigma_j(2k+1)} + \delta_{\pi_j(2k+1)},\\
G_{j,k+1} &:= F_{j,k} + g(E_{j,k} + M_{\sigma_j(2k+1)}) \oplus f(E_{j,k} + M_{\sigma_j(2k+1)} + \delta_{\pi_j(2k+1)}),\\
H_{j,k+1} &:= G_{j,k} + ROL^9(g(E_{j,k} + M_{\sigma_j(2k+1)}) \oplus ROL^5(E_{j,k} + M_{\sigma_j(2k+1)} + \delta_{\pi_j(2k+1)}),
\end{aligned}$$

where functions f and g are defined as

$$f(x) = x + \left(ROL^7(x) \oplus ROL^{22}(x)\right) \quad , \tag{1}$$

$$g(x) = x \oplus \left(ROL^{13}(x) + ROL^{27}(x)\right) \quad . \tag{2}$$

Constants $\delta_0, \ldots, \delta_{15}$ are defined as the first 32 bits of fractional parts of binary expansions of cube roots of the first 16 primes. Their values are

$$\begin{aligned}
\delta_0 &= 428a2f98, & \delta_1 &= 71374491, & \delta_2 &= b5c0fbcf, & \delta_3 &= e9b5dba5 \ , \\
\delta_4 &= 3956c25b, & \delta_5 &= 59f111f1, & \delta_6 &= 923f82a4, & \delta_7 &= ab1c5ed5 \ , \\
\delta_8 &= d807aa98, & \delta_9 &= 12835b01, & \delta_{10} &= 243185be, & \delta_{11} &= 550c7dc3 \ , \\
\delta_{12} &= 72be5d74, & \delta_{13} &= 80deb1fe, & \delta_{14} &= 9bdc06a7, & \delta_{15} &= c19bf174 \ .
\end{aligned}$$

Finally, permutations σ_j of message words and permutations π_j of constants are shown in Table 1.

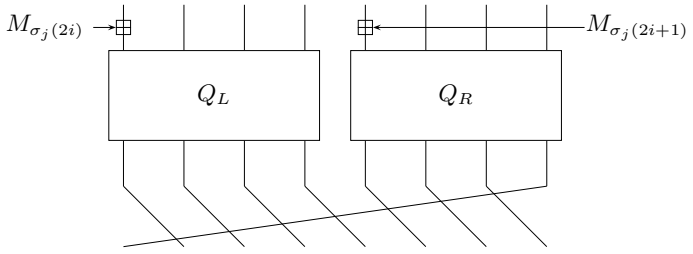
Table 1. Message and constant permutations used in four branches of FORK-256

j	message permutation σ_j	permutation of constants, π_j
1	0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15	0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
2	14 15 11 9 8 10 3 4 2 13 0 5 6 7 12 1	15 14 13 12 11 10 9 8 7 6 5 4 3 2 1 0
3	7 6 10 14 13 2 9 12 11 4 15 8 5 0 1 3	1 0 3 2 5 4 7 6 9 8 11 10 13 12 15 14
4	5 12 1 8 15 0 13 11 3 10 9 2 7 14 4 6	14 15 12 13 10 11 8 9 6 7 4 5 2 3 0 1

2 Analysis of step transformation of FORK-256

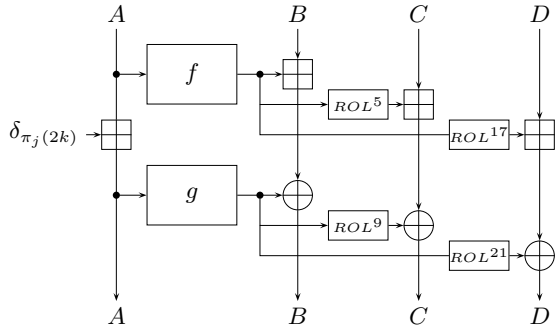
The step transformation described in the previous section can be logically split into three parts: addition of message words, two parallel mixing structures Q_L and Q_R and a final rotation of registers. This view is presented in Fig. 1. The key role is played by the two transformations of four words, Q_L and Q_R as they are the main source of both confusion and diffusion in the compression function. It is clear that if we can find interesting differential characteristics for Q_L and Q_R, we should be able to extend them to the whole branch and maybe also the whole function.

Fig. 1. A high-level structure of step transformation of FORK-256



Let us focus on Q_L, presented in Fig. 2, as Q_R is very similar to Q_L (f and g are swapped and rotation amounts are different) and the arguments we are going to develop work for both of them.

Fig. 2. Q_L-structure of step transformation in FORK-256



Characteristics of the form $(0, \Delta B, \Delta C, \Delta D) \rightarrow (0, \Delta B, \Delta C, \Delta D)$ are not that difficult to get since in each step the difference in registers B, C, D are modified by only one modular addition and one XOR operation. Whether we consider modular or XOR differences, there is only one incompatible operation to deal with.

We can combine such characteristics to get a straightforward differential for up to three steps for each branch.

The difficult part is characteristics of the form $(\Delta A, 0, 0, 0) \rightarrow (\Delta A, 0, 0, 0)$. As far as we could see, there are two ways of finding them. The first method of finding those difficult but desired characteristics is based on the fact that both f and g are not bijective so we can hope that we can find such inputs x, x' that $f(x) = f(x')$ and $g(x + \delta) = g(x' + \delta)$. The second one is aimed at getting zero differences in registers B, C, D in spite of non-zero differences at the outputs of f and g. In next sections we describe both of them in detail.

3 Simultaneous collisions for f and g

For given value δ, we would like to find all x and x' such that $f(x) = f(x')$ and $g(x + \delta) = g(x' + \delta)$. A naive search would require order 2^{64} computations, which is well beyond our computing resources. A less naive method trades time for memory. Below we describe this tradeoff in a way that involves order 2^{32} computations and 2^{32} memory for the particular functions f and g used in FORK-256. Again, we focus on Q_L, i.e. f is applied before g.

Step 1: We determine which inputs x have more than one preimage. This is done by initializing an array of 2^{32} entries to zero, and then incrementing

$f(x)$ within the array for all 2^{32} inputs x. Those values that accumulate 2 or larger are then output. There are about 2^{30} of these. In fact, this step is not necessary for our algorithm, but it may help in practice since it reduces memory requirements for the next step.

Step 2: Read in the values of $f(x)$ from Step 1, i.e. the values that have more than one preimage. Then, for each input value, build a linked list of all preimages of that value. This is done similar to Step 1: compute all 2^{32} values of $f(x)$, and for each value that matches one of the inputs from Step 1 (this can be checked quickly with a hash table), add it to the corresponding linked list. The longest linked list for f has 12 preimages.

Step 3: Process the linked lists from Step 2. For each linked list, consider the set of values that map to the same preimage, $x_1, \ldots, x_k$. See if there is any x_i and x_j in that list such that $g(x_i + \delta) = g(x_j + \delta)$, and if so, output the pair as a solution to simultaneous collisions of f and g when the additive constant is δ.

The running time of Step 3 depends upon the number of combinations of 2 there are of preimages that map to the same image. According to our computations, this is $2134351185 < 2^{31}$.

There are many potential tricks to reduce the search space and/or memory requirements further, but the above algorithm was sufficient for us to determine the following solutions:

$$\begin{aligned}
x &= \texttt{4b4d2a05},\ x' = \texttt{6ff2f3e9},\ \text{for } \delta_1 = \texttt{71374491},\\
x &= \texttt{06def69a},\ x' = \texttt{aeb691e5},\ \text{for } \delta_2 = \texttt{b5c0fbcf},\\
x &= \texttt{27a61343},\ x' = \texttt{67eac4d8},\ \text{for } \delta_3 = \texttt{e9b5dba5},\\
x &= \texttt{04549cdc},\ x' = \texttt{20d331a5},\ \text{for } \delta_7 = \texttt{ab1c5ed5},
\end{aligned}$$

for Q_L and

$$\begin{aligned}
x &= \texttt{445c5563},\ x' = \texttt{d73bc777},\ \text{for } \delta_{10} = \texttt{243185be},\\
x &= \texttt{be452586},\ x' = \texttt{edfd4d5b},\ \text{for } \delta_{14} = \texttt{9bdc06a7}.
\end{aligned}$$

for Q_R.

4 Microcollisions in Q_L and Q_R

In this section we concentrate on an alternative way of finding characteristics of the form $(\Delta A, 0, 0, 0) \rightarrow (\Delta A, 0, 0, 0)$ in Q_L and show that it works for Q_R as well. The idea is to look for pairs of inputs to the register A such that output differences in registers B, C, D are equal to zero in spite of non-zero differences at the outputs of functions f and g. Such a situation is possible if we have three simultaneous *microcollisions* : differences in g cancel out differences from f in all three registers B, C, D (cf. Fig. 2).

4.1 Necessary and sufficient condition for microcollisions

Let us denote $y = f(x)$, $y' = f(x')$ and $z = g(x + \delta)$, $z' = g(x' + \delta)$. We have a microcollision in the first line if the following equation is satisfied

$$(y + B) \oplus z = (y' + B) \oplus z' \tag{3}$$

for given y, y', z, z' and some constant B. Our aim is to find the set of all constants B for which (3) is satisfied.

Let us first introduce three different representations of differences between two numbers $x, x' \in \mathbb{Z}_{2^{32}}$. We will use certain relationships between them in our analysis.

The first kind of representation useful for us is the usual XOR difference. We will treat it as a vector of 32 digits representing bits of $x \oplus x'$ and denote it $\Delta^{\oplus}(x, x') \in \{0, 1\}^{32}$.

The second one is plain integer difference. For two numbers x, x', we define the integer difference ∂x simply as the result of the subtraction of two operands, i.e. $\partial x = x - x'$, $-2^{32} < \partial x < 2^{32}$.

Another kind of representation we will be using is the signed binary representation. It uses three digits, 1, 0, -1, and a pair x, x' has signed binary representation $\Delta^{\pm}(x, x') = (x_0 - x'_0, x_1 - x'_1, \ldots, x_{31} - x'_{31})$, i.e. the i-th component is the result of the subtraction of corresponding bits of x and x' at position i.

A simple but important observation is that if a difference has signed representation $(r_0, r_1, \ldots, r_{31})$ than the corresponding XOR difference is $(|r_0|, |r_1|, \ldots, |r_{31}|)$, i.e. the XOR difference has ones in those places where the signed difference has a non-zero digit, either -1 or 1.

The relationship between integer and signed binary representations is more interesting. An integer difference ∂x corresponds to a signed binary representation $(r_0, \ldots, r_{31})$ if $\partial x = \sum_{i=0}^{31} 2^i \cdot r_i$ where $r_i \in \{-1, 0, 1\}$. Of course this correspondence is one-to-many because of the value–preserving transformations of signed representations, $(*, 0, 1, *) \leftrightarrow (*, 1, -1, *)$ and $(*, 0, -1, *) \leftrightarrow (*, -1, 1, *)$, that can stretch or shrink chunks of ones. To see this on a small example, let us assume words of 4 bits and consider $\Delta^{\pm}(11, 2) = (1, 0, 0, 1)$, $\Delta^{\pm}(14, 5) = (1, 0, 1, -1)$ and $\Delta^{\pm}(12, 3) = (1, 1, -1, -1)$. All these binary signed representations correspond to the integer difference $\partial x = 9$. Note that we can go from one pair of values to another by adding an appropriate constant, e.g. $(12, 3) = (11 + 1, 2 + 1)$. This addition preserves the integer difference but can modify the signed binary representation.

After this introductory part we are equipped with the necessary tools and can go back to our initial problem. Rewriting (3) as

$$(y + B) \oplus (y' + B) = z \oplus z' \ . \tag{4}$$

we can easily see that the signed difference $\Delta^{\pm}(y + B, y' + B)$ can have non-zero digits only in those places where the XOR difference $\Delta^{\oplus}(z, z')$ has ones. This narrows down the set of all possible signed binary representations that can "fit" into XOR difference of a particular form to $2^{h_w(\Delta^{\oplus}(z,z'))}$. But since a single signed

binary representation corresponds to a unique integer difference, there are also only $2^{h_w(\Delta^{\oplus}(z,z'))}$ integer differences ∂y that "fit" into the given XOR difference $\Delta^{\oplus}(z,z')$ and what is important, integer differences are preserved when adding a constant B.

Thus, to check whether a particular difference $\partial y = y - y'$ may "fit" into XOR difference we need to solve the following problem: having $\partial y = y - y'$, $-2^{32} < \partial y < 2^{32}$ and a set of positions $I = \{k_0, k_1, \ldots, k_m\} \subset \{0, \ldots, 31\}$ (that is determined by non-zero bits of $\Delta^{\oplus}(z,z')$) decide whether it is possible to find a binary signed representation $r = (r_0, \ldots, r_{31})$ corresponding to ∂y such that

$$\partial y = \sum_{i=0}^{m} 2^{k_i} \cdot r_{k_i} \quad \text{where } r_{k_i} \in \{-1, 1\} \ . \tag{5}$$

Substituting $t_i = (r_{k_i} + 1)/2$ we can rewrite the above equation in the equivalent form

$$\partial y + \sum_{i=0}^{m} 2^{k_i} = 2^{k_0+1} t_0 + 2^{k_1+1} t_1 + \cdots + 2^{k_m+1} t_m \ , \tag{6}$$

where $t_i \in \{0, 1\}$. Deciding if there are numbers t_i that satisfy (6) is an instance of the knapsack problem and since it is superincreasing (because weights are powers of two), we can do this very efficiently.

This gives us a computationally efficient necessary condition for microcollision in a line: if $\partial y = y - y'$ cannot be represented as (5), no constant B can help us and there is no solution of (3).

Moreover, we can also show that this is as well a sufficient condition: if we can find a solution to the problem (5), there exist a constant B that modifies the signed difference in such a way that it "fits" the prescribed XOR pattern.

First of all, observe that since the solution of the superincreasing knapsack problem (6) is unique, so is the solution of the equivalent problem (5). This means that we know the unique signed representation $\Delta^{\pm}(u, u+\partial y) = (r_0, \ldots, r_{31})$ that is compatible with the XOR difference $\Delta^{\oplus}(z,z')$ and yields the integer difference ∂y. However, a unique signed representation corresponds to a number of concrete pairs $(u, u+\partial y)$. If at a particular position $j \in I$ we have $r_j = -1$, we know that in this position the value of j-th bit of u has to change from 1 to 0. Similarly, if we have $r_j = 1$, the j-th bit of u should change from 0 to 1. The rest of the bits of u (corresponding to positions with zeros in $\Delta^{\pm}(u, u+\partial y)$) can be arbitrary. That way we can easily determine the set $\mathcal{U}$ of all such values u. It is clear that $\mathcal{U}$ always contains at least one element.

Now, since $u = y + B$ for all $u \in \mathcal{U}$, the set $\mathcal{B}$ of all constants B satisfying (3) is simply $\mathcal{B} = \{u - y : u \in \mathcal{U}\}$.

This reasoning shows also that if we can have a microcollision in a line, there are $|\mathcal{B}| = 2^{32-h_w(z \oplus z')}$ constants that yield the microcollision if the most significant bit of $z \oplus z'$ is zero and $2^{32-h_w(z \oplus z')+1}$ if the MSB of $z \oplus z'$ is one. The difference is caused by the fact that if $31 \in I$, we don't need to change u_{31} in a particular way (i.e. either $1 \to 0$ or $0 \to 1$), any change is fine since we don't introduce carries anyway.

Finally, since we didn't use any properties of functions f and g, the same line of argument applies not only to microcollisions in Q_R but also to the same structure with any functions in places of f and g.

4.2 Estimation of probabilities of microcollisions

From the practical point of view, we are very interested in the probability that a random pair of values (A, A') may lead to simultaneous microcollisions and what is the overall probability of characteristics of the form $(\Delta A, 0, 0, 0) \rightarrow (\Delta A, 0, 0, 0)$ when we cannot manipulate the values of registers A, B, C, D.

We conducted some experiments for Q_L and Q_R with different constants δ. Our results indicate that the probability that a random pair of inputs (A, A') may lead to simultaneous microcollisions in all three lines is around 2^{-23} with probability for a single line close to 2^{-13}.

The probability that random constants B, C, D adjust the difference in $f(x)$ properly depends on Hamming weights of $\Delta^{\oplus}(z, z')$. One example of such distribution of weights obtained by testing 2^{32} random pairs[1] is presented in Table 2.

Table 2. Distribution of Hamming weights of $\Delta^{\oplus}(g(x+\delta_0), g(x'+\delta_0))$ corresponding to potential simultaneous microcollisions after testing 2^{32} random pairs x, x'

h_w	0	1	...	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32
count	1	0	...	0	1	6	18	29	59	78	74	90	56	39	14	1	0	0

We can see a clear peak around weights 24–26, so, according to the formula describing the size of the set of constants from the previous subsection, we can expect $2^6 \sim 2^8$ "good" constants in each of the sets $\mathcal{B}$, $\mathcal{C}$, $\mathcal{D}$ and thus the probability that a random constant falls into that set is around $2^{-24} \sim 2^{-26}$. Of course to get a result for all three branches we need to cube that number.

Using the above results, we can try to estimate the probability that a set of three simultaneous microcollisions occurs if we have no control of any values A, A', B, C, D. Multiplying 2^{-23} by $2^{-72} \sim 2^{-78}$ we get an estimation of $2^{-95} \sim 2^{-101}$. It shows that such differentials are not immediately useful, but if we can force specific values of registers to desired values, they may be used to construct collisions for at least simplified variants of FORK, as presented in next sections.

5 Finding high-level differential paths in FORK-256

If we can avoid mixing introduced by the structures Q_L and Q_R (i.e. we know how to get differentials $(\Delta A, 0, 0, 0) \rightarrow (\Delta A, 0, 0, 0)$ and $(\Delta E, 0, 0, 0) \rightarrow (\Delta E, 0, 0, 0)$)

[1] In all experiments we were using Mersenne Twister [5] as the source of pseudorandom numbers

and we can assume that differences in the registers B, C, D and F, G, H remain unchanged, the only places where differences can change are registers A and E, after the addition of a message word difference. Thus, the values of registers in steps are simple linear functions of registers of the initial vector and message words. If we denote $\Delta X_0 + \Delta M_{\sigma_j(a)}$ by [X,a] and $\Delta X_0 + \Delta M_{\sigma_j(a)} + \Delta M_{\sigma_j(b)}$ by [X,a,b], where σ_j is the permutation of message words used in branch $j = 1, 2, 3, 4$, we can write this down concisely in a tabular form presented in Table 3.

Table 3. If no mixing through Q_L and Q_R occurs, differences in registers are combinations of differences in initial vectors and message words. [X,i] stands for $\Delta X_0 + \Delta M_{\sigma_j(i)}$ and [X,a,b] stands for $\Delta X_0 + \Delta M_{\sigma_j(a)} + \Delta M_{\sigma_j(b)}$

	registers							
step	ΔA	ΔB	ΔC	ΔD	ΔE	ΔF	ΔG	ΔH
1	[A,0]	[B]	[C]	[D]	[E,1]	[F]	[G]	[H]
2	[H,2]	[A,0]	[B]	[C]	[D,3]	[E,1]	[F]	[G]
3	[G,4]	[H,2]	[A,0]	[B]	[C,5]	[D,3]	[E,1]	[F]
4	[F,6]	[G,4]	[H,2]	[A,0]	[B,7]	[C,5]	[D,3]	[E,1]
5	[E,1,8]	[F,6]	[G,4]	[H,2]	[A,0,9]	[B,7]	[C,5]	[D,3]
6	[D,3,10]	[E,1,8]	[F,6]	[G,4]	[H,2,11]	[A,0,9]	[B,7]	[C,5]
7	[C,5,12]	[D,3,10]	[E,1,8]	[F,6]	[G,4,13]	[H,2,11]	[A,0,9]	[B,7]
8	[B,7,14]	[C,5,12]	[D,3,10]	[E,1,8]	[F,6,15]	[G,4,13]	[H,2,11]	[A,0,9]
output	[A,0,9]	[B,7,14]	[C,5,12]	[D,3,10]	[E,1,8]	[F,6,15]	[G,4,13]	[H,2,11]

It is clear that differences in registers at any particular step are combinations of differences introduced in the initial vector $(A_0, \ldots, H_0)$ and differences in message words $M_0, \ldots, M_{15}$.

If we consider the simplest case and assume (very optimistically) that any two differences can cancel each other (this is the case with XOR differences), we are in fact working over $\mathbb{F}_2$ and differences in all registers are $\mathbb{F}_2$-linear combinations of differences $\Delta A_0, \ldots, \Delta H_0$ and $\Delta M_0, \ldots, \Delta M_{15}$ (which are now seen as elements of $\mathbb{F}_2$). Now output differences of the whole compression function (including feed-forward) are also linear combinations of differences from $S = (\Delta A_0 \ldots, \Delta H_0, \Delta M_0, \ldots, \Delta M_{15})$ and we can represent this map as an $\mathbb{F}_2$-linear function, $(\Delta A, \ldots, \Delta H) = L_{out}(S)$. This means we can easily find the set $\mathcal{S}_c$ of all vectors $S = (\Delta A_0 \ldots, \Delta H_0, \Delta M_0, \ldots, \Delta M_{15})$ that yield zero output differences at the end of the function simply as the kernel of this map, $\mathcal{S}_c = \ker(L_{out})$.

To minimize the complexity of the attack, we want to find high-level paths as short as possible. Since each register difference in each step is a linear function of differences $\Delta A_0 \ldots, \Delta H_0, \Delta M_0, \ldots, \Delta M_{15}$ and there are only 2^{24} of them, the straightforward approach is to enumerate them all and for any desirable subset of registers (e.g. for collisions in two or three branches) count the number of registers containing non-zero differences and pick those differences S that give

the smallest one. This straightforward process can be improved. If we denote by V the vector of register states we are interested in, there is a matrix Ψ such that $V = S \cdot \Psi$. The matrix Ψ can be seen as a generator matrix of a linear code over $\mathbb{F}_2$. Minimum words of that code correspond to register states with minimal weight. To find collisions (or other restricted paths), the appropriate generating matrix is $Basis(\ker L_{out})) \cdot \Psi$ (or $Basis(\ker(L)) \cdot \Psi$ where L is the linear map describing those registers we want to be zero). Here $Basis(A)$ denotes the basis matrix of a linear space A. Using systems like MAGMA [2], finding minimum words in such codes takes only a fraction of a second.

Our computations show that

- Minimal *collision* path in branches 1-2 uses differences in M_0 and M_9,
- Minimal *collision* path in branches 3-4 uses differences in M_{14} and M_{15},
- Minimal *collision* path for all four branches requires differences in message words M_6 and M_{12},
- Minimal *unrestriced* path for all branches has differences in the message M_{12} only

However, differences in registers other than A and E don't contribute to the complexity of the attack that much. The measure based on the number of differences in registers A and E only corresponds more closely to the number of "difficult" differentials we need to handle that require finding microcollisions. Considering this, we also conducted experiments for different variants of FORK-256 counting only differences in registers A and E.

Table 4. Minimal numbers of sets of simultaneous microcollisions in Q_L and Q_R necessary in different attack scenarios on variants of FORK-256

Scenario	Branches	Number of simult. microcollisions	Differences in
Collisions	1,2	2	M_0, M_9
Collisions	3,4	2	M_{14}, M_{15}
Collisions	1,3	3	M_5
Collisions	1,4	3	M_2
Collisions	2,3	3	M_3
Collisions	2,4	3	M_9
Pseudo-collisions	1,2,3	6	B_0
Pseudo-collisions	1,2,4	6	B_0
Pseudo-collisions	1,3,4	6	B_0
Pseudo-collisions	2,3,4	6	B_0
Collisions	1,2,3,4	12	M_6, M_{12}
Free path	1,2,3,4	6	M_{12}

The results are presented in Table 4. The first column specifies whether we are interested in collision, pseudo-collisions (differences also appear in the initial vector) of just a free path – no specific conditions on differences are imposed.

The third column gives the minimal number of Q–structures that require special differentials and thus also microcollisions in registers B,C,D or F,G,H. The last column gives an example of message and/or chaining variables differences that induce the high-level path with the given number of sets of microcollisions.

6 Collisions for two branches of FORK

We can use the minimal path for branches 1&2 to get collisions for these two branches of FORK-256. The idea is to find two related simultaneous microcollisions, the first one of type f - δ_0 - g (f is followed by δ_0 and then by g) to be used in the left part of the first step of branch 1 and the other one of type g - δ_{12} - f to be used in step 2 of branch 2.

If we can find a pair of values (x, x') that yields f - δ_0 - g microcollisions and a pair (y, y') that yields g - δ_{12} - f microcollisions such that the values satisfy the condition $x - x' = y' - y$, we can construct a collision for branches 1&2 by preserving differences $\partial x = x - x'$ in steps 2, 3, 4 of branch 1 and $\partial y = y - y'$ in steps 3, 4, 5 of branch 2.

The algorithm works as follows:

1. find a pair of values x, x' that produce f - δ_0 - g simultaneous microcollisions and determine the three compatible constants ρ_1, ρ_2, ρ_3, (this step requires around 2^{23} tests of random pairs x, x')
2. for the fixed difference $\partial x = x - x'$ test pairs of the form $y, y' = y + \partial x$ until a simultaneous microcollision of type g - δ_{12} - f is found. Determine compatible constants τ_1, τ_2, τ_3. (Again, experiments suggest that the complexity of that step is 2^{23} tests)
3. set $IV[1] := \rho_1$, $IV[2] := \rho_2$, $IV[3] := \rho_3$,
4. compute $M_0 := x - IV[0]$, $M'_0 := x' - IV[0]$,
5. set both M_{15} and M'_{15} to $\tau_1 - IV[4] - \delta_{14}$,
6. compute initial values $IV[5]$ and $IV[6]$ as follows

$$IV[5] := (\tau_2 \oplus f(IV[4] + M_{15} + \delta_{14})) - g(IV[4] + M_{15}),$$
$$IV[6] := (\tau_3 \oplus ROL^5(f(IV[4] + M_{15} + \delta_{14}))) - ROL^9(g(IV[4] + M_{15}))$$

7. compute the values $M_9 := y - E_1^{(2)}$ and $M'_9 := y' - E_1^{(2)}$, where

$$E_1^{(2)} = ((IV[3] + ROL^{17}(f(IV[0] + M_{14}))) \oplus ROL^{21}(g(IV[0] + M_{14} + \delta_{15}))),$$

   is the value of register E after step 1 in branch 2.
8. preserve the difference ∂x by forcing the value of g to zero in steps 2, 3, 4 (XOR-ing with zero doesn't change the modular difference)
   - set $M'_2 := M_2 := -A_1^{(1)} - \delta 2$,
   - set $M'_4 := M_4 := -A_2^{(1)} - \delta 4$,
   - set $M'_6 := M_6 := -A_3^{(1)} - \delta 6$,
9. similarly, preserve the difference ∂y by forcing the value of f to zero in steps 3, 4, 5 of branch 2

- set $M'_{10} := M_{10} := -E_2^{(2)} - \delta_{10}$,
  - $\diamond$ in step 3 we cannot modify the value of M_4 as it is already fixed by correction done in branch 1. However, we can modify freely the value of M_8 (and M'_8) which indirectly influences the value of $E_3^{(2)}$ we need to adjust. We do this until the difference in $H_4^{(2)}$ is equal to the difference at the beginning of the step, i.e. in $G_3^{(2)}$. If we exhaust all possible values of M_8, we can modify the value of M_{11} and go to step 9 or pick another constant ρ_1 and start over from step 3.
- set $M'_{13} := M_{13} := -E_4^{(2)} - \delta_6$,

The complexity of the attack on branches 1 and 2 depends on the effort to find suitable pair of microcollisions and the amount of work necessary to find the appropriate value of M_8 in step 9.$\diamond$. Microcollisions can be precomputed using around 2^{23} evaluations of functions f, g. The only part we need to deal with during the attack is 9.$\diamond$. In our experiment we had to test ≈ 10000 values of M_8 to find the right one. Since one test is roughly equivalent to computing single step in one branch of FORK (1/32 of the whole function), we can estimate the complexity of 9.$\diamond$ to be less than 2^9 evaluations of the compression function.

This algorithm (partially) uses the following variables: $IV[1]$, $IV[2]$, $IV[3]$, $IV[5]$, $IV[6]$, M_0, M_2, M_4, M_6, M_8, M_9, M_{10}, M_{13}, M_{15}. The following variables can have arbitrary values: $IV[0]$, $IV[4]$, $IV[7]$, M_1, M_3, M_5, M_7, M_{11}, M_{12}, M_{14}.

Finally, we present an example of a collision:

```
IV={6a09e667, ff03f03a, f7da19f9, a19f937d,
    510e527f, d1075199, c4bba02c, 00000000}
 M={97770819, 00000000, 90e31bf1, 00000000,
    e9b1a3b9, 00000000, 36ca5a85, 00000000,
    000024a1, 6ff47b82, 3f7bfaf6, 00000000,
    00000000, 014b4e3b, 00000000, 980100ed}
MM={b479fad2, 00000000, 90e31bf1, 00000000,
    e9b1a3b9, 00000000, 36ca5a85, 00000000,
    000024a1, 52f188c9, 3f7bfaf6, 00000000,
    00000000, 014b4e3b, 00000000, 980100ed}
```

Collisions for branches 3 and 4 can be obtained using exactly the same method by introducing appropriate differences in message words M_{14} and M_{15}.

7 Conclusions and future work

In this paper we presented a preliminary analysis of the compression function of FORK-256. We showed that having enough freedom we can easily find differentials with differences in registers A and E that propagate through step transformation without affecting other registers. This somehow violates the design principles that expect structures Q_L and Q_R to introduce high diffusion of differences. Our algorithm allows us to find such pathological situations very efficiently. Using described techniques we were able to easily construct actual collisions for variants of FORK-256 reduced to only two branches, either 1 and

2 or 3 and 4. We presented directions how to extend these results to variants with more branches. The open question remains if those methods can be used to attack more than three branches of FORK-256. The key question seems to be whether we have enough freedom to apply those techniques to the full function and the key challenge seems to be controlling values in specific registers simultaneously in all branches. We hope that our further research will answer some of these questions.

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