

# A Note on Signature Transformation Attacks and Confirmer Signatures

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**Abstract.** Camenisch and Michels in Eurocrypt 2000 introduced the signature transformation attack on designated confirmer signatures (DCS). We apply this attack on Gentry, et al. Asiacrypt 2005's DCS, and then repair it. We also further optimize their confirmation and disavowal efficiencies.

## 1 The result

Chaum [4] introduced the DCS (Designated Confirmer Signature). The signature verification requires the interaction with a confirmer who was designated by the signer when the signature was created. The motivation was to split the power to sign and the power to confirm in order to mitigate the overpower of the signer. Several applications benefit from such a power splitting [4, 2].

T. Okamoto [7] gave a formal security model for DCS, and a polynomial equivalence reduction between DCS and public-key encryption. Camenisch and Michels [3] presented an upgraded DCS security model which included the *signature transformation attacker* who can query the confirmation oracle with adaptively designed signer public key which is not obtained by the given key generation protocol.

Goldwasser and Waisbard [6] and Gentry, et al. [5] presented DCS without random oracles. [5]'s DCS has  $O(1)$ -size and the state-of-the-art efficiency of costing 10 (resp. 41) exponentiations in confirm (resp. disavow).

**Contributions** We apply Camenisch and Michels [3]'s signature transformation attack on Gentry, et al. [5]'s DCS, and then repair it. We also further optimize their confirmation and disavowal efficiencies. In this brief note, we do not include the security model or other definitions of terminologies. Consult the original references for details [4, 7, 3, 6, 5].

*Review:* [5]’s DCS is  $\sigma' = (\sigma^*, \phi, c)$ ,

$$\begin{aligned}\phi &= \text{Commit}(m, r) = g^m h^r \in QR_{n^2} \\ c &= \text{Enc}(\text{pk}_C, r) = (u_1, u_1, u_3, u_4) = (g_1^\rho, g_2^\rho, d_3^\rho g_0^r, (d_1 d_2^\alpha)^\rho) \in QR_{n^2}^4 \\ \sigma^* &= \text{Sign}(\text{sk}_S, (\phi, c, \text{pk}_S))\end{aligned}$$

where  $\alpha = \text{Hash}(u_1, u_2, u_3)$ . The commitment is Pedersen’s commitment. The base  $g_0 = n + 1$  allows the confirmer to compute the *partial discrete logarithm* in the Paillier system, and thus decrypt  $r$ .  $\text{Sign}$  is any secure signature without random oracles, with signer private key  $\text{sk}_S$ . The confirmer public key  $\text{pk}_C$  consists of  $d_1 = g_1^{x_1} g_2^{x_2}$ ,  $d_2 = g_1^{y_1} g_2^{y_2}$ ,  $d_3 = g_1^z$ . Its private key is  $\text{sk}_C = (x_1, x_2, y_1, y_2, z)$ .

The *signature transformation attack*: Generate the transformed signature using  $c' = c$ ,  $r' = r$ ,  $m' = m + 1$ ,  $\phi' = \phi g$ , and a new signature using attacker’s knowledge of  $\text{sk}_S$  which is granted in the security model. The transformed DCS has the same validity/invalidity as the pre-transformation DCS. Interacting with the  $CVerC$  oracle yields the validity/invalidity of the transformed DCS, and therefore the validity/invalidity of the original pre-transformation DCS.

*Repair:* Change  $\alpha$  above to

$$\alpha = \text{Hash}(u_1, u_2, u_3, \phi, \text{pk}_S, \text{pk}_C, m)$$

When queried with anything other than the  $(\text{DCS}, \text{pk}_S, m)$  in gauntlet, the confirmation oracle will not yield any non-negligible advantage on the invisibility of the validity the DCS [3].

We note that [5]’s DCS remains secure in their own model. However, after the repair above, they can explicitly embellish their model to state that attacker-designed signer public keys not sampled from the model-given key generation protocol are allowed in the confirmation oracle inputs. We also optimize [5]’s four-move concurrent zero-knowledge confirmation/disavowal protocol below.

We omit the straightforward confirmation protocol  $CZK\{r : \phi g^{-m} = h^r\}$ . To disavow, prove either of the following:

$$\begin{aligned}CZK\{(x_1, x_2, y_1, y_2) : d_1 = g_1^{x_1} \wedge d_2 = g_1^{y_1} g_2^{y_2} \\ \wedge u_4 \neq g_1^{x_1 + \alpha y_1} g_2^{x_2 + \alpha y_2}\} \\ CZK\{(z, \bar{r}) : d_3 = g_1^z \wedge u_3 = u_1^z g_0^{\bar{r}} \wedge \phi g^{-m} \neq h^{\bar{r}}\}\end{aligned}$$

They are equivalent to, respectively,

$$\begin{aligned}
& CZK\{(x_1, x_2, y_1, y_2, s_0, s_1 = s_0x_1, s_2 = s_0y_1, s_3 = s_0x_2, s_4 = s_0y_2) : \\
& d_1 = g_1^{x_1} g_2^{x_2} \wedge d_2 = g_1^{y_1} g_2^{y_2} \wedge T = u_4^{-s_0} g_1^{s_1 + \alpha s_2} g_2^{s_3 + \alpha s_4} \\
& \wedge 1 = d_1^{s_0} g_1^{-s_1} g_2^{-s_3} \wedge 1 = d_2^{s_0} g_1^{-s_2} g_2^{-s_4}\} \text{ with } T \neq 1 \\
& CZK\{(z, \bar{r}, s_0, s_1 = s_0\bar{r}) : d_3 = g_1^z \wedge u_3 = u_1^z g_0^{\bar{r}} \wedge T = (\phi^{-1} g^m)^{s_0} g^{s_1} \\
& \wedge T_4 = g_4^{s_0} \wedge 1 = T_4^{\bar{r}} g_4^{-s_1}\} \text{ with } T \neq 1
\end{aligned}$$

The confirmation costs 4 moves totalling 3 exponentiations. The disavow costs 4 moves totally at most 32 exponentiations. In comparison, [5]'s confirmation (resp. disavowal) costs 4 moves and 10 exponentiations (resp. 16 moves and 41 exponentiations).

*Generalizations:* Other DCS schemes employing encryption as a black-box building block, e.g. those in [6, 5] and others, also risk signature transformation attacks possibly beyond their security models. Our results suggest they can open the black box slightly and add more parameters to the hash inputs or other *tag* [1] generating mechanisms.

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