A Provable ID-Based Explicit Authenticated Key Agreement Protocol without Random Oracle

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Abstract: This paper gives out an identity-based key agreement protocol and a modified proof model. The protocol can be proved secure in the proof model. The random oracle is never used in the model thanks to an encryption scheme proposed by Gentry in EuroCrypt 2006. Our main idea is the construction of a key agreement protocol from an encryption scheme, which is converse to the traditional construction of an ElGamal encryption scheme from Diffie-Hellman key agreement protocol. The modified model is based on the widely used model proposed by Bellare and Rogaway in 1993. The different is that the model here refines the ability of an adversary and the security goals of a protocol. The refinement captures more security properties and facilitates the proof of reduction to contradiction.
Keywords: Identity-based Protocol, Key Agreement, Security Model, Random Oracle

1. Introduction

We explain some concepts about explicit authenticated key agreement protocol according to [1]. An explicit authenticated key agreement protocol is a key agreement protocol which provides explicit key authentication. A key agreement protocol or mechanism is a key establishment technique in which a shared secret is derived by two (or more) parties as a function of information contributed by, or associated with, each of these, (ideally) such that no party can predetermine the resulting value. And key establishment is a process or protocol whereby a shared secret becomes available to two or more parties, for subsequent cryptographic use. Explicit key authentication is the property obtained when both implicit key authentication and key confirmation hold. Implicit key authentication is the property whereby one party is assured that no other party aside from a specifically identified second party (and possibly additional identified trusted parties) may access to a particular secret key. And key confirmation is the property whereby one party is assured that a second party (possibly unidentified) actually has possession of a particular secret key.

A key agreement protocol is said to be identity-based (ID-based) if identity information of the party involved is used as the party's public key. ID-based protocols need no public key infrastructure, and can obtain explicit key authentication using only key indexed hash functions. After Shamir proposed the idea of identity-based asymmetric key pairs [2], a few identity-based key agreement protocols based on Shamir's idea have been developed, such as [3], [4], [5] etc. However the practical ID-based protocols boomed after appeared the work of [6] and [7] based on paring techniques. Some of the protocols are [8], [9], [10], [11], [12], [13], [14], [15], and [16] etc. The practical protocols enjoy some security properties, such as partially forward security, key control resistance etc.

Usually, some security properties are used to evaluate the security of key agreement protocols,

including known session key security, perfect forward secrecy, key-compromise impersonation resilience, unknown key-share resilience, and key control resilience etc. By known session key security, we mean that the compromise of one session key should not compromise the keys established in other sessions. Perfect forward security in the two-party case usually means that if their private keys are compromised, the secrecy of session keys previously established by the two parties should not be affected. If the condition is relaxed to only one principle, it is called partially forward security. If the condition is restricted by adding the loss of the third trusted party's master key in the ID-based scenario, it is called master-key forward security [15]. By key-compromise impersonation resilience, we mean that the compromise of party A's long-term private key should not enable the adversary to impersonate other parties to A. Unknown key-share resilience means that party A should not be able to be coerced into sharing a key with party C when in fact A thinks that she/he is sharing the key with some party B. By key control resilience, we mean that one single party should not be able to decide the formation of the session key. It is desirable that the above security properties are captured in a security model, and that a protocol satisfies the security goals defined in the security model, so as to conclude a protocol enjoys the security properties.

To the best of our knowledge, there are some models are used to prove ID-based protocols, at least including BR model [17], BRP model [18], BCP model [19], CK model [20] etc. Most ID-based protocols are proved in some variant models of BR model, such as protocols in [12], [13], [14], [15], and [16]. Usually, an adversary in a BR style model is powered by some kinds of queries, such as Send, Reveal, Corrupt queries etc. The execution of a protocol is described as oracle responses to the adversary's queries. After polynomial bounded times queries, the adversary is expected to win a test game with a non-negligible probability. If the adversary cannot win the game and the transcript between oracles and adversary does satisfy some properties, it is believed that the protocol is secure in the defined model. Roughly all BR style models are defined and used in the above fashion.

In a BR style model, a security property may be captured according to the definitions and usage of the model. For example, if the Reveal query in a model is defined as session key revealing to an adversary, then a secure protocol in the model enjoys the known session key security. Otherwise, if a key compromise of one session s can compromise another session t, the adversary can simply select the session t as the test session and reveal the session s to obtain the answer of the test query. Another example is about perfect forward security which is not captured by current used BR style models for ID-based protocols. If the Corrupt query in a model is defined as long term private key disclosure and the query is not allowed to corrupt the tested session even after the Test query, the model can not capture the perfect forward security. But if the adversary is relaxed to corrupt any session after the Test query provided the response to Corrupt query is only the long term private key, the perfect forward security property can be captured.

Another interesting point about current BR style models are the use of random oracle. The random oracle are used in that the distribution of session key is defined as uniform distribution in $\{0,1\}^k$, where k is related to the security parameter. Usually, a session key is an output of a hash function in a protocol, which is modeled as random oracle so as to satisfy the requirement of uniform distribution. Another reason to use random oracle is due to the public key generation phase of ID-based protocols. Usually, a hash function is used to derive public key from ID, which is modeled as random oracle. However random oracle model is criticized for its inability of instantiation. Canetti et al [21] provided a contrived example and Bellare et al [22] provided a

more natural example to show that a secure scheme in ROM is not secure in real world.

Our contributions include a security model and an ID-based protocol. The modified model is still a BR style model. However, we refined the model as follows. At first, Extract query is used for private key disclosure. Then Corrupt query serves as a method to obtain oracle's internal variables. This ability reflects the adversary's power to dump target's memory. Next the Extract query can be use to query any session after Test query. And then an authentication condition similar with that in the original BR model served as one security goal. That goal assures that a secure protocol in our model enjoys authentication property. Finally, session key distribution is demanded uniformly only in the session key sample space. With these refinements, perfect forward security can be captured. Key-compromise impersonation resilience can be captured by the authentication security goal. Additionally, the direct usage of reduction to contradiction method is enabled, which opens the way to reduce the security of protocol directly to mathematical hard problems.

Our ID-based protocol is derived naturally. Gentry in EuroCrypt 2006 proposed an IND-CPA ID-based encryption scheme [23], which can be proved without random oracle with short public parameters. The IND-CPA scheme is similar with the famous ElGamal encryption scheme except that the random part is carried by two group elements. We notice that the random part in the original ElGamal encryption scheme is in fact a Diffie-Hellman public value. So we take the two group elements in Gentry's scheme as a whole serving as a Diffie-Hellman public key. Then we obtain an ID-based Diffie-Hellman key agreement scheme with almost the same property of the original Diffie-Hellman scheme. Then advantage of the ID-based version over the original one is that the ID-based one does not need signature primitive to prevent man in the middle attack. The indexed hash function is enough. So a three pass ID-based protocol with explicit key authentication property can be obtained using only indexed hash functions and group operations.

The security model, introduction of bilinear maps, and complexity assumption of our protocol are placed in Section 2. Section 3 is our ID-EAKA protocol. The proof of the protocol is in the Section 4. The last is the Conclusion.

2. Preliminaries

Below, we give the security model for an ID-based key agreement protocol. We also review the definition of a bilinear map and discuss the complexity assumption on which the security of our protocol is based.

2.1 Security Model

Our security model is based on Bellare and Rogaway [17] security model for key agreement protocols with several modifications. In our model, the protocol determines how principles behave in response to input signals from their environment. Each principle may execute the protocol multiple times with the same or different partners. This is modeled by allowing each principle to have different instances that execute the protocol. An oracle $\Pi_{i,j}^{s}$ models the behavior of the principle ID_i carrying out a protocol session in the belief that it is

communicating with the principle ID_j for the sth time. One instance is used only for one time, which maintains a variable *view* consisting of the protocol transcripts so far.

The adversary is modeled by a probabilistic polynomial time Turing machine that is assumed to have complete control over all communication links in the network and to interact with the principles via oracle accesses to $\Pi_{i,j}^{s}$. The adversary A is allowed to execute any of the following queries:

- Instantiate (i, j, s). The adversary A lets party i to communicate with party j in the sth session. The system will sets up a new oracle $\Pi_{i,j}^{s}$ as response.
- Extract (i). This allows the adversary to get the long term private key for the principle whose identity string ID is i.
- Send ($\Pi_{i,j}^{s}$, X). The adversary sends message X to the oracle $\Pi_{i,j}^{s}$. The system will give the output of $\Pi_{i,j}^{s}$ to the adversary as response. If $X = \lambda$, the principle ID_i is asked to initiate a session s with ID_j, where λ is the empty string.
- Reveal ($\Pi_{i,j}^{s}$). This asks the oracle $\Pi_{i,j}^{s}$ to reveal whatever session key it currently holds.
- Corrupt ($\Pi_{i,j}^{s}$). This allows the adversary obtains all internal variables of the oracle $\Pi_{i,j}^{s}$, such as temporal keys of oracle $\Pi_{i,j}^{s}$.

An oracle $\Pi_{i,j}^{s}$ exists in one of the following several possible states:

- Accepted: an oracle has accepted if it decides to accept, holding a session key, after receipt of properly formulated messages.
- Rejected: an oracle has rejected if it decides not to establish a session key and to abort the protocol.
- Unsettled: an oracle is unsettled if it has not made any decision to accept or reject.
- Opened: an oracle is opened if it has answered a reveal query.
- Corrupted: an oracle is corrupted if it has answered in a corrupt query.
- Extracted: an oracle is extracted if it has involved in a extract query
- Fresh: an oracle $\Pi_{i,j}^{s}$ is fresh if it is accepted and not opened, not corrupted and not extracted, the matching oracle (if any) $\Pi_{j,i}^{s}$ not opened, not corrupted, and not extracted.

By $\Pi_{j,i}^{s'}$, matching oracle of $\Pi_{i,j}^{s}$, we mean that every message that $\Pi_{i,j}^{s}$ sends out is subsequently delivered to $\Pi_{j,i}^{s'}$, with the response of $\Pi_{j,i}^{s'}$ to this message being returned to the $\Pi_{i,j}^{s}$ as the next message.

The adversary is allowed to make a Test query to receive a value to guess.

- Test ($\Pi_{i,j}^{s}$). If the oracle $\Pi_{i,j}^{s}$ is fresh, the adversary can make a test query to it. The adversary receives either a real session key or a random value as the response with an equal priori probability.

After the test query, the adversary can continue making Instantiation, Extract, Send, Reveal, Corrupt queries to the oracles, except that the adversary cannot corrupt the oracles involved in the test query.

The adversary is demanded to output one bit to show its advantage in winning the game.

- Output. The adversary output "0" to identify the real session key or "1" to identify the random value. The advantage of adversary in winning the game is

$$Adv_A = |\Pr[0 \mid \text{real session key}] - \Pr[0 \mid \text{random value}]|.$$

To define an explicit authenticated key agreement protocol, we should prove the protocol satisfying the following goals:

1. If two oracles are matching, then both of them are accepted and have a same session key which is distributed uniformly in the session key sample space.
2. If the oracle $\Pi_{I,J}^{s}$ accepts, there is only one oracle $\Pi_{J,I}^{s'}$ whose *view* is identical to the *view* of $\Pi_{I,J}^{s}$ just before the oracle $\Pi_{I,J}^{s}$ accepts.
3. The Adv_A in the defined model is negligible.

Remark: Our model has some modifications comparing to the current BR style proof models.

The adversary is more powerful and reasonable. Instantiate query shows the basic principle of passive players and positive adversary about the security model. Send query shows the adversary network control power. Extract query is specifically designed for ID-based system. And we relax the query to allow our adversary to use this power at any time. Corrupt query is modified to give the internal states to the adversary but not the long term private key because the Extract query has given the power of obtaining long term key to the adversary. If an oracle is corrupt, all outputs of the oracle are controlled by the adversary. The query gives a chance for an adversary to participant in a protocol. The reveal query will give the session key to the adversary. Note that even the session key has been computed before an oracle accepted, the reveal query may output nothing if the protocol sets the computed session key as a real session key in an oracle only after the oracle accepts. In this case, before an oracle accepts, the computed temporal session key can only be obtained by an adversary through the Corrupt query.

The definition of Test query is more flexible. We do not define the detail method to produce response to our adversary but to restrict the two possible responses must be equal priori probability. This flexibility gives us a possible way to lay the security of a protocol directly on a decisional mathematical hard problem. The Adv_A is defined according to the Test query, which is a normal definition for a distinguisher to distinguish two different distributions.

The security model captures the perfect forward security notion. After the Test query, and before the last output of an adversary, the adversary can still make most queries, including the Extract query. The adversary can obtain the long term private keys related the tested oracle after the Test query.

The security goals in our security model are more natural. The first goal captures that if a

protocol is run as designed, both honest parties can accept and hold a same session key. The second goal is about authentication, which means that if the party *I* accepts with intended party *J*, then before that point, the party *J* must have executed the protocol with intended party *I* as designed. Note that there is no restriction on the role of party *I* or *J* in the second goal. The third goal is about session key secrecy. Apparently, key-compromise impersonation resilience is captured. Otherwise, an adversary can extract principle *I*, corrupt an oracle $\Pi_{J,I}^{s'}$ and make oracle $\Pi_{I,J}^{s}$ accepts, which does not satisfy the second security goal.

2.2 Bilinear Maps

Basic notations are as follows.

1. G and G_T are two (multiplicative) cyclic groups of prime order p;
2. g is a generator of G;
3. e: $G \times G \rightarrow G_T$ is a bilinear map.

Let G and G_T be two groups as above. A bilinear map is a map e: $G \times G \rightarrow G_T$ with the following properties:

1. Bilinear: for all $u, v \in G$ and $a, b \in Z$, we have $e(u^a, v^b) = e(u, v)^{ab}$;
2. Non-degenerate: $e(g, g) \neq 1$.

We say that G is a bilinear group if the group action in G can be computed efficiently and there exists a group G_T and an efficiently computable bilinear map e: $G \times G \rightarrow G_T$ as above. Note that $e(,)$ is symmetric since $e(g^a, g^b) = e(g, g)^{ab} = e(g^b, g^a)$.

2.3 Complexity Assumptions

The security of our protocol is based on a complexity assumption that is called a truncated version of the decisional augmented bilinear Diffie-Hellman exponent assumption [23] (decisional ABDHE). The problem is defined as follows.

Given a vector of q+3 elements

$$(g', g'^{(\alpha^{q+2})}, g, g^{\alpha}, g^{(\alpha^2)}, ..., g^{(\alpha^q)}) \in G^{q+3}$$

as input, outputs $e(g, g')^{(\alpha^{q+1})} \in G_T$. We use g_i and g_i' to denote $g^{(\alpha^i)}$ and $g'^{(\alpha^i)}$ below. An algorithm A has advantage ε in solving truncated q-ABDHE if

$$\Pr[A(g', g'_{q+2}, g, g_1, ..., g_q) = e(g_{q+1}, g')] \geq \varepsilon$$

where the probability is over the random choice of generators g, g' in G, the random choice of α in Z_p, and the random bits used by A. The assumption is that there is no such an probability polynomial time (p.p.t) algorithm A has a non-negligible advantage ε.

The decisional version of truncated q-ABDHE is defined as one would expect. An algorithm A that outputs $b \in \{0, 1\}$ has advantage ε in solving truncated decision q-ABDHE if

$$|\Pr[A(g', g'_{q+2}, g, g_1, ..., g_q, e(g_{q+1}, g') = 0] - \Pr[A(g', g'_{q+2}, g, g_1, ..., g_q, Z) = 0]| \geq \varepsilon$$

where the probability is over the random choice of generators g, g' in G, the random choice of a in Z_p, the random choice of $Z \in G_T$, and the random bits consumed by A.

The truncated (decision) (t, ε, q)-ABDHE assumption holds in G if no t-time algorithm has advantage at least ε in solving the truncated (decision) q-ABDHE problem in G.

3. The ID-EAKA Protocol

The message flow of our protocol is described in Fig.1.

$$Alice -> Bob : (g_1 g^{-ID_B})^x \,||\, g_T^x$$
$$Bob -> Alice : (g_1 g^{-ID_A})^y \,||\, g_T^y \,||\, M_{23}$$
$$Alice -> Bob : M_3$$

Fig.1. Message flow of ID-EAKA protocol

M_{23} and M_3 in Fig.1 will be defined below. The detail of our protocol is defined as follow.

Setup: The PKG picks random generators g, $h \in G$ and random $\alpha \in Z_p$. It sets $g_1 = g^\alpha \in G$ and $g_T = e(g,g) \in G_T$. The public parameters and private master-key are given by

parameters = (g, g_1, h, g_T, H) master-key = α

where H is a public hash function, H: $G_T \times G \times G_T \times G \times G_T \rightarrow \{0,1\}^k$, k is a security parameter.

Extract: To generate a private key for identity $ID \in Z_p$, the PKG generates random $r_{ID} \in Z_p$, and outputs the private key

$$d_{ID} = (r_{ID}, h_{ID}), \text{ where } h_{ID} = (hg^{-r_{ID}})^{1/(\alpha - ID)}.$$

If $ID = \alpha$, the PKG aborts.

Protocol flow: For two parties Alice and Bob whose identification strings are ID_A and ID_B, the algorithm proceeds as follows.

1. Alice selects $x \in_R Z_p$, computes $M_{11} = (g_1 g^{-ID_B})^x$ and $M_{12} = g_T^x$. Alice sends $M_1 = M_{11} \,||\, M_{12}$ to Bob, where symbol "||" denotes concatenation.
2. Bob selects $y \in_R Z_p$, computes $M_{21} = (g_1 g^{-ID_A})^y$, $M_{22} = g_T^y$, $K_{BA} = e(g,h)^{xy}$, $M_{23} = H(K_{BA} || M_{11} || M_{12} || M_{21} || M_{22})$. Alice sends $M_2 = M_{21} \,||\, M_{22} \,||\, M_{23}$ to Bob. K_{BA} is computed as follows:

$$K_{BA} = e((M_{11})^y, h_{ID_B})((M_{12})^y)^{r_{ID_B}}$$

3. Alice computes $K_{AB} = e((M_{21})^x, h_{ID_A})((M_{22})^x)^{r_{ID_A}}$ and $V\_M_{23} = H(K_{AB} || M_{11} || M_{12} || M_{21} || M_{22})$. If $M_{23} \neq V\_M_{23}$, Alice rejects and aborts the protocol. Else if $M_{23} = V\_M_{23}$, Alice accepts, sets K_{AB} as the session key, computes $M_3 = H(K_{AB} || M_{21} || M_{22} || M_{11} || M_{12})$, and sends M_{31} to Bob.
4. Bob computes $V\_M_3 = H(K_{BA} || M_{21} || M_{22} || M_{11} || M_{12})$. If $M_3 \neq V\_M_3$, Bob rejects and aborts the protocol. Else if $M_{23} = V\_M_{23}$,Bob accepts and sets K_{BA} as the session key.

4. Security analysis

To prove security of the ID-EAKA protocol, we prove our protocol achieves the three security goals in the security model. The first goal is about correctness of protocol. The second goal is about authentication property. The last goal is about secrecy property.

TH 1: If two oracles are matching, then both of them are accepted and have a same session key which is distributed uniformly at random on the session key space.

Proof. Suppose two oracles $\Pi_{A,B}^{s}$ and $\Pi_{B,A}^{s'}$. Assume the oracle $\Pi_{A,B}^{s}$ receives the Send ($\Pi_{A,B}^{s}$, λ) query. Then the oracle $\Pi_{A,B}^{s}$ acts as initiator and $\Pi_{B,A}^{s'}$ as responder. Before the initiator accepts, the initiator has a *view* (M_1, M_2) which is identical to the *view* of responder because the initiator and responder are matching. At that point,

$$K_{BA} = e((M_{11})^{y}, h_{ID_B})((M_{12})^{y})^{r_{ID_B}} = e((g_1 g^{-ID_B})^{xy}, h_{ID_B})((g_T)^{xy})^{r_{ID_B}} = e(g,h)^{xy}$$

$$= e((g_1 g^{-ID_A})^{yx}, h_{ID_B})((g_T)^{yx})^{r_{ID_A}} = e((M_{21})^{x}, h_{ID_A})((M_{22})^{x})^{r_{ID_A}} = K_{AB}$$

and the initiator and responder has identical (M_{11}|| M_{12}|| M_{21}|| M_{22}), so the equality $M_{23} = V\_M_{23}$ holds. The initiator will accept according to the protocol and give the last message to the responder. Before the responder accepts, the responder has a *view* (M_1, M_2, M_3) which is identical to the *view* of initiator. Obviously, the responder will also accept.

The session key is $e(g,h)^{xy}$, where $e(g,h)$ can be determined by public parameters. The session key is distributed uniformly in G_T since the exponent x and y are selected randomly during the protocol execution. □

TH 2: If an oracle $\Pi_{A,B}^{s}$ accepts, there is only one oracle $\Pi_{B,A}^{s'}$ whose *view* is identical to the *view* of $\Pi_{A,B}^{s}$ just before the oracle $\Pi_{A,B}^{s}$ accepts.

Proof. We divided the proof into two parts according to oracle roles.

Case 1: Suppose oracle $\Pi_{A,B}^{s}$ receives the Send ($\Pi_{A,B}^{s}$, λ) query as an initiator. If oracle $\Pi_{A,B}^{s}$ accepts, according to the protocol, the equation $M_{23} = V\_M_{23}$ holds. Before computing V_M_{23}, the initiator has a *view* of (M_1, M'_2), where M_1 is produced by the initiator and M'_2 is received by the initiator. $V\_M_{23} = H(K_{AB}|| M_1 || M'_{21} || M'_{22})$ is computed locally by the initiator after the M'_{23} is received. If M'_{23} is not computed as the same as V_M_{23}, there is another p.p.t. algorithm to produce a target of the H function firstly, and a collision of the function secondly, which violates the collision resistance property of H function. This gives us the conclusion that M'_{23} is computed as the same as V_M_{23}, such that $M'_{23} = H(K_{AB}|| M_1 || M'_{21} || M'_{22})$. So the M'_2

producer must have access to all the elements involved in the M'_{23} computing. Since ID_A is included in the M'_2, the M'_2 producer must have made its intended communication peer as A. since ID_B is included in M_1 and ID_A is included in M'_2, we claim that the K_{AB} can be computed only by A or B using their private keys. Suppose that there is a p.p.t. algorithm to produce K_{AB} without private keys of A or B with input M_1 and $M_{21}||M_{22}$ where ID_A and ID_B is included as the protocol's specification, then the algorithm can be used as a oracle to help an IND-CPA attacker to attack the IND-CPA encryption algorithm in [23]. The IND-CPA attacker can demand the encryption oracle in the IND-CPA game to encrypt a message for A or B. Then the IND-CPA attacker takes the first two elements of the challenge message as M_1 or $M_{21}||M_{22}$. Next the IND-CPA attacker selects a random value y or x in Z_p to produce another input $M_{21}||M_{22}$ or M_1. Even more, the IND-CPA attacker can give the random selected value as an extra input for the algorithm. Now if the algorithm gives out K_{AB}, the IND-CPA attacker can compute $K_{AB}^{x^{-1}}$ or $K_{AB}^{y^{-1}}$, using which the IND-CPA attacker can compute the encrypted message and win the game with the same advantage as the suppose algorithm. The M'_2 producer has used the K_{AB} before M'_2 is received by A. So the K_{AB} is not computed using the private key of A when M'_2 is created. So the M'_2 producer must be B. Since the K_{AB} involves random value y, the probability of another oracle $\Pi_{B,A}^{t}$ selecting the same value y is only 1/p. So if there is q_I times Instantiate queries, the maximal probability of y-collision is $(q_I - 2)/p$. So the lemma holds with a probability $1 - ((q_I - 2)/p)$ at least.

Case 2: Suppose oracle $\Pi_{B,A}^{s}$ acts as a responder. If oracle $\Pi_{B,A}^{s}$ accepts, according to the protocol, the equation $M'_3 = V\_M_3$ holds. Before computing V_M_3, the responder has a *view* of (M'_1, M_2, M'_3), where M_2 is produced by the responder and other messages are received by the responder. $V\_M_3 = H(K_{BA} || M_{21}|| M_{22}|| M'_1)$ is computed locally by the responder after the M'_3 is received. As in case 1, M'_3 must be computed as the same as V_M_3, such that $M'_3 = H(K_{BA} || M_{21}|| M_{22}|| M'_1)$. So the M'_3 producer must have access to all the elements involved in the M'_3 computing. As in case 1, the K_{AB} can be computed only by A or B using their private keys. The K_{AB} involves random values x and y, where the value y is selected by the responder. But before M'_3 is received, the responder only used the K_{AB} in the M_{23} computation, which was

equal to M_3' with probability less than 1/p (if ID_A=ID_B and x=y). For oracle $\Pi_{B,X}^t$ as imitator selecting x to produce M_3', the probability of $x = y$ is only 1/p for one Instantiate query. For oracle $\Pi_{B,X}^t$ as other responder selecting y to produce M_3', the probability is less than 1/p^2 for one Instantiate query. So M_3' is produced by A with probability 1–2/p–1/p^2 for one Instantiate query. We claim that oracle $\Pi_{A,X}^t$ that produced M_3' must have selected the random value x appears in M_1'. Since the random value y is selected by the responder, the oracle $\Pi_{A,X}^t$ does not know the value y with probability 1–1/p. If the value x in M_1' is not selected by the oracle $\Pi_{A,X}^t$, then the probability of oracle $\Pi_{A,X}^t$ knowing value x is only 1/p. So with probability 1–2/p, $\Pi_{A,X}^t$ do not know x or y if x is not selected by $\Pi_{A,X}^t$. The oracle $\Pi_{A,X}^t$ can access to A's private key by default. It can also access public messages M_{21}|| M_{22}|| M_1'. We enhance the ability of $\Pi_{A,X}^t$ by giving the private key of B to this oracle. Then $\Pi_{A,X}^t$ knows $e(g,h)^x$, $e(g,h)^y$. If $\Pi_{A,X}^t$ produced M_3', $\Pi_{A,X}^t$ knows $e(g,h)^{xy}$. However, $\Pi_{A,X}^t$ does not know x or y with probability 1–2/p. This implies a contradiction to CDH problem in the G_T group. So the M_3' producer also produced M_1'. Since ID_B is included in M_1', $\Pi_{A,X}^t$ should be $\Pi_{A,B}^t$. If there is q_I times Instantiate queries, the overall probability of the view (M_1', M_2, M_3') belonging to the oracle $\Pi_{A,B}^t$ is (1–(1+ q_I) /p – q_I /p^2)(1–2 /p) at least.

To sum, the conclusion is that if an initiator oracle $\Pi_{A,B}^s$ accepts, there is only one responder oracle $\Pi_{B,A}^{s'}$ whose *view* was identical to the *view* of $\Pi_{A,B}^s$ just before the oracle $\Pi_{A,B}^s$ accepts with a probability 1– ((q_I −2)/p); if an responder oracle $\Pi_{B,A}^s$ accepts, there is only one initiator oracle $\Pi_{A,B}^{s'}$ whose *view* was identical to the *view* of $\Pi_{B,A}^s$ just before the oracle $\Pi_{B,A}^s$ accepts with a probability (1–(1+ q_I) /p– q_I /p^2)(1–2 /p). □

TH 3: The Adv_A with (q−1) times Extract queries in the defined model is negligible if the truncated decision q-ABDHE problem is hard.

Proof. Let A be an adversary who has non-negligible Adv_A in the defined model. We construct

an algorithm B solves the truncated decisional q-ABDHE problem.

B takes as input a random truncated decision q-ABDHE challenge (g', g'_{q+2}, g, g_1 ,…, g_q, Z), where Z is either e(g_{q+1}, g') or a random element of G_T. Algorithm B proceeds as follows.

Setup: B generates a random polynomial $f(z) \in Z_p[z]$ of degree q. It sets $h=g^{f(\alpha)}$, computing h from (g, g_1, … , g_q). Other public parameters g_T and H is defined as the protocol usual definition. The public parameters are (g, g_1, h, g_T, H). There is no master-key belonging to B.

Queries:

- Instantiate (i, j, s): B sets up a new oracle $\Pi_{i,j}^{s}$.
- Extract (i): If $i = \alpha$, B uses α to solve truncated decision q-ABDHE immediately. Else, let $F_i(z)$ denote the (q −1) degree polynomial ($f(z) - f(i)$)/($z - i$). B computes (r_i, h_i) to be ($f(i)$, $g^{F_i(\alpha)}$). This is a valid private key for i, since $g^{F_i(\alpha)} = g^{(f(z) - f(i))/(z - i)} = (hg^{-f(i)})^{1/(\alpha - i)}$ as required. B gives (r_i, h_i) to the adversary as response. Since the number of Extract queries is less than (q −1) and $f(z)$ is random selected, the generated private key has identical distribution as in a real protocol context.
- Send ($\Pi_{i,j}^{s}$, X). Suppose that B guesses the oracle $\Pi_{I,J}^{s}$ is to be tested. The matching oracle of $\Pi_{I,J}^{s}$ is $\Pi_{J,I}^{t}$ that receives the first message sent by $\Pi_{I,J}^{s}$ or sends the first message received by $\Pi_{I,J}^{s}$. Generally, suppose that $\Pi_{I,J}^{s}$ is the initiator. B will compute M_1, M_2 and M_3 as follows for the two oracles when needed.

  Let $f_2(z) = z^{q+2}$ and let $F_{2,J}(z) = (f_2(z) - f_2(J))/(z - J)$, which is a polynomial of degree q +1.

  $M_{11} = g'^{(f_2(\alpha) - f_2(J)) \cdot x}$ and $M_{12} = Z^x \cdot e(g', \prod_{l=0}^{q} g^{F_{2,J,l}\alpha^l})^x$ where random value x is selected as in our protocol definition.

  $M_{21} = (g_1 g^{-I})^y$, $M_{22} = g_T^y$, $M_{23} = H(K_{JI} \| M_{11} \| M_{12} \| M_{21} \| M_{22})$, where random value y is selected as in our protocol definition and K_{JI} is calculated as follows:

$$K_{JI} = e((M_{11})^y, h_J)((M_{12})^y)^{r_J}$$
$$= e(g'^{(f_2(\alpha) - f_2(J))}, g^{(f(x) - f(J))/(x - J)})^{xy} (Z \cdot e(g', \prod_{l=0}^{q} g^{F_{2,J,l}\alpha^l}))^{xyf(J)}.$$

  $M_3 = H(K_{JI} \| M_{21} \| M_{22} \| M_{11} \| M_{12})$ where K_{JI} is obtained from the oracle $\Pi_{J,I}^{t}$.

  B will set the status of oracle $\Pi_{I,J}^{s}$ as accepted if B checks that the *view* of $\Pi_{I,J}^{s}$ is exactly the specially produced messages just before the special M_3 is sent out. B will set the status of oracle $\Pi_{J,I}^{t}$ as accepted if B checks that the *view* of $\Pi_{J,I}^{t}$ is exactly the specially produced messages after the special M_3 is received. For any other Send queries that are not

related to the above two oracles, *B* will act exactly according to the protocol specification.

- Reveal ($\Pi_{i,j}^{s}$). If the query is to reveal the session key of $\Pi_{I,J}^{s}$ or $\Pi_{J,I}^{t}$, the guessed oracle to be tested or its matching oracle if any, *B* will stop the game with a Fail output. Else, *B* gives the session key hold by the oracle $\Pi_{i,j}^{s}$. Note that our protocol sets session key after an accept decision is made. Before the accept decision, the Reveal query will be responded by a λ symbol.
- Corrupt ($\Pi_{i,j}^{s}$). If the query is to corrupt $\Pi_{I,J}^{s}$ or $\Pi_{J,I}^{t}$, *B* will stop the game with a Fail output. Else, *B* gives all internal variables of $\Pi_{i,j}^{s}$ to the adversary.
- Test ($\Pi_{i,j}^{s}$). If *B* made a wrong guess, *B* stops the game with a Fail output. Else, *B* gives the K_{JI} to the adversary.

Output:

B will forward the output of our adversary to the truncated decision *q*-ABDHE challenger as a response.

Analysis:

If *B* does not stop before the output event, the simulation is indistinguishable. First, from the viewpoint of the adversary, the only chance to distinguish the simulation is to analyze the messages M_1, M_2 and M_3. The reason is that the output of Extract query has identical distribution as in a real protocol context, and that the outputs of other queries are generated according to the protocol specification or the model rules. Next let's focus on the doubtable messages. Assume $s = (\log_g g')F_{2,J}(\alpha)$, then $M_{11} = g^{xs(\alpha - J)}$. If $Z = e(g_{q+1}, g')$, $M_{12} = g_T^{xs}$ and $K_{JI} = e(g,h)^{xsy}$. So the messages M_1, M_2 and M_3 have identical distribution as in a real protocol context in this case. If $Z \neq e(g_{q+1}, g')$, then the distribution of M_{12}, M_{23} and M_3 are not the same as in a real protocol context. If the adversary can distinguish the two distributions, then the adversary can distinguish the value *Z* from $e(g_{q+1}, g')$, which contradicts the truncated decision q-ABDHE assumption.

If the simulation is indistinguishable, the adversary should give a qualified output. So if B does not stop before the output event, the adversary will give "0" to identify the real session key or "1" to identify the random value. Since what the adversary obtained from the Test query is just K_{JI}, which is a real session key if $Z = e(g_{q+1}, g')$ or a random value if not, *B* can use adversary's output to solve the truncated decision q-ABDHE problem with an identical advantage.

Now we calculate the probability that *B* does not stop. If *B* made a right guess, then there is no stop event before output because a right guess means that the guessed oracle is fresh before the Test query and it is limited not to be revealed or corrupted before output and after test query. Due to the definition of fresh, we know that the adversary has not reveal, corrupt the guessed query or

its matching oracle and that has not Extract the private key of I or J before Test. So the probability that B does not stop equals to the probability of right guess, which is at least $1/q$. □

5. Conclusion

We proposed a modified BR style proof model and an ID-based protocol. The modified model can capture more security properties and facilitate the direct reduction to contradictions proof method in protocol security proof. The ID-based protocol is an explicit authenticated key agreement protocol without signatures and with a standard model proof.

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