

# Security Against Covert Adversaries: Efficient Protocols for Realistic Adversaries\*

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## Abstract

In the setting of secure multiparty computation, a set of mutually distrustful parties wish to securely compute some joint function of their private inputs. The computation should be carried out in a secure way, meaning that no coalition of corrupted parties should be able to learn more than specified or somehow cause the result to be “incorrect”. Typically, corrupted parties are either assumed to be semi-honest (meaning that they follow the protocol specification) or malicious (meaning that they may deviate arbitrarily from the protocol). However, in many settings, the assumption regarding semi-honest behavior does not suffice and security in the presence of malicious adversaries is excessive and expensive to achieve.

In this paper, we introduce the notion of *covert adversaries*, which we believe faithfully models the adversarial behavior in many commercial, political, and social settings. Covert adversaries have the property that they may deviate arbitrarily from the protocol specification in an attempt to cheat, but do not wish to be “caught” doing so. We provide a definition of security for covert adversaries and show that it is possible to obtain highly efficient protocols that are secure against such adversaries. We stress that in our definition, we quantify over all (possibly malicious) adversaries and do not assume that the adversary behaves in any particular way. Rather, we guarantee that if an adversary deviates from the protocol in a way that would enable it to “cheat” (meaning that it can achieve something that is impossible in an ideal model where a trusted party is used to compute the function), then the honest parties are guaranteed to detect this cheating with good probability. We argue that this level of security is sufficient in many settings.

## 1 Introduction

### 1.1 Background

In the setting of secure multiparty computation, a set of parties with private inputs wish to jointly compute some functionality of their inputs. Loosely speaking, the security requirements of such a computation are that (i) nothing is learned from the protocol other than the output (privacy), (ii) the output is distributed according to the prescribed functionality (correctness), and (iii) parties cannot make their inputs depend on other parties’ inputs. Secure multiparty computation forms the basis for a multitude of tasks, including those as simple as coin-tossing and agreement, and as complex as electronic voting, electronic auctions, electronic cash schemes, anonymous transactions, remote game playing (a.k.a. “mental poker”), and privacy-preserving data mining.

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The security requirements in the setting of multiparty computation must hold even when some of the participating parties are adversarial. It has been shown that, with the aid of suitable cryptographic tools, *any* two-party or multiparty function can be securely computed [27, 14, 12, 4, 7], even in the presence of very strong adversarial behavior. However, the efficiency of the computation depends dramatically on the adversarial model considered. Classically, two main categories of adversaries have been considered:

1. *Malicious adversaries*: these adversaries may behave arbitrarily and are not bound in any way to following the instructions of the specified protocol. Protocols that are secure in the malicious model provide a very strong security guarantee as honest parties are “protected” irrespective of the adversarial behavior of the corrupted parties.
2. *Semi-honest adversaries*: these adversaries correctly follow the protocol specification, yet may attempt to learn additional information by analyzing the transcript of messages received during the execution. Security in the presence of semi-honest adversaries provides only a weak security guarantee, and is not sufficient in many settings. Semi-honest adversarial behavior primarily models inadvertent leakage of information, and is suitable only where participating parties essentially trust each other, but may have other concerns.

Secure computation in the semi-honest adversary model can be carried out very efficiently, but, as mentioned, provides weak security guarantees. Regarding malicious adversaries, it has been shown that, under suitable cryptographic assumptions, *any* multiparty probabilistic polynomial-time functionality can be securely computed for any number of *malicious* corrupted parties [14, 12]. However, this comes at a price. These feasibility results of secure computation typically do not yield protocols that are efficient enough to actually be implemented and used in practice (particularly if standard *simulation-based security* is required). Their importance is more in telling us that it is perhaps worthwhile searching for other efficient protocols, because we at least know that a solution exists in principle. However, the unfortunate state of affairs today – many years after these feasibility results were obtained – is that very few truly efficient protocols exist for the setting of malicious adversaries. Thus, we believe that some middle ground is called for: an adversary model that accurately models adversarial behavior in the real world, on the one hand, but for which efficient, secure protocols can be obtained, on the other.

## 1.2 Our Work – Covert Adversaries

In this work, we introduce a new adversary model that lies between the semi-honest and malicious models. The motivation behind the definition is that in many real-world settings, adversaries are willing to actively cheat (and as such are not semi-honest), but only if they are not caught (and as such they are not arbitrarily malicious). This, we believe, is the case in many business, financial, political and diplomatic settings, where honest behavior cannot be assumed, but where the companies, institutions and individuals involved cannot afford the embarrassment, loss of reputation, and negative press associated with being *caught* cheating. It is also the case, unfortunately, in many social settings, e.g. elections for a president of the country-club. Finally, in remote game playing, players may also be willing to actively cheat, but would try to avoid being caught, or else they may be thrown out of the game. In all, we believe that this type of *covert* adversarial behavior accurately models many real-world situations. Clearly, with such adversaries, it may be the case that the risk of being caught is weighed against the benefits of cheating, and it cannot be assumed that players would avoid being caught at any price and under all circumstances. Accordingly, our definition explicitly models the probability of catching adversarial behavior; a probability that can

be tuned to the specific circumstances of the problem. In particular, we do not assume that adversaries are only willing to risk being caught with negligible probability, but rather allow for much higher probabilities.

**The definition.** Our definition of security is based on the classical *ideal/real simulation paradigm*,<sup>1</sup> and provides the guarantee that if the adversary cheats, then it will be caught by the honest parties (with some probability). In order to understand what we mean by this, we have to explain what we mean by “cheating”. Loosely speaking, we say that an adversary successfully cheats if it manages to do something that is impossible in the ideal model. Stated differently, successful cheating is behavior that cannot be simulated in the ideal model. Thus, for example, an adversary who learns more about the honest parties’ inputs than what is revealed by the output has cheated. In contrast, an adversary who uses pseudorandom coins instead of random coins (where random coins are what are specified in the protocol) has not cheated.

We are now ready to informally describe the guarantee provided by our definition. Let  $0 < \epsilon \leq 1$  be a value (called the *deterrence factor*). Then, any attempt to cheat by a real adversary  $\mathcal{A}$  is detected by the honest parties with probability at least  $\epsilon$ . This is formalized by allowing the ideal-model simulator  $\mathcal{S}$  to sometimes “fail” (meaning that the output distribution of the execution cannot be generated in the standard ideal model for secure computation), with the requirement that in a real execution with  $\mathcal{A}$  the honest parties would detect cheating with probability that is at least  $\epsilon$  times the probability that the simulator fails. Note that when an adversary follows a strategy that can result in a successful cheat with some probability  $p$ , the honest parties are guaranteed to catch the adversary cheating with probability at least  $\epsilon \cdot p$ . Thus, provided that  $\epsilon$  is sufficiently large, an adversary that wishes not to be caught cheating, will refrain from *attempting* to cheat, lest it be caught doing so. Clearly, the higher the value of  $\epsilon$ , the greater the probability adversarial behavior is caught and thus the greater the *deterrent* to cheat. We therefore call our notion security in the presence of covert adversaries with  $\epsilon$ -deterrent. Note that the security guarantee does not preclude successful cheating. Indeed, if the adversary decides to cheat then it may gain access to the other parties’ private information or bias the result of the computation. The only guarantee is that if it attempts to cheat, then there is a fair chance that it will be caught doing so. This is in contrast to standard definitions, where absolute privacy and security are guaranteed, for the given type of adversary. We remark that by setting  $\epsilon = 1$ , our definition can be used to capture a requirement that cheating parties are always caught.

**Protocol constructions.** As mentioned, the aim of this work is to provide a definition of security for which it is possible to construct highly efficient protocols. We demonstrate this by presenting a generic protocol for secure two-party computation in our model that is only mildly less efficient than the protocol of Yao [27], which is secure only for semi-honest adversaries. The first step of our construction is a protocol for oblivious transfer that is based on homomorphic encryption schemes.<sup>2</sup> Highly efficient protocols under this assumption are known [2, 20]. However, these protocols do not achieve *simulation-based* security. Rather, only privacy is guaranteed (with the plus that privacy

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<sup>1</sup>According to this paradigm, security is formalized by comparing the execution of a real protocol to an ideal execution where a trusted party receives the parties’ inputs, computes the function and returns the outputs. More formally, a protocol is secure if for every real-model adversary  $\mathcal{A}$  attacking the protocol there exists an ideal-model adversary/simulator  $\mathcal{S}$  (interacting in a world where a trusted party computes the function) such that the output distribution of the honest parties and  $\mathcal{S}$  in an ideal execution is computationally indistinguishable from the output distribution of the honest parties and  $\mathcal{A}$  in a real execution of the protocol. See Section 2 for more details.

<sup>2</sup>We remark that there is no need to show “feasibility” here because any protocol that is secure in the presence of malicious adversaries is secure in the presence of covert adversaries (with any  $\epsilon$ ). Thus, our focus is on constructing protocols that are highly efficient and not on using general assumptions.

is preserved even in the presence of fully malicious adversaries). We prove the following informally stated theorem:

**Theorem 1.1** *Let  $\epsilon = 1 - \frac{1}{k}$  where  $k = \text{poly}(n)$  and  $n$  is the security parameter. Assuming the existence of homomorphic encryption schemes, there exists an oblivious transfer protocol that is secure in the presence of covert adversaries with  $\epsilon$ -deterrent, has four rounds of communication and requires  $\mathcal{O}(k)$  homomorphic encryption operations.*

We remark that the constant hidden inside the  $\mathcal{O}$  notation for  $\mathcal{O}(k)$  is very small (to be exact, the protocol requires the generation of  $2k$  pairs of encryption keys, and carrying out  $2k$  encryptions, 2 homomorphic operations and one decryption). When setting  $\epsilon = 1/2$  we have  $k = 2$  and thus the protocol is highly efficient. (To compare, the analogous protocols that achieve only privacy without simulation require generating one encryption key, carrying out one encryption and one decryption, and computing two homomorphic operations. Thus our protocol is about four times slower.)

Having constructed an oblivious transfer protocol that meets our definition, we use it in the protocol of Yao [27] in order to obtain efficient general two-party computation. We modify Yao’s protocol so that a number  $\ell$  of garbled circuits are sent, and then all but one are opened in order to check that they were constructed correctly (this follows the folklore cut-and-choose methodology for boosting the security of Yao’s protocol for adversaries that may not be semi-honest). In addition, as it was pointed out in [22], when dealing with malicious adversaries it is necessary to modify the circuit so that each input bit is “split” into a number of random shares (see Section 6 for a full explanation as to why this is necessary). This modification has a significant effect on efficiency because an oblivious transfer is needed for every input bit. Thus, when each bit is split into  $m$  shares, we have that  $m$  oblivious transfers are needed for each input bit. We present a protocol for general secure two-party computation for which different values of  $\ell$  and  $m$  can be plugged in (recall that  $\ell$  denotes the number of garbled circuits that are constructed and sent, and  $m$  denotes the number of oblivious transfers per input bit). Our protocol achieves  $\epsilon$ -deterrent for  $\epsilon = (1 - \ell^{-1})(1 - 2^{-m+1})$ . Thus, in order to achieve a deterrent of  $\epsilon = 1/2$  it suffices to take  $\ell = m = 3$ . For a higher deterrent of  $\epsilon \approx 9/10$  it is possible to take  $\ell = m = 10$ . We prove the following informally stated theorem:

**Theorem 1.2** *Assume the existence of one-way functions and secure oblivious transfer. Then, for every probabilistic polynomial-time function  $f$  there exists a protocol  $\pi$  that securely computes  $f$  in the presence of covert adversaries with  $\epsilon$ -deterrent for  $\epsilon = (1 - \ell^{-1})(1 - 2^{-m+1})$ . Furthermore, the protocol  $\pi$  has a constant number of rounds, requires  $m$  oblivious transfers per input bit, and has communication complexity  $\mathcal{O}(\ell \cdot n \cdot |C|)$ , where  $|C|$  is the size of the circuit computing  $f$  and  $n$  is the security parameter.*

It is sufficient for the oblivious transfer protocol referred to in Theorem 1.2 to be secure in the presence of covert adversaries (with the same  $\epsilon$  achieved by protocol  $\pi$ ). Thus, a protocol for general two-party computation with  $\epsilon = 1/2$  can be constructed by combining Theorems 1.1 and 1.2, and the result is a protocol that is only a *constant factor* slower than the original protocol of Yao that is only secure for semi-honest adversaries. (Note that the protocol of Yao [27] has communication complexity  $\mathcal{O}(n|C|)$  and requires one oblivious transfer per input bit.)

We view our constructions as a “proof of concept” that highly efficient protocols are achievable in this model, and leave the construction of such protocols for specific tasks of interest for future work.

### 1.3 Related Work

The idea of allowing the adversary to cheat as long as it will be detected was first considered by [11] who defined a property called *t-detectability*; loosely speaking, a protocol fulfilling this property provides the guarantee that no coalition of  $t$  parties can cheat without being caught. The work of [11] differs to ours in that (a) they consider the setting of an honest majority, and (b) their definition is not simulation based. Another closely related work to ours is that of [6] that considers *honest-looking adversaries*. Such adversaries may deviate arbitrarily from the protocol specification, but only if this deviation cannot be detected. Our definition differs from that of [6] in a number of important ways. First, our definition provides security guarantees even for adversaries that are willing to be caught with high probability. Thus, we do not assume anything about the adversary’s willingness or lack of willingness to be caught. Second, we place the onus of detecting any cheating by an adversary on the *protocol*. This is of importance because the fact that an adversary generates messages that are distributed differently to an honest party does not mean that the honest parties can or will detect this. (In order to see this, first note that the honest parties may not have the appropriate distinguisher. Second, the result of any single execution may not be enough to detect cheating. For example, if the protocol tells an honest party to send a random bit and the adversary always sends the bit 1, then the honest parties cannot deduce that the adversary is cheating in any given execution because an honest party also sends the bit 1 with probability  $1/2$ .) Thus, in our formulation, the protocol specification itself has instructions that include outputting the fact that “party  $P_i$  has cheated”. We remark also that our motivation and that of [6] is completely different: they considered a more stringent setting where all parties are either malicious or honest-looking. In contrast, we consider a *relaxation* of the adversary model (where parties are either fully honest or covert) with the aim of obtaining more efficient protocols.

The idea of allowing an adversary to cheat with non-negligible probability as long as it will be caught with good probability has been mentioned many times in the literature; see [18, 24] for just two examples. We stress, however, that none of these works formalized this idea. Furthermore, our experience in proving our protocol secure is that simple applications of cut-and-choose do not meet our definition (and there are actual attacks that can be carried out on the cut-and-choose technique used in [24], for example). Another approach to obtaining efficient protocols is to consider definitions of security that are weaker in the sense that they do not follow the simulation paradigm; see [9] for one example. In contrast, our approach is to remain within the ideal/real simulation paradigm, thereby preserving the well-known advantages of this definitional paradigm.

We conclude by remarking that the works on covert secure two-party and multiparty computation of [1, 8] have no connection with this work; those works consider steganographic secure computation and so it is the computation that is covert, whereas in our work it is the adversarial behavior that is covert.

### 1.4 Organization

In Section 2 we review the standard definitions of secure two-party computation and in Section 3 we present formal definitions for the notion of security in the presence of covert adversaries. We present three formulations of this notion and prove relations between the different formulations. In addition, we show that our definitions fall in between malicious and semi-honest security (i.e., security in the presence of malicious adversaries implies security in the presence of covert adversaries for any  $\epsilon$ , and security in the presence of covert adversaries with  $\epsilon > 1/\text{poly}(n)$  implies security in the presence of semi-honest adversaries). An important property of protocols that are proven according to the simulation paradigm is that they can be “composed” and used as subprotocols,

with security being guaranteed via a general composition theorem (like that of [5]). In Section 4 we prove composition theorems for all three of our formulations of security in the presence of covert adversaries. We then proceed to construct efficient protocols under the strongest of our three definitions. In Section 5 we construct protocols for oblivious transfer (the basic protocol is given in Section 5.1 and extensions in Section 5.2). Then in Section 6 we present our protocol for general two-party computation and prove its security.

## 2 Preliminaries and Standard Definitions

### 2.1 Preliminaries

A function  $\mu(\cdot)$  is *negligible in  $n$* , or just *negligible*, if for every positive polynomial  $p(\cdot)$  and all sufficiently large  $n$ 's it holds that  $\mu(n) < 1/p(n)$ . A **probability ensemble**  $X = \{X(a, n)\}_{a \in \{0,1\}^*; n \in \mathbb{N}}$  is an infinite sequence of random variables indexed by  $a$  and  $n \in \mathbb{N}$ . (The value  $a$  will represent the parties' inputs and  $n$  the security parameter.) Two distribution ensembles  $X = \{X(a, n)\}_{n \in \mathbb{N}}$  and  $Y = \{Y(a, n)\}_{n \in \mathbb{N}}$  are said to be **computationally indistinguishable**, denoted  $X \stackrel{c}{\equiv} Y$ , if for every non-uniform polynomial-time algorithm  $D$  there exists a negligible function  $\mu(\cdot)$  such that for every  $a \in \{0, 1\}^*$ ,

$$|\Pr[D(X(a, n)) = 1] - \Pr[D(Y(a, n)) = 1]| \leq \mu(n)$$

All parties are assumed to run in time that is polynomial in the security parameter. (Formally, each party has a security parameter tape upon which that value  $1^n$  is written. Then the party is polynomial in the input on this tape.)

### 2.2 Secure Multiparty Computation – Standard Definition

In this section we briefly present the standard definition for secure multiparty computation and refer to [12, Chapter 7] for more details and motivating discussion. The following description and definition is based on [12], which in turn follows [15, 25, 3, 5].

**Multiparty computation.** A multiparty protocol problem is cast by specifying a random process that maps sets of inputs to sets of outputs (one for each party). We refer to such a process as a **functionality** and denote it  $f : (\{0, 1\}^*)^m \rightarrow (\{0, 1\}^*)^m$ , where  $f = (f_1, \dots, f_m)$ . That is, for every vector of inputs  $\bar{x} = (x_1, \dots, x_m)$ , the output-vector is a random variable  $\bar{y} = (f_1(\bar{x}), \dots, f_m(\bar{x}))$  ranging over vectors of strings. The  $i^{\text{th}}$  party  $P_i$ , with input  $x_i$ , wishes to obtain  $f_i(\bar{x})$ . We sometimes denote such a functionality by  $(\bar{x}) \mapsto (f_1(\bar{x}), \dots, f_m(\bar{x}))$ . Thus, for example, the oblivious transfer functionality is denoted by  $((x_0, x_1), \sigma) \mapsto (\lambda, x_\sigma)$ , where  $(x_0, x_1)$  is the first party's input,  $\sigma$  is the second party's input, and  $\lambda$  denotes the empty string (meaning that the first party has no output).

**Adversarial behavior.** Loosely speaking, the aim of a secure multiparty protocol is to protect honest parties against dishonest behavior by other parties. In this section, we present the definition for *malicious adversaries* who control some subset of the parties and may instruct them to arbitrarily deviate from the specified protocol. We also consider *static corruptions*, meaning that the set of corrupted parties is fixed at the onset.

**Security of protocols (informal).** The security of a protocol is analyzed by comparing what an adversary can do in a real protocol execution to what it can do in an ideal scenario that is secure by definition. This is formalized by considering an *ideal* computation involving an incorruptible *trusted third party* to whom the parties send their inputs. The trusted party computes the functionality on

the inputs and returns to each party its respective output. Loosely speaking, a protocol is secure if any adversary interacting in the real protocol (where no trusted third party exists) can do no more harm than if it was involved in the above-described ideal computation. One technical detail that arises when considering the setting of no honest majority is that it is impossible to achieve fairness or guaranteed output delivery. That is, it is possible for the adversary to prevent the honest parties from receiving outputs. Furthermore, it may even be possible for the adversary to receive output while the honest parties do not. We consider malicious adversaries and static corruptions in all of our definitions in this paper.

**Execution in the ideal model.** As we have mentioned, some malicious behavior cannot be prevented (for example, early aborting). This behavior is therefore incorporated into the ideal model. Let the set of parties be  $P_1, \dots, P_m$  and let  $I \subseteq [m]$  denote the indices of the corrupted parties, controlled by an adversary  $\mathcal{A}$ . An ideal execution proceeds as follows:

**Inputs:** Each party obtains an input; the  $i^{\text{th}}$  party's input is denoted  $x_i$ . The adversary  $\mathcal{A}$  receives an auxiliary input denoted  $z$ .

**Send inputs to trusted party:** Any honest party  $P_j$  sends its received input  $x_j$  to the trusted party. The corrupted parties controlled by  $\mathcal{A}$  may either abort (by replacing the input  $x_i$  with a special  $\text{abort}_i$  message), send their received input, or send some other input of the same length to the trusted party. This decision is made by  $\mathcal{A}$  and may depend on the values  $x_i$  for  $i \in I$  and its auxiliary input  $z$ . Denote the vector of inputs sent to the trusted party by  $\bar{w}$  (note that  $\bar{w}$  does not necessarily equal  $\bar{x}$ ).

If the trusted party receives an input of the form  $\text{abort}_i$  for some  $i \in I$ , it sends  $\text{abort}_i$  to all parties and the ideal execution terminates. Otherwise, the execution proceeds to the next step.

**Trusted party sends outputs to adversary:** The trusted party computes  $(f_1(\bar{w}), \dots, f_m(\bar{w}))$  and sends  $f_i(\bar{w})$  to party  $P_i$ , for all  $i \in I$  (i.e., to all corrupted parties).

**Adversary instructs trusted party to continue or halt:**  $\mathcal{A}$  sends either  $\text{continue}$  or  $\text{abort}_i$  to the trusted party (for some  $i \in I$ ). If it sends  $\text{continue}$ , the trusted party sends  $f_j(\bar{w})$  to party  $P_j$ , for all  $j \notin I$  (i.e., to all honest parties). Otherwise, if it sends  $\text{abort}_i$ , the trusted party sends  $\text{abort}_i$  to all parties  $P_j$  for  $j \notin I$ .

**Outputs:** An honest party always outputs the message it obtained from the trusted party. The corrupted parties output nothing. The adversary  $\mathcal{A}$  outputs any arbitrary (probabilistic polynomial-time computable) function of the initial inputs  $\{x_i\}_{i \in I}$ , the auxiliary input  $z$ , and the messages  $\{f_i(\bar{w})\}_{i \in I}$  obtained from the trusted party.

This ideal model is different from that of [12] in that in the case of an “abort”, the honest parties output  $\text{abort}_i$  and not a  $\perp$  symbol. This means that the honest parties *know* the identity of the corrupted party who causes the abort. This is achieved by most multiparty protocols, including that of [14], but not all (e.g., the protocol of [16] does not meet this requirement).

Let  $f : (\{0, 1\}^*)^m \rightarrow (\{0, 1\}^*)^m$  be an  $m$ -party functionality, where  $f = (f_1, \dots, f_m)$ , let  $\mathcal{A}$  be a non-uniform probabilistic polynomial-time machine, and let  $I \subseteq [m]$  be the set of corrupted parties. Then, the ideal execution of  $f$  on inputs  $\bar{x}$ , auxiliary input  $z$  to  $\mathcal{A}$  and security parameter  $n$ , denoted  $\text{IDEAL}_{f, \mathcal{A}(z), I}(\bar{x}, n)$ , is defined as the output vector of the honest parties and the adversary  $\mathcal{A}$  from the above ideal execution.

**Execution in the real model.** We next consider the real model in which a real  $m$ -party protocol  $\pi$  is executed (and there exists no trusted third party). In this case, the adversary  $\mathcal{A}$  sends all messages in place of the corrupted parties, and may follow an arbitrary polynomial-time strategy. In contrast, the honest parties follow the instructions of  $\pi$ .

Let  $f$  be as above and let  $\pi$  be an  $m$ -party protocol for computing  $f$ . Furthermore, let  $\mathcal{A}$  be a non-uniform probabilistic polynomial-time machine and let  $I$  be the set of corrupted parties. Then, the real execution of  $\pi$  on inputs  $\bar{x}$ , auxiliary input  $z$  to  $\mathcal{A}$  and security parameter  $n$ , denoted  $\text{REAL}_{\pi, \mathcal{A}(z), I}(\bar{x}, n)$ , is defined as the output vector of the honest parties and the adversary  $\mathcal{A}$  from the real execution of  $\pi$ .

**Security as emulation of a real execution in the ideal model.** Having defined the ideal and real models, we can now define security of protocols. Loosely speaking, the definition asserts that a secure party protocol (in the real model) emulates the ideal model (in which a trusted party exists). This is formulated by saying that adversaries in the ideal model are able to simulate executions of the real-model protocol. We will consider executions where all inputs are of the same length (see discussion in [12]), and will therefore say that a vector  $\bar{x} = (x_1, \dots, x_m)$  is *balanced* if for every  $i$  and  $j$  it holds that  $|x_i| = |x_j|$ .

**Definition 2.1** (secure multiparty computation): *Let  $f$  and  $\pi$  be as above. Protocol  $\pi$  is said to securely compute  $f$  with abort in the presence of malicious adversaries if for every non-uniform probabilistic polynomial-time adversary  $\mathcal{A}$  for the real model, there exists a non-uniform probabilistic polynomial-time adversary  $\mathcal{S}$  for the ideal model, such that for every  $I \subseteq [m]$ , every balanced vector  $\bar{x} \in (\{0, 1\}^*)^m$ , and every auxiliary input  $z \in \{0, 1\}^*$ :*

$$\left\{ \text{IDEAL}_{f, \mathcal{S}(z), I}(\bar{x}, n) \right\}_{n \in \mathbb{N}} \stackrel{c}{\equiv} \left\{ \text{REAL}_{\pi, \mathcal{A}(z), I}(\bar{x}, n) \right\}_{n \in \mathbb{N}}$$

We note that the above definition assumes that the parties (and adversary) know the input lengths (this can be seen from the requirement that  $\bar{x}$  is balanced and so all the inputs in the vector of inputs are of the same length).<sup>3</sup> We remark that some restriction on the input lengths is unavoidable, see [12, Section 7.1] for discussion.

### 2.3 Functionalities that Provide Output to a Single Party

In the standard definition of secure computation, both parties receive output and these outputs may be *different*. However, the presentation of our two-party protocol is far simpler if we assume that only party  $P_2$  receives output. We will show now that this suffices for the general case. That is, we claim that any protocol that can be used to securely compute *any* efficient functionality  $f(x, y)$  where only  $P_2$  receives output, can be used to securely compute *any* efficient functionality  $f = (f_1, f_2)$  where party  $P_1$  receives  $f_1(x_1, x_2)$  and party  $P_2$  receives  $f_2(x_1, x_2)$ . For simplicity, we will assume that the length of the output of  $f_1(x_1, x_2)$  is at most  $n$ , where  $n$  is the security parameter. This can be achieved by simply taking  $n$  to be larger in case it is necessary.

Let  $f = (f_1, f_2)$  be a functionality. We wish to construct a secure protocol in which  $P_1$  receives  $f_1(x_1, x_2)$  and  $P_2$  receives  $f_2(x_1, x_2)$ . As a building block we use a protocol for computing any efficient functionality with the limitation that only  $P_2$  receives output. Let  $r, a, b \in_R \{0, 1\}^n$  be randomly chosen strings. Then, in addition to  $x_1$ , party  $P_1$ 's input includes the elements  $r$ ,  $a$  and  $b$ . Furthermore, define a functionality  $g$  (that has only a single output) as follows:

$$g((r, a, b, x_1), x_2) = (\alpha, \beta, f_2(x_1, x_2))$$

<sup>3</sup>In the case that no parties are corrupted, we assume that the adversary receives the length of the inputs as part of its auxiliary input  $z$ .



where  $\alpha = r + f_1(x_1, x_2)$ ,  $\beta = a \cdot \alpha + b$ , and the arithmetic operations are defined over  $GF[2^n]$ . Note that  $\alpha$  is a one-time pad encryption of  $P_1$ 's output  $f_1(x, y)$ , and  $\beta$  is an information-theoretic message authentication tag of  $\alpha$  (specifically,  $a\alpha + b$  is a pairwise-independent hash of  $\alpha$ ). Now, the parties compute the functionality  $g$ , using a secure protocol in which only  $P_2$  receives output. Following this,  $P_2$  sends the pair  $(\alpha, \beta)$  to  $P_1$ . Party  $P_1$  checks that  $\beta = a \cdot \alpha + b$ ; if yes, it outputs  $\alpha - r$ , and otherwise it outputs `abort`<sub>2</sub>.

It is easy to see that  $P_2$  learns nothing about  $P_1$ 's output  $f_1(x_1, x_2)$ , and that it cannot alter the output that  $P_1$  will receive (beyond causing it to abort), except with probability  $2^{-n}$ . We remark that it is also straightforward to construct a simulator for the above protocol. Applying the composition theorem of [5] (for standard security) or Theorems 4.1 and 4.2 (for covert adversaries – to be defined below), we have the following proposition:

**Proposition 2.2** *Assume that there exists a protocol for securely computing any functionality in which only a single party receives output. Then, there exists a protocol for securely computing any functionality in which both parties receive output. This holds also for security in the presence of covert adversaries for any of Definitions 3.2, 3.4 and 3.5.*

We remark that the circuit for computing  $g$  is only mildly larger than that for computing  $f$ . Thus, the construction above is also efficient and has only a mild effect on the complexity of the secure protocol (assuming that the complexity of the original protocol, where only  $P_2$  receives output, is proportional to the size of the circuit computing  $f$  as is the case for our protocol below).

## 3 Definitions – Secure Computation with Covert Adversaries

### 3.1 Motivation

The standard definition of security (see Definition 2.1) is such that all possible (polynomial-time) adversarial behavior is simulatable. Here, in contrast, we wish to model the situation that parties may *successfully* cheat. However, if they do so, they are likely to be caught. There are a number of ways of defining this notion. In order to motivate ours, we begin with a somewhat naive implementation of the notion, and show its shortcoming.

**First attempt:** Define an adversary to be *covert* if the distribution over the messages that it sends during an execution is computationally indistinguishable from the distribution over the messages that an honest party would send. Then, quantify over all covert adversaries  $\mathcal{A}$  for the real world (rather than all adversaries).<sup>4</sup> A number of problems arise with this definition.

- The fact that the distribution generated by the adversary can be distinguished from the distribution generated by honest parties does not mean that the honest parties can detect this in any specific execution. Consider for example a coin-tossing protocol where the honest distribution gives even probabilities to 0 and 1, while the adversary gives double the probability to the 1 outcome. Clearly, the distributions differ. However, in any given execution, even an outcome of 1 does not provide the honest players with sufficient evidence of any wrong-doing. Thus, it is not sufficient that the *distributions* differ. Rather, one needs to be able to detect cheating in each adversarial execution.

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<sup>4</sup>We remark that this is the conceptual approach taken by [6], and that there are important choices that arise when attempting to formalize the approach. In any case, as we have mentioned, the work of [6] differs greatly because their aim was to model all parties as somewhat adversarial.

- The fact that the distributions differ does not necessarily imply that the honest parties have an efficient distinguisher. Furthermore, in order to guarantee that the honest parties detect the cheating, they would have to analyze all traffic during an execution. However, this analysis *cannot* be part of the protocol because then the distinguishers used by the honest parties would be known (and potentially bypassed).
- Another problem is that, as mentioned in the introduction, adversaries may be willing to risk being caught with more than negligible probability, say  $10^{-6}$ . With such an adversary, the definition would provide no security guarantee. In particular, the adversary may be able to always learn all parties' inputs, and risk being caught in one run in a million.

**Second attempt.** To solve the aforementioned problems, we first require that the protocol itself be responsible for detecting cheating. Specifically, in the case that a party  $P_i$  attempts to cheat, the protocol may instruct the honest parties to output a message saying that “party  $P_i$  has cheated” (we require that this only happens if  $P_i$  indeed cheated). This solves the first problem. To solve the second problem, we explicitly quantify the probability that an adversary is caught cheating. Roughly, given a parameter  $\epsilon$ , a protocol is said to be **secure against covert adversaries with  $\epsilon$ -deterrent** if any cheating adversary will necessarily be caught with probability at least  $\epsilon$ .

This definition captures the spirit of what we want, but is still problematic. To illustrate the problem, consider an adversary that plays honestly with probability 0.99, and cheats otherwise. Such an adversary can only ever be caught with probability 0.01 (because otherwise it is honest). But  $\epsilon = 1/2$  for example, then such an adversary must be caught with probability 0.5, which is impossible. We therefore conclude that an *absolute* parameter cannot be used, and the probability of catching the adversary must be related to the probability that it cheats.

**Final definition.** We thus arrive at the following approach. First, as mentioned, we require that the protocol itself be responsible for detecting cheating. That is, if a party  $P_i$  successfully cheats, then with good probability ( $\epsilon$ ), the honest parties in the protocol will all receive a message that “ $P_i$  cheated”. Second, we do not quantify only over adversaries that are covert (i.e., those that are not detected cheating by the protocol). Rather, we allow all possible adversaries, even completely malicious ones. Then, we require either that this malicious behavior can be successfully simulated (as in Definition 2.1), or that the honest parties will receive a message that cheating has been detected, and this happens with probability at least  $\epsilon$  times the probability that successful cheating takes place. We stress that in the when the adversary chooses to cheat, it may actually learn secret information or cause some other damage. However, since it is guaranteed that such a strategy will likely be caught, there is strong motivation to refrain from doing so.

As it turns out, the above intuition can be formalized in three different ways, which form a hierarchy of security guarantees. In practice, the implementor should choose the formulation that best suites her needs, and for which sufficiently efficient protocols exists. All three definitions are based on the ideal/real simulation paradigm, as presented in Section 2. We now present the definitions in order of security, starting with the weakest (least secure) one.

### 3.2 Version 1: Failed Simulation Formulation

The first formulation we present is based on allowing the simulator to fail sometimes, where by “fail” we mean that its output distribution is not indistinguishable from the real one. This corresponds to an event of successful cheating. However, we guarantee that the probability that the adversary is caught cheating is at least  $\epsilon$  times the probability that the simulator fails. The details follow.

Recall that we call a vector *balanced* if all of its items are of the same length. In addition, we denote the output vector of the honest parties and adversary  $\mathcal{A}$  in an ideal execution of  $f$  by  $\text{IDEAL}_{f,\mathcal{A}(z),I}(\bar{x},n)$ , where  $\bar{x}$  is the vector of inputs,  $z$  is the auxiliary input to  $\mathcal{A}$ ,  $I$  is the set of corrupted parties, and  $n$  is the security parameter, and denote the analogous outputs in a real execution of  $\pi$  by  $\text{REAL}_{\pi,\mathcal{A}(z),I}(\bar{x},n)$ . We begin by defining what it means to “detect cheating”:

**Definition 3.1** *Let  $\pi$  be an  $m$ -party protocol, let  $\mathcal{A}$  be an adversary, and let  $I$  be the index set of the corrupted parties. A party  $P_j$  is said to detect cheating in  $\pi$  if its output in  $\pi$  is  $\text{corrupted}_i$ ; this event is denoted  $\text{OUTPUT}_j(\text{REAL}_{\pi,\mathcal{A}(z),I}(\bar{x})) = \text{corrupted}_i$ . The protocol  $\pi$  is called detection accurate if for every  $j, k \notin I$ , the probability that  $P_j$  outputs  $\text{corrupted}_k$  is negligible.*

We require that all protocols be detection accurate (meaning that only corrupted parties can be “caught cheating”). This is crucial because otherwise a party that is detected cheating can just claim that it is due to a protocol anomaly and not because it really cheated. The definition follows:

**Definition 3.2** (security – failed simulation formulation): *Let  $f$  and  $\pi$  be as in Definition 2.1, and let  $\epsilon : \mathbb{N} \rightarrow [0, 1]$  be a function. Protocol  $\pi$  is said to securely compute  $f$  in the presence of covert adversaries with  $\epsilon$ -deterrent if it is detection accurate and if for every non-uniform probabilistic polynomial-time adversary  $\mathcal{A}$  for the real model, there exists a non-uniform probabilistic polynomial-time adversary  $\mathcal{S}$  for the ideal model such that for every  $I \subseteq [m]$ , every balanced vector  $\bar{x} \in (\{0, 1\}^*)^m$ , every auxiliary input  $z \in \{0, 1\}^*$ , and every non-uniform polynomial-time distinguisher  $D$ , there exists a negligible function  $\mu(\cdot)$  such that,*

$$\begin{aligned} & \Pr \left[ \exists i \in I \forall j \notin I : \text{OUTPUT}_j(\text{REAL}_{\pi,\mathcal{A}(z),I}(\bar{x},n)) = \text{corrupted}_i \right] \\ & \geq \epsilon(n) \cdot \left| \Pr \left[ D(\text{IDEAL}_{f,\mathcal{S}(z),I}(\bar{x},n)) = 1 \right] - \Pr \left[ D(\text{REAL}_{\pi,\mathcal{A}(z),I}(\bar{x},n)) = 1 \right] \right| - \mu(n) \end{aligned}$$

The parameter  $\epsilon$  indicates the probability that successful adversarial behavior is detected (observe that when such a detection occurs, *all* honest parties must detect the same corrupted party). Clearly, the closer  $\epsilon$  is to one, the higher the deterrence to cheat, and hence the level of security, assuming covert adversaries. Note that the adversary can decide to never be detected cheating, in which case the IDEAL and REAL distributions are guaranteed to be *computationally indistinguishable*, as in the standard definition of security. In contrast, it can choose to cheat with some noticeable probability, in which case the IDEAL and REAL output distribution may be distinguishable (while guaranteeing that the adversary is caught with good probability). This idea of allowing the ideal and real models to not be fully indistinguishable in order to model “allowed cheating” was used in [13].

We stress that the definition does not *require* the simulator to “fail” with some probability. Rather, it is *allowed* to fail with a probability that is at most  $1/\epsilon$  times the probability that the adversary is caught cheating. As we shall see, this is what enables us to construct highly efficient protocols. We also remark that due to the required detection accuracy, the simulator cannot fail when the adversary behaves in a fully honest-looking manner (because in such a case, no honest party will output  $\text{corrupted}_i$ ). Thus, security is always preserved in the presence of adversaries that are willing to cheat arbitrarily, as long as their cheating is not detected.

**Cheating and aborting.** It is important to note that according to the above definition, a party that halts mid-way through the computation may be considered a “cheat”. Arguably, this may be undesirable due to the fact that an honest party’s computer may crash (such unfortunate events may not even be that rare). Nevertheless, we argue that as a basic definition it suffices. This is

due to the fact that it is possible for all parties to work by storing their input and random-tape on disk before they begin the execution. Then, before sending any message, the incoming messages that preceded it are also written to disk. The result of this is that if a party’s machine crashes, it can easily reboot and return to its previous state. (In the worst case the party will need to request a retransmit of the last message if the crash occurred before it was written.) We therefore believe that honest parties cannot truly hide behind the excuse that their machine crashed (it would be highly suspicious that someone’s machine crashed in an irreversible way that also destroyed their disk at the critical point of a secure protocol execution).

Despite the above, it is possible to modify the definition so that honest halting is never considered cheating. This modification only needs to be made to the notion of “detection accuracy” and uses the notion of a **fail-stop party** who acts semi-honestly, except that it may halt early.

**Definition 3.3** *A protocol  $\pi$  is non-halting detection accurate if it is detection accurate as in Definition 3.1 and if for every honest party  $P_j$  and fail-stop party  $P_k$ , the probability that  $P_j$  outputs corrupted <sub>$k$</sub>  is negligible.*

The definition of security in the presence of covert adversaries can then be modified by requiring non-halting detection accuracy. We remark that although this strengthening may be desirable, it may also be prohibitive. For example, we are able to modify our main protocol so that it meets this stronger definition. However, in order to do so, we need to assume fully secure oblivious transfer, for which highly efficient (fully simulatable) protocols are not really known.

### 3.3 Version 2: Explicit Cheat Formulation

The drawback of Definition 3.2 is that it allows the adversary to decide whether to cheat as a function of the honest parties’ inputs or of the output. This is undesirable since there may be honest parties’ inputs for which it is more “worthwhile” for the adversary to risk being caught. We therefore wish to force the adversary to make its decision about whether to cheat *obliviously* of the honest parties’ inputs. This brings us to an alternate definition, which is based on redefining the ideal functionality so as to explicitly include the option of cheating. Aside from overcoming the input dependency problem this alternate formulation has two additional advantages. First, it makes the security guarantees that are achieved more explicit. Second, it makes it easy to prove a sequential composition theorem (see below).

We modify the ideal model in the following way. Let  $\epsilon : \mathbb{N} \rightarrow [0, 1]$  be a function. Then, the ideal execution with  $\epsilon$  proceeds as follows:

**Inputs:** Each party obtains an input; the  $i^{\text{th}}$  party’s input is denoted by  $x_i$ ; we assume that all inputs are of the same length, denoted  $n$ . The adversary receives an auxiliary-input  $z$ .

**Send inputs to trusted party:** Any honest party  $P_j$  sends its received input  $x_j$  to the trusted party. The corrupted parties, controlled by  $\mathcal{A}$ , may either send their received input, or send some other input of the same length to the trusted party. This decision is made by  $\mathcal{A}$  and may depend on the values  $x_i$  for  $i \in I$  and the auxiliary input  $z$ . Denote the vector of inputs sent to the trusted party by  $\bar{w}$ .

**Abort options:** If a corrupted party sends  $w_i = \text{abort}_i$  to the trusted party as its input, then the trusted party sends  $\text{abort}_i$  to all of the honest parties and halts. If a corrupted party sends  $w_i = \text{corrupted}_i$  to the trusted party as its input, then the trusted party sends  $\text{corrupted}_i$  to all of the honest parties and halts.

**Attempted cheat option:** If a corrupted party sends  $w_i = \text{cheat}_i$  to the trusted party as its input, then the trusted party sends to the adversary all of the honest parties' inputs  $\{x_j\}_{j \notin I}$ . In addition,

1. With probability  $\epsilon$ , the trusted party sends  $\text{corrupted}_i$  to the adversary and all of the honest parties.
2. With probability  $1 - \epsilon$ , the trusted party sends  $\text{undetected}$  to the adversary. Following this, the adversary sends the trusted party output values  $\{y_j\}_{j \notin I}$  of its choice for the honest parties. Then, for every  $j \notin I$ , the trusted party sends  $y_j$  to  $P_j$ .

The ideal execution then ends at this point.

If no  $w_i$  equals  $\text{abort}_i$ ,  $\text{corrupted}_i$  or  $\text{cheat}_i$ , the ideal execution continues below.

**Trusted party answers adversary:** The trusted party computes  $(f_1(\bar{w}), \dots, f_m(\bar{w}))$  and sends  $f_i(\bar{w})$  to  $\mathcal{A}$ , for all  $i \in I$ .

**Trusted party answers honest parties:** After receiving its outputs, the adversary sends either  $\text{abort}_i$  for some  $i \in I$ , or  $\text{continue}$  to the trusted party. If the trusted party receives  $\text{continue}$  then it sends  $f_j(\bar{w})$  to all honest parties  $P_j$  ( $j \notin I$ ). Otherwise, if it receives  $\text{abort}_i$  for some  $i \in I$ , it sends  $\text{abort}_i$  to all honest parties.

**Outputs:** An honest party always outputs the message it obtained from the trusted party. The corrupted parties output nothing. The adversary  $\mathcal{A}$  outputs any arbitrary (probabilistic polynomial-time computable) function of the initial inputs  $\{x_i\}_{i \in I}$ , the auxiliary input  $z$ , and the messages obtained from the trusted party.

The output of the honest parties and the adversary in an execution of the above ideal model is denoted by  $\text{IDEALC}_{f, \mathcal{S}(z), I}^\epsilon(\bar{x}, n)$ .

Notice that there are two types of “cheating” here. The first is the classic  $\text{abort}$ , except that unlike in Definition 2.1, the honest parties here are informed as to who caused the abort. Thus, although it is not possible to guarantee fairness here, we do achieve that an adversary who aborts after receiving its output is “punished” in the sense that its behavior is always detected.<sup>5</sup> The other type of cheating in this ideal model is more serious for two reasons: first, the ramifications of the cheat are greater (the adversary may learn all of the parties' inputs and may be able to determine their outputs), and second, the cheating is only guaranteed to be detected with probability  $\epsilon$ . Nevertheless, if  $\epsilon$  is high enough, this may serve as a deterrent. We stress that in the ideal model the adversary must decide whether to cheat obliviously of the honest-parties inputs and before it receives any output (and so it cannot use the output to help it decide whether or not it is “worthwhile” cheating). We define:

**Definition 3.4** (security – explicit cheat formulation): *Let  $f$ ,  $\pi$  and  $\epsilon$  be as in Definition 3.2. Protocol  $\pi$  is said to securely compute  $f$  in the presence of covert adversaries with  $\epsilon$ -deterrent if for every non-uniform probabilistic polynomial-time adversary  $\mathcal{A}$  for the real model, there exists a non-uniform probabilistic polynomial-time adversary  $\mathcal{S}$  for the ideal model such that for every  $I \subseteq [m]$ , every balanced vector  $\bar{x} \in (\{0, 1\}^*)^m$ , and every auxiliary input  $z \in \{0, 1\}^*$ :*

$$\left\{ \text{IDEALC}_{f, \mathcal{S}(z), I}^\epsilon(\bar{x}, n) \right\}_{n \in \mathbb{N}} \stackrel{c}{\equiv} \left\{ \text{REAL}_{\pi, \mathcal{A}(z), I}(\bar{x}, n) \right\}_{n \in \mathbb{N}}$$

<sup>5</sup>Note also that there are two types of abort: in one the honest parties receive  $\text{abort}_i$  and in the second they receive  $\text{corrupted}_i$ . This is included to model behavior by the real adversary that results in it being caught cheating with probability greater than  $\epsilon$  (and not with probability exactly  $\epsilon$  as when the ideal adversary sends a  $\text{cheat}_i$  message).

**Definition 3.4 and detection accuracy.** We note that in Definition 3.4 it is not necessary to explicitly require that  $\pi$  be detection accurate because this is taken care of in the ideal model (in an ideal execution, only a corrupted party can send a  $\text{cheat}_i$  input). However, if *non-halting detection accuracy* is desired (as in Definition 3.3), then this should be explicitly added to the definition.

### 3.4 Version 3: Strong Explicit Cheat Formulation

The third, and strongest version follows the same structure and formulation of the previous version (Version 2). However, we make the following slight, but important change to the ideal model. In the case of an attempted cheat, if the trusted party sends  $\text{corrupted}_i$  to the honest parties and the adversary (an event which happens with probability  $\epsilon$ ), then the adversary does *not* obtain the honest parties' inputs. Thus, if cheating is detected, the adversary does not learn anything and the result is essentially the same as a regular abort. This is in contrast to Version 2, where a detected cheat may still be successful. Formally, we modify the “attempted cheat option” in the ideal model as follows:

**Attempted cheat option:** If a corrupted party sends  $w_i = \text{cheat}_i$  to the trusted party as its input, then the trusted party works as follows:

1. With probability  $\epsilon$ , the trusted party sends  $\text{corrupted}_i$  to the adversary and all of the honest parties.
2. With probability  $1 - \epsilon$ , the trusted party sends  $\text{undetected}$  to the adversary along with the honest parties' inputs  $\{x_j\}_{j \notin I}$ . Following this, the adversary sends the trusted party output values  $\{y_j\}_{j \notin I}$  of its choice for the honest parties. Then, for every  $j \notin I$ , the trusted party sends  $y_j$  to  $P_j$ .

Everything else in the ideal model remains the same. We denote the resultant ideal model by  $\text{IDEALSC}_{f,S(z),I}^\epsilon(\bar{x}, n)$  and have the following definition:

**Definition 3.5** (security – strong explicit cheat formulation): *Let  $f$ ,  $\pi$  and  $\epsilon$  be as in Definition 3.2. Protocol  $\pi$  is said to securely compute  $f$  in the presence of covert adversaries with  $\epsilon$ -deterrent if for every non-uniform probabilistic polynomial-time adversary  $\mathcal{A}$  for the real model, there exists a non-uniform probabilistic polynomial-time adversary  $\mathcal{S}$  for the ideal model such that for every  $I \subseteq [m]$ , every balanced vector  $\bar{x} \in (\{0, 1\}^*)^m$ , and every auxiliary input  $z \in \{0, 1\}^*$ :*

$$\left\{ \text{IDEALSC}_{f,S(z),I}^\epsilon(\bar{x}, n) \right\}_{n \in \mathbb{N}} \stackrel{c}{\equiv} \left\{ \text{REAL}_{\pi,\mathcal{A}(z),I}(\bar{x}, n) \right\}_{n \in \mathbb{N}}$$

The difference between the regular and strong explicit cheat formulations is perhaps best exemplified in the case that  $\epsilon = 1$ . In both versions, any potentially successful cheating attempt is detected. However, in the regular formulation, the adversary may learn the honest parties' private inputs (albeit, while being detected). In the strong formulation, in contrast, the adversary learns nothing when it is detected. Since it is always detected, this means that full security is achieved.

### 3.5 Relations Between Security Models

**Relations between covert security definitions.** The three security definitions for covert adversaries constitute a strict hierarchy, with version 1 being strictly weaker than version 2, which is strictly weaker than version 3.

**Proposition 3.6** *Let  $\pi$  be a protocol that securely computes some functionality  $f$  in the presence of covert adversaries with  $\epsilon$ -deterrent by Definition 3.4. Then,  $\pi$  securely computes  $f$  in the presence of covert adversaries with  $\epsilon$ -deterrent by Definition 3.2.*

**Proof:** Let  $f$ ,  $\pi$  and  $\epsilon$  be as in the proposition. Then, we first claim that  $\pi$  is detection accurate. This is due to the fact that in the ideal model of Definition 3.4, honest parties only output  $\text{corrupted}_i$  for  $i \in I$ . Therefore, this must hold also in the real model, except with negligible probability (as required by Definition 3.1). Now, let  $\mathcal{A}$  be an adversary and let  $\mathcal{S}$  be the simulator that is guaranteed to exist for IDEALC by Definition 3.4. We claim that the simulator  $\mathcal{S}$  also works for Definition 3.2. In order to see this, let  $\Delta$  be the probability that  $\mathcal{S}$  sends  $\text{corrupted}_i$  or  $\text{cheat}_i$  for input for some  $i \in I$  (this probability depends only on  $\mathcal{A}$ , the corrupted parties' inputs and the auxiliary input  $z$ ). Now, when  $\mathcal{S}$  sends input  $\text{corrupted}_i$ , the honest parties all output  $\text{corrupted}_i$  with probability 1. In addition, when  $\mathcal{S}$  sends input  $\text{cheat}_i$ , the honest parties all output  $\text{corrupted}_i$  with probability  $\epsilon$  in the ideal model. It follows that the honest parties output  $\text{corrupted}_i$  with probability *at least*  $\epsilon \cdot \Delta$ . It remains, therefore, to show that the IDEAL and REAL distributions can be distinguished with probability at most  $\Delta$  (because then the probability that the adversary is caught cheating is at least  $\epsilon$  times the maximum distinguishing “gap” between the IDEAL and REAL distributions). However, this follows immediately from the fact that if  $\mathcal{S}$  does *not* send any input of the form  $\text{corrupted}_i$  or  $\text{cheat}_i$ , then the ideal execution is the same as in the standard definitions (and so the same as in Definition 3.2). Thus, in the event that  $\mathcal{S}$  does not send  $\text{corrupted}_i$  or  $\text{cheat}_i$ , the IDEAL and REAL of Definition 3.2 are computationally indistinguishable. Since  $\mathcal{S}$  sends  $\text{corrupted}_i$  or  $\text{cheat}_i$  with probability  $\Delta$ , we obtain that the IDEAL distribution can be distinguished from the REAL one with probability at most  $\Delta + \mu(n)$  as desired. ■

The following proposition is straightforward and is therefore stated without a proof:

**Proposition 3.7** *Let  $\pi$  be a protocol that securely computes some functionality  $f$  in the presence of covert adversaries with  $\epsilon$ -deterrent by Definition 3.5. Then,  $\pi$  securely computes  $f$  in the presence of covert adversaries with  $\epsilon$ -deterrent by Definition 3.4.*

**Relation to the malicious and semi-honest models.** As a sanity check regarding our definitions, we present two propositions that show the relation between security in the presence of covert adversaries and security in the presence of malicious and semi-honest adversaries.

**Proposition 3.8** *Let  $\pi$  be a protocol that securely computes some functionality  $f$  with abort in the presence of malicious adversaries, as in Definition 2.1. Then,  $\pi$  securely computes  $f$  in the presence of covert adversaries with  $\epsilon$ -deterrent, for any of the three formulations (Definitions 3.2, 3.4, and 3.5) and for every  $0 \leq \epsilon \leq 1$ .*

This proposition follows from the simple observation that according to Definition 2.1, there exists a simulator that always succeeds in its simulation. Thus, Definition 3.2 holds even if the probability of detecting cheating is 0. Likewise, for Definitions 3.4 and 3.5 the same simulator works (there is simply no need to ever send a `cheat` input).

Next, we consider the relation between covert and semi-honest adversaries. We remark that security for malicious adversaries only implies security for semi-honest adversaries if the semi-honest adversary is allowed to modify its input before the execution begins [17]. Calling such an adversary *augmented semi-honest*, we have the following:

**Proposition 3.9** *Let  $\pi$  be a protocol that securely computes some functionality  $f$  in the presence of covert adversaries with  $\epsilon$ -deterrent, for any of the three formulations and for  $\epsilon(n) \geq 1/\text{poly}(n)$ . Then,  $\pi$  securely computes  $f$  in the presence of augmented semi-honest adversaries.*

This proposition follows from the fact that due to the requirement of detection accuracy, no party outputs corrupted, when the adversary is semi-honest. Since  $\epsilon \geq 1/\text{poly}(n)$  this implies that the REAL and IDEAL distributions can be distinguished with at most negligible probability, as required. We stress that if  $\epsilon = 0$  (or is negligible) then the definition of covert adversaries requires nothing, and so the proposition does not hold for this case.

We conclude that, as one may expect, security in the presence of covert adversaries with  $\epsilon$ -deterrent lies in between security in the presence of malicious adversaries and security in the presence of semi-honest adversaries. If  $1/\text{poly}(n) \leq \epsilon(n) \leq 1 - 1/\text{poly}(n)$  then it can be shown that the definition of security for covert adversaries is strictly different to the semi-honest and malicious models. We remark that for Definitions 3.2 and 3.4 this holds for any  $\epsilon \geq 1/\text{poly}(n)$ . For Definition 3.5 and the case of  $\epsilon(n) = 1 - \mu(n)$ , see below.

**Strong explicit cheat formulation and the malicious model.** The following proposition shows that the strong explicit cheat formulation “converges” to the malicious model as  $\epsilon$  approaches 1.

**Proposition 3.10** *Let  $\pi$  be a protocol and  $\mu$  a negligible function. Then  $\pi$  securely computes some functionality  $f$  in the presence of covert adversaries with  $\epsilon(n) = 1 - \mu(n)$  under Definition 3.5 if and only if it securely computes  $f$  with abort in the presence of malicious adversaries.*

This is true since, by definition, either the adversary does not attempt cheating, in which case the ideal execution is the same as in the regular ideal model, or it attempts cheating, in which case it is caught with probability that is negligibly close to 1 and the protocol is aborted. In both cases, the adversary gains no advantage, and the outcome can be simulated in the standard ideal model. We stress that Proposition 3.10 does not hold for Definitions 3.2 and 3.4 because in these definitions the adversary may learn the honest parties’ private inputs even when it is caught (something that is not allowed in the malicious model).

## 4 Modular Sequential Composition

Sequential composition theorems for secure computation are important for two reasons. First, they constitute a security goal within themselves. Second, they are useful tools that help in writing proofs of security. As such, we believe that when presenting a new definition, it is of great importance to also prove an appropriate composition theorem for that definition. In our case, we obtain composition theorems that are analogous to that of [5] for all three of our definitions.

The basic idea behind these composition theorems is that it is possible to design a protocol that uses an ideal functionality as a subroutine, and then analyze the security of the protocol when a trusted party computes this functionality. For example, assume that a protocol is constructed that uses oblivious transfer as a subroutine. Then, first we construct a protocol for oblivious transfer and prove its security. Next, we prove the security of the protocol that uses oblivious transfer as a subroutine, in a model where the parties have access to a trusted party computing the oblivious transfer functionality. The composition theorem then states that when the “ideal calls” to the trusted party for the oblivious transfer functionality are replaced by real executions of a secure protocol computing this functionality, the protocol remains secure.



**The  $f$ -hybrid model.** We consider a *hybrid model* where parties both interact with each other (as in the real model) and use trusted help (as in the ideal model). Specifically, the parties run a protocol  $\pi$  that contains “ideal calls” to a trusted party computing a functionality  $f$ . These ideal calls are just instructions to send an input to the trusted party. Upon receiving the output back from the trusted party, the protocol  $\pi$  continues. We stress that honest parties do not send messages from  $\pi$  between the time that they send input to the trusted party and the time that they receive back output (this is because we consider *sequential composition* here). Of course, the trusted party may be used a number of times throughout the  $\pi$ -execution. However, each time is independent (i.e., the trusted party does not maintain any state between these calls). We call the regular messages of  $\pi$  that are sent amongst the parties **standard messages** and the messages that are sent between parties and the trusted party **ideal messages**.

Let  $f$  be a functionality and let  $\pi$  be an  $m$ -party protocol that uses ideal calls to a trusted party computing  $f$ . Furthermore, let  $\mathcal{A}$  be a non-uniform probabilistic polynomial-time machine and let  $I$  be the set of corrupted parties. Then, the  $f$ -hybrid execution of  $\pi$  on inputs  $\bar{x}$ , auxiliary input  $z$  to  $\mathcal{A}$  and security parameter  $n$ , denoted  $\text{HYBRID}_{\pi, \mathcal{A}(z), I}^f(\bar{x})$ , is defined as the output vector of the honest parties and the adversary  $\mathcal{A}$  from the hybrid execution of  $\pi$  with a trusted party computing  $f$ .

**Sequential modular composition.** Let  $f$  and  $\pi$  be as above, and let  $\rho$  be a protocol. Consider the real protocol  $\pi^\rho$  that is defined as follows. All standard messages of  $\pi$  are unchanged. When a party  $P_i$  is instructed to send an ideal message  $x$  to the trusted party, it begins a real execution of  $\rho$  with input  $x$  instead. When this execution of  $\rho$  concludes with output  $y$ , party  $P_i$  continues with  $\pi$  as if  $y$  was the output received by the trusted party (i.e. as if it were running in the  $f$ -hybrid model). A special case of the composition theorem of [5] for malicious adversaries states that if  $\rho$  securely computes  $f$ , and  $\pi$  securely computes some functionality  $g$  in the  $f$ -hybrid model, then  $\pi^\rho$  securely computes  $g$  (in the real model). Here, we prove an analogous theorem for covert adversaries with  $\epsilon$ -deterrent. Since our protocols here are for the two-party case, we prove the theorem only for this special case. We also consider the simplified scenario where  $\pi$  contains only a single call to  $f$ ; the more general case can be proven in a similar way (with the addition of a standard hybrid argument). Finally, we assume that the lengths of the inputs to  $\rho$  can be derived given the input to  $\pi$  and the security parameter. All of these assumptions/simplifications are true for our protocols in this paper, and so suffice. A more general theorem can be derived in a straightforward manner.

We prove sequential modular composition theorems for all of our definitions of security in the presence of covert adversaries. The proof for Definition 3.2 is based on the ideas in the composition theorem of [5] while making necessary changes due to the difference in the models. The proofs for Definitions 3.4 and 3.5 are an almost direct corollary of the theorem of [5] (after casting the models of Definitions 3.4 and 3.5 in a different, yet equivalent, model). We present all theorems because, to the best of our knowledge, none can be used to derive another.

## 4.1 Composition for Definition 3.2

In this section we prove a modular sequential composition theorem for the (weaker) Definition 3.2.

**Theorem 4.1** *Let  $f$  be a two-party probabilistic polynomial-time functionality and let  $\rho$  be a protocol that securely computes  $f$  in the presence of covert adversaries with  $\epsilon_1$ -deterrent. Let  $g$  be a two-party functionality and let  $\pi$  be a protocol that securely computes  $g$  in the  $f$ -hybrid model (using a single call to  $f$ ) in the presence of covert adversaries with  $\epsilon_2$ -deterrent. Then,  $\pi^\rho$  securely computes  $g$  in the presence of covert adversaries with  $\epsilon$ -deterrent, where  $\epsilon = \min\{\epsilon_1, \epsilon_2\}$ . The above all refer to Definition 3.2.*

**Proof Sketch:** By the assumption in the theorem, we have that for every non-uniform probabilistic polynomial-time adversary  $\mathcal{A}_\rho$  attacking  $\rho$  in the real model, there exists a non-uniform probabilistic polynomial-time adversary  $\mathcal{S}_\rho$  for the ideal model with  $f$  such that for every  $I \subseteq [2]$ , every balanced vector  $\bar{x} \in (\{0, 1\}^*)^2$ , every  $z \in \{0, 1\}^*$  and every non-uniform polynomial-time  $D$ , there exists a negligible function  $\mu$  such that:

$$\Pr \left[ \exists i \in I \forall j \notin I : \text{OUTPUT}_j(\text{REAL}_{\rho, \mathcal{A}_\rho(z), I}(\bar{x}, n)) = \text{corrupted}_i \right] \geq \epsilon_1(n) \cdot \left| \Pr[D(\text{IDEAL}_{f, \mathcal{S}_\rho(z), I}(\bar{x}, n)) = 1] - \Pr[D(\text{REAL}_{\rho, \mathcal{A}_\rho(z), I}(\bar{x}, n)) = 1] \right| - \mu(n) \quad (1)$$

Furthermore, for every non-uniform probabilistic polynomial-time adversary  $\mathcal{A}_\pi$  attacking  $\pi$  in the  $f$ -hybrid model, there exists a non-uniform probabilistic polynomial-time adversary  $\mathcal{S}_\pi$  for the ideal model with  $g$  such that for every  $I \subseteq [2]$ , every balanced vector  $\bar{x} \in (\{0, 1\}^*)^2$ , every  $z \in \{0, 1\}^*$  and every non-uniform polynomial-time  $D$ , there exists a negligible function  $\mu'$  such that:

$$\Pr \left[ \exists i \in I \forall j \notin I : \text{OUTPUT}_j(\text{HYBRID}_{\pi, \mathcal{A}_\pi(z), I}^f(\bar{x}, n)) = \text{corrupted}_i \right] \geq \epsilon_2(n) \cdot \left| \Pr[D(\text{IDEAL}_{g, \mathcal{S}_\pi(z), I}(\bar{x}, n)) = 1] - \Pr[D(\text{HYBRID}_{\pi, \mathcal{A}_\pi(z), I}^f(\bar{x}, n)) = 1] \right| - \mu'(n) \quad (2)$$

We need to show that under the above assumptions, the real protocol  $\pi^\rho$  securely computes  $g$  in the presence of covert adversaries with  $\epsilon$ -deterrent, where  $\epsilon = \min\{\epsilon_1, \epsilon_2\}$ . In the case that  $I = \phi$  or  $I = \{1, 2\}$ , the proof is straightforward (if  $I = \phi$  then no parties are corrupted and the proof is like in the semi-honest case; if  $I = \{1, 2\}$  then both parties are corrupted and nothing needs to be simulated). We will therefore focus on the case that  $I = \{1\}$  or  $I = \{2\}$ . The proof is the same in both cases and so we will assume that  $I = \{1\}$ ; we take this concrete case because it simplifies notation.

Let  $\mathcal{A}$  be an adversary that controls party  $P_1$  and attacks the real protocol  $\pi^\rho$ , and let  $x_1$  be  $P_1$ 's input and  $z$  the auxiliary input for  $\mathcal{A}$ . We begin by modifying  $\mathcal{A}$  to  $\mathcal{A}'$  in the following way:

1. Adversary  $\mathcal{A}'$  invokes  $\mathcal{A}$  on its own input and forwards all  $\pi$ -messages between  $\mathcal{A}$  (controlling  $P_1$ ) and the honest party  $P_2$ .
2. When  $\mathcal{A}'$  reaches the point in the execution where  $\rho$  begins, it defines an adversary  $\mathcal{A}_\rho$  who receives auxiliary-input  $z_\rho$  that consists of an internal state of a machine, and input  $0^k$  where  $n$  is the length of the input to  $\rho$  in  $\pi$ . (Recall that we assume that the length of the inputs to  $\rho$ , or equivalently to  $f$ , are determined from the input to  $\pi$  and the security parameter.) The machine  $\mathcal{A}_\rho$  runs the machine  $\mathcal{A}$  from the initial state  $z_\rho$  and ignores its input entirely (the input is needed for a technicality that machines have input of the appropriate length). In addition, at the end of the execution of  $\rho$ , adversary  $\mathcal{A}_\rho$  outputs the current state of  $\mathcal{A}$ .
3. After  $\rho$  concludes, adversary  $\mathcal{A}'$  takes the state output by  $\mathcal{A}_\rho$  and continues running  $\mathcal{A}$  from this state by forwarding the messages of  $\pi$  unmodified between  $\mathcal{A}$  and the honest  $P_2$ .
4. At the end of the execution of  $\pi^\rho$ , adversary  $\mathcal{A}'$  outputs whatever  $\mathcal{A}$  does.

It is immediate that the output distribution from a real execution of  $\pi^\rho$  with  $\mathcal{A}'$  and an honest  $P_2$ , is identical to the output distribution of an execution of  $\pi^\rho$  with  $\mathcal{A}$  and an honest  $P_2$ . It is also clear that  $\mathcal{A}_\rho$  is a real adversary for the protocol  $\rho$ . Therefore, by the assumption in the theorem, there exists a simulator  $\mathcal{S}_\rho$  as described above in Eq. (1).

We now use  $\mathcal{A}'$  and  $\mathcal{S}_\rho$  to construct an adversary  $\mathcal{A}_\pi$  for the  $f$ -hybrid execution of  $\pi$ . Adversary  $\mathcal{A}_\pi$  receives auxiliary-input  $z$  and input  $x_1$  and invokes  $\mathcal{A}'$  on these same inputs. However, when  $\mathcal{A}'$

reaches the point that it invokes  $\mathcal{A}_\rho$  upon auxiliary-input  $z_\rho$ , adversary  $\mathcal{A}_\pi$  invokes the simulator  $\mathcal{S}_\rho$  upon auxiliary-input  $z_\rho$  instead. Ideal messages that  $\mathcal{S}_\rho$  wishes to send to the trusted party computing  $f$  are sent by  $\mathcal{A}_\pi$  to its trusted party computing  $f$ . Likewise, outputs from the trusted party are handed by  $\mathcal{A}_\pi$  back to  $\mathcal{S}_\rho$ . When  $\mathcal{S}_\rho$  concludes its execution, its output is interpreted by  $\mathcal{A}_\pi$  as an internal state of  $\mathcal{A}$ . The adversary  $\mathcal{A}_\pi$  then continues by running  $\mathcal{A}'$  from this internal state.

We claim that for every polynomial-time distinguisher  $D$  and every  $\bar{x}$  and  $z$ , there exists a negligible function  $\mu$  such that

$$\begin{aligned} & \Pr \left[ \text{OUTPUT}_2^{\rho}(\text{REAL}_{\pi\rho, \mathcal{A}(z), I}(\bar{x}, n)) = \text{corrupted}_1 \right] \\ & \geq \epsilon_1(n) \cdot \left| \Pr[D(\text{HYBRID}_{\pi, \mathcal{A}_\pi(z), I}^f(\bar{x}, n)) = 1] - \Pr[D(\text{REAL}_{\pi\rho, \mathcal{A}(z), I}(\bar{x}, n)) = 1] \right| - \mu(n) \end{aligned} \quad (3)$$

where  $\text{OUTPUT}_2^{\rho}$  refers to the output of  $P_2$  within the subprotocol  $\rho$  only (i.e., here we consider the probability that  $P_2$  outputs  $\text{corrupted}_1$  within  $\rho$ ). Eq. (3) holds because otherwise this contradicts the security of  $\rho$ . Specifically, the only difference between the real execution with  $\mathcal{A}$  and the hybrid execution with  $\mathcal{A}_\pi$  is that  $\mathcal{A}$  runs  $\mathcal{A}_\rho$  in the real execution of  $\rho$  whereas  $\mathcal{A}_\pi$  runs  $\mathcal{S}_\rho$  instead. Thus, if Eq. (3) does not hold, then  $\mathcal{S}_\rho$  does not “simulate” for  $\mathcal{A}_\rho$  as it should. More formally, if Eq. (3) does not hold when the probabilities are taken over the random tapes of the parties (and thus the entire execution), then there exists a prefix of the execution up until the point that  $\rho$  begins such that Eq. (3) does not hold, even conditioned on this prefix. Such an “execution prefix” is obtained by fixing the portion of the random-tape of  $P_2$  that is used until this point in the protocol and by fixing the random-tape of  $\mathcal{A}$ . (Of course, these tapes can only be found non-uniformly.) Now, this prefix defines an auxiliary-input  $z_\rho$  for  $\mathcal{A}_\rho$  in  $\rho$  that consists of  $\mathcal{A}$ ’s internal state after this execution prefix. Furthermore, it defines the input  $x_2$  of the honest party  $P_2$  to  $\rho$  (the input  $x_1$  of  $P_1$  is implicitly defined by  $\mathcal{A}$  and so can be set to an arbitrary value in order to obtain  $\bar{x}$ ). Fix this input vector  $\bar{x}$  and auxiliary input  $z_\rho$ . Next, we define a distinguisher  $D'$  that receives the outputs of the adversary and  $P_2$  and tries to determine if the execution was  $\text{IDEAL}$  or  $\text{REAL}$ .  $D'$  works by internally simulating the execution until the end: it can do this because it has  $\mathcal{A}'$ ’s state after the execution of  $\rho$  and its random-tape, and also has the input and output of  $P_2$  (the initial input of  $P_2$  needed for this internal simulation can be provided to  $D'$  as auxiliary input). Then, at the end of the internal simulation,  $D'$  applies  $D$  to the output of  $\mathcal{A}'$  and  $P_2$  and outputs whatever  $D$  outputs. It follows that if there exists a polynomial-time distinguisher  $D$  for which Eq. (3) does not hold, then there exists a polynomial-time distinguisher  $D'$  and inputs  $\bar{x}$  and  $z_\rho$  such that for every negligible function  $\mu(n)$

$$\begin{aligned} & \Pr \left[ \text{OUTPUT}_2(\text{REAL}_{\rho, \mathcal{A}_\rho(z_\rho), I}(\bar{x}, n)) = \text{corrupted}_1 \right] \\ & \leq \epsilon_1(n) \cdot \left| \Pr[D(\text{IDEAL}_{f, \mathcal{S}_\rho(z_\rho), I}(\bar{x}, n)) = 1] - \Pr[D(\text{REAL}_{\rho, \mathcal{A}_\rho(z_\rho), I}(\bar{x}, n)) = 1] \right| - \mu(n) \end{aligned}$$

in contradiction to the security of  $\rho$ .

Next, we claim that by Eq. (2) it holds that for every polynomial-time distinguisher  $D$  and every  $\bar{x}$  and  $z$ , there exists a negligible function  $\mu'$  such that

$$\begin{aligned} & \Pr \left[ \text{OUTPUT}_2(\text{HYBRID}_{\pi, \mathcal{A}_\pi(z), I}^f(\bar{x}, n)) = \text{corrupted}_1 \right] \\ & \geq \epsilon_2(n) \cdot \left| \Pr[D(\text{IDEAL}_{f, \mathcal{S}(z), I}(\bar{x}, n)) = 1] - \Pr[D(\text{HYBRID}_{\pi, \mathcal{A}_\pi(z), I}^f(\bar{x}, n)) = 1] \right| - \mu'(n) \end{aligned} \quad (4)$$

This follows directly from the fact that  $\mathcal{A}_\pi$  is an  $f$ -hybrid adversary.

By combining Equations (3) and (4) we obtain that for every polynomial-time distinguisher  $D$  and every  $\bar{x}$  and  $z$ ,

$$\begin{aligned}
& \Pr \left[ \text{OUTPUT}_2(\text{REAL}_{\pi^\rho, \mathcal{A}(z), I}(\bar{x}, n)) = \text{corrupted}_1 \right] \\
& \geq \Pr \left[ \text{OUTPUT}_2(\text{HYBRID}_{\pi, \mathcal{A}_\pi(z), I}^f(\bar{x}, n)) = \text{corrupted}_1 \right] \\
& \quad + \Pr \left[ \text{OUTPUT}_2^\rho(\text{REAL}_{\pi^\rho, \mathcal{A}(z), I}(\bar{x}, n)) = \text{corrupted}_1 \right] \\
& \geq \epsilon_2(n) \cdot \left| \Pr[D(\text{IDEAL}_{f, \mathcal{S}(z), I}(\bar{x}, n)) = 1] - \Pr[D(\text{HYBRID}_{\pi, \mathcal{A}_\pi(z), I}^f(\bar{x}, n)) = 1] \right| - \mu'(n) \\
& \quad + \epsilon_1(n) \cdot \left| \Pr[D(\text{HYBRID}_{\pi, \mathcal{A}_\pi(z), I}^f(\bar{x}, n)) = 1] - \Pr[D(\text{REAL}_{\pi^\rho, \mathcal{A}(z), I}(\bar{x}, n)) = 1] \right| - \mu(n) \\
& \geq \min\{\epsilon_1, \epsilon_2\}(n) \cdot \left| \Pr[D(\text{IDEAL}_{f, \mathcal{S}(z), I}(\bar{x}, n)) = 1] - \Pr[D(\text{REAL}_{\pi^\rho, \mathcal{A}(z), I}(\bar{x}, n)) = 1] \right| - \mu'(n) - \mu(n)
\end{aligned}$$

where the last inequality is due to the fact that  $|a - b| + |b - c| \geq |a - c|$ , and replacing  $\epsilon_1$  and  $\epsilon_2$  with  $\min\{\epsilon_1, \epsilon_2\}$  only makes the result smaller. We therefore conclude that  $\pi^\rho$  securely computes  $g$  in the presence of covert adversaries with  $\epsilon$ -deterrent, where  $\epsilon = \min\{\epsilon_1, \epsilon_2\}$ . ■

## 4.2 Composition for Definitions 3.4 and 3.5

In this section, we prove an analogous modular sequential composition theorem for the stronger Definitions 3.4 and 3.5. Before doing so, we define an  $(f, \epsilon)$ -hybrid model that is the same as the regular hybrid model except that the trusted party is as in  $\text{IDEALC}^\epsilon$  (when considering Definition 3.4) or as in  $\text{IDEALSC}^\epsilon$  (when considering Definition 3.5).

**Theorem 4.2** *Let  $\epsilon : \mathbb{N} \rightarrow [0, 1]$  be a function, let  $f$  be a multiparty probabilistic polynomial-time functionality and let  $\rho$  be a secure protocol for computing  $f$  in the presence of covert adversaries with  $\epsilon$ -deterrent. Furthermore, let  $g$  be a multiparty functionality and let  $\pi$  be a secure protocol for computing  $g$  in the  $(f, \epsilon)$ -hybrid model in the presence of covert adversaries with  $\epsilon$ -deterrent. Then,  $\pi^\rho$  is a secure protocol for computing  $g$  in the presence of covert adversaries with  $\epsilon$ -deterrent. The above holds for Definitions 3.4 and 3.5 by taking the appropriate ideal model in each case.*

**Proof Sketch:** Theorem 4.2 can be derived as an almost immediate corollary from the composition theorem of [5] in the following way. First, define a special functionality interface that follows the instructions of the trusted party in Definition 3.4 (respectively, in Definition 3.5). That is, define a *reactive functionality* that receives inputs and write outputs (this functionality is modelled by an interactive Turing machine). The appropriate reactive functionality here acts exactly like the trusted party (e.g., if it receives a  $\text{cheat}_i$  message, then it tosses coins and with probability  $\epsilon$  outputs  $\text{corrupted}_i$  to all parties and with probability  $1 - \epsilon$  gives the adversary all of the honest parties' inputs and lets it chooses their outputs). Next, consider the standard ideal model of Definition 2.1 with functionalities of the above form. It is easy to see that a protocol securely computes some functionality  $f$  under Definition 3.4 (resp., under Definition 3.5) *if and only if* it is securely computes the appropriately defined reactive functionality under Definition 2.1. This suffices because the composition theorem of [5] can be applied to Definition 2.1, yielding the result.<sup>6</sup> ■

<sup>6</sup>Two remarks are in place here. First, the composition theorem of [5] is formally proven for standard (non-reactive) functionalities and the case of an honest majority. Nevertheless, the proof can be extended to these cases in a straightforward way with almost no changes. Second, the composition theorem of [5] assumes a strict polynomial-time simulator. This is fine because we also required this in our definitions.

We note that it is possible to generalize Theorem 4.2 so that  $\rho$  and  $\pi$  have different values of  $\epsilon$ . However,  $\pi$  must be proven secure with the  $\epsilon$ -value of  $\rho$  in mind. That is, we can state the following theorem: If  $\rho$  is a secure protocol for computing  $f$  in the presence of covert adversaries with  $\epsilon'$ -deterrent, and  $\pi$  is a secure protocol for computing  $g$  in the  $(f, \epsilon')$ -hybrid model in the presence of covert adversaries with  $\epsilon$ -deterrent, then  $\pi^\rho$  is a secure protocol for computing  $g$  in the presence of covert adversaries with  $\epsilon$ -deterrent.

## 5 Oblivious Transfer

In the oblivious transfer functionality [26, 10], a sender has two inputs  $(x_0, x_1)$  and a receiver has an input bit  $\sigma$ . The sender receives no output (and, in particular, learns nothing about the receiver's bit), while the receiver learns  $x_\sigma$  (but learns nothing about  $x_{1-\sigma}$ ). This variant of oblivious transfer is often called **1-out-of-2 oblivious transfer**.

In this section we will construct an efficient oblivious transfer protocol that is secure in the presence of covert adversaries with  $\epsilon$ -deterrent. We will first present the basic scheme that considers a single oblivious transfer and  $\epsilon = 1/2$ . We will then extend this to enable the simultaneous execution of many oblivious transfers and also higher values of  $\epsilon$ . Our constructions all rely on the existence of secure homomorphic encryption schemes.

**Homomorphic encryption.** Let  $(G, E, D)$  be a public-key encryption scheme that has indistinguishable encryptions under chosen-plaintext attacks. We say that  $(G, E, D)$  is homomorphic if it has the following homomorphic property:

1. The plaintext is taken from a finite Abelian group determined by the public key. For notational convenience, we assume here that the group is an “additive” group  $Z_q$ ; however, the same construction works for “multiplicative” groups as well.
2. Given *any* public-key  $pk$  generated by the key generation algorithm  $G$  and *any* two ciphertexts  $c_1 = E_{pk}(m_1)$  and  $c_2 = E_{pk}(m_2)$  under that key, it is possible to efficiently compute a *random* encryption  $E_{pk}(m_1) +_E E_{pk}(m_2) = E_{pk}(m_1 + m_2)$ . Consequently, it is also possible to efficiently compute  $E_{pk}(m_1 \cdot \alpha)$  for any known integer  $\alpha$ . We denote this operation by  $\alpha \cdot_E E_{pk}(m)$ . (This follows because repeated squaring – or addition – can be used.)

We also assume that  $(G, E, D)$  has *no decryption errors*. Such encryption schemes can be constructed under the quadratic-residuosity,  $N$ -residuosity, decisional Diffie-Hellman (DDH) and other assumptions; see [2, 20] for some references.

### 5.1 The Basic Protocol

**Protocol 5.1** (oblivious transfer from errorless homomorphic encryption):

- **Inputs:** *The sender  $S$  has a pair of strings  $(x_0, x_1)$  for input; the receiver  $R$  has a bit  $\sigma$ . Both parties have the security parameter  $1^n$  as auxiliary input. (In order to satisfy the constraints that all inputs are of the same length, it is possible to define  $|x_0| = |x_1| = k$  and give the receiver  $(\sigma, 1^{2k-1})$ .)*
- **Assumption:** *We assume that the group determined by the homomorphic encryption scheme with security parameter  $n$  is large enough to contain all strings of length  $k$ . Thus, if the homomorphic encryption scheme only works for single bits, we will only consider  $k = 1$  (i.e., bit oblivious transfer).*

• **The protocol:**

1. The receiver  $R$  chooses two sets of two pairs of keys:
  - (a)  $(pk_1^0, sk_1^0), (pk_2^0, sk_2^0) \leftarrow G(1^n)$  using random coins  $r_G^0$ , and
  - (b)  $(pk_1^1, sk_1^1), (pk_2^1, sk_2^1) \leftarrow G(1^n)$  using random coins  $r_G^1$

$R$  sends  $(pk_1^0, pk_2^0)$  and  $(pk_1^1, pk_2^1)$  to the sender  $S$ .
2. Key-generation challenge:
  - (a)  $S$  chooses a random coin  $b \in_R \{0, 1\}$  and sends  $b$  to  $R$ .
  - (b)  $R$  sends  $S$  the random-coins  $r_G^b$  that it used to generate  $(pk_1^b, pk_2^b)$ .
  - (c)  $S$  checks that the public keys output by the key-generation algorithm  $G$  when given input  $1^n$  and the appropriate portions of the random-tape  $r_G^b$  equal  $pk_1^b$  and  $pk_2^b$ . If this does not hold, or if  $R$  did not send any message here,  $S$  outputs `corruptedR` and halts. Otherwise, it proceeds.  
Denote  $pk_1 = pk_1^{1-b}$  and  $pk_2 = pk_2^{1-b}$ .
3.  $R$  chooses two random bits  $\alpha, \beta \in_R \{0, 1\}$ . Then:
  - (a)  $R$  computes
 
$$\begin{aligned} c_0^1 &= E_{pk_1}(\alpha) & c_0^2 &= E_{pk_2}(1 - \alpha) \\ c_1^1 &= E_{pk_1}(\beta) & c_1^2 &= E_{pk_2}(1 - \beta) \end{aligned}$$

using random coins  $r_0^1, r_0^2, r_1^1$  and  $r_1^2$ , respectively.
  - (b)  $R$  sends  $(c_0^1, c_0^2)$  and  $(c_1^1, c_1^2)$  to  $S$ .
4. Encryption-generation challenge:
  - (a)  $S$  chooses a random bit  $b' \in_R \{0, 1\}$  and sends  $b'$  to  $R$ .
  - (b)  $R$  sends  $r_{b'}^1$  and  $r_{b'}^2$  to  $S$  (i.e.,  $R$  sends an opening to the ciphertexts  $c_{b'}^1$  and  $c_{b'}^2$ ).
  - (c)  $S$  checks that one of the ciphertexts  $\{c_{b'}^1, c_{b'}^2\}$  is an encryption of 0 and the other is an encryption of 1. If not (including the case that no message is sent by  $R$ ),  $S$  outputs `corruptedR` and halts. Otherwise, it continues to the next step.
5.  $R$  sends a “re-ordering” of the ciphertexts  $\{c_{1-b'}^1, c_{1-b'}^2\}$ . Specifically, if  $\sigma = 0$  then it sets  $c_0$  to be the ciphertext that is an encryption of 1, and sets  $c_1$  to be the ciphertext that is an encryption of 0. Otherwise, if  $\sigma = 1$  then it sets  $c_0$  to be the encryption of 0, and  $c_1$  to be the encryption of 1. (Only the ordering needs to be sent and not the actual ciphertexts. Furthermore, this can be sent together with the openings in Step 4b.)
6.  $S$  uses the homomorphic property and  $c_0, c_1$  as follows.
  - (a)  $S$  computes  $\tilde{c}_0 = x_0 \cdot_E c_0$  (this operation is relative to the key  $pk_1$  or  $pk_2$  depending if  $c_0$  is an encryption under  $pk_1$  or  $pk_2$ )
  - (b)  $S$  computes  $\tilde{c}_1 = x_1 \cdot_E c_1$  (this operation is relative to the key  $pk_1$  or  $pk_2$  depending if  $c_1$  is an encryption under  $pk_1$  or  $pk_2$ )

$S$  sends  $\tilde{c}_0$  and  $\tilde{c}_1$  to  $R$ . (Notice that one of the ciphertexts is encrypted with key  $pk_1$  and the other is encrypted with key  $pk_2$ .)
7. If  $\sigma = 0$ , the receiver  $R$  decrypts  $\tilde{c}_0$  and outputs the result (if  $\tilde{c}_0$  is encrypted under  $pk_1$  then  $R$  outputs  $x_0 = D_{sk_1}(\tilde{c}_0)$ ; otherwise it outputs  $x_0 = D_{sk_2}(\tilde{c}_0)$ ). Otherwise, if  $\sigma = 1$ ,  $R$  decrypts  $\tilde{c}_1$  and outputs the result.

8. If at any stage during the protocol,  $S$  does not receive the next message that it expects to receive from  $R$  or the message it receives is invalid and cannot be processed, it outputs  $\text{abort}_R$  (unless it was already instructed to output  $\text{corrupted}_R$ ). Likewise, if  $R$  does not receive the next message that it expects to receive from  $S$  or it receives an invalid message, it outputs  $\text{abort}_S$ .

We remark that the reordering message of Step 5 can actually be sent by  $R$  together with the message in Step 4b. Furthermore, the messages of the key-generation challenge can be piggybacked with later messages, as long as they conclude before the final step. We therefore have that the number of rounds of communication can be exactly *four* (each party sends two messages).

Before proceeding to the proof of security, we present the intuitive argument showing why Protocol 5.1 is secure. We begin with the case that the receiver is corrupt. First note that if the receiver follows the instructions of the protocol, it learns only a single value  $x_0$  or  $x_1$ . This is because one of  $c_0$  and  $c_1$  is an encryption of 0. If it is  $c_0$ , then  $\tilde{c}_0 = x_0 \cdot_E c_0 = E_{pk}(0 \cdot x_0) = E_{pk}(0)$  (where  $pk \in \{pk_1, pk_2\}$ , and so nothing is learned about  $x_0$ ; similarly if it is  $c_1$  then  $\tilde{c}_1 = E_{pk}(0)$  and so nothing is learned about  $x_1$ . However, in general, the receiver may not generate the encryptions  $c_0^1, c_1^1, c_0^2, c_1^2$  properly (and so it may be that at least one of the pairs  $(c_0^1, c_0^2)$  and  $(c_1^1, c_1^2)$  are *both* encryptions of 1, in which case the receiver could learn both  $x_0$  and  $x_1$ ). This is prevented by the encryption-generation challenge. That is, if the receiver tries to cheat in this way then it is guaranteed to be caught with probability at least  $1/2$ . The above explains why a malicious receiver can learn only one of the outputs, unless it is willing to be caught cheating with probability  $1/2$ . This therefore demonstrates that “privacy” holds. However, we actually need to prove security via simulation, which involves showing how to *extract* the receiver’s implicit input and how to *simulate* its view. Extraction works by first providing the corrupted receiver with the encryption-challenge bit  $b' = 0$  and then rewinding it and providing it with the challenge  $b' = 1$ . If the corrupted receiver replies to both challenges, then the simulator can construct  $\sigma$  from the opened ciphertexts and the reordering provided. Given this input, the simulation can be completed in a straightforward manner; see the proof below. A crucial point here is that if the receiver does not reply to both challenges then an honest sender would output  $\text{corrupted}_R$  with probability  $1/2$ , and so this corresponds to a  $\text{cheat}_R$  input in the ideal world.

We now proceed to discuss why the protocol is secure in the presence of a corrupt sender. In this case, it is easy to see that such a sender cannot learn anything about the receiver’s input because the encryption scheme is semantically secure (and so a corrupt sender cannot determine  $\sigma$  from the unopened ciphertexts). However, as above, we need to show how extraction and simulation works. Extraction here works by providing encryptions so that in one of the pairs  $(c_0^1, c_0^2)$  or  $(c_1^1, c_1^2)$  both of the encrypted values are 1. If this pair is the one used (and not the one opened), then we have that  $\tilde{c}_0$  is an encryption of  $x_0$  and  $\tilde{c}_1$  is an encryption of  $x_1$ . An important point here is that unlike a real receiver, the simulator can do this without being “caught”. Specifically, the simulator generates the ciphertexts so that for a random  $b' \in_R \{0, 1\}$  it holds that  $c_{1-b'}^1$  and  $c_{1-b'}^2$  are both encryptions of 1, whereas  $c_{b'}^1$  and  $c_{b'}^2$  are general correctly, one being an encryption of 0 and the other an encryption of 1. Then, the simulator “hopes” that the corrupted sender asks it to open the ciphertexts  $c_{b'}^1$  and  $c_{b'}^2$  which look as they should. In such a case, the simulator proceeds and succeeds in extracting both  $x_0$  and  $x_1$ . However, if the corrupted sender asks the simulator to open the other ciphertexts (that are clearly invalid), the simulator just rewinds the corrupted sender and tries again. Thus, extraction can be achieved. Regarding the simulation of the sender’s view, this follows from the fact that the only differences between the above and a real execution are the values encrypted in the ciphertexts  $c_0^1, c_0^2, c_1^1, c_1^2$ . These distributions are therefore indistinguishable by the semantic security of the encryption scheme.

We now formally prove that Protocol 5.1 meets Definition 3.5 with  $\epsilon = \frac{1}{2}$  (of course, this immediately implies security under Definitions 3.2 and 3.4 as well).

**Theorem 5.2** *Assuming that  $(G, E, D)$  constitutes a semantically secure homomorphic encryption scheme (with errorless decryption), Protocol 5.1 securely computes the oblivious transfer functionality  $((x_0, x_1), \sigma) \mapsto (\lambda, x_\sigma)$  in the presence of covert adversaries with  $\epsilon$ -deterrent for  $\epsilon = \frac{1}{2}$ , under Definition 3.5.*

**Proof:** We will separately consider the case that no parties are corrupted, the case that the receiver is corrupted and the case that the sender is corrupted (the case that both parties are corrupted is trivial). We note that although we construct three different simulators (one for each corruption case), a single simulator as required by the definition can be constructed by simply combining the three simulators into one machine, and working appropriately given the corruption set  $I$ .

**No corruptions.** We first consider the case that no parties are corrupted (i.e.,  $I = \phi$ ). In this case, the real adversary  $\mathcal{A}$ 's view can be generated by a simulator  $\text{Sim}$  that simply runs  $S$  and  $R$  honestly, with inputs  $x_0 = x_1 = 0^k$  and  $\sigma = 0$  (recall that in this case we assume that the adversary's auxiliary input contains the input length  $k$ ). The fact that this simulation is indistinguishable from a real execution (with the honest parties' real inputs) follows from the indistinguishability property of encryption scheme. The proof is straightforward and is therefore omitted. We remark that in order to show that the REAL and IDEAL outputs are indistinguishable, we also have to show that the honest parties' outputs in a real execution are correct (because this is the case in the ideal world). The sender's output is defined as  $\lambda$  and so this clearly holds. Regarding the receiver, recall that  $\tilde{c}_0 = x_0 \cdot_E c_0$  and  $\tilde{c}_1 = x_1 \cdot_E c_1$ . Thus, if  $\sigma = 0$  it holds  $c_0$  is an encryption of 1 and so  $\tilde{c}_0 = E_{pk_1}(x_0 \cdot 1) = E_{pk_1}(x_0)$ ; likewise, if  $\sigma = 1$  then  $c_1$  is an encryption of 1 and so  $\tilde{c}_1 = E_{pk_1}(x_1)$ . This implies that the receiver correctly obtains  $x_\sigma$  as required.

**Corrupted receiver:** Let  $\mathcal{A}$  be a real adversary that controls the receiver  $R$ . We construct a simulator  $\text{Sim}$  that works as follows:

1.  $\text{Sim}$  receives  $(\sigma, 1^{2k-1})$  and  $z$  as input and invokes  $\mathcal{A}$  on this input.
2.  $\text{Sim}$  plays the honest sender with  $\mathcal{A}$  as receiver.
3. When  $\text{Sim}$  reaches the key-generation challenge step, it first sends  $b = 0$  and receives back  $\mathcal{A}$ 's response. Then,  $\text{Sim}$  rewinds  $\mathcal{A}$ , sends  $b = 1$  and receives back  $\mathcal{A}$ 's response.
  - (a) If neither of the responses from  $\mathcal{A}$  are valid (where by validity we mean a response that would not cause  $S$  to output  $\text{corrupted}_R$  in a real execution),  $\text{Sim}$  sends  $\text{corrupted}_R$  to the trusted party, simulates the honest  $S$  aborting due to detected cheating, and outputs whatever  $\mathcal{A}$  outputs.
  - (b) If  $\mathcal{A}$  sends back exactly one valid response, then  $\text{Sim}$  sends  $\text{cheat}_R$  to the trusted party.
    - i. If the trusted party replies with  $\text{corrupted}_R$ , then  $\text{Sim}$  rewinds  $\mathcal{A}$  and hands it the query for which  $\mathcal{A}$ 's response was not valid.  $\text{Sim}$  then simulates the honest  $S$  aborting due to detected cheating, and outputs whatever  $\mathcal{A}$  outputs.
    - ii. If the trusted party replies with  $\text{undetected}$  and the honest  $S$ 's input pair  $(x_0, x_1)$ , then  $\text{Sim}$  plays the honest sender with input  $(x_0, x_1)$  in the remainder of the execution with  $\mathcal{A}$  as the receiver. At the conclusion,  $\text{Sim}$  outputs whatever  $\mathcal{A}$  outputs.
  - (c) If  $\mathcal{A}$  sends back two valid responses, then  $\text{Sim}$  rewinds  $\mathcal{A}$ , gives it a random  $b'$  and proceeds as below.



4. Sim receives ciphertexts  $c_0^1, c_0^2, c_1^1, c_1^2$  from  $\mathcal{A}$ .
5. Next, in the encryption-generation challenge step, Sim first sends  $b' = 0$  and receives back  $\mathcal{A}$ 's response, which includes the reordering of the ciphertexts (recall that the reordering message are actually sent together with the ciphertext openings). Then, Sim rewinds  $\mathcal{A}$ , sends  $b' = 1$  and receives back  $\mathcal{A}$ 's response.
  - (a) If neither of the responses from  $\mathcal{A}$  are valid (where by validity we mean a response that would not cause  $S$  to output  $\text{corrupted}_R$  in a real execution), Sim sends  $\text{corrupted}_R$  to the trusted party, simulates the honest  $S$  aborting due to detected cheating, and outputs whatever  $\mathcal{A}$  outputs.
  - (b) If  $\mathcal{A}$  sends back exactly one valid response, then Sim sends  $\text{cheat}_R$  to the trusted party.
    - i. If the trusted party replies with  $\text{corrupted}_R$ , then Sim rewinds  $\mathcal{A}$  and hands it the query for which  $\mathcal{A}$ 's response was not valid. Sim then simulates the honest  $S$  aborting due to detected cheating, and outputs whatever  $\mathcal{A}$  outputs.
    - ii. If the trusted party replies with  $\text{undetected}$  and the honest  $S$ 's input pair  $(x_0, x_1)$ , then Sim plays the honest sender with input  $(x_0, x_1)$  and completes the execution with  $\mathcal{A}$  as the receiver. (Note that the sender has not yet used its input at this stage of the protocol. Thus, Sim has no problem completing the execution like an honest sender.) At the conclusion, Sim outputs whatever  $\mathcal{A}$  outputs.
  - (c) If  $\mathcal{A}$  sends back two valid responses, then Sim uses the reorderings to determine the value of  $\sigma$ . Specifically, Sim chooses a random  $b'$  and takes the reordering that relates to  $c_{1-b'}^1$  and  $c_{1-b'}^2$  (if  $c_{1-b'}^1$  is an encryption of 1, then Sim determines  $\sigma = 0$  and otherwise it determines  $\sigma = 1$ ).  
Sim sends  $\sigma$  to the trusted party and receives back  $x = x_\sigma$ . Simulator Sim then completes the execution playing the honest sender and using  $x_0 = x_1 = x$ .
6. If at any point  $\mathcal{A}$  sends a message that would cause the honest sender to halt and output  $\text{abort}_R$ , simulator Sim immediately sends  $\text{abort}_R$  to the trusted party, halts the simulation and proceeds to the final "output" step.
7. *Output:* At the conclusion, Sim outputs whatever  $\mathcal{A}$  outputs.

This completes the description of Sim. Denoting  $\pi$  as Protocol 5.1 and noting that  $I$  here equals  $\{R\}$  (i.e., the receiver is corrupted), we need to prove that for  $\epsilon = \frac{1}{2}$ ,

$$\left\{ \text{IDEALSC}_{\text{OT}, S(z), I}^\epsilon(((x_0, x_1), \sigma), n) \right\} \stackrel{c}{\equiv} \left\{ \text{REAL}_{\pi, \mathcal{A}(z), I}(((x_0, x_1), \sigma), n) \right\}$$

It is clear that the simulation is perfect if Sim sends  $\text{corrupted}_R$  or  $\text{cheat}_R$  at any stage. This is due to the fact that the probability that an honest  $S$  outputs  $\text{corrupted}_R$  in the simulation is identical to the probability in a real execution (probability 1 in the case that  $\mathcal{A}$  responds incorrectly to both challenges and probability 1/2 otherwise). Furthermore, in the case that Sim sends  $\text{cheat}_R$  and receives back  $\text{undetected}$  it concludes the execution using the true input of the sender. The simulation until the last step is perfect (it involves merely sending random challenges); therefore the completion using the true sender's input yields a perfect simulation. The above is clearly true of  $\text{abort}_R$  as well (because this can only occur before the last step where the sender's input is used).

It remains to analyze the case that Sim does not send  $\text{corrupted}_R$ ,  $\text{cheat}_R$  or  $\text{abort}_R$  to the trusted party. Notice that in this case,  $\mathcal{A}$  responded correctly to both the key-generation challenges and the

encryption-generation challenges. In particular, this implies that the keys  $pk_1$  and  $pk_2$  are correctly generated, and that  $\text{Sim}$  computes  $\sigma$  based on the encrypted values sent by  $\mathcal{A}$  and the reordering.

Now, if  $\sigma = 0$ , then  $\text{Sim}$  hands  $\mathcal{A}$  the ciphertexts  $\tilde{c}_0 = E_{pk}(x_0)$  and  $\tilde{c}_1 = E_{pk}(0)$ , where  $pk, pk' \in \{pk_1, pk_2\}$  and  $pk \neq pk'$ , and if  $\sigma = 1$ , it hands  $\mathcal{A}$  the ciphertexts  $\tilde{c}_0 = E_{pk}(0)$  and  $\tilde{c}_1 = E_{pk'}(x_1)$ . This follows from the instructions of  $\text{Sim}$  and the honest party ( $\text{Sim}$  plays the honest party with  $x_0 = x_1 = x_\sigma$  and so  $\tilde{c}_\sigma$  is an encryption of  $x_\sigma$  and  $\tilde{c}_{1-\sigma}$  is an encryption of 0). The important point to notice is that these messages are distributed identically to the honest sender's messages in a real protocol; the fact that  $\text{Sim}$  does not know  $x_{1-\sigma}$  makes no difference because for every  $x'$  it holds that  $x' \cdot E_{pk}(0) = E_{pk}(0)$ . We note that this assumes that the homomorphic property of the encryption scheme holds, but this is given by the fact that  $pk_1$  and  $pk_2$  are correctly formed. Regarding the rest of the messages sent by  $\text{Sim}$ , these are generated independently of the sender-input and so exactly like an honest sender.

We conclude that the view of  $\mathcal{A}$  as generated by the simulator  $\text{Sim}$  is *identical* to the distribution generated in a real execution. Thus, its output is identically distributed in both cases. (Since the sender receives no output, we do not need to consider the output distribution of the honest sender in the real and ideal executions.) We conclude that

$$\left\{ \text{IDEALSC}_{\text{OT}, S(z), I}^\epsilon(((x_0, x_1), \sigma), n) \right\} \equiv \left\{ \text{REAL}_{\pi, \mathcal{A}(z), I}(((x_0, x_1), \sigma), n) \right\}$$

completing this corruption case.

**Corrupted sender:** Let  $\mathcal{A}$  be a real adversary that controls the sender  $S$ . We construct a simulator  $\text{Sim}$  that works as follows:

1.  $\text{Sim}$  receives  $(x_0, x_1)$  and  $z$  and invokes  $\mathcal{A}$  on this input.
2.  $\text{Sim}$  interacts with  $\mathcal{A}$  and plays the honest receiver until Step 3 of the protocol.
3. In Step 3 of the protocol,  $\text{Sim}$  works as follows:
  - (a)  $\text{Sim}$  chooses random bits  $b, \alpha \in_R \{0, 1\}$
  - (b)  $\text{Sim}$  computes:
$$\begin{aligned} c_b^1 &= E_{pk_1}(\alpha) & c_b^2 &= E_{pk_2}(1 - \alpha) \\ c_{1-b}^1 &= E_{pk_1}(1) & c_{1-b}^2 &= E_{pk_2}(1) \end{aligned}$$
  - (c)  $\text{Sim}$  sends  $c_0^1, c_0^2, c_1^1, c_1^2$  to  $\mathcal{A}$ .
4. In the next step (Step 4 of the protocol),  $\mathcal{A}$  sends a bit  $b'$ . If  $b' = b$ , then  $\text{Sim}$  opens the ciphertexts  $c_b^1$  and  $c_b^2$  as the honest receiver would (note that the ciphertexts are “correctly” constructed). Otherwise,  $\text{Sim}$  returns to Step 3 of the simulation above (i.e., it returns to the beginning of Step 3 of the protocol) and tries again with fresh randomness.<sup>7</sup>
5.  $\text{Sim}$  sends a random reordering of the ciphertexts  $c_{1-b}^1$  and  $c_{1-b}^2$  (the actual order doesn't matter because they are both encryptions of 1).

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<sup>7</sup>This yields an expected polynomial-time simulation because these steps are repeated until  $b' = b$ . A strict polynomial-time simulation can be achieved by just halting after  $n$  attempts. The probability that  $b' \neq b$  in all of these attempts can be shown to be negligible, based on the hiding property of the encryption scheme.

6. The simulator  $\text{Sim}$  receives from  $\mathcal{A}$  the ciphertexts  $\tilde{c}_0$  and  $\tilde{c}_1$ .  $\text{Sim}$  computes  $x_0 = D_{sk_1}(\tilde{c}_0)$  and  $x_1 = D_{sk_2}(\tilde{c}_1)$  (or  $x_0 = D_{sk_2}(\tilde{c}_0)$  and  $x_1 = D_{sk_1}(\tilde{c}_1)$ , depending on which of  $c_0, c_1$  is encrypted with  $pk_1$  and which with  $pk_2$ ), and sends the pair  $(x_0, x_1)$  to the trusted party as  $S$ 's input.
7. If at any stage in the simulation  $\mathcal{A}$  does not respond, or responds with an invalid message that cannot be processed, then  $\text{Sim}$  sends  $\text{abort}_S$  to the trusted party for the sender's inputs. (Such behavior from  $\mathcal{A}$  can only occur before the last step and so before any input  $(x_0, x_1)$  has already been sent to the trusted party.)
8.  $\text{Sim}$  outputs whatever  $\mathcal{A}$  outputs.

Notice that  $\text{Sim}$  never sends  $\text{cheat}_S$  to the trusted party. Thus we actually prove standard security in this corruption case. That is, we prove that:

$$\left\{ \text{IDEAL}_{\text{OT}, \text{Sim}(z), I}((x_0, x_1, \sigma), n) \right\} \stackrel{c}{\equiv} \left\{ \text{REAL}_{\pi, \mathcal{A}(z), I}((x_0, x_1, \sigma), n) \right\} \quad (5)$$

By Proposition 3.8, this implies security for covert adversaries as well. In order to prove Eq. (5), observe that the only difference between the view of the adversary  $\mathcal{A}$  in a real execution and in the simulation by  $\text{Sim}$  is due to the fact that  $\text{Sim}$  does not generate  $c_b^1, c_b^2$  correctly. Thus, intuitively, Eq. (5) follows from the security of the encryption scheme. That is, we begin by showing that if the view of  $\mathcal{A}$  in the real and ideal executions can be distinguished, then it is possible to break the security of the encryption scheme. We begin by showing that the view of  $\mathcal{A}$  when interacting with an honest sender with input  $\sigma = 0$  is indistinguishable from the view of  $\mathcal{A}$  when interacting in a simulation with  $\text{Sim}$ .

Let  $\mathcal{A}'$  be an adversary that attempts to distinguish encryptions under a key  $pk$ .<sup>8</sup> Adversary  $\mathcal{A}'$  receives a key  $pk$ , chooses a random bit  $\gamma \in_R \{0, 1\}$  and a random index  $\ell \in_R \{1, 2\}$  and sets  $pk_\ell^{1-\gamma} = pk$ . It then chooses the keys  $pk_{3-\ell}^{1-\gamma}, pk_1^\gamma$  and  $pk_2^\gamma$  by itself and sends  $\mathcal{A}$  the keys  $(pk_1^0, pk_2^0)$  and  $(pk_1^1, pk_2^1)$ . When  $\mathcal{A}$  replies with a bit  $b$ , adversary  $\mathcal{A}'$  acts as follows. If  $b = \gamma$ , then  $\mathcal{A}'$  opens the randomness used in generating  $(pk_1^b, pk_2^b)$  as the honest receiver would ( $\mathcal{A}'$  can do this because it chose  $(pk_1^\gamma, pk_2^\gamma)$  by itself and  $\gamma = b$ ). If  $b \neq \gamma$ , then  $\mathcal{A}'$  cannot open the randomness as an honest receiver would. Therefore,  $\mathcal{A}'$  just halts. If  $\mathcal{A}$  continues, then it sets  $pk_1 = pk_1^{1-\gamma}$  and  $pk_2 = pk_2^{1-\gamma}$  (and so  $pk_\ell$  is the public-key  $pk$  that  $\mathcal{A}'$  is “attacking”). Now,  $\mathcal{A}'$  computes the ciphertexts  $c_0^1, c_0^2, c_1^1, c_1^2$  in the following way.  $\mathcal{A}'$  chooses  $\alpha$  and  $\beta$  at random, as the honest receiver would. Then, for a random  $\zeta$  adversary  $\mathcal{A}'$  computes  $c_\zeta^1 = E_{pk_1}(\alpha)$ ,  $c_\zeta^2 = E_{pk_2}(1 - \alpha)$ , and  $c_{1-\zeta}^{3-\ell} = E_{pk_{3-\ell}}(1)$ . However,  $\mathcal{A}'$  does not compute  $c_{1-\zeta}^\ell = E_{pk_\ell}(1)$ . Rather, it outputs a pair of plaintexts  $m_0 = 0, m_1 = 1$  and receives back  $c = E_{pk}(m_b) = E_{pk_\ell}(m_b)$  (for  $b \in_R \{0, 1\}$ ). Adversary  $\mathcal{A}'$  sets  $c_{1-\zeta}^\ell = c$  (i.e., to equal the challenge ciphertext) and continues playing the honest receiver until the end. In this simulation,  $\mathcal{A}'$  sets the reordering so that  $c_0$  equals  $c_{1-\zeta}^{3-\ell}$  (that is, it is an encryption of 1). The key point here is that if  $\mathcal{A}'$  does not halt and  $b = 0$ , then the simulation by  $\mathcal{A}'$  is identical to a real execution between  $\mathcal{A}$  and an honest receiver  $R$  who has input  $\sigma = 0$  (because  $c_0 = c_{1-\zeta}^{3-\ell}$  is an encryption of 1 and  $c_1 = c_{1-\zeta}^\ell$  is an encryption of 0, as required). In contrast, if  $\mathcal{A}'$  does not halt and  $b = 1$ , then the simulation by  $\mathcal{A}'$  is identical to the simulation carried out by  $\text{Sim}$  (because in this case they are both encryptions of 1). Finally, note that  $\mathcal{A}'$  halts with probability exactly  $1/2$  in both cases (this is due to the fact that the distribution of the keys is identical for

<sup>8</sup>The game that  $\mathcal{A}'$  plays is that it receives a key  $pk$ , outputs a pair of plaintexts  $m_0, m_1$ , receives back a *challenge ciphertext*  $E_{pk}(m_b)$  for some  $b \in \{0, 1\}$ , and outputs a “guess” bit  $b'$ . An encryption scheme is indistinguishable if the probabilities that  $\mathcal{A}'$  outputs  $b' = 1$  when  $b = 1$  and when  $b = 0$  are negligibly close.

both choices of  $\gamma$ ). Combining the above together, we have that if it is possible to distinguish the view of  $\mathcal{A}$  in the simulation by  $\text{Sim}$  from a real execution with a receiver who has input 0, then it is possible to distinguish encryptions. Specifically,  $\mathcal{A}'$  can just run the distinguisher that exists for these views and output whatever the distinguisher outputs.

The above shows that the view of  $\mathcal{A}$  in the simulation is indistinguishable from its view in a real execution with an honest receiver with input  $\sigma = 0$ . However, we actually have to show that when the honest receiver has input  $\sigma = 0$ , the *joint distribution* of  $\mathcal{A}$  and the honest receiver's outputs in a real execution is indistinguishable from the joint distribution of  $\text{Sim}$  and the honest receiver's outputs in the ideal model. The point to notice here is that the output of the honest receiver in both the real and ideal models is the value obtained by decrypting  $\tilde{c}_0$  using key  $pk_{3-\ell}$ . (In the real model this is what the protocol instructs the honest party to output and in the ideal model this is the value that  $\text{Sim}$  sends to the trusted party as the sender's input  $x_0$ .) However, in this reduction  $\mathcal{A}'$  knows the associated secret-key to  $pk_{3-\ell}$ , because it chose  $pk_{3-\ell}$  itself. Thus,  $\mathcal{A}'$  can append the decryption of  $\tilde{c}_0$  to the view of  $\mathcal{A}$ , thereby generating a joint distribution. It follows that if  $\mathcal{A}'$  received an encryption of  $m_0 = 0$  then it generates the joint distribution of the outputs in the real execution, and if it received an encryption of  $m_1 = 1$  then it generates the joint distribution of the outputs in the ideal execution. By the indistinguishability of the encryption scheme we have the real and ideal distributions are indistinguishable, completing the proof of Eq. (5) for the case that  $\sigma = 0$ . The case for  $\sigma = 1$  follows from an almost identical argument as above. Combining these two cases, we have the output distribution generated by the simulator in the ideal model is computationally indistinguishable from the output distribution of a real execution. It remains to show that  $\text{Sim}$  runs in expected polynomial-time. Note that  $\text{Sim}$  rewinds if in the simulation it holds that  $b' \neq b$ . Now, in the case that the ciphertexts  $c_0^1, c_0^2, c_1^1, c_1^2$  are generated as by the honest party (each pair containing an encryption of 0 and an encryption of 1), the probability that  $b' \neq b$  is exactly  $1/2$  because the value of  $b'$  is information-theoretically hidden. In contrast, in the simulation this is not the case because  $c_b^1, c_b^2$  are "correctly" constructed, whereas  $c_{1-b}^1, c_{1-b}^2$  are both encryptions of 1. Nevertheless, if the probability that  $b' \neq b$  is non-negligibly far from  $1/2$ , then this can be used to distinguish an encryption of 0 from an encryption of 1 (the actual reduction can be derived from the reduction already carried out above and is thus omitted). It follows that the expected number of rewinds is at most slightly greater than 2, implying that the overall simulation runs in expected polynomial-time. As we have mentioned in Footnote 7, the simulation can be made to run in strict polynomial-time by aborting if for  $n$  consecutive trials it holds that  $b' \neq b$ . By the argument given above, such an abort can only occur with negligible probability. This concludes the proof of this corruption case, and thus of the theorem. ■

**Discussion.** The proof of Protocol 5.1 in the case that the receiver is corrupted relies heavily on the fact that the simulator can send `cheat` and therefore does not need to complete a "standard" simulation. Take for example the case that  $\mathcal{A}$  (controlling the receiver) only replies with one valid response to the encryption-generation challenge. In this case, the receiver can learn both  $x_0$  and  $x_1$  with probability  $1/2$ . However, the simulator in the ideal model can never learn both  $x_0$  and  $x_1$ . Therefore, the simulator cannot generate the correct distribution. However, by allowing the simulator to declare a cheat, it can complete the simulation as required. This demonstrates why it is possible to achieve higher efficiency for this definition of security.

**The proof of security for a corrupted sender.** We stress that we have actually proven something stronger. Specifically, we have shown that Protocol 5.1 is secure in the presence of a covert receiver with  $1/2$ -deterrent as stated. However, we have also shown that Protocol 5.1 is (fully) secure with abort in the presence of a *malicious sender*.

**Efficiently recognizable public keys.** We remark that in the case that it is possible to efficiently recognize that a public-key is in the range of the key-generator of the public-key encryption scheme, it is possible to skip the key-generation challenge step in the protocol (the sender can verify for itself if the key is valid).

## 5.2 Extensions

**String oblivious transfer.** In Protocol 5.1,  $x_0$  and  $x_1$  are elements in the group over which the homomorphic encryption scheme is defined. If this group is large, then we can carry out string oblivious transfer. This is important because later we will use Protocol 5.1 to exchange symmetric encryption keys. However, if the group contains only 0 and 1, then this does not suffice. In order to extend Protocol 5.1 to deal with string oblivious transfer, even when the group has only two elements, we only need to change the last two steps of the protocol. Specifically, instead of  $S$  computing a single encryption for  $x_0$  and a single encryption for  $x_1$ , it computes an encryption for each bit. That is, denote the bits of  $x_0$  by  $x_0^1, \dots, x_0^n$ , and likewise for  $x_1$ . Then,  $S$  computes:

$$\tilde{c}_0 = x_0^1 \cdot_E c_0, \dots, x_0^n \cdot_E c_0$$

and

$$\tilde{c}_1 = x_1^1 \cdot_E c_1, \dots, x_1^n \cdot_E c_1 .$$

Note that the receiver can still only obtain one of the strings because if  $\sigma = 0$  then  $\tilde{c}_1$  just contains encryptions to zeroes, and vice versa if  $\sigma = 1$ .

**Simultaneous oblivious transfer.** We will use Protocol 5.1 in Yao’s protocol for secure two-party computation. This means that we will run one oblivious transfer for every bit of the input. In principle, these oblivious transfers can be run in parallel, as long as the protocol being used remains secure under parallel composition. Technically, however, our protocol does not compose in parallel (in the sense that the honest party views each execution independently). Nevertheless, we can modify Protocol 5.1 so that it is possible to simultaneously run many oblivious transfers with a cost that is *less* than running Protocol 5.1 the same number of times in parallel. We call this *simultaneous oblivious transfer* in order to distinguish it from “parallel oblivious transfer” which considers (stateless) parallel composition. The simultaneous oblivious transfer functionality is defined as follows:

$$((x_1^0, x_1^1), \dots, (x_n^0, x_n^1), (\sigma_1, \dots, \sigma_n)) \mapsto (\lambda, (x_1^{\sigma_1}, \dots, x_n^{\sigma_n}))$$

Thus, we essentially have  $n$  oblivious transfers where in the  $i^{\text{th}}$  such transfer, the sender has input  $(x_i^0, x_i^1)$  and the receiver has input  $\sigma_i$ .

The extension to Protocol 5.1 works as follows. First, the same public-key pair  $(pk_1, pk_2)$  can be used in all executions. Therefore, Steps 1 and 2 remain unchanged. Then, Step 3 is carried out *independently* for all  $n$  bits  $\sigma_1, \dots, \sigma_n$ . That is, for every  $i$ , two pairs of ciphertexts encrypting 0 and 1 (in random order) are sent. The important change comes in Step 4. Here, the *same* challenge bit  $b'$  is used for every  $i$ . The sender then replies as it should, opening the  $c_b^1$  and  $c_b^2$  ciphertexts for every  $i$ . The protocol then concludes by the sender computing the  $\tilde{c}_0$  and  $\tilde{c}_1$  ciphertexts for every  $i$ , and the receiver decrypting.

The proof of the above extension is almost identical to the proof of Theorem 5.2. The main point is that since only a single challenge is used for both the key-generation challenge and encryption-generation challenge, the probability of achieving  $b' = b$  (as needed for the simulation) and  $b = \gamma$  (as

needed for the reduction to the security of the encryption scheme) remains one half. Furthermore, the probability that a corrupted  $R$  will succeed in cheating remains the same because if there is any  $i$  for which the encryptions are not correctly formed, then the receiver will be caught with probability one half.

**Higher values of  $\epsilon$ .** Finally, we show how it is possible to obtain higher values of  $\epsilon$  with only minor changes to Protocol 5.1. The basic idea is to increase the probability of catching a corrupted receiver in the case that it attempts to generate an invalid key-pair or send ciphertexts in Step 3 that do not encrypt the same value. Let  $k = \text{poly}(n)$  be an integer. Then, first the receiver generates  $k$  pairs of public-keys  $(pk_1^1, pk_2^1), \dots, (pk_1^k, pk_2^k)$  instead of just two pairs. The sender then asks the receiver to reveal the randomness used in generating all the pairs except for one (the unrevealed key-pair is the one used in the continuation of the protocol). Note that if a corrupted receiver generated even one key-pair incorrectly, then it is caught with probability  $1 - 1/k$ . Likewise, in Step 3, the receiver sends  $k$  pairs of ciphertexts where in each pair one ciphertext is an encryption of 0 and the other an encryption of 1. Then, the sender asks the receiver to open all pairs of encryptions of  $\sigma_i$  except for one pair. Clearly, the sender still learns nothing about  $\sigma$  because the reordering is only sent on the ciphertext pair that is not opened. Furthermore, if the receiver generates even one pair of ciphertexts so that the ciphertexts are not correctly formed, then it will be caught with probability  $1 - 1/k$ . The rest of the protocol remains the same. We conclude that the resulting protocol is secure in the presence of covert adversaries with  $\epsilon$ -deterrent where  $\epsilon = 1 - 1/k$ . Notice that this works as long as  $k$  is polynomial in the security parameter and thus  $\epsilon$  can be made to be very close to 1, if desired. (Of course, this methodology *cannot* be used to make  $\epsilon$  negligibly close to 1, because then  $k$  has to be super-polynomial.)

**Summary.** We conclude with the following theorem, derived by combining the extensions above:

**Theorem 5.3** *Assume that there exist semantically secure homomorphic encryption schemes with errorless decryption. Then, for any  $k = \text{poly}(n)$  there exists a protocol that securely computes the simultaneous string oblivious transfer functionality*

$$((x_1^0, x_1^1), \dots, (x_n^0, x_n^1), (\sigma_1, \dots, \sigma_n)) \mapsto (\lambda, (x_1^{\sigma_1}, \dots, x_n^{\sigma_n}))$$

*in the presence of covert adversaries with  $\epsilon$ -deterrent for  $\epsilon = 1 - \frac{1}{k}$ . Furthermore, the protocol has four rounds of communication, and involves generating  $2k$  encryption keys, carrying out  $2kn$  encryption operations,  $2n$  homomorphic multiplications and  $n$  decryptions.*

Note that the amortized complexity of each oblivious transfer is:  $2k$  encryptions, 2 scalar multiplications with the homomorphic encryption scheme and 1 decryption. (The key generation which is probably the most expensive is run  $2k$  times independently of  $n$ . Therefore, when many oblivious transfers are run, this becomes insignificant.)

## 6 Secure Two-Party Computation

In this section, we show how to securely compute any two-party functionality in the presence of covert adversaries. We present a protocol for the strong explicit cheat formulation, with parameters that can be set to obtain a wide range of values for the  $\epsilon$ -deterrent. Our protocol is based on Yao's protocol for semi-honest adversaries [27]. We will base our description on the write-up of [22] of this protocol, and will assume familiarity with it. Nevertheless, in Appendix A, we briefly describe Yao's garbled circuit construction and present an important lemma regarding it.

**Overview of the protocol.** The original protocol of Yao is not secure when the parties may be malicious. Intuitively, there are two main reasons for this. First, the circuit constructor  $P_1$  may send  $P_2$  a garbled circuit that computes a completely different function. Second, the oblivious transfer protocol that is used when the parties can be malicious must be secure for this case. The latter problem is solved here by using the protocol guaranteed by Theorem 5.3. The first problem is solved by having  $P_1$  send  $P_2$  a number of garbled circuits; denote this number by  $\ell$ . Then,  $P_2$  asks  $P_1$  to open all but one of the circuits (chosen at random) in order to check that they are correctly constructed. This opening takes place before  $P_1$  sends the keys corresponding to its input, so nothing is revealed by opening the circuits. The protocol then proceeds similarly to the semi-honest case. The main point here is that if the unopened circuit is correct, then this will constitute a secure execution that can be simulated. However, if it is not correct, then with probability  $1 - 1/\ell$  party  $P_1$  will have been caught cheating and so  $P_2$  will output `corrupted1` (recall,  $\ell$  denotes the number of circuits sent). While the above intuition forms the basis for our protocol, the actual construction of the appropriate simulator is somewhat delicate, and requires a careful construction of the protocol. We note some of these subtleties hereunder.

First, it is crucial that the oblivious transfers are run before the garbled circuits are sent by  $P_1$  to  $P_2$ . This is due to the fact that the simulator sends a corrupted  $P_2$  a fake garbled circuit that evaluates to the exact output received from the trusted party (and only this output), as described in Lemma A.1. However, in order for the simulator to receive the output from the trusted party, it must first send it the input used by the corrupted  $P_2$ . This is achieved by first running the oblivious transfers, from which the simulator is able to extract the corrupted  $P_2$ 's input.

The second subtlety relates to an issue we believe may be a problem for many other implementations of Yao that use cut-and-choose. The problem is that the adversary can construct (at least in theory) a garbled circuit with two sets of keys, where one set of keys decrypt the circuit to the specified one and another set of keys decrypt the circuit to an incorrect one. This is a problem because the adversary can supply “correct keys” to the circuits that are opened and “incorrect keys” to the circuit that is computed. Such a strategy cannot be carried out without risk of detection for the keys that are associated with  $P_2$ 's input because these keys are obtained by  $P_2$  in the oblivious transfers *before* the garbled circuits are even sent (thus if incorrect keys are sent for one of the circuits,  $P_2$  will detect this if that circuit is opened). However, it is possible for a corrupt  $P_1$  to carry out this strategy for the input wires associated with its own input. We prevent this by having  $P_1$  commit to these keys and send the commitments together with the garbled circuits. Then, instead of  $P_1$  just sending the keys associated with its input, it sends the appropriate decommitments.

A third subtlety that arises is connected to the difference between Definitions 3.2 and 3.4 (where the latter is the stronger definition where the decision by the adversary to cheat is not allowed to depend on the honest parties' inputs or on the output). Consider a corrupted  $P_1$  that behaves exactly like an honest  $P_1$  except that in the oblivious transfers, it inputs an invalid key in the place of the key associated with 0 as the first bit of  $P_2$ . The result is that if the first bit of  $P_2$ 's input is 1, then the protocol succeeds and no problem arises. However, if the first bit of  $P_2$ 's input is 0, then the protocol will always fail and  $P_2$  will always detect cheating. Thus,  $P_1$ 's decision to cheat may depend on  $P_2$ 's private input, something that is impossible in the ideal models of Definitions 3.4 and 3.5. In summary, this means that such a protocol achieves Definition 3.2 (with  $\epsilon = 1/\ell$ ) but not Definition 3.4. In order to solve this problem, we use a circuit that computes the function  $g(x_1, x_2^1, \dots, x_2^m) = f(x_1, \oplus_{i=1}^m x_2^i)$ , instead of a circuit that directly computes  $f$ . Then, upon input  $x_2$ , party  $P_2$  chooses random  $x_2^1, \dots, x_2^{m-1}$  and sets  $x_2^m = (\oplus_{i=1}^{m-1} x_2^i) \oplus x_2$ . This makes no difference to the result because  $\oplus_{i=1}^m x_2^i = x_2$  and so  $g(x_1, x_2^1, \dots, x_2^m) = f(x_1, x_2)$ . However, this modification makes every bit of  $P_2$ 's input uniform when considering any proper subset of  $x_2^1, \dots, x_2^m$ . This helps

because as long as  $P_1$  does not provide invalid keys for all  $m$  shares of  $x_2$ , the probability of failure is independent of  $P_2$ 's actual input (because any set of  $m - 1$  shares is independent of  $x_2$ ). Since  $m - 1$  invalid shares are detected with probability  $1 - 2^{-m+1}$  we have that  $P_2$  detects the cheating by  $P_1$  with this probability, independently of its input value. This method was previously used in [23] (however, there they must set  $m$  to equal the security parameter).

Intuitively, an adversary can cheat by providing an incorrect circuit or by providing invalid keys for shares. However, it is then detected with the probabilities described above. Below, we show that when using  $\ell$  circuits and splitting  $P_2$ 's input into  $m$  shares, we obtain  $\epsilon = (1 - 1/\ell)(1 - 2^{-m+1})$ . This enables us to play around with the values of  $m$  and  $\ell$  in order to optimize efficiency versus  $\epsilon$ -deterrent. For example, if we wish to obtain  $\epsilon = 1/2$  we can use the following parameters:

1. *Set  $\ell = 2$  and  $m = n$ :* This yields  $\epsilon = (1 - 1/2)(1 - 2^{-n+1})$  which is negligibly close to  $1/2$ . However, since in Yao's protocol we need to run an oblivious transfer for every one of  $P_2$ 's input bits, this incurs a blowup of the number of oblivious transfers (and thus exponentiations) by  $n$ . Thus, this setting of parameters results in a considerable computational blowup.
2. *Set  $\ell = 3$  and  $m = 3$ :* This yields  $\epsilon = (1 - 1/3)(1 - 1/4) = 1/2$ . The computational cost incurred here is much less than before because we only need 3 oblivious transfers for each of  $P_2$ 's input bits. Furthermore, the cost of sending 3 circuits is not much greater than 2, and so the overall complexity is much better.

Before proceeding to the protocol, we provide one more example of parameters. In order to achieve  $\epsilon = 9/10$  it is possible to set  $\ell = 25$  and  $m = 5$  (setting  $\ell = m = 10$  gives 0.898 which is very close). This gives a significantly higher value of  $\epsilon$ . We remark that such a setting of  $\epsilon$  also assumes a value of  $\epsilon = 9/10$  for the oblivious transfer protocol. As we have seen, this involves a blowup of 5 times more computation than for oblivious transfer with  $\epsilon = 1/2$ .

We are now ready to describe the actual protocol.

**Protocol 6.1** (two-party computation of a function  $f$ ):

- **Inputs:** Party  $P_1$  has input  $x_1$  and party  $P_2$  has input  $x_2$ , where  $|x_1| = |x_2|$ . In addition, both parties have parameters  $\ell$  and  $m$ , and a security parameter  $n$ . For simplicity, we will assume that the lengths of the inputs are  $n$ .
- **Auxiliary input:** Both parties have the description of a circuit  $C$  for inputs of length  $n$  that computes the function  $f$ . The input wires associated with  $x_1$  are  $w_1, \dots, w_n$  and the input wires associated with  $x_2$  are  $w_{n+1}, \dots, w_{2n}$ .
- **The protocol:**
  1. Parties  $P_1$  and  $P_2$  define a new circuit  $C'$  that receives  $m + 1$  inputs  $x_1, x_2^1, \dots, x_2^m$  each of length  $n$ , and computes the function  $f(x_1, \oplus_{i=1}^m x_2^i)$ . Note that  $C'$  has  $n + mn$  input wires. Denote the input wires associated with  $x_1$  by  $w_1, \dots, w_n$ , and the input wires associated with  $x_2^i$  by  $w_{n+(i-1)m+1}, \dots, w_{n+im}$ , for  $i = 1, \dots, m$ .
  2. Party  $P_2$  chooses  $m - 1$  random strings  $x_2^1, \dots, x_2^{m-1} \in_R \{0, 1\}^n$  and defines  $x_2^m = (\oplus_{i=1}^{m-1} x_2^i) \oplus x_2$ , where  $x_2$  is  $P_2$ 's original input (note that  $\oplus_{i=1}^m x_2^i = x_2$ ). The value  $z_2 \stackrel{\text{def}}{=} x_2^1, \dots, x_2^m$  serves as  $P_2$ 's new input of length  $mn$  to  $C'$ . (The input wires associated with  $P_2$ 's new input are  $w_{n+1}, \dots, w_{n+mn}$ .)



3. For each  $i = 1, \dots, mn$  and  $\beta = 0, 1$ , party  $P_1$  chooses  $\ell$  encryption keys by running  $G(1^n)$ , the key generator for the encryption scheme,  $\ell$  times. The  $j^{\text{th}}$  key associated with a given  $i$  and  $\beta$  is denoted  $k_{w_{n+i},\beta}^j$ ; note that this is the key associated with the bit  $\beta$  for the input wire  $w_{n+i}$  in the  $j^{\text{th}}$  circuit. The result is an  $\ell$ -tuple, denoted:

$$[k_{w_{n+i},\beta}^1, \dots, k_{w_{n+i},\beta}^\ell]$$

(This tuples constitutes the keys that are associated with the bit  $\beta$  for the input wire  $w_{n+i}$  in all  $\ell$  circuits.)

4.  $P_1$  and  $P_2$  run  $mn$  executions of an oblivious transfer protocol, as follows. In the  $i^{\text{th}}$  execution, party  $P_1$  inputs the pair

$$\left( [k_{w_{n+i},0}^1, \dots, k_{w_{n+i},0}^\ell], [k_{w_{n+i},1}^1, \dots, k_{w_{n+i},1}^\ell] \right)$$

and party  $P_2$  inputs the bit  $z_2^i$  ( $P_2$  receives the keys  $[k_{w_{n+i},z_2^i}^1, \dots, k_{w_{n+i},z_2^i}^\ell]$  as output). The executions are run using a simultaneous oblivious transfer functionality, as in Theorem 5.3. If a party receives a `corruptedi` or `aborti` message as output from the oblivious transfer, it outputs it and halts.

5. Party  $P_1$  constructs  $\ell$  garbled circuits  $GC_1, \dots, GC_\ell$  using independent randomness (the circuits are garbled versions of  $C'$  described above). The keys for the input wires  $w_{n+1}, \dots, w_{n+mn}$  in the garbled circuits are taken from above (i.e., in  $GC_j$  the keys associated with  $w_{n+i}$  are  $k_{w_{n+i},0}^j$  and  $k_{w_{n+i},1}^j$ ). The keys for the inputs wires  $w_1, \dots, w_n$  are chosen randomly, and are denoted in the same way.

$P_1$  sends the  $\ell$  garbled circuits to  $P_2$ .

6.  $P_1$  commits to the keys associated with its inputs. That is, for every  $i = 1, \dots, n$ ,  $\beta = 0, 1$  and  $j = 1, \dots, \ell$ , party  $P_1$  computes

$$c_{w_i,\beta}^j = \text{Com}(k_{w_i,\beta}^j; r_{i,\beta}^j)$$

where  $\text{Com}$  is a perfectly-binding commitment scheme,  $\text{Com}(x; r)$  denotes a commitment to  $x$  using randomness  $r$ , and  $r_{i,\beta}^j$  is a random string of sufficient length to commit to a key of length  $n$ .

$P_1$  sends all of the above commitments. The commitments are sent as  $\ell$  vectors of pairs (one vector for each circuit); in the  $j^{\text{th}}$  vector the  $i^{\text{th}}$  pair is  $\{c_{w_i,0}^j, c_{w_i,1}^j\}$  in a random order (the order is randomly chosen independently for each pair).

7. Party  $P_2$  chooses a random index  $\gamma \in_R \{1, \dots, \ell\}$  and sends  $\gamma$  to  $P_1$ .
8.  $P_1$  sends  $P_2$  all of the keys for the input wires in all garbled circuits except for  $GC_\gamma$  (this enables a complete decryption of the garbled circuit), together with the associated mappings and the decommitment values. (I.e. for every  $i = 1, \dots, n + mn$  and  $j \neq \gamma$ , party  $P_1$  sends the keys and mappings  $(k_{w_i,0}^j, 0), (k_{w_i,1}^j, 1)$ . In addition, for every  $i = 1, \dots, n$  and  $j \neq \gamma$  it sends the decommitments  $r_{i,0}^j, r_{i,1}^j$ .)

9.  $P_2$  checks that everything that it received is in order. That is, it checks:

– That the keys it received for all input wires in circuits  $GC_j$  ( $j \neq \gamma$ ) indeed decrypt the circuits (when using the received mappings), and the decrypted circuits are all  $C'$ .

- That the decommitment values correctly open all of the commitments  $c_{w_i, \beta}^j$  that were received, and these decommitments reveal the keys  $k_{w_i, \beta}^j$  that were sent for  $P_1$ 's input wires.
- That the keys that it received in the oblivious transfers earlier match the appropriate keys that it received in the opening (i.e., if it received  $[k_i^1, \dots, k_i^\ell]$  in the  $i^{\text{th}}$  oblivious transfer, then it checks that  $k_i^j$  from the oblivious transfer equals  $k_{w_{n+i}, z_2^i}^j$  from the opening).

If all the checks pass, it proceeds to the next step. If not, it outputs `corrupted1` and halts. In addition, if  $P_2$  does not receive this message at all, it outputs `corrupted1`.

10.  $P_1$  sends decommitments to the input keys associated with its input for the unopened circuit  $GC_\gamma$ . That is, for  $i = 1, \dots, n$ , party  $P_1$  sends  $P_2$  the key  $k_{w_i, x_i}^\gamma$  and decommitment  $r_{i, x_i}^\gamma$ , where  $x_i$  is the  $i^{\text{th}}$  bit of  $P_1$ 's input.
11.  $P_2$  checks that the values received are valid decommitments to the commitments received above. If not, it outputs `abort1`. If yes, it uses the keys to compute  $C'(x_1, z_2) = C'(x_1, x_2^1, \dots, x_2^m) = C(x_1, x_2)$ , and outputs the result. If the keys are not correct (and so it is not possible to compute the circuit), or if  $P_2$  doesn't receive this message at all, it outputs `abort1`.

Note that steps 7–10 are actually a single step of  $P_1$  sending a message to  $P_2$ , followed by  $P_2$  carrying out a computation.

If any party fails to receive a message as expected during the execution, it outputs `aborti` (where  $P_i$  is the party who failed to send the message). This holds unless the party is explicitly instructed above to output `corruptedi` instead (as in Step 9).

We have motivated the protocol construction above and thus proceed directly to prove its security.

**Theorem 6.2** *Let  $\ell$  and  $m$  be parameters in the protocol that are both upper-bound by  $\text{poly}(n)$ , and set  $\epsilon = (1 - 1/\ell)(1 - 2^{-m+1})$ . Let  $f$  be any probabilistic polynomial-time function. Assume that the encryption scheme used to generate the garbled circuits has indistinguishable encryptions under chosen-plaintext attacks (and has an elusive and efficiently verifiable range), and that the oblivious transfer protocol used is secure in the presence of covert adversaries with  $\epsilon$ -deterrent according to Definition 3.5. Then, Protocol 6.1 securely computes  $f$  in the presence of covert adversaries with  $\epsilon$ -deterrent according to Definition 3.5.*

**Proof:** Our analysis of the security of the protocol is in the hybrid model, where the parties are assumed to have access to a trusted party computing the oblivious transfer functionality; see Section 4. Thus the simulator that we describe will play the trusted party in the oblivious transfer, when simulating for the adversary. We separately consider the different corruption cases (when no parties are corrupted, and when either one of the parties is corrupted). In the case that no parties are corrupted, the security reduces to the semi-honest case which has already been proven in [22] (the additional steps in Protocol 6.1 don't make a difference here).

**Party  $P_2$  is corrupted.** Intuitively, the security in this case relies on the fact that  $P_2$  can only learn a single set of keys in the oblivious transfers and thus can decrypt the garbled circuit to only a single value as required. Formally, let  $\mathcal{A}$  be an adversary controlling  $P_2$ . The simulator  $\mathcal{S}$  works as follows:

1.  $\mathcal{S}$  chooses  $\ell$  sets of  $mn$  random keys as  $P_1$  would.
2.  $\mathcal{S}$  plays the trusted party for the oblivious transfers with  $\mathcal{A}$  as the receiver.  $\mathcal{S}$  receives the input that  $\mathcal{A}$  sends to the trusted party (as its input as receiver to the oblivious transfers):
  - (a) If the input is  $\text{abort}_2$  or  $\text{corrupted}_2$ , then  $\mathcal{S}$  sends  $\text{abort}_2$  or  $\text{corrupted}_2$  (respectively) to the trusted party computing  $f$ , simulates  $P_1$  aborting and halts (outputting whatever  $\mathcal{A}$  outputs).
  - (b) If the input is  $\text{cheat}_2$ , then  $\mathcal{S}$  sends  $\text{cheat}_2$  to the trusted party. If it receives back  $\text{corrupted}_2$ , then it hands  $\mathcal{A}$  the message  $\text{corrupted}_2$  as if it received it from the trusted party, simulates  $P_1$  aborting and halts (outputting whatever  $\mathcal{A}$  outputs). If it receives back  $\text{undetected}$  (and thus  $P_1$ 's input  $x_1$  as well), then  $\mathcal{S}$  works as follows. First, it hands  $\mathcal{A}$  the string  $\text{undetected}$  together with the  $nm$  random keys that it chose (note that  $\mathcal{A}$  expects to receive the inputs of  $P_1$  to the oblivious transfers in the case of  $\text{undetected}$ ). Next,  $\mathcal{S}$  uses the input  $x_1$  of  $P_1$  that it received in order to perfectly emulate  $P_1$  in the rest of the execution. That is, it runs  $P_1$ 's honest strategy with input  $x_1$  while interacting with  $\mathcal{A}$  playing  $P_2$  for the rest of the execution. Let  $y_1$  be the output for  $P_1$  that it receives.  $\mathcal{S}$  sends  $y_1$  to the trusted party (for  $P_1$ 's output) and outputs whatever  $\mathcal{A}$  outputs. The simulation ends here in this case.
  - (c) If the input is a series of bits  $z_2^1, \dots, z_2^{mn}$ , then  $\mathcal{S}$  hands  $\mathcal{A}$  the keys from above that are "chosen" by the  $z_2^i$  bits, and proceeds with the simulation below.
3.  $\mathcal{S}$  defines  $x_2 = \bigoplus_{i=0}^{m-1} (z_2^{i \cdot n + 1}, \dots, z_2^{i \cdot n + n})$  and sends  $x_2$  to the trusted party computing  $f$ .  $\mathcal{S}$  receives back some output  $y$ .
4.  $\mathcal{S}$  chooses a random value  $\zeta$  and computes the garbled circuits  $GC_j$  for  $j \neq \zeta$  correctly (using the appropriate input keys from above as  $P_1$  would). However, for the garbled circuit  $GC_\zeta$ , the simulator  $\mathcal{S}$  does not use the true circuit for computing  $f$  but rather a circuit  $\widetilde{GC}$  that always evaluates to  $y$  (the value it received from the trusted party), using Lemma A.1.  $\mathcal{S}$  uses the appropriate input keys from above also in generating  $GC_\zeta$ .  $\mathcal{S}$  also computes commitments to the keys associated with  $P_1$ 's input in an honest way.
5.  $\mathcal{S}$  sends  $GC_1, \dots, GC_\ell$  and the commitments to  $\mathcal{A}$  and receives back an index  $\gamma$ .
6. If  $\gamma \neq \zeta$  then  $\mathcal{S}$  rewinds  $\mathcal{A}$  and returns to Step 4 above (using fresh randomness).  
Otherwise, if  $\gamma = \zeta$ , then  $\mathcal{S}$  opens all the commitments and garbled circuits  $GC_j$  for  $j \neq \gamma$ , as the honest  $P_1$  would, and proceeds to the next step.
7.  $\mathcal{S}$  hands  $\mathcal{A}$  arbitrary keys associated with the input wires of  $P_1$ . That is, for  $i = 1, \dots, n$ ,  $\mathcal{S}$  hands  $\mathcal{A}$  an arbitrary one of the two keys associated with the input wire  $w_i$  in  $GC_\gamma$  (one key per wire), together with its correct decommitment.
8. If at any stage,  $\mathcal{S}$  does not receive a response from  $\mathcal{A}$ , it sends  $\text{abort}_2$  to the trusted party (resulting in  $P_1$  outputting  $\text{abort}_2$ ). If the protocol proceeds successfully to the end,  $\mathcal{S}$  sends continue to the trusted party and outputs whatever  $\mathcal{A}$  outputs.

Denoting  $\pi$  as Protocol 6.1 and  $I = \{2\}$  (i.e., party  $P_2$  is corrupted), we prove that:

$$\left\{ \text{IDEALSC}_{f, \mathcal{S}(z), I}^c((x_1, x_2), n) \right\} \stackrel{c}{=} \left\{ \text{HYBRID}_{\pi, \mathcal{A}(z), I}^{\text{OT}}((x_1, x_2), n) \right\} \quad (6)$$

In order to prove Eq. (6) we separately consider the cases of `abort` (including a “corrupted” input), `cheat` or neither. If  $\mathcal{A}$  sends `abort`<sub>2</sub> or `corrupted`<sub>2</sub> as the oblivious transfer input, then  $\mathcal{S}$  sends `abort`<sub>2</sub> or `corrupted`<sub>2</sub> (respectively) to the trusted party computing  $f$ . In both cases the honest  $P_1$  outputs the same (`abort`<sub>2</sub> or `corrupted`<sub>2</sub>) and the view of  $\mathcal{A}$  is identical. Thus, the IDEAL and HYBRID output distributions are identical. The exact same argument is true if  $\mathcal{A}$  sends `cheat`<sub>2</sub> and the reply to  $\mathcal{S}$  from the trusted party is `corrupted`<sub>2</sub>. In contrast, if  $\mathcal{A}$  sends `cheat`<sub>2</sub> and  $\mathcal{S}$  receives back the reply `undetected`, then the execution does not halt immediately. Rather,  $\mathcal{S}$  plays the honest  $P_1$  with its input  $x_1$ . Since  $\mathcal{S}$  follows the exact same strategy as  $P_1$ , and the output received by  $P_1$  from the execution is the same  $y_1$  that  $\mathcal{S}$  receives from the protocol execution, it is clear that once again the output distributions are identical (recall that in the ideal model,  $P_1$  outputs the same  $y_1$  obtained by  $\mathcal{S}$ ). We remark that the probability of the trusted party answering `corrupted`<sub>2</sub> or `undetected` is the same in the hybrid and ideal executions (i.e.,  $\epsilon$ ), and therefore the output distributions in the cases of `abort`, `corrupted` or `cheat` are identical. We denote the event that  $\mathcal{A}$  sends an `abort`, `corrupted` or `cheat` message in the oblivious transfers by `bad`<sub>OT</sub>. Thus, we have shown that

$$\left\{ \text{IDEALSC}_{f,\mathcal{S}(z),I}^{\epsilon}((x_1, x_2), n) \mid \text{bad}_{\text{OT}} \right\} \equiv \left\{ \text{HYBRID}_{\pi,\mathcal{A}(z),I}^{\text{OT}}((x_1, x_2), n) \mid \text{bad}_{\text{OT}} \right\}$$

We now show that the IDEAL and HYBRID distributions are computationally indistinguishable in the case that  $\mathcal{A}$  sends valid input in the oblivious transfer phase (i.e., in the event  $\neg\text{bad}_{\text{OT}}$ ). In order to show this, we consider a modified simulator  $\mathcal{S}'$  who is also given the honest party  $P_1$ 's real input  $x_1$ . Simulator  $\mathcal{S}'$  works exactly as  $\mathcal{S}$  does, except that it constructs  $GC_{\zeta}$  honestly, and not as  $\widehat{GC}$  from Lemma A.1. Furthermore, in Step 7 it sends the keys associated with  $P_1$ 's input  $x_1$  and not arbitrary keys. It is straightforward to verify that the distribution generated by  $\mathcal{S}'$  is identical to the distribution generated by  $\mathcal{A}$  in an execution of the real protocol. This is due to the fact that all  $\ell$  circuits received by  $\mathcal{A}$  are honestly constructed and the keys that it receives from  $\mathcal{S}'$  are associated with  $P_1$ 's real input. The only difference is the rewinding. However, since  $\zeta$  is chosen uniformly, this has no effect on the output distribution. Thus:

$$\left\{ \text{IDEALSC}_{f,\mathcal{S}'(z,x_1),I}^{\epsilon}((x_1, x_2), n) \mid \neg\text{bad}_{\text{OT}} \right\} \equiv \left\{ \text{HYBRID}_{\pi,\mathcal{A}(z),I}^{\text{OT}}((x_1, x_2), n) \mid \neg\text{bad}_{\text{OT}} \right\}$$

Next we prove that the distributions generated by  $\mathcal{S}$  and  $\mathcal{S}'$  are computationally indistinguishable. That is,

$$\left\{ \text{IDEAL}_{f,\mathcal{S}(z),I}((x_1, x_2), n) \mid \neg\text{bad}_{\text{OT}} \right\} \stackrel{c}{\equiv} \left\{ \text{IDEAL}_{f,\mathcal{S}'(z,x_1),I}^{\epsilon}((x_1, x_2), n) \mid \neg\text{bad}_{\text{OT}} \right\}$$

In order to see this, notice that the only difference between  $\mathcal{S}$  and  $\mathcal{S}'$  is in the construction of the garbled circuit  $GC_{\zeta}$ . By Lemma A.1 it follows immediately that these distributions are computationally indistinguishable. (Note that we do not need to consider the joint distribution of  $\mathcal{A}$ 's view and  $P_1$ 's output because  $P_1$  has no output from Protocol 6.1.) This yields the above equation. In order to complete the proof of Eq. (6), note that the probability that the event `bad`<sub>OT</sub> happens is *identical* in the IDEAL and HYBRID executions. This holds because the oblivious transfer is the first step of the protocol and  $\mathcal{A}$ 's view in this step with  $\mathcal{S}$  is identical to its view in a protocol execution with a trusted party computing the oblivious transfer functionality. Combining this fact with the above equations we derive Eq. (6).

We remark that the simulator  $\mathcal{S}$  described above runs in expected polynomial-time. In order to see this, note that by Lemma A.1, a fake garbled circuit is indistinguishable from a real one. Therefore, the probability that  $\gamma = \zeta$  is at most negligibly far from  $1/\ell$  (otherwise, this fact alone can be used to distinguish a fake garbled circuit from a real one). It follows that the expected

number of attempts by  $\mathcal{S}$  is close to  $\ell$ , and so its expected running-time is polynomial (by the assumption on  $\ell$ ). By our definition,  $\mathcal{S}$  needs to run in strict polynomial-time. However, this is easily achieved by having  $\mathcal{S}$  halt if it fails after  $n\ell$  rewinding attempts. Following the same argument as above, such a failure can occur with at most negligible probability.

We conclude that  $\mathcal{S}$  meets the requirements of Definition 3.5. (Note that  $\mathcal{S}$  only sends `cheat2` due to the oblivious transfer. Thus, if a “fully secure” oblivious transfer protocol were to be used, the protocol would meet the standard definition of security for malicious adversaries for the case that  $P_2$  is corrupted.)

**Party  $P_1$  is corrupted.** The proof of security in this corruption case is considerably more complex. Intuitively, security relies on the fact that if  $P_1$  does not construct the circuits correctly or does not provide the same keys in the oblivious transfers and circuit openings, then it will be caught with probability at least  $\epsilon$ . In contrast, if it does construct the circuits correctly and provide the same keys, then its behavior is effectively the same as an honest party and so security is preserved. Formally, let  $\mathcal{A}$  be an adversary controlling  $P_1$ . The simulator  $\mathcal{S}$  works as follows:

1.  $\mathcal{S}$  invokes  $\mathcal{A}$  and plays the trusted party for the oblivious transfers with  $\mathcal{A}$  as the sender.  $\mathcal{S}$  receives the input that  $\mathcal{A}$  sends to the trusted party (as its input to the oblivious transfers):
  - (a) If the input is `abort1` or `corrupted1`, then  $\mathcal{S}$  sends `abort1` or `corrupted1` (respectively) to the trusted party computing  $f$ , simulates  $P_2$  aborting and halts (outputting whatever  $\mathcal{A}$  outputs).
  - (b) If the input is `cheat1`, then  $\mathcal{S}$  sends `cheat1` to the trusted party. If it receives back `corrupted1`, then it hands  $\mathcal{A}$  the message `corrupted1` as if it received it from the trusted party, simulates  $P_2$  aborting and halts (outputting whatever  $\mathcal{A}$  outputs). If it receives back `undetected` (and thus  $P_2$ 's input  $x_2$  as well), then  $\mathcal{S}$  works as follows. First, it hands  $\mathcal{A}$  the string `undetected` together with the input string  $z_2$  that an honest  $P_2$  upon input  $x_2$  would have used in the oblivious transfers (note that  $\mathcal{A}$  expects to receive  $P_2$ 's input to the oblivious transfers in the case of `undetected`). We remark that  $\mathcal{S}$  can compute  $z_2$  by simply following the instructions of an honest  $P_2$  with input  $x_2$  from the start (nothing yet has depended on  $P_2$ 's input so there is no problem of consistency). Next,  $\mathcal{S}$  uses the derived input  $z_2$  that it computed above in order to perfectly emulate  $P_2$  in the rest of the execution. That is, it continues  $P_2$ 's honest strategy with input  $z_2$  while interacting with  $\mathcal{A}$  playing  $P_1$  for the rest of the execution. Let  $y_2$  be the output for  $P_2$  that it receives.  $\mathcal{S}$  sends  $y_2$  to the trusted party (for  $P_2$ 's output) and outputs whatever  $\mathcal{A}$  outputs. The simulation ends here in this case.
  - (c) If the input is a series of  $mn$  pairs of  $\ell$ -tuples of keys

$$\left( [k_{w_{n+i},0}^1, \dots, k_{w_{n+i},0}^\ell], [k_{w_{n+i},1}^1, \dots, k_{w_{n+i},1}^\ell] \right)$$

for  $i = 1, \dots, mn$ , then  $\mathcal{S}$  proceeds below.

2.  $\mathcal{S}$  receives from  $\mathcal{A}$  a message consisting of  $\ell$  garbled circuits  $GC_1, \dots, GC_\ell$  and a series of commitments.
3. For  $j = 1, \dots, \ell$ , simulator  $\mathcal{S}$  sends  $\mathcal{A}$  the message  $\gamma = j$ , receives its reply and rewinds  $\mathcal{A}$  back to the point before  $\mathcal{A}$  receives  $\gamma$ .
4.  $\mathcal{S}$  continues the simulation differently, depending on the validity of the circuit openings. In order to describe the cases, we introduce some terminology.

**Legitimate circuit:** We say that a garbled circuit  $GC_j$  is *legitimate* if in at least one of its openings, in response to a challenge  $\gamma \neq j$ , it is decrypted to the auxiliary input circuit  $C'$ . Note that if a circuit is legitimate then in all valid decryptions of the circuit (for all  $\gamma \neq j$ ) it decrypts to  $C'$ .

**Inconsistent key:** This notion relates to the question of whether the keys provided by  $P_1$  in the oblivious transfers are the same as those provided in the circuit opening. We say that a key  $k_{w_i, \beta}^j$  received in an oblivious transfer is *inconsistent* if it is different from the analogous key provided by  $P_1$  when opening the garbled circuit  $GC_j$ .

**Inconsistent wire:** A wire  $w_i$  is *inconsistent* if there *exists* a circuit  $GC_j$  such that either  $k_{w_i, 0}^j$  or  $k_{w_i, 1}^j$  is an inconsistent key.

**Totally inconsistent input:** An original input bit  $x_2^i$  is *totally inconsistent* if all of the wires associated with the shares of  $x_2^i$  are inconsistent (recall that  $x_2^i$  is split over  $m$  input wires). Note that the different inconsistent wires need not be inconsistent in the same circuit, nor need they be inconsistent with respect to the same value (0 or 1). Note that the determination that a wire is inconsistent is independent of  $\gamma$  because the oblivious transfers and commitments to keys take place before  $\gamma$  is sent.

$\mathcal{S}$  works according to the follows cases:

- (a) *Case 1 – at least one circuit is illegitimate:* Let  $GC_{j_0}$  be the first illegitimate circuit. Then,  $\mathcal{S}$  sends  $w_1 = \text{cheat}_1$  to the trusted party. By the definition of the ideal model, with probability  $\epsilon = (1 - 1/\ell)(1 - 2^{-m+1})$  it receives the message  $\text{corrupted}_1$ , and with probability  $1 - \epsilon$  it receives the message  $\text{undetected}$  together with  $P_2$ 's input  $x_2$ :
- i. If  $\mathcal{S}$  receives the message  $\text{corrupted}_1$  from the trusted party, then it chooses  $\gamma \neq j_0$  at random and sends  $\gamma$  to  $\mathcal{A}$ . Then,  $\mathcal{S}$  receives back  $\mathcal{A}$ 's opening for the circuits, including the illegitimate circuit  $GC_{j_0}$ , and simulates  $P_2$  aborting due to detected cheating.  $\mathcal{S}$  then outputs whatever  $\mathcal{A}$  outputs and halts.
  - ii. If  $\mathcal{S}$  receives the message  $\text{undetected}$  from the trusted party (together with  $P_2$ 's input  $x_2$ ), then with probability  $p = \frac{\ell-1}{1-\epsilon}$  it sets  $\gamma = j_0$ , and with probability  $1 - p$  it chooses  $\gamma \neq j_0$  at random. It then sends  $\gamma$  to  $\mathcal{A}$ , and continues to the end of the execution emulating the honest  $P_2$  with the input  $x_2$  it received from the trusted party. (When computing the circuit,  $\mathcal{S}$  takes the keys from the oblivious transfer that  $P_2$  would have received when using input  $x_2$  and when acting as the honest  $P_2$  to define the string  $z_2$ .) Let  $y_2$  be the output that  $\mathcal{S}$  received when playing  $P_2$  in this execution.  $\mathcal{S}$  sends  $y_2$  to the trusted party (to be the output of  $P_2$ ) and outputs whatever  $\mathcal{A}$  outputs). Note that if the output of  $P_2$  in this emulated execution would have been  $\text{corrupted}_1$  then  $\mathcal{S}$  sends  $y_2 = \text{corrupted}_1$  to the trusted party.<sup>9</sup> (We remark that below we will show below that the above probabilities result in  $\gamma$  being uniformly distributed in  $\{1, \dots, \ell\}$ .)
- (b) *Case 2 – All circuits are legitimate but there is a totally inconsistent input:* Let  $x_2^i$  be the totally inconsistent input and, for brevity, assume that the inconsistent keys are all for the 0-value on the wires (i.e. there are inconsistent keys  $k_{w_{n+(i-1)m+1}, 0}^{j_1}, \dots, k_{w_{n+im}, 0}^{j_m}$  for some  $j_1, \dots, j_m \in \{1, \dots, \ell\}$ ). In this case,  $\mathcal{S}$  sends  $w_1 = \text{cheat}_1$  to the trusted

<sup>9</sup>We remark that  $P_2$  may output  $\text{corrupted}_1$  with probability that is higher than  $\epsilon$ . This possibility is dealt with by having  $\mathcal{S}$  play  $P_2$  and force a  $\text{corrupted}_1$  output if this would have occurred in the execution.

party. With probability  $\epsilon$  it receives the message `corrupted1`, and with probability  $1 - \epsilon$  it receives the message `undetected` together with  $P_2$ 's input  $x_2$ :

- i. If  $\mathcal{S}$  receives the message `corrupted1` from the trusted party, then it chooses random values for the bits on the wires  $w_{n+(i-1)m+1}, \dots, w_{n+im-1}$ , subject to the constraints that not all are 1; i.e. at least one of these wires gets a value with an inconsistent key.<sup>10</sup> Let  $w_{n+(i-1)m+t}$  be the first of these that is 0, and let  $G_{j_0}$  be the first circuit for which the key of this wire is inconsistent.  $\mathcal{S}$  chooses  $\gamma \neq j_0$  at random and sends it to  $\mathcal{A}$ . Among other things,  $\mathcal{S}$  receives back  $\mathcal{A}$ 's opening of  $GC_{j_0}$ , and simulates  $P_2$ 's aborting due to detected cheating. (Note that the probability that a real  $P_2$  will make these two choices – choose the values for the first  $m - 1$  wires so that not all are 1, and choose  $\gamma \neq j_0$  – is exactly  $\epsilon$ .)  $\mathcal{S}$  then outputs whatever  $\mathcal{A}$  outputs and halts.
- ii. If  $\mathcal{S}$  receives the message `undetected` (and thus the real input  $x_2$  of  $P_2$ ) from the trusted party, it first determines the values for the shares of  $x_2^i$  and for the value  $\gamma$ , as follows:
  - With probability  $p = \frac{2^{-m+1}}{1-\epsilon}$ , for all  $t = 1, \dots, m - 1$  it sets the value on the wire  $w_{n+(i-1)m+t}$  to equal 1 (corresponding to *not* choosing the inconsistent keys), and the value on the wire  $w_{n+im}$  to equal the XOR of  $x_2^i$  with the values set on the wires  $w_{n+(i-1)m+1}, \dots, w_{n+(i-1)m+m-1}$ . The value  $\gamma$  is chosen at random (out of  $1, \dots, \ell$ ).
  - With probability  $1 - p$ , for all  $t = 1, \dots, m - 1$  it sets the value on the wire  $w_{n+(i-1)m+t}$  to a random value, subject to the constraint that not all are 1 (i.e. at least one of the shares has an inconsistent key), and it sets the value on the wire  $w_{n+im}$  to equal the XOR of  $x_2^i$  with the values set on the wires  $w_{n+(i-1)m+1}, \dots, w_{n+(i-1)m+m-1}$ . Let  $w_{n+(i-1)m+t}$  be the first wire that is 0, and let  $j_0$  be the first circuit for which the key of this share is inconsistent. Then  $\mathcal{S}$  sets  $\gamma = j_0$ .

The values for shares of all other input bits are chosen at random (subject to the constraint that their XOR is the input value obtained from the trusted party, as an honest  $P_2$  would choose).  $\mathcal{S}$  now sends  $\gamma$  to  $\mathcal{A}$ , and completes the execution emulating an honest  $P_2$  using these shares and  $\gamma$ . It outputs whatever  $\mathcal{A}$  would output, and sets  $P_2$ 's output, as explained in step 4(a)ii above (i.e., if the emulated  $P_2$  would receive `corrupted1` then  $\mathcal{S}$  causes  $P_2$  to receive `corrupted1` in the ideal model).

- (c) *Case 3 – All circuits are legitimate and there is no totally inconsistent input:* For each inconsistent wire (i.e. a wire for which there *exists* an inconsistent key), if there are any,  $\mathcal{S}$  chooses a random value, and checks whether the value it chose corresponds to an inconsistent key. There are two cases:
  - i. *Case 3a –  $\mathcal{S}$  chose values with inconsistent keys:* In this case,  $\mathcal{S}$  sends  $w_1 = \text{cheat}_1$  to the trusted party. With probability  $\epsilon$  it receives the message `corrupted1`, and with probability  $1 - \epsilon$  it receives the message `undetected` together with  $P_2$ 's input  $x_2$ . Let  $w_{i_0}$  be the first of the wires for which the value chosen has an inconsistent key, and let  $GC_{j_0}$  be the first circuit in which the key is inconsistent:

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<sup>10</sup>Recall that the input wires associated with  $P_2$ 's input bit  $x_2^i$  are  $w_{n+(i-1)m+1}, \dots, w_{n+im}$ . Thus, the simulator here fixes the values on all the wires except the last (recall also that the first  $m - 1$  values plus  $P_2$ 's true input bit fully determine the value for the last wire  $w_{n+im}$ ).

- A. If  $\mathcal{S}$  receives the message `corrupted1` from the trusted party, then it chooses  $\gamma \neq j_0$  at random and sends it to  $\mathcal{A}$ .  $\mathcal{S}$  then simulates  $P_2$  aborting due to detected cheating.  $\mathcal{S}$  then outputs whatever  $\mathcal{A}$  outputs and halts.
- B. If  $\mathcal{S}$  receives the message `undetected`, together with  $x_2 = (x_2^1, \dots, x_2^n)$ , from the trusted party, then first it chooses values for the remaining (consistent) shares at random, subject to the constraint that for any input bit  $x_2^i$ , the XOR of all its shares equals the value of this bit, as provided by the trusted party. In addition:
- With probability  $p = \frac{\ell-1}{1-\epsilon}$ , simulator  $\mathcal{S}$  sets  $\gamma = j_0$ .
  - With probability  $1 - p$ , simulator  $\mathcal{S}$  chooses  $\gamma \neq j_0$  at random.
- In both cases,  $\mathcal{S}$  sends  $\gamma$  to  $\mathcal{A}$  and completes the execution emulating an honest  $P_2$  using the above choice of shares, and outputting the values as explained in step 4(a)ii above (in particular, if the output of the emulated  $P_2$  is `corrupted1`, then  $\mathcal{S}$  causes this to be the output of  $P_2$  in the ideal model).
- ii. *Case 3b –  $\mathcal{S}$  chose only values with consistent keys:*  $\mathcal{S}$  reaches this point of the simulation if all garbled circuits are legitimate and if either all keys are consistent or it is simulating the case that no inconsistent keys were chosen. Thus, intuitively, the circuit and keys received by  $\mathcal{S}$  from  $\mathcal{A}$  are the same as from an honest  $P_1$ . The simulator  $\mathcal{S}$  begins by choosing a random  $\gamma$  and sending it to  $\mathcal{A}$ . Then,  $\mathcal{S}$  receives the opening of the other circuits, as before. In addition,  $\mathcal{S}$  receives from  $\mathcal{A}$  the set of keys and decommitments (for the wires  $w_1, \dots, w_n$ ) for the unopened circuit  $GC_\gamma$ . If anything in this process is invalid (i.e. any of the circuits is not correctly decrypted, or the decommitments are invalid, or the keys cannot be used in the circuit), then  $\mathcal{S}$  sends `abort1` or `corrupted1` to the trusted party causing  $P_2$  to output `abort1` or `corrupted1`, respectively (the choice of whether to send `abort1` or `corrupted1` is according to the protocol description and what causes  $P_2$  to output `abort1` and what causes it to output `corrupted1`). Otherwise,  $\mathcal{S}$  uses the opening of the circuit  $GC_\gamma$  obtained above, together with the keys obtained in order to derive the input  $x'_1$  used by  $\mathcal{A}$ . This input is derived by comparing the keys for the wires associated with  $P_1$ 's inputs as received by  $\mathcal{S}$  from  $\mathcal{A}$  with the keys provided by  $\mathcal{A}$  in a legitimate opening of the circuit. This legitimate opening provides the association of each key to a bit, and the input bits of  $x'_1$  are derived using this association.  $\mathcal{S}$  sends the trusted party  $x'_1$  (and `continue`) and outputs whatever  $\mathcal{A}$  outputs.

This concludes the description of  $\mathcal{S}$ . Denote by `badOT` the event that  $\mathcal{A}$  sends `abort1`, `corrupted1` or `cheat1` in the oblivious transfers. The analysis of the event `badOT` is identical to the case that  $P_2$  is corrupted and so denoting  $\pi$  as Protocol 6.1 and  $I = \{1\}$  (i.e., party  $P_1$  is corrupted), we have that:

$$\left\{ \text{IDEALSC}_{f, \mathcal{S}(z), I}^\epsilon((x_1, x_2), n) \mid \text{bad}_{\text{OT}} \right\} \equiv \left\{ \text{HYBRID}_{\pi, \mathcal{A}(z), I}^{\text{OT}}((x_1, x_2), n) \mid \text{bad}_{\text{OT}} \right\}$$

It remains to analyze the case that  $\neg \text{bad}_{\text{OT}}$  (i.e., the oblivious transfer is not aborted). We will prove the case following the same case analysis as in the description of the simulator. Before doing so, notice that the only messages that  $\mathcal{A}$  receives in a protocol execution are in the oblivious transfers and the challenge value  $\gamma$ . Thus, when analyzing Protocol 6.1 in a hybrid model with a trusted party computing the oblivious transfer functionality, its view consists only of the value  $\gamma$ . Thus, in order to show that  $\mathcal{A}$ 's view in the simulation is indistinguishable from its view in a real execution, it suffices to show that the value  $\gamma$  that  $\mathcal{S}$  hands  $\mathcal{A}$  is (almost) uniformly distributed in  $\{1, \dots, \ell\}$ . We stress that this is not the case when considering the *joint distribution* including  $P_2$ 's output



(because cheating by  $\mathcal{A}$  can cause  $P_2$  to output an incorrect value). The focus of the proof below is thus to show that the distribution over the challenge value  $\gamma$  sent by  $\mathcal{S}$  during the simulation is uniform, and that the joint distribution over  $\mathcal{A}$ 's view and the output of  $P_2$  in the simulation is statistically close to a real execution.

1. *Case 1 – at least one circuit is illegitimate:* We first show that the value  $\gamma$  sent by  $\mathcal{S}$  in the simulation is uniformly distributed over  $\{1, \dots, \ell\}$ , just like the value sent by  $P_2$  in a real execution. In order to see this, we distinguish between the case that  $\mathcal{S}$  receives **corrupted<sub>1</sub>** and the case that it receives **undetected**. We first prove that  $\gamma = j_0$  with probability  $1/\ell$ :

$$\begin{aligned} \Pr[\gamma = j_0] &= \Pr[\gamma = j_0 \mid \text{corrupted}_1] \Pr[\text{corrupted}_1] + \Pr[\gamma = j_0 \mid \text{undetected}] \Pr[\text{undetected}] \\ &= 0 \cdot \Pr[\text{corrupted}_1] + \frac{\ell^{-1}}{1 - \epsilon} \cdot \Pr[\text{undetected}] \\ &= \frac{1}{\ell} \cdot \frac{1}{1 - \epsilon} \cdot (1 - \epsilon) = \frac{1}{\ell} \end{aligned}$$

where the second equality is by the simulator's code, and the third follows from the fact that  $\Pr[\text{undetected}] = 1 - \epsilon$ , by definition. We now proceed to prove that for every  $j \neq j_0$  it also holds that  $\Pr[\gamma = j] = 1/\ell$ . For every  $j = 1, \dots, \ell$  with  $j \neq j_0$ :

$$\begin{aligned} \Pr[\gamma = j] &= \Pr[\gamma = j \mid \text{corrupted}_1] \Pr[\text{corrupted}_1] + \Pr[\gamma = j \mid \text{undetected}] \Pr[\text{undetected}] \\ &= \Pr[\gamma = j \mid \text{corrupted}_1] \cdot \epsilon + \Pr[\gamma = j \mid \text{undetected}] \cdot (1 - \epsilon) \\ &= \left( \frac{1}{\ell - 1} \right) \cdot \epsilon + \left( \left( 1 - \frac{1}{\ell(1 - \epsilon)} \right) \cdot \frac{1}{\ell - 1} \right) \cdot (1 - \epsilon) \\ &= \frac{1}{\ell - 1} \cdot \left( \epsilon + (1 - \epsilon) - \frac{1 - \epsilon}{\ell(1 - \epsilon)} \right) \\ &= \frac{1}{\ell - 1} \cdot \left( 1 - \frac{1}{\ell} \right) = \frac{1}{\ell} \end{aligned}$$

where, once again, the second equality is by the code of the simulator. We now proceed to show that the joint distribution of  $\mathcal{A}$ 's view and  $P_2$ 's output in a real execution (or more exactly, a hybrid execution where the oblivious transfers are computed by a trusted party) is identical to the joint distribution of  $\mathcal{S}$  and  $P_2$ 's output in an ideal execution. First, note that when a real  $P_2$  chooses  $\gamma \neq j_0$ , then it always outputs **corrupted<sub>1</sub>**. Likewise, in an ideal execution where the trusted party sends **corrupted<sub>1</sub>** to  $P_2$ , the simulator  $\mathcal{S}$  chooses  $\gamma \neq j_0$ . Recalling from above that  $\gamma$  as chosen by  $\mathcal{S}$  is uniformly distributed, we have that the distributions in this case are identical. Next, observe that when  $\mathcal{S}$  receives **undetected**, it emulates a real execution between  $P_2$  and  $\mathcal{A}$  by using the real input that  $P_2$  holds. Furthermore, the output of  $P_2$  in the ideal execution is the output that  $\mathcal{S}$  receives in this emulation (playing  $P_2$ ). Thus, the output distributions are again identical. (Note that  $P_2$  may output **corrupted<sub>1</sub>** even when  $\mathcal{S}$  receives **undetected**. However, what we have shown is that when the trusted party sends **corrupted<sub>1</sub>**, the simulation by  $\mathcal{S}$  results in execution transcripts that would result in  $P_2$  certainly outputting **corrupted<sub>1</sub>**. Furthermore, when the trusted party sends **undetected**, the simulation fills in the rest of the cases that are not included in the  $\epsilon$  probability of  $P_2$  outputting **corrupted<sub>1</sub>** that were covered when the trusted party sends **corrupted<sub>1</sub>**.) This completes this case.

2. *Case 2 – All circuits are legitimate but there is a totally inconsistent input:* We analyze this case in an analogous way to above. Let 'all=1' denote the case that in a real execution all of the  $m - 1$  first wires associated with the totally inconsistent input are given value 1 (and so

the inconsistent keys determined for those wires are not revealed). Since the values on these wires are chosen by  $P_2$  uniformly, we have that  $\Pr[\text{'all=1'}] = 2^{-m+1}$ . Noting also that  $\gamma$  is chosen by  $P_2$  independently of the values on the wires, we have that:

$$\Pr[\gamma \neq j_0 \ \& \ \neg\text{'all=1'}] = \left(1 - \frac{1}{\ell}\right) \left(1 - \frac{1}{2^{m-1}}\right) = \epsilon$$

where the second equality is by the definition of  $\epsilon$  (recall that  $j_0$  is the index of the first circuit for which an inconsistent key is chosen by  $\mathcal{S}$ ). Now, the trusted party sends  $\text{corrupted}_1$  with probability exactly  $\epsilon$ . Furthermore, in this case,  $\mathcal{S}$  generates a transcript for which the event  $\gamma \neq j_0 \ \& \ \neg\text{'all=1'}$  holds, and such an event in a real execution results in  $P_2$  certainly outputting  $\text{corrupted}_1$ . We thus have that the  $\text{corrupted}_1$  event in the ideal model is mapped with probability exactly  $\epsilon$  to a sub-distribution over the real transcripts in which  $P_2$  outputs  $\text{corrupted}_1$ .

Next we analyze the case that not all values on the wires are 1, but  $\gamma = j_0$ . In a real execution, we have that this event occurs with the following probability:

$$\Pr[\gamma = j_0 \ \& \ \neg\text{'all=1'}] = \frac{1}{\ell} \cdot \left(1 - 2^{-m+1}\right)$$

By the description of  $\mathcal{S}$ , this occurs in the simulation with probability  $(1 - \epsilon)(1 - p)$  where  $p = 2^{-m+1}/(1 - \epsilon)$ ; see the second bullet of Case (2), subitem (ii). Now,

$$\begin{aligned} (1 - \epsilon)(1 - p) &= (1 - \epsilon) \cdot \left(1 - \frac{2^{-m+1}}{1 - \epsilon}\right) \\ &= 1 - \epsilon - 2^{-m+1} \\ &= 1 - \left(1 - 2^{-m+1}\right) \left(1 - \ell^{-1}\right) - 2^{-m+1} \\ &= 1 - \left(1 - \frac{1}{\ell} - 2^{-m+1} + \frac{2^{-m+1}}{\ell}\right) - 2^{-m+1} \\ &= \frac{1}{\ell} - \frac{2^{-m+1}}{\ell} \\ &= \frac{1}{\ell} \cdot \left(1 - 2^{-m+1}\right). \end{aligned}$$

Thus, the probability of this event in the simulation by  $\mathcal{S}$  is exactly the same as in a real execution. Furthermore, the transcript generated by  $\mathcal{S}$  in this case (and the output of  $P_2$ ) is identical to in a real execution, because  $\mathcal{S}$  runs an emulation using  $P_2$ 's real input.

Combining the above, we have shown that in the case that not all wires are 1, the simulation by  $\mathcal{S}$  is perfect. It remains to consider the case that all  $m - 1$  wires do equal 1; this case is covered by the simulation in the first bullet of Case (2), subitem (ii). In a real execution, this case occurs with probability  $2^{-m+1}$ . Likewise, in the simulation,  $\mathcal{S}$  reaches subitem (ii) with probability  $1 - \epsilon$  and then proceeds to the first bullet with probability  $p = 2^{-m+1}/(1 - \epsilon)$ . Therefore, this case appears with overall probability  $2^{-m+1}$  exactly as in a real execution. Furthermore, as above, the simulation by  $\mathcal{S}$  is perfect because it emulates using  $P_2$ 's real input.

We conclude that the simulation by  $\mathcal{S}$  in this case is perfect (meaning that when all circuits are legitimate but there is a totally inconsistent input, the joint output distribution of  $\mathcal{S}$  and  $P_2$  in an ideal execution is identical to the joint output distribution of  $\mathcal{A}$  and  $P_2$  in a hybrid execution of the protocol where a trusted party is used for the oblivious transfers).

3. *Case 3 – all circuits are legitimate and there is no totally inconsistent input:* We have the following subcases:

(a) *Case 3a –  $\mathcal{S}$  chose values with inconsistent keys:* First observe that  $\mathcal{S}$  chooses values with inconsistent keys with exactly the same probability as  $P_2$  in a real execution. This holds because there are no totally inconsistent values and thus the choice of values on the wires with inconsistent keys is uniform. (Note that  $P_2$ 's strategy for choosing values is equivalent to choosing any subset of  $m - 1$  values uniformly and then choosing the last value so that the XOR equals the associated input bit.) We therefore fix the choice of values for the wires and proceed to analyze the transcripts generated by the simulator, *conditioned on this choice of keys.*

In a real execution in which  $P_2$  chose inconsistent keys, it outputs `corrupted1` if the circuit in which the inconsistent keys were chosen is opened (it may also output `corrupted1` if the circuit is opened but this is not relevant here). Now, if the trusted party sends `corrupted1`, then the simulator ensures that the circuit in which the inconsistent keys were chosen is opened (it does this by chooses  $\gamma$  uniformly under the constraint that  $\gamma \neq j_0$ ; see subitem (A) of subitem (i) in Case 3a). In contrast, if the trusted party sends `undetected`, then  $\mathcal{S}$  runs a perfect emulation using  $P_2$ 's real input; the two subcases (with probability  $p$  and  $1 - p$ ) are to ensure that  $\gamma$  is chosen uniformly. Thus, it remains to show that in this case, for every  $j = 1, \dots, \ell$  we have  $\Pr[\gamma = j] = 1/\ell$ . As above, we separately analyze the probability for  $j = j_0$  and  $j \neq j_0$ . The computation is almost the same as in Case 1 above and we are therefore brief:

$$\begin{aligned} \Pr[\gamma = j_0] &= \Pr[\gamma = j_0 \mid \text{corrupted}_1] \cdot \epsilon + \Pr[\gamma = j_0 \mid \text{undetected}] \cdot (1 - \epsilon) \\ &= 0 \cdot \epsilon + \frac{\ell^{-1}}{1 - \epsilon} \cdot (1 - \epsilon) = \frac{1}{\ell}. \end{aligned}$$

In addition, for all  $j \neq j_0$ :

$$\begin{aligned} \Pr[\gamma = j] &= \Pr[\gamma = j \mid \text{corrupted}_1] \cdot \epsilon + \Pr[\gamma = j \mid \text{undetected}] \cdot (1 - \epsilon) \\ &= \left(\frac{1}{\ell - 1}\right) \cdot \epsilon + \left(\left(1 - \frac{1}{\ell(1 - \epsilon)}\right) \cdot \frac{1}{\ell - 1}\right) \cdot (1 - \epsilon) = \frac{1}{\ell} \end{aligned}$$

Thus, in this case,  $\mathcal{S}$  chooses  $\gamma$  uniformly in  $\{1, \dots, \ell\}$ . Furthermore, the transcript in each subcase is exactly as in a real execution, as required.

(b) *Case 3b –  $\mathcal{S}$  chose only values with consistent keys:* As above, the probability that  $\mathcal{S}$  chose only values with consistent keys is identical to the probability that a real  $P_2$  chooses only values with consistent keys. Now, in such a case, all circuits are legitimate, and in addition, all keys that are retrieved by  $P_2$  are consistent (this includes the keys for the opened circuits and for the circuit that is computed). This means that the computation of the circuit using the keys retrieved by  $P_2$  is identical to the computation of an honestly generated circuit. (Note that  $P_2$  may abort or output `corrupted1` in this case. However, here we are interested in the result of the computation of the circuit  $G_\gamma$ , if it is computed by  $P_2$ .) We also note that the keys provided by  $P_1$  that are associated with its own input are provided via decommitments. Thus,  $P_1$  can either not provide valid decommitments, or must provide decommitments that yield keys that result in the circuit being decrypted correctly. This also means that the associations made by  $\mathcal{S}$  between the input keys of  $P_1$  and the string  $x'_1$  that it sends to the trusted party are

correct. We conclude that in this case, the joint output of  $\mathcal{A}$  and the real  $P_2$  in a real execution is identical to the joint output of  $\mathcal{S}$  and  $P_2$  in an ideal execution, as required.

This completes the proof.  $\blacksquare$

## 6.1 Non-Halting Detection Accuracy

It is possible to modify Protocol 6.1 so that it achieves *non-halting detection accuracy*; see Definition 3.3. Before describing how we do this, notice that the reason that we need to recognize a halting-abort as cheating in Protocol 6.1 is that if  $P_1$  generates one faulty circuit, then it can always just refuse to continue (i.e., abort) in the case that  $P_2$  asks it to open the faulty circuit. This means that if aborting is not considered cheating, then a corrupted  $P_1$  can form a strategy whereby it is never detected cheating, but succeeds in actually cheating with probability  $1/\ell$ . In order to solve this problem, we construct a method whereby  $P_1$  does not know if it will be caught or not. We do so by having  $P_2$  receive the circuit openings via a fully secure 1-out-of- $\ell$  oblivious transfer protocol, rather than having  $P_1$  send it explicitly. This forces  $P_1$  to either abort before learning anything, or to risk being caught with probability  $1 - 1/\ell$ . In order to describe this in more detail, we restate the circuit opening stage of Protocol 6.1 as follows:

1. Party  $P_1$  sends  $\ell$  garbled circuits  $GC_1, \dots, GC_\ell$  to party  $P_2$ .
2.  $P_2$  sends a random challenge  $\gamma \in_R \{1, \dots, \ell\}$ .
3.  $P_1$  opens  $GC_j$  for all  $j \neq \gamma$  by sending decommitments, keys and so on. In addition, it sends the keys associated with its own input in  $GC_\gamma$ .
4.  $P_2$  checks the circuits  $GC_j$  for  $j \neq \gamma$  and computes  $GC_\gamma$  (using the keys from  $P_1$  in the previous step and the keys it obtained earlier in the oblivious transfers).  $P_2$ 's output is defined to be the output of  $GC_\gamma$ .

Notice that  $P_2$  only outputs `corrupted1` if the checks from the circuit that is opened do not pass. As we have mentioned, there is no logical reason why an adversarial  $P_1$  would ever actually reply with an invalid opening; rather it would just abort. Consider now the following modification:

1. Party  $P_1$  sends  $\ell$  garbled circuits  $GC_1, \dots, GC_\ell$  to party  $P_2$ .
2.  $P_1$  and  $P_2$  participate in a (fully secure) 1-out-of- $\ell$  oblivious transfer with the following inputs:
  - (a)  $P_1$  defines its inputs  $(x_1, \dots, x_\ell)$  as follows. Input  $x_i$  consists of the opening of circuits  $GC_j$  for  $j \neq i$  together with the keys associated with its own input in  $GC_i$ .
  - (b)  $P_2$ 's input is a random value  $\gamma \in_R \{1, \dots, \ell\}$ .
3.  $P_2$  receives an opening of  $\ell - 1$  circuits together with the keys needed to compute the other and proceeds as above.

Notice that this modified protocol is essentially equivalent to Protocol 6.1 and thus its proof of security is very similar. However, in this case, an adversarial  $P_1$  who constructs one faulty circuit must decide *before* the oblivious transfer if it wishes to abort (in which case there is no successful cheating) or if it wishes to proceed (in which case  $P_2$  will receive an explicitly invalid opening). Note that due to the security of the oblivious transfer,  $P_1$  cannot know what value  $\gamma$  party  $P_2$  inputs, and so cannot avoid being detected.

The price of this modification is that of one fully secure 1-out-of- $\ell$  oblivious transfer and the replacement of all of the original oblivious transfer protocols with fully secure ones. (Of course, we could use oblivious transfer protocols that are secure in the presence of covert adversaries with non-halting detection accuracy, but we do not know how to construct such a protocol more efficiently than a fully secure one.) Fully-secure oblivious transfer is rather expensive, but reasonably efficient protocols do exist [21]. (We remark that one should not be concerned with the lengths of  $x_1, \dots, x_\ell$  in  $P_1$ 's input to the oblivious transfer. This is because  $P_1$  can send them encrypted ahead of time with independent symmetric keys  $k_1, \dots, k_\ell$ . Then the oblivious transfer takes place only on the keys.)

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## A Yao’s Protocol for Semi-Honest Adversaries

We now describe Yao’s protocol for secure two-party computation (in the presence of semi-honest adversaries) which is proven secure in [22]. Yao’s protocol is based on the following “garbled-circuit” construction.

**The garbled circuit construction.** Let  $C$  be a Boolean circuit that receives two inputs  $x_1, x_2 \in \{0, 1\}^n$  and outputs  $C(x_1, x_2) \in \{0, 1\}^n$  (for simplicity in this description, we assume that the input length, output length and the security parameter are all of the same length  $n$ ). We also assume that  $C$  has the property that if a circuit-output wire comes from a gate  $g$ , then gate  $g$  has no wires that are input to other gates.<sup>11</sup> (Likewise, if a circuit-input wire is itself also a circuit-output, then it is not input into any gate.) The reduction uses a private key encryption scheme  $(G, E, D)$  that has indistinguishable encryptions for multiple messages, and also a special property called an *elusive efficiently verifiable range*; see [22].<sup>12</sup>

We begin by describing the construction of a single garbled gate  $g$  in  $C$ . The circuit  $C$  is Boolean, and therefore any gate is represented by a function  $g : \{0, 1\} \times \{0, 1\} \rightarrow \{0, 1\}$ . Now, let the two input wires to  $g$  be labelled  $w_1$  and  $w_2$ , and let the output wire from  $g$  be labelled  $w_3$ . Furthermore, let  $k_1^0, k_1^1, k_2^0, k_2^1, k_3^0, k_3^1$  be six keys obtained by independently invoking the key-generation algorithm  $G(1^n)$ ; for simplicity, assume that these keys are also of length  $n$ . Intuitively, we wish to be able to compute  $k_3^{g(\alpha, \beta)}$  from  $k_1^\alpha$  and  $k_2^\beta$ , without revealing any of the other three values  $k_3^{g(1-\alpha, \beta)}, k_3^{g(\alpha, 1-\beta)}, k_3^{g(1-\alpha, 1-\beta)}$ . The gate  $g$  is defined by the following four values

$$\begin{aligned} c_{0,0} &= E_{k_1^0}(E_{k_2^0}(k_3^{g(0,0)})) \\ c_{0,1} &= E_{k_1^0}(E_{k_2^1}(k_3^{g(0,1)})) \\ c_{1,0} &= E_{k_1^1}(E_{k_2^0}(k_3^{g(1,0)})) \\ c_{1,1} &= E_{k_1^1}(E_{k_2^1}(k_3^{g(1,1)})) \end{aligned}$$

The actual gate is defined by a *random permutation* of the above values, denoted as  $c_0, c_1, c_2, c_3$ ; from here on we call them the *garbled table* of gate  $g$ . Notice that given  $k_1^\alpha$  and  $k_2^\beta$ , and the values  $c_0, c_1, c_2, c_3$ , it is possible to compute the output of the gate  $k_3^{g(\alpha, \beta)}$  as follows. For every  $i$ , compute  $D_{k_2^\beta}(D_{k_1^\alpha}(c_i))$ . If more than one decryption returns a non- $\perp$  value, then output abort. Otherwise, define  $k_3^\gamma$  to be the only non- $\perp$  value that is obtained. (Notice that if only a single non- $\perp$  value is obtained, then this will be  $k_3^{g(\alpha, \beta)}$  because it is encrypted under the given keys  $k_1^\alpha$  and  $k_2^\beta$ . By the properties of the encryption scheme, it can be shown that except with negligible probability, only one non- $\perp$  value is indeed obtained.)

We are now ready to show how to construct the entire garbled circuit. Let  $m$  be the number of *wires* in the circuit  $C$ , and let  $w_1, \dots, w_m$  be labels of these wires. These labels are all chosen uniquely with the following exception: if  $w_i$  and  $w_j$  are both output wires from the same gate  $g$ , then  $w_i = w_j$  (this occurs if the fan-out of  $g$  is greater than one). Likewise, if an input bit enters more than one gate, then all circuit-input wires associated with this bit will have the same label. Next, for every label  $w_i$ , choose two independent keys  $k_i^0, k_i^1 \leftarrow G(1^n)$ ; we stress that all of these keys are chosen independently of the others. Now, given these keys, the four garbled values of each gate are computed as described above and the results are permuted randomly. Finally, the output or decryption tables of the garbled circuit are computed. These tables simply consist of the values  $(0, k_i^0)$  and  $(1, k_i^1)$  where  $w_i$  is a *circuit-output wire*. (Alternatively, output gates can just

<sup>11</sup>This requirement is due to our labelling of gates described below, that does not provide a unique label to each wire (see [22] for more discussion). We note that this assumption on  $C$  increases the number of gates by at most  $n$ .

<sup>12</sup>Loosely speaking, an encryption scheme has an elusive range if without knowing the key, it is hard to generate a ciphertext that falls in the range. An encryption scheme has a verifiable range if given the key and a ciphertext, it is easy to verify that the ciphertext is in the range. Such encryption schemes can be constructed using pseudorandom functions by encrypting the message together with  $n$  zeroes. It is easy to see that this provides both an elusive range and an efficiently verifiable one. We denote by  $\perp$  the result of decrypting a value not in the range.

compute 0 or 1 directly. That is, in an output gate, one can define  $c_{\alpha,\beta} = E_{k_1^\alpha}(E_{k_2^\beta}(g(\alpha,\beta)))$  for every  $\alpha, \beta \in \{0, 1\}$ .)

The entire garbled circuit of  $C$ , denoted  $G(C)$ , consists of the garbled table for each gate and the output tables. We note that the structure of  $C$  is given, and the garbled version of  $C$  is simply defined by specifying the output tables and the garbled table that belongs to each gate. This completes the description of the garbled circuit.

Let  $x_1 = x_1^1 \cdots x_1^n$  and  $x_2 = x_2^1 \cdots x_2^n$  be two  $n$ -bit inputs for  $C$ . Furthermore, let  $w_1, \dots, w_n$  be the input labels corresponding to  $x_1$ , and let  $w_{n+1}, \dots, w_{2n}$  be the input labels corresponding to  $x_2$ . It is shown in [22] that given the garbled circuit  $G(C)$  and the strings  $k_1^{x_1^1}, \dots, k_n^{x_1^n}, k_{n+1}^{x_2^1}, \dots, k_{2n}^{x_2^n}$ , it is possible to compute  $C(x_1, x_2)$ , except with negligible probability.

**Yao's protocol.** Yao's protocol works by designating one party, say  $P_1$ , to be the circuit constructor.  $P_1$  builds a garbled circuit to compute  $f$  and hands it to  $P_2$ . In addition,  $P_1$  sends  $P_2$  the keys  $k_1^{x_1^1}, \dots, k_n^{x_1^n}$  that are associated with its input  $x_1$ . Finally,  $P_2$  obtains the keys  $k_{n+1}^{x_2^1}, \dots, k_{2n}^{x_2^n}$  associated with its input via (semi-honest) oblivious transfer. That is, for every  $i = 1, \dots, n$ , parties  $P_1$  and  $P_2$  run an oblivious transfer protocol. In the  $i^{\text{th}}$  execution,  $P_1$  plays the sender with inputs  $(k_{n+i}^0, k_{n+i}^1)$  and  $P_2$  plays the receiver with input  $x_2^i$ . Following this,  $P_2$  has the keys  $k_1^{x_1^1}, \dots, k_n^{x_1^n}, k_{n+1}^{x_2^1}, \dots, k_{2n}^{x_2^n}$  and so, as stated above, it can compute the circuit to obtain  $C(x_1, x_2)$ . Furthermore, since it has only these keys, it cannot compute the circuit for any other input.

**A Lemma.** In our proof of security, we will use the following lemma:

**Lemma A.1** *Given a circuit  $C$  with inputs wires  $w_1, \dots, w_{2n}$  and an output value  $y$  (of the same length as the output of  $C$ ) it is possible to efficiently construct a garbled circuit  $\widetilde{GC}$  such that:*

1. *The output of  $\widetilde{GC}$  is always  $y$ , regardless of the garbled values that are provided for  $P_1$  and  $P_2$ 's input wires, and*
2. *If  $y = f(x_1, x_2)$ , then no non-uniform probabilistic polynomial-time adversary  $\mathcal{A}$  can distinguish between the distribution ensemble consisting of  $\widetilde{GC}$  and a single arbitrary key for every input wire, and the distribution ensemble consisting of a real garbled version of  $C$ , together with the keys  $k_1^{x_1^1}, \dots, k_n^{x_1^n}, k_{n+1}^{x_2^1}, \dots, k_{2n}^{x_2^n}$ .*

**Proof Sketch:** The proof of this lemma is taken from [22] (it is not stated in this way there, but is proven). We sketch the construction of  $\widetilde{GC}$  here for the sake of completeness, and refer the reader to [22] for a full description and proof. The first step in the construction of the fake circuit  $\widetilde{GC}$  is to choose two random keys  $k_i$  and  $k'_i$  for every wire  $w_i$  in the circuit  $C$ . Next, the gate tables of  $C$  are computed: let  $g$  be a gate with input wires  $w_i, w_j$  and output wire  $w_\ell$ . The table of gate  $g$  contains encryptions of the single key  $k_\ell$  that is associated with wire  $w_\ell$ , under *all four combinations* of the keys  $k_i, k'_i, k_j, k'_j$  that are associated with the input wires  $w_i$  and  $w_j$  to  $g$ . (This is in contrast to a real construction of the garbled circuit that involves encrypting both  $k_\ell$  and  $k'_\ell$ , depending on the function that the gate in question computes.) That is, the following values are computed:

$$\begin{aligned} c_{0,0} &= E_{k_i}(E_{k_j}(k_\ell)) \\ c_{0,1} &= E_{k_i}(E_{k'_j}(k_\ell)) \\ c_{1,0} &= E_{k'_i}(E_{k_j}(k_\ell)) \\ c_{1,1} &= E_{k'_i}(E_{k'_j}(k_\ell)) \end{aligned}$$



The gate table for  $g$  is then just a random ordering of the above four values. This process is carried out for all of the gates of the circuit. It remains to describe how the output decryption tables are constructed. Denote the  $n$ -bit output  $y$  by  $y_1 \cdots y_n$ , and denote the circuit-output wires by  $w_{m-n+1}, \dots, w_m$ . In addition, for every  $i = 1, \dots, n$ , let  $k_{m-n+i}$  be the (single) key encrypted in the gate whose output wire is  $w_{m-n+i}$ , and let  $k'_{m-n+i}$  be the other key (as described above). Then, the output decryption table for wire  $w_{m-n+i}$  is given by:  $[(0, k_{m-n+i}), (1, k'_{m-n+i})]$  if  $y_i = 0$ , and  $[(0, k'_{m-n+i}), (1, k_{m-n+i})]$  if  $y_i = 1$ . This completes the description of the construction of the fake garbled circuit  $\widetilde{GC}$ .

Notice that by the above construction of the circuit, the output keys (or garbled values) obtained by  $P_2$  for *any* set of input keys (or garbled values), equals  $k_{m-n+1}, \dots, k_m$ . Furthermore, by the above construction of the output tables, these keys  $k_{m-n+1}, \dots, k_m$  decrypt to  $y = y_1 \cdots y_n$  exactly. Thus, property (1) of the lemma trivially holds. The proof of property (2) follows from a hybrid argument in which the gate construction is changed one at a time from the real construction to the above fake one (indistinguishability follows from the indistinguishability of encryptions). The construction and proof of this hybrid are described in full in [22]. ■