How to Derive Lower Bound on Oblivious Transfer Reduction

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Abstract

Suppose that we are given an ideal oblivious transfer protocol (OT). We wish to construct a larger OT by using the above OT as a blackbox. Then how many instances of the given ideal OT should be invoked ? For this problem, some lower bounds were derived using entropy. In this paper, we show more tight lower bounds by using combinatorial techniques. Roughly speaking, our lower bounds are two times larger than the previous bounds.

Keywords: Oblivious Transfer, Reduction, Lower bound, Combinatorial Approach

1 Introduction

1.1 OT reduction

In the ideal model of 1-bit $(1, 2)$-Oblivious Transfer (OT), the sender (Alice) has secret two bits, s_0 and s_1, and sends them to the trusted third party (TTP). The receiver (Bob) has a choice bit c and sends it to the TTP.

Finally, the TTP sends s_c to Bob. A two-party 1-bit $(1,2)$-OT is a two-party protocol which implements the ideal 1-bit $(1,2)$-OT without TTP. A two-party L-bit $(1,2)$-OT is a generalization such that s_0 and s_1 are L-bit strings.

Such OT is a fundamental primitive in cryptography. Most notably, any secure multiparty computation can be based on OT [11, 8, 9].

It is known that L-bit $(1,2)$-OT is equivalent to 1-bit $(1,2)$-OT. OT reduction is to construct a two-party L-bit $(1,2)$-OT by using the ideal model of 1-bit $(1,2)$-OT t times, where t should be small. [1] The resulting L-bit $(1,2)$-OT must satisfy receivers's privacy and sender's privacy.

- Receiver's privacy means that any infinitely powerful sender A^* learns no information on c.
- Sender's privacy means that any infinitely powerful receiver B^* learns no information on s_{1-c}.

Sender's privacy can be further defined in two ways.

- Strong privacy: B^* learns no information s_{1-c}.
- Weak privacy: B^* learns almost no information s_{1-c}.

Then some OT reductions are known as follows. (See Table 1.)

1. Brassard, Cŕepeau and Santha used zigzag functions [2]. Their L-bit $(1,2)$-OT satisfies strong privacy and it never aborts. In this construction, it is known that $t/L \geq 3.5277$.
2. Brassard and Crépeau used privacy amplificaiton technique [1]. The resulting protocol satisfies weak privacy and it never aborts. In this protocol, $t/L = 2 + \epsilon$, where ϵ is a negligible factor.
3. Crépeau and Savvides used interactive hashing [6]. The resulting protocol satisfies weak privacy and it aborts with small probability. In this protocol, $t/L = 1 + \epsilon$, where ϵ is a negligible factor.

[1] The other dirctsion is easy.

Table 1: Known OT-Reductions

	Privacy	Abort	t/L
Zigzag function [2]	strong	no	≥ 3.5277
Privacy amplificaiton [1]	weak	no	$2+\epsilon$
Interactive hashing [6]	weak	yes	$1+\epsilon$

Table 2: Lower Bound on OT-Reduction

	Lower bound	technique
Dodis and Micali [7]	$t \geq L$	entropy
This paper	$t \geq 2 \times L$	combinatorial technique

1.2 Lower bound on OT reduction

Then what is a lower bound on t/L ? For strong privacy with no abort, Dodis and Micali showed a lower bound such that $t \geq L$ by using entropy [7].

In this paper, we show a better lower bound such that $t \geq 2 \times L$ by using a combinarotial technique for strong privacy with no abort. (See Table 2.) Our lower bound implies that there exists a clear separation between strong privacy with no abort and weak privacy with abort. (See table 3.)

1.3 Generalization

Our result can be generalized as follows. $(1, n)$-OT is a generalization of $(1, 2)$-OT such that Alice has n strings $s_0, \cdots, s_{n-1}$, and Bob receives one

Table 3: Implication of Our Bound

Privacy	abort	known reduction	our bound
strong	no	$t/L \geq 3.5277$	$t/L \geq 2$
weak	no	$t/L = 2+\epsilon$	?
weak	yes	$t/L = 1+\epsilon$	?

Table 4: Generalization

	Lower bound	technique
Dodis and Micali [7]	$t \geq L/\ell$	entropy
This paper	$t \geq 2 \times L/\ell$	combinatorial technique

of them. Suppose that the ideal ℓ-bit $(1, n)$-OT is used t times to construct an L-bit $(1, n)$-OT. Then what is a lower bound on t ? Dodis and Micali showed that $t \geq L/\ell$ by using entropy. Roughly speaking, we show that $t \geq 2 \times L/\ell$ by using a combinatorial technique. See table 4.

1.4 Related work

Wolf and Wullschleger presented another lower bound on the reduction of L-bit $(1, N)$-OT to ℓ-bit $(1, n)$-OT [10]. However, their bound is the same as the one of Dodis and Micali for $N = n$.

2 Oblivious Transfer (OT)

As an ℓ-bit $(1, n)$-Oblivious Transfer, imagine an ideal world as follows. Alice has n secret strings of ℓ bits $s_0, s_1, \cdots, s_{n-1} \in \{0, 1\}^\ell$, and Bob has a secret $c \in \{0, 1, \cdots, n-1\}$.

1. First, Alice sends $s_0, s_1, \cdots, s_{n-1}$ to a trusted third party (TTP), and Bob sends c to TTP.

2. Next TTP sends s_c to Bob.

We say that the above three party protocol (Alice, Bob, TTP) is the ideal ℓ-bit $(1, n)$-Oblivious Transfer.

By using the above ideal ℓ-bit $(1, n)$-Oblivious Transfer as a building block, we are interested in to construct a *two*-party L-bit $(1, N)$-Oblivious Transfer protocol (Alice, Bob) which satisfies the following three conditions, where $L \geq \ell$ and $N \geq n$.

Completeness. If Alice and Bob follow the protocol, then Bob receives s_c.

Receiver's privacy. For any infinitely powerful $\tilde{A}$, $\tilde{A}$ learns no information on c when $(\tilde{A}, B)$ is executed.

Sender's privacy. For any infinitely powerful $\tilde{B}$, $\tilde{B}$ learns no information on $s_0, s_1, \cdots, s_{N-1}$ other than some s_c when $(A, \tilde{B})$ is executed.

More formally, sender's privacy is defined as follows. For $i = 0, 1, \cdots, N-1$, let S_i denote the random variable induced by $s_i \in \{0,1\}^L$. For each i, we assume that

$$\Pr(S_i = \alpha) > 0$$

for any $\alpha \in \{0,1\}^L$. We also assume that each S_i is independent each other.

Let *view* denote the view of Bob (receiver) which consists of his random coin tosses and the messages that he received from Alice. Let $View$ denote the random variable induced by *view*.

Definition 2.1 *(Sender's privacy) We say that (strong) sender's privacy is satisfied if for any infinitely powerful $\tilde{B}$ and his any possible view, there exists $c \in \{0, 1, \cdots, N-1\}$ such that for any $i \neq c$,*

$$\Pr(S_i = \alpha \mid View = view) = \Pr(S_i = \alpha) > 0$$

for any $\alpha \in \{0,1\}^L$.

For two strings R_0 and R_1, let $R_0||R_1$ denote the concatenation.

3 Previous Lower Bounds

Suppose that we want to construct an L-bit $(1,2)$-OT from t instances of the ideal ℓ-bit $(1,2)$-OT. Dodis and Micali showed the first lower bound on t as follows [7].

Proposition 3.1 *Suppose that there exists an L-bit $(1,N)$-OT which invokes t instances of the ideal ℓ-bit $(1,n)$-OT. Then we have*

$$t \geq \frac{L}{\ell} \times \frac{N-1}{n-1}.$$

Wolf and Wullschleger presented another lower bound as follows [10].

Proposition 3.2 *Suppose that there exists an L-bit $(1, N)$-OT which invokes t instances of the ideal ℓ-bit $(1, n)$-OT. Then we have*

$$t \geq \log N / \log n, \tag{1}$$
$$t \geq L/\ell. \tag{2}$$

In particular, for $N = n = 2$ and $\ell = 1$, we have the following corollary from Proposition 3.1 and Proposition 3.2. This is the most tight bound known so far for $N = n = 2$ and $\ell = 1$.

Corollary 3.1 *Suppose that there exists an L-bit $(1, 2)$-OT which invokes t instances of the ideal 1-bit $(1, 2)$-OT. Then we have $t \geq L$.*

Also, eq.(2) is the best known bound for $L \geq \ell$ and $N < 2n - 1$. All the above bounds were derived by using entropy.

4 Our First Lower Bound

In this section, we derive our lower bounds by using a simple counting argument (while the previous bounds were derived by using entropy). We consider the reduction of L-bit $(1, 2)$-OT to 1-bit $(1, 2)$-OT first, and then the reduction of L-bit $(1, n)$-OT to ℓ-bit $(1, n)$-OT. Our bounds are more tight than the previous bounds. See Sec.2 for the definition of ideal OT.

4.1 Lower Bound for $(1, 2)$-OT

Theorem 4.1 *Suppose that there exists an L-bit $(1, 2)$-OT which invokes t instances of the ideal 1-bit $(1, 2)$-OT. Then we have*

$$t \geq 2L - 1.$$

(Proof) Suppose that there exists an L-bit $(1, 2)$-OT which invokes t instances of the ideal 1-bit $(1, 2)$-OT. In the L-bit $(1, 2)$-OT protocol,

- Alice has two secret strings $s_0, s_1 \in \{0, 1\}^L$ and Bob has a choice bit c.
- At the end, Bob receives s_c.

We denote by $Alice(R_A; s_0, s_1)$ Alice who has R_A as her random tape and s_0, s_1 as her input, where $s_0, s_1 \in \{0,1\}^L$. We also denote by $Bob(R_B; c)$ Bob who has R_B as his random tape and c as his input, where $c \in \{0,1\}$. Let $com(Alice(R_A; s_0, s_1), Bob(R_B; c))$ denote the communication sequence between $Alice(R_A; s_0, s_1)$ and $Bob(R_B; c)$ other than the t invocations of the ideal 1-bit $(1,2)$-OT.

Fix R_A, s_0 and s_1 arbitrarily. For any R_0 and $c = 0$, let

$$\mathbf{com_0} = com(Alice(R_A; s_0, s_1), Bob(R_0; 0)). \tag{3}$$

Since Alice learns no information on c, there exists some R_1 for $c = 1$ such that

$$\mathbf{com_0} = com(Alice(R_A; s_0, s_1), Bob(R_1; 1)). \tag{4}$$

Fix the above R_0, com_0 and R_1. Then all the inputs to Alice are fixed. Therefore, her input to the ideal 1-bit $(1,2)$-OTs are determined. Suppose that (x_i, y_i) is her input to the ith OT for $i = 1, \cdots, t$.

Without loss of generality, we can assume that Bob behaves as follows.

- $Bob(R_0; 0)$ receives $(x_1, \cdots, x_t)$ and finally computes s_0.
- $Bob(R_1; 1)$ receives $(y_1, \cdots, y_t)$ and finally computes s_1. [2]

First suppose that $t = even$. Consider a malicious $\tilde{B}$ who has $R_0||R_1$ and receives $Z = (x_1, \cdots, x_{t/2}, y_{(t/2)+1}, \cdots, y_t)$. $\tilde{B}$ behaves in the same way as $Bob(R_0; 0)$ except for that it receives Z. Alternatively, we can say that $\tilde{B}$ behaves in the same way as $Bob(R_1; 1)$ except for that it receives Z. Hence com_0 is the communication sequence between $Alice(R_A; s_0, s_1)$ and $\tilde{B}$ other than the t invocations of the ideal 1-bit $(1,2)$-OT.

Everything is fixed here. In particular, The view of $\tilde{B}$ is fixed, where the view is given by $\mathbf{view'} = (R_0||R_1, Z, \mathbf{com_0})$.

Now fixing the above $\mathbf{view'}$, we do not fix R_A, s_0 and s_1 any more. (In other words, we consider conditional probability distribution on R_A, s_0 and s_1 given $\mathbf{view'}$.) Then $\tilde{B}$ has no information on either s_0 or s_1 from Sender's privacy. Without loss of generality, suppose that $\tilde{B}$ has no information on

[2] Suppose that $Bob(R_1; 1)$ receives some x_i. It is an easy exercise to show that our bound holds even in this case.

s_0. This means that s_0 can be any L-bit string because for any L-bit string $\alpha \in \{0,1\}^L$,

$$\Pr(S_0 = \alpha \mid View = \mathbf{view}') = \Pr(S_0 = \alpha) > 0.$$

Here it is helpful to note the following: (honest) Bob is an interactive Turing machine. But there exists a (usual) algorithm (based on Bob) such that

- it outputs s_0 on input $(R_0, (x_1, \cdots, x_t), \mathbf{com_0})$, and
- it outputs s_1 on input $(R_1, (y_1, \cdots, y_t), \mathbf{com_0})$.

By using this algorithm (which is essentially Bob), $\tilde{B}$ can compute

- s_0 on input $(R_0||R_1, (x_1, \cdots, x_t), \mathbf{com_0})$, and
- s_1 on input $(R_0||R_1, (y_1, \cdots, y_t), \mathbf{com_0})$.

Now $(x_{t/2+1}, \cdots, x_t)$ are not included in the $\mathbf{view}'$. This means that $(x_{t/2+1}, \cdots, x_t) \in \{0,1\}^{t/2}$ uniquely determine $s_0 \in \{0,1\}^L$. In other words, there exists an onto mapping $F : \{0,1\}^{t/2} \to \{0,1\}^L$. This implies that $t/2 \geq L$. Hence

$$t \geq 2L. \tag{5}$$

Next suppose that $t = odd$. Let $t_0 = \lfloor t/2 \rfloor$ and $t_1 = \lceil t/2 \rceil$. Consider malicious $\tilde{B}$ who receives $(x_1, \cdots, x_{t_0}, y_{t_1}, \cdots, y_t)$ in the t invocations of the ideal $(1,2)$-OT. Then by using the same argument as above, we obtain that $t_0 \geq L$ or $t_1 \geq L$. Hence $t_1 \geq L$. This means that $t_0 = t_1 - 1 \geq L - 1$. Therefore,

$$t = t_0 + t_1 \geq L + (L-1) = 2L - 1. \tag{6}$$

From eq.(5) and eq.(6), we obtain that $t \geq 2L - 1$.

Q.E.D.

4.2 Generalization to $(1,n)$-OT

Theorem 4.2 *Suppose that there exists an L-bit $(1,n)$-OT which invokes t instances of the ideal ℓ-bit $(1,n)$-OT. Then we have*

$$t \geq 2\lceil L/\ell \rceil - 1.$$

(Proof) Suppose that there exists an L-bit $(1,n)$-OT which invokes t instances of the ideal ℓ-bit $(1,n)$-OT. In the L-bit $(1,n)$-OT protocol,

- Alice has n secret strings $s_0, \cdots, s_{n-1} \in \{0,1\}^L$ and Bob has a secret $c \in \{0, \cdots, n-1\}$.
- At the end, Bob receives s_c.

We use the same notation and the same argument as shown in the proof of Theorem 4.1. Although $c \in \{0, \cdots, n-1\}$, we consider $Bob(R_0; 0)$ for $c = 0$ and $Bob(R_1; 1)$ for $c = 1$.

First suppose that $t = even$. Then similarly to the proof of Theorem 4.1, there exists an onto mapping $F : \{0,1\}^{\ell t/2} \to \{0,1\}^L$. This implies that $\ell t/2 \geq L$. Hence we have

$$t \geq \lceil 2L/\ell \rceil. \tag{7}$$

Next suppose that $t = odd$. Then similarly to the proof of Theorem 4.1, we have

$$t = t_0 + t_1 \geq \lceil L/\ell \rceil - 1 + \lceil L/\ell \rceil = 2\lceil L/\ell \rceil - 1. \tag{8}$$

From eq.(7) and eq.(8), we obtain that $t \geq 2\lceil L/\ell \rceil - 1$. Q.E.D.

5 Improved Bounds

In this section, we improve our lower bounds by using orthogonal arrays for large L.

5.1 Orthogonal Array

We define orthogonal arrays as follows.

Definition 5.1 *An orthogonal array $OA(m,k,d)$ is a $k \times m^d$ matrix of m symbols such that in any d rows, every one of the possible m^d tuples of symbols appears exactly once.*

For example, the following matrix is an $OA(2,3,2)$. That is, in any two rows, each of $(00)^T, \cdots, (11)^T$ appears exactly once.

$$\begin{pmatrix} 0011 \\ 1001 \\ 0101 \end{pmatrix}.$$

Then Bush bound is known as follows [3, 5].

Proposition 5.1 (Bush bound) *An orthogonal array* $OA(m, k, d)$ *with* $d > 1$ *exists only if*

$$k \leq \begin{cases} m + d - 1 & \text{if } m \text{ even and } d \leq m, \\ m + d - 2 & \text{if } m \text{ odd and } 3 \leq d \leq m, \\ d + 1 & \text{if } d \geq m. \end{cases}$$

In particular, the thirs bound tells that for $OA(2, k, d)$, it must be that $k \leq d + 1$ if $d \geq 2$.

5.2 Improvement of Theorems 4.1 and 4.2

By using Bush bound, we can improve Theorems 4.1 and 4.2 as shown below.

Theorem 5.1 *For* $L \geq 3$*, suppose that there exists an* L*-bit* $(1, 2)$*-OT which invokes* t *instances of the ideal* 1*-bit* $(1, 2)$*-OT. Then we have*

$$t \geq 2L.$$

Theorem 5.2 *Let* L/ℓ *be an integer such that* $L/\ell \geq 2^\ell + 1$*. Suppose that there exists an* L*-bit* $(1, n)$*-OT which invokes* t *instances of the ideal* ℓ*-bit* $(1, n)$*-OT. Then we have*

$$t \geq 2L/\ell.$$

5.3 Proof of Theorem 5.1

From Theorem 4.1, it holds that $t \geq 2L - 1$. Suppose that $t = 2L - 1$. We use the same notation as in the proof of Theorem 4.1, and fix $R_0, R_1, \mathbf{com_0}$ as in the proof of Theorem 4.1.

Let Y_0 be the set of all $(y_1, \cdots, y_t)$ such that

$$\Pr(\text{ Bob receives } s_1 = 0^L \text{ }) > 0.$$

Let P be a $t \times |Y_0|$ matrix which consists of all $(y_1, \cdots, y_t)^T \in Y_0$. We will show that P is an $\mathrm{OA}(2, t, L - 1)$.

Similarly to the proof of Theorem 4.1, consider malicious $\tilde{B}$ who receives

$$Z = (x_1, \cdots, x_L, y_{L+1}, \cdots, y_{2L-1})$$

in the t instances of the ideal 1-bit $(1,2)$-OT. It must be that $\tilde{B}$ has no information on either s_0 or s_1. Suppose that $\tilde{B}$ has no information on s_0. Then similarly to deriving eq.(5), we obtain that $L-1 \geq L$. However, this is a contradiction.

Therefore, $\tilde{B}$ has no information on s_1. In this case, there must exist an onto mapping $F : \{(y_1, \cdots, y_L)\} \to \{s_1\}$. This means that there exists a bijection between $\{(y_1, \cdots, y_L)\}$ and the set of all l-bit strings. Hence

$$\Pr((y_1, \cdots, y_L) = 0^L) > 0.$$

Therefore, we have

$$\Pr((y_1, \cdots, y_{L-1}) = 0^{L-1}) > 0.$$

Now for $(y_1, \cdots, y_{L-1}) = 0^{L-1}$, it is to easy to show that there exists a bijection between $\{(y_L, \cdots, y_{2L-1})\}$ and the set of s_1 such that $(y_1, \cdots, y_{L-1}, y_L, \cdots, y_{2L-1}) = (0^{L-1}, \beta)$ determines s_1 uniquely.

In particular, there exists a unique $\beta_0 \in \{0,1\}^L$ such that $(y_1, \cdots, y_{2L-1}) = (0^{L-1}, \beta)$ determines $s_1 = 0^L$. This means that $(0^{L-1}, \beta_0)^T$ is a column of P and 0^{L-1} appears exactly once in the first $L-1$ rows. Similarly, for some β_1, $(1^{L-1}, \beta_1)^T$ is a column of P and 1^{L-1} appears exactly once in the first $L-1$ rows. By the same argument, in the first $L-1$ rows, each $L-1$ bit string appears exactly once. That is,

$$P = \begin{pmatrix} (0\cdots 0)^T & \cdots & (1\cdots 1)^T \\ \beta_0 & \cdots & \beta_1 \end{pmatrix}.$$

The above observation holds in any $L-1$ rows. Hence P is an $\mathrm{OA}(2, t, L-1)$. Then from Bush bound, it must be that

$$t \leq (L-1) + 1 = L$$

because $L \geq 3 > 2$. However, this is impossible because $t = 2L-1$.

Hence $t \neq 2L-1$. Therefore, it must be that $t \geq 2L$.

5.4 Proof of Theorem 5.2

From our assumption, $\eta = L/\ell$ is an integer. From Theorem 4.2, it holds that $t \geq 2L/\ell - 1 = 2\eta - 1$. Suppose that $t = 2\eta - 1$. We use the same notation as in the proof of Theorem 4.2. Fix $R_0, R_1, \mathbf{com_0}$.

Let Y_0 be the set of all $(y_1, \cdots, y_t)$ such that

$$\Pr(\text{ Bob receives } s_1 = 0^L \text{ }) > 0.$$

Let P be a $t \times |Y_0|$ matrix which consists of all $(y_1, \cdots, y_t)^T \in Y_0$. We will show that P is an $\mathrm{OA}(2^\ell, t, \eta - 1)$.

Similarly to the proof of Theorem 4.1, consider malicious $\tilde{B}$ who receives

$$Z = (x_1, \cdots, x_\eta, y_{\eta+1}, \cdots, y_{2\eta-1})$$

in the t $(= 2\eta - 1)$ instances of the ideal ℓ-bit $(1, n)$-OT. It must be that $\tilde{B}$ has no information on either s_0 or s_1. Suppose that $\tilde{B}$ has no information on s_0. Then similarly to deriving eq.(7), we obtain that $\ell(\eta - 1) \geq L$. Since $\ell\eta = L$, it implies $L - \ell \geq L$. However, this is a contradiction.

Therefore, $\tilde{B}$ has no information on s_1. In this case, there must exist an onto mapping $F : \{(y_1, \cdots, y_\eta)\} \to \{s_1\}$. This means that there exists a bijection between $\{(y_1, \cdots, y_\eta)\}$ and the set of s_1 because $|\{(y_1, \cdots, y_\eta)\}| = |\{0, 1, \ldots, 2^\ell - 1\}^\eta| = 2^{\ell\eta} = 2^L$ and $|\{s_1\}| = |\{0, 1\}^L| = 2^L$. Hence for any $\gamma \in \{0, 1, \ldots, 2^\ell - 1\}^\eta$,

$$\Pr((y_1, \cdots, y_\eta) = \gamma) > 0.$$

In particular, we have

$$\Pr((y_1, \cdots, y_{\eta-1}) = 0^{\eta-1}) > 0.$$

Now for $(y_1, \cdots, y_{\eta-1}) = 0^{\eta-1}$, we can see that there exists a bijection between $\{(y_\eta, \cdots, y_{2\eta-1})\}$ and the set of s_1 such that $(y_1, \cdots, y_{\eta-1}, y_\eta, \cdots, y_{2\eta-1}) = (0^{\eta-1}, \beta)$ determines s_1 uniquely.

In particular, there exists a unique $\beta \in \{0, 1, \ldots, 2^\ell - 1\}^\eta$ such that $(y_1, \cdots, y_{2\eta-1}) = (0^{\eta-1}, \beta)$ determines $s_1 = 0^L$. This means that $(0^{\eta-1}, \beta)^T$ is a column of P and $0^{\eta-1}$ appears exactly once in the first $\eta - 1$ rows. By the same argument, in the first $\eta - 1$ rows, each $\beta \in \{0, 1, \ldots, 2^\ell - 1\}^{\eta-1}$ appears exactly once.

The above observation holds in any $\eta-1$ rows. Hence P is an $\mathrm{OA}(2^\ell, t, \eta-1)$. Then from Bush bound, it must be that

$$t \leq (\eta - 1) + 1 = \eta$$

because $\eta - 1 \geq 2^\ell$ from our assumption. However, this is impossible because $t = 2\eta - 1$.

Hence it must be that $t \geq 2\eta$.

6 Discussion

The following table shows a comparison of our bounds with the best known bounds. It is clear that our bounds are more tight.

Reduction	L-bit $(1,2)$-OT to 1-bit $(1,2)$-OT	L-bit $(1,n)$-OT to ℓ-bit $(1,n)$-OT
Previous	$t \geq L$ (Corollary 3.1)	$t \geq L/\ell$ (eq.(2))
This paper (1)	$t \geq 2L-1$ (Theorem 4.1)	$t \geq 2\lceil L/\ell \rceil - 1$ (Theorem 4.2)
This paper (2)	$t \geq 2L$ if $L \geq 3$ (Theorem 5.1)	$t \geq 2L/\ell$ if $\eta = L/\ell$ is an integer and $\eta \geq 2^\ell + 1$ (Theorem 5.2)

Our bounds hold as far as there exists $c \in \{0, 1, \cdots, n-1\}$ such that for any $i \neq c$,

$$\Pr(S_i = \alpha \mid View = view) > 0$$

for any $\alpha \in \{0,1\}^L$.

We derived our bounds by using our combinatorial techniques while the previous bounds [7, 10] were derived by using entropy. We believe that our approach gives a new insight into more understanding of oblivious transfer.

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