# Efficient Hierarchical Identity Based Signature in the Standard Model 

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#### Abstract

The only known constructions of Hierarchical Identity Based Signatures that are proven secure in the strongest model without random oracles are based on the approach of attaching certificate chains or hierarchical authentication tree with one-time signature. Both construction methods lead to schemes that are somewhat inefficient and leave open the problem of efficient direct construction. In this paper, we propose the first direct construction of Hierarchical Identity Based Signature scheme that is proven under the strongest model without relying on random oracles and using more standard $q$-SDH assumption. It is computationally efficient and the signature size is constant. When the number of hierarchical level is set to be one, our scheme is a normal identity based signature scheme. It enjoys the shortest size in public parameters and signatures when compare with others in the literature, with the same security level.


## 1 Introduction

Identity based (ID-based) cryptosystem [19] is a public key cryptosystem where the public key can be represented as an arbitrary string such as an email address. The concept was proposed in 1984. However, practical ID-based encryption (IBE) schemes were not found until the work of Boneh and Franklin [6] in 2001. It requires a central authority called the Public Key Generator (PKG) to use a master key to issue private keys to identities that request them. It is provable secure in the random oracle model. Several IBE schemes [8, 2, 14] are later proposed which are secure without random oracles but under a weaker "selective-ID" model [8]. [3] and [20] proposed IBE schemes which are provably
secure without random oracles under the model of [6]. On the other side, several direct constructions of ID-based signature (IBS) can be found in literature, such as $[17,15]$.

Hierarchical ID-based cryptography was first proposed in [12] and [16] in 2002. It is a generalization of IBE that mirrors an organizational hierarchy. It allows a root PKG to distribute the workload by delegating private key generation and identity authentication to lower-level PKGs. In a hierarchical ID-based encryption (HIBE) scheme, a root PKG only needs to generate private keys for domain-level PKGs, who in turn generate private keys for their users in the domains of the lower level. To encrypt a message to Bob, Alice only needs to obtain the public parameters of Bob's root PKG and his identity. It is especially useful in large companies or e-government structure where there are hierarchical administrative issues needed to be taken care. Another application of HIBE is to construct forward secure encryption, as suggested by Canetti, Halevi and Katz [8]. It allows users to periodically update their private keys so that a message encrypted at period $n$ cannot be read using a private key from period $n^{\prime}>n$. HIBE provides one of the most direct and practical solutions to the key exposure problem in daily life public key infrastructure applications.

Recently, Boneh et al. [5] (preliminary papers [9, 7]) suggested some methods to construct chosen ciphertext secure (CCA) $\ell$-level HIBE scheme from a chosen plaintext secure (CPA) $(\ell+1)$-level HIBE scheme. Several HIBE without random oracles are proposed in $[2,3,20,4]$ using this result. However, They are all secure in the selective-ID model only. Transforming of selective-ID model into the model of [6] introduces a loss factor of about $2^{160}$ in the reduction $[2,4]$.

In parallel to HIBE, the idea of hierarchical ID-based signature (HIBS) scheme was first proposed by Gentry and Silverberg [12] in 2002 while the first provable secure HIBS scheme was proposed by Chow et al. [10]. It requires the random oracle to prove its security. It is observed that HIBS can be constructed by using hierarchical authentication tree and one-time signature [13, 21], but it is inefficient. Bellare, Namprempre and Neven [1] suggested that IBS without random oracles can be constructed by certificate chaining. Yuen and Wei [21] remarked that some of the certificate chaining instantiations [9, 7] bear a striking resemblance to the multi-level certificate chaining structure in HIBS. User identity can be certified by his parent, by signing an IBS on the user's identity. The parent's identity can be certified again by one level higher, and the process repeats up until the root. If in each level, the certification of user identity is secure in the standard model, and finally the lowest level user signature is secure against adaptive chosen message attack in the standard model, then the entire HIBS scheme is full adaptive chosen identity and message attack in the standard model. However, this solution will increase the signature size by the level of hierarchy. Yuen and Wei also provided a direct construction where the size of the signature is independent to the number of levels in the same paper. Although their scheme can be proven without random oracles, it is either provable secure under a even weaker model called the "gauntlet-ID model" or require a specially designed strong assumption, the OrcYW assumption.

We also noted, attributed to Gentry and Silverberg [12], that IBS schemes can be constructed from a HIBE scheme. Similarly, HIBS scheme can be obtained from HIBE scheme, where signing identities are part of a hierarchy having one level less.

Contributions. In this paper, we propose the first direct construction of HIBS scheme that is secure in the strongest model of [6] without using random oracles and its security is proven using the more standard $q$-SDH assumption. The size of the signature is a constant while the size of public parameters is independent to the number of bit representing an identity. It is more efficient than the generic constructions of using certificate chain or hierarchical authentication tree with one-time signature.

Our scheme is based on a hierarchical extension of Gentry's IBE scheme [11], and we convert it into a HIBS scheme. However, the conversion is not straightforward. Several techniques have to be suitably combined to obtain the required proof.

When we set the number of hierarchical level to be 1 , our scheme becomes a normal IBS. When compare to the IBS scheme in [18], we have a significant improvement in space efficiency. The size of public parameter is constant in our proposed scheme while they are growing linear with the number of bit of identity representation. Our signature size is just two group elements while they need three group elements to achieve the same security level as ours.

Organization. The rest of the paper is organized as follow. Some mathematical preliminaries are given in Section 2. Security definition is given in Section 3. Our proposed HIBS scheme is presented in Section 4. The paper is concluded in Section 5.

## 2 Preliminaries

### 2.1 Pairings

We briefly review bilinear pairing. Let $\mathbb{G}$ and $\mathbb{G}_{T}$ be two multiplicative cyclic groups of prime order $p$. Let $g$ be a generator of $\mathbb{G}$, and $e$ be a bilinear map such that $e: \mathbb{G} \times \mathbb{G} \rightarrow \mathbb{G}_{T}$ with the following properties:

1. Bilinearity: For all $u, v \in \mathbb{G}$, and $a, b \in \mathbb{Z}, e\left(u^{a}, v^{b}\right)=e(u, v)^{a b}$.
2. Non-degeneracy: $e(g, g) \neq 1$.
3. Computability: It is efficient to compute $e(u, v)$ for all $u, v \in \mathbb{G}$.

### 2.2 Intractability Assumption

The security of our HIBS scheme is based on $q$-SDH assumption, which is defined as follow:

Definition 1 ( $q$-Strong Diffie-Hellman Assumption ( $q$-SDH)). The $q$ Strong Diffie-Hellman ( $q-S D H$ ) problem in $\mathbb{G}$ is defined as follow: On input a $(q+2)$-tuple $\left(g_{0}, h_{0}, h_{0}^{x}, h_{0}^{x^{2}}, \cdots, h_{0}^{x^{q}}\right) \in \mathbb{G}^{q+2}$, output a pair $(A, c)$ such that $A^{(x+c)}=g_{0}$ where $c \in \mathbb{Z}_{p}^{*}$. We say that the $(t, \epsilon, q)$-SDH assumption holds in $\mathbb{G}$ if no t-time algorithm has advantage at least $\epsilon$ in solving the $q-S D H$ problem in $\mathbb{G}$.

## 3 Security Model

An $\ell$-level HIBS scheme consists of four algorithms: (Setup, Extract, Sign, Verify). They are specified as follows:

- Setup: On input a security parameter $1^{\lambda_{s}}$, the TA generates $\langle m s k$, param〉 where $m s k$ is the randomly generated master secret key and param is the corresponding public parameter.
- Extract: On input an identity vector $I D$ (where $|I D|<\ell$ ), its associated secret key $S K_{I D}$, and a string r, it returns the corresponding private key $S K_{I D . r}$ (corresponds to param).
- Sign: On input the private key of the signer $I D, S K_{I D}$ and a message $M$, it outputs a signature $\sigma$ corresponding to param.
- Verify: On input the signer identity vector $I D$, a message $M$ and signature $\sigma$, it outputs $\top$ if $\sigma$ is a valid signature of $M$ corresponding to $I D$, param. Otherwise, it outputs $\perp$.

The security of a HIBS consists of two requirements, namely Correctness and Existential Unforgeability. They are defined as follows:

Correctness. We require that $\top \leftarrow \operatorname{Verify}\left(I D, M, \operatorname{Sign}\left(S K_{I D}, M\right)\right)$ for any message $M$, any private key $S K_{I D}$ and its corresponding identity $I D$.

Existential Unforgeability. We define the existential unforgeability against adaptive identity and adaptive chosen message attack for HIBS (EU-ID-CMA), as in the following game. We define the following oracles:

- $\mathcal{K} \mathcal{E} \mathcal{O}(I D)$ : The Key Extraction Oracle with input $I D$ (where $|I D| \leq \ell$ ) will output the secret key $S K_{I D}$ corresponding to $m s k$.
- $\mathcal{S O}(I D, M)$ : The Signing Oracle with input signer $I D$ (where $|I D| \leq \ell$ ) and message $M$ outputs a signature $\sigma$ such that $\operatorname{Verify}(I D, M, \sigma)=\top$.

The Game is defined as follows:

1. (Phase 1.) Simulator $\mathcal{S}$ generates system parameter param and gives it to Adversary $\mathcal{A}$.
2. (Phase 2.) $\mathcal{A}$ queries $\mathcal{K E O}(I D)$ and $\mathcal{S O}(I D, M)$, in arbitrary interleaf.
3. (Phase 3.) $\mathcal{A}$ delivers a signature $\sigma^{*}$ for signer identity $I D^{*}$ (where $\left|I D^{*}\right| \leq \ell$ ) and message $M^{*}$. $I D^{*}$ or its prefix have never been input to a $\mathcal{K E \mathcal { O }}$ and $\left(I D^{*}, M^{*}\right)$ has never been input to a $\mathcal{S O}$.
$\mathcal{A}$ wins if he completes the Game with $\top=\operatorname{Verify}\left(I D^{*}, M^{*}, \sigma^{*}\right)$. Its advantage is its probability of winning.
Definition 2. The HIBS scheme is $\left(t, \epsilon, q_{e}, q_{s}\right)$-EU-ID-CMA secure if no $t$-time adversary $\mathcal{A}$ has an advantage at least $\epsilon$ in the EU-ID-CMA game using $q_{e}$ queries to $\mathcal{K} \mathcal{E} \mathcal{O}$ and $q_{s}$ queries to $\mathcal{S O}$.

## 4 The Proposed HIBS scheme

### 4.1 Construction of a $\ell$-HIBS scheme

Let $\mathbb{G}$ and $\mathbb{G}_{T}$ be groups of order $p$, and let $e: \mathbb{G} \times \mathbb{G} \rightarrow \mathbb{G}_{T}$ be the bilinear map. We use a multiplicative notation for the operation in $\mathbb{G}$ and $\mathbb{G}_{T}$.

Setup: The PKG selects a random generator $g \in \mathbb{G}$ and randomly chooses $h_{0}, \ldots, h_{\ell+1} \in_{R} \mathbb{G}, \alpha \in_{R} \mathbb{Z}_{p}$ and $r_{0} \in_{R} \mathbb{Z}_{p}^{*}$. It sets $g_{1}=g^{\alpha}$. Let $\mathbb{I D}_{0} \in \mathbb{Z}_{p}^{*}$ be the identity of the PKG. The public parameters param and master secret key $m s k$ are given by

$$
\operatorname{param}=\left(g, g_{1}, r_{0}, h_{0}, \ldots, h_{\ell+1}, \mathrm{ID}_{0}\right) \quad m s k=\alpha
$$

Extract: To generate a private key for identity $\left(\mathrm{ID}_{1}, \ldots, \mathrm{ID}_{i}\right) \in \mathbb{Z}_{p}^{* i}$, the PKG computes:

$$
a_{0}=\left(h_{0} g^{-r_{0}}\right)^{1 /\left(\alpha-\mathrm{ID}_{0}\right)}
$$

The PKG generates random $r_{i} \in_{R} \mathbb{Z}_{p}$ and computes:
$a_{i}=a_{0}\left(\prod_{k=1}^{i} h_{k}^{\mathrm{ID}_{k}}\right)^{r_{i}}, \quad b_{i}=\left(g_{1} g^{-\mathrm{ID}_{0}}\right)^{r_{i}}, \quad c_{i, i+1}=h_{i+1}^{r_{i}}, \quad \ldots, \quad c_{i, \ell+1}=h_{\ell+1}^{r_{i}}$
The private key is $\left(a_{i}, b_{i}, c_{i, i+1}, \ldots, c_{i, \ell+1}\right)$.
The private key can also be generated by its parent $\left(\mathrm{ID}_{1}, \ldots, I \mathrm{D}_{i-1}\right)$ having the secret key $a_{i-1}, b_{i-1}, c_{i-1, i}, \ldots, c_{i-1, \ell+1}$. He generates random $t \in \in_{R} \mathbb{Z}_{p}$ and computes:

$$
\begin{array}{cc}
a_{i}=a_{i-1} \cdot c_{i-1, i}^{\mathrm{ID}_{i}} \cdot\left(\prod_{k=1}^{i} h_{k}^{\mathrm{ID}_{k}}\right)^{t}, & b_{i}=b_{i-1} \cdot\left(g_{1} g^{-\mathrm{ID}_{0}}\right)^{t}, \\
c_{i, i+1}=c_{i-1, i+1} \cdot h_{i+1}^{t}, & \ldots,
\end{array} c_{i, \ell+1}=c_{i-1, \ell+1} \cdot h_{\ell+1}^{t} .
$$

This private key is a properly distributed private key for $r_{i}=r_{i-1}+t$.
Sign: To sign a message $m \in \mathbb{Z}_{p}^{*}$ using identity $\left(\mathrm{ID}_{1}, \ldots, \mathrm{ID}_{i}\right) \in \mathbb{Z}_{p}^{* i}$ with secret key $\left(a_{i}, b_{i}, c_{i, i+1}, \ldots, c_{i, \ell+1}\right)$, the signer randomly chooses $s \in_{R} \mathbb{Z}_{p}$ and constructs the signature

$$
\sigma_{1}=a_{i} \cdot c_{i, i+1}^{m} \cdot\left(h_{i+1}^{m} \prod_{k=1}^{i} h_{k}^{\mathrm{ID}_{k}}\right)^{s}, \quad \sigma_{2}=b_{i} \cdot\left(g_{1} g^{-\mathrm{ID}_{0}}\right)^{s}
$$

The signature is $\left(\sigma_{1}, \sigma_{2}\right)$
Verify: To verify the signature $\left(\sigma_{1}, \sigma_{2}\right)$ for message $m$ and identity $\left(\mathrm{ID}_{1}, \ldots, \mathrm{ID}_{i}\right)$, he compares if

$$
e\left(g_{1} g^{-\mathrm{ID}_{0}}, \sigma_{1}\right) \stackrel{?}{=} e\left(g, h_{0}\right) \cdot e(g, g)^{-r_{0}} \cdot e\left(\sigma_{2}, h_{i+1}^{m} \prod_{k=1}^{i} h_{k}^{\mathrm{ID}_{k}}\right)
$$

### 4.2 Security Analysis

Correctness. The correctness is as follows:

$$
\begin{aligned}
e\left(g_{1} g^{-\mathrm{ID}_{0}}, \sigma_{1}\right) & =e\left(g^{\alpha-\mathrm{ID}_{0}}, a_{i} \cdot c_{i, i+1}^{m} \cdot\left(h_{i+1}^{m} \prod_{k=1}^{i} h_{k}^{\mathrm{ID}_{k}}\right)^{s}\right) \\
& =e\left(g^{\alpha-\mathrm{ID}_{0}}, a_{1}\right) \cdot e\left(g^{\alpha-\mathrm{ID}_{0}},\left(\prod_{k=1}^{i} h_{k}^{\mathrm{ID}_{k}}\right)^{r_{i}} \cdot h_{i+1}^{m r_{i}} \cdot\left(h_{i+1}^{m} \prod_{k=1}^{i} h_{k}^{\mathrm{ID}_{k}}\right)^{s}\right) \\
& =e\left(g^{\alpha-\mathrm{ID}_{0}},\left(h_{0} g^{-r_{0}}\right)^{1 /\left(\alpha-\mathrm{ID}_{0}\right)}\right) \cdot e\left(\sigma_{2}, h_{i+1}^{m} \prod_{k=1}^{i} h_{k}^{\mathrm{ID}_{k}}\right) \\
& =e\left(g, h_{0}\right) \cdot e(g, g)^{-r_{0}} \cdot e\left(\sigma_{2}, h_{i+1}^{m} \prod_{k=1}^{i} h_{k}^{\mathrm{ID}_{k}}\right)
\end{aligned}
$$

Theorem 1. The scheme is $\left(t^{\prime}, \epsilon^{\prime}, q_{e}, q_{s}\right)$-EU-ID-CMA secure if the $(t, \epsilon, q)-S D H$ assumption holds, with

$$
t^{\prime}=t-\mathcal{O}\left(t_{\text {exp }} \cdot q\left(q_{e}+q_{s}\right)\right), \quad \epsilon^{\prime}=\epsilon+q / p
$$

where $t_{\text {exp }}$ is the time required to exponentiate in $\mathbb{G}$.
Proof. Assume there is a $\left(t, \epsilon, q_{e}, q_{s}\right)$-adversary $\mathcal{A}$ exists. We are going to construct another PPT $\mathcal{B}$ that makes use of $\mathcal{A}$ to solve the $q$-SDH problem.
$\mathcal{B}$ takes as input a random $q$ - SDH challenge $\left(g, g_{1}, \ldots, g_{q}\right.$ ) (recall that $g_{i}=$ $\left.g^{\alpha^{i}}\right)$. In order to use $\mathcal{A}$ to solve for the problem, $\mathcal{B}$ needs to simulate the oracles for $\mathcal{A}$.
$\underline{\text { Setup. } \mathcal{B} \text { generates a random polynomial } f(x) \in \mathbb{Z}_{p}[x] \text { of degree } q \text {. It sets } h_{0}=; ~=~=~}$ $\overline{g^{f(\alpha)}}$, which can be computed from $\left(g, g_{1}, \ldots, g_{q}\right) . \mathcal{B}$ randomly picks $\mathrm{ID}_{0} \in_{R} \mathbb{Z}_{p}^{*}$. If $\mathrm{ID}_{0}=\alpha, \mathcal{B}$ uses $\alpha$ to solve the $q$-SDH problem immediately. Otherwise, it picks random $\mu_{i} \in R \mathbb{Z}_{p}^{*}$ and sets $h_{i}=\left(g^{\alpha}\right)^{\mu_{i}} g^{-\mathrm{ID}_{0} \mu_{i}}=g^{\mu_{i}\left(\alpha-\mathrm{ID}_{0}\right)}$ for $i=1, \ldots, \ell+1$.
$\mathcal{B}$ computes a polynomial $g(x) \in \mathbb{Z}_{p}[x]$ of degree $q-1$ and $r_{0} \in \mathbb{Z}_{p}^{*}$ such that

$$
f(x)=g(x)\left(x-\mathrm{ID}_{0}\right)+r_{0}
$$

Note that if $r_{0}=0, \mathcal{B}$ restarts again by picking another $I D_{0}$.

Now we have

$$
a_{0}=\left(h_{0} g^{-r_{0}}\right)^{1 /\left(\alpha-\mathrm{ID}_{0}\right)}=g^{\left(f(\alpha)-r_{0}\right) /\left(\alpha-\mathrm{ID}_{0}\right)}
$$

It sends the param $=\left(g, g_{1}, r_{0}, h_{0}, \ldots, h_{\ell+1}, \mathrm{ID}_{0}\right)$ to $\mathcal{A}$. We can see that param is uniformly random and the public key has a distribution identical to that in the real world.

Oracles Simulation. $\mathcal{B}$ simulates the extraction oracle as follow:
(Extraction oracle.) Upon receiving a query for a private key of an identity $\left(\bar{I}_{1}, \ldots, \bar{I}_{i}\right)$ for some $i \in\{1, \ldots, \ell\}, \mathcal{B}$ randomly picks $\bar{r} \in \mathbb{Z}_{p}^{*}$ and computes:

$$
\begin{aligned}
a_{i} & =g^{g(\alpha)+\bar{r} \sum_{k=1}^{i} \mu_{k} \bar{I}_{k}}, & \quad b_{i}=g^{\bar{r}}, \\
c_{i, i+1} & =g^{\mu_{i+1} \bar{r}}, \quad \ldots, & c_{i, \ell+1}=g^{\mu_{\ell+1} \bar{r}}
\end{aligned}
$$

This is a valid secret key since we set a random $r=\bar{r} /\left(\alpha-\mathrm{ID}_{0}\right)$ :

$$
\begin{aligned}
b_{i} & =g^{r\left(\alpha-\mathrm{ID}_{0}\right)}=\left(g_{1} g^{-\mathrm{ID}_{0}}\right)^{r} \\
a_{i} & =g^{g(\alpha)+\bar{r} \sum_{k=1}^{i} \mu_{k} \bar{I}_{k}} \\
& =g^{\frac{f(\alpha)-r_{0}}{\alpha-1 \mathrm{D}_{0}}+\bar{r} \sum_{k=1}^{i} \mu_{k} \bar{I}_{k}} \\
& =\left(h_{0} g^{-r_{0}}\right)^{1 /\left(\alpha-\mathrm{ID}_{0}\right)} \cdot\left(\prod_{k=1}^{i} h_{k}^{\bar{I}_{k}}\right)^{r} \\
c_{i, j} & =g^{r \mu_{j}\left(\alpha-\mathrm{ID}_{0}\right)}=h_{j}^{r}
\end{aligned}
$$

Notice that $\mathcal{B}$ records the input and output of the extraction oracle, and return the same output for duplicate inputs.
(Signing oracle.) Upon receiving a query for a signature for users $\left(I_{1}, \ldots, I_{i}\right)$ and message $m, \mathcal{B}$ runs the extraction oracle for identity $\left(I_{1}, \ldots, I_{i}\right)$ and runs Sign algorithm to generate a signature for $m$.

Output Calculation. Finally, $\mathcal{A}$ outputs a signature $\left(\sigma_{1}^{*}, \sigma_{2}^{*}\right)$ for message $m^{*}$ and signer $I D^{*}=\left(I_{1}^{*}, \ldots, I_{i}^{*}\right)$ for some $i \in\{1, \ldots, \ell\}$.

Recall $g(x)$ as the $(q-1)$-degree polynomial $\left(f(x)-r_{0}\right) /\left(x-\mathrm{ID}_{0}\right)$. Then we have

$$
g(\alpha)=\sum_{k=0}^{q-1} A_{k} \alpha^{k}+\frac{A_{-1}}{\alpha-\mathrm{ID}_{0}}
$$

$\mathcal{B}$ computes $A_{-1}, A_{0}, \ldots, A_{q-1}$. If $A_{-1}=0, \mathcal{B}$ aborts. Otherwise, we have:

$$
\sigma_{1}^{*}=a_{i} \cdot c_{i, i+1}^{m^{*}} \cdot\left(h_{i+1}^{m^{*}} \cdot \prod_{k=1}^{i} h_{k}^{I_{k}^{*}}\right)^{s}
$$

$$
\begin{aligned}
& =a_{0} \cdot\left(\prod_{k=1}^{i} h_{k}^{I_{k}^{*}}\right)^{r_{i}} \cdot\left(h_{i+1}^{r_{i}}\right)^{m^{*}} \cdot\left(h_{i+1}^{m^{*}} \cdot \prod_{k=1}^{i} h_{k}^{I_{k}^{*}}\right)^{s} \\
& =a_{0} \cdot\left(h_{i+1}^{m^{*}} \cdot \prod_{k=1}^{i} h_{k}^{I_{k}^{*}}\right)^{r_{i}+s} \\
& =g^{\left(f(\alpha)-r_{0}\right) /\left(\alpha-\mathrm{ID}_{0}\right)} \cdot\left(g^{\mu_{i+1}\left(\alpha-\mathrm{ID}_{0}\right) m^{*}} \cdot \prod_{k=1}^{i} g^{\mu_{k}\left(\alpha-\mathrm{ID}_{0}\right) I_{k}^{*}}\right)^{r_{i}+s} \\
& =g^{\left(f(\alpha)-r_{0}\right) /\left(\alpha-\mathrm{ID}_{0}\right)} \cdot\left(g^{\mu_{i+1} m^{*}} \cdot \prod_{k=1}^{i} g^{\mu_{k} I_{k}^{*}}\right)^{\left(r_{i}+s\right)\left(\alpha-\mathrm{ID}_{0}\right)} \\
\sigma_{2}^{*} & =g^{\left(\alpha-\mathrm{ID}_{0}\right)\left(r_{i}+s\right)}
\end{aligned}
$$

Therefore $\mathcal{B}$ can compute:

$$
\begin{aligned}
W & =\frac{\sigma_{1}^{*}}{\sigma_{2}^{* \mu_{i+1} m^{*}+\sum_{k=1}^{i} \mu_{k} I_{k}^{*}}} \\
& =g^{\left(f(\alpha)-r_{0}\right) /\left(\alpha-\mathrm{ID}_{0}\right)} \\
& =g^{\sum_{k=0}^{q-1} A_{k} \alpha^{k}+A_{-1} /\left(\alpha-\mathrm{ID}_{0}\right)}
\end{aligned}
$$

Finally $\mathcal{B}$ computes:

$$
\left(\frac{W}{g^{\sum_{k=0}^{q-1} A_{k} \alpha^{k}}}\right)^{1 / A_{-1}}=g^{1 /\left(\alpha-\mathrm{ID}_{0}\right)}
$$

Then $\mathcal{B}$ returns $\left(g^{1 /\left(\alpha-I D_{0}\right)}, \mathrm{ID}_{0}\right)$ as the solution to the $q$-SDH problem.
Probability Analysis. As $f(x)$ is a uniformly random polynomial of degree $q$ and $\overline{\mathrm{I}}_{0}$ is random from $\mathbb{Z}_{p}^{*}$, the values $g(x)$ and $r_{0}$ are uniformly random. Therefore the keys issued by $\mathcal{B}$ are appropriately distributed.
$\mathcal{B}$ aborts if $A_{-1}=0$. As $f$ is a randomly distributed polynomial, it happens with probability $q / p$ (as there is at most $q$ roots of the polynomial in $\mathbb{Z}_{p}^{*}$ ).

Therefore $\mathcal{B}$ aborts with probability $q / p$.
Time Complexity Analysis. $\mathcal{B}$ 's overhead is dominated by computing $g^{g(\alpha)}$ in the extraction oracle queries. Each such computation requires $\mathcal{O}(q)$ exponentiations in $\mathbb{G}$. Since $\mathcal{A}$ makes at most $q_{s}+q_{e}$ queries, $t=t^{\prime}+\mathcal{O}\left(t_{\text {exp }} \cdot q\left(q_{e}+q_{s}\right)\right)$.

Remarks. We remark that the proof is not a straight-forward extension from Gentry's IBE scheme. In our proposed scheme, we set ID ${ }_{0}$ as the identity of the PKG. It is different from other HIBE or HIBS schemes. In our first attempt, we do not have this "dummy" identity level while allowing the first level to be a "real" user. However, in this way $r_{0}$ is different from every user. In other words, $r_{0}$ is different from every "family" but should be the same within the same
"family". Moreover, $r_{0}$ is needed in the signature verification. The only way we can publish everyone's $r_{0}$ is to put $r_{0}$ as a part of the signature.

In this preliminary attempt, we have encountered a problem in the proof. We first want to follow Gentry's proof to set $r_{0}=f\left(I_{1}\right)$ for identity $I_{1}$. That is, all children of $I_{1}$ bear the same $r_{0}$. If the adversary asks for a signature of $I_{1}$ or his children, the simulator outputs this $r_{0}$ as part of the signature. Unfortunately, if the adversary chooses $I_{1}$ as the challenged identity and output a forged signature containing the same $r_{0}$, we cannot compute the required reduction result. The reason is as follow. Let $g^{\prime}(x)$ be the $(q-1)$-degree polynomial $\left(f(x)-r_{0}\right) /\left(x-I_{1}\right)$ in the output calculation part of the proof. Then we have

$$
g^{\prime}(\alpha)=\sum_{k=0}^{q-1} A_{k} \alpha^{k}+\frac{A_{-1}}{\alpha-I_{1}}
$$

If $r_{0}=f\left(I_{1}\right),\left(x-I_{1}\right)$ can fully divide $\left(f(x)-r_{0}\right)$. Then we have $A_{-1}=0$ and cannot carry out the further reduction.

We solve this problem by introducing a dummy level, $\mathrm{ID}_{0}$ as the identity of the PKG.

## 5 Conclusion

In this paper, we proposed a HIBS scheme with adaptive chosen identity and message security with a constant signature size. It is proven secure in the standard mode, using the $q$-SDH assumption. It is the first direct implementation in the literature to achieve these efficiency and security level, regardless of the scheme in [21] which requires a non-standard assumption. When we set the number of hierarchical level is set to be one, the resulting IBS scheme enjoys significant space efficiency improvement over the scheme in [18] with the same security level.

We believe our implementation is far more efficient than the generic approach by using certificate chains or hierarchical authentication tree with one-time signature.

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