Edon–$\mathcal{R}(256, 384, 512)$ – an Efficient Implementation of Edon–$\mathcal{R}$ Family of Cryptographic Hash Functions

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Abstract. We have designed three fast implementations of recently proposed family of hash functions Edon–$\mathcal{R}$. They produce message digests of length 256, 384 and 512 bits. We have defined huge quasigroups of orders 2^{256}, 2^{384} and 2^{512} by using only bitwise operations on 32 bit values (additions modulo 2^{32}, XORs and left rotations) and achieved processing speeds of the Reference C code of 16.18 cycles/byte, 24.37 cycles/byte and 32.18 cycles/byte on x86 (Intel and AMD microprocessors). In this paper we give their full description, as well as an initial security analysis.

Key words: hash function, Edon–$\mathcal{R}$, quasigroup

1 Introduction

On Second NIST Hash Workshop a family of hash functions Edon–$\mathcal{R}$ was proposed [11]. The initial design was by general quasigroups of relatively small order (up to 256), and the approach was without concrete realization of those hash functions. No concrete measurements about the speed of those hash functions were given, although the authors admitted that computational speed of their design is slow.

In this paper we will describe three concrete realizations of Edon–$\mathcal{R}$ that will produce hash outputs of 256, 384 and 512 bits. We have used bitwise operations on 32 bit values (additions modulo 2^{32}, XORs and left rotations) to construct quasigroups of huge order (2^{256}, 2^{384} and 2^{512}) and then we have used those quasigroups as a basis for implementing the compression function of Edon–$\mathcal{R}$. We will show that the designed quasigroups of huge order lack some of the laws that are satisfied in groups such as commutativity and associativity. That is similar to the approach in the original proposal for the Edon–$\mathcal{R}$ family of cryptographic hash functions. Thus, we are relying our claims about the security of our concrete realization of Edon–$\mathcal{R}$ hash functions on the difficulty of solving general quasigroup equations.

The organization of the paper is as follows: In Section 2 we give some basic mathematical definitions, a definition of a general compression function of Edon–$\mathcal{R}$ with only three blocks, and a definition of three huge quasigroups of orders 2^{256}, 2^{384} and 2^{512}, in Section 3 we define three hash functions Edon–$\mathcal{R}(256, 384, 512)$, in Section 4 we give a design rationale, in Section 5 we give some implementation characteristics, in Section 6 we give an initial security analysis of the proposed hash functions and we conclude the paper by Section 7.

2 Mathematical preliminaries and notation

In this section we need to repeat some parts of the definition of the class of one-way candidate functions $\mathcal{R}_1$ recently defined in [11, 12]. For that purpose we will need also several brief definitions for quasigroups and quasigroup string transformations.

A quasigroup $(Q, *)$ is an algebraic structure consisting of a nonempty set Q and a binary operation $* : Q^2 \to Q$ with the property each of the equations

$$\begin{aligned} a * x &= b \\ y * a &= b \end{aligned} \tag{1}$$

to have unique solutions x and y in Q. Closely related combinatorial structures to finite quasigroups are Latin squares, since the main body of the multiplication table of a quasigroup is just a Latin square. More detailed information about theory of quasigroups, quasigroup string processing, Latin squares and hash functions you can find in [1, 19–21].

For the description of the algorithm we will use the following definitions:

Definition 1. ([12] **Quasigroup reverse string transformation** $\mathcal{R}_1 : Q^r \to Q^r$)

Let r be a positive integer, let $(Q, *)$ be a quasigroup and $a_j, b_j \in Q$. For each fixed $m \in Q$ define first the transformation $Q_m : Q^r \to Q^r$ by

$$Q_m(a_0, a_1, \dots, a_{r-1}) = (b_0, b_1, \dots, b_{r-1}) \iff b_i := \begin{cases} m * a_0, & i = 0 \\ b_{i-1} * a_i, & 1 \le i \le r-1. \end{cases}$$

Then define $\mathcal{R}_1$ as composition of transformations of kind Q_m, for suitable choices of the indexes m, as follows:

$$\mathcal{R}_1(a_0, a_1, \dots, a_{r-1}) := Q_{a_0}(Q_{a_1} \dots (Q_{a_{r-1}}(a_0, a_1, \dots, a_{r-1}))).$$

	a_0	a_1	a_2
a_2	$x_0^{(1)}$	$x_1^{(1)}$	$x_2^{(1)}$
a_1	$x_0^{(2)}$	$x_1^{(2)}$	$x_2^{(2)}$
a_0	b_0	b_1	b_2

a.

	x_0	x_1	x_2
x_2	$x_0^{(1)}$	$x_1^{(1)}$	$x_2^{(1)}$
x_1	$x_0^{(2)}$	$x_1^{(2)}$	$x_2^{(2)}$
x_0	b_0	b_1	b_2

b.

Table 1. a. Schematic presentation of the function $\mathcal{R}_1$ for $r = 3$, **b.** Conjectured one-wayness of $\mathcal{R}_1$ comes from the difficulty to solve a system of three equations where b_0, b_1 and b_2 are given, and $a_0 = x_0$, $a_1 = x_1$ and $a_2 = x_2$ are indeterminate variables.

It was conjectured in [11, 12] that $\mathcal{R}_1$ is one-way function (under some assumptions about the underlying quasigroup $(Q, *)$) and that the complexity of its inverting is exponential i.e. that inverting $\mathcal{R}_1$ has a complexity $O(|Q|^{\frac{r}{3}})$, where $|Q|$ is the size of the set Q.

In our construction of Edon–$\mathcal{R}(n), n = 256, 384, 512$, we will use the function $\mathcal{R}_1$ with $r = 3$. The transformation can be schematically presented by the Table 1a.

The conjectured one-wayness of $\mathcal{R}_1$ can be explained by Table 1b. Namely, let us take that only the values b_0, b_1 and b_2 are given. Then, in order to find pre-image values $a_0 = x_0$, $a_1 = x_1$ and $a_2 = x_2$ we can use the Definition 1 and we will obtain the following equalities for the elements of Table 1b:
$x_0^{(1)} = x_2 * x_0$; $x_1^{(1)} = (x_2 * x_0) * x_1$; $x_2^{(1)} = ((x_2 * x_0) * x_1) * x_2$; $x_0^{(2)} = x_1 * (x_2 * x_0)$; $x_1^{(2)} = (x_1 * (x_2 * x_0)) * ((x_2 * x_0) * x_1)$; $x_2^{(2)} = \big((x_1 * (x_2 * x_0)) * ((x_2 * x_0) * x_1)\big) * \big(((x_2 * x_0) * x_1) * x_2\big)$.

From them, we can obtain the following system of quasigroup equations with indeterminate x_0, x_1, x_2:

$$\begin{cases} b_0 = x_0 * (x_1 * (x_2 * x_0)) \\ b_1 = b_0 * \big((x_1 * (x_2 * x_0)) * ((x_2 * x_0) * x_1)\big) \\ b_2 = b_1 * \Big(\big((x_1 * (x_2 * x_0)) * ((x_2 * x_0) * x_1)\big) * \big(((x_2 * x_0) * x_1) * x_2\big)\Big). \end{cases}$$

One can show that for any given $a_0 = x_0 \in Q$ either there are values of $a_1 = x_1$ and $a_2 = x_2$ as a solution or there is no solution. However, if the quasigroup operation is non-commutative and non-associative, and if the size of the quasigroup is very big (for example 2^{256}, 2^{384} or 2^{512}) then solving this simple system of three quasigroup equations is hard. Actually there is no known efficient method for solving such systems of quasigroup equations.

Of coarse, one inefficient method for solving that system would be to try every possible value for $a_0 = x_0 \in Q$ until obtaining other two indeterminates $a_1 = x_1$ and $a_2 = x_2$. That brute force method would require in average $\frac{1}{2}|Q|$ attempts to guess $a_0 = x_0 \in Q$ before solving the system.

2.1 Definition of quasigroups of huge order

In this section we will describe the construction of quasigroups of huge orders (2^{256}, 2^{384} and 2^{512}). We will use the following notation: Q is a set of cardinality 2^n, and elements $x \in Q$ will be represented in their bitwise form as n-bit words

$$x \equiv (\overline{x}_0, \overline{x}_1, \ldots, \overline{x}_{n-2}, \overline{x}_{n-1}) \equiv \overline{x}_0 \cdot 2^{n-1} + \overline{x}_1 \cdot 2^{n-2} + \ldots + \overline{x}_{n-2} \cdot 2 + \overline{x}_{n-1}$$

where $\overline{x}_i \in \{0, 1\}$.

Let us start first with the following simple (and obvious) proposition:

Proposition 1. *For any finite set Q of cardinality 2^n the operation "Bitwise eXlusive OR" $\oplus_n : Q^2 \to Q$ is quasigroup operation.* □

Let $\pi_1, \pi_2, \pi_3 : Q \to Q$ be three permutations on the set Q.

Proposition 2. *For any finite set Q of cardinality 2^n the operation $* : Q^2 \to Q$ defined as:*

$$a * b \equiv \pi_1(\pi_2(a) \oplus_n \pi_3(b))$$

is a quasigroup operation. □

Proposition 3. *If permutations π_2 and π_3 are not equal, then the quasigroup $(Q, *)$ is non-commutative.* □

Let us denote by $Q_{256} = \{0, 1\}^{256}$, $Q_{384} = \{0, 1\}^{384}$ and $Q_{512} = \{0, 1\}^{512}$ the corresponding sets of 256–bit, 384–bit and 512–bit words. Since our intention is to define Edon–$\mathcal{R}$ by bitwise operations on 32 bit values, we will introduce the following convention: elements $X \in Q_{256}$ will be represented as $X = (X_0, X_1, \ldots, X_7)$, elements $X \in Q_{384}$ will be represented as $X = (X_0, X_1, \ldots, X_{11})$, and elements $X \in Q_{512}$ will be represented as $X = (X_0, X_1, \ldots, X_{15})$, where X_i are 32–bit words.

Further, let us denote by $ROTL(Y, k)$ left rotation of a 32–bit word Y by k positions, by $Y \oplus Z$ ordinary bitwise XOR operations between two 32–bit words Y and Z, and by $Y + Z$ addition modulo 2^{32}.

We will give the formal definitions for the following permutations: $\pi_{1,256}$, $\pi_{2,256}$, $\pi_{3,256}$, $\pi_{1,384}$, $\pi_{2,384}$, $\pi_{3,384}$, $\pi_{1,512}$, $\pi_{2,512}$, $\pi_{3,512}$ where the corresponding three digit index (256, 384 or 512) denotes the cardinality of the set Q over which they are defined.

Definition 2. *Transformation* $\pi_{1,256} : Q_{256} \to Q_{256}$ *is defined as:*

$$\pi_{1,256}(X_0, X_1, X_2, X_3, X_4, X_5, X_6, X_7) = (X_5, X_6, X_7, X_0, X_1, X_2, X_3, X_4)$$

Lemma 1. *Transformation* $\pi_{1,256}$ *is permutation.* $\square$

Definition 3. *Transformation* $\pi_{2,256} : Q_{256} \to Q_{256}$ *is defined as:*

$$\pi_{2,256}(X_0, X_1, X_2, X_3, X_4, X_5, X_6, X_7) = (Y_0, Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7)$$

where

$$\left\{ \begin{array}{l} T_0 = ROTL((X_1 + X_2 + X_4 + X_6 + X_7), 1); \\ T_1 = ROTL((X_0 + X_1 + X_3 + X_4 + X_7), 3); \\ T_2 = ROTL((X_0 + X_1 + X_2 + X_6 + X_7), 4); \\ T_3 = ROTL((X_1 + X_3 + X_4 + X_5 + X_6), 5); \\ T_4 = ROTL((X_0 + X_3 + X_4 + X_5 + X_6), 7); \\ T_5 = ROTL((X_0 + X_2 + X_4 + X_5 + X_7), 8); \\ T_6 = ROTL((X_0 + X_1 + X_2 + X_3 + X_5), 10); \\ T_7 = ROTL((X_2 + X_3 + X_5 + X_6 + X_7), 13); \end{array} \right. \quad \textit{and} \quad \left\{ \begin{array}{l} Y_0 = ROTL((T_0 \oplus T_3 \oplus T_5), 1); \\ Y_1 = ROTL((T_2 \oplus T_5 \oplus T_6), 4); \\ Y_2 = ROTL((T_3 \oplus T_4 \oplus T_5), 8); \\ Y_3 = ROTL((T_0 \oplus T_2 \oplus T_7), 9); \\ Y_4 = ROTL((T_1 \oplus T_2 \oplus T_7), 10); \\ Y_5 = ROTL((T_1 \oplus T_3 \oplus T_6), 12); \\ Y_6 = ROTL((T_4 \oplus T_6 \oplus T_7), 13); \\ Y_7 = ROTL((T_0 \oplus T_1 \oplus T_4), 14); \end{array} \right.$$

Lemma 2. *Transformation* $\pi_{2,256}$ *is permutation.*

The proof is given in the Appendix.

Definition 4. *Transformation* $\pi_{3,256} : Q_{256} \to Q_{256}$ *is defined as:*

$$\pi_{3,256}(X_0, X_1, X_2, X_3, X_4, X_5, X_6, X_7) = (Y_0, Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7)$$

where

$$\left\{ \begin{array}{l} T_0 = ROTL((X_1 + X_4 + X_5 + X_6 + X_7), 2); \\ T_1 = ROTL((X_1 + X_2 + X_4 + X_6 + X_7), 5); \\ T_2 = ROTL((X_0 + X_1 + X_3 + X_6 + X_7), 6); \\ T_3 = ROTL((X_0 + X_2 + X_3 + X_5 + X_6), 7); \\ T_4 = ROTL((X_2 + X_3 + X_4 + X_5 + X_7), 8); \\ T_5 = ROTL((X_0 + X_3 + X_4 + X_5 + X_6), 10); \\ T_6 = ROTL((X_0 + X_1 + X_2 + X_3 + X_4), 11); \\ T_7 = ROTL((X_0 + X_1 + X_2 + X_5 + X_7), 14); \end{array} \right. \quad \textit{and} \quad \left\{ \begin{array}{l} Y_0 = ROTL((T_0 \oplus T_2 \oplus T_3), 3); \\ Y_1 = ROTL((T_0 \oplus T_3 \oplus T_5), 4); \\ Y_2 = ROTL((T_2 \oplus T_4 \oplus T_5), 6); \\ Y_3 = ROTL((T_1 \oplus T_4 \oplus T_7), 8); \\ Y_4 = ROTL((T_0 \oplus T_1 \oplus T_6), 9); \\ Y_5 = ROTL((T_1 \oplus T_2 \oplus T_7), 11); \\ Y_6 = ROTL((T_5 \oplus T_6 \oplus T_7), 12); \\ Y_7 = ROTL((T_3 \oplus T_4 \oplus T_6), 13); \end{array} \right.$$

Lemma 3. *Transformation* $\pi_{3,256}$ *is permutation.* $\square$

The proof is similar to the proof for $\pi_{2,256}$.

Theorem 1. *Operation* $*_{256} : Q_{256}^2 \to Q_{256}$ *defined as:*

$$a *_{256} b = \pi_{1,256}(\pi_{2,256}(a) \quad \oplus_{256} \quad \pi_{3,256}(b))$$

is a non-commutative and non-associative quasigroup operation that is not a loop.

The proof is given in the Appendix.

Analogously, for $n = 384$ and $n = 512$ we will define quasigroup operations $*_{384} : Q_{384}^2 \to Q_{384}$ and $*_{512} : Q_{512}^2 \to Q_{512}$. Their concrete definitions are given in the Appendix.

Having defined three quasigroup operations $*_{256}$, $*_{384}$ and $*_{512}$ we will define three one-way functions $\mathcal{R}_{1,256}$, $\mathcal{R}_{1,384}$ and $\mathcal{R}_{1,512}$ as follows:

Definition 5. *1.* $\mathcal{R}_{1,256} : Q_{256}^3 \to Q_{256}^3 \equiv \mathcal{R}_1$ *where* $\mathcal{R}_1$ *is defined as in Definition 1 over* Q_{256} *with the quasigroup operation* $*_{256}$.

2. $\mathcal{R}_{1,384} : Q_{384}^3 \to Q_{384}^3 \equiv \mathcal{R}_1$ *where* $\mathcal{R}_1$ *is defined as in Definition 1 over* Q_{384} *with the quasigroup operation* $*_{384}$.

3. $\mathcal{R}_{1,512} : Q_{512}^3 \to Q_{512}^3 \equiv \mathcal{R}_1$ *where* $\mathcal{R}_1$ *is defined as in Definition 1 over* Q_{512} *with the quasigroup operation* $*_{512}$.

3 Edon–$\mathcal{R}(256, 384, 512)$ hash algorithm

Having one-way quasigroup functions $\mathcal{R}_{1,256}$, $\mathcal{R}_{1,384}$ and $\mathcal{R}_{1,512}$, we now define three hash algorithms Edon–$\mathcal{R}(256)$, Edon–$\mathcal{R}(384)$ and Edon–$\mathcal{R}(512)$ that map a messages M of arbitrary length of l bits ($l \leq 2^{128}$) into a hash value of 256, 384 or 512 bits.

3.1 Padding

Padding of the messages M of arbitrary length of l bits is done by the standard Merkle-Damgård strengthening. Let us shortly denote all three hash functions as Edon–$\mathcal{R}(n)$ where the parameter n can take the values 256, 384 or 512.

The padding of a message M that is long l bits by Edon–$\mathcal{R}(n)$ is done by the following procedure:

1. Append the bit 1 at the end of the message.
2. Append the smallest amount l_1 of zero bits, such that $l + 1 + l_1 + 128 \equiv 0 \pmod{n}$.
3. Represent the original length l of the message M as an 128–bit number and append it at the end of the message. The length of the appended message M' becomes multiple of n bits. Let represent the appended message as $M' = M_1 M_2 \ldots M_N$ where M_i is n–bit long block.

3.2 Initial predetermined values

The definition of Edon–$\mathcal{R}(n)$ hash function includes one initial string H_0 of length $2n$ bits. That initial string is given as follows (represented in hexadecimal as concatenation of 32-bits chunks):

1. For $n = 256$, $H_0 =$ `0x01020304, 0x05060708, 0x090A0B0C, 0x0D0E0F10, 0x11121314, 0x15161718, 0x191A1B1C,`
   `0x1D1E1F20, 0x21222324, 0x25262728, 0x292A2B2C, 0x2D2E2F30, 0x31323334,`
   `0x35363738, x393A3B3C, 0x3D3E3F40.`
2. For $n = 384$, $H_0 =$ `0x01020304, 0x05060708, 0x090A0B0C, 0x0D0E0F10, 0x11121314, 0x15161718, 0x191A1B1C,`
   `0x1D1E1F20, 0x21222324, 0x25262728, 0x292A2B2C, 0x2D2E2F30, 0x31323334,`
   `0x35363738, 0x393A3B3C, 0x3D3E3F40, 0x41424344, 0x45464748, 0x494A4B4C, 0x4D4E4F50,`
   `0x51525354, 0x55565758, 0x595A5B5C, 0x5D5E5F60.`
3. For $n = 512$, $H_0 =$ `0x01020304, 0x05060708, 0x090A0B0C, 0x0D0E0F10, 0x11121314, 0x15161718, 0x191A1B1C,`
   `0x1D1E1F20, 0x21222324, 0x25262728, 0x292A2B2C, 0x2D2E2F30, 0x31323334,`
   `0x35363738, 0x393A3B3C, 0x3D3E3F40, 0x41424344, 0x45464748, 0x494A4B4C, 0x4D4E4F50,`
   `0x51525354, 0x55565758, 0x595A5B5C, 0x5D5E5F60, 0x61626364, 0x65666768, 0x696A6B6C,`
   `0x6D6E6F70, 0x71727374, 0x75767778, 0x797A7B7C, 0x7D7E7F80.`

The initial values are obtained by concatenation of the 8–bit representation of the numbers $1, 2, \ldots, 128$.

3.3 Edon–$\mathcal{R}(n)$ hash function

Input: n and M, where: n is 256, 384 or 512, and M is the message to be hashed.
Output: A hash of length n bits.

1. **Pad** the message M, so the length of the padded message M' is multiple of n–bit words i.e. $|M'| = N \times n$.
2. **Initialize** H_0.
3. **Compute** the hash with the following iterative procedure:

$$\text{For } i = 1 \text{ to } N \text{ do}$$
$$H_i = \mathcal{R}_{1,n}(H_{i-1}||M_i) \ \texttt{mod}\ 2^{2n};$$

Output:

$$Edon\text{-}\mathcal{R}(n)(M) = H_N \ \texttt{mod}\ 2^n$$

Since the one-way functions $\mathcal{R}_{1,n}$ are considered as transformations $\{0,1\}^{3n} \rightarrow \{0,1\}^{3n}$ for obtaining the intermediate value H_i, we apply the operation $\texttt{mod}\ 2^{2n}$ that takes the last two n–bit words from the result of $\mathcal{R}_{1,n}$. Finally, since the requested output from the hash function is n bits, we take just the last n–bit word from the H_N and that is denoted as the operation $\texttt{mod}\ 2^n$.

4 Design rationale

4.1 Choosing basic 32–bit operations

We have decided to choose 32–bit operations of addition modulo 2^{32}, XOR-ing and left rotations as an optimum choice that can be efficiently implemented both on low-end 8–bit and 16–bit processors, as well as on modern 32–bit and 64–bit CPUs. In the past several cryptographic primitives have been designed following the same rationale as well, such as: Salasa20 [2], The Tiny Encryption Algorithm [26], or IDEA [15] - to name a few.

4.2 Choosing permutations π_1, π_2 and π_3

Our goal was to design a structure that is non-commutative and non-associative quasigroup of huge orders ($2^{256}, 2^{384}$ and 2^{512}) in order to apply the principles of the hash family Edon–$\mathcal{R}$. We have found a way how to construct such a structure by applying some basic permutations π_1, π_2 and π_3 on the sets $\{0,1\}^{256}, \{0,1\}^{384}$ and $\{0,1\}^{512}$.

The permutations $\pi_{1,256}$, $\pi_{1,384}$ and $\pi_{1,512}$ are simple rotations on 256, 384 or 512–bit words. They can be effectively realized just by appropriate referencing of the 32–bit variables (after performing permutations π_2 and π_3). While the permutations π_2 and π_3 do the work of diffusion and nonlinear mixing separately on the first and the second argument of the quasigroup operations, after their outputs are XORed, the permutations π_1 introduce additional diffusion on the whole n–bit word. That diffusion then have influence on the next application of the quasigroup operation $*_n$ (since we apply three such operations in every row).

For the choice of the permutations π_2 and π_3 we had plenty of possibilities. However, since our design is based on quasigroups, it was natural choice to use Latin squares in the construction of those permutations. Actually there is a long history of using Latin squares in the randomized experimental design (see for example [10]) as well as in cryptography [24, 23, 4–6].

Since for the permutations $\pi_{2,256}$ and $\pi_{3,256}$ we wanted bijectively to mix eight 32–bit variables we have used the following 8×8 Latin squares:

$$L_1 = \left(\begin{array}{cccccccc} 2&1&7&6&3&4&0&5 \\ 4&3&2&5&0&7&1&6 \\ 7&0&1&4&6&2&5&3 \\ 6&7&0&1&4&5&3&2 \\ 1&4&6&3&5&0&2&7 \\ \hline 0&6&5&2&1&3&7&4 \\ 5&2&3&0&7&6&4&1 \\ 3&5&4&7&2&1&6&0 \end{array}\right) = \begin{pmatrix} L_{1,1} \\ L_{1,2} \end{pmatrix} \qquad L_2 = \left(\begin{array}{cccccccc} 5&7&0&3&4&6&1&2 \\ 6&2&1&0&7&3&4&5 \\ 7&1&3&6&5&4&2&0 \\ 4&6&7&5&2&0&3&1 \\ 1&4&6&2&3&5&0&7 \\ \hline 2&5&4&1&0&7&6&3 \\ 3&0&5&4&1&2&7&6 \\ 0&3&2&7&6&1&5&4 \end{array}\right) = \begin{pmatrix} L_{2,1} \\ L_{2,2} \end{pmatrix}$$

Then we have split L_1 and L_2 on two (upper and lower) Latin rectangles $L_{1,1}$, $L_{1,2}$, $L_{2,1}$ and $L_{2,2}$. We used the columns of upper rectangles as index sets (block designs) for the variables that are bijectively transformed by addition modulo 2^{32} and the columns of lower rectangles as index sets (block designs) for the variables that are bijectively transformed by XORing of 32–bit variables. More concretely:

$$L_{1,1} \Rightarrow A_{1,1} = \begin{pmatrix} 0&1&1&0&1&0&1&1 \\ 1&1&0&1&1&0&0&1 \\ 1&1&1&0&0&0&1&1 \\ 0&1&0&1&1&1&1&0 \\ 1&0&0&1&1&1&1&0 \\ 1&0&1&0&1&1&0&1 \\ 1&1&1&1&0&1&0&0 \\ 0&0&1&1&0&1&1&1 \end{pmatrix} \qquad L_{1,2} \Rightarrow A_{1,2} = \begin{pmatrix} 1&0&0&1&0&1&0&0 \\ 0&0&1&0&0&1&1&0 \\ 0&0&0&1&1&1&0&0 \\ 1&0&1&0&0&0&0&1 \\ 0&1&1&0&0&0&0&1 \\ 0&1&0&1&0&0&1&0 \\ 0&0&0&0&1&0&1&1 \\ 1&1&0&0&1&0&0&0 \end{pmatrix}$$

As we mentioned in Section 2.1 matrix $A_{1,1}$ is nonsingular in $(\mathbb{Z}_{2^{32}}, +)$ and matrix $A_{1,2}$ is nonsingular in $GF(2)$. Similarly from Latin rectangles $L_{2,1}$ and $L_{2,2}$ we got the nonsingular matrices $A_{2,1}$ and $A_{2,2}$.

$$L_{2,1} \Rightarrow A_{2,1} = \begin{pmatrix} 0&1&0&0&1&1&1&1 \\ 0&1&1&0&1&0&1&1 \\ 1&1&0&1&0&0&1&1 \\ 1&0&1&1&0&1&1&0 \\ 0&0&1&1&1&1&0&1 \\ 1&0&0&1&1&1&1&0 \\ 1&1&1&1&1&0&0&0 \\ 1&1&1&0&0&1&0&1 \end{pmatrix} \qquad L_{2,2} \Rightarrow A_{2,2} = \begin{pmatrix} 1&0&1&1&0&0&0&0 \\ 1&0&0&1&0&1&0&0 \\ 0&0&1&0&1&1&0&0 \\ 0&1&0&0&1&0&0&1 \\ 1&1&0&0&0&0&1&0 \\ 0&1&1&0&0&0&0&1 \\ 0&0&0&0&0&1&1&1 \\ 0&0&0&1&1&0&1&0 \end{pmatrix}$$

Analogously, we have chosen two Latin squares L_3 and L_4 of order 12×12 for Edon–$\mathcal{R}(384)$ and two Latin squares L_5 and L_6 of order 16×16 for Edon–$\mathcal{R}(512)$. They are given in the Appendix.

5 Implementation characteristics of Edon–$\mathcal{R}(256, 384, 512)$

We have initial implementation of all three functions Edon–$\mathcal{R}(256, 384, 512)$ in C. We have run tests compiling both on Microsoft Visual Studio 2005 Pro and Intel C++ 9.1 for Windows. The code was tested only for x86 processors in 32–bit mode. Intel compiler was producing 8.5% – 17.8% faster code. However, in both cases we did not use 64 or 128 bit SSE and SSE2 registers as well as their SIMD capabilities. The initial processing speeds (in cycles/byte) are given in the Table 2.

n	MSVS 2005 Pro	Intel C++ 9.1
256	17.56	16.18
384	28.64	24.37
512	37.91	32.18

Table 2. Speed (cycles/byte) of the Reference C code for Edon–$\mathcal{R}(n)$ on x86 platforms in 32–bit mode obtained from Microsoft Visual Studio 2005 Pro and Intel C++ 9.1 for Windows.

We project that significant improvements (at least twofold increasing) in the speed can be achieved by using SIMD instructions and capabilities of modern CPUs.

On the other hand, measuring of the performances of Edon–$\mathcal{R}(256, 384, 512)$ on 8–bit platforms still have to be done, but we hope that the speeds will be relatively fast due to the fact that we are using only basic 32–bit operations such as addition modulo 2^{32}, eXlusive OR and rotations.

By careful analysis of the order of operations performed in Edon–$\mathcal{R}(256, 384, 512)$ one can notice that there are two types of parallelism of operations:

1. Operations inside the permutations π_2 and π_3 can be executed in parallel.
2. Pipelining of quasigroup operations: after the first quasigroup operation in the firs row, two quasigroup operations can be performed in parallel (one on the first row and one on the second row), and then similarly three quasigroup operations (in all three rows) can be performed in parallel.

This property can lead to hardware implementation of Edon–$\mathcal{R}(256, 384, 512)$ that can achieve even higher speeds.

6 Security analysis of the algorithm

The design of Edon–$\mathcal{R}(n)$ is based on Merkle-Damgård iterating principles [8, 9, 22]. In the light of latest attacks with multi-collisions, the design of Edon–$\mathcal{R}$ has incorporated the suggestions of Lucks [18] and Coron et al. [7]. Namely, by setting the size of the internal memory of the iterated compression function to be twice as much as the output length, weaknesses against generic attacks of Joux [13], and Kelsy and Schneier [14] are eliminated.

Doubling of the internal memory in our design is done by the fact that in every iterative step of its compression function, the strings of length $3n$ bits are mapped to strings of length $3n$ bits and then only the last significant $2n$ bits are kept for the next iterative step.

6.1 Natural resistance of Edon–$\mathcal{R}(n)$ against generic length extension attacks

Generic length extension attacks on iterated hash function based upon Merkle-Damgård iterating principles works as follows:

Let $M = M_1||M_2||\dots||M_N$ be a message consisting of exactly N blocks that will be iteratively digested by some compression function $C(A, B)$ according to the Merkle-Damgård iterating principles, and where A and B are messages (input parameters for the compression function) that has same length as the final message digest. Let P_M is the padding block of M obtained according to the Merkle-Damgård strengthening. Then, the digest H of the message M, is computed as

$$H(M) = C(\dots C(C(IV, M_1), M_2)\dots, P_M),$$

where IV is the initial fixed value for the hash function.

Now suppose that the attacker does not know the message M but knows (or can easily guess the length of the message M). So the attacker actually knows the padding block P_M. Now, the attacker can construct a new message $M' = P_M||M'_1$ such that he knows the hash digest of the message $M||M'$. Namely,

$$H(M||M') = C(H(M), M'_1).$$

Edon–$\mathcal{R}(n)$ has natural resistance against this generic attack due to the fact that it is iterated with the chaining variables that has length that is two times wider that the final digest value (see also the work of Lucks [18]).

6.2 Testing avalanche properties of Edon–$\mathcal{R}(n)$

First we will show the the avalanche propagation of the initial one bit differences of the compression function of Edon–$\mathcal{R}(n)$ during their evolution in all 9 quasigroup operations $*_n$, $(n = 256, 384, 512)$.

We have used two experimental settings:

1. Examining the propagation of the initial 1–bit difference in a message consisting of all zeroes
2. Examining the propagation of the initial 1–bit difference in a randomly generated messages of n–bits.

The results for $n = 256$ are shown in Table 3. Notice that the level of Hamming distance equal to $\frac{1}{2}n = 128$ which would be expected in theoretical models of ideal random functions is achieved after applying quasigroup operations that lie on the down-right half of the tables (in bold).

$Min = 15$ $Avr = 15$ $Max = 15$	$Min = 86$ $Avr = 108.44$ $Max = 133$	$Min = 107$ **Avr=127.43** $Max = 153$
$Min = 80$ $Avr = 110.84$ $Max = 142$	$Min = 103$ **Avr=128.17** $Max = 160$	$Min = 100$ **Avr=127.43** $Max = 151$
$Min = 103$ **Avr=127.54** $Max = 148$	$Min = 102$ **Avr=127.25** $Max = 146$	$Min = 105$ **Avr=127.86** $Max = 148$

a.

$Min = 15$ $Avr = 26.59$ $Max = 74$	$Min = 76$ $Avr = 113.68$ $Max = 149$	$Min = 102$ **Avr=128.11** $Max = 154$
$Min = 73$ $Avr = 115.93$ $Max = 155$	$Min = 103$ **Avr=128.09** $Max = 158$	$Min = 95$ **Avr=127.75** $Max = 155$
$Min = 101$ **Avr=128.07** $Max = 153$	$Min = 100$ **Avr=128.01** $Max = 154$	$Min = 95$ **Avr=127.67** $Max = 155$

b.

Table 3. a. Avalanche propagation of the Hamming distance between two 256–bit words M_1 and M_2 that initially differs in one bit and where $M_1 = 0$ (minimum, average and maximum) **b.** Avalanche propagation of the Hamming distance between two 256–bit words M_1 and M_2 that initially differs in one bit (minimum, average and maximum)

Similar results can be obtained for $n = 384$ and $n = 512$, and they are given in the Appendix.

6.3 Description of all possible collision paths in the compression function $\mathcal{R}_1$ and infeasibility of finding local collisions

Although the general design of Edon–$\mathcal{R}(n)$ follows Merkle-Damgård iterating principles, the design of the compression function $\mathcal{R}_1$ is pretty different than the design of compression functions of known hash function that are designed from scratch. While other compression functions have 64, 80 or even more iterating steps, $\mathcal{R}_1$ has 9 steps. So far, all successful attacks against the MDx and SHA families of hash functions exploited local collisions in the processing of the data block. Local collisions are collisions that can be found within few steps of the compression function.

$*_n$	$B_1 = \{b_1\}$	$B_2 = \{b_1, b_2\}$
$A_1 = \{a_1\}$	$C_1 = \{c_1\}$ where $a_1 *_n b_1 = c_1$	$C_2 = \{c_1, c_2\}$ where $a_1 *_n b_1 = c_1$ and $a_1 *_n b_2 = c_2$
$A_2 = \{a_1, a_2\}$	$C_2 = \{c_1, c_2\}$ where $a_1 *_n b_1 = c_1$ and $a_2 *_n b_1 = c_2$	$C_2 = \{c_1, c_2\}$ where $a_1 *_n b_1 = c_1$ and $a_2 *_n b_2 = c_2$ or $C_1 = \{c_1\}$ where $a_1 *_n b_1 = c_1$ and $a_2 *_n b_2 = c_1$

Table 4. Definition of quasigroup operation between one or two-element sets.

	$\{a_0\}$	$\{a_1\}$	$\{x_1, x_2\}$
$\{x_1, x_2\}$	$\{c_1, c_2\}$	$\{c_3, c_4\}$	$\{c_9, c_{10}\}$
$\{a_1\}$	$\{c_5, c_6\}$	$\{c_{11}, c_{12}\}$	$\{c_{13}, c_{14}\}$
$\{a_0\}$	$\{c_7, c_8\}$	$\{c_{15}, c_{16}\}$	$\{c_{17}\}$

a.

	$\{a_0\}$	$\{a_1\}$	$\{x_1, x_2\}$
$\{x_1, x_2\}$	$\{c_1, c_2\}$	$\{c_3, c_4\}$	$\{c_9, c_{10}\}$
$\{a_1\}$	$\{c_5, c_6\}$	$\{c_{11}, c_{12}\}$	$\{c_{13}\}$
$\{a_0\}$	$\{c_7, c_8\}$	$\{c_{14}\}$	$\{c_{15}\}$

b.

	$\{a_0\}$	$\{a_1\}$	$\{x_1, x_2\}$
$\{x_1, x_2\}$	$\{c_1, c_2\}$	$\{c_3, c_4\}$	$\{c_9, c_{10}\}$
$\{a_1\}$	$\{c_5, c_6\}$	$\{c_{11}\}$	$\{c_{12}, c_{13}\}$
$\{a_0\}$	$\{c_7, c_8\}$	$\{c_{14}, c_{15}\}$	$\{c_{16}\}$

c.

	$\{a_0\}$	$\{a_1\}$	$\{x_1, x_2\}$
$\{x_1, x_2\}$	$\{c_1, c_2\}$	$\{c_3, c_4\}$	$\{c_9\}$
$\{a_1\}$	$\{c_5, c_6\}$	$\{c_{10}, c_{11}\}$	$\{c_{12}, c_{13}\}$
$\{a_0\}$	$\{c_7, c_8\}$	$\{c_{14}, c_{15}\}$	$\{c_{16}\}$

d.

Table 5. Description of all possible differential paths in the compression function $\mathcal{R}_1$ that can give collisions.

The small number of steps in the compression function $\mathcal{R}_1$ as well as the algebraic properties of quasigroup operations will allow us to describe all possible collision paths within the compression function.

In order to track the collision paths for the compression function $\mathcal{R}_1$ we will introduce a definition for quasigroup operation between sets of cardinality one and two.

Definition 6. *Let* $A_1 = \{a_1\}, A_2 = \{a_1, a_2\}, B_1 = \{b_1\}, B_2 = \{b_1, b_2\}, C_1 = \{c_1\}, C_2 = \{c_1, c_2\}$ *be sets of cardinality one or two and where* a_i, b_i *and* $c_i \in Q_n (n = 256, 384, 512)$*. The operation of quasigroup multiplication* $*_n$ *between these sets is defined by the Table 4:*

Following directly by the properties of unique solutions of equations of type (1) it is easy to prove the following two propositions:

Proposition 4. *If* $b_1 \neq b_2$ *then* $\{a_1\} *_n \{b_1, b_2\} = \{c_1, c_2\}$ *such that* $c_1 \neq c_2$*.* □

Proposition 5. *If* $a_1 \neq a_2$ *then* $\{a_1, a_2\} *_n \{b_1\} = \{c_1, c_2\}$ *such that* $c_1 \neq c_2$*.* □

However if both $a_1 \neq a_2$ and $b_1 \neq b_2$ then $\{a_1, a_2\} *_n \{b_1, b_2\}$ can be either $\{c_1, c_2\}$ or $\{c_1\}$ and that is formulated in the following proposition:

Proposition 6. *If* $a_1 \neq a_2$ *and* $b_1 \neq b_2$ *then* $\{a_1, a_2\} *_n \{b_1, b_2\}$ *can be either* $\{c_1, c_2\}$ *(where* $c_1 \neq c_2$*) or* $\{c_1\}$*.* □

We will formalize the notion of collisions for the compression function $\mathcal{R}_1$ by the following definition:

Definition 7. *Let* $(a_0, a_1, x_1), (a_0, a_1, x_2) \in Q_n \times Q_n \times Q_n$ *where* a_0 *and* a_1 *are initial constants defined in Subsection 3.2. If* $\mathcal{R}_1(a_0, a_1, x_1) = (c_0, c_1, y)$ *and* $\mathcal{R}_1(a_0, a_1, x_2) = (d_0, d_1, y)$ *then we say that the pair* $\{x_1, x_2\}$ *is a collision for* $\mathcal{R}_1$.

Using the Definition 6 and Definition 7 we can trace all possible paths that can produce collisions in the compression function $\mathcal{R}_1$. That is formulated in the following theorem:

Theorem 2. *If* $x_1 \neq x_2$ *are two values in* Q_n*, then all possible differential paths starting with the set* $\{x_1, x_2\}$ *that can produce collisions in the compression function* $\mathcal{R}_1$ *are described in Table 5.* □

$$\left\{\begin{array}{l}
c_{17} = c_{15} *_n c_{13} \\
c_{17} = c_{16} *_n c_{14} \\
c_{15} = c_7 *_n c_{11} \\
c_{13} = c_{11} *_n c_9 \\
c_{16} = c_8 *_n c_{12} \\
c_{14} = c_{12} *_n c_{10} \\
c_7 = a_0 *_n c_5 \\
c_{11} = c_5 *_n c_3 \\
c_9 = c_3 *_n x_1 \\
c_8 = a_0 *_n c_6 \\
c_{12} = c_6 *_n c_4 \\
c_{10} = c_4 *_n x_2 \\
c_5 = a_1 *_n c_1 \\
c_3 = c_1 *_n a_1 \\
c_6 = a_1 *_n c_2 \\
c_4 = c_2 *_n a_1 \\
c_1 = x_1 *_n a_0 \\
c_2 = x_2 *_n a_0
\end{array}\right.$$

a.

$$\left\{\begin{array}{l}
c_{15} = c_{14} *_n c_{13} \\
c_{14} = c_7 *_n c_{11} \\
c_{14} = c_8 *_n c_{12} \\
c_{13} = c_{11} *_n c_9 \\
c_{13} = c_{12} *_n c_{10} \\
c_7 = a_0 *_n c_5 \\
c_{11} = c_5 *_n c_3 \\
c_8 = a_0 *_n c_6 \\
c_{12} = c_6 *_n c_4 \\
c_9 = c_3 *_n x_1 \\
c_{10} = c_4 *_n x_2 \\
c_5 = a_1 *_n c_1 \\
c_3 = c_2 *_n a_1 \\
c_6 = a_1 *_n c_2 \\
c_4 = c_2 *_n a_1 \\
c_1 = x_1 *_n a_0 \\
c_2 = x_2 *_n a_0
\end{array}\right.$$

b.

$$\left\{\begin{array}{l}
c_{16} = c_{14} *_n c_{12} \\
c_{16} = c_{15} *_n c_{13} \\
c_{14} = c_7 *_n c_{11} \\
c_{12} = c_{11} *_n c_9 \\
c_{15} = c_8 *_n c_{11} \\
c_{13} = c_{11} *_n c_{10} \\
c_7 = a_0 *_n c_5 \\
c_{11} = c_5 *_n c_3 \\
c_{11} = c_6 *_n c_4 \\
c_9 = c_3 *_n x_1 \\
c_8 = a_0 *_n c_6 \\
c_{10} = c_4 *_n x_2 \\
c_5 = a_1 *_n c_1 \\
c_3 = c_1 *_n a_1 \\
c_6 = a_1 *_n c_2 \\
c_4 = c_2 *_n a_1 \\
c_1 = x_1 *_n a_0 \\
c_2 = x_2 *_n a_0
\end{array}\right.$$

c.

$$\left\{\begin{array}{l}
c_{16} = c_{14} *_n c_{12} \\
c_{16} = c_{15} *_n c_{13} \\
c_{14} = c_7 *_n c_{10} \\
c_{12} = c_{10} *_n c_9 \\
c_{15} = c_8 *_n c_{11} \\
c_{13} = c_{11} *_n c_9 \\
c_7 = a_0 *_n c_5 \\
c_{10} = c_5 *_n c_3 \\
c_9 = c_3 *_n x_1 \\
c_9 = c_4 *_n x_2 \\
c_8 = a_0 *_n c_6 \\
c_{11} = c_6 *_n c_4 \\
c_5 = a_1 *_n c_1 \\
c_3 = c_1 *_n a_1 \\
c_4 = c_2 *_n a_1 \\
c_6 = a_1 *_n c_2 \\
c_1 = x_1 *_n a_0 \\
c_2 = x_2 *_n a_0
\end{array}\right.$$

d.

Table 6. Concrete systems of quasigroup equations that can give collisions in the compression function $\mathcal{R}_1$

From Table 5 it is clear that for the collision in Table 5a., there are no local collisions. For the other three cases there are local collisions $\{c_{13}\}$ and $\{c_{14}\}$ in Table 5b., $\{c_{11}\}$ in Table 5c. and $\{c_9\}$ in Table 5d. In Table 6 we give four systems of quasigroup equations that are following directly from collision paths described in Table 5. From the complexity of the given quasigroup equations we can say that in this moment we see that it is infeasible even to find local collisions. As a support for that claim we can point out that the position of all local collisions lie in the areas that are reaching the level of randomness that is characteristic for a random Boolean functions (see bolded parts in Table 3, 8 and 9 and a position of local collisions in Table 5b., 5c. and 5d.).

6.4 Fix points for the compression function $\mathcal{R}_1$

From the definition of the permutations π_1, π_2 and π_3 over Q_{256}, Q_{384} and Q_{512} it is clear that 0 is the fixed point of the compression function $\mathcal{R}_1$, i.e. $\mathcal{R}_1(0) = 0$ where $0 \in Q_{256}$ or $0 \in Q_{384}$ or $0 \in Q_{512}$.

We had (and still have) a dilemma should we put some constants in π_2 and π_3 that will have an effect that $\mathcal{R}_1(0) \neq 0$.

In this moment we do not see any argument how the fact that $\mathcal{R}_1(0) = 0$ jeopardize the security of the whole hash function Edon–$\mathcal{R}(n)$, i.e., how can it be used to find collisions, preimages and second preimages.

Of coarse there is always concern that the property of the compression function $\mathcal{R}_1(0) = 0$ is not a "typical" random behavior, and hash functions are often used as random functions. A counter argument for this can be that there is clear distinction between the whole hash function (in this case Edon–$\mathcal{R}(n)$ which seems to act as a random function) and its compression function.

6.5 Getting all the additions to behave as XORs

Having a compression function $\mathcal{R}_1$ defined only by additions modulo 2^{32}, XORs and left rotations, it is a natural idea to try to find values for which additions in $\mathcal{R}_1$ behave as XORs [25].

In such a case, one would have a completely linear system in $GF(2)^n$ for which collisions, preimages and second preimages can easily be found. However, getting all the additions to behave as XORs is a challenge.

Here we can point out several significant works that are related with analysis of differential probabilities of operations that combine additions modulo 2^n, XORs and left rotations. In 1993 Berson have made a differential cryptanalysis of addition modulo 2^{32} and applied it on MD5 [3], in 2001 Lipmaa and Moriai, have constructed efficient algorithms for computing differential properties of addition modulo 2^n [16], and Lipmaa, Wallén and Dumas in 2004 have constructed linear-time algorithm for computing the additive differential probability of exclusive-or [17].

All of these works are determining the additive differential probability of exclusive-or:

$$Pr[((x + \alpha) \oplus (y + \beta)) - (x \oplus y) = \gamma]$$

and exclusive-or differential probability of addition:

$$Pr[((x \oplus \alpha) + (y \oplus \beta)) \oplus (x + y) = \gamma]$$

where probability is computed for all pairs $(x, y) \in \mathbb{Z}_{2^n} \times \mathbb{Z}_{2^n}$ and for any predetermined triplet $(\alpha, \beta, \gamma) \in \mathbb{Z}_{2^n} \times \mathbb{Z}_{2^n} \times \mathbb{Z}_{2^n}$.

In the case of Edon–$\mathcal{R}(n)$, instead of simple combination of two 32-bit variables once by additions modulo 2^n then by xoring, we have a linear transformation of 8, 12 or 16 32-bit variables described by transformations defined in Definition 3, 4, 10, 11, 13, 14. Additionally, having in mind that $\mathcal{R}_1 : \{0,1\}^{3n} \rightarrow \{0,1\}^{3n}$, in this moment we don't see how these results will help in finding concrete values of arguments for the function $\mathcal{R}_1$ for which additions will behave as XORs.

6.6 Infeasibility of going backward and infeasibility of finding free start collisions

According to the conjectured one-wayness of the function $\mathcal{R}_1$, iterating backward Edon–$\mathcal{R}(n)$ is infeasible. The conjecture is again based on the infeasibility of solving nonlinear quasigroup equations in non-commutative and non-associative quasigroups. From this it follows that the workload for finding preimages and second-preimages for any hash function of the family Edon–$\mathcal{R}(n)$ is 2^n hash computations.

Moreover, inverting one-way function $\mathcal{R}_1$ would imply that finding free start collisions is feasible for the whole function Edon–$\mathcal{R}(n)$. Consequently, we base our conjecture that it is infeasible to find free start collisions for Edon–$\mathcal{R}(n)$ on the infeasibility of inverting the one-way function $\mathcal{R}_1$.

We will elaborate our claims more concretely by the following discussion:

Definition 8. *Let* $(a_0, a_1, x_1), (b_0, b_1, x_2) \in Q_n \times Q_n \times Q_n$. *If* $\mathcal{R}_1(a_0, a_1, x_1) = (c_0, c_1, y)$ *and* $\mathcal{R}_1(b_0, b_1, x_2) = (d_0, d_1, y)$ *then we say that the pair* $((a_0, a_1, x_1), (b_0, b_1, x_2))$ *is a free start collision for Edon–*$\mathcal{R}(n)$.

	a_0	a_1	x_1
x_1	$x_0^{(1)}$	$x_1^{(1)}$	$x_2^{(1)}$
a_1	$x_0^{(2)}$	$x_1^{(2)}$	$x_2^{(2)}$
a_0	c_0	c_1	y

a.

	b_0	b_1	x_2
x_2	$y_0^{(1)}$	$y_1^{(1)}$	$y_2^{(1)}$
b_1	$y_0^{(2)}$	$y_1^{(2)}$	$y_2^{(2)}$
b_0	d_0	d_1	y

b.

Table 7. a. Schematic presentation of the function $\mathcal{R}_1(a_0, a_1, x_1) = (c_0, c_1, y)$, **b.** Schematic presentation of the function $\mathcal{R}_1(b_0, b_1, x_2) = (d_0, d_1, y)$.

The free start collision situation is described in the Table 7.

In this moment we see two ways how to find free start collisions for Edon–$\mathcal{R}(n)$:

1. Generate a random $y \in Q_n$. Construct vectors (c_0, c_1, y) and (d_0, d_1, y) where $c_0, c_1, d_0, d_1 \in Q_n$ are randomly chosen. Try to find $\mathcal{R}_1^{-1}(c_0, c_1, y)$ and $\mathcal{R}_1^{-1}(d_0, d_1, y)$.
2. Generate a random (a_0, a_1, x_1) and compute $\mathcal{R}_1(a_0, a_1, x_1) = (c_0, c_1, y)$. Construct vector (d_0, d_1, y) where $d_0, d_1 \in Q_n$ are randomly chosen. Try to find $\mathcal{R}_1^{-1}(d_0, d_1, y)$.

Both ways need inversion of $\mathcal{R}_1$ and as we already said we see that as an infeasible task.

7 Conclusions

We have designed a concrete realization of the family of hash functions Edon–$\mathcal{R}$ with message digests of 256, 384 and 512 bits by defining huge non-commutative and non-associative quasigroups that are not loops of orders 2^{256}, 2^{384} and 2^{512}. The definition of quasigroups involve 32–bit operations of addition modulo 2^{32}, bitwise XORing and left rotations. Those operations are very fast on most modern microprocessors but they can be also efficiently realized on low-end 8–bit and 16–bit processors. By our reference C code implementation on x86 platforms we have achieved processing speeds of 16.18 cycles/byte, 24.37 cycles/byte and 32.18 cycles/byte.

In the forthcoming period we will do additional security analysis and we will try to develop some optimized implementations for different platforms.

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Appendix

Proof of Lemma 2.

Proof. It is elementary exercise to check that the matrix $A_{1,1} = \begin{pmatrix} 0\,1\,1\,0\,1\,0\,1\,1 \\ 1\,1\,0\,1\,1\,0\,0\,1 \\ 1\,1\,1\,0\,0\,0\,1\,1 \\ 0\,1\,0\,1\,1\,1\,1\,0 \\ 1\,0\,0\,1\,1\,1\,1\,0 \\ 1\,0\,1\,0\,1\,1\,0\,1 \\ 1\,1\,1\,1\,0\,1\,0\,0 \\ 0\,0\,1\,1\,0\,1\,1\,1 \end{pmatrix}$ which correspond to the additions for obtaining temporal variables T_i is nonsingular in $(\mathbb{Z}_{2^{32}}, +)$. Thus the operations of additions are permutations over Q_{256}.

Similarly, the matrix $A_{1,2} = \begin{pmatrix} 1\,0\,0\,1\,0\,1\,0\,0 \\ 0\,0\,1\,0\,0\,1\,1\,0 \\ 0\,0\,0\,1\,1\,1\,0\,0 \\ 1\,0\,1\,0\,0\,0\,0\,1 \\ 0\,1\,1\,0\,0\,0\,0\,1 \\ 0\,1\,0\,1\,0\,0\,1\,0 \\ 0\,0\,0\,0\,1\,0\,1\,1 \\ 1\,1\,0\,0\,1\,0\,0\,0 \end{pmatrix}$ which correspond to the bitwise XoRs for obtaining final values Y_i is nonsingular in $GF(2)$, so the operations of XoRs are permutations over Q_{256}.

Since the left rotations are also permutations, by composition of all permutations we get that the transformation $\pi_{2,256}$ is permutation. $\square$

The proof of Theorem 1.

Proof. The proof that the operation $*_{256}$ is quasigroup operation follows immediately from the previous propositions and lemmas. The non-associativity can be easily checked. Namely,

$$(1 *_{256} 2) *_{256} 3 \neq 1 *_{256} (2 *_{256} 3)$$

where 1, 2 and 3 are represented as 256–bit words.

The only non-obvious part is to show that $*_{256}$ is not a loop i.e. that there is no element $e \in Q_{256}$ such that for every $a \in Q_{256}$, $a *_{256} e = a = e *_{256} a$. Let us suppose that there is a neutral element $e \in Q_{256}$. Let us first put

$$\pi_{2,256}(e) \oplus_{256} \pi_{3,256}(e) = Const_e$$

where $Const_e \in Q_{256}$ is a constant element.

If we apply concrete definition of the quasigroup operation $*_{256}$ for the neutral element e we will get:

$$\pi_{1,256}(\pi_{2,256}(e) \oplus_{256} \pi_{3,256}(a)) = \pi_{1,256}(\pi_{2,256}(a) \oplus_{256} \pi_{3,256}(e))$$

Since $\pi_{1,256}$ is a permutation we can remove it from the last equation and we will get:

$$\pi_{2,256}(e) \oplus_{256} \pi_{3,256}(a) = \pi_{2,256}(a) \oplus_{256} \pi_{3,256}(e)$$

and if we rearrange the last equation we will get:

$$\pi_{2,256}(a) \oplus_{256} \pi_{3,256}(a) = \pi_{2,256}(e) \oplus_{256} \pi_{3,256}(e) = Const_e$$

The last equation states that for every $a \in Q_{256}$ the expression $\pi_{2,256}(a) \oplus_{256} \pi_{3,256}(a)$ is a constant and it is not true (for example $\pi_{2,256}(1) \oplus_{256} \pi_{3,256}(1) \neq \pi_{2,256}(2) \oplus_{256} \pi_{3,256}(2)$. Thus we conclude that $*_{256}$ is not a loop. □

Definition 9. *Transformation* $\pi_{1,384} : Q_{384} \to Q_{384}$ *is defined as:*

$$\pi_{1,384}(X_0, X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}, X_{11}) = (X_7, X_8, X_9, X_{10}, X_{11}, X_0, X_1, X_2, X_3, X_4, X_5, X_6)$$

Lemma 4. *Transformation* $\pi_{1,384}$ *is permutation.* □

Definition 10. *Transformation* $\pi_{2,384} : Q_{384} \to Q_{384}$ *is defined as:*

$$\pi_{2,384}(X_0, X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}, X_{11}) = (Y_0, Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8, Y_9, Y_{10}, Y_{11})$$

where

$$\begin{cases} T_0 = ROTL((X_1 + X_2 + X_3 + X_7 + X_8 + X_{10} + X_{11}), 1); \\ T_1 = ROTL((X_0 + X_1 + X_4 + X_5 + X_6 + X_8 + X_{10}), 3); \\ T_2 = ROTL((X_0 + X_2 + X_4 + X_5 + X_9 + X_{10} + X_{11}), 4); \\ T_3 = ROTL((X_1 + X_3 + X_4 + X_5 + X_6 + X_7 + X_{11}), 5); \\ T_4 = ROTL((X_0 + X_1 + X_3 + X_7 + X_8 + X_9 + X_{11}), 7); \\ T_5 = ROTL((X_0 + X_2 + X_4 + X_5 + X_6 + X_7 + X_8), 8); \\ T_6 = ROTL((X_0 + X_1 + X_2 + X_3 + X_4 + X_9 + X_{10}), 10); \\ T_7 = ROTL((X_0 + X_2 + X_5 + X_6 + X_8 + X_9 + X_{11}), 13); \\ T_8 = X_3 + X_4 + X_5 + X_7 + X_8 + X_9 + X_{10}; \\ T_9 = X_0 + X_3 + X_4 + X_6 + X_7 + X_{10} + X_{11}; \\ T_{10} = X_1 + X_2 + X_5 + X_6 + X_7 + X_9 + X_{10}; \\ T_{11} = X_1 + X_2 + X_3 + X_6 + X_8 + X_9 + X_{11}; \end{cases} \quad \textit{and} \quad \begin{cases} Y_0 = ROTL((T_0 \oplus T_4 \oplus T_5 \oplus T_6 \oplus T_9), 1); \\ Y_1 = ROTL((T_2 \oplus T_3 \oplus T_7 \oplus T_9 \oplus T_{11}), 4); \\ Y_2 = ROTL((T_1 \oplus T_3 \oplus T_6 \oplus T_7 \oplus T_8), 8); \\ Y_3 = ROTL((T_0 \oplus T_2 \oplus T_8 \oplus T_9 \oplus T_{10}), 9); \\ Y_4 = ROTL((T_2 \oplus T_4 \oplus T_5 \oplus T_6 \oplus T_{10}), 10); \\ Y_5 = ROTL((T_1 \oplus T_3 \oplus T_9 \oplus T_{10} \oplus T_{11}), 12); \\ Y_6 = ROTL((T_5 \oplus T_6 \oplus T_7 \oplus T_8 \oplus T_{11}), 13); \\ Y_7 = ROTL((T_1 \oplus T_3 \oplus T_4 \oplus T_7 \oplus T_{10}), 14); \\ Y_8 = T_0 \oplus T_1 \oplus T_2 \oplus T_6 \oplus T_{11}; \\ Y_9 = T_1 \oplus T_2 \oplus T_5 \oplus T_8 \oplus T_9; \\ Y_{10} = T_0 \oplus T_3 \oplus T_4 \oplus T_8 \oplus T_{11}; \\ Y_{11} = T_0 \oplus T_4 \oplus T_5 \oplus T_7 \oplus T_{10}; \end{cases}$$

Lemma 5. *Transformation* $\pi_{2,384}$ *is permutation.* □

Definition 11. *Transformation* $\pi_{3,384} : Q_{384} \to Q_{384}$ *is defined as:*

$$\pi_{3,384}(X_0, X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}, X_{11}) = (Y_0, Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8, Y_9, Y_{10}, Y_{11})$$

where

$$\begin{cases} T_0 = ROTL((X_4 + X_5 + X_6 + X_7 + X_9 + X_{10} + X_{11}), 2); \\ T_1 = ROTL((X_1 + X_2 + X_3 + X_7 + X_8 + X_{10} + X_{11}), 5); \\ T_2 = ROTL((X_0 + X_1 + X_2 + X_3 + X_7 + X_9 + X_{10}), 6); \\ T_3 = ROTL((X_0 + X_2 + X_4 + X_5 + X_7 + X_8 + X_9), 7); \\ T_4 = ROTL((X_1 + X_3 + X_4 + X_6 + X_7 + X_9 + X_{11}), 8); \\ T_5 = ROTL((X_0 + X_1 + X_2 + X_5 + X_6 + X_9 + X_{10}), 10); \\ T_6 = ROTL((X_0 + X_1 + X_4 + X_6 + X_8 + X_{10} + X_{11}), 11); \\ T_7 = ROTL((X_0 + X_2 + X_3 + X_4 + X_8 + X_9 + X_{11}), 14); \\ T_8 = X_0 + X_1 + X_5 + X_6 + X_7 + X_8 + X_{11}; \\ T_9 = X_0 + X_3 + X_5 + X_6 + X_8 + X_9 + X_{10}; \\ T_{10} = X_2 + X_3 + X_4 + X_5 + X_6 + X_7 + X_8; \\ T_{11} = X_1 + X_2 + X_3 + X_4 + X_5 + X_{10} + X_{11}; \end{cases} \quad \textit{and} \quad \begin{cases} Y_0 = ROTL((T_0 \oplus T_1 \oplus T_2 \oplus T_3 \oplus T_8), 3); \\ Y_1 = ROTL((T_0 \oplus T_4 \oplus T_5 \oplus T_6 \oplus T_9), 4); \\ Y_2 = ROTL((T_4 \oplus T_5 \oplus T_6 \oplus T_8 \oplus T_{11}), 6); \\ Y_3 = ROTL((T_1 \oplus T_3 \oplus T_6 \oplus T_{10} \oplus T_{11}), 8); \\ Y_4 = ROTL((T_0 \oplus T_2 \oplus T_5 \oplus T_8 \oplus T_{10}), 9); \\ Y_5 = ROTL((T_3 \oplus T_4 \oplus T_7 \oplus T_8 \oplus T_{11}), 11); \\ Y_6 = ROTL((T_2 \oplus T_3 \oplus T_5 \oplus T_7 \oplus T_9), 12); \\ Y_7 = ROTL((T_1 \oplus T_5 \oplus T_6 \oplus T_7 \oplus T_{10}), 13); \\ Y_8 = T_2 \oplus T_3 \oplus T_4 \oplus T_9 \oplus T_{10}; \\ Y_9 = T_1 \oplus T_2 \oplus T_4 \oplus T_7 \oplus T_{11}; \\ Y_{10} = T_0 \oplus T_1 \oplus T_9 \oplus T_{10} \oplus T_{11}; \\ Y_{11} = T_0 \oplus T_6 \oplus T_7 \oplus T_8 \oplus T_9; \end{cases}$$

Lemma 6. *Transformation* $\pi_{3,384}$ *is permutation.* □

Theorem 3. *Operation* $*_{384} : Q_{384}^2 \to Q_{384}$ *defined as:*

$$a *_{384} b = \pi_{1,384}(\pi_{2,384}(a) \quad \oplus_{384} \quad \pi_{3,384}(b))$$

is a non-commutative and non-associative quasigroup operation that is not a loop. $\square$

Definition 12. *Transformation* $\pi_{1,512} : Q_{384} \to Q_{384}$ *is defined as:*

$$\begin{aligned}\pi_{1,384}(X_0, X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}, X_{11}, X_{12}, X_{13}, X_{14}, X_{15}) = \\ (X_9, X_{10}, X_{11}, X_{12}, X_{13}, X_{14}, X_{15}, X_0, X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8)\end{aligned}$$

Lemma 7. *Transformation* $\pi_{1,512}$ *is permutation.* $\square$

Definition 13. *Transformation* $\pi_{2,512} : Q_{512} \to Q_{512}$ *is defined as:*

$$\begin{aligned}\pi_{2,512}(X_0, X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}, X_{11}, X_{12}, X_{13}, X_{14}, X_{15}) = \\ (Y_0, Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8, Y_9, Y_{10}, Y_{11}, Y_{12}, Y_{13}, Y_{14}, Y_{15})\end{aligned}$$

where

$$\begin{cases} T_0 = ROTL((X_0 + X_2 + X_3 + X_4 + X_6 + X_9 + X_{10} + X_{13} + X_{15}), 1); \\ T_1 = ROTL((X_0 + X_1 + X_3 + X_4 + X_6 + X_8 + X_{10} + X_{13} + X_{15}), 3); \\ T_2 = ROTL((X_0 + X_1 + X_3 + X_6 + X_7 + X_{10} + X_{11} + X_{12} + X_{14}), 4); \\ T_3 = ROTL((X_1 + X_3 + X_4 + X_6 + X_8 + X_{10} + X_{11} + X_{14} + X_{15}), 5); \\ T_4 = ROTL((X_0 + X_1 + X_2 + X_4 + X_7 + X_8 + X_9 + X_{11} + X_{13}), 7); \\ T_5 = ROTL((X_1 + X_2 + X_4 + X_5 + X_7 + X_8 + X_9 + X_{10} + X_{12}), 8); \\ T_6 = ROTL((X_0 + X_5 + X_7 + X_9 + X_{10} + X_{11} + X_{12} + X_{13} + X_{14}), 10); \\ T_7 = ROTL((X_0 + X_3 + X_5 + X_8 + X_9 + X_{11} + X_{12} + X_{13} + X_{14}), 13); \\ T_8 = X_0 + X_1 + X_2 + X_3 + X_4 + X_5 + X_9 + X_{13} + X_{15}; \\ T_9 = X_0 + X_1 + X_2 + X_3 + X_5 + X_6 + X_8 + X_9 + X_{15}; \\ T_{10} = X_1 + X_2 + X_6 + X_8 + X_{11} + X_{12} + X_{13} + X_{14} + X_{15}; \\ T_{11} = X_3 + X_5 + X_7 + X_{10} + X_{11} + X_{12} + X_{13} + X_{14} + X_{15}; \\ T_{12} = X_2 + X_3 + X_5 + X_6 + X_7 + X_9 + X_{12} + X_{13} + X_{14}; \\ T_{13} = X_0 + X_4 + X_5 + X_6 + X_7 + X_9 + X_{11} + X_{12} + X_{15}; \\ T_{14} = X_2 + X_4 + X_5 + X_6 + X_7 + X_8 + X_{10} + X_{11} + X_{14}; \\ T_{15} = X_1 + X_2 + X_4 + X_7 + X_8 + X_{10} + X_{12} + X_{14} + X_{15}; \end{cases} \quad and \quad \begin{cases} Y_0 = ROTL((T_1 \oplus T_5 \oplus T_7 \oplus T_8 \oplus T_{11} \oplus T_{12} \oplus T_{14}), 1); \\ Y_1 = ROTL((T_2 \oplus T_5 \oplus T_7 \oplus T_9 \oplus T_{11} \oplus T_{12} \oplus T_{14}), 4); \\ Y_2 = ROTL((T_2 \oplus T_4 \oplus T_5 \oplus T_8 \oplus T_9 \oplus T_{13} \oplus T_{15}), 8); \\ Y_3 = ROTL((T_0 \oplus T_2 \oplus T_5 \oplus T_7 \oplus T_9 \oplus T_{12} \oplus T_{13}), 9); \\ Y_4 = ROTL((T_3 \oplus T_5 \oplus T_6 \oplus T_{10} \oplus T_{12} \oplus T_{14} \oplus T_{15}), 10); \\ Y_5 = ROTL((T_0 \oplus T_3 \oplus T_6 \oplus T_{11} \oplus T_{13} \oplus T_{14} \oplus T_{15}), 12); \\ Y_6 = ROTL((T_1 \oplus T_2 \oplus T_3 \oplus T_4 \oplus T_6 \oplus T_8 \oplus T_{15}), 13); \\ Y_7 = ROTL((T_1 \oplus T_2 \oplus T_4 \oplus T_6 \oplus T_7 \oplus T_{10} \oplus T_{15}), 14); \\ Y_8 = T_6 \oplus T_7 \oplus T_8 \oplus T_{10} \oplus T_{11} \oplus T_{12} \oplus T_{14}; \\ Y_9 = T_4 \oplus T_7 \oplus T_{10} \oplus T_{11} \oplus T_{12} \oplus T_{13} \oplus T_{14}; \\ Y_{10} = T_0 \oplus T_3 \oplus T_4 \oplus T_5 \oplus T_7 \oplus T_9 \oplus T_{10}; \\ Y_{11} = T_0 \oplus T_1 \oplus T_2 \oplus T_4 \oplus T_6 \oplus T_8 \oplus T_9; \\ Y_{12} = T_0 \oplus T_1 \oplus T_4 \oplus T_8 \oplus T_{10} \oplus T_{11} \oplus T_{15}; \\ Y_{13} = T_1 \oplus T_2 \oplus T_3 \oplus T_8 \oplus T_{10} \oplus T_{13} \oplus T_{14}; \\ Y_{14} = T_0 \oplus T_1 \oplus T_3 \oplus T_9 \oplus T_{12} \oplus T_{13} \oplus T_{15}; \\ Y_{15} = T_0 \oplus T_3 \oplus T_5 \oplus T_6 \oplus T_9 \oplus T_{11} \oplus T_{13}; \end{cases}$$

Lemma 8. *Transformation* $\pi_{2,512}$ *is permutation.* $\square$

Definition 14. *Transformation* $\pi_{3,512} : Q_{512} \to Q_{512}$ *is defined as:*

$$\begin{aligned}\pi_{3,512}(X_0, X_1, X_2, X_3, X_4, X_5, X_6, X_7, X_8, X_9, X_{10}, X_{11}, X_{12}, X_{13}, X_{14}, X_{15}) = \\ (Y_0, Y_1, Y_2, Y_3, Y_4, Y_5, Y_6, Y_7, Y_8, Y_9, Y_{10}, Y_{11}, Y_{12}, Y_{13}, Y_{14}, Y_{15})\end{aligned}$$

where

$$\begin{cases} T_0 = ROTL((X_1 + X_2 + X_3 + X_4 + X_8 + X_{10} + X_{11} + X_{13} + X_{14}), 2); \\ T_1 = ROTL((X_0 + X_1 + X_3 + X_6 + X_7 + X_{11} + X_{13} + X_{14} + X_{15}), 5); \\ T_2 = ROTL((X_0 + X_3 + X_5 + X_6 + X_8 + X_{10} + X_{12} + X_{14} + X_{15}), 6); \\ T_3 = ROTL((X_0 + X_2 + X_4 + X_6 + X_9 + X_{10} + X_{11} + X_{12} + X_{15}), 7); \\ T_4 = ROTL((X_0 + X_3 + X_4 + X_5 + X_7 + X_9 + X_{10} + X_{12} + X_{13}), 8); \\ T_5 = ROTL((X_2 + X_4 + X_5 + X_6 + X_8 + X_{10} + X_{12} + X_{14} + X_{15}), 10); \\ T_6 = ROTL((X_1 + X_3 + X_4 + X_7 + X_8 + X_9 + X_{11} + X_{12} + X_{14}), 11); \\ T_7 = ROTL((X_2 + X_4 + X_5 + X_7 + X_9 + X_{11} + X_{13} + X_{14} + X_{15}), 14); \\ T_8 = X_1 + X_5 + X_6 + X_7 + X_9 + X_{10} + X_{11} + X_{13} + X_{14}; \\ T_9 = X_0 + X_1 + X_2 + X_3 + X_7 + X_8 + X_9 + X_{10} + X_{15}; \\ T_{10} = X_0 + X_1 + X_2 + X_3 + X_5 + X_6 + X_{11} + X_{12} + X_{14}; \\ T_{11} = X_0 + X_1 + X_5 + X_6 + X_7 + X_8 + X_{11} + X_{12} + X_{13}; \\ T_{12} = X_1 + X_3 + X_4 + X_5 + X_8 + X_{10} + X_{12} + X_{13} + X_{15}; \\ T_{13} = X_0 + X_1 + X_2 + X_4 + X_6 + X_7 + X_8 + X_9 + X_{10}; \\ T_{14} = X_0 + X_2 + X_7 + X_8 + X_9 + X_{12} + X_{13} + X_{14} + X_{15}; \\ T_{15} = X_2 + X_3 + X_4 + X_5 + X_6 + X_9 + X_{11} + X_{13} + X_{15}; \end{cases} \quad and \quad \begin{cases} Y_0 = ROTL((T_0 \oplus T_5 \oplus T_6 \oplus T_7 \oplus T_9 \oplus T_{12} \oplus T_{15}), 3); \\ Y_1 = ROTL((T_2 \oplus T_4 \oplus T_5 \oplus T_8 \oplus T_9 \oplus T_{10} \oplus T_{12}), 4); \\ Y_2 = ROTL((T_1 \oplus T_2 \oplus T_4 \oplus T_7 \oplus T_9 \oplus T_{11} \oplus T_{13}), 6); \\ Y_3 = ROTL((T_1 \oplus T_3 \oplus T_5 \oplus T_7 \oplus T_8 \oplus T_{13} \oplus T_{14}), 8); \\ Y_4 = ROTL((T_1 \oplus T_2 \oplus T_6 \oplus T_8 \oplus T_{11} \oplus T_{14} \oplus T_{15}), 9); \\ Y_5 = ROTL((T_0 \oplus T_1 \oplus T_3 \oplus T_7 \oplus T_9 \oplus T_{11} \oplus T_{13}), 11); \\ Y_6 = ROTL((T_0 \oplus T_2 \oplus T_5 \oplus T_6 \oplus T_{10} \oplus T_{13} \oplus T_{15}), 12); \\ Y_7 = ROTL((T_0 \oplus T_1 \oplus T_3 \oplus T_6 \oplus T_8 \oplus T_{10} \oplus T_{12}), 13); \\ Y_8 = T_0 \oplus T_2 \oplus T_3 \oplus T_4 \oplus T_8 \oplus T_{12} \oplus T_{15}; \\ Y_9 = T_4 \oplus T_5 \oplus T_6 \oplus T_{11} \oplus T_{12} \oplus T_{13} \oplus T_{14}; \\ Y_{10} = T_4 \oplus T_7 \oplus T_8 \oplus T_9 \oplus T_{10} \oplus T_{13} \oplus T_{15}; \\ Y_{11} = T_2 \oplus T_3 \oplus T_4 \oplus T_9 \oplus T_{10} \oplus T_{14} \oplus T_{15}; \\ Y_{12} = T_0 \oplus T_2 \oplus T_6 \oplus T_7 \oplus T_9 \oplus T_{11} \oplus T_{14}; \\ Y_{13} = T_3 \oplus T_5 \oplus T_{11} \oplus T_{12} \oplus T_{13} \oplus T_{14} \oplus T_{15}; \\ Y_{14} = T_1 \oplus T_3 \oplus T_4 \oplus T_5 \oplus T_6 \oplus T_{10} \oplus T_{11}; \\ Y_{15} = T_0 \oplus T_1 \oplus T_7 \oplus T_8 \oplus T_{10} \oplus T_{12} \oplus T_{14}; \end{cases}$$

Lemma 9. *Transformation* $\pi_{3,512}$ *is permutation.* $\square$

Theorem 4. *Operation* $*_{512} : Q_{512}^2 \to Q_{512}$ *defined as:*

$$a *_{512} b = \pi_{1,512}(\pi_{2,512}(a) \quad \oplus_{512} \quad \pi_{3,512}(b))$$

is a non-commutative and non-associative quasigroup operation that is not a loop. $\square$

$$L_3 = \left(\begin{array}{cccccccccccc}
11 & 0 & 9 & 6 & 3 & 4 & 10 & 8 & 5 & 7 & 1 & 2 \\
3 & 10 & 2 & 11 & 8 & 7 & 1 & 6 & 4 & 0 & 5 & 9 \\
1 & 5 & 0 & 7 & 9 & 8 & 4 & 11 & 10 & 3 & 2 & 6 \\
2 & 4 & 10 & 1 & 7 & 5 & 0 & 9 & 8 & 11 & 6 & 3 \\
10 & 1 & 11 & 5 & 0 & 6 & 3 & 2 & 9 & 4 & 7 & 8 \\
7 & 8 & 5 & 4 & 1 & 2 & 9 & 0 & 3 & 6 & 10 & 11 \\
8 & 6 & 4 & 3 & 11 & 0 & 2 & 5 & 7 & 10 & 9 & 1 \\
\hline
0 & 9 & 6 & 10 & 5 & 3 & 7 & 1 & 2 & 8 & 11 & 4 \\
6 & 2 & 3 & 8 & 10 & 1 & 5 & 4 & 11 & 9 & 0 & 7 \\
4 & 11 & 7 & 2 & 6 & 9 & 8 & 10 & 0 & 1 & 3 & 5 \\
5 & 7 & 1 & 9 & 4 & 10 & 11 & 3 & 6 & 2 & 8 & 0 \\
9 & 3 & 8 & 0 & 2 & 11 & 6 & 7 & 1 & 5 & 4 & 10
\end{array}\right)
\qquad
L_4 = \left(\begin{array}{cccccccccccc}
11 & 10 & 9 & 5 & 7 & 0 & 4 & 8 & 1 & 6 & 2 & 3 \\
4 & 7 & 0 & 8 & 11 & 2 & 10 & 9 & 6 & 5 & 3 & 1 \\
9 & 1 & 3 & 2 & 4 & 5 & 6 & 0 & 8 & 10 & 7 & 11 \\
5 & 11 & 1 & 9 & 6 & 10 & 8 & 3 & 7 & 0 & 4 & 2 \\
6 & 8 & 2 & 7 & 3 & 1 & 11 & 4 & 0 & 9 & 5 & 10 \\
7 & 3 & 10 & 4 & 1 & 9 & 0 & 2 & 11 & 8 & 6 & 5 \\
10 & 2 & 7 & 0 & 9 & 6 & 1 & 11 & 5 & 3 & 8 & 4 \\
\hline
2 & 9 & 11 & 1 & 8 & 7 & 3 & 5 & 10 & 4 & 0 & 6 \\
8 & 0 & 4 & 6 & 5 & 11 & 9 & 10 & 3 & 2 & 1 & 7 \\
3 & 6 & 5 & 10 & 0 & 8 & 2 & 1 & 4 & 7 & 11 & 9 \\
1 & 5 & 8 & 3 & 10 & 4 & 7 & 6 & 2 & 11 & 9 & 0 \\
0 & 4 & 6 & 11 & 2 & 3 & 5 & 7 & 9 & 1 & 10 & 8
\end{array}\right)$$

$$L_5 = \left(\begin{array}{cccccccccccccccc}
4 & 10 & 11 & 1 & 2 & 5 & 7 & 3 & 13 & 0 & 8 & 14 & 9 & 12 & 6 & 15 \\
0 & 15 & 1 & 10 & 8 & 7 & 13 & 12 & 9 & 3 & 14 & 11 & 6 & 5 & 2 & 4 \\
15 & 3 & 6 & 4 & 1 & 9 & 10 & 14 & 0 & 2 & 11 & 12 & 13 & 7 & 5 & 8 \\
6 & 1 & 3 & 11 & 0 & 2 & 14 & 8 & 5 & 9 & 15 & 13 & 7 & 4 & 10 & 12 \\
10 & 4 & 0 & 6 & 9 & 8 & 12 & 13 & 1 & 5 & 2 & 15 & 3 & 11 & 14 & 7 \\
13 & 0 & 14 & 3 & 4 & 10 & 9 & 11 & 15 & 8 & 1 & 5 & 12 & 6 & 7 & 2 \\
2 & 13 & 7 & 8 & 11 & 12 & 5 & 9 & 3 & 15 & 6 & 10 & 14 & 0 & 4 & 1 \\
3 & 6 & 10 & 15 & 13 & 4 & 11 & 0 & 2 & 1 & 12 & 7 & 5 & 9 & 8 & 14 \\
9 & 8 & 12 & 14 & 7 & 1 & 0 & 5 & 4 & 6 & 13 & 3 & 2 & 15 & 11 & 10 \\
\hline
5 & 9 & 15 & 2 & 12 & 14 & 8 & 6 & 11 & 4 & 7 & 1 & 10 & 13 & 3 & 0 \\
14 & 5 & 13 & 9 & 10 & 15 & 6 & 7 & 8 & 11 & 4 & 0 & 1 & 2 & 12 & 3 \\
1 & 11 & 5 & 13 & 14 & 0 & 2 & 4 & 7 & 12 & 3 & 6 & 8 & 10 & 15 & 9 \\
8 & 12 & 2 & 7 & 5 & 11 & 3 & 10 & 14 & 13 & 9 & 4 & 15 & 1 & 0 & 6 \\
11 & 7 & 8 & 5 & 3 & 6 & 1 & 15 & 12 & 10 & 0 & 2 & 4 & 14 & 9 & 13 \\
12 & 14 & 9 & 0 & 15 & 13 & 4 & 2 & 6 & 7 & 10 & 8 & 11 & 3 & 1 & 5 \\
7 & 2 & 4 & 12 & 6 & 3 & 15 & 1 & 10 & 14 & 5 & 9 & 0 & 8 & 13 & 11
\end{array}\right)
\qquad
L_6 = \left(\begin{array}{cccccccccccccccc}
3 & 14 & 8 & 12 & 4 & 15 & 7 & 11 & 6 & 10 & 0 & 5 & 1 & 2 & 13 & 9 \\
1 & 3 & 5 & 0 & 10 & 4 & 9 & 7 & 11 & 2 & 14 & 12 & 13 & 6 & 8 & 15 \\
2 & 11 & 6 & 9 & 12 & 5 & 8 & 14 & 10 & 3 & 1 & 13 & 15 & 7 & 0 & 4 \\
4 & 13 & 10 & 11 & 9 & 14 & 3 & 15 & 1 & 7 & 2 & 6 & 8 & 0 & 12 & 5 \\
11 & 0 & 15 & 10 & 7 & 6 & 14 & 4 & 13 & 1 & 12 & 8 & 5 & 9 & 2 & 3 \\
8 & 15 & 12 & 6 & 0 & 2 & 4 & 13 & 5 & 9 & 3 & 7 & 10 & 1 & 14 & 11 \\
13 & 7 & 0 & 2 & 3 & 10 & 1 & 9 & 14 & 8 & 5 & 11 & 12 & 4 & 15 & 6 \\
10 & 1 & 14 & 4 & 5 & 12 & 11 & 2 & 9 & 15 & 6 & 0 & 3 & 8 & 7 & 13 \\
14 & 6 & 3 & 15 & 13 & 8 & 12 & 5 & 7 & 0 & 11 & 1 & 4 & 10 & 9 & 2 \\
\hline
7 & 9 & 11 & 3 & 1 & 13 & 2 & 6 & 15 & 4 & 8 & 14 & 0 & 12 & 5 & 10 \\
12 & 10 & 7 & 5 & 2 & 3 & 13 & 8 & 0 & 11 & 9 & 4 & 14 & 15 & 6 & 1 \\
9 & 5 & 13 & 8 & 11 & 7 & 6 & 0 & 4 & 12 & 15 & 10 & 2 & 3 & 1 & 14 \\
0 & 4 & 2 & 14 & 15 & 1 & 5 & 12 & 8 & 6 & 10 & 3 & 9 & 13 & 11 & 7 \\
5 & 8 & 4 & 1 & 6 & 9 & 0 & 10 & 2 & 13 & 7 & 15 & 11 & 14 & 3 & 12 \\
6 & 12 & 9 & 13 & 14 & 0 & 15 & 1 & 3 & 5 & 4 & 2 & 7 & 11 & 10 & 8 \\
15 & 2 & 1 & 7 & 8 & 11 & 10 & 3 & 12 & 14 & 13 & 9 & 6 & 5 & 4 & 0
\end{array}\right)$$

Above: Latin squares L_3 and L_4 of order 12×12 for Edon–$\mathcal{R}(384)$ and Latin squares L_5 and L_6 of order 16×16 for Edon–$\mathcal{R}(512)$.

Examining the avalanche properties for Edon–$\mathcal{R}(384)$ and Edon–$\mathcal{R}(512)$

The results for $n = 384$ are shown in Table 8. Notice again that the level of Hamming distance equal to $\frac{1}{2}n = 192$ which would be expected in theoretical models of ideal random functions is achieved after applying quasigroup operations that lie on the down-right half of the tables (in bold), but some close values are obtained also after the second quasigroup operation (in italic).

The results for $n = 512$ are shown in Table 9. There also the level of Hamming distance equal to $\frac{1}{2}n = 256$ which would be expected in theoretical models of ideal random functions is achieved after applying quasigroup operations that lie on the down-right half of the tables (in bold), but some close values are obtained also after the second quasigroup operation (in italic).

One possible explanation about the reasons why Edon–$\mathcal{R}(384)$ and Edon–$\mathcal{R}(512)$ come slightly faster to the level of ideal random function than Edon–$\mathcal{R}(256)$ may lie in the fact that permutations π_2 and π_3 for $n = 384, 512$ are defined by bigger Latin squares of order 12×12 and 16×16 (see the Section 4). Thus they are more complex then corresponding permutations π_2 and π_3 for $n = 256$.

$Min = 23$ $Avr = 30.33$ $Max = 35$	$Min = 162$ *Avr=190.28* $Max = 255$	$Min = 166$ **Avr=190.89** $Max = 219$
$Min = 162$ *Avr=190.87* $Max = 218$	$Min = 166$ **Avr=192.17** $Max = 218$	$Min = 160$ **Avr=192.40** $Max = 222$
$Min = 162$ **Avr=191.40** $Max = 225$	$Min = 168$ **Avr=192.11** $Max = 223$	$Min = 160$ **Avr=192.15** $Max = 221$

a.

$Min = 23$ $Avr = 52.54$ $Max = 103$	$Min = 157$ *Avr=191.69* $Max = 227$	$Min = 163$ **Avr=192.31** $Max = 222$
$Min = 166$ *Avr=192.17* $Max = 225$	$Min = 164$ **Avr=191.41** $Max = 222$	$Min = 166$ **Avr=191.88** $Max = 222$
$Min = 166$ **Avr=192.68** $Max = 217$	$Min = 160$ **Avr=191.90** $Max = 216$	$Min = 167$ **Avr=191.99** $Max = 218$

b.

Table 8. a. Avalanche propagation of the Hamming distance between two 384–bit words M_1 and M_2 that initially differs in one bit and where $M_1 = 0$ (minimum, average and maximum) **b.** Avalanche propagation of the Hamming distance between two 384–bit words M_1 and M_2 that initially differs in one bit (minimum, average and maximum)

$Min = 27$ $Avr = 39.50$ $Max = 51$	$Min = 199$ *Avr=252.46* $Max = 289$	$Min = 222$ **Avr=256.031** $Max = 296$
$Min = 220$ *Avr=254.93* $Max = 293$	$Min = 222$ **Avr=255.25** $Max = 283$	$Min = 227$ **Avr=257.01** $Max = 288$
$Min = 224$ **Avr=256.36** $Max = 287$	$Min = 222$ **Avr=255.54** $Max = 290$	$Min = 227$ **Avr=255.89** $Max = 295$

a.

$Min = 27$ $Avr = 73.00$ $Max = 142$	$Min = 209$ *Avr=254.54* $Max = 288$	$Min = 222$ **Avr=255.34** $Max = 288$
$Min = 214$ *Avr=255.49* $Max = 287$	$Min = 226$ **Avr=255.85** $Max = 290$	$Min = 226$ **Avr=256.50** $Max = 287$
$Min = 217$ **Avr=255.35** $Max = 286$	$Min = 225$ **Avr=256.38** $Max = 288$	$Min = 221$ **Avr=256.402** $Max = 297$

b.

Table 9. a. Avalanche propagation of the Hamming distance between two 512–bit words M_1 and M_2 that initially differs in one bit and where $M_1 = 0$ (minimum, average and maximum) **b.** Avalanche propagation of the Hamming distance between two 512–bit words M_1 and M_2 that initially differs in one bit (minimum, average and maximum)