A public key encryption based on Oracle Diffie-Hellman assumption with short ciphertext

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Abstract

We construct a practical public key encryption scheme that is provably secure against adaptive chosen ciphertext attacks in the standard model based on the Oracle Diffie-Hellman assumption. The ciphertext of the new scheme is as short as the basic Elgamal scheme. Compared to DHIES it is nearly the same efficient in computation, and more efficient in bandwidth.

Keywords: DHIES, PKE, CCA, standard model

1 Introduction

There are several ElGamal-based [1] public key encryption schemes that provably secure against adaptive chosen ciphertext attacks(CCA secure) in standard model. Cramer and Shoup describe the first provably CCA secure and practical ElGamal-based scheme [5]. It led to a variety of constructions [6, 8, 9, 10, 11]. These schemes are more costly than the ElGamal scheme in terms of key sizes, computation and bandwidth. The most efficient scheme in these schemes is the hybrid scheme proposed by K. Kurosawa and Y. Desmedt [10].

M. Abdalla, M. Bellare and P. Rogaway propose an efficient Diffie-Hellman Integrated Encryption Scheme(DHIES)[7]. DHIES is now embodied in three(draft) standards [2, 3, 4]. It is a natural extension of the ElGamal scheme, and enhanced ElGamal in a couple of ways important to cryptographic practice. First, it provide the capability of encrypting arbitrary bit strings while ElGamal requires that message be a group element. Second, it is secure against chosen ciphertext attack , while ElGamal is secure against chosen plaintext attack. Most importantly DHIES realized the above two goals without increasing the number of group operations for encryption and decryption, and without increasing key sizes relative to ElGamal. The CCA security of DHIES relies on the Oracle Diffie-Hellman assumption(ODH).

Recently, Kiltz proposed a practical KEM with simple and intuitive design concept [12]. It is proved to be CCA secure in the standard model under a new assumption, the Gap Hashed Diffie-Hellman(GHDH) assumption. Compared to the previously most efficient scheme by Kurosawa and Desmedt [15] it has 128 bits shorter ciphertexts, between 25-50% shorter public/secret keys, and it is slightly more efficient in terms of encryption/decryption speed.

1.1 Our Contributions

DHIES is the most efficient scheme in all the previous CCA secure ElGamal-based scheme. It is nearly as efficient as the ElGamal scheme in computation and slightly less efficient in bandwidth. We construct a practical ElGamal-based public key encryption scheme that is provably secure against adaptive chosen ciphertext attacks in the standard model based on the ODH assumption. The ciphertext of the new scheme is as short as the basic Elgamal scheme. Compared to DHIES it is nearly the same efficient in computation, and more efficient in bandwidth.

Our new scheme can be seen as a simplification of DHIES. We remove the message authentication codes in DHIES and there is no redundancy in the ciphertext of the new scheme. It means that all ciphertexts are valid.

2 Preliminaries

We review the standard definitions of public key encryption scheme(PKE) and symmetric key encryption scheme(SKE). This is followed by the definition of ODH assumption.

In describing probabilistic processes, we write $x \stackrel{R}{\leftarrow} X$ to denote the action of assigning to the variable x a value sampled according to the distribution X. If S is a finite set, we simply write $s \stackrel{R}{\leftarrow} S$ to denote assignment to s of an element sampled from uniform distribution on S. If A is a probabilistic algorithm and x an input, then $A(x)$ denotes the output distribution of A on in put x. Thus, we write $y \stackrel{R}{\leftarrow} A(x)$ to denote of running algorithm A on input x and assigning the output to the variable y.

We write

$$Pr[x_1 \stackrel{R}{\leftarrow} X_1, x_2 \stackrel{R}{\leftarrow} X_2, \ldots, x_n \stackrel{R}{\leftarrow} X_n : \phi(x_1, \ldots, x_n)]$$

to denote the probability that when x_1 is drawn from a certain distribution X_1, and x_2 is drawn from a certain distribution $X_2(x_1)$, possibly depending on the particular choice of x_1, and so on, all the way to x_n, the predicate $\phi(x_1, ..., x_n)$ is true. We allow the predicate ϕ to involve the execution of probabilistic algorithms.

2.1 Public-Key Encryption

Definition 1 *A public key encryption scheme PKE is a triple of PPT (probabilistic polynomial time) algorithms :*

- $PKE.KeyGen(1^k)$: The randomized key generation algorithm takes as input a security parameter (1^k) and outputs a public key PK and secret key SK. We write $(PK, SK) \leftarrow PKE.KeyGen(1^k)$
- $PKE.Encrypt(PK, m)$: The randomized encryption algorithm takes as input a public key PK and a message m, and outputs a ciphertext C. We write $C \leftarrow PKE.Encrypt(PK, m)$
- $PKE.Decrypt(SK, C)$: The decryption algorithm takes as input a ciphertext C and secret key SK. It returns a message or the distinguished symbol $\perp$. We write $m \leftarrow PKE.Decrypt(SK, C)$.

We require that for all PK, SK output by $PKE.KeyGen$, all $m \in \{0,1\}^*$, and all C output by $PKE.Encrypt(PK, m)$ we have $PKE.Decrypt(SK, C) = m$.

We recall the standard definition of security for public key encryption schemes against adaptive chosen ciphertext attacks.

Definition 2 *A PKE scheme is secure against adaptive chosen ciphertext attacks if the advantage of any PPT adversary A in the following game is negligible in the security parameter k:*

1. $PKE.KeyGen(1^k)$ outputs PK, SK. Adversary A is given 1^k and PK.
2. The adversary may make a sequence of queries to a decryption oracle $PKE.Decrypt(SK, \cdot)$.
3. At some point, A outputs two messages m_0, m_1 with $|m_0| = |m_1|$. A bit b is randomly chosen and the adversary is given a "challenge ciphertext" $C^* \leftarrow PKE.Encrypt(PK, m_b)$.
4. A may continue to query its decryption oracle $PKE.Decrypt(SK, \cdot)$ except that it may not request the decryption of C^*.
5. Finally, A outputs a guess b' .

   We call the game above IND-CCA2 game of PKE. We say A succeeds if $b' = b$, and denote the probability of this event by $Pr_{A,PK}[Succ]$. The adversary's advantage is defined as $AdvCCA_A = |Pr_{A,PK}[Succ] - 1/2|$.

2.2 Symmetric key encryption scheme

Definition 3 *A symmetric key encryption scheme SKE consists of two algorithms:*

- SKE.Encrypt(k, m): The deterministic, polynomial-time encryption algorithm takes as input a key k, and a message m, and outputs a ciphertext χ. We write $\chi \leftarrow SKE.Encrypt(k, m)$
- SKE.Decrypt(k, χ): The deterministic, polynomial-time decryption algorithm takes as input a key k, and a ciphertext χ, and outputs a message m or the special symbol $reject$. We write $m \leftarrow SKE.Decrypt(k, \chi)$

We require that for all $kLen \in N$, for all $k \in \{0,1\}^{kLen}$,$kLen$ denotes the length of the key of SKE, and for all $m \in \{0,1\}^*$,we have:

$$SKE.Decrypt(k, SKE.Encrypt(k, m)) = m.$$

We recall the definition of security for symmetric key encryption scheme against passive attacks.

1. The challenger randomly generates an appropriately sized key k.
2. A queries an encryption oracle with two messages m_0, m_1 , $|m_0| = |m_1|$. A bit b is randomly chosen and the adversary is given a "challenge ciphertext" $\chi^* \leftarrow SKE.Encrypt(k, m_b)$.
3. Finally, A outputs a guess b' .

We call the game above IND-PA game.

Definition 4 *A SKE scheme is secure against passive attacks if the advantage of any PPT adversary A in the IND-PA game is negligible in the security parameter kLen:*

We define $AdvPA_{SKE,A}(kLen)$ to be $|Pr[b = b'] - 1/2|$ in the IND-PA game. We say that SKE is secure against passive attack if for all probabilistic, polynomial-time oracle query machines A, the function $AdvPA_{SKE,A}(kLen)$ grows negligibly in $kLen$.

2.3 The Oracle Diffie-Hellman Problem

Now we review the definition of oracle Diffie-Hellman assumption[7].Let G be a group of large prime order q, $H : \{0,1\}^* \to \{0,1\}^{hLen}$ be a cryptographic hash function and consider the following two experiments:

experiments $\text{Exp}_{H,A}^{odh-real}$:

$$u \overset{R}{\leftarrow} Z_q^*; U \leftarrow g^u; v \overset{R}{\leftarrow} Z_q^*; V \leftarrow g^v; W \leftarrow H(g^{uv})$$

$$\mathcal{H}_v(X) \overset{def}{=} H(X^v); b \leftarrow A^{\mathcal{H}_v(\cdot)}(U, V, W); \text{return } b$$

experiments $\text{Exp}_{H,A}^{odh-rand}$:

$$u \overset{R}{\leftarrow} Z_q^*; U \leftarrow g^u; v \overset{R}{\leftarrow} Z_q^*; V \leftarrow g^v; W \leftarrow \{0,1\}^{hLen}$$

$$\mathcal{H}_v(X) \overset{def}{=} H(X^v); b \leftarrow A^{\mathcal{H}_v(\cdot)}(U, V, W); \text{return } b$$

Now define the advantage of the A in violating the oracle Diffie-Hellman assumption as

$$Adv_{H,A}^{odh} = \Pr[\text{Exp}_{H,A}^{odh-real} = 1] - \Pr[\text{Exp}_{H,A}^{odh-rand} = 1]$$

Here A is allowed to make oracle queries that depend on the target g^u with the sole restriction of not being allowed to query g^u itself. When it is the $\text{Exp}_{H,A}^{odh-rand}$ experiment we say $(g, U, V, W) \in R$, otherwise $(g, U, V, W) \in D$.

3 New scheme

Now we describe our new scheme.

- $PKE.KeyGen(1^k)$: Assume that G is group of order q where q is large.

$$g \overset{R}{\leftarrow} G; x, y \overset{R}{\leftarrow} Z_q^*; c \leftarrow g^x; d \leftarrow g^y$$

$$PK = (g, c, d, TCR, H, SKE); SK = (x, y)$$

Here TCR is target collision resistant hash function [9].$H : \{0,1\}^* \to \{0,1\}^{kLen}$ is a cryptographic hash function used in ODH oracle, SKE is a symmetric key encryption scheme secure against passive attack.

- $PKE.Encrypt(PK, m)$: Given a message m, the encryption algorithm runs as follows.

$$r \overset{R}{\leftarrow} Z_q^*; k \leftarrow H(g^r); c_1 \leftarrow SKE.Encrypt(k, m); a \leftarrow TCR(c_1); c_2 \leftarrow c^r d^{ra}$$

$$C \leftarrow (c_1, c_2)$$

- $PKE.Decrypt(SK, C)$: Given a cipher-text $C = (c_1, c_2)$, the decryption algorithm runs as follows.

$$a \leftarrow TCR(c_1); k \leftarrow H(c_2^{1/(x+ya)}); m \leftarrow SKE.Decrypt(k, c_1);$$

Before the formal security proof we give some intuition to show that the new scheme is secure against active attacks. The ODH assumption guarantees that different ciphertexts will yield different keys independent to each other. So the adversary can not get the information of b from the decryption oracle. The ODH assumption also assures that the adversary can not get the information of b from the challenge ciphertext(the output of the encryption oracle). Finally we have that the new scheme is CCA secure based on the ODH assumption.

4 Security

Now we give the formal proof of the new scheme.

Theorem 1 *The new scheme is secure against adaptive chosen ciphertext attack assuming that (1) the oracle Diffie-Hellman problem is hard in group G, (2) TCR is a target collision resistant hash function, (3)SKE is a IND-PA secure symmetric key encryption scheme.*

To prove the theorem, we will assume that there is an adversary that can break the cryptosystem, and TCR is a target collision resistant hash function, SKE is a IND-PA secure symmetric key encryption scheme and show how to use this adversary to construct a statistical test for the ODH problem.

For the statistical test, we are given $(\hat{g}, U, V, W)$ coming from either the distribution R or D. At a high level, our construction works as follows. We build a simulator that simulates the joint distribution consisting of adversary's view in its attack on the cryptosystem, and the hidden bit b generated by the generated oracle (which is not a part of the adversary's view). We will show that if the input comes from D, the simulation will be nearly perfect, and so the adversary will have a non-negligible advantage in guessing the hidden bit b. We will also show that if the input comes from R, then the adversary's view is essentially independent of b, and therefore the adversary's advantage is negligible. This immediately implies a statistical test distinguishing R from D: run the simulator and adversary together, and if the simulator outputs b and the adversary outputs b', the distinguisher outputs 1 if $b = b'$, and 0 otherwise.

We now give the details of the simulator. The input to the simulator is $(\hat{g}, U, V, W)$. The simulator runs the following key generation algorithm, using the given $(\hat{g}, U)$. The simulator chooses

$$x, y \overset{R}{\leftarrow} Z_q^*$$

and set

$$g \leftarrow U; c \leftarrow \hat{g}^x; d \leftarrow \hat{g}^y;$$

The public key that the adversary sees is (g, c, d, H, TCR, SKE), where TCR is target collision resistant hash function [9]. $H : \{0,1\}^* \to \{0,1\}^{kLen}$ is a cryptographic hash function used in ODH oracle, SKE is a symmetric key encryption scheme secure against passive attack. The simulator knows $(x, y, \hat{g})$.

First we describe the simulation of the encryption oracle. Given m_0, m_1, the simulator chooses $b \in \{0,1\}$ at random, and computes

$$k \leftarrow W; c_1 \leftarrow SKE.Encrypt(k, m_b); a \leftarrow TCR(c_1); c_2 \leftarrow V^{x+ya}$$

and outputs

$$(c_1, c_2)$$

We now describe the simulation of the decryption oracle. Given (c_{1i}, c_{2i}), the simulator runs as follow:

$$a \leftarrow TCR(c_{1i}); k_i \leftarrow \mathcal{H}_v(c_{2i}^{1/(x+ya)}); m_i \leftarrow SKE.Decrypt(k_i, c_{1i})$$

Finally the simulator outputs m_i.

That completes the description of the simulator. As we will see, when the input to the simulator comes from D, the output of the encryption oracle is a perfectly legitimate ciphertext; however, when the input to the simulator comes from R, the output of the decryption oracle will not be legitimate, in the sense that the corresponding plaintext of the ciphertext is not m_1 or m_0. This is not a problem, and indeed, it is crucial to the proof of security.

The theorem now follows immediately from the following two lemmas.

Lemma 1 *When the simulator's input comes from D, the joint distribution of the adversary's view and the hidden bit b is statistically indistinguishable from that in the actual attack.*

Consider the joint distribution of the adversary's view and the bit b when the input comes from the distribution D. Say $U = \hat{g}^u, V = \hat{g}^v, W = H(\hat{g}^{uv})$.

It is clear in this case that the output of the encryption oracle has the right distribution, since:

$$k = W = H(\hat{g}^{uv}) = H(g^v)$$

$$c_2 = V^{x+ya} = \hat{g}^{v(x+ya)} = \hat{g}^{vx}\hat{g}^{vya} = c^v d^{va}$$

To complete the proof, we need to argue that the output of the decryption oracle has the right distribution. Given (c_{1i}, c_{2i}), we have:

$$\mathcal{H}_v(c_{2i}^{1/(x+ya_i)}) = \mathcal{H}_v((c^{r_i} d^{r_i a_i})^{1/(x+ya_i)}) = \mathcal{H}_v(\hat{g}^{r_i}) = H(\hat{g}^{ur_i}) = H(g^{r_i}) = k_i$$

$$SKE.Decrypt(k_i, c_{1i}) = m_i$$

therefore, the decryption oracle outputs m_i just as it should. So the joint distribution of the adversary's view and the hidden bit b is just the same as that in the actual attack.

Lemma 2 *When the simulator's input comes from R, the distribution of the hidden bit b is (essentially) independent from the adversary's view.*

Let $U = \hat{g}^u, V = \hat{g}^v, W = H(\hat{g}^w), w \neq uv$. It is clear that the decryption oracle will not leak any information of $k = W = H(\hat{g}^w)$. So the distribution of the hidden bit b is independent from the adversary's view. Since the corresponding plaintext to the challenge ciphertext is not m_1 or m_0, the lemma follows immediately from the following claim:

Claim 1 *The adversary can not find that the challenge ciphertext is not legitimate.*

There are three cases we need to consider:

Case 1: $c_1 = c_{1i}, c_2 \neq c_{2i}$. In this case we have $a = a_i, k_i = \mathcal{H}_v(c_{2i}^{1/(x+ya_i)}) = H(g^{r_i}) \neq \mathcal{H}_v(c_2^{1/(x+ya)}) = H(g^v) = k$. According to the ODH assumption the output of the ODH oracle $H(g^{r_i})$ will not help the adversary to get the information of $H(g^v)$.

Case 2: $c_1 \neq c_{1i}, c_2 = c_{2i}$ In this case we have $a \neq a_i, k_i = \mathcal{H}_v(c_{2i}^{1/(x+ya_i)}) = H(g^{r_i}) \neq \mathcal{H}_v(c_2^{1/(x+ya)}) = H(g^v) = k$. According to the ODH assumption the output of the ODH oracle $H(g^{r_i})$ will not help the adversary to get the information of $H(g^v)$.

Case 3: $c_1 \neq c_{1i}, c_2 \neq c_{2i}$ In this case we have $a \neq a_i$. If $c_2^{1/(x+ya)} = c_{2i}^{1/(x+ya_i)}$ then we can get:

$$\left(\frac{c_2}{c_{2i}}\right)^{1/(a-a_i)} = \left(\frac{c^v d^{va}}{c^v d^{a_i v}}\right)^{1/(a-a_i)} = \left(d^{(a-a_i)v}\right)^{1/(a-a_i)} = d^v$$

That is to say we can work out the computation Diffie-Hellman problem: given $d, cd^a, (cd^a)^v$ get d^v. So we have $c_2^{1/(x+ya)} \neq c_{2i}^{1/(x+ya_i)}$ and $H(g^v) \neq H(g^{r_i})$. According to the ODH assumption the output of the ODH oracle $H(g^{r_i})$ will not help the adversary to get the information of $H(g^v)$.

Now we get that the adversary can not find that the challenge ciphertext is not legitimate.

5 Efficiency Analysis

Table 1: Efficiency comparison

	Encryption(exp)	Decryption(exp)	Cipher-text overhead(bit)	Assumption
DHIES	2(2exp+0mexp)	1(1exp+0mexp)	$\|q\| + \|t\|$	ODH
KD04	3.5(2exp+1mexp)	1.5(0exp+1mexp)	$2\|q\| + \|t\|$	DDH
Kiltz07	3.5(2exp+1mexp)	1.5(0exp+1mexp)	$2\|q\|$	GHDH
NEW	2.5 (1exp+1mexp)	1(1exp+0mexp)	$\|q\|$	DDH

Where NEW is our new scheme, KD04 is the scheme in [10], Kiltz07 is the first scheme in [12], DHIES is the scheme in [7]. When tabulating computational efficiency hash function and block cipher evaluations are ignored, multi-exponentiation (*mexp*) is counted as 1.5 exponentiations (*exp*). Ciphertext overhead represents the difference between the ciphertext length and the message length, and $|q|$ is the length of a group element,$|t|$ is the length of the tag in KD04 and DHIES.

It clear that the new scheme is more efficient than all previous schemes in bandwidth, and it is the same efficient in decryption as the previous most efficient scheme DHIES. The new scheme is slightly less efficient than DHIES in encryption.

6 Conclusion

We construct a practical ElGamal-based public key encryption scheme that is provably secure against adaptive chosen ciphertext attacks in the standard model based on the ODH assumption. The new scheme is more efficient than all previous CCA Secure ElGamal-based schemes in bandwidth, and it is nearly the same efficient in computation as DHIES. The new scheme can be seen as a simplification of DHIES. There is no redundancy in the ciphertext of the new scheme, all ciphertexts are valid.

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