# A Verifiable Voting Protocol based on Farnel <br> (Extended Abstract) 

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#### Abstract

Farnel is a voting system proposed in 2001 in which each voter signs a ballot. It uses two ballot boxes to avoid the association between a voter and a vote. In this paper we first point out a flaw in the ThreeBallot system proposed by Rivest that seems to have gone unnoticed so far: it reveals statistical information about who is winning the election. Then, trying to resolve this and other flaws, we present a new, voter-verifiable version of the Farnel voting system in which voters retain copies of ballot IDs as receipts.


## 1 Introduction

Secure voting systems are a cornerstone of modern democratic societies. They can prevent or detect frauds or faults, and so provide accurate results. To increase transparency in such systems, researchers have been designing voter-verifiable schemes. These schemes allow the voter to verify whether her vote was taken into account in the result, but without violating the vote privacy.

Different strategies have been used to design voter-verifiable schemes. Almost all solutions described in the literature uses cryptography as basis, but the resulting protocols are often hard to grasp by a common person. Recently, a new kind of scheme with verification property was proposed by Rivest [9] - the ThreeBallot voting system. His proposal attempts to satisfy the voter verifiability without employing cryptography. Many drawbacks, though, have been reported for this scheme and improvements were incorporated in its newer versions.

In 2001, Custódio et al. [4] proposed a protocol, called Farnel ${ }^{1}$, in which uses two ballot boxes and the voters sign ballots. In fact, Rivest uses the concept of the Farnel to sidestep a flaw in his scheme.

[^0]This paper presents a modified version of Farnel, which is voter-verifiable. Also, it points out another flaw in the ThreeBallot scheme which seems to have gone unnoticed so far; it leaks information. We do this as follows: Section 2 describes the original Farnel protocol. Section 3 shows how the ThreeBallot protocol leaks information. Section 4 describes the modified Farnel protocol; it inherits some interesting characteristics that can be incorporated to obtain a verifiable voting system. Section 5 presents a simple scheme that requires only a conventional ballot and that is verifiable as the modified Farnel. Section 6 presents an electronic version of our protocol. Finally, Section 7 presents our conclusions.

## 2 The original Farnel scheme

Farnel [3] was conceived to address the problems of a conventional ballot box. This paper-based scheme requires each voter to sign one ballot. However, in order to avoid an association between the voter and her ballot, the voter does not sign her own ballot, but another one chosen at random, as explained below. This way it is possible to know who the voters were, and any attempt to add, modify, or delete votes, after the voting period, can be detected.

Initialization phase Farnel uses two ballot boxes. Before voting starts, the first ballot box is publicly initialized with ballots filled out and signed by a ballot authority. This set of ballots must represent, with an equal probability, all possible ballots. The second ballot box starts empty.

Voting phase In order to vote, the voter receives a blank valid ballot (signed by the ballot authority), makes her choice and casts the ballot into the first (pre-initialized) ballot box. Then, through manual or mechanical shuffling, the first ballot box presents a ballot chosen randomly from
its current set of votes to the voter. After receiving the ballot, the voter signs and deposits it into the second box. This ends the voting process for the voter.

Tallying phase After the voting period has finished, the ballot authority opens and signs a second time all the votes of the first ballot box and adds them into the second box. Then the second box is opened and all ballots are counted. From this result the ballots from the initialization step are discounted.

Properties of Farnel Farnel gives warranties to the voter that her ballot will be counted, and that the exclusion or the addition of new votes is not possible after the voting phase. Anyone can, for example, verify that all ballots are signed, either by the voters or by the precinct. Moreover, everybody can check who voted without needing the list of voters. The scheme, however, is not voter verifiable.

## 3 Information leakage in the ThreeBallot voting system

We give a brief description of Rivest's ThreeBallot voting scheme [9]. It gets its name from the fact that each ballot consists of three columns, each representing a full ballot. Each row of the ballot has a candidate name, and a ballot must have exactly one of the three cells following the candidate name marked. However, the candidate that gets chosen will have two cells marked. For instance, in the example ballot of Figure 1 the voter chose candidate 1.

| candidate 1 | X |  | X |
| :---: | :---: | :---: | :---: |
| candidate 2 |  | X |  |
| candidate 3 |  |  | X |

Figure 1: A ballot for candidate 1.

Then the ballot is cut vertically, separating the three columns. One of the columns is copied; this is the voter's receipt. All three columns end up in the ballot box.

When the voting phase has completed all votes are tallied. Obviously each candidate gets one free vote per ballot, so these votes must be subtracted to obtain the final tally.

There is a flaw with this scheme which is not mentioned in the latest version dated October 1,

2006; information about the contents of the ballot box is leaking before the election has finished.

When reading the ThreeBallot paper superficially it may appear that the secrecy of the ballot is perfect, i.e., that no information leaks. However, each receipt in fact does reveal a tiny bit of information, so little that it cannot be used against the voter. But in a large set of receipts statistical pattern do emerge.

The issue is best explained using an extreme example: suppose that candidate 1 gets all the votes and the other two none (we are assuming 3 candidates). Furthermore, suppose that all voters behave uniformly random with regard to where they put the marks and which column they choose as a receipt. Finally, suppose that all voters are willing to show their receipt to some organization who are at the polling station awaiting people who have just voted.

Counting the number of marks for each candidate (row) on the receipts reveals information on who is winning the election at that particular polling place. In this example, the winning candidate can expect $2 / 3$ mark per receipt, whereas all the others can expect only $1 / 3$ mark per receipt. The information is of a statistical nature.

To show the effect we wrote a small simulation program. Table 1 shows ten simulations for an election with three candidates, where 100 receipts have been collected and candidate 1 gets all the votes. The lines show the number of marks for each candidate, leaving no doubt at all about who is winning already while voting is still going on.

| 69 | 73 | 61 | 65 | 65 | 64 | 65 | 65 | 68 | 61 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 34 | 39 | 32 | 37 | 29 | 32 | 30 | 31 | 29 | 34 |
| 43 | 34 | 31 | 37 | 30 | 37 | 37 | 28 | 26 | 27 |

Table 1: A simulation of ten elections where every voter votes for candidate 1 and 100 receipts are collected.

In fact we are dealing with two $(p, n)$-Bernoulli distributions: one with $p=2 / 3$, and the others with $p=1 / 3$. In both cases $n=\#$ receipts. Observe that adding candidates (rows) to the ballot does not help. Adding columns does, because it flattens the distributions $(p=1 / 4$ vs. $p=2 / 4$; $p=1 / 5$ vs. $p=2 / 5$ etc.), but this is undesirable for practical reasons. Observe also that a statistical analysis is more difficult if the voters do not behave randomly and the original scheme is used: the voter chooses which column to copy.

The flaw in the ThreeBallot system is debatable. It is true that the information obtained from the receipts has the same effect as exit polls. But
there is a difference: not every country has or allows exit polls, and in addition voters can lie about how they voted, whereas in the threeballot system the receipts reveal actual information. In an election where the difference of votes among two candidates is small, for example, the information obtained from the receipts can certainly influence voters while the election is going on.

## 4 A variant of the Farnel scheme

As presented in section 2, Farnel does not provide individual voter verification; it just assures, through signatures, that after the voting phase votes cannot be excluded and that new votes cannot be added. In this section we present a new paper-based scheme inspired on Farnel. It also uses two ballot boxes, but does not depend on signatures. It provides a receipt to the voter, but without leaking information during the voting phase.

### 4.1 Prerequisites

The ballot form The ballot form used is composed of two halves. The first half is not much different from the layout traditionally used in elections. It is composed of a list of voting options where next to each option there is a space to select it. It also contains a unique identification number (ID) which identifies the ballot uniquely and associates it to the election. The second half contains only the same ID (see section 4.4 for a discussion about the IDs). The halves are separated by a perforation to allow separation by the voter and the IDs are covered by scratch surfaces (see also Figure 2).

The ballot boxes Two ballot boxes are used. One of them is a conventional box; however, it must be initialized with filled out, fake ballots (i.e. just the part that contains the options) before the voting starts (otherwise the first voters would not have a set of random IDs to choose from).

The second ballot box is able to receive a slip containing an ID, add it to a set of already received slips, and to copy $l$ randomly chosen IDs from this set. To this end we assume that the box has some mechanical device, and that copies are made in a memoryless way. The shuffling mechanism, for example, could be similar to those used to shuffle cards in card games [10].

### 4.2 The protocol

Initialization phase In this phase the ballot authority establishes the following voting parameters: a number $x$ of initial votes and a number $l$ of IDs that should be printed on the receipts (see section 4.4 for a discussion about these parameters). Moreover, it initializes publicly the ballot boxes. Let's say that there are $v$ eligible voters in the election.
Before initializing the ballot boxes, the ballot authority performs a cut-and-choose process to prove the correct formation of the ballots. For a number $2 x$ of blank ballots, it takes $x$ ballots at random, detaches their protective layers, and publishes them; these ballots are no more used. After that, the authority holds the other (entire) $x$ blank ballots and tears each of them in two along the perforation. Next, it marks an option on each of the parts containing the options, detaches their layers, and cast them into the first ballot box. The options can be selected at random, but each of them should have at least one vote. The authority then scratches away the layers of the other parts (the slips that contain copies of the IDs) and cast them into the second ballot box. The total number of fake votes for each option is published. Finally, the authority seals both boxes until the voting begins. Note that neither the authority nor third parties should record or remember the IDs of the initial votes.

Voting phase After proving her eligibility to the voting authorities, the voter receives a blank ballot form. The following steps are performed to cast a vote and to obtain the receipt (see also Figure 2).

1. (Verifying and filling out the ballot form) The voter scratches away the layer covering the IDs and matches them (a). If they are equal, she makes her vote by marking one of the options available (b). We assume that the voter cannot record or remember the ballot's ID.
2. (Casting the vote) The voter separates the two parts (c) of the ballot form, casting the part containing the ballot ID and the options into the first ballot box (d). The other part, showing only the ID, is cast into the second ballot box (e).
3. (Obtaining the receipt) The second ballot box is shuffled (f) and $l$ copies are produced of IDs
which are printed as a receipt to the voter (g).


Figure 2: Main voting steps of the new paper-based scheme.

Tally phase In a public session the talliers open the two ballot boxes and publish their contents on the bulletin board. To compute the results of the election, all votes are tallied. The fake ballots cast in the initialization phase are subtracted from the sums yielding the final result.

Ballot verification Anyone can check on the bulletin board whether each ballot from the first ballot box has a corresponding ID in the second ballot box. In addition, the voters confirm whether their receipts (i.e. the IDs) match to ballots on the bulletin board. If one ballot and its ID were not published, the voter can complain by showing her receipt to an voting authority.

### 4.3 Security requirements and the new paper-based scheme

Here we sketch an analysis of our scheme based on security requirements normally found in the literature. In this analysis, we supposed that the bulletin board cannot be compromised.

Accuracy In our scheme duplication, elimination, substitution, and addition of votes can be detected. The detection is accomplished by checking the information published on the bulletin board. Duplicates can be identified by checking if the IDs of the votes published are unique. Anyone can also detect elimination and substitution of votes. Every vote on the board should have a corresponding ID published. Moreover, the voters can independently match their receipts (i.e. the IDs) to the votes on the board. Note, though, that the detection is probabilistic since not all votes will have their IDs printed on the receipts. The addition of votes can be detected through the total number of votes published. The total should be the sum of the number of initial votes and of the number of voters that cast their votes.

Privacy The voter privacy in our scheme is assured even if she wants to violate it as follow. An adversary could try to violate the privacy by: obtaining an ID of a specific vote or extracting information about votes from the receipts.

In the first case, a voter or a voting authority could attempt to remember or record the ID of a ballot. In order to prevent this, the ballots in our scheme have their IDs covered by scratch surfaces. We suppose, though, that the voter cannot remember or record the $I D$ of her own ballot (see also section 4.4).

In the second case, an adversary could ask a voter to point the ID of her vote on her receipt. As the receipt is composed of a set of IDs chosen at random, the voter can only try to guess an ID related to her option. Again, we consider that the voter cannot remember the ID of her own ballot.

Alternatively, the adversary could collect the receipts of most of the voters and try to determine the votes of the first voters; we call this the attack of collecting receipts. She could explore the fact that the IDs of the first voters are more probable to appear on the receipts than the IDs of the last ones. To attempt to determine the votes, the adversary would check the most repeated IDs on the receipts and match them later to the votes on the board. Note, though, that the IDs cast before the election are as probable as the IDs of the first voters and that the adversary cannot distinguish among them at least as long as there is one initial vote for each option.

Verifiability Our scheme can be verified by the voters and by third parties. The IDs on second ballot box aim at verifying the votes on the first ballot box. The publication of the IDs and of the votes allows anyone to verify the exactness of the voting results.

The voter verifiability in our proposal is different from the normally found in the literature. Instead of verifying if her own vote is in the final tally, the voter verifies a small subset of all votes. This is accomplished by matching the IDs on the receipt to the votes on the board. Note that here the verifiability depends on the voting parameters.

Voter-verifiable election schemes usually take into account that the voters will use their receipts to verify their ballots. Karlof et al. [7], though, pointed out that some voters can discard their receipts. Consequently, an adversary could take advantage of this and replace votes without being detected. Our scheme employs two approaches to
mitigate this problem. First, each receipt is composed of a set of IDs and these IDs (or some of them) can be printed in others receipts. This way, even if a voter discards her IDs, others voters can possibly verify the votes related to these IDs. Second, the second ballot box maintains the IDs of all votes and they are also published. This way, it adds redundancy to the verification process.

### 4.4 Discussion

The IDs The IDs on the ballot form should be easy to compare and difficult to remember. The voter should compare the IDs to detect a possible malformation of her ballot (i.e. different IDs) and should not remember it afterwards. However, some voters could not perform the comparison or ignore the malformation of their ballots. Moreover, the properties of the IDs are contradictory and difficult to implement. Barcodes, for example, could be used to encode IDs and prevent the voters to recall them, but they cannot be easily compared.

A better solution is to avoid the voter comparing the IDs of her own ballot. This way, the ID could be just a long alphanumeric string. The drawback is that malformed ballots would not be detected. To mitigate this, we employ a cut-andchoose process for auditing the ballots: before receiving a blank ballot, the voter chooses some random ballots and detaches their scratches to verify their IDs; these ballots are no more used. As the voter does not remove the scratches of her own ballot, the second ballot box now needs a special mechanism to remove the scratch of the slip; the other scratch can be removed in the tally phase.

Chain voting This is a real threat to our scheme. An adversary could obtain a valid blank ballot and corrupt a voter to use it; the voter then returns to the adversary the blank ballot received from the authority, afterwards. The new blank ballot is used to corrupt another voter. Here the adversary could also obtain a slip containing an ID and corrupt a voter to use it; the voter returns the ID of her own ballot and the adversary uses it to confirm the voter vote in the tally phase. In both cases the security of the scheme would be compromised.

In order to prevent chain voting, we modify our ballot form. We add a serial number to the ballot such as Jones [6]. The number is printed over the scratch surface that cover the ID on the slip. We
also change the position of the other ID to allow the ballot to be folded showing just the scratch surfaces; now it is printed on the back of the ballot. In addition to the ballot form, some voting steps must be modified. For example, the authority should record the serial number of the ballot before gives it to the voter and should confirm the number before the voter casts the vote.

The voting parameters As describe before, the voting parameters are composed of the number of initial votes $x$ and the number of IDs $l$ printed on the receipts. These parameters as well as the number of voters $v$ affect the voter verifiability. Also, they are related to the privacy, that is, they can facilitate or not the attack of collecting receipts (see section 4.3). From these remarks, we noted the following:

Considering $x$ much bigger than the number of voters $v$ (e.g. 10 times bigger), the IDs cast by the voters will be almost statistically indistinguishable from the initial IDs. As a result, if $l$ is small (e.g. $l=1$ ), an adversary cannot violate the voters privacy (particularly the privacy of the first voters) by distinguishing among the voters IDs and the initial IDs from the receipts. A small $l$, though, affects the voter verifiability as the chance of detecting problems in the tally decreases.

A $v$ bigger than $x$, on the other hand, results in more IDs of voters on the receipts even if $l$ is small. Consequently, the adversary have more chance to distinguish among the voters IDs and the initial IDs. For example, if $v=500, x=2$, and $l=4$, the information leaked could be used to violate the privacy as the voters IDs will appear more on the receipts than the initial ones.

Certainly, the voter verifiability and the privacy in our scheme are related. To measure the relation of these properties, we performed experiments considering a fix number of voters $v=500$ for different $l$ and $x$. The experiments show the probability of the ID cast by the first voter to appear on at least one of the 500 receipts. Also, they show the chance of detecting the elimination of a vote in the tally through the receipts. The Table 2 presents some results of these experiments.

## 5 A scheme with preconstructed receipts

The variant of Farnel presented in the last Section requires a special ballot box that shuffles and

| x | 100 |  | 300 |  | 1000 |  |  | 1500 |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $l$ | 1 | 2 | 1 | 2 | 3 | 4 | 5 | 4 | 5 | 6 |
| 1st ID \% | 83 | 97 | 62 | 86 | 70 | 80 | 87 | 68 | 76 | 82 |
| Detection \% | 49 | 66 | 43 | 63 | 60 | 69 | 76 | 61 | 68 | 74 |

Table 2: $x$ - number of initial votes. $l$ - number of IDs on each receipt; 1st ID - prob. of the 1st ID to appear on the receipts. Detection - prob. of detecting the elimination of a vote in the tally.
outputs ballots; this ballot box should also make copy of ballots (i.e. the parts that contain the IDs). These requirements, though, are strong and cannot be implemented easily.

The Farnel's variant introduced a new way of voter verifiability: the voter verifies multiple votes (i.e. a small subset of all votes) instead of verifying her own vote. To perform this, though, the scheme depends on the special ballot box. We present here a simple scheme that relies on the construction of receipts in advance as an alternative to the special ballot box. The scheme employs only a conventional ballot box and allows the verification of multiple votes as the Farnel's variant.

### 5.1 Prerequisites

The ballot form The ballot form has a special format and includes a receipt that the voter takes home. As the Farnel's variant of Section 4, the form here has two halves that are separated by a perforation to allow detachment. The first half is identical to the Farnel's variant, that is, it has a list of voting options and has a unique identification number (ID) covered by a scratch surface. Let $I$ be the set of IDs printed in the first half of all ballot forms.

The second half is the receipt that the voter holds. It contains a list of IDs (see also Figure 3) chosen uniformly at random from the set $I$. This list is covered by a scratch surface. Let $L$ be the set of the lists of IDs printed in the second half of all ballots, each ID in $I$ should appear at least in one of the lists in $L$. Every list in $L$ has the same number of IDs and this number is defined according to the total number of ballots.

All ballots with elements in the sets $I$ and $L$ composes a unique set of blank ballot forms.

The ballot box The scheme employs a conventional ballot box. This ballot box, differently from the scheme presented in Section 4, begins the election empty.

### 5.2 The protocol

Initialization phase Supposing that there are two sets of blank ballot forms made as described before. The ballot authority selects at random one of these sets to be opened. The authority then detaches the scratch surfaces of all ballots of the set selected and publish them.

This cut-and-choose process is public. It aims at showing anyone that the IDs printed in the first part of the forms correspond to the IDs listed on their second parts (i.e. the receipts). If the ballots opened were correctly formed, the other set (not opened) will be used in the voting phase. Note that the cut-and-choose process can be performed with more sets of ballots to increase the chance of detecting malformed ballots. In addition, the ballots opened should not be used anymore.

Voting phase After the voter proves her eligibility to the voting authorities, the authorities select at random a blank ballot from the set of ballots available and give it to the voter. The voter performs the following steps to cast a vote and to obtain the receipt (see also Figure 3).

1. (Filling out the ballot form) As usual, the voter marks one of the options available in her ballot form (a).
2. (Casting the vote) After marking her ballot, the voter separates it in two parts (b). The voter casts the first part (i.e. the part that contains the options) in the ballot box (c). This step as the next one should be performed in the presence of the voting authorities without revealing the first part of the ballot.
3. (Obtaining the receipt) The voter then holds the second part of the ballot as her receipt (d); this part contains a list of IDs. She can now remove the scratch surface on her receipt.

The scheme can have an additional step in the voting phase to add redundancy to the verification process. Before the voter leaves the voting place, she copies her receipt and cast the copy in a second conventional ballot box. Thus, in the tally phase, the receipts of the second ballot box can be published to allow the verification of all votes.

After the voting, the blank ballots not used are immediately invalidated (e.g. by stamping "not used" on the ballots) and published without their scratch surfaces on the bulletion board.


Figure 3: The voting steps of the scheme with preconstructed receipts.

Tally phase In the tally phase, the authorities open publicly the ballot box and detach the scratches of all votes. Then she count the votes and publish them on the bulletion board.

Ballot verification In order to verify the votes, the voter matches the IDs on their receipts to the votes on the bulletin board. If we use a second ballot box and copy the voters' receipts as an additional step, anyone can match the IDs on the receipts to the IDs on the votes.

## 6 An electronic version of the paper-based scheme

We now introduce an electronic voting scheme of the scheme presented in the previous section. It uses commitments as the IDs, which are constructed by a voting machine (DRE). Also, it uses a special ballot box which accepts a ballot ID and hands out copies of other ballot IDs.

### 6.1 Building blocks

Threshold ElGamal Cryptosystem As a basis for the scheme we employ the ElGamal public key cryptosystem [5] under a subgroup of order $q$ of $Z_{p}^{*}$, where $p$ and $q$ are large primes and $q \mid p-1$. More specifically, we utilize a threshold variant, as described by Cramer et al. [2].

We utilize the following notation: $T$ is an ElGamal public key corresponding to a secret key $\widehat{T}$, while $E_{T}(i, s)$ is the ElGamal encryption of a message $i$ constructed with $T$ and a random number $s \in_{R} Z_{q}$, and $D_{\widehat{T}}(i)$ is the ElGamal decryption of $i$.

Mix Net In order to make encryptions anonymous during the tallying, we employ a mixnet. This primitive was introduced by Chaum [1] and further improved by many others authors. Specifically, we require a verifiable reencryption mixnet such as the proposal of Neff [8].

Commitment scheme Another cryptographic primitive is a commitment scheme, which must be homomorphic and will be used to commit to the voting options. We use the ElGamal cryptosystem for this purpose.

Cut-and-choose We employ a cut-and-choose process to prove the voter that her vote was correctly formed by the voting machine. This is accomplished in a similar way to Lee et al. [11]. Especially, the voting machine makes some encryptions with ElGamal and presents them to the voter; the voter selects some of them for verification and the machine opens them by revealing the random numbers employed.

### 6.2 Prerequisites

The ballot The ballot is constructed by the voting machine and is presented to the voter. It contains each possible option and some encrypted stuff next to it: each option $i$ is associated to two commitments, as follows: $\left\langle i, \operatorname{commit}\left(i, r_{i 1}\right), \operatorname{commit}\left(i, r_{i 2}\right)\right\rangle$ for $r_{i 2}, r_{i 1} \quad \in_{R}$ $Z_{q}$. In particular, each commitment is represented by: commit $(i, r)=\left\langle E_{T}\left(i, s_{1}\right), E_{T}\left(r, s_{2}\right)\right.$, $\left.E_{M}\left(i r, s_{3}\right)\right\rangle$ for $r, s_{1}, s_{2}, s_{3} \in_{R} Z_{q}$. Here $r$ is chosen uniformly at random from $Z_{q}$, and $i r$ is the product of $i$ and $r$, while $(T, \widehat{T})$ stands for the ElGamal keys of the talliers, and $(M, \widehat{M})$ stands for the keys of the special ballot box which uses the same ElGamal modulus.

The special ballot box The paper-based scheme from the previous section required a ballot box that received a ballot ID, shuffled its contents, and output copies of randomly chosen IDs. Here our special ballot box is initialized with a set of encrypted IDs which it keeps in a private list, $L$. It receives an enciphered ID, adds it to $L$, decrypts some random elements from $L$ and prints the result on the receipt. Elements selected are not deleted from the list, though.

There are four parties involved in our scheme:

Voters The voters cast votes and receive receipts for check data later. Each receipt is composed of three parts: the ballot (with commitments and hidden commitments) and some decommitments, the commitment of the option chosen, and some plaintexts from the list $L$.
Voting machine The voting machine generates ballots, makes the first two parts of the re-
ceipts, and publishes commitments on the bulletin board.
Special ballot box It holds a private list of encryptions $L$ and acts as described before. It has a barcodes reader and receives new encryptions through this reader. It also prints the last part of the receipt.
Tallying authorities These authorities are responsible for running a mixnet, and for decrypting and counting the votes. They also define the number of initial votes and generates them; in addition, they define the number of votes that each voter verifies. They hold the keys $T, \widehat{T}$. We suppose that a subset of the talliers are trustworthy.

### 6.3 The protocol

Initialization phase In this phase, the following parameters of the voting are established and published on the bulletin board: the voting options (or candidates), the number of initial votes, the number of IDs that should be printed on the receipt. Let's say that there are $m$ options $i$ $(i=1, \cdots, m)$, a number $x$ of initial votes, and a number $l$ of IDs printed on the receipt.

The talliers generate the initial votes according to $x$ and to the commitment scheme explained before. Then the talliers publish commitments of the form: $\operatorname{commit}(i, r)=\left\langle E_{T}\left(i, s_{1}\right), E_{T}\left(r, s_{2}\right), E_{M}\left(i r, s_{3}\right)\right\rangle$ for different $r, s_{1}, s_{2}, s_{3} \in_{R} Z_{q}$ in each commitment. The values $E_{M}\left(i r, s_{3}\right)$ are handled by the special ballot box as its private list $L$ of encryptions.

Voting phase After proving her eligibility to the voting authorities, the voter is allowed to interact with the voting machine in the voting booth. The following steps are executed to cast a vote and to obtain the receipt. Figures 4 and 5 exemplify the scheme for three voting options.

1. (Generating the ballot) For each option $i(i=1, \cdots, n)$, the machine generates the triple: $\left\langle i, \operatorname{commit}\left(i, r_{i 1}\right), \operatorname{commit}\left(i, r_{i 2}\right)\right\rangle$ for $r_{i 2}, r_{i 1} \quad \in_{R} \quad Z_{q}$. As described before, each commitment is composed of: $\quad\left\langle E_{T}\left(i, s_{1}\right), E_{T}\left(r, s_{2}\right), E_{M}\left(i r, s_{3}\right)\right\rangle \quad$ for $r, s_{1}, s_{2}, s_{3} \in_{R} Z_{q}$. After that, the machine prints the ballot on the receipt (a). Here as well as in the next two steps the ballot is not shown to the voter.
2. (Opening some commitments)

The voter informs the machine to open the first or the second commitment for each option $i$. After this, the machine opens the corresponding commitments (b). In other words, for a commitment $\left\langle E_{T}\left(i, s_{1}\right), E_{T}\left(r, s_{2}\right), E_{M}\left(i r, s_{3}\right)\right\rangle$ already printed on the receipt, the machine prints $r, s_{1}, s_{2}, s_{3}$ on the receipt as decommitment.
3. (Voting)

In order to vote, the voter informs her option $i$ and the machine prints the corresponding, not opened, commitment on the receipt (c). In particular, the machine prints $\left\langle E_{T}\left(i, s_{1}\right), E_{T}\left(r, s_{2}\right), E_{M}\left(i r, s_{3}\right)\right\rangle$.
4. (Verifying the ballot)

Now, the machine shows the receipt (A) to the voter. The voter should verify if the commitments selected were opened and if the commitment corresponding to her vote was printed. If the receipt is correct, the voter confirms her vote. The machine then prints a stripe on the not-open commitments of the ballot to erase them. She also prints the barcode of $E_{M}\left(i r, s_{3}\right)$ of the voter's vote and adds a digital signature to the receipt; the voter holds this receipt (B). The other elements of the vote, $\left\langle E_{T}\left(i, s_{1}\right), E_{T}\left(r, s_{2}\right)\right\rangle$, are sent to the bulletin board.


Figure 4: Parts I and II of the receipt. A - the receipt that the voter verifies; B - the receipt that the voter holds.
5. (Obtaining the last part of the receipt) Using the barcodes reader, the voter adds $E_{M}\left(i r, s_{3}\right)$ to the special ballot box (d). The box writes $E_{M}\left(i r, s_{3}\right)$ to its private list $L$. Then, it chooses $y$ elements at random from $L$, decrypts them, and prints the results on the receipt (e). In doing so the elements are not deleted from $L$. Figure 5 illustrates this step.


Figure 5: The last part of the receipt.

Tally phase After the election, the talliers send all pairs of encryptions published on the bulletin board, $\left\langle E_{T}\left(i, s_{1}\right), E_{T}\left(r, s_{2}\right)\right\rangle$, to a mixnet. The mixnet shuffles the pairs and publishes them on the bulletin board. After this, the talliers cooperate to decrypt the pairs to obtain the options $i$ and the random numbers $r$. The talliers then multiply $i$ and $r$, and publish the triples $\langle i, r, i r\rangle$ on the bulletin board. To compute the elections results, all votes from the voting phase (i.e. the $i$ 's) are counted and from this result are subtracted the fake votes generated in the setup phase.

Ballot verification The voter receives a receipt composed of three parts. The first part, which contains the ballot (with commitments and hidden commitments) and the decommitments, is used to verify the construction of the ballot. This may be accomplished by a helper organization through a computer. It constructs the commitments from the decommitment values of the receipt and then checks if the resulting commitments match the commitments printed on the receipt.

The second part of the receipt, which contains the commitment of the option chosen, is used to check if the commitment of the receipt appears on the bulletin board.

The third part of the receipt contains a list of $i r$ to verify if the values ir match the values published by the talliers on the board.

## 7 Conclusion

We presented a modified version of the Farnel voting system; a paper-based scheme and an electronic one. In addition, we showed a flaw in the ThreeBallot voting system.

The schemes introduce a new way to verify votes: the voter does not verify her own vote, but copies of a subset of votes cast so far. More precisely, the voter receives copies of some ballot IDs. These are used later to compare with the IDs of the ballots published on the bulletin board.

The paper based version uses a simple ballot form. It just requires the voter to compare IDs
and to mark her option. However, the scheme relies on a ballot box that can shuffle and copy IDs. Also, the security of the scheme depends on the voting parameters.

We used the paper version to model the electronic version. The scheme works as expected, but is not very practical and has several drawbacks. It requires a verifiable mix net in the tally phase and the special ballot box must perform correctly. Moreover, the voter must compare a lot of information. We believe, though, that this scheme can be improved and are working in this direction.

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[^0]:    ${ }^{1}$ Farnel means basket in Portuguese.

