Merging Prêt-à-Voter and PunchScan

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Abstract. *We describe a variation of Prêt-à-Voter that keeps the same ballot layout but borrows and slightly modifies the underlying cryptographic primitives from Punchscan, substituting the mix network for bit commitments.We also suggest a limited-resources solution to get to unconditional privacy and unconditional integrity, and propose ways to have several races on the Prêt-à-Voter ballot.*

1. Introduction

Over the last few years we have seen a sequence of papers on voter-verifiable elections. The idea of such systems is that the voter takes home a receipt which allows him to verify that her vote is included in the tally without revealing any usefull information about her vote. Though this idea is not new, Chaum's paper [2] arguably gave a new impetus to this line of research (see also [1]).

Chaum's paper was improved upon in two significant ways. First there is a protocol called Prêt-à-Voter (PaV), as described in [3], which has several advantages over [2], such as a simpler ballot lay-out, pre-printed ballots on which the voter marks his preferences with a pen thus insuring that the voting machine (DRE) does not learn the vote, etc. However, PaV still uses traditional mixing. Inspired by this, Chaum developed PunchScan (PS). See the site *www.punchscan.org* for fancy demos. For a detailed protocol description we refer to [7] and [5]. PS differs from PaV in several aspects: *(1)* in each ballot both the top and bottom layer are permuted; *(2)* a mark is placed on both layers; *(3)* the voter gets to choose which layer he keeps and which gets destroyed; *(4)* no mixing takes place; the only cryptographic primitive needed is a Bit Commitment scheme.

In this paper we obtain a new protocol by merging PaV and PS as follows: we maintain PaV's ballot lay-out but we borrow the underlying cryptographic primitives from PS. Apart from giving us a thorough understanding of the similarities and differences between the two protocols, the final result seems superior to both because compared to PaV it disposes of mixing, while compared to PS it results in a simpler ballot lay-out.

The outline of this paper is as follows: we start with a high-level description of the PaV ballot, but instead of using mixing we describe how to apply the underlying cryptographic ideas used in PS to PaV. We also propose some improvements to the protocol, and provide a very brief description of the cryptography of Punchscan. We assume that the reader is familiar with the general setting and the terminology of voting protocols.

2. The Prêt-à-Voter Ballot

The ballots used in PaV are described in detail in [3], section 4, using an example with 4 candidates. A base canonical ordering of candidates is defined: 0: Anarchist, 1:

Alchemist, 2: Nihilist, 3: Buddhist. An example ballot looks like this (section 4.1):

3: Buddhist	X
0: Anarchist	
1: Alchemist	
2: Nihilist	
(Offset $x = 1$*)*	`Qqkr3c`

The left part contains a cyclic permutation (shift) of the candidates; in this case the offset $x = 1$. The right part is empty except for the last row, and the voter votes by putting an “X” in one of its first four cells. The magic string `Qqkr3c` (in reality probably longer) is an encryption of x, encrypted with the public keys of the mixes.

Casting the vote consists of separating the left and the right columns, destroying the left column and scanning the right column. Either manually or through OCR the row containing the X and the encryption of the offset are associated to the ballot image. The voter can take the right column home as a receipt. At the end of the day, all ballots will enter the mix process. That is, each mix contributes in decrypting the shift and shuffling all the ballots; see section 6 of [3].

3. Using bit commitments instead of mixing

Mixing is a tedious process and has several disadvantages: it is difficult to explain to the average person, the privacy of the ballot is only computational, it is computationally intensive, etc. A protocol that uses bit commitment does not have these disadvantages: pieces of papers in an enveloppe serve as an excellent explanation for BCs, unconditionally hiding bit commitment schemes exist, and they are certainly not less efficient than mixing. Therefore our purpose here is to develop a variant of PaV using BCs.

Since Punchscan uses two permutatons, both the top and the bottom layer, a straightforward idea is to break the offset value x in two, i.e. to choose x_1 and x_2 random such that $x = x_1 + x_2 (\operatorname{mod} m)$. We let the Election Authority(EA) commit to x_1 and x_2. We write these BCs at the bottom of the right column on the ballot (like in PaV) or, alternatively, we have the EA commit to these values publicly and use a unique ballot id number to establish the link between the two BCs published and the printed ballot.

Furthermore we use the following notation: x is the offset; y is the number of the row marked by the voter, counting from 0 to $m - 1$; v is the actual vote, that is, the row chosen in the canonical representation. Obviously, $y = x + v (\operatorname{mod} m)$, where the modulus m is the number of candidates on the ballot; in the example $m = 4$.

Let us now describe the table to be created by the Election Authority (EA) before the election which is a simplification of Punchscan’s. Note the hats on the symbols for some columns; these mean that each cell in that column is a bit commitment. The columns labelled y, $y - x_1$ and v will remain empty until the counting of the votes, as we will see below.

i	y	$\widehat{j}$	$\widehat{x_1}$	$y-x_1$	$\widehat{x_2}$	$\widehat{\pi_2(j)}$	k	v
1								
$\cdots$								
$\cdots$								
$2R$								

Observe that the table is divided in a left, middle and right part, with i, j and k as index, resp. Let π_1 be the permutation between the rows of the first and the second part, and π_2 between the rows of the second and the third part. Then the columns labelled $\widehat{j}$ and $\widehat{\pi_2(j)}$ are used to define and to verify these two permutations. In particular, j should be the pre-image π_1^{-1} under π_1 for row j, whereas k contains the image $\pi_2(j)$ under π_2 for row j.

Auditing the ballot construction Let there be $2R$ rows. The set of rows is divided randomly in an audit set A and election set E both of size R. The EA is now required to open all bit commitments related to A: it must open all rows i in the left part of the table if $i \in A$ and all rows with index j in the middle part of the table if $j = \pi_1(i)$ and $i \in A$. The right part contains no commitments. Scrutineers should check that all bit commitments were created honestly. After the audit, the EA prints the unopened ballots with index $i \in E$.

The election The voter casts her vote as described earlier, and for each vote the value y_i is determined. Since EA also knows x_1 and x_2 he can compute the corresponding values $y_j - x_{j1}$ and v_k.

Publishing the results After the election, the EA publishes y_i for each $i \in E$, $y_j - x_{j1}$ for each $j \in \pi_1(E)$ and v_k for each $k \in \pi_2(\pi_1(E))$. From the column labelled v he calculates the tally, which can be verified by anybody.

Auditing the votes published The EA could try to cheat by modifying the values v_k. We therefore first define the following *naive* approach: for each j in the middle part of the table a random bit is created out of EA's control: Left or Right, which has the following sematics:

Left The EA opens $\widehat{x_{j1}}$ and it is verified whether $y_{\pi_1^{-1}(j)} - x_{j1} = (y - x_1)_j$ holds.

Right The EA opens $\widehat{x_{j2}}$ and it is verified whether $(y - x_1)_j = x_{j2} + v_{\pi_2(j)}$ holds. Observe that this equation should be satisfied because $y = x + v = x_1 + x_2 + v$ so $y - x_1 = x_2 + v$.

Using this approach we catch a cheating EA with probability $1/2$ for each vote v_k he modifies. However, too much information is revealed about the overall permutation $\pi = \pi_2 \oplus \pi_1$ between the left and the right part of the table, violating voter privacy. We can think of three possible ways out:

(1) We do K versions of this protocol in parallel, each with different bit commitments and one Left/Right choices for all rows in each parallel version. Then the probability of EA getting away is 2^{-K}. This is the solution adopted by Punchscan ([5], section 5.4).

(2) Instead of using two permutations π_1 and π_2, we use four. We also split x in four parts: $x = x_1 + x_2 + x_3 + x_4 (\mathrm{mod}\, m)$. Then we use Chaum´s improvement [2] of the mixing protocol proposed in [6]. See [1] for a detailed description.

(3) We use a special kind of bit commitment scheme that has a homomorphic property: we assume that the multiplication of two bit commitments is equivalent to the addition $(\mathrm{mod}\ m)$ of their contents. BCs with this property can be constructed from from homomorphic encryption schemes. This variant does not trivially generalize to elections in which a mere cyclic shift will not do and full permutations are needed.

4. A brief description of Punchscan

The header of the table used by Punchscan is as follows:

i	$\widehat{x_1}$	$\widehat{x_2}$	y	$\widehat{j}$	$\widehat{t_1}$	$y - t_1$	$\widehat{t_2}$	$\widehat{\pi_2(j)}$	k	v
	$P_{.1}$	$P_{.2}$	$P_{.3}$	$D_{.1}$	$D_{.2}$	$D_{.3}$	$D_{.4}$	$D_{.5}$		$R_{.1}$
$\ldots$										

The first row shows the notation introduced in this paper, whereas the second row shows the notation of [5] and [7]. Observe that where they use x, y, z as the indices of the left (P), middle (D) and right (R) part of the table, we use i, j, k, so that when they we write (x, P_3) we would write y_i, etc. Also, the description of Punchscan uses $m = 2$, so that adding $1(\mathrm{mod}\ 2)$ is called "flipping" or "inverting" the bit.

Simplifying this table by defining $x_1 = t_1$; $x_2 = t_2$ is tempting but leads to an *insecure* protocol because of the following difference between PaV and PS. In PaV the offset (or offsets, in the new protocol) is (are) kept secret: the left side of the ballot is destroyed, and the value on the right side is protected by a bit commitment. But in Punchscan the offset from the top (x_1) or bottom (x_2) layer can be deduced from the printed ballot. One layer gets destroyed but the other has its scanned image published, so this information, combined with the information about the destroyed layer revealed during the post-election audit, compromises the ballot security, which happens with $p = 1/2$. Therefore x_1, x_2, t_1 and t_2 are chosen randomly satisfying $x_1 + x_2 = t_1 + t_2(\mathrm{mod}\ m)$.

5. About unconditional integrity and unconditional privacy

The only cryptographic primitive used in PunchScan, and now in Prêt-à-Votertoo, is a bit commitment scheme. Instead of using encryptions as bit commitments (a proposed in PS), it is interesting to consider bit commitments that are unconditionally hiding and computationally binding. One possible example is the BC scheme described in [6] based on hash functions; another example are BCs based on the discrete log problem.

The reason that using unconditional BCs is interesting is the following: it is perfectly conceivable to have one authority (or group of several authorities) create these BCs using specific cryptographic hardware in a controled environment, the operation of which is controlled with smart cards kept in sealed and signed enveloppes and which is used only in cerimonies witnessed by all the political parties, etc. The keys of al the BCs would remain inside this equipment, during official cerimonies all BCs that need to be opened are opened, and when the election is over the keys to the not-opened BCs are destroyed. From that moment on it is impossible to prove how the ballots were constructed.

In this way one obtains unconditional, that is, everlasting privacy, whereas upto now Prêt-à-Voterand PunchScan only provide computational privacy, which will be broken eventually. This is why substituting (Chaum-like) mixing and using BC instead is so important: with mixing there was no hope to get to everlasting privacy, but with BCs it possible

The only reason to prefer BCs that are unconditionally binding is that one gets unconditionally integrity of the election outcome and only computational privacy of the vote. But in order to change the outcome the authority would have to break the computational assumption *while the election is going on.* Though it is very hard to estimate how difficult it is to break a system 40 years from know, it very well possible to create a system based on a computational assumption that wont be broken in the next three months.

We can even narrow down this idea even more. Suppose one uses a Hardware Security Module (or HSM, basically the equivalent of a smart card but with more security features and higher processing capicity) which is connected to a trusted computer to create the keys that open the bit commitments.

Before the elections you a cerimony is held to create the BCs. The smart cards that activate the HSM are then held by the authorities (or maybe in transparent boxes next to the HSM). After this pre-election cerimony computer and HSM are kept in a ”transparent” room so that any observer can see it sitting there. Apart from being guarded by heavily armed people, political parties are allowed access to the ”viewing” room and can organize a 24 hour vigilance checking that nobody is touching the thing.

After the election is over, a post-election cerimony is held in which half of the remaining BCs are opened and the other halfs are destroyed.

Now the computer inside the transparent room cannot rig the election because, apart from all the physical protection involved that makes it impossible for the computer to run unauthorized software, it has insufficient computational power to break the computational assumption that guarantees the binding property of the BCs. And even the NSA cannot rig the election since it cannot access the data inside the HSM and/or the trusted computer.

Compared to overall cost of elections this solution is not outlandish; in fact it is very reasonable. And I believe people and political parties will love it: the security of the system is concentrated in one very well-identifiable place with one very well-defined task: looking at the damned thing and see that nobody touches it. You can even broadcast the image on tv if you want, though it would be rather boring.

By the way, note that the observers do not have to verify the correctness of the software that is being used. As long as they verify that (1) nobody can access the machines between the two cerimonies, ans (2) the bit commitments that are opened are correct, they can trust the outcome of the election. (But the authorities of course have an interest in the correctness of the software to make sure that the election completes.)

Note that most cryptographers usually don’t like special equipments (and it is their job not to like them) but they should not forget that all of PKI is based on it: VeriSign stores its root certificates in an HSM held in a well-protected strong-room in a well-protected building And banks use HSMs to store their master keys, so this is known technology that could be applied to voting.

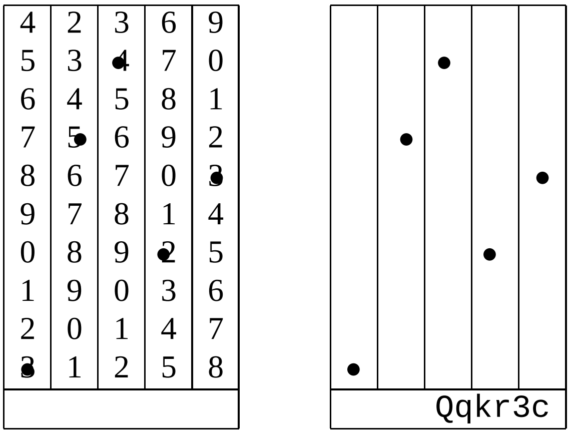
Of course, from a cryptographic point of view a computationally hiding verifiable secret sharing scheme may be preferable here. But from a psychological point of view, with an HSM-like solution it may be easier to win the trust of the lay-man. When designing systems that need to be trusted by a large majority of the population such arguments shouldn't be overlooked. A simple, understandable, robust method of making a system secure may be preferable over a superior cryptographic protocol that only a tiny fraction of the population can hope to understand. (If I can't explain it to my colleagues in the computer science department over a cup of coffee, I think it is doomed.)

6. Adapting the Pret-a-Voter ballot lay-out to the Brazilian system

A peculiarity of the Brazilian election system is that each political party has a two-digit number. And in races were more than one candidate per party are allowed, additional digits are added, resulting in a possible 5-digit number for the representatives in the federal house.

With PaV it is fairly easy to think of a way to present several digits on one ballot. Let us first describe how to transform the ballot of Section 2 in this new format. Instead of a left and a right column, we suppose two equal-sized pieces of paper. What was the left column now goes on top, while the right column constitutes the bottom, and we suppose that the voter casts his vote by perforating a hole through both layers simultaneously. As before, the top layer contains a permuted version of the candidate list and will be destroyed aftrewards, while the bottom layer contains an encryption of this permutation and will be retained. This constitutes a completely equivalent (isomorphic) ballot.

The only observation now is that instead of having 1 column with 4 candidates, one can have 5 columns with 10 numbers (where we suppose that the cyclic permutations on each column are independently chosen). In order to cast a valid vote the voter must perforate 5 numbers, one in each column; in all other respects the protocol is identical. The idea is outlined in the following diagram, which showes the upper and lower layer of a vote for candidate 35423:



4	2	3	6	9
5	3	4●	7	0
6	4	5	8	1
7	5●	6	9	2
8	6	7	0	3●
9	7	8	1	4
0	8	9	2●	5
1	9	0	3	6
2	0	1	4	7
3●	1	2	5	8

Note the similarity with PunchScan: there, one would have the right part, with 10 holes per columns, put on top of the left part, and a dauber for marking the vote on both layers simultaneously.

7. Conclusion

This paper started as a study the similarities and differences between Pret-a-Voter and Punchscan. Surprisingly we found a merge which seems an improvement on both. The Section 4 shows that cryptographically PS is actually slightly more complicated than the PaV variant presented in Section 3. The fact that *both* the top and the bottom layer can be flipped seems to complicate matters, without any real benefit. In particular, the choice between top and bottom layers does not seem to add to the security of Punchscan, unlike in [2]. Also in terms of ballot lay-out there seems little difference between the two schemes.

History and shortcoming of this version

This paper is the result of a long-overdue task I had set myself: going to the cryptographic core of both Prêt-à-Voter and PunchScan. This effort resulted in Sections 1 thru 4 which, together with the references, were submitted as a short paper to the Simpósio Brasileiro de Segurança. Drafts of Sections 5 and 6 were written at the same time, but they did not fit; they are included now for completeness.

Intended as a short paper (4 pages only), I am aware of the short-comings of the current version: if you do not know the subject already (or without reading the cited papers) it is almost incomprehensible, and there are not many references. Furthermore, a more detailed comparison of the differences (e.g. encryption printed on the ballot vs. ballot ids printed + bit commitment elsewhere), including a table listing them, is still missing.

Acknowledgments

Ben Hosp and Stefan Popovenuic very kindly provided me with [5], and the first clarified some other doubts that remained. Peter Ryan avoided that I make a fool of myself in Section 6 when I proposed to drill two equidistant holes simultaneously in the left and right part, instead of superimposing them. With David Chaum I had various discussions about PunchScan (ballot lay-out with arrows; internet solutions), though we never went over the crypto aspects.

References

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