General Certificateless Encryption and Timed-Release Encryption

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Abstract. Recent non-interactive timed-release encryption (TRE) schemes can be viewed as being supported by a certificateless encryption (CLE) mechanism. However, the security models of CLE and TRE differ and there is no generic transformation that turns a CLE into a TRE. In this paper, we give a generalized model for CLE that is also sufficient to fulfill the requirements of TRE.
Our model is secure against an adversary with adaptive trapdoor extraction capabilities for arbitrary identifiers (instead of selective identifiers), decryption capabilities for arbitrary public keys (as considered in strongly-secure CLE) and partial decryption capabilities (as considered in security-mediated certificateless encryption, or SMCLE).
Our model also supports hierarchical identities and recipient anonymity, which are not considered formally in the paradigms of TRE and CLE. We propose a concrete scheme under our generalized model and prove it secure without random oracles. Our proposal yields the first strongly-secure (hierarchical) SMCLE and the first TRE in the standard model.

Key words: security-mediated certificateless encryption, timed-release encryption

1 Introduction

The distinguished feature of an identity-based encryption (IBE) scheme (e.g. [6, 10, 15, 16, 27–29, 42]) is that a public key can be derived from any arbitrary string that acts as an identifier (ID). There exists a trusted authority, called a key generation center (KGC), which is responsible for the generation of the ID-based private key on demand.

Since the birth of practical constructions of IBE, we see many cryptographic schemes borrowing the idea of IBE for other security goals (e.g. broadcast encryption [8], searchable encryption [9] and oblivious transfer [29]). This paper studies two of them, which are certificateless encryption (CLE) [1–3, 17, 19, 21, 22, 34, 39] and timed-release encryption (TRE) [5, 12–14, 18, 20, 23, 30, 32]. Both of them have undergone quite rapid development.

CLE is an intermediary between IBE and traditional public key encryption (PKE). Generally speaking, CLE is constructed from a combination of IBE and PKE, such that the ability of the KGC to generate any ID-based private key cannot help decrypting the ciphertext due to the existence of the PKE component, in which the KGC does not know the corresponding private key.

TRE is a public key encryption scheme that the sender encrypts the message under a public key and a time, so the knowledge of both the matching private key and a time-dependent trapdoor are necessary for decryption. A time-server is trusted to keep a time-dependent trapdoor confidential until at an appointed time, which means the recipient cannot decrypt the ciphertext prior to a certain instant in time. A feature of modern TRE schemes is that the sender is not required to interact with the time-server other than retrieving the system parameter once.

⋆ This research is done while the author was a research intern of FX Palo Alto Laboratory.

1.1 Relationship between CLE and TRE

A practical TRE requires the system parameter size to be small compared with the number of supported time periods. This is where the idea of IBE (e.g. [6, 10, 27]) comes to the play. By treating the identities as time periods, IBE gives raise to a time-based unlock mechanism (e.g. [6, 37, 38]). However, this approach only supports a universal disclosure of encrypted documents since one trapdoor can decrypt all ciphertexts for a specific time. In other words, the inherent key-escrow property of IBE prohibits the encryption for a designated receiver.

Since CLE is an "escrow-free version" of IBE, and both TRE and CLE are a kind of double-encryption, it is natural to think CLE is what we are looking for to realize a TRE. Despite of the similarities in syntax and functionality one may imagine, it has been pointed out in [12] that a generic transformation from CLE to TRE is unlikely to be provable secure.

In CLE, each user is determined by a combination of an identity and a public key, which means an identity is only associated to a certain public key. Difficulty in reducing the confidentiality of TRE to that of CLE arises when the adversary is a "curious" time-server. In CLE, a curious KGC is not allowed to replace the public key associated with an identifier (otherwise, decryption of the ciphertext will be trivial since it holds both pieces of secrets). On the other hand, a time identifier is never bound to any public key in TRE, which means that the public key associated with a time identifier can be replaced. Thus, there is no way to simulate this implicit public key replacement when the CLE is viewed as a black box. We will show three examples of CLE [3, 33, 39] which cannot be trivially extended to TRE in Section 2.2.

Nevertheless, most of the recent non-interactive TRE schemes can be seen as converted from a corresponding implicit CLE mechanism.

1.2 Our Contributions

While the observation in [12] is true for a restricted definition of CLE, this work gives a generalized model for CLE that is also sufficient to fulfill the requirements of TRE. Our model is secure against an adversary with adaptive trapdoor extraction capabilities for arbitrary identifiers (instead of selective identifiers, e.g. [39]), decryption capabilities for arbitrary public keys (as considered in strongly-secure CLE) and partial decryption capabilities (as considered in security-mediated certificateless encryption, or SMCLE). Our model also supports hierarchical identities and recipient anonymity, which are not considered formally in the paradigms of TRE and CLE.

We propose a concrete construction under our generalized model. All existing concrete TRE schemes [5, 12–14, 18, 20, 23, 30, 32] and the only concrete SMCLE scheme [19] are proven in the random oracle model. It is true that the generic construction of SMCLE [19] can be instantiated by an IBE and a PKE without random oracles, nevertheless, the resulting scheme is not strongly-secure. Our proposal yields the first strongly-secure (hierarchical) SMCLE and the first TRE in the standard model.

2 Related Work

2.1 Timed-Release Encryption

The concept of timed-release cryptographic protocols is suggested by May [36] in 1993. It is further studied by many researchers, such as "price via processing" by Dwork and Naor [25], timed key escrow by Bellare and Goldwasser [4], timed commitments by Boneh and Noar [7], and time capsule signature by Dodis and Yum [24].

Early TRE schemes require interaction with the time-server. For examples, Rivest *et al.*'s idea [40] requires the senders to reveal their identities and the messages' release-time in their interactions with the server. In Di Crescenzo *et al.*'s scheme [20], the job of interacting with the time-server is moved from the sender to the receiver since a "conditional oblivious transfer protocol" will be executed between the server and the receiver. Such a protocol ensures that if the release-time is not less than the current time (the condition),

the receiver learns nothing (obliviousness). However, this protocol [20] is computationally intensive and thus the whole idea is vulnerable to denial-of-service attacks.

The first attempt to construct a non-interactive and user-anonymous TRE was made in [5]. A concrete construction is provided, but not supported by a formal security model and any claimed security properties are only argued for heuristically. The formal security model of message confidentiality is later considered independently by Cheon *et al.* [18] and Cathalo-Libert-Quisquater [12]. The former focuses on authenticated TRE and the latter is claimed to have a stronger model than the implicit non-authenticated version of [18]. Cathalo-Libert-Quisquater [12] also formalizes the release-time confidentiality, but not recipient-anonymity.

The recovery of past time-dependent trapdoors from a current trapdoor is studied in [14] and [38], which employs a hash chain and a tree structure [11] respectively. The study of the pre-open capability by the sender is initiated in [32] and later improved by [23].

Recently, Chalkias et al. proposed an efficient TRE scheme [13]. Without formal analysis, it is claimed that their scheme supports anonymous recipient. They also claim that their scheme is the most computationally efficient one for unknown recipients. However, it can be shown that the confidentiality of their scheme can be broken by a curious time-server. A plausible fix makes their scheme less efficient and the purported comparative advantage is lost.

Finally, we note that there is another way to realize the idea of TRE without employing a trusted server, which is known as time-lock puzzle approach [40]. The delayed release is made possible by the fact that the recipient has to invest a significant computational effort in a non-stop manner to solve a difficult problem. However, it is computationally expensive and the release-time is not precisely controllable.

Apart from the obvious application of delayed release of information, the need for sending a ciphertext into the future also appears in many scenarios. They can be broadly classified into two categories.

Rapid Dissemination of Information. The size of the time-dependent trapdoor is small when compared with the ciphertext (even of text message, due to the inherent ciphertext expansion in probabilistic encryption). With TRE, one can send the bulky ciphertext beforehand, without worrying the leakage of the confidential information. When it should be made public, a small trapdoor can be made available to a potentially large set of recipients. This avoids the problem of any network impedance at the release time. Examples of applicable scenarios are abundant, such as stock market values, strategic business plans, news agencies timed publications, licensed software updates, scheduled payments, or "casual" applications like internet contests, where participants should not get any information about the challenge before the designated time.

Commitment of Confidential Information. Commitment of confidential information is needed in many scenarios, such as sealed-bid auction, electronic lotteries, legal will, certified e-mail [32] etc. One can view the ciphertext as a kind of commitment made by the sender. In TRE, the decryption algorithm deterministically recovers the message from the ciphertext by a time-dependent trapdoor and the user's private key. Once the ciphertext is sent, there is no chance for the message sender to change the message that will be obtained from the decryption by the recipient later.

A special class of TRE scheme supports pre-open capability [23, 32], which means that the sender can help the recipient to decrypt the ciphertext by publishing a pre-open key. Since the pre-open key is given by the sender, it may give an opportunity to the sender to somehow control what message will be given by the decryption algorithm by manipulating the pre-open key. Using a TRE with pre-open capability as a way to commit some confidential information requires the TRE scheme to be binding (to be defined in Section 5.3).

2.2 Certificateless Encryption

The concept of certificateless cryptography has been suggested by Al-Riyami and Paterson [1] in 2003. Before we delve into the related work, we need a basic understanding of the security model to see the contribution of different proposals. Two types of adversaries are considered in certificateless cryptography. A Type-I adversary models coalitions of rogue users without the master secret. Due to the lack of a certificate, the

adversary is allowed to replace the public keys of users at will. A Type-II adversary models a curious KGC who has the master key but cannot replace the public keys of any users. In Al-Riyami and Paterson's security model for the encryption [1], a Type-I adversary can ask for the decryption of a ciphertext under a replaced public key. Schemes secure against such a class of attack are called "strongly-secure" [22]. A weaker type of adversary, termed Type-I$^-$, can only obtain a correct plaintext if the ciphertext is submitted along with the corresponding private key.

In security-mediated certificateless encryption [19] (SMCLE) introduced by Chow, Boyd and González Nieto, there is a security-mediator (SEM) who performs partial decryption for the user per request. This idea gives another variant for the decryption queries in the CLE paradigm, such that the adversary can ask for the partial decryption results under either the SEM trapdoor generated by the KGC or the user private key. Intuitively, the notion of SMCLE is more general than that of CLE since two partial decryption algorithms can always be combined into a single one, but the converse is not necessary true (see also Section 3.4). A concrete construction in the random oracle model and a generic construction in the standard model are proposed in [19].

The first CLE scheme by Al-Riyami and Paterson [1] is secure against both Type-I and Type-II adversary in the random oracle model. This scheme is also the basis for the hierarchical CLE described in [1]. However, neither a security model nor a security proof are given for this hierarchical extension. The authors later proposed a more efficient CLE scheme in [2], which has been shown to be insecure [17, 44]. A CLE without using pairing is proposed in [3]. However, the reduction used in the security proof does not hold up if the public key associated with the challenge ciphertext has been replaced. The later proposal of CLE without pairing [33] uses ideas similar to those in [3], and there is no formal evidence showing that the scheme is secure under the public key replacement attack. The reason why the two CLE schemes described in [3] and [33] are pairing-free is that part of the user's public key is dependent on the identity-specific trapdoor given by the KGC, which also means that TRE schemes cannot be trivially obtained from these constructions.

Many generic constructions of CLE from IBE and PKE exists, some are later shown to be insecure in [26, 34, 39], some of them [17, 34] actually rely on the random oracle heuristics.

It is believed [33, 35, 39] that [35] gives the first CLE in the standard model. However, it is possible to instantiate a prior generic construction in [19] with a PKE and an IBE in the standard model to obtain a secure CLE without random oracles. Both [35] and the instantiation of [19] are only secure against Type-I$^-$ attacks. Based on [27], a selective-ID secure CLE without random oracles is proposed in [39]. This scheme cannot be trivially extended to a TRE since the user's public key is dependent on the identity, but a user's public key is never coupled with a single time-identifier in TRE. Recently, the first strongly-secure CLE secure against Type-I adversaries in the standard model is proposed in [22].

In summary, there is no strongly-secure SMCLE (i.e. a CLE with partial decryption queries) proven secure in the standard model. We are not ware of any literature with formal work on hierarchical CLE, particularly none proven secure in the standard model. We are also not aware of any literature about recipient-anonymity in CLE. An extensive survey of CLE can be found in [21].

3 General Security-Mediated Certificateless Encryption

We propose a new definition of the (security-mediated) certificateless encryption. We will also highlight the relationship between our definition and existing definitions.

3.1 Notations

We use an ID-vector $\overrightarrow{ID} = (\mathsf{ID}_1, \mathsf{ID}_2, \cdots, \mathsf{ID}_L)$ to denote a hierarchy of identifiers $(\mathsf{ID}_1, \mathsf{ID}_2, \cdots, \mathsf{ID}_L)$. The length of $\overrightarrow{ID}$ is denoted by $|\overrightarrow{ID}| = L$. Let $\overrightarrow{ID}||\mathsf{ID}_r$ denote the vector $(\mathsf{ID}_1, \mathsf{ID}_2, \cdots, \mathsf{ID}_L, \mathsf{ID}_r)$ of length $|\overrightarrow{ID}|+1$. We say that $\overrightarrow{ID}$ is a prefix of $\overrightarrow{ID'}$ if $|\overrightarrow{ID}| \leq |\overrightarrow{ID'}|$ and $\mathsf{ID}_i = \mathsf{ID}'_i$ for $i \leq |\overrightarrow{ID}|$. We use $\emptyset$ to denote an empty ID-vector where $|\emptyset| = 0$ and $\emptyset||\mathsf{ID}_r = \mathsf{ID}_r$. Finally, we use the notation $(\{0,1\}^n)^{\leq h}$ to denote the set of vectors of length less than or equals to h, where each component is a n-bit long bit-string.

3.2 Syntax

From a natural extension of the definition of a 1-level SMCLE scheme [19], an h-level SMCLE scheme for identifiers of length n (where h and n are polynomially-bounded functions) is defined by the following sextuple of PPT algorithms:

- Setup (run by the server) is a probabilistic algorithm which takes a security parameter 1^λ, outputs a master secret key Msk, which can also be denoted as $d_\emptyset$, and the global parameters Pub. We assume that λ, $h = h(\lambda)$ and $n = n(\lambda)$ are implicit in Pub and all other algorithms take Pub implicitly as an input.
- $\mathsf{Extract}$ (run by the server or the one who hold a trapdoor) is a possibly probabilistic algorithm which an ID-vector $\overrightarrow{ID} \in (\{0,1\}^n)^{\leq h}$, its associated trapdoor $d_{\overrightarrow{ID}}$, and a string $\mathsf{ID}_r \in \{0,1\}^n$, outputs a trapdoor key $d_{\overrightarrow{ID}||\mathsf{ID}_r}$ associated with the ID-vector $\overrightarrow{ID}||\mathsf{ID}_r$.
- KeyGen (run by a user) is a probabilistic algorithm which generates a public/private key pair $(\mathsf{pk}_u, \mathsf{sk}_u)$.
- Enc (run by a sender) is a probabilistic algorithm which takes a message m from some implicit message space, an identifier $\overrightarrow{ID} \in (\{0,1\}^n)^{\leq h}$, and the receiver's public key pk_u as input, returns a ciphertext C.
- Dec^S (run by the one who hold the trapdoor, either a SEM in SMCLE or a receiver in CLE) is a possibly probabilistic algorithm which takes a ciphertext C and the trapdoor key $d_{\overrightarrow{ID}}$ as input, returns either a token D which can be seen as a partial decryption result of C, or an invalid flag $\perp$ (which is not in the message space).
- Dec^U (run by a receiver) is a possibly probabilistic algorithm which takes the ciphertext C, the receiver's private key sk_u and a token D as input, returns either the plaintext, an invalid flag $\perp_D$ denoting D is an invalid token, or an invalid flag $\perp_C$ denoting the ciphertext is invalid.

Different invalid flags will be returned by Dec^U to distinguish the case that the token from the SEM is invalid or the ciphertext is invalid. This is not captured by the original SMCLE model in [19]. We remark that it is possible to incorporate such feature into the concrete scheme in [19] by an interactive proof-of-knowledge.

For correctness, we require that $\mathsf{Dec}^U(C, \mathsf{sk}, \mathsf{Dec}^S(C, \mathsf{Extract}(\mathsf{Msk}, \overrightarrow{ID}))) = m$ for all $\lambda \in \mathbb{N}$, all $(\mathsf{Pub}, \mathsf{Msk}) \xleftarrow{\$} \mathsf{Setup}(1^\lambda)$, all $(\mathsf{pk}, \mathsf{sk}) \xleftarrow{\$} \mathsf{KeyGen}$, all message m, all ID-vector $\overrightarrow{ID}$ in $(\{0,1\}^n)^{\leq h}$ and all $C \xleftarrow{\$} \mathsf{Enc}(m, \overrightarrow{ID}, \mathsf{pk})$.

3.3 Security

Each adversary has access to the following oracles:

1. An $\mathsf{Extract}$ oracle that takes an ID-vector $\overrightarrow{ID} \in (\{0,1\}^n)^{\leq h}$ as input and returns its trapdoor $d_{\overrightarrow{ID}}$.
2. An Dec^S oracle that takes a ciphertext C and an ID-vector $\overrightarrow{ID}$, and outputs $\mathsf{Dec}^S(C, d_{\overrightarrow{ID}})$. Note that C may or may not be encrypted under $\overrightarrow{ID}$.
3. An Dec^U oracle that takes a ciphertext C, a public key pk and a token D, and outputs $\mathsf{Dec}^U(C, \mathsf{sk}, D)$ where sk is the secret key that matches pk.
4. An Dec oracle that takes a ciphertext C, an ID-vector $\overrightarrow{ID}$, and a public key pk, and outputs $\mathsf{Dec}^U(C, \mathsf{sk}, D)$ where sk is the secret key that matches pk and $D = \mathsf{Dec}^S(C, d_{\overrightarrow{ID}})$. Note that C may or may not be encrypted under $\overrightarrow{ID}$ and pk.

Following common practice, we consider the two kinds of adversaries.

1. A Type-I adversary that models any coalition of rogue users, and who aims to break the confidentiality of another user's ciphertext.
2. A Type-II adversary that models a curious KGC, who aims to break the confidentiality of an user's ciphertext. (We do not explicitly consider a rogue SEM since this type of adversary is weaker than the Type-II adversary.)

We use the common security model in which the adversary plays a two-phased game against a challenger. The game is modeled by the experiment below, for $\mathsf{X} \in \{\mathrm{I}, \mathrm{II}\}$, denoting whether an PPT adversary $\mathcal{A} = (\mathcal{A}_{\mathsf{find}}, \mathcal{A}_{\mathsf{guess}})$ is of Type-I or Type-II. The allowed oracle queries and the auxiliary information Aux depends on X.

Definition 1. Experiment $\mathbf{Exp}_{\mathcal{A}}^{\mathsf{CCA-X}}(\lambda)$

$(\mathsf{Pub}, \mathsf{Msk}) \xleftarrow{\$} \mathsf{Setup}(1^\lambda)$
$(m_0, m_1, \mathsf{pk}^*, \overrightarrow{ID}^*, \mathsf{state}) \xleftarrow{\$} \mathcal{A}_{\mathsf{find}}^{\mathcal{O}}(\mathsf{Pub}, \mathsf{Aux})$
$b \xleftarrow{\$} \{0,1\}$, $C^* \xleftarrow{\$} \mathsf{Enc}(m_b, \overrightarrow{ID}^*, \mathsf{pk}^*)$
$b' \xleftarrow{\$} \mathcal{A}_{\mathsf{guess}}^{\mathcal{O}}(C^*, \mathsf{state})$
If $b \neq b'$ *then return 0 else return 1*

where $\mathcal{O}$ *refers to a set of four oracles* $\mathsf{Extract}(\cdot), \mathsf{Dec}^S(\cdot,\cdot), \mathsf{Dec}^U(\cdot,\cdot,\cdot), \mathsf{Dec}(\cdot,\cdot,\cdot)$.

Those variables marked with * are basically about the challenge of the adversary. The adversary chooses a public key pk^* and a ID-vector $\overrightarrow{ID}^*$ to be challenged with, and the challenger returns C^* to the adversary as the challenge ciphertext. The two definitions below basically prohibit the adversary from trivially cheating by using the oracles to query for the answer to (parts of) the challenge.

Definition 2. *A hierarchical security-mediated certificateless encryption scheme is* (t, q_E, q_D, ϵ) *IND-CCA secure against a Type-I adversary if* $|\Pr[\mathbf{Exp}_{\mathcal{A}}^{\mathsf{CCA-I}}(\lambda) = 1] - \frac{1}{2}| \leq \epsilon$ *for all* t*-time adversary* $\mathcal{A}$ *making at most* q_E *extraction queries and* q_D *decryption queries (of any type), subjects to the following constraints:*

1. $\mathsf{Aux} = \emptyset$*, i.e. no auxiliary information is given to the adversary.*
2. *No* $\mathsf{Extract}(\overrightarrow{ID}')$ *query throughout the game, where* $\overrightarrow{ID}'$ *is a prefix of* $\overrightarrow{ID}^*$.
3. *No* $\mathsf{Dec}^S(C^*, \overrightarrow{ID}^*)$ *query throughout the game.*
4. *No* $\mathsf{Dec}(C^*, \overrightarrow{ID}^*, \mathsf{pk}^*)$ *query throughout the game.*

In existing models, $\mathsf{Extract}$ query on $(\overrightarrow{ID}')$ where $\overrightarrow{ID}'$ is a prefix of $\overrightarrow{ID}^*$ is allowed; but if such a query is issued, the challenge public key pk^* can no longer chosen by the adversary. In our discussion, we try to separate this from Type-I model and consider this type of adversarial behavior as a weaker variant of, and hence covered by, a Type-II adversary.

Note that we do allow the decryption of the ciphertext under $\overrightarrow{ID}'$ which is a prefix of $\overrightarrow{ID}^*$. This is stronger than the hierarchical IBE model in [28].

Definition 3. *A hierarchical security-mediated certificateless encryption scheme is* (t, q_E, q_D, ϵ) *IND-CCA secure against a Type-II adversary if* $|\Pr[\mathbf{Exp}_{\mathcal{A}}^{\mathsf{CCA-II}}(\lambda) = 1] - \frac{1}{2}| \leq \epsilon$ *for all* t*-time adversary* $\mathcal{A}$ *making at most* q_K *public key queries,* q_E *extraction queries and* q_D *decryption queries (of any type), subjects to the following conditions:*

1. $\mathsf{Aux} = (\mathsf{Msk}, \{\mathsf{pk}_1^*, \cdots, \mathsf{pk}_{q_K}^*\})$*, i.e. the master secret key and a set of challenge public key* pk^* *is given to the adversary.*
2. $\mathsf{pk}^* \in \{\mathsf{pk}_1^*, \cdots, \mathsf{pk}_{q_K}^*\}$*, i.e. the challenge public key must be among the set given by the challenger.*
3. *No* $\mathsf{Dec}^U(C^*, \mathsf{pk}^*, D)$ *query throughout the game, where* D *is obtained from* $\mathsf{Dec}^S(C^*, \overrightarrow{ID}^*)$.
4. *No* $\mathsf{Dec}(C^*, \overrightarrow{ID}^*, \mathsf{pk}^*)$ *query throughout the game.*

3.4 Relationship with Existing Models

Having two separated decryption oracles in the SMCLE model gives a more general notion than CLE. This can be justified as follows:

1. Partial decryption result cannot be made available in the CLE model.
2. Since the decryption oracle is separated into two, the SMCLE model does not have the notion of a "full" private key which is present in previous CLE models (a full private key is a single secret for the complete decryption of the ciphertext). On the ground that separated secrets can always be concatenated into a single full one, this simplification has already been adopted in more recent models [31].

For our attempt of generalizing CLE, we do not have an oracle for replacing the public key corresponding to an identifier, which is present in the existing model for CLE. This may make a difference in the following.

1. The adversary's choice of the victim user it wishes to be challenged with,
2. The choice of user in decryption oracle queries.

Our model still allows the adversary to choose which identifier/public key it wants to attack. For the decryption queries, the adversary can just supply different combination of identifier and public key to the Dec^S and Dec^U oracles. In this way, implicitly replacement is done. In other words, the security model is not weakened, but generalized to cover other applications of CLE such as TRE.

4 Our Proposed Construction

4.1 Preliminaries

Let $\mathbb{G}$ be a multiplicative group of prime order p and $\mathbb{G}_T$ be a multiplicative group also of order p. We assume the existence of an efficiently computable bilinear map $\hat{e} : \mathbb{G} \times \mathbb{G} \rightarrow \mathbb{G}_T$ such that

1. *Bilinearity*: For all $u, v \in \mathbb{G}$ and $r, s \in \mathbb{Z}_p$, $\hat{e}(u^r, v^s) = \hat{e}(u, v)^{rs}$.
2. *Non-degeneracy*: $\hat{e}(u, v) \neq 1_{\mathbb{G}_T}$ for all $u, v \in \mathbb{G} \setminus \{1_{\mathbb{G}}\}$.

We also assume the following problems are intractable in such groups.

Definition 4. *The* Decision **3**-Party Diffie-Hellman Problem *(3-DDH) in* $\mathbb{G}$ *is to decide if* $T = g^{\beta\gamma\delta}$ *given* $(g, g^\beta, g^\gamma, g^\delta, T) \in \mathbb{G}^5$. *Formally, defining the advantage of a* PPT *algorithm* $\mathcal{D}$, $Adv_{\mathcal{D}}^{3-\mathrm{DDH}}(k)$, *as*

$$|\Pr[1 \xleftarrow{\$} \mathcal{D}(g, g^\beta, g^\gamma, g^\delta, T) | T \leftarrow g^{\beta\gamma\delta} \wedge \beta, \gamma, \delta \xleftarrow{\$} \mathbb{Z}_p^*] - \Pr[1 \xleftarrow{\$} \mathcal{D}(g, g^\beta, g^\gamma, g^\delta, T) | T \xleftarrow{\$} \mathbb{G} \wedge \beta, \gamma, \delta \xleftarrow{\$} \mathbb{Z}_p^*]|.$$

We say 3-DDH is intractable if the advantage is a negligible function for all PPT *algorithms* $\mathcal{D}$.

Compared with the Bilinear Diffie-Hellman (BDH) problem, the problem instance of 3-DDH is purely in $\mathbb{G}$ while that of BDH contains an element $\hat{t} \in \mathbb{G}_T$. If BDH problem is solvable, one can solve 3-DDH by feeding $(g, g^\beta, g^\gamma, g^\delta, \hat{e}(g, T))$ to a BDH oracle. Apart from CLE [22], the above assumption has been employed in other advanced pairing-based cryptographic schemes such as [8].

We introduce a variant of the weak Bilinear Diffie-Hellman Inversion (BDHI) assumption [6] below in the favor of 3-DDH. The original h-wBDHI problem in $(\mathbb{G}, \mathbb{G}_T)$ [6] is to decide whether $\hat{t} = \hat{e}(g, g^\gamma)^{\alpha^{h+1}}$. The naming of "inversion" comes from the equivalence to the problem of deciding whether $\hat{t} = \hat{e}(g, g^\gamma)^{1/\alpha}$.

Definition 5. *The* Modified h-Weak Bilinear-Diffie-Hellman Inversion Problem *(h-wBDHI') in* $\mathbb{G}$ *is to decide if* $T = g^{\gamma\alpha^{h+1}}$ *given* $(g, g^\gamma, g^\alpha, g^{\alpha^2}, \cdots, g^{\alpha^h}, T) \in \mathbb{G}^{h+3}$. *Formally, defining the advantage of a* PPT *algorithm* $\mathcal{D}$, $Adv_{\mathcal{D}}^{h-\mathrm{wBDHI}'}(k)$, *as*

$$\begin{aligned} &|\Pr[1 \xleftarrow{\$} \mathcal{D}(g, g^\gamma, g^\alpha, g^{\alpha^2}, \cdots, g^{\alpha^h}, T) | T \leftarrow g^{\gamma\alpha^{h+1}} \wedge \alpha, \gamma \xleftarrow{\$} \mathbb{Z}_p^*] \\ &- \Pr[1 \xleftarrow{\$} \mathcal{D}(g, g^\gamma, g^\alpha, g^{\alpha^2}, \cdots, g^{\alpha^h}, T) | T \xleftarrow{\$} \mathbb{G} \wedge \alpha, \gamma \xleftarrow{\$} \mathbb{Z}_p^*]|. \end{aligned}$$

We say h-wBDHI' is intractable if the advantage is a negligible function for all PPT *algorithms* $\mathcal{D}$.

We require a hash function H drawn from a family of collision resistant hash functions too.

Definition 6. *A hash function* $H \xleftarrow{\$} \mathcal{H}(k)$ *is* collision resistant *if for all* PPT *algorithms* $\mathcal{C}$ *the advantage*

$$Adv_{\mathcal{C}}^{\mathrm{CR}}(k) = \Pr[H(x) = H(y) \wedge x \neq y | (x, y) \xleftarrow{\$} \mathcal{C}(1^k, H) \wedge H \xleftarrow{\$} \mathcal{H}(k)]$$

is negligible as a function of the security parameter k.

4.2 Proposed Construction

$\mathsf{Setup}(1^\lambda, n)$: Let $\mathbb{G}$, $\mathbb{G}_T$ be two multiplicative groups with a bilinear map $\hat{e}$ as defined before. They are of the same order p, which is a prime and $2^\lambda < p < 2^{\lambda+1}$.

- **Encryption key:** choose two generators $g, g_2 \in_R \mathbb{G}$.
- **Master public key:** choose an exponent $\alpha \in_R \mathbb{Z}_p$ and set $g_1 = g^\alpha$.
- **Hash key for identity-based public key derivation:** choose h many $(\ell+1)$-length vectors $\vec{U}_1, \cdots, \vec{U}_h \in_R \mathbb{G}^{\ell+1}$, where each $\vec{U}_j = (u'_j, u_{j,1}, \cdots, u_{j,\ell})$, $1 \le j \le h$.
  Each vector $\vec{U}_j$ $(1 \le j \le h)$ corresponds to the j-th level of the hierarchy. We use the notation $\overrightarrow{ID} = (\mathsf{ID}_1, \cdots, \mathsf{ID}_k)$ to denote an identity which is a hierarchy of different identities at different levels. Each ID_j is an n-bit string. We write ID_j as ℓ blocks each of length n/ℓ bits $(\mathsf{ID}_{j,1}, \cdots, \mathsf{ID}_{j,\ell})$. We define $F_{\vec{U}_j}(\mathsf{ID}_j) = u'_j \prod_{i=1}^{\ell} u_{j,i}^{\mathsf{ID}_{j,i}}$.
- **Hash key for ciphertext validity:** choose an $(n+1)$-length vector $\vec{V} = (v', v_1, \cdots, v_n) \in_R \mathbb{G}^{n+1}$ This vector defines the hash function $F_{\vec{V}}(w) = v' \prod_{j=1}^{n} {v_j}^{b_j}$ where w is a n-bit string $b_1 b_2 \cdots b_n$.
- **Hash function:** pick a function $H : \{0,1\}^* \to \{0,1\}^n$ from a family of collision-resistant hash functions.

The public parameters Pub and the master secret key Msk are given by

$$\mathsf{Pub} = (\mathbb{G}, \mathbb{G}_T, \hat{e}(\cdot,\cdot), n, g, g_1, g_2, \vec{U}_1, \cdots, \vec{U}_h, \vec{V}, H(\cdot)), \qquad \mathsf{Msk} = g_2^\alpha.$$

$\mathsf{Extract}(\mathsf{Msk}, \overrightarrow{ID})$: Given an identity $\overrightarrow{ID} = (\mathsf{ID}_1, \cdots, \mathsf{ID}_k)$ for $k \le h$, pick $r \in_R \mathbb{Z}_p^*$, return

$$d_{\overrightarrow{ID}} = (a_1, a_2, \vec{z}_{k+1}, \cdots, \vec{z}_h) = (g_2^\alpha \cdot (\prod_{j=1}^{k} F_{\vec{U}_j}(\mathsf{ID}_j))^r, g^r, (\vec{U}_{k+1})^r, \cdots, (\vec{U}_h)^r),$$

where $(\vec{U}_{k+1})^r = ((u'_{k+1})^r, (u_{k+1,1})^r, \cdots, (u_{k+1,\ell})^r)$.

Note that (a_1, a_2) is sufficient for decryption, while $\vec{z}_{k+1}, \cdots, \vec{z}_h$ can help the derivation of the trapdoor for $(\mathsf{ID}_1, \cdots, \mathsf{ID}_k, \mathsf{ID}_{k+1})$ for any n-bit string ID_{k+1} and $k+1 \le h$. $d_{\overrightarrow{ID}}$ becomes shorter as the length of ID increases.

$\mathsf{KeyGen}()$: Pick $\mathsf{sk} \in_R \mathbb{Z}_p^*$, return sk as the secret key and $\mathsf{pk} = (X, Y) = (g^{\mathsf{sk}}, g_1^{\mathsf{sk}})$ as the public key.

$\mathsf{Enc}(m, \overrightarrow{ID}, \mathsf{pk})$: To encrypt $m \in \mathbb{G}_T$ for $\overrightarrow{ID} = (\mathsf{ID}_1, \cdots, \mathsf{ID}_k)$ where $k \le h$, parse pk as (X, Y), then check that it is a valid public key by verifying that $e(X, g_1) = e(g, Y)$. If equality holds, pick $s \in_R \mathbb{Z}_p^*$ and compute

$$\begin{aligned} C &= (C_1, C_2, \tau, \sigma) \\ &= (m \cdot \hat{e}(Y, g_2)^s, \prod_{j=1}^{k} F_{\vec{U}_j}(\mathsf{ID}_j)^s, g^s, F_{\vec{V}}(w)^s) \end{aligned}$$

where $w = H(C_1, C_2, \tau, \overrightarrow{ID}, \mathsf{pk})$.

$\mathsf{Dec}^S(C, d_{\overrightarrow{ID}})$: Parse C as (C_1, C_2, τ, σ), and $d_{\overrightarrow{ID}}$ as $(a_1, a_2, \cdots)$. First check if $\hat{e}(\tau, \prod_{j=1}^{k} F_{\vec{U}_j}(\mathsf{ID}_j) \cdot F_{\vec{V}}(w')) = \hat{e}(g, C_2 \cdot \sigma)$ where $w' = H(C_1, C_2, \tau, \overrightarrow{ID}, \mathsf{pk})$. Return $\perp$ if inequality holds or any parsing is not possible, otherwise pick $t \in_R \mathbb{Z}_p^*$ and return

$$\begin{aligned} D &= (D_1, D_2, D_3) \\ &= (a_1 \cdot F_{\vec{V}}(w')^t, a_2, g^t) \end{aligned}$$

$\mathsf{Dec}^U(C, \mathsf{sk}, D)$: Parse C as (C_1, C_2, τ, σ) and check if $\hat{e}(\tau, \prod_{j=1}^{k} F_{\vec{U}_j}(\mathsf{ID}_j) \cdot F_{\vec{V}}(w')) = \hat{e}(g, C_2 \cdot \sigma)$ where $w' = H(C_1, C_2, \tau, \overrightarrow{ID}, \mathsf{pk})$. If equality does not hold or parsing is not possible, return $\perp_C$. Next, parse D as

(D_1, D_2, D_3) and check if $\hat{e}(g, D_1) = \hat{e}(g_1, g_2)\hat{e}(D_2, \prod_{j=1}^{k} F_{\vec{U}_j}(\mathsf{ID}_j))\hat{e}(D_3, F_{\vec{V}}(w'))$. If equality does not hold or parsing is not possible, return $\perp_D$. Otherwise, return

$$m \leftarrow C_1 \cdot \left(\frac{\hat{e}(C_2, D_2)\hat{e}(\sigma, D_3)}{\hat{e}(\tau, D_1)} \right)^{\mathsf{sk}}.$$

5 Special Features for Timed-Release Encryption

5.1 Security-Mediator in Timed-Release Encryption

We introduce the concept of security-mediator in the TRE paradigm, which gives a new model for the operation of the time-server. If the time-server is going charge for each decryption, instead of releasing a system-wide time-dependent trapdoor, the time-server can decrypt a ciphertext partially by the time-dependent trapdoor per request.

5.2 Time Hierarchy

Each identifier corresponds to a single time period, which means that the server has to publish t private keys on a bulletin board after t time-periods have passed. Given a hierarchical CLE, the amount data on the bulletin board can be reduced by using CHK forward secure encryption scheme [11] in reverse, as suggested in [6]. Suppose the CHK framework is setup as a tree of depth $\log_2 T$ which is for a total of T time periods. To encrypt a message for time $t < T$, the time identifier is the CHK identifier for time period $T - t$. Release of trapdoor is done in the same manner, the private key for the time period $T - t$ is released on the t^{th} time period. This single private key enables anyone to derive the private keys for CHK time periods $T - t, T - t + 1, \cdots, T$, which means the user can obtain the trapdoors for time in the range of $1, \cdots, t$. By using this trick, the server only needs to publish a single private key comprising $O(\log^2 T)$ group elements at any time.

5.3 Pre-open Capability

In many applications of TRE, it is desirable to have a pre-open mechanism that the sender can enable the recipient to decrypt the ciphertext before the pre-specified release-time, without re-sending the plaintext. In a TRE with such a pre-open capability [32], the sender gets hold of a pre-open key that can functionally substitute the role of the system's time-dependent trapdoor, for the ciphertext prepared by him/her.

The concept of pre-open capability is introduced to the TRE paradigm by [32]. However, the scheme of [32] does not consider the security threat that the sender can give a pre-open key which opens the ciphertext to another message that is different from the one originally being encrypted. This deficiency is pointed out by [23], where the property of binding is formally defined and a scheme with binding pre-open key is proposed.

Model. We chose not to cover pre-open capability in the our general model because we do not think it makes a good sense in the context of CLE. The syntactic changes for pre-open capability include:

- Enc (run by a sender) is a probabilistic algorithm which takes a message m from some implicit message space, an identifier $\overrightarrow{ID} \in (\{0,1\}^n)^{\leq h}$, and the receiver's public key pk_u as input, returns a ciphertext C and its pre-open key D_C.
- $\mathsf{PreOpen}$ (run by a receiver) is a possibly probabilistic algorithm which takes the ciphertext C, the receiver's private key sk_u and a pre-open key D_C as input, returns either the plaintext, an invalid flag $\perp_D$ denoting D_C is an invalid pre-open key, or an invalid flag $\perp_C$ denoting the ciphertext is invalid.

Correctness requires $\mathsf{PreOpen}(C, \mathsf{sk}, D_C) = m$ for all $\lambda, n \in \mathbb{N}$, all Pub given by $\mathsf{Setup}(1^\lambda, n)$, all $(\mathsf{pk}, \mathsf{sk})$ given by KeyGen, all message m, all time $\overrightarrow{ID}$ in $(\{0,1\}^n)^{\leq h}$ and all (C, D_C) given by $\mathsf{Enc}(m, \overrightarrow{ID}, \mathsf{pk})$.

Binding requires the following probability is negligible for all PPT algorithm $\mathcal{A}$.

$$\Pr[(C^*, \mathsf{ID}^*, D_C^* \xleftarrow{\$} \mathcal{A}(\mathsf{Pub}) \mid (\mathsf{Pub}, \mathsf{Msk}) \xleftarrow{\$} \mathsf{Setup}(1^\lambda) \\ \wedge \mathsf{PreOpen}(C^*, \mathsf{sk}, D_C^*) \notin \{\perp_D, \perp_C\} \\ \wedge \mathsf{PreOpen}(C^*, \mathsf{sk}, D_C^*) \neq \mathsf{Dec}^U(C^*, \mathsf{sk}, \mathsf{Dec}^S(C^*, \mathsf{Extract}(\mathsf{Msk}, \overrightarrow{ID^*})))]$$

Construction. To make our concrete scheme supports pre-open capability, Enc just outputs $D_C = g_1^s$ as the pre-open key, where s is the random factor chosen in Enc. The pre-open mechanism is defined as below.

$\mathsf{PreOpen}(C, \mathsf{sk}, D_C)$: Firstly, check if the pre-open key is valid by $\hat{e}(D_C, g) = \hat{e}(g_1, \tau)$, returns $\perp_D$ if the equality does not hold. Otherwise, parse C as (C_1, C_2, τ, σ) and check if $\hat{e}(\tau, \prod_{j=1}^{k} F_{\vec{U}_j}(\mathsf{ID}_j) \cdot F_{\vec{V}}(w')) = \hat{e}(g, C_2 \cdot \sigma)$ where $w' = H(C_1, C_2, \tau, \overrightarrow{ID}, \mathsf{pk})$. Return $\perp_C$ if parsing is not possible or the equality does not hold. Otherwise, return $m \leftarrow C_1 / \hat{e}(D_C, g_2)^{\mathsf{sk}}$.

Security. A Type-I adversary is not entitled to have the pre-open key. We show that the addition of pre-open key will not compromise the confidentiality of the scheme against a time-server adversary. Using the knowledge of α (which is known to a Type-II adversary), the pre-open key of the challenge ciphertext can be easily computed by $(g^\gamma)^\alpha = {g_1}^\gamma$.

Next, we need to show it is binding. Given $(\overrightarrow{ID}, \mathsf{pk})$, the random factor in a valid ciphertext validity is uniquely fixed. From the pre-open key validity checking $\hat{e}(D_C, g) = \hat{e}(g_1, \tau)$ and the bilinearity, D_C must be in a correct form. Hence, the probability for breaking the binding property is zero.

5.4 Release-time Confidentiality

Release-time confidentiality protects the ciphertext release-time from being known to anyone but the recipient. In the context of CLE, this property means recipient-ID anonymity.

Naturally, one will consider an anonymous IBE (e.g. [9, 10, 27]), where the ciphertext does not reveal any information about its intended recipient. However, we can leverage the fact that a kind of double-encryption is done in TRE.

In the context of TRE, the release-time should be sent to the intended recipient. So we add into our framework another algorithm called GetID for this purpose.

Model. The algorithm GetID is one executed by the intended recipient who holds the user private key and the ciphertext, but not the time-dependent trapdoor. This allows the intended recipient to get the time-identifier from the ciphertext by using the user secret key. The correctness requirement is as follows.

- $\mathsf{GetID}(\mathsf{Enc}(m, \overrightarrow{ID}, \mathsf{pk}), \mathsf{sk}) = \overrightarrow{ID}$ for all $\ell, n \in \mathbb{N}$, all Pub given by $\mathsf{Setup}(1^\ell, n)$, all $(\mathsf{pk}, \mathsf{sk})$ given by KeyGen, all message m, and all identifier $\overrightarrow{ID}$ in $(\{0,1\}^n)^{\leq h}$.

The formal security requirement is similar to that in [12], but on top of that we need to add an GetID oracle that takes a ciphertext and a public key of the adversary's choice. Basically, the challenge ciphertext is not encrypting one of the two messages under the fixed identifier all given by the adversary, but encrypting a fixed message under one of the two identifiers. The adversary's goal is to tell which random identifier is employed by the challenger.

Definition 7. Experiment $\mathbf{Exp}_{\mathcal{A}}^{\mathsf{RTC-II}}(\lambda)$

$(\mathsf{Pub}, \mathsf{Msk}) \xleftarrow{\$} \mathsf{Setup}(1^\lambda)$
$(m, \mathsf{pk}^*, \overrightarrow{ID}_0^*, \overrightarrow{ID}_1^*, \mathsf{state}) \xleftarrow{\$} \mathcal{A}_{\mathsf{find}}^{\mathcal{O}}(\mathsf{Pub}, \mathsf{Aux})$
$b \xleftarrow{\$} \{0,1\}$, $C^* \xleftarrow{\$} \mathsf{Enc}(m, \overrightarrow{ID}_b^*, \mathsf{pk}^*)$
$b' \xleftarrow{\$} \mathcal{A}_{\mathsf{guess}}^{\mathcal{O}}(C^*, \mathsf{state})$
If $b \neq b'$ then return 0 else return 1

where $\mathcal{O}$ refers to a set of five oracles $\mathsf{Extract}(\cdot), \mathsf{GetID}(\cdot,\cdot), \mathsf{Dec}^S(\cdot,\cdot), \mathsf{Dec}^U(\cdot,\cdot,\cdot), \mathsf{Dec}(\cdot,\cdot,\cdot)$.

Definition 8. *A hierarchical security-mediated certificateless encryption scheme is (t, q_E, q_D, ϵ) RTC-CCA secure against a Type-II adversary if $|\Pr[\mathbf{Exp}_{\mathcal{A}}^{\mathsf{RTC-II}}(\lambda) = 1] - \frac{1}{2}| \leq \epsilon$ for all t-time adversary $\mathcal{A}$ making at most q_K public key queries, q_E extraction queries and q_D decryption queries (of any type), subjects to the following conditions:*

1. $\mathsf{Aux} = (\mathsf{Msk}, \{\mathsf{pk}_1^*, \cdots, \mathsf{pk}_{q_K}^*\})$, *i.e. the master secret key and a set of challenge public key* pk^* *is given to the adversary.*
2. $\mathsf{pk}^* \in \{\mathsf{pk}_1^*, \cdots, \mathsf{pk}_{q_K}^*\}$, *i.e. the challenge public key must be among the set given by the challenger.*
3. *No* $\mathsf{GetID}(C^*, \mathsf{pk}^*)$ *query throughout the game.*
4. *No* $\mathsf{Dec}^U(C^*, \mathsf{pk}^*, D)$ *query throughout the game, where D is obtained from* $\mathsf{Dec}^S(C^*, \overrightarrow{ID}_0^*)$ *or* $\mathsf{Dec}^S(C^*, \overrightarrow{ID}_1^*)$.
5. *No* $\mathsf{Dec}(C^*, \overrightarrow{ID}^*, \mathsf{pk}^*)$ *query throughout the game, where* $\overrightarrow{ID}^* \in \{\overrightarrow{ID}_0^*, \overrightarrow{ID}_1^*\}$.

Construction. We need the help of a key-derivation function $K : \mathbb{G}_T \to \{0,1\}^{n \cdot h + k + 1}$. We also assume an implicit one-to-one mapping between $\mathbb{G}$ and $\{0,1\}^{k+1}$, i.e. the public parameters Pub is given by

$$\mathsf{Pub} = (\mathbb{G}, \mathbb{G}_T, \hat{e}(\cdot,\cdot), n, g, g_1, g_2, \vec{U}_1, \cdots, \vec{U}_h, \vec{V}, H(\cdot), K(\cdot))$$

The modified encryption algorithm Enc and the GetID algorithm are described as follows.
$\mathsf{Enc}(m, \overrightarrow{ID}, \mathsf{pk})$: To encrypt $m \in \mathbb{G}_T$ for $\overrightarrow{ID} = (\mathsf{ID}_1, \cdots, \mathsf{ID}_k)$ where $k \leq h$, parse pk as (X, Y), then check that it is a valid public key by $e(X, g_1) = e(g, Y)$. If equality holds, pick $s \in_R \mathbb{Z}_p^*$ and compute

$$\begin{aligned} C &= (C_1, C_2, \tau, \sigma) \\ &= (m \cdot \hat{e}(Y, g_2)^s, (\overrightarrow{ID} || \prod_{j=1}^{k} F_{\vec{U}_j}(\mathsf{ID}_j)^s) \oplus K(\hat{e}(X, g_2)^s), g^s, F_{\vec{V}}(w)^s) \end{aligned}$$

where $w = H(C_1, C_2, \tau, \overrightarrow{ID}, \mathsf{pk}, K(\hat{e}(X, g_2)^s))$.

$\mathsf{GetID}(C, \mathsf{sk})$: Parse C as (C_1, C_2, τ, σ), $C_2 \oplus K(\hat{e}(\tau, g_2)^{\mathsf{sk}})$ as $(\overrightarrow{ID'} || f')$, and $\overrightarrow{ID'}$ as $(\mathsf{ID}_1, \cdots, \mathsf{ID}_k)$; check if $\hat{e}(\tau, \prod_{j=1}^{k} F_{\vec{U}_j}(\mathsf{ID}_j) \cdot F_{\vec{V}}(w')) = \hat{e}(g, f'\sigma)$ where $w' = H(C_1, C_2, \tau, \overrightarrow{ID'}, \mathsf{pk}, K(\hat{e}(\tau, g_2)^{\mathsf{sk}}))$. Return $\perp$ if inequality holds or any parsing is not possible, otherwise return $\overrightarrow{ID'}$.

Security. Intuitively, the malleable XOR cipher can be used in C_2 since the decryption algorithms checks the σ term, which is computed from the hash taking C_2 as part of the input.

Security is defined against a Type-II adversary since the intended recipient should know the release-time, but not any other party including the time-server. The new things in the security proof are the simulation of the challenge ciphertext in a new format, all decryption oracles including GetID, and the new well-formness checking of the ciphertext (since w is the output of the hash H which now takes $\hat{e}(X, g_2)^s$ as part of the input, but not just public information).

Note that the padding for encrypting the message is $\hat{e}(Y, g_2)^s$ while the padding we introduced for hiding all the information related to the identifier is $\hat{e}(X, g_2)^s$. By the relationship that $Y = X^{\mathsf{Msk}}$, $\mathcal{S}$ can easily obtain $\hat{e}(X, g_2)^s$ by $(\hat{e}(Y, g_2)^s)^{\frac{1}{\mathsf{Msk}}}$ from $\hat{e}(Y, g_2)^s$, which we have shown how to compute already. The simulation of the new ciphertext can be done in this way. SEM partial decryption is trivial for a Type-II adversary. For user partial decryption, $\mathcal{S}$ computes $\hat{e}(Y, g_2)^s$ as $\hat{e}(Y, (\sigma/\tau^{K_v(w)})^{\frac{1}{J_v(w)}})$, $\hat{e}(X, g_2)^s$ can thus easily computed as $\hat{e}(X, (\sigma/\tau^{K_v(w)})^{\frac{1}{J_v(w)}})$. The same is true for computing w for well-formness checking.

5.5 Recipient Anonymity

Release-time is not the only dimension about which key can be used to decrypt the ciphertext. It may be possible that information about the intended recipient is leaked from the ciphertext too, which against the requirement of recipient anonymity. In the context of CLE, release-time confidentiality and recipient anonymity ensures that no one can tell who is the intended recipient of a CLE-encrypted ciphertext.

The formal security requirement is similar to the release-time confidentiality. The adversary chooses two public keys to be challenged with. The challenger encrypts a message chosen by the adversary under a random key among the two. The adversary's goal is to tell which key is employed by the challenger.

Definition 9. Experiment $\mathbf{Exp}_{\mathcal{A}}^{\mathsf{RKA-II}}(\lambda)$

$(\mathsf{Pub}, \mathsf{Msk}) \xleftarrow{\$} \mathsf{Setup}(1^\lambda)$
$(m, \mathsf{pk}'_0, \mathsf{pk}'_1, \overrightarrow{ID}^*, \overrightarrow{ID}^*, \mathsf{state}) \xleftarrow{\$} \mathcal{A}^{\mathcal{O}}_{\mathsf{find}}(\mathsf{Pub}, \mathsf{Aux})$
$b \xleftarrow{\$} \{0,1\}$, $C^* \xleftarrow{\$} \mathsf{Enc}(m, \overrightarrow{ID}^*, \mathsf{pk}'_b)$
$b' \xleftarrow{\$} \mathcal{A}^{\mathcal{O}}_{\mathsf{guess}}(C^*, \mathsf{state})$
If $b \neq b'$ *then return 0 else return 1*

where $\mathcal{O}$ *refers to a set of four oracles* $\mathsf{Extract}(\cdot), \mathsf{GetID}(\cdot,\cdot), \mathsf{Dec}^S(\cdot,\cdot), \mathsf{Dec}^U(\cdot,\cdot,\cdot), \mathsf{Dec}(\cdot,\cdot,\cdot)$.

Definition 10. *A hierarchical security-mediated certificateless encryption scheme is* (t, q_E, q_D, ϵ) *RKA-CCA secure against a Type-II adversary if* $|\Pr[\mathbf{Exp}_{\mathcal{A}}^{\mathsf{RKA-II}}(\lambda) = 1] - \frac{1}{2}| \leq \epsilon$ *for all* t*-time adversary* $\mathcal{A}$ *making at most* q_K *public key queries,* q_E *extraction queries and* q_D *decryption queries (of any type), subjects to the following conditions:*

1. $\mathsf{Aux} = (\mathsf{Msk}, \{\mathsf{pk}^*_1, \cdots, \mathsf{pk}^*_{q_K}\})$*, i.e. the master secret key and a set of challenge public key* pk^* *is given to the adversary.*
2. $\mathsf{pk}'_0, \mathsf{pk}'_1 \in \{\mathsf{pk}^*_1, \cdots, \mathsf{pk}^*_{q_K}\}$*, i.e. the challenge pair of public keys must be among the set given by the challenger.*
3. *No* $\mathsf{GetID}(C^*, \mathsf{pk}')$ *query throughout the game, where* $\mathsf{pk}' \in \{\mathsf{pk}'_0, \mathsf{pk}'_1\}$.
4. *No* $\mathsf{Dec}^U(C^*, \mathsf{pk}', D)$ *query throughout the game, where* D *is obtained from* $\mathsf{Dec}^S(C^*, \overrightarrow{ID}^*)$ *and where* $\mathsf{pk}' \in \{\mathsf{pk}'_0, \mathsf{pk}'_1\}$.
5. *No* $\mathsf{Dec}(C^*, \overrightarrow{ID}^*, \mathsf{pk}')$ *query throughout the game, where* $\mathsf{pk}' \in \{\mathsf{pk}'_0, \mathsf{pk}'_1\}$.

Security. The recipient anonymity of our scheme can be easily seen in the Game 7 and game 8 of our proof. The intended recipient of the ciphertext is uniquely determined by θ_i. When T is a random element, θ_i is perfectly hidden.

6 Conclusions

In the study of cryptography, we always seek for the strongest definition and try to achieve it. The current model of certificateless encryption (CLE) is restrictive and cannot give the desired security properties when it is instantiated as timed-release encryption (TRE). We give a generalized model for CLE that is also sufficient to fulfill the requirements of TRE. All future CLE proposals in our general model automatically gives a secure TRE scheme.

Our model is defined against full-identifier extraction, decryption under arbitrary public key, and partial decryption, which incorporates the strongest properties one may desire. We also give the first formal study of hierarchical identities and recipient anonymity in the context of TRE and CLE. Our concrete scheme is the first strongly-secure (hierarchical) security-mediated CLE and the first TRE in the standard model.

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A Formal Security Proof

We now define a series of games where each one is an interactive game between a simulator $\mathcal{S}$ and an adversary $\mathcal{A}$, which is either an insider attacker or a curious server, depending on the allowed queries. The skeleton of the proof is based on the proof given in [22].

Game 1 (The Original Game). This game is the one played between a simulator $\mathcal{S}$ and an adversary $\mathcal{A}$ as specified in the experiment $\mathbf{Exp}^{\mathsf{CCA-X}}$. We use the following notations: For the queries, let $\mathcal{T} = \{\overrightarrow{ID}_1, \cdots, \overrightarrow{ID}_{q_E}\}$ denote the trapdoors extraction queries and $\mathcal{W} = \{w_1, \cdots, w_{q_D}\}$ be the set of strings involved in decryption queries where $w_j = H(C_1, C_2, \tau, \overrightarrow{ID}_j, \mathsf{pk}_j)$. For the challenges, let $\overrightarrow{ID}^*$ and pk^* denote the challenge identifier and the challenge public key respectively, and let $C^* = (C_1^*, C_2^*, \tau^*, \sigma^*)$ be the returned challenge ciphertext and let $w^* = H(C_1^*, C_2^*, \tau^*, \overrightarrow{ID}^*, \mathsf{pk}^*)$. The random bit ι is chosen by $\mathcal{S}$ in order to select which message is encrypted.

Game 2 (Change of Public Parameters). Let $Z_i = (g)^{\alpha^i}$, $1 \leq i \leq h+1$. This game is the same as Game 1 except that the generation of the parameters is changed. $\mathcal{S}$ picks $\alpha, \beta \in_R \mathbb{Z}_p$, and set $g_1 = Z_1$, $g_2 = Z_h \cdot g^\beta$.

The simulator also changes the vectors as follows. Let ρ_u and ρ_v be integers such that $\rho_u(n+1) < p$ and $\rho_v(n+1) < p$. The exact choices of ρ_u and ρ_u will be determined later. The simulator randomly selects

- $\kappa_{u_1}, \cdots, \kappa_{u_n}, \kappa_v$ from $\{0, \cdots, \ell(n^{1/\ell} - 1)\}$,
- h many $(\ell+1)$-length vectors $\overrightarrow{x}_1, \cdots, \overrightarrow{x}_h$ from $\mathbb{Z}_{\rho_u}$, where each $\overrightarrow{x}_j = (x'_j, x_{j,1}, \cdots, x_{j,\ell})$.
- h many $(\ell+1)$-length vectors $\overrightarrow{y}_1, \cdots, \overrightarrow{y}_h$ from $\mathbb{Z}_p$, where each $\overrightarrow{y}_j = (y'_j, y_{j,1}, \cdots, y_{j,\ell})$.
- $(x'_v, x_{v,1}, \cdots, x_{v,n})$ from $\mathbb{Z}_{\rho_v}^{n+1}$
- $(y'_v, y_{v,1}, \cdots, y_{v,n})$ from $\mathbb{Z}_p^{n+1}$.

The hash keys for the identity-based key derivation, for $1 \leq j \leq h$, are set as:

$$u'_j = Z_{h-j+1}^{(p+\rho_u\kappa_j - x'_j)} \cdot g^{y'_j}, \quad u_{j,i} = Z_{h-j+1}^{-x'_{j,i}} \cdot g^{y_{j,i}} \text{ for } 1 \leq i \leq \ell.$$

The hash key for the ciphertext validity is set as (note that $g_2 = Z_h \cdot g^\beta$):

$$v' = g_2^{(p+\rho_v\kappa_v - x'_v)} \cdot g^{y'_v}, \qquad v_i = g_2^{-x_{v,i}} g^{y'_{v,i}} \text{ for } 1 \leq i \leq n.$$

Define the following functions

$$J_{u_1}(\mathsf{ID}_1) = p + \rho_u\kappa_1 - x'_1 - \sum_{i=1}^{\ell} x_{1,i}\mathsf{ID}_{1,i},\ K_{u_1}(\mathsf{ID}_1) = y'_1 + \textstyle\sum_{1=1}^{\ell} y_{1,i}\mathsf{ID}_{h,i},$$

$$\vdots \qquad\qquad \vdots$$

$$J_{u_h}(\mathsf{ID}_h) = p + \rho_u\kappa_h - x'_h - \sum_{i=1}^{\ell} x_{h,i}\mathsf{ID}_{h,i},\ K_{u_h}(\mathsf{ID}_h) = y'_h + \textstyle\sum_{1=1}^{\ell} y_{h,i}\mathsf{ID}_{h,i},$$

$$J_v(w) = p + \rho_v\kappa_v - x'_v - \sum_{i=1}^{\ell} x_{v,j}b_j, \qquad K_v(w) = y'_v + \textstyle\sum_{1=1}^{\ell} y_{v,j}b_j,$$

that take as input $\mathsf{ID}_j = (\mathsf{ID}_{j,1}, \cdots, \mathsf{ID}_{j,\ell})$ or $w = b_1 \cdots b_n$. The settings above give

$$F_{\vec{U}_j}(\mathsf{ID}_j) = u'_j \prod_{i=1}^{\ell} u_{j,i}^{\mathsf{ID}_{j,i}} = Z_{h-j+1}^{J_{u_j}(\mathsf{ID}_j)} \cdot g^{K_{u_j}(\mathsf{ID}_j)}, j \in \{1, \cdots, h\}$$

$$F_{\vec{V}}(w) = v' \prod_{j=1}^{n} {v_j}^{b_j} = g_2^{J_v(w)} \cdot g^{K_v(w)}$$

These changes do not change the distribution of the public parameters, so we have $\Pr[S_2] = \Pr[S_1]$.

Game 3 (Elimination of Hash Collisions). The simulator aborts and assumes $\mathcal{A}$ outputs a random bit in this game if $\mathcal{A}$ submits a decryption query $(C, \overrightarrow{ID}, \mathsf{pk} = (g^{\mathsf{sk}}, g_1^{\mathsf{sk}}))$ for a well-formed ciphertext $C = (C_1, C_2, \tau, \sigma)$ where $w = H(C_1, C_2, \tau, \overrightarrow{ID}, \mathsf{pk})$ is either equal to the same value as a previously submitted ciphertext or w^* of the challenge ciphertext.

For such a decryption query to be legal, we have $C \neq C^*$ or $(\overrightarrow{ID}, \mathsf{pk}) \neq (\overrightarrow{ID}^*, \mathsf{pk}^*)$. In either case, this implies a collision for H, which means we can construct an adversary $\mathcal{C}$ against the collision resistance of H such that $|\Pr[S_3] - \Pr[S_2]| \leq Adv_{\mathcal{C}}^{\mathrm{CR}}(k)$.

Game 4 (Preparation for the Simulation of the Challenge Ciphertext). Let $\overrightarrow{ID}^* = (\mathsf{ID}_1^*, \cdots, \mathsf{ID}_k^*)$ where $k \leq h$. This time $\mathcal{S}$ aborts if $J_{u_j}(\mathsf{ID}_j^*) \neq 0 \bmod p$ for any $j \in \{1, \cdots, k\}$ or $J_v(w^*) \neq 0 \bmod p$.

Since the values determining these functions are information theoretically hidden from $\mathcal{A}$, such ID^* and w^* can only be produced by chance. Therefore

$$\begin{aligned} &\Pr[J_v(w^*) = 0 \bmod p] \\ &= \Pr[J_v(w^*) = 0 \bmod p | J_v(w^*) = 0 \bmod \rho_v] \cdot \Pr[J_v(w^*) = 0 \bmod \rho_v] \\ &= \frac{1}{\rho_v(n+1)} \end{aligned}$$

Unless $\mathcal{S}$ aborts, Game 3 and Game 4 are identical and we have $|\Pr[S_4] - \Pr[S_3]| \leq \frac{1}{(\rho_u)^h \rho_v (\ell+1)^{h+1}}$ by a similar computation ($n \geq \ell$). The significance of this extra abort condition will be manifested in Game 7.

Game 5 (Artificial Abort for Consistent View of Adversary). Now $\mathcal{S}$ aborts if $J_{u_1}(\mathsf{ID}'_1) = \cdots = J_{u_k}(\mathsf{ID}'_k) = 0 \bmod \rho_u$ for some $\overrightarrow{ID}' = (\mathsf{ID}'_1, \cdots, \mathsf{ID}'_k) \in \mathcal{T}$ or $J_v(w') = 0 \bmod \rho_v$ for some $w' \in \mathcal{W}$.

Since $\mathcal{A}$'s power is dependent on the extraction and decryption queries, the above abort event is not independent of S_4, and we cannot relate the probability of S_4 and S_5 in a similar way as before.

This problem can be circumvented by the "re-normalization" technique due to Waters [42], such that "artificial aborts" are added to make sure that the probability of aborts is exactly equal to some negligible upper bound for the probability that E occurs for any set of oracle queries.

Conditioning on $\Pr[J_v(w^*) = 0 \bmod p]$ the theoretical lower bound of $\Pr[J_v(w^*) \neq 0 \bmod p]$ is $(1 - \frac{q_D}{\rho_v})$. Setting $\rho_v = 2q_D$ and will make it bounded by $1/2$. On the other hand, a lower bound on the probability for the first event is $\frac{1}{2(4\ell q_E 2^{n/\ell})^h}$ by setting $\rho_u = 4q_E$ [15].

We estimate the probability that $\mathcal{A}$'s oracle queries will cause $\mathcal{S}$ to abort by repeatedly sampling values determining $J_{u_1}(\cdot), \cdots, J_{u_h}(\cdot), J_v(\cdot)$. This would not involve re-running $\mathcal{A}$ as $\mathcal{A}$'s view (of the public parameters) remains unchanged by assuming y's are changing accordingly. Waters [42] has shown that a polynomial number of trials is sufficient to give an estimate of the abort probability η to within a negligible error term.

If $\mathcal{S}$ did not abort, we force an artificial abort with probability $(\eta - 1/(4(4\ell q_E 2^{n/\ell})^h))/\eta$, and $\mathcal{S}$ will abort with probability sufficiently close to $\frac{1}{4(4\ell q_E 2^{n/\ell})^h}$. Now we can say $\Pr[S_5] = \Pr[S_4]/4(4\ell q_E 2^{n/\ell})^h$. An exposition of Waters' technique can be found at [16].

Game 6 (Simulation of Extraction and Decryption). This game changes the simulation of all $\mathcal{A}$'s queries for trapdoor extractions, partial decryptions, and complete decryptions. We will have $\Pr[S_6] = \Pr[S_5]$.

Trapdoor extraction: For trapdoor key extraction query of $\overrightarrow{ID} = (\mathsf{ID}_1, \cdots, \mathsf{ID}_k)$ where $k \leq h$. Let $j' \in \{1, \cdots, k\}$ be a minimum one such that $J_{u_{j'}}(\mathsf{ID}_{j'}) \neq 0$. There exists such a j' or $\mathcal{S}$ has aborted in Game 5. $\mathcal{S}$ needs to return $d_{\overrightarrow{ID}} = (a_1, a_2, \vec{z}_{k+1}, \cdots, \vec{z}_h)$.

We first show how to compute $a_{1|j'}$, a "trapdoor for only $\mathsf{ID}_{j'}$" (without any appearance of any elements from other levels); then we will show how to compute a trapdoor $(a_1, a_2, \vec{z}_{k+1}, \cdots, \vec{z}_h)$ that matches the same implicit random factor used in $a_{1|j'}$. Recall that $F_{\overrightarrow{U}_{j'}}(\mathsf{ID}_{j'}) = Z_{h-j'+1}^{J_{u_{j'}}(\mathsf{ID}_{j'})} \cdot g^{K_{u_{j'}}(\mathsf{ID}_{j'})}$. $\mathcal{S}$ picks $r \in \mathbb{Z}_p^*$ and computes

$$a_{1|j'} = (Z_1^{\beta} \cdot Z_{j'}^{-\frac{K_{u_{j'}}(\mathsf{ID}_{j'})}{J_{u_{j'}}(\mathsf{ID}_{j'})}}) \cdot (Z_{h-j'+1}^{J_{u_{j'}}(\mathsf{ID}_{j'})} \cdot g^{K_{u_{j'}}(\mathsf{ID}_{j'})})^r$$

The second component of $a_{1|j'}$ is only for randomization. We will show the first component of $a_{1|j'}$ is in the form of $g_2^{\alpha}(F_{\overrightarrow{U}_{j'}}(\mathsf{ID}_{j'}))^{-\frac{\alpha^{j'}}{J_{u_{j'}}(\mathsf{ID}_{j'})}}$, which means $a_{1|j'}$ is in the form of $g_2^{\alpha}(F_{\overrightarrow{U}_{j'}}(\mathsf{ID}_{j'}))^{\tilde{r}}$ where $\tilde{r} = r - \frac{\alpha^{j'}}{J_{u_{j'}}(\mathsf{ID}_{j'})}$.

$$\begin{aligned}
& g_2^{\alpha}(F_{\overrightarrow{U}_{j'}}(\mathsf{ID}_{j'}))^{-\frac{\alpha^{j'}}{J_{u_{j'}}(\mathsf{ID}_{j'})}} \\
&= (Z_h \cdot g^{\beta})^{\alpha} (Z_{h-j'+1}^{J_{u_{j'}}(\mathsf{ID}_{j'})} \cdot g^{K_{u_{j'}}(\mathsf{ID}_{j'})})^{-\frac{\alpha^{j'}}{J_{u_{j'}}(\mathsf{ID}_{j'})}} \\
&= Z_{h+1} \cdot Z_1^{\beta} \cdot Z_{h+1}^{-\frac{J_{u_{j'}}(\mathsf{ID}_{j'})}{J_{u_{j'}}(\mathsf{ID}_{j'})}} \cdot Z_{j'}^{-\frac{K_{u_{j'}}(\mathsf{ID}_{j'})}{J_{u_{j'}}(\mathsf{ID}_{j'})}} \\
&= Z_1^{\beta} \cdot Z_{j'}^{-\frac{K_{u_{j'}}(\mathsf{ID}_{j'})}{J_{u_{j'}}(\mathsf{ID}_{j'})}}
\end{aligned}$$

To compute $a_1 = g_2^{\alpha} \cdot (\prod_{j=1}^{k} F_{\overrightarrow{U}_j}(\mathsf{ID}_j))^{\tilde{r}}$, $\mathcal{S}$ needs to compute $F_{\overrightarrow{U}_j}(\mathsf{ID}_j)^{\tilde{r}} = (Z_{h-j+1}^{J_{u_j}(\mathsf{ID}_j)})^{\tilde{r}} \cdot (g^{K_{u_j}(\mathsf{ID}_j)})^{\tilde{r}}$ for $j \neq j'$. We would like to compute it without knowing α and Z_{h+1}, but with the help of $(Z_1, \cdots, Z_h)$. Now the only difficulty comes from the fact that $\alpha^{j'}$ in $\tilde{r}$ is unknown. Note that the second term $(g^{K_{u_j}(\mathsf{ID}_j)})^{\alpha^{j'}}$ can be computed from $Z_{j'}$. We can see how the first term can be obtained by considering two different cases.

1. $j < j'$: $J_{u_j}(\mathsf{ID}_j) = 0$ by the choice of j'.
2. $j > j'$: $Z_{h-j+1}^{\alpha^{j'}} = Z_{h+1-(j-j')}$, note that $1 \leq j - j' \leq h - 1$.

By similar reasoning, since $k+1 > j'$, it is easy to see that $\vec{z}_{k+1}, \cdots, \vec{z}_h$ can also be computed from $(Z_1, \cdots, Z_h)$. This completes the simulation of the trapdoor queries.

SEM partial decryption: $\mathcal{S}$ performs the usual validity checking to reject any invalid ciphertext C that is purported to be encrypted under $\overrightarrow{ID}$ and pk. For decrypting a valid ciphertext with hash w by the trapdoor of $\overrightarrow{ID}$, if $d_{\overrightarrow{ID}} = (a_1, a_2, \cdots)$ is computable by $\mathcal{S}$, it is easy to generate $(a_1 F_{\vec{V}}(w)^t, a_2, g^t)$ for a random $t \in \mathbb{Z}_p^*$.

$\mathcal{S}$ cannot generate the trapdoor for $d_{\overrightarrow{ID}}$ only if $J_{u_1}(\mathsf{ID}_1) = \cdots = J_{u_k}(\mathsf{ID}_k) = 0 \bmod \rho_u$. Note that $J_v(w) \neq 0 \bmod \rho_v$ or $\mathcal{S}$ has aborted in Game 5. Under this condition, $\mathcal{S}$ can generate the token similar to the generation of the trapdoor before. Recall that $F_{\vec{V}}(w) = (Z_h \cdot g^{\beta})^{J_v(w)} \cdot g^{K_v(w)}$, we have

$$\begin{aligned}
& g_2^{\alpha} (F_{\vec{V}}(w))^{-\frac{\alpha}{J_v(w)}} \\
&= (Z_h \cdot g^{\beta})^{\alpha} (Z_h^{J_v(w)} \cdot (g^{\beta})^{J_v(w)} \cdot g^{K_v(w)})^{-\frac{\alpha}{J_v(w)}} \\
&= Z_{h+1} \cdot Z_1^{\beta} \cdot Z_{h+1}^{-\frac{J_v(w)}{J_v(w)}} \cdot Z_1^{-\beta \frac{J_v(w)}{J_v(w)}} \cdot Z_1^{-\frac{K_v(w)}{J_v(w)}} \\
&= Z_1^{-\frac{K_v(w)}{J_v(w)}}
\end{aligned}$$

This means $Z_1^{-\frac{K_v(w)}{J_v(w)}}$ gives a token with the implicit random factor equals to $-\frac{\alpha}{J_v(w)}$. Randomization can be done easily by multiplying the above term by $(F_{\vec{V}}(w))^r$ where $r \in \mathbb{Z}_p^*$. Since $J_{u_1}(\mathsf{ID}_1) = \cdots = J_{u_k}(\mathsf{ID}_k) = 0 \bmod \rho_u$, all $\frac{\alpha}{J_v(w)}$ power terms appear in the construction of the token can be computed from Z_1.

User partial decryption: $\mathcal{A}$ queries $\mathcal{S}$'s oracle $\mathsf{Dec}^U(C, \mathsf{pk}, D)$. $\mathcal{S}$ performs the usual ciphertext validity checking to reject any invalid ciphertext C that is purported to be encrypted under $\overrightarrow{ID}$ and pk, and the token validity checking to reject any invalid token D that is purported to be a partial decryption of C.

For decrypting a valid ciphertext (C_1, C_2, τ, σ) with hash w, we have $\tau = g^s$ and $\sigma = F_{\vec{V}}(w)^s$ for some $s \in \mathbb{Z}_p^*$, i.e. $\sigma = g_2^{s \cdot J_v(w)} \cdot (g^s)^{K_v(w)}$. $\mathcal{S}$ can compute $\hat{e}(Y, g_2)^s$ by $\hat{e}(Y, (\sigma / \tau^{K_v(w)})^{\frac{1}{J_v(w)}})$. Note that the secret key sk that matches pk is never explicitly used.

Complete decryption: After validity checking, $\mathcal{S}$ returns $m = C_1 / \hat{e}(Y, (\sigma / \tau^{K_v(w)})^{\frac{1}{J_v(w)}})$ for valid ciphertext.

Game 7 (Simulation of the Ciphertext / Embedding of the Problem Instance). Depending on whether the adversary is an insider or the server, we have different modes of simulations. Now $\mathcal{S}$ introduces a variable $\gamma \in_R \mathbb{Z}_p^*$ and sets $\tau^* = g^{\gamma}$.

If *mode* = I, g_1 is set to $Z_1 = g^{\alpha}$. $\mathcal{A}$ chooses an identifier $\overrightarrow{ID}^* = (\mathsf{ID}_1^*, \cdots, \mathsf{ID}_k^*)$, a public key $\mathsf{pk}^* = (X^*, Y^*)$ to be challenged with. $\mathcal{S}$ proceeds if $\hat{e}(Y^*, g) = \hat{e}(X^*, g_1)$. Let $T = (g^{\alpha^{h+1}})^{\gamma}$, $\mathcal{S}$ computes C_1^* by

$$\begin{aligned}
& m_{\iota} \cdot \hat{e}(X^*, T) \cdot (Y^*, g^{\gamma})^{\beta} \\
&= m_{\iota} \cdot \hat{e}(X^*, (g^{\alpha^{h+1}})^{\gamma}) \cdot (Y^*, g^{\gamma})^{\beta} \\
&= m_{\iota} \cdot \hat{e}(Y^*, g^{\alpha^h})^{\gamma} \cdot (Y^*, g^{\beta})^{\gamma} \\
&= m_{\iota} \cdot \hat{e}(Y^*, Z_h \cdot g^{\beta})^{\gamma} \\
&= m_{\iota} \cdot \hat{e}(Y^*, g_2)^{\gamma}
\end{aligned}$$

Note that it is the first time in the simulation that β is used directly (i.e. not in the form of g^{β}).

If *mode* = II, $\mathcal{S}$ introduces a variable $\delta \in_R \mathbb{Z}_p^*$. Since $\mathcal{A}$ is a Type-II adversary, it can only choose a public key from $\{\mathsf{pk}_1^*, \cdots \mathsf{pk}_{q_K}^*\}$ given in *aux* to attack. $\mathcal{A}$ chooses an identifier $\overrightarrow{ID}^*$ and a public key $\mathsf{pk}_i^* = (X_i^*, Y_i^*) = ((g^{\delta})^{\theta_i}, (g^{\delta})^{\theta_i \alpha})$ to be challenged with. Note that the choice of $\theta_i \in_R \mathbb{Z}_p^*$ is known to $\mathcal{S}$

since it was $\mathcal{S}$ who prepared it. Let $T = g^{\beta\gamma\delta}$, $\mathcal{S}$ computes C_1^* by

$$\begin{aligned}
& m_\iota \cdot (\hat{e}(g^\delta, g^\gamma)^{\alpha^{h+1}} \cdot \hat{e}(g^\alpha, T))^{\theta_i} \\
&= m_\iota \cdot (\hat{e}(g^\delta, g^{\alpha^{h+1}})^\gamma \cdot \hat{e}(g^\alpha, g^{\beta\gamma\delta}))^{\theta_i} \\
&= m_\iota \cdot \hat{e}(g^{\delta\theta_i\alpha}, g^{\alpha^h})^\gamma \cdot \hat{e}(g^{\delta\theta_i\alpha}, g^\beta)^\gamma \\
&= m_\iota \cdot \hat{e}(Y^*, Z_h \cdot g^\beta)^\gamma \\
&= m_\iota \cdot \hat{e}(Y^*, g_2)^\gamma
\end{aligned}$$

Note that it is the first time in the simulation that α is used directly (i.e. not in the form of $g^\alpha, \cdots, g^{\alpha^h}$).

In both modes, $\mathcal{S}$ sets $C_2^* = \prod_{j=1}^k (g^\gamma)^{K_{u_j}(\mathsf{ID}_j^*)}$, $\sigma^* = (g^\gamma)^{K_v(w^*)}$ where $w^* = H(C_1^*, C_2^*, \tau^*, \overrightarrow{ID}^*, \mathsf{pk}^*)$ for the rest of the challenge, which is a perfect simulation if $\mathcal{S}$ did not abort in Game 4. We have $\Pr[S_7] = \Pr[S_6]$.

Game 8 (The Indistinguishability Cards). If $mode = \text{I}$, $\mathcal{S}$ forgets (α, γ). If $mode = \text{II}$, $\mathcal{S}$ forgets (β, γ, δ). Note that $\mathcal{S}$ can simulate the game in both modes as long as $(g^\alpha, \cdots, g^{\alpha^h}, g^\gamma)$ are known for $mode = \text{I}$ or $(g^\beta, g^\gamma, g^\delta)$ are known for $mode = \text{II}$, except computing the term T. Now $\mathcal{S}$ just picks a $T \in_R \mathbb{G}$. The transition from Game 7 to Game 8 is based on the intractability of either h-wBDHI' or 3-DDH. Both games are equal unless there exists a PPT algorithm $\mathcal{D}$ that distinguishes T from random. Therefore, we have $|\Pr[S_8] - \Pr[S_7]| \leq Adv_{\mathcal{D}}^{\text{X}}(k)$ where X is either h-wBDHI' or 3-DDH. Finally, C_1^* perfectly hides m_ι from $\mathcal{A}$, we have $\Pr[S_8] = 1/2$.