Attacking Reduced Round SHA-256

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14th January, 2008

Abstract. The SHA-256 hash function has started getting attention recently by the cryptanalysis community due to the various weaknesses found in its predecessors such as MD4, MD5, SHA-0 and SHA-1. At FSE '06, Mendel et al. presented a message pair which they claimed is a colliding message pair for 18-step SHA-256 with standard IV and differential path for 19-step near collision for SHA-256. We note that the message pair presented by Mendel et al. is not a valid colliding pair for 18 step SHA-256 with the standard IV. We make two contributions in this work. First we describe message modification techniques and use them to obtain an algorithm to generate message pairs which collide for the actual SHA-256 reduced to 18 steps. Our second contribution is to present differential paths for 19, 20, 21, 22 and 23 steps of SHA-256. We construct parity check equations in a novel way to find these characteristics. Further, the 19-step differential path presented here is constructed by using only 15 local collisions, as against the previously known 19-step near collision differential path which consists of interleaving of 23 local collisions. Our 19-step differential path can also be seen as a single local collision at the message word level.

1 Introduction

Cryptanalysis of hash functions has been an area of intense interest to the research community since past decade and a half. Many hash functions were broken in this time, most notable among them are MD4, MD5, SHA-0 and theoretical break of SHA-1. This has directed the attention of the cryptology community to the SHA-2 family of hash functions.

Known Results for the SHA-2 Family: Gilbert and Handschuh (GH) [7] were the first to study local collisions in the SHA-2 family. They reported a 9-round local collision and estimated the probability of the differential path to be 2^{-66}. This probability estimate was later improved by [12] and [8]. Sanadhya and Sarkar [17] recently presented 16 new 9-round local collisions for SHA-2 family of hash functions. The message expansion of SHA-256 was studied by Mendel et al [12], who reported (i) a message pair which they claimed is a colliding pair for 18-step SHA-256 with standard IV, (ii) a differential path for 1-bit near collision for 19-step SHA-256 and (iii) a message pair which gives rise to a pseudo collision for 22-step SHA-256. We note that the message pairs provided for (i) and (iii) are not valid. One of the authors of [12], Christian Rechberger, has in [15] provided us a modified IV for (iii). We note that no correction for (i) has been provided. See Section D. Further, only the differential path (and not the message pair) is provided for the 19-step near collision. An earlier work [11] studied a very simplified variant of SHA-256. The encryption mode of SHA-256 is analyzed in [23] and is not relevant to collision search attacks.

Our Contributions: We make two independent contributions in this work :

1. We construct a 18-step collision characteristic using one of the local collisions from [17]. We describe message modification techniques to find messages following this differential characteristic. Using these techniques, we provide an algorithm to generate pairs of messages which collide for 18 step SHA-256 with the standard IV. We show two such pairs of messages.
2. We show multiple differential paths for attacking up to 23-step SHA-256. In obtaining these differential paths, we use coding theoretic methods in a novel way. There were no colliding differential paths known for SHA-256 beyond 18 rounds. Previously known best differential path was for 19-step SHA-256 which used 23 local collisions and gave rise to a near collision. In contrast, our 19-step characteristic uses only 15 local collisions and is an exact collision path. All the 15 local collisions start in the same word and therefore this differential path can also be seen as consisting of a single local collision with the starting word difference having a weight of 15

bits. In addition there are no impossible conditions caused by the f_{IF} and f_{MAJ} functions for the differential paths reported here. Therefore the search for actual colliding message pairs following these paths is likely to be easier.

We also show that neutral bit technique may not be of much help in finding actual colliding pair of messages while message modification methods seem to hold much more promise.

Note : We have recently noted that a work titled "Collisions for step-reduced SHA-256" has been accepted at FSE 2008. We have contacted one of the authors of this work but have not received a response till the date (14th Jan, 2008) of submission of this paper. Consequently, we are not aware of the results or the methodology used there. For the 18-step collision in the present work, we use a local collision presented in [17] and for longer round differential paths, we use two different local collisions. In most of the previous works on SHA-256, the local collision by Gilbert and Handschuh [7] has been used. If the FSE 2008 paper also uses the GH local collision and reports 18-round collisions, then our first contribution reporting 18-round collisions is of independent interest. Regarding our second contribution, we expect our technique of forming parity check equations and results on longer round differential paths to be new.

2 Notation

In this paper we use the following notation:

- $m_i \in \{0,1\}^{32}$, $W_i \in \{0,1\}^{32}$, $W'_i \in \{0,1\}^{32}$ for any i.
- The colliding message pair is: $\{m_0, m_1, m_2, \ldots m_{15}\}$ and $\{m'_0, m'_1, m'_2, \ldots m'_{15}\}$.
- The expanded message pair is: $\{W_0, W_1, W_2, \ldots W_{63}\}$ and $\{W'_0, W'_1, W'_2, \ldots W'_{63}\}$.
- $\oplus$: bitwise XOR.
- $+$: addition modulo 2^{32}.
- $\Delta W_i = W_i \oplus W'_i$
- $\mathrm{ROTR}^n(x)$: Right rotation of a 32 bit quantity x by n bits.
- $\mathrm{SHR}^n(x)$: Right shift of a 32 bit quantity x by n bits.

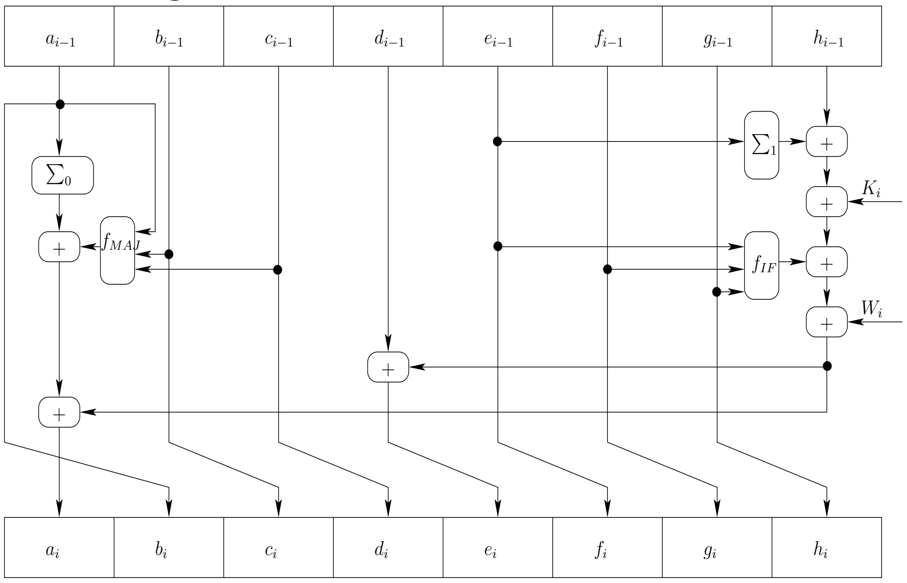
3 The SHA-256 Hash Function

The newest members of SHA family of hash functions were standardized by US NIST in 2002 [18]. There are 2 differently designed functions in this standard: the SHA-256 and SHA-512. In addition, the standard also specifies 2 truncated versions of these two functions: the SHA-224 and SHA-384. The number in the name of the hash function refers to the length of message digest produced by that function. In this work we are interested in reduced round collision attacks against SHA-256. Since SHA-224 is obtained by truncating last 32 bits from the output of SHA-256, all the reduced round attack against SHA-256 are also valid for SHA-224. Next we describe SHA-256 in detail.

The round function of SHA-256 hash function is shown in Figure 1. Eight registers are used in the evaluation of SHA-256. The initial value in the registers is specified by an 8x32 bit IV. In Step i, the 8 registers are updated from $(a_{i-1}, b_{i-1}, c_{i-1}, d_{i-1}, e_{i-1}, f_{i-1}, g_{i-1}, h_{i-1})$ to $(a_i, b_i, c_i, d_i, e_i, f_i, g_i, h_i)$ according to the following equations:

$$\left.\begin{aligned}
a_i &= \Sigma_0(a_{i-1}) + f_{MAJ}(a_{i-1}, b_{i-1}, c_{i-1}) + \Sigma_1(e_{i-1}) \\
&\quad + f_{IF}(e_{i-1}, f_{i-1}, g_{i-1}) + h_{i-1} + K_i + W_i \\
b_i &= a_{i-1} \\
c_i &= b_{i-1} \\
d_i &= c_{i-1} \\
e_i &= d_{i-1} + \Sigma_1(e_{i-1}) + f_{IF}(e_{i-1}, f_{i-1}, g_{i-1}) \\
&\quad + h_{i-1} + K_i + W_i \\
f_i &= e_{i-1} \\
g_i &= f_{i-1} \\
h_i &= g_{i-1}
\end{aligned}\right\} \tag{1}$$

Fig. 1. Round function of SHA-256 hash function



The f_{IF} and the f_{MAJ} are three variable boolean functions defined as:

$$\begin{aligned} f_{IF}(x,y,z) &= (x \wedge y) \oplus (\neg x \wedge z) \\ f_{MAJ}(x,y,z) &= (x \wedge y) \oplus (y \wedge z) \oplus (z \wedge x) \end{aligned}$$

The functions Σ_0 and Σ_1 are defined as:

$$\begin{aligned} \Sigma_0(x) &= ROTR^2(x) \oplus ROTR^{13}(x) \oplus ROTR^{22}(x) \\ \Sigma_1(x) &= ROTR^6(x) \oplus ROTR^{11}(x) \oplus ROTR^{25}(x) \end{aligned}$$

Round i uses a 32 bit word W_i which is derived from the message and a constant word K_i. There are 64 rounds in all. The hash function operates on a 512 bit message specified as 16 words of 32 bits. Given the message words m_0, m_1, $\ldots m_{15}$, the W_i 's are computed using the equation:

$$W_i = \begin{cases} m_i & \text{for } 0 \le i \le 15 \\ \sigma_1(m_{i-2}) + m_{i-7} + \sigma_0(m_{i-15}) + m_{i-16} & \text{for } 16 \le i \le 63 \end{cases} \tag{2}$$

The functions σ_0 and σ_1 are defined as:

$$\begin{aligned} \sigma_0(x) &= ROTR^7(x) \oplus ROTR^{18}(x) \oplus SHR^3(x) \\ \sigma_1(x) &= ROTR^{17}(x) \oplus ROTR^{19}(x) \oplus SHR^{10}(x) \end{aligned}$$

The IV $= (a_{-1}, b_{-1}, c_{-1}, d_{-1}, e_{-1}, f_{-1}, g_{-1}, h_{-1})$ is defined as (0x6a09e667, 0xbb67ae85, 0x3c6ef372, 0xa54ff53a, 0x510e527f, 0x9b05688c, 0x1f83d9ab, 0x5be0cd19). All additions in Equations 1 and 2 are modulo 2^{32}.

The output hash value of a one block (512 bit) message is obtained by chaining the IV with the register values at the end of the final round as per the Merkle-Damgård construction [6,13]. A similar strategy is used for multi-block messages, where the IV for next block is taken as the hash output of the previous block.

4 Collision Attacks Against Hash Functions

The aim of a hash function attack is to produce two different messages both of which map to the same hash output. This is done by employing differential attack against the hash function in question. First a suitable difference of messages is found such that a pair of messages having that difference is likely to collide to the same hash value with high probability. For example, if a given message differential $\{\Delta W_0, \Delta W_1, \ldots \Delta W_{15}\}$ is likely to generate colliding pairs with probability $\frac{1}{2^8}$ then one needs to try roughly 2^8 different pairs $\{W_0, W_1, \ldots, W_{15}\}$ and $\{W'_0, W'_1, \ldots W'_{15}\}$ having the given difference to get a colliding pair of messages.

However, the probability of the specified differential to generate a collision is likely to be very low for most of the practical hash functions. Hence some sophisticated methods are used to search for the right (colliding) pair, rather than generating them at random. Message modification techniques [22,20] and neutral bit technique [1] are the two widely used methods to find colliding message pairs.

We next describe some terms and techniques used in the differential attack on hash functions.

4.1 Local Collision

To analyze a hash function, it is advantageous to consider its linearized version. That is, all the non-linear components in the hash function design are first replaced by their linear approximations. Since the resulting design is linear, we have $h(x) \oplus h(x \oplus \Delta x) = h(\Delta x)$ where h is one step of the hash function evaluation. Therefore we can look at the behavior of the differential of the messages Δx on the linearized hash function rather than considering a pair of messages x and $x \oplus \Delta x$ operated on by a step of linearized hash function, and then taking their differential.

The simplified version of the hash function is then analyzed for a small number of steps. The technique is to introduce a small perturbation in the message at the first step and then correct it by suitable differences in the messages in next few steps. After introducing some message differences for few rounds it is possible to completely cancel the effect of the original perturbation. A sequence of message word differences which cancel their own effect locally are said to construct a "*local collision*". All the message words are considered to be unrestricted during the search for local collisions. Finally many local collisions are interleaved to obtain (reduced round) colliding pair of messages.

Attacks on the SHA-0 and SHA-1 hash functions use the 5-step local collision obtained by Chabaud and Joux [4]. For SHA-256, a 9-step local collision by Gilbert and Handschuh [7] and sixteen 9-step local collisions by Sanadhya and Sarkar [17] are known. We discuss the case of SHA-256 local collisions in more detail.

4.2 Local Collisions in SHA-256

Let the first step in SHA-2 be denoted by Step 0. If a 9-step local collision is started at step i, it defines the 9 word differences $W_j \oplus W'_j$ for $i \leq j \leq i+8$. We use two types of local collisions in the present work. The first is due to Gilbert and Handschuh [7] and the second is one of the 16 local collisions presented in [17]. From among the 16, we choose the 5^{th} local collision because of the following two reasons :

1. It is one of the 4 which are suitable for getting 18-step collision, as explained later (the others being 7^{th}, 14^{th} and 16^{th}).
2. It has the highest probability among these 4.

We call the two local collisions the GH local collision and the SS_5 local collision respectively. The other three local collisions from [17] are denoted by SS_7, SS_{14} and SS_{16}.

The following approximations are used in these local collisions :

1. Operator $+$ is approximated by $\oplus$.
2. In GH, f_{IF} and f_{MAJ} are approximated by zero function. This causes certain impossible conditions while searching for the message pair following this differential path, as has been observed in [12].
3. In SS_5, f_{IF} and f_{MAJ} are approximated by their middle arguments. These linear approximations avoid two types of impossible conditions encountered when using GH local collision.

See [17] for details on other local collisions.

All the local collisions mentioned above are:

- GH : $\{x, \Sigma_0(x) \oplus \Sigma_1(x), \Sigma_0(\Sigma_1(x)), 0, x, \Sigma_0(x) \oplus \Sigma_1(x), 0, 0, x\}$
- SS_5 : $\{x, \Sigma_0(x) \oplus \Sigma_1(x), \Sigma_0(\Sigma_1(x)), \Sigma_0(x) \oplus \Sigma_1(x), 0, \Sigma_0(x) \oplus \Sigma_1(x), 0, 0, x\}$
- SS_7 : $\{x, x \oplus \Sigma_0(x) \oplus \Sigma_1(x), \Sigma_0(x) \oplus \Sigma_0(\Sigma_1(x)), x \oplus \Sigma_0(x) \oplus \Sigma_1(x), 0, x \oplus \Sigma_0(x) \oplus \Sigma_1(x), 0, 0, x\}$
- SS_{14} : $\{x, x \oplus \Sigma_0(x) \oplus \Sigma_1(x), x \oplus \Sigma_1(x) \oplus \Sigma_0(\Sigma_1(x)), \Sigma_1(x), \Sigma_0(x) \oplus \Sigma_1(x), \Sigma_0(x) \oplus \Sigma_1(x), 0, 0, x\}$
- SS_{16} : $\{x, \Sigma_0(x) \oplus \Sigma_1(x), \Sigma_0(x) \oplus \Sigma_1(x) \oplus \Sigma_0(\Sigma_1(x)), x \oplus \Sigma_1(x), x \oplus \Sigma_0(x) \oplus \Sigma_1(x), x \oplus \Sigma_0(x) \oplus \Sigma_1(x), 0, 0, x\}$

Note that in all the above local collisions, Σ_0 and Σ_1 are used as operators on 32 bit quantities, and x is any 32 bit message word difference. Once a starting message difference x is chosen, next 8 words must have the difference in accordance with the local collision.

4.3 Differential Path

The introduction of the message differentials causes the internal registers of hash function to differ too. The propagation of the differences in the registers creates a "*differential path*". For SHA-256, the differential path for k steps can be completely specified by differences in the 8 registers for these k steps along with the k message differences corresponding to the same steps. The term "*linear characteristic*" is also widely used in literature to refer to the differential path. We also use it interchangeably with differential path.

If the differential path includes steps in which the message expansion is involved, then the message words for some steps are computed on the basis of previous message words. In such a case, the message expansion is also linearized. If the register differences are all zero at the last step of the differential path then such a path is said to be a "*colliding differential path*". The differential of the messages for a colliding differential path are called as "*colliding message differential*".

The differential path holds for the linearized version of the hash function with probability 1, but with much less probability for the actual hash function.

5 Attacking 18 rounds of SHA-256

It is possible to get up to 18 step reduced round collisions for SHA-256 using a single local collision. Such an idea has already been used in [12] and mentioned in [17]. We describe this for clarity of exposition.

First of all, note that any local collision under consideration spans 9 steps and the message expansion of SHA-256 does not play any role in the first 16 steps. Therefore if a local collision spans from Step i to Step $(i+8)$, and if we take $\mathit{\Delta} W_0 = \mathit{\Delta} W_1 = \ldots = \mathit{\Delta} W_{i-1} = \mathit{\Delta} W_{i+9} = \mathit{\Delta} W_{i+10} = \ldots = \mathit{\Delta} W_{15} = 0$, we get a differential path for 16-step collision for SHA-256.

The issue of message expansion is not considered in obtaining the 16 step colliding differential path described above. Next we tackle two steps of the message expansion.

Message expansion rule for W_{16} and W_{17} are given by :

$$W_{16} = \sigma_1(W_{14}) + W_9 \;\; + \sigma_0(W_1) + W_0 \tag{3}$$
$$W_{17} = \sigma_1(W_{15}) + W_{10} + \sigma_0(W_2) + W_1 \tag{4}$$

Let a local collision $\mathcal{L}$ start at Step 3 and hence end at Step 11. This local collision defines the 9 word differences $\mathit{\Delta} W_3, \mathit{\Delta} W_4, \ldots \mathit{\Delta} W_{11}$. The first step of the local collision corresponds to $\mathit{\Delta} W_3$ and the 9^{th} step corresponds to $\mathit{\Delta} W_{11}$. Taking the differentials of all the message words outside the span of the local collision to be zero, the differential path for $\mathcal{L}$ will have $\mathit{\Delta} W_0 = \mathit{\Delta} W_1 = \mathit{\Delta} W_2 = \mathit{\Delta} W_{12} = \mathit{\Delta} W_{13} = \mathit{\Delta} W_{14} = \mathit{\Delta} W_{15} = 0$.

Note that $\mathit{\Delta} W_i = 0$ means that $W_i = W'_i$. Since $\mathit{\Delta} W_0 = \mathit{\Delta} W_1 = \mathit{\Delta} W_{14} = 0$ for $\mathcal{L}$, from Equation 3, W_{16} and W'_{16} may be different only due to the differences in W_9 and W'_9.

$\mathit{\Delta} W_9$ corresponds to the 7^{th} step word difference for $\mathcal{L}$. If $\mathcal{L}$ is chosen such that it's 7^{th} step word difference is zero, then $W_9 = W'_9$. Therefore even after the message expansion recursion is used, we will have $W_{16} = W'_{16}$. This results in a 17-step differential path for SHA-256.

Similarly, if the 8^{th} message word difference for $\mathcal{L}$ is zero, then by Equation 4, $W_{17} = W'_{17}$. This results in a 18-step differential path for SHA-256.

Both the 17 and the 18 Step paths discussed above use just one local collision. To increase the probability of this differential path for the case of real SHA-256, we can take starting messages differing in only 1 bit.

All the local collisions listed in the previous section have the 7^{th} and the 8^{th} message word differences zero. Therefore any one of them can be used to obtain the 18 step colliding differential path for SHA-256. We list one of these differential paths in Table 1. This 18-step colliding path is also a 17-step colliding path for SHA-256.

Further, it can be seen that it is not possible to obtain a differential path for 19 or more steps with a single local collision where the weight of the perturbation in first word is just 1-bit. This impossibility arises due to the message expansion of SHA-256, and because there are no local collisions in which 3 consecutive word differences are zero. We discuss the case of more than 18 steps in later sections.

Table 1. 18 step linear characteristic for SHA-256. Only 1 SS$_5$ local collision is used to build this path.

Step i	ΔW_i	Δa_i	Δb_i	Δc_i	Δd_i	Δe_i	Δf_i	Δg_i	Δh_i
0-2	0	0	0	0	0	0	0	0	0
3	0x80000000	0x80000000	0	0	0	0x80000000	0	0	0
4	0x22140240	0	0x80000000	0	0	0x20040200	0x80000000	0	0
5	0x42851098	0	0	0x80000000	0	0x80000000	0x20040200	0x80000000	0
6	0x22140240	0	0	0	0x80000000	0	0x80000000	0x20040200	0x80000000
7	0	0	0	0	0	0x80000000	0	0x80000000	0x20040200
8	0x22140240	0	0	0	0	0	0x80000000	0	0x80000000
9	0	0	0	0	0	0	0	0x80000000	0
10	0	0	0	0	0	0	0	0	0x80000000
11	0x80000000	0	0	0	0	0	0	0	0
12-17	0	0	0	0	0	0	0	0	0

6 Message Modification Techniques for SHA-256

We have used XOR differences for registers and message words in the differential path for reduced round SHA-256. The differential path in Table 1 is obtained by using linearized SHA-256. However our aim is to obtain a pair of messages which follows this differential path for real SHA-256. The probability for this to happen for random messages is 2^{-49} for 18-step SHA-256. If the message-pair satisfies certain conditions then the probability of the differential path can be increased significantly. We list conditions on the registers and the message words which help in finding messages following the 18 step differential path shown in Table 1 when actual SHA-256 is used. These conditions try to ensure that the functions f_{IF} and f_{MAJ} both behave like their middle arguments, and that $+$ behaves like $\oplus$. These conditions are shown in Table 2. Sufficient conditions for 9 step SHA-256 collision have also been given in [9], Table 3. We next highlight the advantages of our conditions with those in [9].

1. The conditions in [9] are for only 9-step collision in SHA-256. Our conditions are for 18-step collision in SHA-256.
2. The GH local collision is used in [9] whereas we use SS$_5$ local collision. Further, no explanation is provided in [9] on how these conditions are derived whereas we provide complete details about our conditions. It is now possible to use the method described in this work to derive conditions for 18-step SHA-256 collision using any other local collision.
3. In [9] the conditions are claimed to be "sufficient" but it is not clear if satisfying them will immediately lead to a collision. The conditions that we identify are not claimed to be sufficient. We only note that satisfying them will increase the probability of finding colliding message pairs.

6.1 Explanation of Conditions in Table 2

$\Delta W_k = 0$ for steps k=0, 1 and 2 and hence there are no restrictions due to these steps. In Step 3, although $\Delta W_3 \neq 0$, the difference is only in the most significant bit. The $+$ and $\oplus$ behave the same with probability 1 for a difference in MSB, so even Step 3 does not impose any restrictions. Hence conditions are needed to tackle the proper differential behavior for the message pair only from Step 4 onwards.

Conditions Due to f_{MAJ} and f_{IF} : In Step 4, f_{MAJ} has inputs a_3, b_3 and c_3 with $\Delta a_3 =$ 0x80000000. In SS$_5$ local collision f_{MAJ} is approximated by it's middle argument, which will happen if $b_3^{31} = c_3^{31}$. Similarly the f_{IF} function having arguments e_3, f_3 and g_3 will behave like it's middle argument if $f_3^{31} = g_3^{31}$.

Conditions Due to Register a_4 : Once the two boolean functions are approximated by their middle arguments, register a_4 is evaluated for both the messages as follows :

$$a_4 = \Sigma_0(a_3) + b_3 + \Sigma_1(e_3) + f_3 + h_3 + K_4 + W_4 \text{ and}$$
$$a'_4 = \Sigma_0(a'_3) + b'_3 + \Sigma_1(e'_3) + f'_3 + h'_3 + K_4 + W'_4$$

Registers a_3 and a'_3 (resp. e_3 and e'_3) differ in their MSB, and the operator Σ_0 (resp. Σ_1) expands this difference to 3 bit positions 6, 20 and 25 (resp. 9, 18 and 29). The word difference ΔW_4 at this step has been chosen to differ in these 6 bit positions (namely 6, 20, 25, 9, 18 and 29) with the aim of cancelling these differences.

The cancellation will happen as desired if :

1. The difference of words W_4 and W'_4 is opposite to the difference in words $\Sigma_0(a_3)$ and $\Sigma_0(a'_3)$ on bit positions 9, 18 and 29. For example, if $(\Sigma_0(a_3))^i = 1$ and $(\Sigma_0(a'_3))^i = 0$, then we would like $(W_4)^i = 0$ and $(W'_4)^i = 1$ so that $W_4 + \Sigma_0(a_3)$ and $W'_4 + \Sigma_0(a'_3)$ are equal at the i^{th} bit position; $i = 9$, 18 and 29.
2. Similarly, $(W_4)^i$ and $(W'_4)^i$ have difference opposite to the difference in $(\Sigma_1(e_3))^i$ and $(\Sigma_1(e'_3))^i$ at bit positions $i = 6$, 20 and 25.

All the 6 bit differences will be cancelled if the conditions shown in Table 2, Step 4, column a_k are met. Note that this is not a necessary way of cancelling the differences, other possibilities exist when the sum of the terms in a_4 and a'_4 may behave as desired. In particular, we do not use bit carries in addition modulo 2^{32} to cancel these type of differences like Wang et. al do for SHA-1 [21]. We use XOR differences only, unlike [21] where modular differences are used.

Conditions due to register e_4 : Having cancelled the 6 bit differences to obtain $\Delta(a_4) = 0$, it can be seen that 3 bits from $\Delta(W_4)$ will certainly propagate into $\Delta(e_4)$ because there is no Σ_0 term in calculating e_4 and e'_4. If the differential path is to be followed, then these 3 differing bits in W_4 and W'_4 should not carry forward to other positions. Carry propagation to other bits will cause problems in adjusting the register differences in next steps since any single bit difference in a or e register is expanded into 3 bit differences by the operators Σ_0 and Σ_1. We have chosen the word differences in next steps considering these positions by following the linear (XOR) characteristics. It is possible to allow some bit carries here but it seems that it will only reduce the probability of the differential path.

To complete the analysis of step 4, we finally look at the difference $\Delta(e_4)$. The registers e_4 and e'_4 are computed as follows:

$$\begin{aligned} & e_4 = d_3 + \Sigma_1(e_3) + f_{IF}(e_3, f_3, g_3) + h_3 + K_4 + W_4, \\ \text{and} \quad & e'_4 = d'_3 + \Sigma_1(e'_3) + f_{IF}(e'_3, f'_3, g'_3) + h'_3 + K_4 + W'_4. \end{aligned}$$

In these two computations, bits 6, 20 and 25 corresponding to Σ_1 rotations of the differing bit 31 in e_3 have already been taken care of while considering a_4. Bit numbers 9, 18 and 29 are the places where W_4 and W'_4 differ and these differences are required to be propagated to Δe_4. Since $d_3 = d'_3$, $h_3 = h'_3$ and $f_{IF}(e_3, f_3, g_3) = f_{IF}(e'_3, f'_3, g'_3)$;

$$\begin{aligned} \text{if we write} \quad & \mathit{rest} = \Sigma_1(e_3) + f_{IF}(e_3, f_3, g_3) + h_4 + K_4, \\ \text{then} \quad & e_4 = \mathit{rest} + W_4, \\ \text{and} \quad & e'_4 = \mathit{rest} + W'_4. \end{aligned}$$

If the i^{th} bit of rest is 0 and there is no carry into the i^{th} bit while addition with W_4 takes place, then the XOR difference $W_4 \oplus W'_4$ will propagate into $e_4 \oplus e'_4$ as desired. Alternately, if the i^{th} bit of rest is 1 and there is a carry into the i^{th} bit while addition with W_4 takes place, then too the XOR difference $W_4 \oplus W'_4$ will propagate into $e_4 \oplus e'_4$.

Thus either we would like no carry propagation in e_4 and e'_4 at bits 6, 20 and 25 if rest is 0 at these bit positions or we would like carry propagation in both these registers if rest is 1 at these bits. We do not have a deterministic way to ensure this since we do not have complete freedom to choose the registers and the message words as desired at this stage. However, the probability of the carries to happen as desired can be increased if we we set other free bits of W_4 and W'_4 according to the following conditions :

1. if rest^9 is 0 then $W_4^7 = W_4^8 = 0$.
2. if rest^9 is 1 then $W_4^7 = W_4^8 = 1$.
3. if rest^{18} is 0 then $W_4^{10} = W_4^{11} = \ldots = W_4^{17} = 0$.
4. if rest^{18} is 1 then $W_4^{10} = W_4^{11} = \ldots = W_4^{17} = 1$.
5. if rest^{29} is 0 then $W_4^{26} = W_4^{27} = W_4^{28} = 0$.
6. if rest^{29} is 1 then $W_4^{26} = W_4^{27} = W_4^{28} = 1$.

In setting these conditions, we have used the bits between 6, 9, 20 and 9, 18 and 29 which are not restricted.

Similarly we have set conditions for other steps so that the messages follow the differential path as desired.

6.2 Method to Satisfy Conditions in Table 2

First 4 words in the differential path are free and hence we choose them randomly. Thereafter, many conditions in Table 2 are easy to fulfill as they depend only on word W_k in step k. Some of the conditions on registers can be tackled by suitably choosing the word W_k at that step which we can choose as desired. However, there may be instances when a previously selected message word causes impossible condition at a later step. As an example, we may not get the bit carry conditions for register e_4 as described previously. Also we wish to have e_6^{31} following a particular pattern at step 8 whereas this bit has been set at step 6 itself. In such contradicting cases, we choose another message word randomly at the previous step where the condition was breaking down. Then we apply message modification techniques from that step onwards and continue the search process for further steps. We search incrementally proceeding further only when all the conditions at a step are fulfilled and the differential path is as desired. The differential path in Table 1 holds with probability 2^{-49}, but with the procedure described above, we are able to get a much higher probability. In fact, Steps 0 to 7 become very easy to fulfill with the message modification and we are able to satisfy all the conditions till Step 7 in about a minute on an ordinary PC. The only difficult conditions are those imposed due to a_8. We could find a colliding message pair following exact differential characteristic in time varying from about 40 minutes to a couple of hours on an ordinary PC. Repeatedly running the program we could generate many such pairs. We show two such colliding pairs of messages.

Table 2. Conditions for the 18 step differential path in Table 1. x^i denotes i^{th} bit of a 32 bit quantity x. $\overline{x}$ denotes the bitwise negation of x which can be a 32 bit or a 1 bit quantity. Operator + is addition modulo 2^{32} and operator * is multiplication of 2 single bits. Both these operators are used in steps 6 and 8.

Step k	Due to f_{MAJ}	Pr.	Due to f_{IF}	Pr.	Due to a_k	Pr.	Due to e_k	Pr.	Step Pr.
0-3	-	-	-	-	-	-	-	-	1
4	$b_3^{31} = c_3^{31}$	$\frac{1}{2}$	$f_3^{31} = g_3^{31}$	$\frac{1}{2}$	$\overline{W_4^i} = (\Sigma_0(a_3))^i;\ i = 9, 18, 29$ $\overline{W_4^i} = (\Sigma_1(e_3))^i;\ i = 6, 20, 25$	$\frac{1}{2^6}$	bit differences $\Delta W_4^i; i = 9, 18, 29$ propagate into e_4	$\frac{1}{2^3}$	$\frac{1}{2^{11}}$
5	$a_4^{31} = \overline{c_4^{31}}$	$\frac{1}{2}$	$e_4^{31} = 1$, $e_3^i = f_3^i;\ i = 9, 18, 29$	$\frac{1}{2^4}$	$\overline{W_5^i} = (\Sigma_1(e_4))^i$; $i = 3, 4, 7, 12, 16, 18, 23, 25, 30$	$\frac{1}{2^9}$	-	-	$\frac{1}{2^{14}}$
6	$a_5^{31} = b_5^{31}$	$\frac{1}{2}$	$e_5^{31} = 1;\ i = 9, 18, 29$ $e_4^{31} = \overline{e_5^{31}} * e_3^{31}$	$\frac{1}{2^4}$	$\overline{W_5^i} = (\Sigma_1(e_5) + f_5)^i$; $i = 6, 9, 18, 20, 25, 29$	$\frac{1}{2^6}$	-	-	$\frac{1}{2^{11}}$
7	-	-	$e_6^i = 1$; $i = 9, 18, 29, 31$	$\frac{1}{2^4}$	-	-	-	-	$\frac{1}{2^4}$
8	-	-	$e_6^{31} = \overline{e_7^{31}} * e_5^{31}$	$\frac{1}{2}$	$\overline{W_8^i} = (\Sigma_1(e_7) + h_7)^i$; $i = 6, 9, 18, 20, 25, 29$	$\frac{1}{2^6}$	-	-	$\frac{1}{2^7}$
9	-	-	$e_8^{31} = 1$	$\frac{1}{2}$	-	-	-	-	$\frac{1}{2}$
10	-	-	$e_9^{31} = 1$	$\frac{1}{2}$	-	-	-	-	$\frac{1}{2}$
11-17	-	-	-	-	-	-	-	-	1
								Prob.	$\frac{1}{2^{49}}$

6.3 Colliding Message Pairs for 18-Step SHA-256

Tables 3 and 4 show the message pairs found using the techniques described previously. All 18 words of the messages are given in the tables. First 16 words can be used to compute the last two words using the message expansion of SHA-256. Similar method can be used for finding 9-round pseudo collisions for SHA-256 as well. Since we can already find message pairs colliding for 18-step SHA-256 with the standard IV, the only utility for such an exercise would be to see how easy it becomes to find these pseudo collisions due to the benefits of relaxing the IV conditions. However, we found that the time required to find a 9-round pseudo collision is only marginally less than the time required to find an 18-step collision. We give an example of such a pseudo collision in Section A.

It seems possible to use neutral bits to increase the efficiency of the search for finding message pairs following the given differential path. We experimented with this idea and found that the gains are not significant. More details about our experiments with neutral bits are available in Section B.

Table 3. Colliding message pair for 18 step SHA-256 with standard IV. All 18 words of the two messages are given here. Last 2 words can be obtained by message recursion involving the first 16 words. These two messages follow the differential path given in Table 1.

M_1	0-8	0xccea5c17	0x53ad1a2d	0x141db23c	0xb6acfaa8	0x5ee7fe4d	0x53c5b764	0x2bf20d44	0x87d63bf6	0x63a07869
	9-17	0xf305fdea	0x26ee271f	0xb973b91c	0xd0f87828	0xb724a487	0xa295fa2a	0x0a67c97a	0x18c2a801	0xae50a6fd
M_2	0-8	0xccea5c17	0x53ad1a2d	0x141db23c	0x36acfaa8	0x7cf3fc0d	0x1140a7fc	0x09e60f04	0x87d63bf6	0x41b47a29
	9-17	0xf305fdea	0x26ee271f	0x3973b91c	0xd0f87828	0xb724a487	0xa295fa2a	0x0a67c97a	0x18c2a801	0xae50a6fd

Table 4. Another colliding message pair for 18 step SHA-256 with standard IV. These two messages also follow the differential path given in Table 1.

M_1	0-8	0xed919421	0xaa75e4fe	0x8548d0e0	0x9c1888f7	0x1da3fc3d	0xa11f7a02	0xbb463b64	0xe9b28365	0x323ecf28
	9-17	0x8097e497	0x4343b78b	0xdc484e91	0xbf588b4b	0x8401140a	0x42499da1	0xf88a3e2e	0xfcc3918d	0x2c41c953
M_2	0-8	0xed919421	0xaa75e4fe	0x8548d0e0	0x1c1888f7	0x3fb7fe7d	0xe39a6a9a	0x99523924	0xe9b28365	0x102acd68
	9-17	0x8097e497	0x4343b78b	0x5c484e91	0xbf588b4b	0x8401140a	0x42499da1	0xf88a3e2e	0xfcc3918d	0x2c41c953

7 Using Coding Theoretic Methods to Find Linear Differential Paths for Reduced Round SHA-256

In [16] and [14] coding theoretic techniques were used to search for differential paths in SHA-1. Extension to SHA-2 was mentioned in [12]. We describe a new way of forming parity check equations and then find low weight codewords for the corresponding generator matrix. Each of these codewords can be used to build a differential characteristic for reduced round SHA-256. This method results in tackling up to 23-step reduced SHA-256.

7.1 A New Way of Constructing Parity Check Equations

Tackling message expansion in SHA-2 can be a problem. A non-zero value of ΔW_i for $i \geq 16$ necessitates tackling the recursion for message expansion. So one way to avoid this is to ensure that $\Delta W_i = 0$ for $i \geq 16$. Clearly, this cannot work for full SHA-2. But, for reduced round versions, one can find differential paths using this approach, as we describe below.

The technique described below assumes a local collision $\mathcal{L}$. The description is not for any particular local collision. It holds for any local collision. Obtaining a particular local collision requires certain linear approximations of the constituents of the SHA-256 round function. This converts the round function into a linear map based on which we define our linear code. We note that the linear code is not straightforwardly obtained from the linear map.

A message consists of 16 32-bit words for a total of 512 bits. We use the Chabaud-Joux [4] type disturbance vector approach. Let DV $= \{d_0, d_1, d_2, \ldots, d_{255}\}$ be a 256-bit disturbance vector. If $d_i = 1$ then the two initial messages differ in their i^{th} bit, and further message bits differ as per the local collision.

We do not consider a 512-bit DV for the following reason. A local collision defines the differences of 9 words of messages and only the first 16 words of SHA-256 are unrestricted. Thereafter the message words are calculated using the message expansion recurrence. This implies that a local collision can not be started after first 8 steps without affecting the message expansion.

Let us now describe the linear code that we require. This is done in two steps. In the first step, we express ΔW_i ($i \geq 16$) in terms of $d_0, \ldots, d_{255}$. In the second step, we define the parity check equations for the code by setting $\Delta W_i = 0$ for $i \geq 16$. Thus, any DV $(d_0, \ldots, d_{255})$ which satisfies these parity check equations is a codeword. Our task then is to look for a low weight codeword as this gives a differential path with a small number of local collisions.

It is clear that such codes can be formed as long as there are less than 256 parity check equations. If we apply this procedure up to N rounds (corresponding to step $N-1$), then we will obtain $32(N-16)$ parity check equations. Thus, the maximum N that we can use with this method is $N = 23$. The minimum value of N is clearly 17. Since we already report 18-round collision, we do not consider $N = 17$ and 18. Instead we report differential paths from 19 to 23 rounds.

The first task is to express ΔW_i ($i \geq 16$) in terms of $d_0, \ldots, d_{255}$. We describe how this is done. For any local collision $\mathcal{L}$, the first word determines the next eight words. Consider the 32-bit vector $(d_0, 0, \ldots, 0)$, where d_0 is treated as a (binary) variable. Then $\mathcal{L}$ defines the next 8 32-bit words. At this point, the first 9 words have been defined. The rest 7 are taken to be zero. For $i \geq 16$, ΔW_i is now obtained using the message recursion. This expresses all the ΔW_is ($i \geq 16$) as linear function of d_0. Next consider the 32-bit vector $(0, d_1, 0, \ldots, 0)$; use $\mathcal{L}$ to obtain the next eight words and the message expansion recursion to express ΔW_is ($i \geq 16$) as linear function of d_1. Now, for the 32-bit vector $(d_0, d_1, 0, \ldots, 0)$, we can express ΔW_is ($i \geq 16$) as linear function of d_0 and d_1 by XORing the separate linear functions corresponding to d_0 and d_1. Clearly, the procedure can be extended to the entire DV $(d_0, \ldots, d_{255})$. The exact details are given in Table 5.

Methods presented in [3], [10] and [19] are used to search for low weight codewords from the check-matrices (and the corresponding generator matrices) obtained using the algorithm in Table 5. Codewords of least weight found and the linear differential path for that codeword are shown in Section 8.

Table 5. Algorithm for generating parity check equations for linearized N step SHA-256.

```
external LC($x$) : accepts a 32 bit input $x$ and returns 9 words of 32 bits conforming to the local
collision chosen.

Set $\Delta W_{final} := (U_0, U_1, \ldots U_{N-1})$   $U_i \in \{0,1\}^{32}$
Set $\delta W_{cur} := (V_0, V_1, \ldots V_8)$   $V_i \in \{0,1\}^{32}$
Initialize $\Delta W_{final}$ and $\delta W_{cur}$ to all zeros.

For($i = 0$ to 8){
    For($j = 0$ to 31){
        set $D := (0, 0, \ldots, d_{32i+j}, 0, \ldots, 0)$;   /* The $j^{th}$ bit of $D$ is given by $d_{32i+j}$.
              Each $d_n \in \{0,1\}$ is the component of the disturbance vector and $D \in \{0,1\}^{32}$ */
        set $\delta W_{cur} :=$ LC($D$);
        For($k = i$ to $i + 8$){
            $\Delta W_{final}[k] = \Delta W_{final}[k] \oplus \delta W_{cur}[k - i]$;
        }
    }
}
/* At this point the $\Delta W_{final}$ list contains $W_i \oplus W_i'$ for $0 \leq i < 16$ */

Obtain $\Delta W_i$ for $16 \leq i < N$ using linearized message expansion of SHA-256.
Equate all 32 bits of $\Delta W_i$ for $i \geq 16$ to zero to get $32 * (N - 16)$ parity check equations.
```

8 Results and Comparison to Previous Work

Low weight disturbance vectors are searched for reduced round SHA-256 by using the probabilistic methods described in [3], [10] and [19]. The minimum weights of codewords found are listed in Table 6. For 19-step SHA-256 the weight of the codeword found is 15 for both GH and SS_5 local collision. This means that 15 local collisions are interleaved to obtain the 19-step characteristic. Interestingly, all the 15 local collisions start at the same word for both GH and SS_5. Thus the case of 19-step characteristic can be considered as consisting of a single local collision starting at Step 3 where the initial message difference is a word with weight 15 bits. There is no colliding differential path known before this work. The best known 19-step differential path is for a near collision consisting of 23 GH local collisions [12]. As has already been noted in [12], the GH local collision causes certain impossible conditions in the search for actual colliding pairs. The use of SS_5 local collision ensures that we do not face two types of impossible conditions.

For 20 to 23 steps, no differential path is known so far. We provide the first differential paths for these cases. For 23-step SHA-256, the size of the corresponding generator matrix is 32×256, i.e. there are only 32 codewords of length 256. It is possible to do exhaustive search on this size, hence we did not use the probabilistic methods for this case. For the 23-step case, the reported codeword weight is actually the best possible. All these differential paths are reported in Section C.

Table 6. Summary of results. Least weight of the codeword found using different local collisions. For 23 step case, the codeword weight is obtained by exhaustive search. For all other cases, methods described in [3], [10] and [19] are used.

Step i	Size of Check matrix	using GH	using SS_5
18	-	1	1
19	96×256	15	15
20	128×256	33	31
21	160×256	45	45
22	192×256	59	60
23	224×256	79	75

Acknowledgements

We would like to thank Christian Rechberger for carefully reading an earlier version of this paper and giving helpful suggestions.

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A 9-round Pseudo Collision for SHA-256

This is a 9 round pseudo-collision for SHA-256.

Table 7. IV and the messages for the 9 round pseudo collision. These two messages follow the differential path given in Table 1 starting from Step 3 till Step 11 (i.e. Step 3 in Table 1 is Step 1 for this message pair).

registers	a	b	c	d	e	f	g	h
IV	0x1b309331	0x1d07d226	0x190c04e9	0x0b413baf	0x7c11cf34	0x2d035d09	0x76e27935	0x0fb234e2

M_1	0-8	0x5ce03f69	0xa36bfc0b	0xf99332c1	0x590db302	0x7c1cd4df	0x163c2f4b	0x2077b003	0x29ab9330	0xefb8306e
M_2	0-8	0xdce03f69	0x817ffe4b	0xbb162259	0x7b19b142	0x7c1cd4df	0x34282d0b	0x2077b003	0x29ab9330	0x6fb8306e

B Using Neutral Bits to Search for the Colliding Pairs

Once a linear differential path is obtained, message modification or neutral bit technique are used to find message pairs following that characteristic. It is suggested in [12] that using neutral bits will bring the probability of finding the message pairs to close to 1. We have used message modification technique to find the message pairs, since the number of neutral bits was too low for random messages pairs that were generated during the search for the right pair. To see the usefulness of neutral bit technique for this 18 step case, we next detail the number of 1-neutral bits and their distribution in various words of the message pairs for the two colliding message pairs shown in Tables 3 and 4. This distribution is shown in Tables 8 and 9. The entry in the j^{th} column of the i^{th} row in the two tables denotes the number of 1-neutral bits for the j^{th} message word at step i. The entry '-' means that the particular message word is not available at that step.

As can be expected, all the bits in the first 4 words in both the tables are 1-neutral. However, as soon as the fifth word is chosen, the number of 1-neutral bits comes down to about 30 and surprisingly almost all the bits of first 4 words do not remain 1-neutral anymore. If we assume that :
(1) All the 1-neutral bits are also 2-neutral. And
(2) The set of neutral bits is about $1/8^{th}$ the size of the set of 2-neutral bits (as in SHA-0),
Then we are likely to get about 10 neutral bits at step 4. The probability of success of this step is 2^{-11}. Similarly the next two steps are also likely to have about 10 neutral bits whereas the probability of success of these steps is 2^{-14} and 2^{-11} respectively. The number of neutral bits at a step and the order of the probability of that step is close enough and we may get a message pair which will follow the characteristic. However, note that both the assumptions about the neutral bits made above are not likely to hold for SHA-256. In particular, the number of 2-neutral bits are going to be much less than the number of 1-neutral bits and the size of the neutral set is likely to be smaller than that in the case of SHA-0. Further, the set of 1-neutral bits shown are for the messages which are actually colliding at 18 step. When we started the search from random message pairs, in most of the cases we could not get even these many 1-neutral bits. This suggests that the application of neutral bits alone in SHA-256

Table 8. Count of 1-neutral bits and their distribution for the message pair shown in Table 3. Steps 11 to 15 are not shown here since they do not put any restrictions on the messages and hence have all bits neutral.

Step i	0	1	2	3	4	5	6	7	8	9	10	Total
0	32	-	-	-	-	-	-	-	-	-	-	32
1	32	32	-	-	-	-	-	-	-	-	-	64
2	32	32	32	-	-	-	-	-	-	-	-	96
3	32	32	32	32	-	-	-	-	-	-	-	128
4	0	0	0	5	23	-	-	-	-	-	-	28
5	0	0	0	0	4	23	-	-	-	-	-	27
6	0	0	0	0	0	15	26	-	-	-	-	41
7	0	0	0	0	0	2	23	32	-	-	-	57
8	0	0	0	0	0	0	3	21	26	-	-	50
9	0	0	0	0	0	0	2	12	23	32	-	69
10	0	0	0	0	0	0	1	7	17	30	32	87

may not help us finding colliding message pairs. A combination of message modification and neutral bit technique may be required to attack longer number of rounds in SHA-2 family.

Table 9. Count of 1-neutral bits and their distribution for the message pair shown in Table 4. Steps 11 to 15 are not shown here since they do not put any restrictions on the messages and hence have all bits neutral.

Step i	0	1	2	3	4	5	6	7	8	9	10	Total
0	32	-	-	-	-	-	-	-	-	-	-	32
1	32	32	-	-	-	-	-	-	-	-	-	64
2	32	32	32	-	-	-	-	-	-	-	-	96
3	32	32	32	32	-	-	-	-	-	-	-	128
4	0	0	0	5	26	-	-	-	-	-	-	31
5	0	0	0	0	8	23	-	-	-	-	-	31
6	0	0	0	0	0	13	26	-	-	-	-	39
7	0	0	0	0	0	3	22	32	-	-	-	57
8	0	0	0	0	0	1	6	19	26	-	-	52
9	0	0	0	0	0	1	3	15	25	32	-	76
10	0	0	0	0	0	1	1	9	19	31	32	93

C Differential Paths for Reduced Round SHA-256

Differential paths for 19, 20, 21, 22 and 23 step SHA-256 are provided in Table 10 to Table 19. All the differential paths have been obtained by using two different local collisions, namely the GH and the SS$_5$ local collision.

D Non Valid Message Pairs from [12]

The colliding message pair given in Table 7 in [12] for 18-step SHA-256 gives rise to the differential path given in Table 20.

For the 22-step pseudo collision, the messages in Table 9 in [12] are claimed to be colliding such that $H(M, IV_{new}) = H(M^{*}, IV^{*})$. The two IV's have a difference $\Delta(IV) = IV_{new} \oplus IV^{*}$. New IV is obtained by $IV_{new} = IV_{standard} \oplus IV_{Corr}$. The calculations for new IV are shown in Table 21.

We contacted the authors of [12] regarding the non valid message pairs. Christian Rechberger, in a personal communication [15], sent a new value for the starting IV for register A as IV_{new} = 0xb5f4d225. This, as we can compute, results in the modified correction for register A as IV_{Corr} = 0xdffd3442. With these changes the messages as reported in [12] do follow the 22-step pseudo collision differential path.

We note that no message pair which collides for the 18-step SHA-256 with the standard IV has been provided.

Table 10. 19 step linear characteristic for SHA-256. There are 15 SS_5 local collisions. This characteristic can also be seen as a single SS_5 local collision in which weight of ΔW_3 is 15 bits.

Step i	ΔW_i	Δa_i	Δb_i	Δc_i	Δd_i	Δe_i	Δf_i	Δg_i	Δh_i
0-2	0	0	0	0	0	0	0	0	0
3	0x27f42515	0x27f42515	0	0	0	0x27f42515	0	0	0
4	0xbde9c6f8	0	0x27f42515	0	0	0xb1c0627b	0x27f42515	0	0
5	0x4180045d	0	0	0x27f42515	0	0x27f42515	0xb1c0627b	0x27f42515	0
6	0xbde9c6f8	0	0	0	0x27f42515	0	0x27f42515	0xb1c0627b	0x27f42515
7	0	0	0	0	0	0x27f42515	0	0x27f42515	0xb1c0627b
8	0xbde9c6f8	0	0	0	0	0	0x27f42515	0	0x27f42515
9	0	0	0	0	0	0	0	0x27f42515	0
10	0	0	0	0	0	0	0	0	0x27f42515
11	0x27f42515	0	0	0	0	0	0	0	0
12-18	0	0	0	0	0	0	0	0	0

Table 11. 19 step linear characteristic for SHA-256. There are 15 GH local collisions. This characteristic can also be seen as a single GH local collision in which weight of ΔW_3 is 15 bits. Note that this characteristic differs from the one in Table 10 at several places. For example in message words and registers at 7^{th} step.

Step i	ΔW_i	Δa_i	Δb_i	Δc_i	Δd_i	Δe_i	Δf_i	Δg_i	Δh_i
0-2	0	0	0	0	0	0	0	0	0
3	0x27f42515	0x27f42515	0	0	0	0x27f42515	0	0	0
4	0xbde9c6f8	0	0x27f42515	0	0	0xb1c0627b	0x27f42515	0	0
5	0x4180045d	0	0	0x27f42515	0	0	0xb1c0627b	0x27f42515	0
6	0	0	0	0	0x27f42515	0	0	0xb1c0627b	0x27f42515
7	0x27f42515	0	0	0	0	0x27f42515	0	0	0xb1c0627b
8	0xbde9c6f8	0	0	0	0	0	0x27f42515	0	0
9	0	0	0	0	0	0	0	0x27f42515	0
10	0	0	0	0	0	0	0	0	0x27f42515
11	0x27f42515	0	0	0	0	0	0	0	0
12-18	0	0	0	0	0	0	0	0	0

Table 12. 20 step linear characteristic for SHA-256. There are 31 SS_5 local collisions.

Step i	ΔW_i	Δa_i	Δb_i	Δc_i	Δd_i	Δe_i	Δf_i	Δg_i	Δh_i
0	0x00210808	0x00210808	0	0	0	0x00210808	0	0	0
1	0xf4ec270b	0x0100c000	0x00210808	0	0	0xc568a30a	0x00210808	0	0
2	0xfd5cb0fc	0x00000204	0x0100c000	0x00210808	0	0x0361320e	0xc568a30a	0x00210808	0
3	0x0f1c9c64	0x12080000	0x00000204	0x0100c000	0x00210808	0x0320d081	0x0361320e	0xc568a30a	0x00210808
4	0xe38e0ddb	0x40080208	0x12080000	0x00000204	0x0100c000	0x64ab980c	0x0320d081	0x0361320e	0xc568a30a
5	0x91de6968	0x00040600	0x40080208	0x12080000	0x00000204	0x3344e7c2	0x64ab980c	0x0320d081	0x0361320e
6	0x552353b0	0x00006000	0x00040600	0x40080208	0x12080000	0x601161ac	0x3344e7c2	0x64ab980c	0x0320d081
7	0x9b7f2ad2	0x26239208	0x00006000	0x00040600	0x40080208	0x35af8c0b	0x601161ac	0x3344e7c2	0x64ab980c
8	0x694d62fd	0	0x26239208	0x00006000	0x00040600	0x5789970e	0x35af8c0b	0x601161ac	0x3344e7c2
9	0x3c97a984	0	0	0x26239208	0x00006000	0x26279408	0x5789970e	0x35af8c0b	0x601161ac
10	0x85cea813	0	0	0	0x26239208	0x00006000	0x26279408	0x5789970e	0x35af8c0b
11	0x13b8198f	0	0	0	0	0x26239208	0x00006000	0x26279408	0x5789970e
12	0x27dcb927	0	0	0	0	0	0x26239208	0x00006000	0x26279408
13	0x00040600	0	0	0	0	0	0	0x26239208	0x00006000
14	0x00006000	0	0	0	0	0	0	0	0x26239208
15	0x26239208	0	0	0	0	0	0	0	0
16-19	0	0	0	0	0	0	0	0	0

Table 13. 20 step linear characteristic for SHA-256. There are 33 GH local collisions.

Step i	ΔW_i	Δa_i	Δb_i	Δc_i	Δd_i	Δe_i	Δf_i	Δg_i	Δh_i
0	0x11000200	0x11000200	0	0	0	0x11000200	0	0	0
1	0x921ea8c4	0x46110000	0x11000200	0	0	0x525988c4	0x11000200	0	0
2	0x71813ea9	0x01000002	0x46110000	0x11000200	0	0x54867192	0x525988c4	0x11000200	0
3	0xd8d10826	0x60a14800	0x01000002	0x46110000	0x11000200	0xe0f14804	0x54867192	0x525988c4	0x11000200
4	0xb9f28f71	0x1e420280	0x60a14800	0x01000002	0x46110000	0xd2495608	0xe0f14804	0x54867192	0x525988c4
5	0x8f9b9016	0x00a00200	0x1e420280	0x60a14800	0x01000002	0x5d2b70c9	0xd2495608	0xe0f14804	0x54867192
6	0x6d415897	0x00004084	0x00a00200	0x1e420280	0x60a14800	0x91204504	0x5d2b70c9	0xd2495608	0xe0f14804
7	0xc7276fd3	0x000000a0	0x00004084	0x00a00200	0x1e420280	0x65834883	0x91204504	0x5d2b70c9	0xd2495608
8	0x0b152ad9	0	0x000000a0	0x00004084	0x00a00200	0x1b4082a8	0x65834883	0x91204504	0x5d2b70c9
9	0x08044ede	0	0	0x000000a0	0x00004084	0x00a00200	0x1b4082a8	0x65834883	0x91204504
10	0x8123d10c	0	0	0	0x000000a0	0x00004084	0x00a00200	0x1b4082a8	0x65834883
11	0x65230b89	0	0	0	0	0x000000a0	0x00004084	0x00a00200	0x1b4082a8
12	0x8f40d2aa	0	0	0	0	0	0x000000a0	0x00004084	0x00a00200
13	0x00a00200	0	0	0	0	0	0	0x000000a0	0x00004084
14	0x00004084	0	0	0	0	0	0	0	0x000000a0
15	0x000000a0	0	0	0	0	0	0	0	0
16-19	0	0	0	0	0	0	0	0	0

Table 14. 21 step linear characteristic for SHA-256. There are 45 SS$_5$ local collisions.

Step i	ΔW_i	Δa_i	Δb_i	Δc_i	Δd_i	Δe_i	Δf_i	Δg_i	Δh_i
0	0x0100000a	0x0100000a	0	0	0	0x0100000a	0	0	0
1	0x23510596	0x0a050090	0x0100000a	0	0	0x8a152096	0x0100000a	0	0
2	0x926e15c7	0xc2041880	0x0a050090	0x0100000a	0	0xd10748ae	0x8a152096	0x0100000a	0
3	0x6b6f5fa5	0x22a00008	0xc2041880	0x0a050090	0x0100000a	0xcc401590	0xd10748ae	0x8a152096	0x0100000a
4	0x4bcb749e	0x00035100	0x22a00008	0xc2041880	0x0a050090	0x4bee7c02	0xcc401590	0xd10748ae	0x8a152096
5	0x1b692042	0x8480040c	0x00035100	0x22a00008	0xc2041880	0x2961d0ce	0x4bee7c02	0xcc401590	0xd10748ae
6	0x2c749ed0	0x26422000	0x8480040c	0x00035100	0x22a00008	0xe5117e91	0x2961d0ce	0x4bee7c02	0xcc401590
7	0x78a6949f	0x05014062	0x26422000	0x8480040c	0x00035100	0xa230feee	0xe5117e91	0x2961d0ce	0x4bee7c02
8	0x70cf2020	0	0x05014062	0x26422000	0x8480040c	0xa1108106	0xa230feee	0xe5117e91	0x2961d0ce
9	0x3c408d06	0	0	0x05014062	0x26422000	0x8181446e	0xa1108106	0xa230feee	0xe5117e91
10	0xb375fdee	0	0	0	0x05014062	0x26422000	0x8181446e	0xa1108106	0xa230feee
11	0x023c7a57	0	0	0	0	0x05014062	0x26422000	0x8181446e	0xa1108106
12	0x83a6352d	0	0	0	0	0	0x05014062	0x26422000	0x8181446e
13	0x8480040c	0	0	0	0	0	0	0x05014062	0x26422000
14	0x26422000	0	0	0	0	0	0	0	0x05014062
15	0x05014062	0	0	0	0	0	0	0	0
16-20	0	0	0	0	0	0	0	0	0

Table 15. 21 step linear characteristic for SHA-256. There are 45 GH local collisions.

Step i	ΔW_i	Δa_i	Δb_i	Δc_i	Δd_i	Δe_i	Δf_i	Δg_i	Δh_i
0	0x0002009c	0x0002009c	0	0	0	0x0002009c	0	0	0
1	0xee6bb675	0x80090000	0x0002009c	0	0	0x8cebf037	0x0002009c	0	0
2	0xa91c7e24	0x00201514	0x80090000	0x0002009c	0	0x0426575c	0x8cebf037	0x0002009c	0
3	0xe2c68750	0x42816080	0x00201514	0x80090000	0x0002009c	0x6a7d34c5	0x0426575c	0x8cebf037	0x0002009c
4	0xecac961f	0x4e100262	0x42816080	0x00201514	0x80090000	0x5f324fdf	0x6a7d34c5	0x0426575c	0x8cebf037
5	0x13c4cbce	0x40000200	0x4e100262	0x42816080	0x00201514	0x0096fb20	0x5f324fdf	0x6a7d34c5	0x0426575c
6	0xc742bbcf	0x6c113420	0x40000200	0x4e100262	0x42816080	0x6c3b20b4	0x0096fb20	0x5f324fdf	0x6a7d34c5
7	0x4a23a824	0x04240100	0x6c113420	0x40000200	0x4e100262	0xb872cdb1	0x6c3b20b4	0x0096fb20	0x5f324fdf
8	0x8f8f731c	0	0x04240100	0x6c113420	0x40000200	0xd71d2312	0xb872cdb1	0x6c3b20b4	0x0096fb20
9	0xa701e563	0	0	0x04240100	0x6c113420	0x40000200	0xd71d2312	0xb872cdb1	0x6c3b20b4
10	0x2d32209c	0	0	0	0x04240100	0x6c113420	0x40000200	0xd71d2312	0xb872cdb1
11	0xb5551b71	0	0	0	0	0x04240100	0x6c113420	0x40000200	0xd71d2312
12	0xe50db794	0	0	0	0	0	0x04240100	0x6c113420	0x40000200
13	0x40000200	0	0	0	0	0	0	0x04240100	0x6c113420
14	0x6c113420	0	0	0	0	0	0	0	0x04240100
15	0x04240100	0	0	0	0	0	0	0	0
16-20	0	0	0	0	0	0	0	0	0

Table 16. 22 step linear characteristic for SHA-256. There are 60 SS_5 local collisions.

Step i	ΔW_i	Δa_i	Δb_i	Δc_i	Δd_i	Δe_i	Δf_i	Δg_i	Δh_i
0	0x20088104	0x20088104	0	0	0	0x20088104	0	0	0
1	0x7723d081	0x61404100	0x20088104	0	0	0x43677185	0x20088104	0	0
2	0x974e080e	0x10042040	0x61404100	0x20088104	0	0x215bba83	0x43677185	0x20088104	0
3	0xc03afd79	0x05a14180	0x10042040	0x61404100	0x20088104	0x726188f1	0x215bba83	0x43677185	0x20088104
4	0x301bc5d8	0x420111c8	0x05a14180	0x10042040	0x61404100	0xfa63cdf0	0x726188f1	0x215bba83	0x43677185
5	0xa13271fd	0x43008748	0x420111c8	0x05a14180	0x10042040	0xbd64f2ba	0xfa63cdf0	0x726188f1	0x215bba83
6	0xa3795c7f	0x1d044014	0x43008748	0x420111c8	0x05a14180	0x679e6946	0xbd64f2ba	0xfa63cdf0	0x726188f1
7	0x5d086433	0x398a1838	0x1d044014	0x43008748	0x420111c8	0x69ca76a3	0x679e6946	0xbd64f2ba	0xfa63cdf0
8	0xdbcb0f3a	0	0x398a1838	0x1d044014	0x43008748	0xb8c6fb64	0x69ca76a3	0x679e6946	0xbd64f2ba
9	0x702d2d4f	0	0	0x398a1838	0x1d044014	0x7a8a9f70	0xb8c6fb64	0x69ca76a3	0x679e6946
10	0xb5f25131	0	0	0	0x398a1838	0x1d044014	0x7a8a9f70	0xb8c6fb64	0x69ca76a3
11	0xc3975255	0	0	0	0	0x398a1838	0x1d044014	0x7a8a9f70	0xb8c6fb64
12	0x872fbe4f	0	0	0	0	0	0x398a1838	0x1d044014	0x7a8a9f70
13	0x43008748	0	0	0	0	0	0	0x398a1838	0x1d044014
14	0x1d044014	0	0	0	0	0	0	0	0x398a1838
15	0x398a1838	0	0	0	0	0	0	0	0
16-21	0	0	0	0	0	0	0	0	0

Table 17. 22 step linear characteristic for SHA-256. There are 59 GH local collisions.

Step i	ΔW_i	Δa_i	Δb_i	Δc_i	Δd_i	Δe_i	Δf_i	Δg_i	Δh_i
0	0x40200502	0x40200502	0	0	0	0x40200502	0	0	0
1	0x91474f60	0x10034314	0x40200502	0	0	0x280d4a54	0x40200502	0	0
2	0x01ac1d4e	0x0c800345	0x10034314	0x40200502	0	0x1d2c03da	0x280d4a54	0x40200502	0
3	0xc8319061	0x14021803	0x0c800345	0x10034314	0x40200502	0x4d0768e0	0x1d2c03da	0x280d4a54	0x40200502
4	0x41c0f00e	0x12111224	0x14021803	0x0c800345	0x10034314	0x5f493d66	0x4d0768e0	0x1d2c03da	0x280d4a54
5	0x29db56e8	0x41122608	0x12111224	0x14021803	0x0c800345	0x80fd2155	0x5f493d66	0x4d0768e0	0x1d2c03da
6	0xf592f204	0x82031028	0x41122608	0x12111224	0x14021803	0xe61db37a	0x80fd2155	0x5f493d66	0x4d0768e0
7	0xcaaa1ee6	0xa0340814	0x82031028	0x41122608	0x12111224	0x19b2660d	0xe61db37a	0x80fd2155	0x5f493d66
8	0xcb37951b	0	0xa0340814	0x82031028	0x41122608	0xaa994301	0x19b2660d	0xe61db37a	0x80fd2155
9	0xaac397a4	0	0	0xa0340814	0x82031028	0x41122608	0xaa994301	0x19b2660d	0xe61db37a
10	0x8f02dd86	0	0	0	0xa0340814	0x82031028	0x41122608	0xaa994301	0x19b2660d
11	0xbf223e6e	0	0	0	0	0xa0340814	0x82031028	0x41122608	0xaa994301
12	0xe0899ff0	0	0	0	0	0	0xa0340814	0x82031028	0x41122608
13	0x41122608	0	0	0	0	0	0	0xa0340814	0x82031028
14	0x82031028	0	0	0	0	0	0	0	0xa0340814
15	0xa0340814	0	0	0	0	0	0	0	0
16-21	0	0	0	0	0	0	0	0	0

Table 18. 23 step linear characteristic for SHA-256. There are 75 SS_5 local collisions.

Step i	ΔW_i	Δa_i	Δb_i	Δc_i	Δd_i	Δe_i	Δf_i	Δg_i	Δh_i
0	0x80018c11	0x80018c11	0	0	0	0x80018c11	0	0	0
1	0xe08ba9dd	0x22c18214	0x80018c11	0	0	0x247da71c	0x80018c11	0	0
2	0x9acf831a	0x29105222	0x22c18214	0x80018c11	0	0xb708f831	0x247da71c	0x80018c11	0
3	0x99b76e84	0x81804506	0x29105222	0x22c18214	0x80018c11	0xf95c13bc	0xb708f831	0x247da71c	0x80018c11
4	0xef3d10f6	0x5217b401	0x81804506	0x29105222	0x22c18214	0x72466d77	0xf95c13bc	0xb708f831	0x247da71c
5	0xf2ce8282	0x5421e110	0x5217b401	0x81804506	0x29105222	0x5d3f5ef7	0x72466d77	0xf95c13bc	0xb708f831
6	0x15e6cb63	0x84815301	0x5421e110	0x5217b401	0x81804506	0x65882d39	0x5d3f5ef7	0x72466d77	0xf95c13bc
7	0xee97bfc1	0x64196841	0x84815301	0x5421e110	0x5217b401	0x4dd8ba8f	0x65882d39	0x5d3f5ef7	0x72466d77
8	0x6d60f25f	0	0x64196841	0x84815301	0x5421e110	0xa83a984b	0x4dd8ba8f	0x65882d39	0x5d3f5ef7
9	0x4e6744df	0	0	0x64196841	0x84815301	0x30388951	0xa83a984b	0x4dd8ba8f	0x65882d39
10	0xbf10f8de	0	0	0	0x64196841	0x84815301	0x30388951	0xa83a984b	0x4dd8ba8f
11	0x5b6b267a	0	0	0	0	0x64196841	0x84815301	0x30388951	0xa83a984b
12	0x2db30d74	0	0	0	0	0	0x64196841	0x84815301	0x30388951
13	0x5421e110	0	0	0	0	0	0	0x64196841	0x84815301
14	0x84815301	0	0	0	0	0	0	0	0x64196841
15	0x64196841	0	0	0	0	0	0	0	0
16-22	0	0	0	0	0	0	0	0	0

Table 19. 23 step linear characteristic for SHA-256. There are 79 SS_5 local collisions.

Step i	ΔW_i	Δa_i	Δb_i	Δc_i	Δd_i	Δe_i	Δf_i	Δg_i	Δh_i
0	0x06410482	0x06410482	0	0	0	0x06410482	0	0	0
1	0x5a5956e6	0x4310a0e6	0x06410482	0	0	0xe282dbd7	0x06410482	0	0
2	0x936b23d5	0x22053aa0	0x4310a0e6	0x06410482	0	0xf7709310	0xe282dbd7	0x06410482	0
3	0xc4c16eb1	0x9421149d	0x22053aa0	0x4310a0e6	0x06410482	0x5d4bca94	0xf7709310	0xe282dbd7	0x06410482
4	0x50e078c8	0xb50c2248	0x9421149d	0x22053aa0	0x4310a0e6	0xf6fbb4b5	0x5d4bca94	0xf7709310	0xe282dbd7
5	0xd0d250ef	0x01606240	0xb50c2248	0x9421149d	0x22053aa0	0x4dff4081	0xf6fbb4b5	0x5d4bca94	0xf7709310
6	0xce2982f4	0x4036003e	0x01606240	0xb50c2248	0x9421149d	0xf1e22908	0x4dff4081	0xf6fbb4b5	0x5d4bca94
7	0x6cb9d29e	0x8bc0502c	0x4036003e	0x01606240	0xb50c2248	0x561e3c0e	0xf1e22908	0x4dff4081	0xf6fbb4b5
8	0xe3a3f08f	0	0x8bc0502c	0x4036003e	0x01606240	0x17d8da6e	0x561e3c0e	0xf1e22908	0x4dff4081
9	0x540feff8	0	0	0x8bc0502c	0x4036003e	0x01606240	0x17d8da6e	0x561e3c0e	0xf1e22908
10	0x09d6a48d	0	0	0	0x8bc0502c	0x4036003e	0x01606240	0x17d8da6e	0x561e3c0e
11	0xb3d6fdee	0	0	0	0	0x8bc0502c	0x4036003e	0x01606240	0x17d8da6e
12	0x404eb561	0	0	0	0	0	0x8bc0502c	0x4036003e	0x01606240
13	0x01606240	0	0	0	0	0	0	0x8bc0502c	0x4036003e
14	0x4036003e	0	0	0	0	0	0	0	0x8bc0502c
15	0x8bc0502c	0	0	0	0	0	0	0	0
16-22	0	0	0	0	0	0	0	0	0

Table 20. Differential path with the message pair given in [12] for 18-step collision. The expected differentials at step 4 are $\Delta a_4 = \Delta e_4 = 0$ and the difference of hash value after 18 steps is expected to be zero. The last two message words for both the messages are computed using the message expansion of SHA-256.

Step i	W_i	W'_i	ΔW_i	Δa_i	Δb_i	Δc_i	Δd_i	Δe_i	Δf_i	Δg_i	Δh_i
0	0x02679857	0x02679857	0	0	0	0	0	0	0	0	0
1	0x0183b9a1	0x0183b9a1	0	0	0	0	0	0	0	0	0
2	0x005de4f5	0x005de4f5	0	0	0	0	0	0	0	0	0
3	0x0266ee0c	0x8266ee0c	0x80000000	0x80000000	0	0	0	0	0	0	0x8000000
4	0x0d1442f0	0x2f0040b0	0x22140240	0xcc680000	0x80000000	0	0	0x646c0200	0x80000000	0	0
5	0x06373a71	0x44b22ae9	0x42851098	0xe8c39561	0xcc680000	0x80000000	0	0x7b4d1f1e	0x646c0200	0x80000000	0
6	0xc445dec2	0xc445dec2	0	0xcf5bccf4	0xe8c39561	0xcc680000	0x80000000	0x1fb5af8e	0x7b4d1f1e	0x646c0200	0x80000000
7	12542ec1	0x92542ec1	0x80000000	0x9da5a2ec	0xcf5bccf4	0xe8c39561	0xcc680000	0xb44d09aa	0x1fb5af8e	0x7b4d1f1e	0x646c0200
8	0982b61a	0x2b96b45a	0x22140240	0x75cd1d37	0x9da5a2ec	0xcf5bccf4	0xe8c39561	0xde12fed5	0xb44d09aa	0x1fb5af8e	0x7b4d1f1e
9	205a614c	0x205a614c	0	0x3807615e	0x75cd1d37	0x9da5a2ec	0xcf5bccf4	0xebbb4f53	0xde12fed5	0xb44d09aa	0x1fb5af8e
10	2495a094	0x2495a094	0	0x3a261824	0x3807615e	0x75cd1d37	0x9da5a2ec	0x733615ba	0xebbb4f53	0xde12fed5	0xb44d09aa
11	166ae4ac	0x966ae4ac	0x80000000	0x311882ec	0x3a261824	0x3807615e	0x75cd1d37	0x62387a46	0x733615ba	0xebbb4f53	0xde12fed5
12	15917909	0x15917909	0	0x63a304ec	0x311882ec	0x3a261824	0x3807615e	0x060363a0	0x62387a46	0x733615ba	0xebbb4f53
13	1178f05a	0x1178f05a	0	0x99af8895	0x63a304ec	0x311882ec	0x3a261824	0xd49164ce	0x060363a0	0x62387a46	0x733615ba
14	0aae5a46	0x0aae5a46	0	0xe733bc41	0x99af8895	0x63a304ec	0x311882ec	0x655a938b	0xd49164ce	0x060363a0	0x62387a46
15	178058c6	0x178058c6	0	0x2b3945ae	0xe733bc41	0x99af8895	0x63a304ec	0x24ae94fa	0x655a938b	0xd49164ce	0x060363a0
16	0xb586995e	0xb586995e	0	0x1a1d1b84	0x2b3945ae	0xe733bc41	0x99af8895	0x60127122	0x24ae94fa	0x655a938b	0xd49164ce
17	0xe0cdca9b	0xe0cdca9b	0	0xc2101029	0x1a1d1b84	0x2b3945ae	0xe733bc41	0x9ffdfb0a	0x60127122	0x24ae94fa	0x655a938b

Table 21. Modified IV for the 22-step pseudo collision from [12]. $IV_{new} = IV_{standard} \oplus IV_{corr}$. The two messages W_1 and W_2 use different IV's. (W_1, IV_{new}) and $(W_2, IV_{new} \oplus \Delta(IV))$ are expected to collide after 22 steps.

registers	a	b	c	d	e	f	g	h
$IV_{standard}$	0x6a09e667	0xbb67ae85	0x3c6ef372	0xa54ff53a	0x510e527f	0x9b05688c	0x1f83d9ab	0x5be0cd19
IV_{Corr}	0xdcbd1a68	0x0	0x0	0x00000080	0x60810000	0x0	0x0	0x0
IV_{new}	0xb6b4fc0f	0xbb67ae85	0x3c6ef372	0xa54ff5ba	0x318f527f	0x9b05688c	0x1f83d9ab	0x5be0cd19
$\Delta(IV)$	0x00000200	0x0	0x0	0x50090088	0x0	0x0	0x20880000	0x10080080