ON THE ECONOMIC PAYOFF OF FORENSIC SYSTEMS WHEN USED TO TRACE COUNTERFEITED SOFTWARE AND CONTENT

YACOV YACOBI

Abstract. We analyze how well forensic systems reduce counterfeiting of software and content. We make a few idealized assumptions, and show that if the revenues of the producer before the introduction of forensics (R_o) are non-zero then the payoff of forensics is independent of the overall market size, it declines as the ratio between the penalty and the crime (both monetized) goes up, but that this behavior is reversed if $R_o = 0$. We also show that the payoff goes up as the ratio between success probability with and without forensics grows, however, for typical parameters most of the payoff is already reached when this ratio is 5.

1. Introduction

Unnoticeable marks which are embedded in content and software can be used for forensic applications as well as for screening. Both may help resist piracy and counterfeiting. Unlike the pirate, the counterfeiter pretends to be the legitimate producer, charges the same price as the producer, and in general competes with the producer in the same market (that does not include the market share of the pirate). As such, it is a valid assumption that every copy sold by the counterfeiter is a copy lost to the producer. Such an assumption is not valid when dealing with piracy, where illegal copies are sold for a small fraction of their legal price, and many of the buyers of piracy in poor countries cannot afford the legal price. Another assumption that is valid for counterfeiting but not necessarily for piracy is that counterfeited material is sold mostly in countries where law-enforcement has teeth; offenders can be prosecuted, and penalties can be imposed after succesful prosecution.

We limit the scope of this paper to the economic effects of forensic systems on counterfeiting.

Enforcement mechanisms, such as screening and registration, as well as raising the initial bar (e.g. by adding sophisticated holograms) push down the number of counterfeiters, n, however, forensic systems have no effect on n if the counterfeiters are rational, since, as is shown later, the counterfeiter has positive expected profit for any practical value of the parameters.

We make the following assumptions: (i) audit events are independent of each other, (ii) The probability of false positives is negligible (at the expense of higher probability of false negatives); (iii) Once caught and successfully prosecuted, the

Date: November 4, 2003.

Key words and phrases. Counterfeiting, Piracy, forensics, economics.

magnitude of the whole theft of an independent counterfeiting group is known and the penalty is in some fixed proportion, γ, to the theft.

We show that if the revenues of the producer before the introduction of forensics (R_o) are non-zero then the payoff of forensics is $\lambda np(\frac{1}{\ln(1-q_o)} - \frac{1}{\ln(1-q_w)})$, where λ depends only on γ, n =the number of independent counterfeiting groups, p = the selling price of one copy of the protected object, q_w, q_o are the probability of audit, detection, and successful prosecution of a single illegal copy, with and without forensics, respectively. This payoff is independent of the overall market size, it declines as γ goes up (this behavior is reversed if $R_o = 0$), and goes up as $k = q_w/q_o$ grows, however, for typical parameters most of the potential payoff is already reached when $k = 5$.

In section 2 we develop the payoff function of the counterfeiter, find its optimum, and show that it is always positive. In section 3 we develop the payoff function of the forensic system, and prove its properties. This work was heavily influenced by unpublished manuscripts [HV1,HV2]. An earlier version of section 2 is included in [YY].

2. The economics of the counterfeiter

Let x = the number of copies made by a counterfeiter; F = the penalty when caught. For each copy let the probability to audit an illegal copy, make a correct decision, and successfully prosecute be $q << 1$. Let $\pi(x)$ be the probability to audit an illegal copy, make a correct decision, and successfully prosecute after the counterfeiter has sold x illegal copies. When the audit events are independent (as we henceforth assume) $\pi(x) = 1 - (1-q)^x$. Let p =the price of a single copy (legal and illegal); Assuming zero costs for the counterfeiter, the gain function of the counterfeiter is [HV1]

$$P(x) = (1 - \pi(x))px - \pi(x)F, \tag{2.1}$$

If there are dependencies between audit events this function becomes an upper bound on the gain function. Let x^* denote the value of x for which $P(x)$ reaches its maximum. In reality x takes only integer values, we first pretend that x can have any real value, so that we can use differentials to approximate the behavior of $P(x)$. At the end of this section we get back to the case of integer x [KJ].

Let $L(x)$ be the Lambert function, i.e., that function for which $L(x)e^{L(x)} = x$, and let D denote the market size of the producer and the counterfeiter when the price is p. $\gamma = \frac{F}{xp}$ is assumed constant. Let $\lambda = L\left(e\frac{\gamma}{1+\gamma}\right) - 1$. Since nobody needs more than one copy of each protected object, if each counterfeiter sells the optimum x^* then n counterfeiters sell nx^*, and $nx^* \leq D$

Theorem 1. $x^* = \min\{D/n, \frac{\lambda}{\ln(1-q)}\}$.

Proof. Clearly $x^* \leq D/n$. $P(x) = [1 - \pi(x)(1+\gamma)]px = [(1-q)^x(1+\gamma) - \gamma]px$.

$P'(x) = (1-q)^x (1+\gamma) p[x \ln(1-q) + 1] - \gamma p$. Let $x^* = \frac{L\left(e\frac{\gamma}{1+\gamma}\right)-1}{\ln(1-q)}$. Then $P'(x^*) = 0$. $P"(x^*) = (1-q)^x p(x^* \ln(1-q) + 2)(1+\gamma)\ln(1-q) < 0$ (since $\ln(1-q) < 0$ and $x^* \ln(1-q) + 2 = L\left(e\frac{\gamma}{1+\gamma}\right) + 1 > 0$), implying that $P(x^*)$ is a maximum. □

Note that $\lambda < 0$ and hence $x^* > 0$. There is no value of real $\gamma > 0$ for which $L(e\frac{\gamma}{1+\gamma}) = 1$, although $\lim_{\gamma\to\infty} L(e\frac{\gamma}{1+\gamma}) = L(e) = 1$. In other words, when the penalty goes to infinity the counterfeiter's gain goes to zero as expected.

Theorem 2. $0 < P(x^*)$.

Proof. $P(0) = 0$, $P'(0) = p > 0$ and $0 < x^*$. □

The above is true for the continuous function approximating the realistic $P(x)$, but if $q(1+\gamma) > 1$ then $0 < x^* < 1$, and for all integer x, $P(x) \leq 0$ [KJ]. Note that in reality $q << 1$, and γ is small (e.g. $\gamma = 3$ in triple damage), so $q(1+\gamma) > 1$ doesn't happen. Also, to be precise we have to use the floor notation, ie, $x^* = \min\{\lfloor D/n \rfloor, \left\lfloor \frac{\lambda}{\ln(1-q)} \right\rfloor\}$.

When the counterfeiter has costs $c > 0$ it shifts the $P(x)$ curve down by c without affecting x^*.

3. The economics of the forensic system

We initially analyze a toy problem involving only one counterfeiter ($n = 1$), and later generalize to multiple counterfeiters. We use subscripts w, o, to denote parameter values **w**ith and with**o**ut forensic system, respectively. For $i \in \{w, o\}$, let R_i denote the revenues of the producer. The payoff of the producer due to the introduction of forensics is

$$P_2 = R_w - R_o.$$

3.1. **A Single counterfeiter.** Assuming the counterfeiter maximizes her gain, $R_w = (D - x_w^*)p$, and $R_o = (D - x_o^*)p$, hence $R_w - R_o = p(x_o^* - x_w^*)$. We henceforth write $\frac{\lambda}{\ln(1-q_i)}$ instead of $\left\lfloor \frac{\lambda}{\ln(1-q_i)} \right\rfloor$ (although the purist may want to reinsert the floor notation eg when examining border cases with $D < 1$). In those cases where $D \geq \frac{\lambda}{\ln(1-q_i)}$

$$P_2 = \lambda p\left(\frac{1}{\ln(1-q_o)} - \frac{1}{\ln(1-q_w)}\right)$$

independent of D. Otherwise D is substituted for x_i^*.

We look at $P_2 = R_w - R_o$ in three cases depending on the value of D relative to the interval $[x_w^*, x_o^*]$ (it is always the case that $x_w^* \leq x_o^*$).

(1) If $D < x_w^*$: then the counterfeiter owns the whole market even with forensics, and $P_2 = 0$.
(2) If $x_w^* \leq D < x_o^*$ then $P_2 = (D - \frac{\lambda}{\ln(1-q_w)})p$. which grows with γ. Here the situation is hopeless without forensics, and once we have some forensics the harsher the penalties the higher the gain from forensics.
(3) If $x_w^* \leq x_o^* \leq D$ then $P_2 = \lambda p(\frac{1}{\ln(1-q_o)} - \frac{1}{\ln(1-q_w)})$. which goes down as γ grows. It is a situation where even without forensics we have significant probability (q_o) to detect and punish fraud.

For any fixed value of γ and $q_o = 10^{-6}$ the payoff$=\lambda p(\frac{1}{\ln(1-q_o)} - \frac{1}{\ln(1-kq_o)})$ captures most of its potential value when $k = q_w/q_o \approx 5$ (see Fig. 1 below).

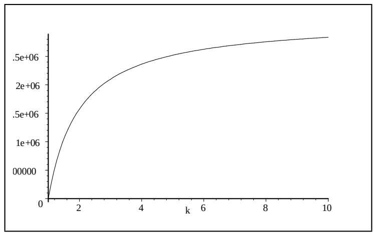

Fig. 1: Payoff as a function of k, for $q_o = 10^{-6}$; $p = 10$; $\gamma = 1$.

3.2. **Many counterfeiters.** Each counterfeiting group sells the optimum $x_i^* = \frac{\lambda}{\ln(1-q_i)}$ copies, as before, independent of n, so n counterfeiting groups sell nx^* (nobody needs more that one copy), and the detection probability for each counterfeiting group is $\pi(x_i^*)$. In the three cases above in the condition clauses we have to substitute D/n for D, and in the payoff expression we need to substitute nx_i^* for x_i^*. The result is:

Theorem 3. *For n counterfeiters: (i) If $D/n < x_w^*$ then $P_2 = 0$, (ii) If $x_w^* \le D/n < x_o^*$: then $P_2 = (D - \frac{n\lambda}{\ln(1-q_w)})p$, which grows with γ, (iii) If $x_w^* \le x_o^* \le D/n$ then $P_2 = \lambda np(\frac{1}{\ln(1-q_o)} - \frac{1}{\ln(1-q_w)})$, which goes down as γ grows.*

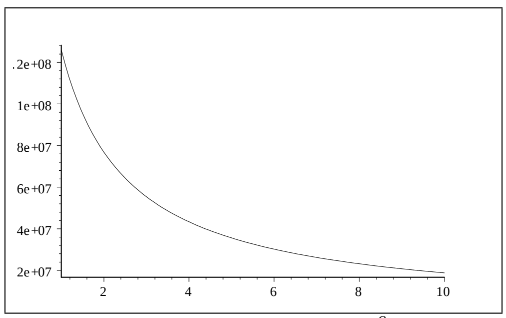

Fig 2: Payoff as a function of γ. For $q_o = 10^{-6}$, $p = 10$, $k = 5$, $n = 50$.

Too many counterfeiters can crowd out the producer and even each other. The average market in which a counterfeiter competes with the producer is of size D/n. When $D < \frac{n\lambda}{\ln(1-q_i)}$ for $i = o, w$ the producer is crowded out. And when $P(D/n) = (1-\pi(D/n))pD/n - \pi(D/n)F = 0$ the counterfeiters crowd out each other and their returns disappear. This happens when $n_{\max} = \frac{D\ln(1-q)}{\ln(\frac{\gamma}{1+\gamma})}$. This can help estimate the legal expenses for prosecuting all the potential counterfeiting groups.

Acknowledgement: Hal R. Varian inspired the direction of this research, and shared with me his yet unpublished insights. Gideon Yaniv brought reference [PS] to my attention, and shared with me his thoughts about the monetization of jail terms. Kamal Jain made an interesting observation that helped clarify Theorem 1. Zoe Krumm, Rich LaMagna, and Alexei Palladin provided insight into the real world of anti-counterfeiting. I am very grateful to all of them.

4. References

[KJ] Kamal Jain, private communication.

[PS] A. Mitchell Polinsky and Steven Shavell: On the Disutility and discounting of Imprisonment and the Theory of Deterrence, *Journal of Legal Studies, vol. XXVIII (January 1999).*

[HV1] Hal R. Varian: Economic Analysis of Copying (unpublished manuscript; July 1996; current version Match 17, 2003).

[HV2] Hal R. Varian: The Social cost of sharing (unpublished manuscript; December 10, 2002).

[YY] Yacov Yacobi: A necessary but insufficient condition for viable protection (a rough draft), submitted to Financial Cryptography'04.

Microsoft Research, One Microsoft Way, Redmond 98052
E-mail address: yacov@microsoft.com