Could The 1-MSB Input Difference Be The Fastest Collision Attack For MD5 ?

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Abstract: So far, two different 2-block collision differentials have been found both with 3-bit input differences for MD5, respectively by Wang etc in 2005 and Xie etc in 2008, and they have been improved later on to generate a collision respectively within around one minute and a half hour on a desktop PC. Can we again find a more efficient collision differential for MD5 ? In this paper, we firstly propose the whole set of 1-bit to 3-bit input difference patterns that are probably qualified to construct a feasible collision differential, and from which a new collision differential with only 1-bit input difference is then developed, finally the performances are compared with the prior two 3-bit collision attacks based on seven criteria. Two-block message, however, is still needed to produce a collision, the first block being only one MSB different while the second block remains the same. Although the differential path appears to be computationally infeasible, most of conditions can be fulfilled by multi-step modifications, and the collision searching efficiency can be much improved further by a specific divide-and-conquer technique, which transforms a multiplicative accumulation of the computational complexities into an addition by properly grouping of the conditional bits. In particular, a tunneling-like technique is also applied to enhance the attack algorithm by introducing some additional conditions. As a result, a currently the fastest attack algorithm is obtained with an averaged computational complexity of $2^{21.3}$ MD5 operations, implying being able to search a collision averagely in one second on a 2.66 Ghz Pentium4 PC for arbitrary random initial values. With a reasonable probability a collision is found within milliseconds, allowing for instancing an attack during the execution of a practical protocol.

Key Words: MD5, Collision Attack, Collision Differentials, Differential Path.

1 Introduction

A hash function is a cryptographic primitive which computes a fixed size message digest from arbitrary size messages. The output value is used usually as the digital digest of the input message, so that a single bit flip in the input would cause averagely a half of the digest bits to change. Therefore, a cryptographic hash function is essentially a type of irreversible one-way functions built with nonlinear operations. MD2, MD4 and MD5 are hash functions that were developed in the early1990's by Ron Rivest at MIT for RSA Data Security. A description of these hash functions can be found in RSA Laboratories Technical Report TR-101 .

This paper mainly focuses on collision attacks on MD5. While it is postulated in RFC [1]that the difficulty of coming up with two messages having the same message digest is on the order of 2^{64} operations, researches on collision attacks have never stopped since the publication of MD5. In 1992, Berson[2] showed that using differential cryptanalysis, it is possible in reasonable time to find two messages that produce the same digest for a single-round MD5. In 1993, Den Boer and Bosselaer[3] found pseudo-collisions for the compression function of MD5 with different initial values but common input. In 1996, Dobbertin[4] constructed collisions of the MD5 compression function, that is, MD5 collisions with a wrong initial value. In 2004, Wang et al.[5,6] succeeded in producing real collisions for the full MD5 hash function as well as collisions in a host of other hash functions including MD4, RIPEMD, and HAVAL-128.This new idea in their approach was to look for a collision after processing not one but two blocks of the message. Again at 2005 CRYPTO conference, Wang et al[7]. detailed the applications of their methods to the hash functions SHA0 and SHA1, with a generated collision for SHA0, and a description on how to obtain collisions in SHA1. Given the variety of hash functions efficiently attacked by Wang et al, it therefore seems worthwhile to seek a complete understanding of how this approach works, how it can be improved, and how it can

be generalized .

Fundamentally, Wang's differential collision attack is a hybrid differential cryptanalysis which takes advantages of both the modular difference and the XOR difference together. Wang et al have found a full two-block collision differential with its full differential path, which is computationally feasible, and for the first time constructed a real collision for MD5. Wang 's attack on MD5 has called its security especially in digital signature into question. Since the publication of [6], quite a number of researchers have worked on the optimization of the differential path and the set of sufficient conditions and hence the collision searching algorithms, resulted in a great improvement on the collision searching efficiency to $2^{24.8}$ MD5 operations as declared in [8], implying that a collision can be found around one minute on a desktop PC. Practical attacks on real protocols and applications based on MD4 family functions have continuously been developed by different applications of Wang's collision. By using an *if-then-else* programming structure, two different Postscript files were created with the same MD5 digest to result in different texts when screening [9], and this attack was extended to other file formats in [10] . By Using Wang's approach to find a near-collision for different IVs and further using different differential paths to absorb the remaining difference, a pair of colliding X.509 certificates for two different distinguished name was found with the same MD5 digest [11]. Other applications of Wang's collision have been proposed to attack HMAC with several hash functions in [12,13].

To date, however, the method used by Wang et al has been fairly difficult to grasp, and furthermore, the lack of some technical details and some small perhaps deliberately made errors (bugs) in the literature [6], might have constituted the appeal to have frustrated other cryptanalysts to grasp their technique. What is really inexplicable consists in that, no new two-block collision differentials have been published to be more efficient since Wang's paper [6], and it seems to remain a supernatural work to find a feasible collision differential and widely considered to be rely on one's experience and intuition. In 2007, an 1-bit input difference was used to construct a new collision attack[14], with a computational complexity of 2^{42} MD5 operations, but no details is known. In the same year, however, we have found the second 3-bit collision differential, which resulted in an initial collision search algorithm with a computational complexity of 2^{36} MD5 operations[15] and an improvement has been made later on to reduce the computational complexity to 2^{30} MD5 operations[16]. The authors of this paper, believed that, nevertheless there must exist other more efficient collision differentials than Wang's. In this paper, however, a whole set of 1-bit to 3-bit input differences is provided for the first time, one of the 1-bit input differences is then present with its full differential path and sufficient conditions. Based on these sufficient conditions, finally a currently the fastest collision searching algorithm is developed with an averaged computational complexity of $2^{21.3}$ MD5 operations, implying that a collision can be found within around one second on a desktop PC.

The rest of this paper is organized as follows: In section 2, the definitions for the XOR difference, the modular difference as well as the signed difference are given, some properties especially with respect to the differential path design and extra condition derivation are presented. In section 3, the basic principle on how to find collision differentials is described, and a whole set of 1-bit to 3-bit input difference patterns is made public for the first time, in which the two published MD5 collision differentials are included, and a new 2-block collision differential with only 1-bit input difference is presented with the design of its full differential path. In section 4, some general and basic principles for differential path design are described, the basic condition derivation rules implicit in the auxiliary functions are presented, some extra conditions for preventing unexpected modular differences are also derived, and a specific divide-and-conquer technique is proposed to reduce the computational complexity. In section 5, a divide-and-conquer based collision searching algorithm is specialized for the 1-bit collision differential, and a tunnel-like technique is applied to enhance the algorithm by introducing some additional conditions. Finally, in section 6, some evaluation criteria on collision differentials are given, and based on these criteria a comparison is made among the three collision differentials, and some suggestions for future researches on hash collision attacks are given. In appendix A, a concise description of the MD5 algorithm is given to help understand this paper.

2 Some Properties of the Signed Difference

In this paper, $<<<^{s}$ denotes a left rotation of a word by s-bit, and '+' denotes an addition $\bmod(2^{32})$, $||$ denotes a concatenation operation, LSB and MSB denote respectively the least and most significant bit of a word.

Let $\mathbf{X}, \mathbf{X}^{\bullet} \in F_2^n$, $\Delta^{\oplus}\mathbf{X}$ denotes the XOR difference defined as a bitwise XOR difference between $\mathbf{X}$ and $\mathbf{X}^{\bullet}$, $\Delta^{+}\mathbf{X}$ denotes the modular difference as a modular integer subtraction between $\mathbf{X}$ and $\mathbf{X}^{\bullet}$, and $\Delta^{\pm}\mathbf{X}$ denotes the signed difference as a bitwise difference between $\mathbf{X}$ and $\mathbf{X}^{\bullet}$. For example, let $n = 10$, $\mathbf{X}$=1001000101, $\mathbf{X}^{\bullet}$=0000111010, $\Delta^{\oplus}\mathbf{X}$, $\Delta^{+}\mathbf{X}$ and $\Delta^{\pm}\mathbf{X}$ are computed, respectively as follows:

$$\Delta^{\oplus}\mathbf{X} = \mathbf{X} \oplus \mathbf{X}^{\bullet} = \Big\|_{i=1}^{n} \mathbf{X}_i \oplus \mathbf{X}_i^{\bullet} = 1001111111;$$

$$\Delta^{+}\mathbf{X} = \left(\mathbf{X} - \mathbf{X}^{\bullet}\right) \bmod\left(2^n\right) = \left(\sum_{i=1}^{n} \mathbf{X}_i 2^{i-1} - \sum_{i=1}^{n} \mathbf{X}_i^{\bullet} 2^{i-1}\right) \bmod\left(2^n\right) = 1000001011;$$

$$\Delta^{\pm}\mathbf{X} = \Big\|_{i=1}^{n} \left(\mathbf{X}_i - \mathbf{X}_i^{\bullet}\right) = 1001\text{-}1\text{-}1\text{-}11\text{-}11.$$

For the sake of simplicity, we omit those "0"s in the signed difference $\Delta^{\pm}\mathbf{X}$, but index the signed difference bits (+1 or -1) with their position identity instead, starting from 0 (the LSB) in $\mathbf{X}$. Using 10-bit word as an example, the signed difference (1001-1-1-11-11) can be indexed as (9,6,-5,-4,-3,2,-1,0).

Theorem 1[17]. Let $\mathbf{X}, \mathbf{X}^{\bullet} \in F_2^n$ with some fixed signed difference, then the XOR difference $\Delta^{\oplus}\mathbf{X}$ and the modular difference $\Delta^{+}\mathbf{X}$ are uniquely determined.

Proof: A more comprehensible proof than that in [17] is given here.

By the definitions of XOR difference $\Delta^{\oplus}\mathbf{X}$ and signed difference $\Delta^{\pm}\mathbf{X}$ between $\mathbf{X}, \mathbf{X}^{\bullet} \in F_2^n$,

$\Delta^{\oplus}\mathbf{X} = \Big\|_{i=1}^{n} \left(\mathbf{X}_i \oplus \mathbf{X}_i^{\bullet}\right) = \Big\|_{i=1}^{n} \left|\mathbf{X}_i - \mathbf{X}_i^{\bullet}\right| = \Big\|_{i=1}^{n} \left|\Delta^{\pm}\mathbf{X}_i\right|$, i.e. the XOR difference $\Delta^{\oplus}\mathbf{X}$ is uniquely determined by $\Delta^{\pm}\mathbf{X}$. For each $\Delta^{\pm}\mathbf{X}_i = \mathbf{X}_i - \mathbf{X}_i^{\bullet}$, $\Delta^{\pm}\mathbf{X}_i$ has three possible cases 0, 1 and -1:

when $\Delta^{\pm}\mathbf{X}_i$=0, we have $\mathbf{X}_i = \mathbf{X}_i^{\bullet}$, which contributes nothing to the modular difference $\Delta^{+}\mathbf{X}$;

when $\Delta^{\pm}\mathbf{X}_i$=1, we have $\mathbf{X}_i = 1$ and $\mathbf{X}_i^{\bullet} = 0$, which contributes 2^{i-1} to $\Delta^{+}\mathbf{X}$;

when $\Delta^{\pm}\mathbf{X}_i$=-1, we have $\mathbf{X}_i = 0$ and $\mathbf{X}_i^{\bullet} = 1$, which contributes - 2^{i-1} to $\Delta^{+}\mathbf{X}$.

Then, we have

$$\Delta^{\pm}\mathbf{X} = \Big\|_{i=1}^{n} \left(\mathbf{X}_i - \mathbf{X}_i^{\bullet}\right) = \Big\|_{i=1}^{n} \Delta^{\pm}\mathbf{X}_i \overset{\bmod\left(2^n\right)}{\equiv} \sum_{i=1}^{n} \Delta^{\pm}\mathbf{X}_i 2^{i-1} \bmod\left(2^n\right) = \Delta^{+}\mathbf{X}.$$

That means, $\Delta^{+}\mathbf{X}$ is uniquely determined by $\Delta^{\pm}\mathbf{X}$. □

Theorem 2. Let $\mathbf{X}, \mathbf{X}^{\bullet} \in F_2^n$, then the modular difference $\Delta^{+}\mathbf{X}$ is equivalent to the signed difference $\Delta^{\pm}\mathbf{X}$ in modulo 2^n, i.e. the $\Delta^{\pm}\mathbf{X}$ is the signed difference representation of the $\Delta^{+}\mathbf{X}$ with the corresponding XOR difference $\Delta^{\oplus}\mathbf{X}$. By formularization, we have

$$\Delta^{+}\mathbf{X}=\left(\mathbf{X}-\mathbf{X}^{\bullet}\right)\bmod\left(2^{n}\right)=\sum_{i=1}^{n}\left(\mathbf{X}_{i}-\mathbf{X}_{i}^{\bullet}\right)2^{i-1}\bmod\left(2^{n}\right)\overset{\bmod\left(2^{n}\right)}{\equiv}\coprod_{i=1}^{n}\left(\mathbf{X}_{i}-\mathbf{X}_{i}^{\bullet}\right)=\Delta^{\pm}\mathbf{X}.$$

Proof: We have the deduction step by step as follows:

$$\Delta^{+}\mathbf{X}=\left(\mathbf{X}-\mathbf{X}^{\bullet}\right)\bmod\left(2^{n}\right)$$

$$=\left(\sum_{i=1}^{n}\mathbf{X}_{i}2^{i-1}-\sum_{i=1}^{n}\mathbf{X}_{i}^{\bullet}2^{i-1}\right)\bmod\left(2^{n}\right)$$

$$=\sum_{i=1}^{n}\left(\mathbf{X}_{i}-\mathbf{X}_{i}^{\bullet}\right)2^{i-1}\bmod\left(2^{n}\right)=\sum_{i=1}^{n}\Delta^{\pm}\mathbf{X}_{i}2^{i-1}\bmod\left(2^{n}\right)\overset{\bmod\left(2^{n}\right)}{\equiv}\Delta^{\pm}\mathbf{X}$$

Thus, theorem 2 is proved! □

Theorem 1 and 2 reveal that a bijective mapping does exist between the signed difference $\Delta^{\pm}\mathbf{X}$ and the XOR difference $\Delta^{\oplus}\mathbf{X}$ plus the modular difference $\Delta^{+}\mathbf{X}$, given $\mathbf{X},\mathbf{X}^{\bullet}\in F_{2}^{n}$.

Proposition 3. Given the modular difference 2^{k} between $\mathbf{X},\mathbf{X}^{\bullet}\in F_{2}^{n}$, there exist $n-k+1$ signed differences that match it; similarly, given the modular difference -2^{k} between $\mathbf{X},\mathbf{X}^{\bullet}\in F_{2}^{n}$, there also exist $n-k+1$ signed differences that match it.

Proof：Consecutively, we can simply have the equivalent transformations as follows:

$$2^{k}=2^{k+1}-2^{k}=2^{k+2}-2^{k+1}-2^{k}=\cdots=2^{n-1}-2^{n-2}-\cdots-2^{k}=-\left(2^{n-1}+2^{n-2}+\cdots+2^{k}\right)\bmod\left(2^{n}\right)$$

$$-2^{k}=-2^{k+1}+2^{k}=-2^{k+2}+2^{k+1}+2^{k}=\cdots=-2^{n-1}+2^{n-2}+\cdots+2^{k}=\left(2^{n-1}+2^{n-2}+\cdots+2^{k}\right)\bmod\left(2^{n}\right)$$

In particular, we always have $2^{n-1}=-2^{n-1}\bmod\left(2^{n}\right)$. Thus, proposition 3 is proved. □

Directly by proposition 3, the following scaling rules for the signed difference notation hold in terms of equivalent modular difference:

$$[k]=[k+1,-k]=[k+2,-(k+1),-k]=\cdots=[n-1,-(n-2),\cdots,-k]=[-(n-1),\cdots,-k];$$

$$[-k]=[-(k+1),k]=[-(k+2),(k+1),k]=\cdots=[-(n-1),n-2,\cdots,k]=[n-1,\cdots,k].$$

For example: 3= -3,4 = -3,-4,5 =-3,-4,-5,-6,-7,-8,9= -3,-4,-5,-6,-7,-8,-9

-7,6,-5,-3,-2,-1,0 = 9,8,7,6,-5,-3,-2,-1,0 =-6,-5,-4,1,0 = -7,4,1,0

Theorem 4. Let $\Delta^{+}\mathbf{X}=2^{k}$, $\Delta^{\pm}\mathbf{X}=[k+l,-(k+l-1),\cdots,-k]$, $1\le l\le n-k-1$. If $k+l+s\le n-1$, then $\left(\Delta^{+}\mathbf{X}\right)^{<<<s}=2^{k+s}\bmod\left(2^{n}\right)$; otherwise, $\left(\Delta^{+}\mathbf{X}\right)^{<<<s}=\left(2^{k+s}+1\right)\bmod\left(2^{n}\right)$.

Similarly, let $\Delta^{+}\mathbf{X}=-2^{k}$, $\Delta^{\pm}\mathbf{X}=[-(k+l),k+l-1,\cdots,k]$, $1\le l\le n-k-1$. If $k+l+s\le n-1$, then $\left(\Delta^{+}\mathbf{X}\right)^{<<<s}=-2^{k+s}\bmod\left(2^{n}\right)$; otherwise, $\left(\Delta^{+}\mathbf{X}\right)^{<<<s}=-\left(2^{k+s}+1\right)\bmod\left(2^{n}\right)$.

Proof: For the first half of this proof, if $k+l+s\le n-1$, then we have

$$\left(\Delta^{+}\mathbf{X}\right)^{<<<s}=[k+l,-(k+l-1),\cdots,-k]^{<<<s}$$

$$=[k+l+s,-(k+l+s-1),\cdots,-(k+s)]\ ;$$

$$=2^{k+s}\bmod\left(2^{n}\right)$$

If $k+l+s>n-1$, then we have

$$\begin{aligned}\left(\Delta^{+}\mathbf{X}\right)^{<<<s} &= \left[k+l,-(k+l-1),\cdots,-k\right]^{<<<s}\\ &= \left[-(n-1),\cdots,-(k+s),k+l+s-n,-(k+l-1+s-n),\cdots,-0\right].\\ &= \left(2^{k+s}+1\right)\bmod\left(2^{n}\right)\end{aligned}$$

Thus, the first half of theorem 4 is proved.

Similarly, the second half can be proved as above. □

Theorem 5. Let $\Delta^{+}\mathbf{X}=2^{k}$, $\Delta^{\pm}\mathbf{X}=\left[-(n-1),\cdots,-k\right]$, then $\left(\Delta^{+}\mathbf{X}\right)^{<<<s}=\left(2^{k+s}-2^{s}+1\right)\bmod\left(2^{n}\right)$.Similarly, let $\Delta^{+}\mathbf{X}=-2^{k}$, $\Delta^{\pm}\mathbf{X}=\left[(n-1),\cdots,k\right]$, then $\left(\Delta^{+}\mathbf{X}\right)^{<<<s}=\left(-2^{k+s}+2^{s}-1\right)\bmod\left(2^{n}\right)$.

Proof: Actually, theorem 5 is the extreme representation of theorem 4.

We have consecutive deduction of the first half as follows:

$$\begin{aligned}\left(\Delta^{+}\mathbf{X}\right)^{<<<s} &= \left[-(n-1),\cdots,-k\right]^{<<<s}\\ &= \left[-(n-1),\cdots,-(k+s),-(s-1),-(s-2),\cdots,-0\right]\\ &= \left(2^{k+s}-2^{s}+1\right)\bmod\left(2^{n}\right)\end{aligned}$$

The second half of theorem 5 can be proved similarly as above. □

Theorem 4 and 5 reveal how the carries due to modular addition or subtraction bring about some unexpected modular difference when bit rotations are applied in the differential path, and from which some extra conditions can be derived to prevent the occurrence of unexpected modular differences.

3 Collision Differential Selection For MD5

3.1 General Principles

Single block or multi-block collision differentials always exist for any iterated hash function based on Merkle-Damgard theory, and the number of collision differentials may be numerous, but finite given the fact that MD5 puts a limit on the length of the message. To carry out a successful collision attack, the first and crucial step is to find an input difference pattern which can be controlled in the differential propagation process, so that the input differences can be eliminated by a single or multiple iterations in the final steps (four steps for MD5).

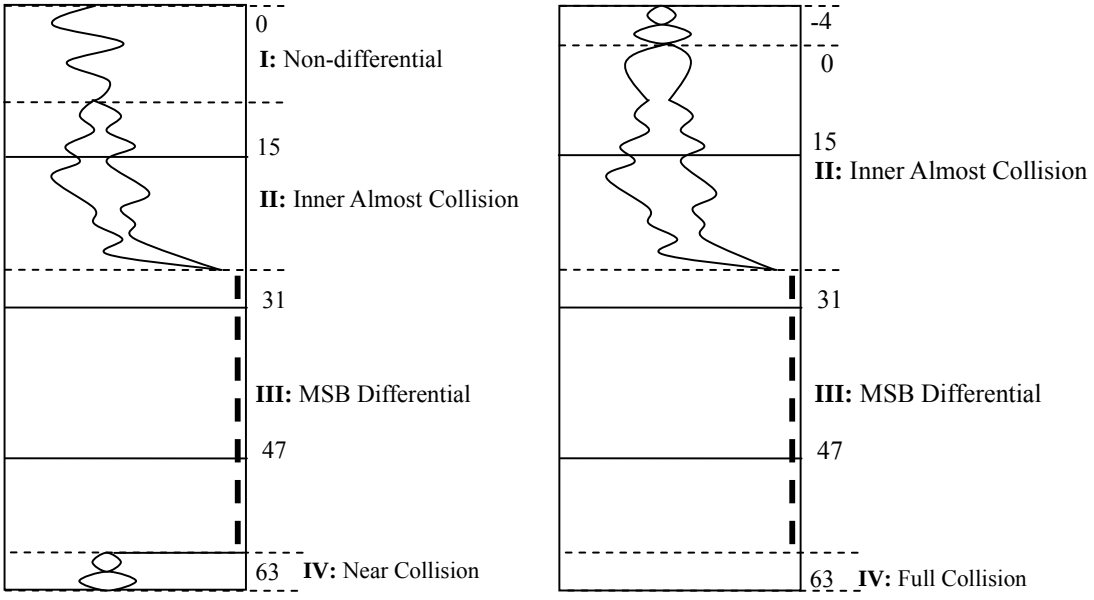
Given an input message difference, if a differential path exists that leads to a collision, then it is called a feasible collision differential, hence a feasible differential path, otherwise an infeasible collision differential and path. If the probability to fulfill the set of necessary conditions that maintain the differential path is computationally feasible, then we call it a computationally feasible collision differential, hence a computationally feasible differential path, otherwise a computationally infeasible collision differential and path. In general, firstly a good collision differential should result in smaller and smaller differences beginning from round 2, so that an elimination of all differentials or most differentials can be achieved in the final round; secondly, the start differences in round 1 should be as far away as possible from the first step to ensure enough free message words in round 1, so that some states in round 2 can also be directly satisfied by these free message words through multi-step modifications. Wang has given the first collision differential [8] which properly meets the principles described above.

3.2 Input Difference Patterns

The first successful attack on the compression function of MD5 was proposed by Dobbertin[4]. The basic idea of Dobbertin's attack, is to describe the whole compression function as a system of equations. As variables these equations include the contents of the registers after various steps and the message words, while the equations are mainly derived from the step operation and the message expansion. Using the concepts of inner collision and inner almost collision, the system of equations can be extremely simplified such that it becomes solvable with some special techniques including

evolutionary approaches. Dobbertin's method can be used to produce real collisions for MD4 and collisions for the compression function of MD5, which is a pseudo collision attack for MD5. Inspired by Dobbertin's attack, Wang developed her new technique of attack on MD4-like hash function, which can produce real collisions for MD5 and other MD4-like hash functions efficiently. The success of both Dobbertin's and Wang's attacks depends on selecting an appropriate input difference pattern so that the number of equations or the avalanche effects of the step operation can be minimized. In Dobbertin's attacks, only 1-bit input difference patterns are considered, while in Wang's attack, 3-bit input difference pattern is used.

By Dobbertin's attack, the system of equations consists of two inner collisions, one starting from step p_0 in the first round and ending at step p_1 in the second round, the other one starting from step p_2 in the third round and ending at step p_3 in the fourth round, both inner collisions are connected by a non-differential chain starting from step $p_1 + 1$ and ending at step $p_2 - 1$. The selection of 1-bit input difference pattern is to minimize the number of equations which depends significantly on the two inner collisions and thus on the choice of the initial bit difference step p_0. Considering the round-wise permutations $\sigma_k(i)$, a complete list of 1-bit difference patterns is given in [17], which shows $p_0 = 15$ would be the best choice in terms of the number of equations to solve, but Dobbertin's choice was actually $m_{14,9}$ with $p_0 = 14$ instead taking into account the inner almost collision.



The First Block The Second Block

Figure1: Overview Of The Differential Collision Attack On MD5

Wang's attack on MD5 completely differs from Dobbertin's in that it uses the first block to produce a near collision which can be further eliminated by the second block to generate a real collision. An input difference pattern should be selected such that in the first block there exist four differential sections, respectively denoted as I,II,III and IV. Section I is a non-differential area without input differences, section II is a near (almost) collision area spanning across the first and second rounds, section III is a non-differential area plus MSB-only differential chain starting from the first input difference of the third round if section II is an inner collision, or a MSB-only differential chain directly derived from section II if section II is an inner almost collision. Section IV is a near collision area consisting of at least the last four consecutive steps, which are summed with the initial variables to be the chain input differences of the second block. Due to the chain input differences, there does not

exist section I in the second block, the chain input differences propagate all the way to the beginning of section II and constitute a lengthened section II starting from step 0, and the section III and IV in the second block correspond to the section III and IV in the first block, except that the differences in the last four consecutive steps are eliminated, i.e. turning a near collision into a full collision. This can be illustrated in figure 1 by an overview of the differential scheme of the collision attack on MD5.

Despite of over 3-bit input differences, if only 1-bit to 3-bit input difference patterns are considered, we can thus make a list of all possible input difference patterns that are probably qualified to construct a feasible collision differential.

Table 1. The Most Probable Input Difference Patterns For Collision Attacks On MD5

Bit Differences	Number	Section I	Section II	Section III	Section IV
$m_{4,20}, m_{7,31}, m_{13,31}$	3-bit	1-4	5-31	32-60	61-64
$m_{6,8}, m_{9,31}, m_{15,31}$	3-bit	1-6	7-25	26-58	59-64
$m_{2,31}, m_{9,27}, m_{12,31}$	3-bit	1-2	3-32	33-63	64-64
$m_{4,31}, m_{11,15}, m_{14,31}$	3-bit	1-4	5-26	27-61	62-64
$m_{2,7}$	1-bit	1-2	3-30	31-62	63-64
$m_{4,20}$	1-bit	1-4	5-24	25-60	61-64
$m_{6,8}$	1-bit	1-6	7-18	19-58	59-64
$m_{9,27}$	1-bit	1-9	10-25	26-63	64-64
$m_{11,15}$	1-bit	1-11	12-19	20-61	62-64
$m_{5,10}, m_{9,27}$	2-bit	1-5	6-25	26-63	64-64
$m_{5,31}, m_{11,31}$	2-bit	1-5	6-21	22-64	
$m_{8,31}, m_{11,21}$	2-bit	1-8	9-31	32-64	
				III + IV = MSB Diff. Chain	
$m_{5,31}$	1-bit	1-5	6-21	22-64	
$m_{8,31}$	1-bit	1-8	9-28	29-64	
$m_{11,31}$	1-bit	1-11	12-19	20-64	
$m_{14,31}$	1-bit	1-14	15-26	27-64	
$m_{5,10}$	1-bit	1-5	6-29	30-64	
$m_{8.25}$	1-bit	1-8	9-30	31-64	
$m_{11,21}$	1-bit	1-11	12-31	32-64	
$m_{14,16}$	1-bit	1-14	15-32	33-64	

1-Bit Input Difference: If only 1-bit input difference is considered, the beginning MSB difference in section III must be derived from section II such that the differential can propagate along all the MSBs in section III to the beginning of section IV. By this specific requirement and some principles in section 3.1, a group of MSB-differences including $m_{5,31}, m_{8,31}, m_{11,31}, m_{14,31}$, and a group of non-MSB differences including $m_{2,7}, m_{4,20}, m_{5,10}, m_{6,8}, m_{8.25}$ $m_{9,27}, m_{11,15}$ and $m_{14,16}$ are probably qualified to construct a feasible collision differential.

2-Bit Input Difference: Since 2-bit input differences can not themselves produce a section III (MSB differential chain) in the third round, at least one MSB difference at the beginning steps in section III must be derived from section II so that 3-bit input differences (1-bit plus 2-bit) can be combined in the third round to form a section III. By this specific requirement and some principles in section 3.1, an

appropriate composition of two 1-bit input difference can be a 2-bit input difference that is probably qualified to construct a feasible collision differential. Basically, there exist three ways of combination which are respective the two 1-bit input differences are selected both from the MSB-difference group, or from the non-MSB difference group, or one from each group. Therefore, many 2-bit input differences can be selected by the way of combination. As for examples, a couple of two MSB-differences $m_{5,31}$ and $m_{11,31}$ or $m_{8,31}$ and $m_{14,31}$, a couple of two non-MSB differences $m_{5,10}$ and $m_{9,27}$, and a couple of MSB-difference $m_{8,31}$ and non-MSB difference $m_{5,10}$ can all be qualified to construct a feasible collision differential. For the sake of limited space, we only give three different compositions in table 1.

3-Bit Input Difference: Since 3-bit input differences can themselves produce a section III (a MSB differential chain) in the third round, no derivation of differences from section II is needed. The 3-bit input differences should be arranged in such a way that, after left rotations the first bit difference in the third round must be a MSB, and this MSB difference is combined with the second and third MSB input differences within four steps to build a consecutive MSB difference, which will propagate along the way to the beginning of section IV. By this specific requirement and some principle in section 3.1, only the corresponding bit differences of $m_2, m_4, m_6, m_9, m_{11}, m_{13}$ and m_{15} in the end of the fourth round can be used to produce the beginning MSB difference in the third round. Due to the round-wise message permutation in the third round, m_2 and m_{15} can not be the beginning MSB difference, while the corresponding bit differences in m_4, m_6, m_9, m_{11} and m_{13} are qualified as the first input difference, namely, $m_{4,20}, m_{6,8}, m_{9,27}, m_{11,15}$ and $m_{13,27}$. Therefore, we have five groups of 3-bit input difference patterns, each consisting of 3 words, each word having 1-bit difference, respectively $m_{4,20}$, $m_{7,31}$ and $m_{13,31}$ constitute the first group, $m_{6,8}$, $m_{9,31}$ and $m_{15,31}$ the second group, $m_{9,27}$, $m_{12,31}$ and $m_{2,31}$ the third group, $m_{11,15}$, $m_{14,31}$ and $m_{4,31}$ the fourth group, and $m_{13,27}, m_{0,31}$ and $m_{6,31}$ the fifth group. Being required of enough free message words in the first round, the fifth group can not be a good collision differential.

For the sake of clarity, we make a list of the whole set of input difference patterns in table 1, that are most probably qualified to construct a feasible collision attack on MD5. In particular, all the collision differentials that have already been published are included in this table. From table 2, we can see that Wang's choice is perhaps the best one in the four groups of 3-bit input differences, the 2-bit input differences are derived from the 1-bit input differences, and can be regarded as compositions of them. What is really noticeable consists in these 1-bit input differences. However, it does not seem obviously which 1-bit input difference is more appropriate for collision differential, before your work is finished.

3.3 Three Feasible Collision Differentials

For comparison, in table 2 we make a list of all the collision differentials that have been published, with their chain output differential together.

Table 2. The Three 2-Block Collision Differentials Published For MD5

Δ^+	No.1 Collision Differential[6]	No.2 Collision Differential[15]	No.3 Collision Differential
$\Delta^+ M_0$	0,0,0,0,2^{31},0,0,0,0,0,0,2^{15},0,0,2^{31},0	0,0,0,0,0,0,-2^8,0,0, 2^{31},0,0,0,0,0, 2^{31}	0,0,0,0,0,0,0,0,2^{31},0,0,0,0,0,0,0
$\Delta^+ M_1$	0,0,0,0,2^{31},0,0,0,0,0,0,-2^{15},0,0,2^{31},0	0,0,0,0,0,0,2^8,0,0, 2^{31},0,0,0,0,0, 2^{31}	0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0
$\Delta^+ H_1$	2^{31}, 2^{31}+2^{25}, 2^{31}+2^{25}, 2^{31}+2^{25}	2^{31}-2^{23}, 2^{31}-2^{23}, 2^{31}-2^{23}, 2^{31}-2^{23}	2^{31}, 2^{31}, 2^{31}, 2^{31}
$\Delta^+ H_2$	0, 0, 0, 0	0, 0, 0, 0	0, 0, 0, 0

4 Design of Differential Paths

4.1 General Principles

When an input difference pattern is found to be probably qualified to construct a feasible collision differential, the next work is to design a feasible differential path which leads to a collision. The basic design criterion is to minimize the Hamming weight of the differential path, which counts the number of bit differences in the differential path, especially the differential section III and IV. In addition, section II is the critical part of the differential path, a successful design of the differential path usually depends on it. When designing a differential path, an intelligent trial-and-error method is necessary in the backward-and-forward construction process. To design a good differential path with as small Hamming weight as possible, the following principles would be benefited from if observed.

1) Deduce a differential path bottom-up in a backward way, starting from the first inner collision in section II (in the second round), up to four or five steps away from the first input difference step in the first round;
2) Deduce the differential up-down from the first input difference step in the first round so that it can link up with the bottom-up differential;
3) In general, the start input difference in section II is applied in the step operation in such a way that, all differences that are needed by the bottom-up differential can be generated within five steps;
4) Employ the properties implicit in the signed difference to extend the signed differences as required in each backward or forward step, and this is the basic rule suitable for any hash functions;
5) Use the generation and elimination rules implicit in the auxiliary functions in each backward or forward step, and these are the special rules derived from the particular hash function.

4.2 MD5 Differential Propagation

A MD5 differential path is composed of 64 consecutive steps of state differences.

Four consecutive signed differences (in order of $\Delta^{\pm}a$, $\Delta^{\pm}d$, $\Delta^{\pm}c$ and $\Delta^{\pm}b$) are employed as inputs to the step operation function to generate the next signed difference $\Delta^{\pm}a^{\bullet}$, we call this computation a step of MD5 differential iteration.

In a MD5 step of differential iteration, the modular differences in the next step can be :

1) Directly derived from the modular difference of $\Delta^{\pm}a$ in state variable a;
2) Directly derived from the modular difference of $\Delta^{\pm}b$ in state variable b;
3) Indirectly generated by the auxiliary function, provided that at least one signed difference exists at the same bit position in the last three state variables b, c and d :
   i) A modular difference can be generated in quite a few ways;
   ii) Actually, modular differences can be generated from the last three state variables b, c and d in an arbitrary way, by utilizing both the properties implicit in the signed difference and the rules implicit in the auxiliary functions;
   iii) Almost all intelligence of differential path designing is focused here, and for the basic differential propagation rules with respect to the four auxiliary functions, please refer to [15].
4) The modular difference generated by the auxiliary function can be used to cancel out those modular differences derived directly from the top or last state variables a and b.

With the properties implicit in the signed difference, in each forward or backward differential iteration step, the critical technique will most probably be, on the one hand, to employ the auxiliary functions to generate those modular differences, required by the next output signed differences $\Delta^{\pm}a^{\bullet}$ but not directly derived from the top or last signed differences $\Delta^{\pm}a$ and $\Delta^{\pm}b$; on the other hand, to employ the auxiliary functions to generate the complementary modular differences for those directly derived from the top and last signed differences $\Delta^{\pm}a$ and $\Delta^{\pm}b$, but not required by the next output signed differences $\Delta^{\pm}a^{\bullet}$, so that two complementary signed differences be eliminated together.

4.3 Basic conditions due to signed differences

A bit that must be specified a value to keep control of the differential path, is called a conditional

bit, a set of bit specifications on all the conditional bits is called sufficient if it will definitely leads to a collision when all are imposed on. In particular, two bits may be relatively specified to include two situations, for example, $a_{i,j} = d_{i,j}$ or $a_{i,j} \neq d_{i,j}$.

All the bit specifications due to the signed difference bits in the state variables are called basic conditions. Every basic condition is incidental to a signed difference bit in a state variable within two steps, in other words, a bit can not become a basic condition if there exist no signed difference bit on the same position in a state variable within two steps. Each state variable works as different component in three consecutive step operations, consequently a bit difference in a state variable will produce at most five basic conditions, which are uniquely determined by the auxiliary function applied. As for the ITE function used in the first round, one condition is the difference bit itself, one or two conditions depend on if there are modular differences derived from when it works as the selection component, two conditions are defined by the bit of being selected or not in the ITE function. For the three different auxiliary functions of MD5, we give the basic condition derivation rules in table 3.

In table 3, each auxiliary function has one bit signed difference at respectively one of the three components, denoted in order as b_i, c_i and d_i. When the component b has signed difference $\Delta^{\pm} b_i$, the other two component bits c_i and d_i must be (or relatively) specified according to the output bit signed difference $\Delta^{\pm} F_i$, $\Delta^{\pm} H_i$ or $\Delta^{\pm} I_i$ as required by the differential path. The condition derivation rules are listed in the columns of every auxiliary function, each having three situations. The '0's in the $\Delta^{\pm} F_i, \Delta^{\pm} H_i$ and $\Delta^{\pm} I_i$ rows represent non-difference output, while the '0's in other rows represent the conditional bit value. The asterisk " * "denotes an arbitrarily specified bit.

Table 3. Basic Condition Derivation Rules For Auxiliary Functions In MD5

$F(b_i, c_i, d_i) = (b_i \wedge c_i) \vee (\overline{b_i} \wedge d_i),\ 0 \le i < 32$												
$\Delta^{\pm} F_i$	0		-1		+1		$\Delta^{\pm} F_i$	0	± 1	$\Delta^{\pm} F_i$	0	± 1
d_i	0	1	0	1	1	0	d_i	*	*	$\Delta^{\pm} d_i$	± 1	± 1
c_i	0	1	1	0	0	1	$\Delta^{\pm} c_i$	± 1	± 1	c_i	*	*
$\Delta^{\pm} b_i$	± 1		-1	+1	-1	+1	b_i	0	1	b_i	1	0
$H(b_i, c_i, d_i) = b_i \oplus c_i \oplus d_i,\ 0 \le i < 32$												
$\Delta^{\pm} H_i$	± 1		∓ 1				$\Delta^{\pm} H_i$	± 1	∓ 1	$\Delta^{\pm} H_i$	± 1	∓ 1
d_i	0	1	0	1			d_i	0 1	0 1	$\Delta^{\pm} d_i$	± 1	
c_i	0	1	1	0			$\Delta^{\pm} c_i$	± 1		c_i	0 1	0 1
$\Delta^{\pm} b_i$	± 1						b_i	0 1	1 0	b_i	0 1	1 0
$I(b_i, c_i, d_i) = c_i \oplus (b_i \vee \overline{d_i}),\ 0 \le i < 32$												
$\Delta^{\pm} I_i$	0		-1		+1		$\Delta^{\pm} I_i$	∓ 1	± 1	$\Delta^{\pm} I_i$	0	± 1 ∓ 1
d_i	0		1		1		d_i	0 0	1 1	$\Delta^{\pm} d_i$	± 1	
c_i	*		0	1	1	0	$\Delta^{\pm} c_i$	± 1		c_i	*	0 1
$\Delta^{\pm} b_i$	± 1		-1	+1	-1	+1	b_i	0 1	1 0	b_i	1	0

By the principles and rules in section 4.1 to 4.3, we give the basic differential paths with respect to the described input bit difference $\Delta^{+} m_8 = 2^{31}$ for two blocks, respectively in table 6 and table 8.

4.4 Extra Conditions Due to Carries and Rotation

Besides the basic conditions that must be fulfilled, some extra conditions must be satisfied to prevent the occurrence of some possible unexpected modular differences due to the carries or overflow. By the theorem 4 and 5 in section 2, unexpected carries and even overflows are always possible when the part $\sum a_i$ of step operation is implemented, since $\sum a_i$ will probably have a much lengthened signed difference representation of a equal modular difference, and probably again the rotation operation will just break off it. Therefore, the set of sufficient conditions must include both the basic conditions and the extra conditions, and fortunately, most of the extra conditions are fulfilled with much high probabilities.

By the theorem 4 and 5, no extra conditions are needed for the differential path of the second block, and the extra conditions for the first block are included with the following groups of equations: $\sum d_{3,6\sim19}=0, \sum a_{4,21\sim24}=0, \sum a_{5,22\sim24}=0, \sum d_{5,29\sim31}=0, \sum c_{5,7\sim17}=0, \sum a_{6,24\sim26}=1,$ $\sum d_{6,11\sim22}=0, \sum c_{6,21\sim31}=0, \sum b_{6,9\sim11}\neq 101, \sum b_{6,15\sim31}=1, \sum a_{7,26}=1$ or $\sum a_{7,24\sim25}=11,$ $\sum c_{7,7\sim17}=0, \sum b_{7,29\sim31}=1$. As for example, $\sum d_{3,6\sim19}=0$ means at least one equation of $\sum d_{3,6}=0, \sum d_{3,7}=0, \ldots, \sum d_{3,19}=0$ must hold.

4.5 Condition Fulfillment: Divide-and-Conquer

There always exist conditions that can not be satisfied by direct modifications, these conditions have to be probabilistically fulfilled through random or brute force search, which compose the computational complexity of collision attack algorithm. For example, if there exist k conditions that can only be probabilistically fulfilled in the round 2, 3 and 4, then the computational complexity will be around 2^k hash operations, which is a multiplicative accumulation on the conditions. One idea is to change the multiplicative accumulation of computational complexities, into an additional accumulation by properly grouping the conditional bits that can not be directly modified, so that the previously fulfilled groups of conditions will not be violated by later searches. This will result in a specific divide-and-conquer technique for hash collision attacks, which will greatly reduce the computational complexity to be determined actually by the maximal group of conditions.

To be more precise, if the k conditions can be divided into p groups, namely $G_1, G_2 \ldots$ and G_p, $\sum_{i=1}^{p}|G_i|=k$ and $G_{\max}=\max\{G_1, G_2, \cdots, G_p\}$, which is the largest group with the most conditions; if groups G_1 to G_i will not be violated by the search of group G_{i+1}'s satisfaction and so on. Then the computational complexity for the k conditions will be reduced to an additive accumulation of the complexities for groups $G_1, G_2 \ldots$ and G_p instead, and the group $G_{\max}$ will be representative of the whole computational complexity, provided that there exist enough free message bits to be searched for each group.

According to the principle that the previously used message words or bits must not be further modified later, these probabilistically satisfied conditions can be grouped mainly by the step orders. In this way, the conditions in round 2,3 and 4 can be divided into three groups, respectively a_5, d_5 and c_5 constitute the first group, b_5, a_6, d_6 and c_6 the second group, finally the state variables from b_6 to the end b_{16} the third group. The first group relies directly on the brute force search on the free bits of a_5, but indirectly on m_1 to fulfill the conditions that can not be satisfied by direct modification in d_5 and c_5. The second group relies directly on the brute force search on the free bits of b_5, but indirectly on m_0 to fulfill the conditions in a_6, d_6 and c_6, which are all probabilistically satisfied. Besides the conditions in a_7 are satisfied directly through a brute force search on the four selected

bits in a_3 but indirectly on m_9, the third group relies mainly on the brute force search directly on the free bits of b_1, but indirectly on m_3, m_4 and m_7 (and m_8, m_9 and m_{12} due to the selected bits search on a_3) to fulfill the conditions in the state variables from b_6 to the end b_{16}.

4.6 Additional Conditions: Change Absorption

Take the first block as an example. In the third group, the brute-force search on the free bits of b_1 will certainly make changes on d_2 and c_2, and indirectly on m_5 and m_6 if the d_2 and c_2 remain the same, this will result in a conflict with the previously used message words m_5 and m_6, since m_5 and m_6 are used respectively in state variables d_5 (in the first group) and a_6 (in the second group). To avoid this type of conflicts, as components of the choose function $F(X,Y,Z)$ the state variables a_2 and d_2 need to be 0x00000000 and 0Xffffffff, respectively, so that the brute force search on the state variable b_1 will be absorbed in the recomputation of d_2 and c_2. For the same reason, the four couples of bits (namely, $d_{3,1}, d_{3,2}, d_{3,17}$ and $d_{3,31}$, $c_{3,1}, c_{3,2}, c_{3,17}$ and $c_{3,31}$) in the state variables d_3 and c_3 need to be 0 and 1, respectively, so that the brute force search on the four free bits of a_3 can be absorbed in the recomputation of d_3 and c_3, resulting in no changes on the previously used m_{10} and m_{11}. The second block is treated in a similar way.

Table 7 and table 9 are obtained by respectively modifying table 6 and table 8 as described above, additional conditions are appended for absorbing the changes due to random or brute-force searches.

5 Collision Search Algorithms

In general, a collision searching algorithm is fundamentally determined by the corresponding differential path, a good differential path will have an intrinsically efficient algorithm. The objective of designing an algorithm for a collision differential path is to reduce the number of probabilistically fulfilled conditions as many as possible, this can be achieved by some methods such as single-step modification, multi-step modification and the tunneling-like techniques. In this paper, by properly grouping of conditional bits, we particularly transform the multiplicative computational complexities into additional accumulation, which is the divide-and conquer technique introduced in section 4.5 and 4.6. As a result, the actual computational complexity is much greatly reduced. The collision searching algorithm, however, is very complex, but we suggest you visit the website (http://www.is.iscas.ac.cn/gnomon) for a personal experience if you are interested in it, where the computational efficiencies of the three collision differentials can be compared on the same machine. For example, a collision pair is given in table 4 with its MD5 digest.

5.1 The Algorithm For The First Block

Step 1: Randomly initialize the state variables c_1 and b_1, set a_2 and d_2 to be 0x00000000 and 0xffffffff, randomly initialize the state variables from c_2 to a_5 but with all the conditions satisfied, check by the extra conditions in section 4.4 if there exist invalid carries in d_3 and a_4, then go to step 1 (do step1 again); otherwise, compute the message words from m_6 to m_{15} according to their corresponding step equations and based on the state variables from c_1 to b_4;

Step 2: Do the brute force search on the free bits of a_5 one binary combination each time, if all binary combinations of the free bits are searched over, then go to step 1;

Step 3: Randomly initialize d_5 but with all its conditions satisfied, compute m_6 according to the d_5 step equation, then make an update of c_2. If there exist conditions unsatisfied in c_2, go to step 2;

Step 4: Compute c_5, if there exist conditions unsatisfied for c_5, then modify b_2 and a_3 or directly c_4 to compute m_{11}, so that the conditions for c_5 can be satisfied from the less significant bit to more significant bits. If $c_{5,20} = 0$ or $c_{5,21} = 1$, go to step 2 since no modifications can be applied; otherwise, initialize b_5 so that its conditions are all satisfied, and compute m_1 according to the a_5 step equation;

Step 5: Do the brute force search on the free bits of b_5 one binary combination each time. Compute m_0 according to the b_5 step equation, then make an update of a_1 and d_1, and compute m_5 according to the d_2 step equation. If all binary combinations of the free bits in b_5 are searched over, go to step 2;

Step 6: Compute a_6, check if there exist any conditions unsatisfied for a_6, then go to step 5;

Step 7: Compute d_6, check if there exist any conditions unsatisfied for d_6, then go to step 5;

Step 8: Compute c_6, check if there exist any conditions unsatisfied for c_6, then go to step 5; otherwise, compute m_2, m_3 and m_4 according to the c_1, b_1 and a_2 step equations;

Step 9: Do the brute force search on the free bits of b_1, compute m_4 according to the a_2 step equation to make an update of b_6. Check if all conditions for b_6 are satisfied, then compute m_3 and m_7; otherwise go to step 9 (do step9 again). If all binary combinations of the free bits in b_1 are searched over, go to step 5;

Step 10: Do the brute force search on the four free bits ($a_{3,1}$, $a_{3,2}$, $a_{3,17}$ and $a_{3,31}$) of a_3 one binary combination each time, compute m_9 according to the d_3 step equation to make an update of a_7. Check if all conditions for a_7 are satisfied, then compute m_8 and m_{12}; otherwise go to step 10. If all binary combinations of the free bits in a_3 are searched over, go to step 9;

Step 11: Compute the next step operation till the last one, check if all the conditions are satisfied, then output the chain variables $a_{16} + a_0$, $b_{16} + b_0$, $c_{16} + c_0$ and $d_{16} + d_0$ to the algorithm for the second block; otherwise, go to step 10.

5.2 The Algorithm For The Second Block

Step 1: Randomly initialize a_1 and b_1 but with all their conditions satisfied; set a_2 and d_2 to be 0x00000000 and 0x7fffffff so that their conditions are satisfied; randomly initialize the state variables from c_2 to a_3 but with all the conditions satisfied; set d_3 and c_3 to be 0x00000000 and 0x7fffffff so that their conditions are satisfied; randomly initialize the state variables from b_3 to c_4 but with all the conditions satisfied. Compute the message words from m_0 to m_{14} according to their corresponding step equations and based on the state variables from a_1 to c_4;

Step 2: Randomly initialize b_4 but with all its conditions satisfied, do the random search on the free bits of b_4 .If the prescribed limit on the number of random search tries is over, then go to step 1;

Step 3: Compute the state variables from a_5 to c_6, check if there exist any conditions unsatisfied for the state variables from a_5 to c_6, then go to step 2;

Step 4: Randomly initialize b_1 but with all its conditions satisfied. Do the brute force search on the free bits of b_1 one binary combination each time, compute m_4 according to the a_2 step equation to make an update of b_6. Check if there exist any conditions unsatisfied for b_6, go to step 4 (do step4 again);

otherwise, compute m_3 and m_7. If the prescribed limit on the number of brute force search tries is over, go to step2;

Step 5: Randomly initialize a_3 but with all its conditions satisfied. Do the brute force search on the free bits of a_3 one binary combination each time, compute m_9 according to the d_3 step equation to make an update of a_7. Check if there exist any conditions unsatisfied for a_7, go to step 5 (do step5 again); otherwise, compute m_8 and m_{12}. If the prescribed limit on the number of brute force search tries is over, go to step4;

Step 6: Compute the next step operation till the last one, check if all the conditions are satisfied, then output the collision blocks; otherwise, go to step 5.

Table 4. A Collision Example With The MD5 Digest (Underlined Bits With Difference)

M_0	0x6f5405b5, 0xb891efe, 0xae153522, 0x3dd541ab, 0x77cfac08, 0xb4ae7077, 0xb14ec779, 0xa7ccf30, 0xf1c56954, 0x70dc3345, 0x5eda46a1, 0xc9fc1730, 0x948b9be, 0x2ef76cad, 0x86149360, 0x3bcecd25
M_1	0x1dea12a, 0x50179204, 0x6a2ab7f9, 0x80e06efa, 0x1da137c9, 0x22032f7e, 0x3af27c94, 0xbfd0dda2, 0x54dd5054, 0xde27de3, 0x328eb6dc, 0x1da31980, 0xf0a9c456, 0x720e6177, 0xe5ac6c8f, 0x15ab7afc
M_0^*	0x6f5405b5, 0xb891efe, 0xae153522, 0x3dd541ab, 0x77cfac08, 0xb4ae7077, 0xb14ec779, 0xa7ccf30, 0x71c56954, 0x70dc3345, 0x5eda46a1, 0xc9fc1730,0x948b9be, 0x2ef76cad, 0x86149360, 0x3bcecd25
M_1^*	0x1dea12a, 0x50179204, 0x6a2ab7f9, 0x80e06efa, 0x1da137c9, 0x22032f7e, 0x3af27c94, 0xbfd0dda2, 0x54dd5054, 0xde27de3, 0x328eb6dc, 0x1da31980, 0xf0a9c456, 0x720e6177, 0xe5ac6c8f, 0x15ab7afc
	MD5 value: 0x281e1404 0x596131cd 0x9cd2262c 0xa5aa822f

5.3 Computational Complexity Analysis

There totally exist 47 and 31 conditions respectively in the first block and the second block starting from the second round, which must be probabilistically fulfilled, the computational complexity would be around 2^{47} and 2^{31} MD5 operations if only multi-step modifications are applied. When the divide-and-conquer technique is applied, these conditions are divided into three groups, each group of conditions are independently and probabilistically fulfilled without violating each other, resulting in a great decrease in the computational complexity. In details, the condition fulfillment in the first block can be divided into four phases as follows:

Phase 1: Phase 1 includes step 1 and step 2. Since in this phase only direct modifications are needed, the computational complexity is a constant defined as C;

Phase 2: Phase 3 includes step 3 and step 4. Direct modifications coexist with probabilistic condition fulfillment in this phase, and only three conditions (one for step d_5 and two for step c_5) are probabilistically fulfilled without violating the previously satisfied conditions in phase 1, resulting in a computational complexity of around 2^3 MD5 operations;

Phase 3: Phase 3 includes step 5 to step 8. In this phase, totally 15 conditions are probabilistically fulfilled without violating the previously satisfied conditions in phase 1 and phase 2, resulting in a computational complexity of around 2^{15} MD5 operations;

Phase 4: Phase 4 includes step 9 to step 11. There totally exist 29 conditions that can only be probabilistically fulfilled without violating the previously satisfied conditions in phase 1, 2 and 3. Since a single try involves (3+3+41) steps of operation, it results in an averaged computational complexity of $2^{29\times\frac{47}{64}} \approx 2^{21.3}$ MD5 operations.

Due to the separation of the four phases above, the total computational complexity for the first block is an additive accumulation of that in all the four phases, which means that the computational complexity is $C + 2^3 + 2^{15} + 2^{21.3}$, instead of a multiplicative accumulation, which would be $C + 2^3 \times 2^{15} \times 2^{29} \approx 2^{47}$. A similar analysis on the second block shows that the averaged computational complexity for the second block is $2^{24\times\frac{47}{64}} \approx 2^{17.6}$ MD5 operations.

6 Summary, Comparison and Suggestions

In this paper, firstly, a whole list of 1-bit to 3-bit eligible input differences is presented, and the supernatural appearance of collision differential selection is thus unclosed. Secondly, a new 1-bit input difference pattern is developed to be the currently fastest collision attack algorithm for MD5, with an averaged computational complexity of $2^{21.3}$ MD5 operations, implying that a common desktop PC can produce a MD5 collision within around one second. Thirdly, a divide-and-conquer technique specific for hash collision attacks is proposed with a concrete application of it. Finally, some technical details related to the derivation of the basic conditions and the extra conditions are presented. This paper will help the cryptology community to further grasp the recent techniques on hash cryptanalysis.

A collision differential can be evaluated according to the following five criteria:

1) Whether the differential path depends on the fixed IVs of hash function or not?
2) The number of message blocks comprising of the collision differential;
3) The number of free words in the message;
4) The number of bit differences in the messages;
5) The number of all sufficient conditions which must be satisfied to make collision;
6) The number of all conditions excluding the first round;
7) The averaged computational complexity of finding a collision.

Considering the real-world cryptanalytic attacks, a differential path which does not rely on the fixed initial IVs will obviously be better than that must rely on it, a collision differential which has more free words, less input differences and sufficient conditions will be more easily used to construct meaningful attacks, a collision differential with less message blocks and probabilistically fulfilled conditions will be more efficient for practical attacks. The less the conditions necessary to maintain the full differential path, the higher the density of collision message will be; the less the average computational complexity of finding a collision, the more feasible an attack on practical protocols based on hash function will be. For the three collision differentials that have been published, we make a comparison in table 5 based on the above criteria. From table 5, the 1-bit input difference exceeds the other two 3-bit input differences, in terms of free message words, bit differences, sufficient conditions and especially the computational complexity.

Table 5. Performance Comparison For The Three Collision Differentials

Items	No.1 [6]	No.2 [16]	No.3	Comments
depend on fixed IVs	not	not	not	IVs are free
number of blocks	2	2	2	2-block collision
number of free words	1~2	1~4	1~6	Steps indexed by 1~64
number of diff. bits	3-bit/3-bit	3-bit/3-bit	1-bit/0-bit	No.3 is specific
number of all conditions	290 / 309	205 / 306	264 / 47	exclude extra conditions
number of prob. conditions	43 / 36	38 / 35	84 / 31	exclude extra conditions
computational complexity	$2^{24.8}$	2^{30}	$2^{21.3}$	averaged
time / a collision (averaged.)	1 min.	30 min.	1 sec.	2.66 GHZ PC

By the seven criteria above, in this paper the 1-MSB input difference $m_{8,32}$ may not be the best choice for the MD5 collision differential, probably there exists better choice from the other input

differences in table 1, as the $m_{8,32}$-based collision differential was developed before the whole set of input differences is found. Hence, a continue work on finding a more efficient input difference from the table 1 is suggested and encouraged, perhaps a collision attack algorithm based on a better input difference can generate hundreds of collisions one second.

While it will no longer be regarded as a more or less supernatural work that mainly relies on one's intuition since this paper, it probably remains a challenging work to design a good differential path. Despite of some initial work in this direction[14,18], it is worth a further and deep study on how to intelligently and automatically design a good differential path, perhaps some heuristic methods like evolutionary approaches may help a lot.

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Appendix A MD5 Function

Practically, a Merkle-Damgard structure-based hash function is iterated by a compression function $Y = f(X)$, which compresses l-bit message block X to a s-bit hash value Y, where $l > s$. For MD5, $l = 512$, $s = 128$. For a padded message M with multiple (t) of l-bit blocks, the iteration process can be described as: $H_{i+1} = f(H_i, M_i)$, $0 \le i \le t-1$, where $M = (M_0, M_1, ..., M_{t-1})$, H_i is the 128-bit chaining variables (including four 32-bit words) which is updated during the processing of each block, H_0 is the prescribed initial value IVs in MD5 algorithm, and the final H_t is the digest that we expect to obtain. The concrete padding rule is omitted here, since it has no influence on our attack.

The whole processing of the ith block $f(H_i, M_i)$ can be defined as follows: $H_{i+1} = f(H_i, M_i) = H_i + II(M_i, HH(M_i, GG(M_i, FF(M_i, H_i))))$, where four round functions FF, GG, HH and II are involved. All round functions are similar to one another in structure. The chaining variable H_i is treated as a four-element shift register, with each element being one 32-bit word wide, referred to as a_0, b_0, c_0 and d_0, respectively. Each 512-bit block M_i is divided into 16 32-bit words, denoted as $M_i = (m_0, m_1, ..., m_{15})$, each round consists of 16 steps of operation, in each step operation the register is used with one word from M_i. The 64 step operations are formulated as a system of equations: $a_{i+1} = b_i + \left((a_i + \Phi_j(b_i, c_i, d_i) + w_j + t_j)^{<<<s_j}\right)$, $0 \le i \le 16$, $1 \le j \le 64$. Where a_i, b_i, c_i and b_i are the internal state variables, with $1 \le i \le 16$; $\Phi_j(X, Y, Z)$ is an auxiliary function which varies from round to round; w_j is a word chosen from $(m_0, m_1, ..., m_{15})$ by a round-wise message permutation $\sigma_k(i)$, $k = 0,1,2,3$, $i = 0,1,...,15$; t_j and s_j are constant parameters associated with step j. Note that each step operation involves four modular additions ($\mathrm{mod}\, 2^{32}$), one auxiliary function and one <<< operation. As the step operation of MD5 is reversible, the compression function $f(H_i, M_i)$ uses a feed-forward operation which adds the initial value H_i of the register to their final values, so that $f(H_i, M_i)$ cannot be inverted.

The auxiliary function and the round-wise permutation $\sigma_k(i)$ for each round are given as follows:

$$\begin{aligned}
&\Phi_j(X,Y,Z) = F(X,Y,Z) = (X \wedge Y) \vee (\overline{X} \wedge Z), 1 \le j \le 16;\\
&\Phi_j(X,Y,Z) = G(X,Y,Z) = (X \wedge Z) \vee (Y \wedge \overline{Z}), 17 \le j \le 32;\\
&\Phi_j(X,Y,Z) = H(X,Y,Z) = X \oplus Y \oplus Z, \ 33 \le j \le 48;\\
&\Phi_j(X,Y,Z) = I(X,Y,Z) = Y \oplus (X \vee \overline{Z}), 49 \le j \le 64.
\end{aligned}$$

$$w_{j+1} = \begin{cases} m_j, & 0 \le j < 16;\\ m_{1+5j \bmod 16}, & 16 \le j < 32;\\ m_{5+3j \bmod 16}, & 32 \le j < 48;\\ m_{7j \bmod 16}, & 48 \le j < 64. \end{cases}$$

Where X, Y, Z are 32-bit words. The auxiliary functions $\Phi_j(X, Y, Z)$ each takes three consecutive 32-bit words from the register of chaining variables and produces one 32-bit word as output. The four words in the chaining variable register are initialized as

$a = 0x67452301$, $b = 0xefcdab89$, $c = 0x98badcfe$, $d = 0x10325476$.

For the sake of understanding how and where some extra conditions are derived from late in section 5, which are used to prevent the possible unexpected modular differences due to the joint effect of both modular addition and left rotation, we define part of the step operation as $\sum a_{i+1} = a_i + \Phi_j(b_i, c_i, d_i) + w_j + t_j$.

For a detailed description of MD5 algorithm, please refer to [1].

Appendix B The Differential Paths

Table 6: The Basic Differential Path Using $\Delta^+ m_8 = 2^{31}$ (Block1).

t	Bits Qt: $a_0 \ldots a_{31}$				#
1~6	********	********	********	********	0
7	******1*	********	********	********	1
8	****0*0*	********	**0*****	****1***	4
9	****1^+^	^^^**0*1	0*0^^^^^	^^^^0^^*	23
10	****01--	--+**1^0	1*++++++	++++++-*	24
11	^**^11+0	001^^+++	-1-00111	0000000*	29
12	+**-++-0	110++000	00+++-1	1000101*	29
13	0**0011*	1**00111	0-10000*	****+01^	22
14	11*1110*	+**10***	*011111*	***00--+	20
15	*1*-0*00	+*******	*1****1*	***01000	13
16	*-1*1*10	0*******	*0000*+*	***-*001	15
17	**0^+*+-	0*******	*11111**	^*******	12
18	*^******	*01****0	*---+1^*	***^****	11
19	****^*^^	*11****0	***01+**	00******	11
20	********	*+****-	*^^10***	11******	9
21	***0****	********	****^**	--******	4
22	***1****	*^^****^	********	*****00*	6
23	***+****	********	********	^^****11*	5
24	********	********	********	*****+**	2
25	***^****	********	*0******	********	2
26	********	********	*1******	*****^^*	3
27	********	********	*+******	********	1
28	********	********	********	********	0
29	********	********	*^******	*******0	2
30	********	********	********	********	0
31~47	********	********	********	*******-	0
48~55	********	********	********	*******-	8
56	********	********	********	*******+	1
57	********	********	********	*******-	1
58	********	********	********	*******+	1
59	********	********	********	*******-	1
60	********	********	********	*******+	1
61	********	********	********	*******-	1
62	********	********	********	*******+	1
63	********	********	********	*******-	1
64	********	********	********	*******+	0

Table 7: The Basic Differential Path Using $\Delta^+ m_i = 0, 0 \le i < 16$ (Block2).

t	Bits Qt: $a_0 \ldots a_{31}$				#
-3	********	********	********	*******+	0
-2	********	********	********	*******+	0
-1	********	********	********	*******+	1
0	********	********	********	*******+	1
1~31	********	********	********	*******+	31
32~47	********	********	********	*******+	0
48~63	********	********	********	*******+	16

Table 8: The Modified Differential Path With Additional Absorbing Bits (Block1).

t	Bits Qt: $a_0 \ldots a_{31}$				#
1~4	********	********	********	********	0
5_t	00000000	00000000	00000000	00000000	32
6_t	11111111	11111111	11111111	11111111	32
7	******1*	********	********	********	1
8	****0*0*	********	**0*****	****1***	4
9	****1^+^	^^^^*0*1	0*0^^^^^	^^^^0^^*	23
10_t	*00*01--	--+**1^0	10++++++	++++++-0	28
11_t	^11^11+0	001^^+++	-1-00111	00000001	32
12	+**-++-0	110++000	00++++-1	1000101*	29
13	0**0011*	1**00111	0-10000*	****+01^	22
14	11*1110*	+**10***	*011111*	***00--+	20
15	*1*-0*00	+*******	*1****1*	***01000	13
16	*-1*1*10	0*******	*0000*+*	***-*001	15
17	**0^+*+-	0*******	*11111**	^*******	12
18	*^******	*01****0	*---+1^*	***^****	11
19	****^*^^	*11****0	***01+**	00******	11
20	********	*++****-	*^^10***	11******	9
21	***0****	********	*****^**	--******	4
22	***1****	*^^****^	********	*****00*	6
23	***+****	********	********	^^****11*	5
24	********	********	********	*****++*	2
25	***^****	********	*0******	********	2
26	********	********	*1******	*****^^*	3
27	********	********	*+******	********	1
28	********	********	********	********	0
29	********	********	*^******	*******0	2
30	********	********	********	********	0
31~47	********	********	********	*******-	0
48~55	********	********	********	*******-	8
56	********	********	********	*******+	1
57	********	********	********	*******-	1
58	********	********	********	*******+	1
59	********	********	********	*******-	1
60	********	********	********	*******+	1
61	********	********	********	*******-	1
62	********	********	********	*******+	1
63	********	********	********	*******-	1
64	********	********	********	*******+	0

Table 9: The Modified Differential Path With Additional Absorbing Bits (Block2).

t	Bits Qt: $a_0 \ldots a_{31}$				#
-3	********	********	********	*******+	0
-2	********	********	********	*******+	0
-1	********	********	********	*******+	1
0	********	********	********	*******+	1
1~4	********	********	********	*******+	4
5_t	00000000	00000000	00000000	0000000+	32
6_t	11111111	11111111	11111111	1111111+	32
7~9	********	********	********	*******+	3
10_t	00000000	00000000	00000000	0000000+	32
11_t	11111111	11111111	11111111	1111111+	32
12~31	********	********	********	*******+	20
32~47	********	********	********	*******+	0
48~63	********	********	********	*******+	16

Notes: In tables 6,7,8 and 9, + denotes a positive flip(0->1), - denotes a negative flip(1->0), 0(1) denote the conditional bit value, ^ denotes the bit equal to the up bit, ! denotes the bit not equal to the up bit, * denotes free bit, t denotes the step, # denotes the number of conditions for each step.