Scalable Compilers for Tree Based Key Establishment - Two/Three Party to Group

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Abstract. In this paper we present the first scalable and efficient generic compilers for the construction of tree based group key exchange(TGKE) protocols from two/three party key exchange(2-KE/3-KE) protocols. Our compiler requires a total of $\mathcal{O}(n \lg n)$ communication as compared to the existing compiler for circular topology where the communication cost is $\mathcal{O}(n^2)$. By combining the compiler presented here with the techniques in [18], a scalable compiler can be obtained that transforms any 2-KE/3-KE protocol, secure against a passive eavesdropper to an authenticated TGKE, secure against an active adversary. As an added advantage our compiler can be used in a setting where there is asymmetric distribution of computing power. We also present a compiler for 3-KE to group key exchange(CGKE) transformation for circular geometry. Finally, we present a constant round authenticated Group key exchange(2-TAGKE) obtained by application of our compiler to Diffie-Hellman protocol. We prove the security of our compilers in Real or Random model and do not assume the existence of random oracles.

Keywords. Group Key Exchange, Compilers, Tree Based Key Exchange, Scalability.

1 Introduction

Secure communication over an insecure channel that might be under the control of an adversary is one of the fundamental goals of cryptography. Encryption and authentication are the tools for achieving this goal. Public key based encryption and signature schemes can be used for achieving this, but these primitives are very costly and may not fit for some applications. As an alternative concerned parties may establish a secret key using key exchange(KE) protocols and then use this key to derive keys for symmetric encryption and message authentication schemes. Group key exchange protocols(GKE) aim at establishment of a secret key among a group of n users over an insecure channel. With the increase in use of applications like encrypted group communication for audio-video conferences, chat systems, computer supported collaborative workflow systems etc., group key exchange protocols are gaining more and more attention. Naive approach for designing these GKE is to have a group leader who will choose the secret key and exchange it with the next user, who will exchange with the next etc., but the round complexity for this is $\mathcal{O}(n)$ which is very inefficient and hence non-scalable. Burmester-Desmedt [5][7][6] gave constant round group key exchange protocols, but the protocols given are unauthenticated.

Compilers are tools used for adding extra features to existing KE schemes. As two-party KE protocols are quite well studied in literature [20–29] and there exist efficient three-party KE protocols [30], compilers for transforming 2-KE and 3-KE to GKE can be quite useful in practice. [1] gives one such compiler for transforming secure 2-KE to secure GKE for circular topology. Tree based topology is more common in practice (Consider the case of an organization with a CEO at the top having some managers under his control who further have many subordinates and so on, and the group wishes to communicate securely), also tree based group key exchange protocols are more efficient as they have lesser communication and computation complexity. To our knowledge, no efficient compiler for transformation of 2-KE/3-KE to tree based GKE exists. In this paper we present the compilers for transforming 2-KE/3-KE to tree based GKE.

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Related work. Several Diffie-Hellman based [5][6][7][8][9] and pairing based [11][12][13][14] group key exchange protocols have been proposed. Bresson et. al. [19] gave a GKE (BCPQ) that required n rounds and $\mathcal{O}(n)$ communication and computation. Boyd-Nieto [32] gave constant round group key exchange protocols (BN PK and BN DH) but the length of messages communicated in their protocols is $\mathcal{O}(n)$. Burmester-Desmedt [5], [7] presented a constant round GKE (BD-I) with constant message size for circular topology. Burmester-Desmedt [6] also gave a protocol (BD-II) for tree based settings. But the protocols of Burmester-Desmedt are not authenticated, also BD-II is not contributory. Katz and Yung in [15] gave a compiler for transforming any secure GKE into an authenticated group key exchange (AGKE) protocol, which requires $\mathcal{O}(n)$ signature verifications per user and adds one more round to the protocol. Katz and Shin [16] gave a compiler for adding UC composibility and mutual authentication, Bresson et. al.[17] gave a compiler which provides authentication as well as mutual authentication. Recently, an improvement of [15] for tree based GKE was proposed by Desmedt et. al. [18]. Most of the tree based key exchange protocols have complexity $\mathcal{O}(\lg n)$ but [15] adds $\mathcal{O}(n)$ computation as each user has to perform $\mathcal{O}(n)$ verifications, while [18] adds only $\mathcal{O}(\lg n)$ computation thereby keeping the complexity class intact. Mayer and Yung [31] first considered the expansion of autheticated 2-party key transport to authenticated group key transport, but the length of messages communicated in [31] is $\mathcal{O}(n)$. Hwang et. al.[1] gave a compiler for transforming any secure 2-KE protocol to a secure GKE assuming that group members are arranged in a circular fashion and each member knows the relative position of all other members. Compiler in [1] requires $\mathcal{O}(n^2)$ communication, besides this the resulting protocol is not authenticated. Authors of [1] suggest the use of compiler in [15] for making the resulting protocol authenticated, which in turn adds $\mathcal{O}(n)$ verification per user. In a similar work for password authenticated key exchange(PAKE), Abdalla et.al. [2] gave a compiler from 2 party PAKE to password authenticated group key exchange(GPAKE). They prove the security of their scheme in a stronger model -'real or random model'(ROR). Wu et. al. [3] extended the work in [2] to consider group dynamism also. Although the protocols resulting from [3] are authenticated, they do not introduce any means for construction of unique session identifier and hence [3] is prone to replay attack that we show in our paper. For avoiding this attack a unique session identifier should be established for every new instance of the protocol similar to [15].

Our Contribution: In this paper we present compilers for transformation of secure 2/3 KE to TGKE secure against passive eavesdroppers. In the existing works only circular topology was considered but it is important to consider such compilers for tree based protocols also, as tree based topology is very common in practice. We prove the security of our compiler in ROR model and do not use any random oracle assumption. Our compiler requires only $\mathcal{O}(n \lg n)$ communication and computation. For achieving authentication we use techniques of Desmedt et. al. [18](it keeps the overall complexity $\mathcal{O}(n \lg n)$ only). We present a replay attack on the protocol obtained by compiler of Wu et. al.[3]. Further we extend the compiler in [1] for converting any 3-KE into GKE for circular topology. We also present a constant round authenticated tree based group key exchange protocol(2-TAGKE) obtained by applying our compiler to Diffie-Hellman protocol. Finally, we compare the performance of our compilers and protocols with the existing ones.

Paper Organization: Rest of the paper is organized as follows. In section 2, we discuss security model and efficiency requirements of group key exchange. In section 3, we give a replay attack on the compiler in [3]. In section 4, we present compilers for transformation of 2/3 KE to tree based GKE(2-TGKE, 3-TGKE and 3-CGKE). In section 5, we prove the security of our compilers in ROR model. In section 6, we present a constant round authenticated tree based group key exchange protocol obtained by applying our compiler to Diffie-Hellman key exchange protocol. Section 7, gives efficiency comparison of our compilers and protocol with the existing ones. Finally we conclude the paper in section 8.

2 Security models for Group key exchange

In this section we give security models and efficiency goals for group key exchange Protocols. We follow the model of Katz and Yung [15] who have used the security model for GKE due to Bresson et al. [19]. However our model differs from [15] as we allow multiple test queries. Allowing multiple test queries is a frequently used technique in password authenticated group key exchange(PAKE) and is commonly referred to as real or random(ROR) model. It should be noted that ROR model is a stronger model as it considers a stronger adversary who is capable of asking many test queries. It can be proved that ROR model is equivalent to the prevalent model with a loss of factor r equal to the number of protocol instances given to adversary. First

we discuss the notations and then we briefly discuss the various oracles which a GKE adversary has access to. Finally we recall the efficiency concerns for the GKE.

Participants and Initialization: We assume that set of participants $\mathcal{P}$ is a polynomial size set, any subset of $\mathcal{P}$ can establish a session key. For the authenticated version of our protocol we further assume that during the initialization phase each participant runs an algorithm $\mathcal{G}(1^k)$ to generate a pair of public and private keys (PK, SK). The secret key is stored by the user and public keys are made available to all the members.

Adversarial Model: Each participant is given access to an unlimited number of instances of the protocol. We denote the instance i of user U as Π_U^i, each instance can be used only once. As in [15] each instance Π_U^i has variables $state_U^i$, $term_U^i$, acc_U^i, $used_U^i$, partner id pid_U^i, session id sid_U^i and session key sk_U^i associated with it. Similar to [18], we define gid and rel_U^i. gid is the group identifier contained in pid_U^i, which identifies all the partners involved in the current execution of the protocol. The set $rel_U^i = \{V_1, V_2, \ldots, V_t\}$ is the set of users whose input is processed by U (including U itself).
Adversary is assumed to have full control over the communication channel. As in [15] and [18] we too do not consider malicious insiders. Different adversarial capabilities are captured by giving him access to following oracles:-

1. **Execute**$(U_1, U_2, \ldots, U_n)$ This oracle models a passive eavesdropper. GKE is executed between the unused instances of $U_1, U_2, \ldots, U_n \in P$ and the transcript of the execution is returned as the output. Adversary has control over the number of players and their identities.

2. **Send**(U_1, i, M) This oracle models an active adversary. It sends message M to the instance $\Pi_{U_1}^i$ and outputs the reply generated by the instance. This oracle can also be used to prompt $\Pi_{U_1}^i$ to initiate protocol with the unused instances of the users $U_2, U_3, \ldots, U_n$ by calling $Send(U_1, i, (U_2, \ldots, U_n))$.

3. **Reveal**(U, i) Secret key sk_U^i is returned as the output.

4. **Corrupt**(U) Long term secret key SK_U of user U is given as output.

5. **Test**(U, i) This query is allowed only if the session key is defined(i.e. $acc_U^i = true$ and $sk_U^i \neq NULL$) and instance Π_U^i is fresh(we define freshness of instance below). Challenger selects a random bit $b \in \{0, 1\}$ prior to the first call. It returns the session key sk_U^i if $b = 0$. Otherwise a uniformly chosen random session key is returned. Similar to [2] we allow an arbitrary number of test queries, but once the test oracle returned a value for an instance Π_U^i, it will return the same value for all instances partnered with Π_U^i(see the definition of partnering below).

A passive adversary is given access to Execute, Reveal, Corrupt and Test oracles, while an active adversary is additionally given access to Send oracle.

Partnering. As in [15] the session ID sid_U^i equals the concatenation of all messages sent and received by Π_U^i during the course of its execution. While for partner ID we follow the approach of [18], partner ID pid_U^i consists of the group identifier and the identities of the player in the group with which Π_U^i exchanges messages during the protocol execution. Two instances Π_U^i and Π_U^j are said to be partnered if (1) session ID of both the instances are equal, (2) each of the player belongs to the partner ID of the other and (3) gid for both the instances is same.

Correctness. For correctness we require that for all partnered instances Π_U^i, $\Pi_{U'}^j$ such that $acc_U^i = acc_{U'}^j = TRUE$, same valid session key $sk_U^i = sk_U^j$ is established.

Freshness. An instance Π_U^i is fresh unless one of the following is true (1) at some point, the adversary queried $Reveal(U, i)$ or $Reveal(U', i')$ for any $\Pi_{U'}^{i'}$ partnered with Π_U^i or (2) a query $Corrupt(V)$ was asked

before a query of the form $Send(U', i', *)$ by V, where V and U' are in pid_U^i.

Security. Let $Succ$ be the event that the adversary queries $Test$ oracle only on fresh instances and guesses correctly the bit b used by the $Test$ oracle. The advantage of adversary $\mathcal{A}$ against protocol P is defined as:

$$Adv_{\mathcal{A},P}\left(1^k\right) = |2.Pr\left[Succ\right] - 1|.$$

A protocol P is a secure GKE if it is secure against a passive adversary, i.e. for any PPT adversary $\mathcal{A}$ the advantage $Adv_{\mathcal{A},P}\left(1^k\right)$ is negligible. Protocol P is a secure authenticated GKE(AGKE) if it is secure against an active adversary. We use $Adv_P^{KE}\left(t, q_{ex}, q_t\right)$ to denote the maximum advantage of any passive adversary attacking P, running in time t, asking q_{ex} $Execute$ queries and q_t $Test$ queries. For the AKE we use $Adv_P^{AKE}\left(t, q_{ex}, q_s, q_t\right)$ where q_s is number of $Send$ queries.

Efficiency Concerns. Number of players in the GKE can be quite large, therefore efficiency is a major concern. For GKE to be scalable it should be constant round, should have minimum possible communication(number of messages exchanged) and computational complexity. We consider both overall complexity and average complexity per user(It should be noted that our protocols are more valuable in the case when a subset of users has access to lesser computational resources).

3 A replay attack:

In this section we give a replay attack on the protocols produced by [3] (although they do not claim the resilience against replay attacks, it is an important requirement and can easily be achieved by establishing a unique session ID for every different run of the protocol). For the description of the protocol we refer the reader to [3]. [3] extends the work of Abdalla et. al. [2] to consider the dynamism also but they do not use the unique randomness as used in [2]. This is why, [2] is secure against replay attacks while [3] is not.
Let P be the underlying 2-KE(or 2-PAKE) which is authenticated and secure. The scheme in [3] do not give any description of the unique session identifier for the GKE protocol, therefore the protocol obtained is prone to replay attack. Let $\mathcal{A}$ be an active adversary. First $\mathcal{A}$ observes the honest run of the protocol between the members $U_1, U_2, \ldots, U_n$ and saves the values $X_1, X_2, \ldots, X_n$ broadcasted by the users. If every member has broadcasted the right value and the protocol is successful then, $X_1 \oplus X_2 \oplus \ldots \oplus X_n = 0$. $\mathcal{A}$ waits for the initiation of another run of the protocol between the same set of users and allows the first round to complete without any interruption. During the second round when U_1 broadcasts $X_1^{'}$, $\mathcal{A}$ replays $X_2^{'}, X_3^{'}, \ldots, X_n^{'}$ on user U_1 and lets all other members operate according to the protocol. Here,

$$X_2^{'} = X_2 \oplus X_1,\ X_n^{'} = X_n \oplus X_1^{'} \text{ and } X_i^{'} = X_i \text{ for all other } i.$$

As $X_1^{'} \oplus X_2^{'} \ldots \oplus X_n^{'} = 0$, replayed values pass the verification test of U_1. U_1 computes the secret key according to replayed values, while all others compute the intended secret key. We show that $\mathcal{A}$ has a non negligible advantage in the security game in ROR model. First $\mathcal{A}$ asks the $Execute(U_1, U_2, \ldots, U_n)$ and stores the transcript which contains all the X_i for $i = 1, \ldots, n$. Then it starts another run of the protocol and replays the messages as above using the proper $Send$ queries. $\mathcal{A}$ then asks $Test$ oracle to U_1 and another fresh instance(Note that instance corresponding to U_1 is also fresh as no $Reveal$ or $Corrupt$ query has been asked on this instance). If the randomly chosen bit b of the simulator is 0 then both of them output different value because session keys computed by both of them are different. Otherwise they output the same random value. Therefore, comparing the output of the $Test$ oracles, $\mathcal{A}$ can guess the correct value of bit b and thus can win the game in all the cases.
This attack can be avoided by having a unique session identifier(session ID) for each new run of the protocol and sending it with all the messages so that messages of one session cannot be replayed in some other session.

4 Efficient Compilers from 2/3 KE to GKE

In this section we present compilers for transformation of secure 2/3 KE into secure GKE. First, we present a compiler(2-TGKE) for transformation of 2-KE to a tree based GKE, then we present a similar compiler(3-

TGKE) which uses 3-KE as the basic protocol. Finally, we give a compiler(3-CGKE) for converting 3-KE to GKE in circular topology. Protocols obtained by our compilers are not authenticated but authenticated protocols can be obtained by using the techniques in [18] for tree based protocols (It should be noted that using the techniques in [18] keeps the complexity of the protocol intact). For circular geometry, compiler in [15] can be used. Our compiler follows the design similar to [1],[2],[3] but it should be noted that we are the first to consider such compilers for tree based settings and 3-party key exchange protocols. We follow the approach of [3] to consider group dyanmism. For tree based protocols we assume that users join and leave at lowermost level only so as to maintain the almost complete structure. Due to space constraint we give *Join* and *Leave* for 2-TGKE only, which can easily be adapted to work for 3-TGKE and 3-CGKE also.

4.1 2-KE to TGKE (2-TGKE)

Let the members who wish to share the key be arranged in an almost complete binary tree (fig. 1). We denote user i by U_i and use $parent(i)$, $left(i)$, $right(i)$ to denote parent of U_i, left child of U_i and right child of U_i respectively. We assume that 1 and 2 are parents of each other. Let 2-P be the underlying two party key exchange protocol secure against passive adversary. Let κ be the security parameter then the group key established belongs to $\{0,1\}^{\kappa}$. Let $G = \langle g \rangle$ be a cyclic group with prime order p. Similar to [3] we choose a collision resistant pseudo random function $\mathcal{F} : \{0,1\}^* \longrightarrow \{0,1\}^l$ and an injective mapping $f : G \longrightarrow \mathbb{Z}_q($ f is introduced to consider the group dynamism). Let $P = \{U_1, U_2, \ldots U_n\}$ be the set of participants. Our compiler works as follows:

1. ***Round 1:*** Each non leaf user U_i establishes 2-P keys with its parent and both children while a leaf node establishes 2-P key with its parent only. Thus non leaf nodes have $K_{i,parent(i)}, K_{i,left(i)}$ and $K_{i,right(i)}$, while leaf nodes have $K_{i,parent(i)}$.
2. ***Round 2:*** Each non-leaf user U_i computes and broadcasts
   $X_{left} = f(g^{K_{i,parent(i)}}) \oplus f(g^{K_{i,left(i)}})$ to it's left descendants and
   $X_{right} = f(g^{K_{i,parent(i)}}) \oplus f(g^{K_{i,right(i)}})$ to it's right descendant

3. Each user i computes

$$N_i = f(g^{K_{i,parent(i)}})$$

$$N_{parent(i)} = X_i \oplus N_i$$

$$\vdots$$

$$N_{1or2} = X_{3or4or5or6} \oplus N_{3or4or5or6} = f(g^{K_{1,2}})$$

   Each user sets $\mathcal{F}\left(f(g^{K_{1,2}}) \parallel P\right)$ as his secret key.

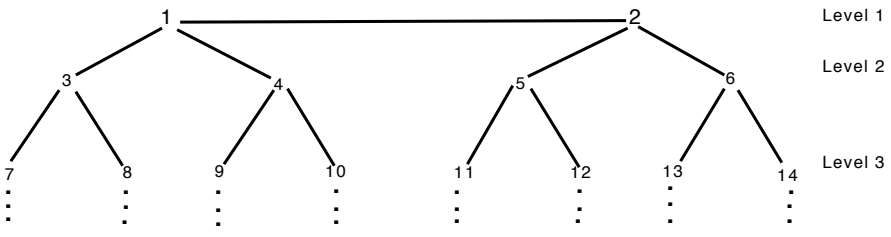

Fig. 1.

Now we give *Leave* and *Join* algorithms for considering group dynamism.

Join: Let $\mathcal{J} = \{U_{n+1}, U_{n+2}, \ldots, U_{n+n'}\}\,(n' \geq 1)$ be the set of new members who wish to join the existing group P. It is required that none of these new members should be able to know the previously established keys. Algorithm works as follows:

- ***Round 1:*** Members of $\mathcal{J}$ are arranged at the bootom of tree so as to maintain the almost complete structure. Each member $U_{n+i}(i = 1, \ldots, n)$ of $\mathcal{J}$ executes 2-P with its parent(and children also if it is non-leaf node) to get the corresponding keys. All other members update their new keys to the square of their old 2-P keys($K_{i,parent(i)}, K_{i,left(i)}, K_{i,right(i)}$ etc.).
- ***Round 2:*** Same as **step 2** of 2-TGKE with group size $n + n'$.
- Same as **step 3** of 2-TGKE with group size $n + n'$.

Intutively, the security depends on the unability of a newly joining member to compute g^x from g^{x^2}.

Leave: Let $\mathcal{L} = \{U_{n-n'+1}, \ldots, U_n\}\,(n' \geq 1)$ be the set of members who wish to leave from the existing group P. It is required that none of these old members should be able to know the future keys of the new group. We assume that the members leave from bottom only, maintaining the tree structure. Algorithm works as follows:

- ***Round 1:*** Old 2-P keys established with the members of $\mathcal{L}$ are expired and the remaining members update their new keys to the square of their old 2-P keys($K_{i,parent(i)}, K_{i,left(i)}, K_{i,right(i)}$ etc.).
- ***Round 2:*** Same as **step 2** of 2-TGKE with group size $n - n'$.
- Same as **step 3** of 2-TGKE with group size $n - n'$.

Security here depends on the unability of a leaving member to compute g^{x^2} from g^x.
Remark 1: Note that leaf nodes don't have to do any broadcast, also leaf nodes have to participate only in one run of the protocol while all non-leaf nodes have to participate in 3-runs of the protocol. The protocol obtained can be used for the settings where some users have lower computational power and can't do highly expensive broadcasts. Such a scene can occur quite frequently, for example consider the case of security agency of some country having a head office in the capital where enough computational power is available, it is also having zonal offices in state capitals and regional offices in other cities with in the state with computing powers in decreasing order. Finally, it is having some local agents in different cities of various countries with mobiles or other hand held devices. Here we require (1) Tree structure as an implicit requirement, also (2) users at the lowest level don't have enough computational power so a GKE given by above compiler will be well suited for this kind of applications.
Remark 2: Note that the protocol obtained is secure against a passive adversary only. To make it secure against an active adversary we use the technique presented in [18].

4.2 3-KE to TGKE(3-TGKE)

Consider the tree in $fig.2$. Let 3-P be a three party key exchange protocol secure against a passive adversary. It can be extended to obtain a GKE secure against a passive adversary as follows:

1. ***Round 1:*** Each non-leaf user U_i (from level 1 onwards) establishes 3-P keys with (1) both children and (2) its parent and siblings while each leaf node establishes a 3-P with its parent and sibling. Thus non-leaf nodes compute $K_{i,left(i),right(i)}$ and $K_{i,parent(i),sibling(i)}$ while leaf nodes compute only $K_{i,parent(i),sibling(i)}$.
2. ***Round 2:*** Each non-leaf user U_i(from level 1 onwards) computes and broadcasts to all its descendants:

$$X_i = f(g^{K_{i,parent(i),sibling(i)}}) \oplus f(g^{K_{i,left(i),right(i)}})$$

3. Each user i $(i \geq 2)$ computes

$$N_i = f(g^{K_{i,parent(i),sibling(i)}})$$

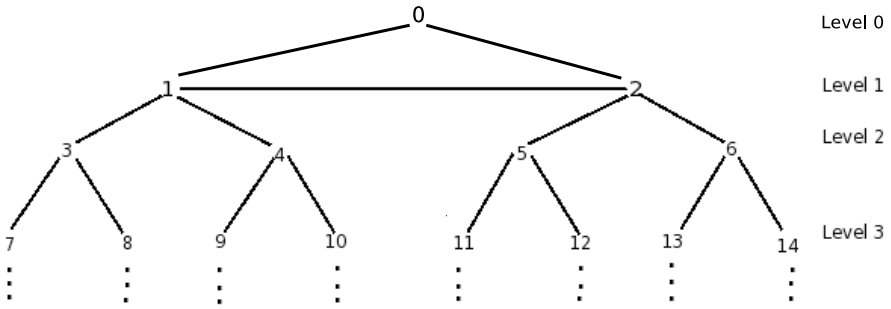

Fig. 2.

$$N_{parent(i)} = X_{parent(i)} \oplus N_i$$

$$\vdots$$

$$N_{1or2} = X_{1or2} \oplus N_{3or4or5or6} = f(g^{K_{0,1,2}})$$

Each user sets $\mathcal{F}\left(f(g^{K_{0,1,2}}) \parallel P\right)$ as his secret key.

4.3 3-KE to GKE for circular topology(3-CKGE)

Consider the users $U_1, U_2, \ldots U_n$ arranged in a circular fashion(i.e. $U_{n+1} = U_1$ etc.). Assuming that the number of members is even($n = 2m$ and $2m + 1 = 1$), the compiler works as follows:

1. ***Round 1:*** Each user U_i for $i = 1, 3, 5, \ldots, 2m-1$ establishes 3-P key with $i-2, i-1$ and $i+1, i+2$ to get $K_{i,i-1,i-2}$ and $K_{i,i+1,i+2}$.

2. ***Round 2:*** Each user U_i for $i = 1, 3, 5, \ldots, 2m-1$ broadcasts $X_i = f(g^{K_{i,i-1,i-2}}) \oplus f(g^{K_{i,i+1,i+2}})$.

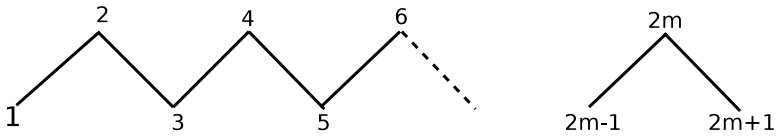

Fig. 3.

3. Each user checks that $X_1 \oplus X_3 \oplus \ldots X_{2m-1} = 0$. If the check fails it sets $acc_U^i = 0$ and terminates the protocol. Otherwise it evaluates(assuming that user U is from the triplet U_i, U_{i+1}, U_{i+2})

$$N_i = f(g^{K_{i,i+1,i+2}})$$

$$N_{i+2} = X_{i+2} \oplus N_i$$

$$\vdots$$

$$N_{i+2m-2} = X_{i+2m-2} \oplus N_{i+2m-4}$$

In this way each user evaluates the secret key of all the triplets. Each user sets $\mathcal{F}(f(g^{K_{1,2,3}}) \parallel f(g^{K_{3,4,5}}) \ldots f(g^{K_{2m-1,2m,1}}) \parallel P)$ as his secret key.

5 Security Proof:

In this section we prove the security of the compilers given in section 4. We prove, if the underlying protocol is secure against a passive adversary then the resulting GKE is secure against a passive adversary. We consider security in ROR model where multiple $Test$ queries are allowed. ROR model is apparently stronger than traditional model where only one $Test$ query is allowed, although both the model can be shown to be equivalent with a loss of factor r equal to the number of protocol instances which the adversary is given access to. Our proof goes on the lines of [4][2] except that we do not give access to $Send$ oracles as we are dealing with a passive adversary. We give the proof of security of first compiler only, which can easily be adapted for other two also. Due to space constraint we do not give proof of security for $Join$ and $Leave$, we have given intution for security with respect to $Join$ and $Leave$ algorithms in section 4.1, for full proof we refer the reader to $Theorem2$ of [3].

Theorem 1. *Let $\mathcal{A}_{GKE}^{ror-KE}$ be a passive adversary having non-negligible advantage against the security of group key exchange produced by $2-TGKE$ compiler, running in time t and asking at most q_{exe}, q_{reveal} and q_t Execute, Reveal and Test queries respectively then there exists a passive adversary $\mathcal{A}_{2-P}^{ror-KE}$ against the security of underlying 2-KE protocol 2-P such that*

$$Adv_{\mathcal{A}_{GKE}}^{ror-KE}(t, q_{exe}, q_{reveal}, q_t) \leq 2Adv_{\mathcal{A}_{2-P}}^{ror-KE}(t, 2nq_{exe}, q_{reveal}, nq_{exe} + 2q_t) +$$

$$\frac{q_{exe}^2}{2q} + \frac{q_{exe}^2}{2^{l+1}}$$

Proof: We prove the security using a sequence of hybrid games, starting with the real attack and ending in a game where adversary's advantage is zero. We use Adv_{G_i} to denote the advantage of $\mathcal{A}_{GKE}^{ror-KE}$ in Game i.
Game 0: This game corresponds to the real attack. By definition we have

$$Adv(G_0) = Adv_{\mathcal{A}_{GKE}}^{ror-KE}$$

Game 1: In this game we replace all the 2-P session keys by random session keys. We show below that difference between the advantages of adversary in Game 0 and Game 1 is at most that of the advantage of $\mathcal{A}_{2-P}^{ror-KE}$ against the security of underlying 2-KE protocol 2-P

$$|Adv(G_1) - Adv(G_0)| = 2Adv_{\mathcal{A}_{2-P}}^{ror-KE}(t, 2nq_{exe}, q_{reveal}, nq_{exe} + 2q_t)$$

We show this by constructing an adversary $\mathcal{A}_{2-P}^{ror-KE}$ using an adversary $\mathcal{A}$, distinguishing Game 0 from Game 1.
$\mathcal{A}_{2-P}^{ror-KE}$ is given access to the simulation of 2-P. To answer its queries, $\mathcal{A}_{2-P}^{ror-KE}$ first associates three instances of 2-P with each non-leaf and a single instance of 2-P with each leaf user according to the specification of 2-TGKE in section 5. Now whenever $\mathcal{A}$ queries a $Corrupt$ query, $\mathcal{A}_{2-P}^{ror-KE}$ answers it by querying its own $Corrupt$ oracle. To answer an $Execute$ query it first queries the $Execute$ oracles of 2-P instances to obtain the transcript for Round 1. To simulate the following rounds, $\mathcal{A}_{2-P}^{ror-KE}$ first queries the $Test$ oracles of instances of 2-P and uses the returned values as the 2-P keys of Round 1 ($K_{i,parent(i)}, K_{i,left(i)}, K_{i,right(i)}$ etc.). It uses these values to construct the $X_i's$ braodcasted in Round 2 and returns the transcript. To simulate a $Reveal$ query $\mathcal{A}_{2-P}^{ror-KE}$ asks its $Reveal$ oracle to the corresponding instance of the user 1 or 2 if it is non-fresh, otherwise, it asks its $Test$ oracle for that instance and uses the 2-P key obtained to construct the session key and returns it. In a similar way it can simulate the $Test$ oracle.
It is easy to observe that view of $\mathcal{A}$ corresponds to Game 0 if $Test$ query reveals the actually exchanged key and to Game 1 if $Test$ returns a random element from the key space. Thus, $\mathcal{A}$ distinguishes the Game 0 from Game 1 with probability at most $2Adv_{\mathcal{A}_{2-P}}^{ror-KE}(t, 2nq_{exe}, q_{reveal}, nq_{exe} + 2q_t)$
Game 2 Now the simulation of $Execute$ is modified at the point of computing the session key. Simulator keeps a list of assignments $(N_1, N_2, \ldots, N_n, i_1, i_2, \ldots, i_n, U_1, \ldots, U_n, sk)$. When a $Reveal$ query is asked simulator first checks whether the secret key for that session was already established and returns that key. Otherwise, a key $sk \in \{0,1\}^l$ is selected uniformly at random. The output of pseudo random function $\mathcal{F}$ is indistinguishable from the random secret key sk except in case of a collision, which occurs with probability at

most $\frac{q_{exe}^2}{2^{l+1}}$. Also a collision might occur in the records $(N_1, N_2, \ldots, N_n)$ but that happens with a probability at most $\frac{q_{exe}^2}{2q}$. Therefore

$$|Adv(G_2) - Adv(G_1)| = \frac{q_{exe}^2}{2q} + \frac{q_{exe}^2}{2^{l+1}}$$

In Game 2 all session keys are chosen uniformly at random and the adversary has no advantage i.e. $Adv(G_2) = 0$. We get the result mentioned by noting that,

$$Adv_{\mathcal{A}_{GKE}}^{ror-KE}(t, q_{exe}, q_{reveal}, q_t) = |Adv(G_0) - Adv(G_2)|$$
$$\leq |Adv(G_1) - Adv(G_0)| + |Adv(G_2) - Adv(G_1)|$$

6 A constant round authenticated tree based group key exchange protocol (2-TAGKE)

In this section we give a constant round authenticated tree based group key exchange protocol(2-TAGKE) obtained by applying a combination of our 2-TGKE compiler and techniques of Desmedt et.al.[18] on the Diffie-Hellman key exchange protocol for two parties. Let $P = \{U_1, U_2, \ldots U_n\}$ be the subset of users who wish to establish a common group key and $rel_U = \{V_1, V_2, \ldots, V_{tu}\}$ be the set of ancestors of U(including U) whose broadcast will be processed by U. Let G be a group of prime order p and $\phi : G \longrightarrow \{0,1\}^\kappa$ be an injective mapping. They proceed in the following way:-

1. During the initialization phase each party $U \in P$ generates the signing / verification key pair (PK_U, SK_U) using the algorithm $\mathcal{G}(1^\kappa)$.

Round 0:

2. Each of $U_1, U_2, \ldots, U_n$ chooses some random nonce $r_i \in \{0,1\}^\kappa$ and broadcasts to its descendants $U_i|0|r_i$. Each instance Π_U^j stores the identities and their per round randomness together with the group ID in $direct_U^j$. Where $direct_U^j = (gid|V_1|r_1 \ldots V_{tu}|r_{tu})$ and stores this as a part of its state information.

Round 1:

3. Each non leaf member U_i sends to its parent and children $U_i|1|g^{a_i}|\sigma$ where $\sigma = Sign_{SK_{U_i}}(1|g^{a_i}|direct_{U_i}^j)$. While leaf members send only to parent.
4. Each non-leaf member U_i on receiving the messages $U_s|k|g^{a_s}|\sigma_s$ for $s = \{parent(i), left(i), right(i)\}$ checks that (1) $U_s \in pid_{U_i}^j$, (2) $k = 1$ and (3) $Verify_{PK_{U_s}}(k|g^{a_s}|direct_{U_s}^j, \sigma_s) = 1$. If any of these checks fails, $\Pi_{U_i}^j$ aborts the protocol and sets $acc_{U_i}^j$ = FALSE and $sk_{U_i}^j$ = NULL. Otherwise, it computes $K_{i,s} = \phi((g^{a_s})^{a_i}) = \phi(g^{a_s \cdot a_i})$ for $s = \{parent(i), left(i), right(i)\}$. While leaf users proceed in a similar manner to compute $K_{i,parent(i)}$ only.

Round 2:

5. Each non-leaf member computes and broadcasts $(U_i|2|X_{left}|\sigma_{left})$, $(U_i|2|\ X_{right}|\sigma_{right})$ to its left descendants and right descendants respectively. Where $X_{left} = f(g^{K_{i,parent(i)}}) \oplus f(g^{K_{i,left(i)}})$, $X_{right} = f(g^{K_{i,parent(i)}}) \oplus f(g^{K_{i,right(i)}})$, $\sigma_{left} = Sign_{SK_{U_i}}(2|X_{left}|direct_{U_i}^j)$ and σ_{right} is equal to $Sign_{SK_{U_i}}(2|X_{right}|$ $direct_{U_i}^j)$.
6. Each member U_i on receiving the messages $U_s|k|X_s|\sigma_s$ where $s \in \left\{rel_{U_i}^j\right\}$ first goes through the verification process as in step 4 and then computes

$$N_i = f(g^{K_{i,parent(i)}})$$

$$N_{parent(i)} = X_i \oplus N_i$$

$$\vdots$$

$$N_{1or2} = X_{3or4or5or6} \oplus N_{3or4or5or6} = f(g^{K_{1,2}})$$

Each user sets $\mathcal{F}\left(f(g^{K_{1,2}}) \parallel P\right)$ as his secret key.

Remark 1: 2-TAGKE runs in 3 rounds. For each run of the protocol $\frac{n}{2}$ non leaf users require 6 exponentiations(E) and $\frac{n}{2}$ leaf users require only 4 E.(note that two extra exponentiations in Step 5 are to consider dynamism). Therefore on an average 5 exponentiations are to be computed per user(3 if dynamism is not considered). On an average each user has to compute 2 signatures(S), $\lg n$ XORs and has to do at most $\mathcal{O}(\lg n)$ signature verifications(V).
Remark 2: 2-TAGKE is efficient communication wise. On an average each user needs to do 1 broadcast(b) to $\mathcal{O}(\lg n)$ users and 2 point to point communication(Overall $\mathcal{O}(n \lg n)$ communication overhead is there as compared to $\mathcal{O}(n^2)$ for [1]). Users at the bottom don't have to do any broadcast, therefore 2-TAGKE is useful in applications where leaf users have lower computational power.
Remark 3: Security of 2-TAGKE follows from the security proof in Section 5 and security proof of [18].
Remark 4: Note that 2-TAGKE is non contributory. In fact, no protocol with computation and communication complexity lower than $\mathcal{O}(n)$ can be fully contributory [18].
In a similar fashion we can obtain 3-TAGKE and 3-CAGKE by application of 3-TGKE and 3-CGKE protocol on joux 3-party protocol [30].

7 Comparison:

In this section we compare our compiler and protocols with the existing ones. We are the first to consider the compiler for transformation of 2/3 KE to GKE in tree based settings. Mayer et.al. [31] considered the transformation of any two party authenticated key transport to group key transport but it is not scalable. Compiler of Hwang et. al. [1] converts a 2-AKE to GKE assuming the circular arrangement. We compare the authenticated version of the 2-GKE obtained by application of compiler in [1](HLGKE) with our 2-TAGKE and 3-TAGKE, it can be observed from $Table1$. that 2-TAGKE is far more efficient than HLGKE. It should be noted that HLGKE is contributory but 2-TAGKE is not. We also compare the efficiency of protocols obtained by applying our compilers to Joux 3-party protocol. If the parameters are chosen appropriately such that pairing computation is fast then these pairing based protocols can give performance comparable to discrete logarithm based protocols.
$Table1$ compares our protocol with the authenticated version of various other existing protocols. We use PM to denote multiplication of points on elliptic curve by a scaler, pa for pairing computation and M for multiplication of field elements, rest of the symbols are same as in *Remark 1* of section 6. $Table2$ compares the properties of various AGKE schemes. Particularly, we compare 2-TAGKE with BD-II, both are non-contributory and have $\mathcal{O}(\lg n)$ computation and communication complexity. But 2-TAGKE is dynamic while BD-II is not, also BD-II requires $\lg n$ multiplications while 2-TAGKE requires $\lg n$ XORs which are easier to compute.

Table 1: Comparison of the costs of few AGKE schemes

	Rounds	Message	Comm	Length	Computation
BCPQ[19]	n	2**b**	$(n-1)$**p**, 2**b**	$\mathcal{O}(1)$	nS, nV, nE, nM
BN PK[32]	1	1**b**	1**b**	$\mathcal{O}(n)$	1S, nV , $2(n-1)$E, $(n-1)$M
BN DH[32]	2	1**p**, 1**b**	$(n1)$**p**, 1**b**	$\mathcal{O}(n)$	2S, nV, nE, $(n-1)$M
BD-I[5, 7]	3	2**p**, 1**b**	4**p**, n**b**	$\mathcal{O}(1)$	2S, nV , 3E, $(2n-1)$M
BD-II[6]	3	3**p**, 1**b**	6**p**, $(log_2 n)$**b**	$\mathcal{O}(1)$	2S, $(log_2 n)$V, 3E, $(log_2 n)$M
BD-II seq [6]	$(log_2 n)$	5**p**	6**p**	$\mathcal{O}(1)$	3S, 4V , 4E, 2M
HLL [1]	3	1**b**, 2**p**	$(n-1)$**b**, 2**p**	$\mathcal{O}(1)$	2S, $(n+1)$V, 3E, $(n-1)$ XOR
2-TGKE	3	1**b**, 3**p**	$\mathcal{O}(\lg n)$**b**, 3**p**	$\mathcal{O}(1)$	2S, $\lg n$V, 5(3)$^*E$, $\lg n$ XOR
3-TGKE	3	1**b**, 4**p**	$\mathcal{O}(\lg n)$**b**, 4**p**	$\mathcal{O}(1)$	1.5S, $\lg n$V, 3E(1.5E), 1 PM, 1.5 pa, $\lg n$ XOR
3-CGKE	3	1**b**, 4**p**	$n-1$**b**, 4**p**	$\mathcal{O}(1)$	1.5S, $\frac{n}{2}$V, 3E(1.5E), 1 PM, 1.5 pa, $(\frac{n}{2}-1)$ XOR

Table 2: Comparison of the properties of few AGKE schemes

	Geometry	Dynamic	Work Distribution	Contributory
BCPQ[19]	Any	-	Asymmetric	√
BN PK[32]	Any	-	Asymmetric	√
BN DH[32]	Any	-	Asymmetric	√
BD-I[5, 7]	Circular	-	Symmetric	√
BD-II[6]	Tree	-	Symmetric	-
BD-II seq [6]	Tree	-	Asymmetric	-
HLL [1]	Circular	-	Symmetric	√
2-TGKE	Tree	√	Asymmetric	-
3-TGKE	Tree	√	Asymmetric	-
3-CGKE	Circular	√	Asymmetric	√

8 Conclusion:

In this paper we presented efficient and scalable compilers for transformation of secure 2/3 KE to secure GKE. We proved the security of our compilers in ROR model without assuming the existence of random oracles. Protocols produced by our compiler have $\mathcal{O}(\lg n)$ computational and communicational complexity which is a great saving as compared to $\mathcal{O}(n)$ computation and communication by protocols of [1]. We also gave a replay attack on [3]. Finally we presented a constant round authenticated Tree based group key exchange protocol with performance comparable to BD-II.
An interesting open problem [18] is to find contributory protocols for tree based setting which are as efficient as the protocols presented here.

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