Cryptanalysis of Dynamic SHA(2)

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Abstract. In this paper, we analyze the hash functions Dynamic SHA and Dynamic SHA2, which have been selected as first round candidates in the NIST hash function competition. These hash functions rely heavily on data-dependent rotations, similar to certain block ciphers, e.g., RC5. Our analysis suggests that in the case of hash functions, where the attacker has more control over the rotations, this approach is less favorable. We present practical, or close to practical, collision attacks on both Dynamic SHA and Dynamic SHA2. Moreover, we present a preimage attack on Dynamic SHA that is faster than exhaustive search.

1 Introduction

New generic cryptanalytic techniques for hash functions [3, 4] and the recent results on MD5 and SHA-1 [1, 11, 12], along with the fact that the SHA-2 family of hash functions was designed with a similar structure, have led to the initiation of the NIST hash function competition [7], a public competition to develop a new hash standard, which will be called SHA-3.

The competition has sparked a great deal of submissions: 64 new hash function proposals were submitted to the competition, of which 51 were accepted as meeting the submission criteria for the first round. Among the 51 candidates, Dynamic SHA and Dynamic SHA2 stand out as a combination of the SHA family design with data-dependent rotations.

The concept of data-dependent rotations has been explored for block ciphers in several constructions, most notably in the RC5 and RC6 block ciphers [8, 9]. The security of such block ciphers has been challenged many times, and a majority of attacks is based on guessing the distances of the rotations. In cryptanalysis of hash functions, however, the internal state is known. The attacker even has control over (parts of) the internal state, including rotations, though sometimes this control is only indirect. For example, Mendel et al. [6] exploited data-dependent rotations to find collisions for the hash function of Shin et al. [10]. Our attacks on Dynamic SHA and Dynamic SHA2 also exploit data-dependent rotations, to find (second) preimages and collisions.

2 Brief Description of Dynamic SHA and Dynamic SHA2

Dynamic SHA and Dynamic SHA2 use similar building blocks, but have different compression functions. This section gives a brief description of these algorithms.

Dynamic SHA and Dynamic SHA2 follow a classical Merkle-Damgård construction, based on a compression function that maps an 8-word chaining value and a 16-word message to a new 8-word chaining value. The 256-bit versions use 32-bit words, and the 512-bit versions use 64-bit words. We will focus on the 256-bit versions, also called Dynamic SHA-256 and Dynamic SHA2-256. See [13,14] for details on the 512-bit versions, Dynamic SHA-512 and Dynamic SHA2-512. The following presents a bottom-up description of the compression function, thus starting with its building blocks.

The symbol $\oplus$ stands for exclusive OR (XOR), $\wedge$ for logical AND, $\vee$ for logical OR, and $+$ for integer addition. Numbers in hexadecimal basis are written in typewriter font (e.g., $255 = \mathtt{FF}$). We count bit indices starting from zero at the least significant bit (LSB). Thus, the first bit of a word w is written as w^0, and more generally we use the notation w^i for the bit i of the word w. The most significant bit (MSB) of w is thus w^{31} for Dynamic SHA-256, and w^{63} for Dynamic SHA-512. Note that the i-th bit of a word corresponds to the bit number $i-1$, since we start counting from zero.

2.1 Building Blocks

The function G takes as input three words x_1, x_2, x_3 and an integer $t \in \{0, 1, 2, 3\}$, and returns one word, computed as follows.

$$G_t(x_1, x_2, x_3) = \begin{cases} x_1 \oplus x_2 \oplus x_3 & \text{if } t = 0 \\ (x_1 \wedge x_2) \oplus x_3 & \text{if } t = 1 \\ (x_1 \wedge x_2) \oplus x_3 \oplus \neg x_1 & \text{if } t = 2 \\ (x_1 \wedge x_2) \oplus x_3 \oplus \neg x_2 & \text{if } t = 3 \end{cases} .$$

Note that this definition is simplified, but equivalent to the original in [13,14].

The function R takes as input eight words $x_1, \ldots, x_8$ and an integer t, and returns one word computed as follows:

$$R(x_1, \ldots, x_8, t) = (((((((x_1 \oplus x_2) + x_3) \oplus x_4) + x_5) \oplus x_6) + x_7) \oplus x_8) \ggg t .$$

The function $R1$ takes as input eight words $x_1, \ldots, x_8$ and returns one word computed as follows (in the 256-bit versions):

$$
\begin{aligned}
&t_0 \leftarrow (((((x_1 + x_2) \oplus x_3) + x_4) \oplus x_5) + x_6) \oplus x_7 \\
&t_1 \leftarrow ((t_0 \gg 17) \oplus t_0) \wedge \mathtt{0001FFFF} \\
&t_2 \leftarrow ((t_1 \gg 10) \oplus t_1) \wedge \mathtt{000003FF} \\
&t_3 \leftarrow ((t_2 \gg\ 5) \oplus t_2) \wedge \mathtt{0000001F} \\
&\textbf{return } x_8 \ggg t_3
\end{aligned}
$$

Finally, the COMP function takes as input eight words $a, \ldots, h$ representing the internal state, eight message words $w_0, \ldots, w_7$, or $w_8, \ldots, w_{15}$, and an integer t. COMP updates the internal state as follows (in the 256-bit versions):

$$
\begin{aligned}
T &\leftarrow R(a, \ldots, h, w_t \bmod 32) & T &\leftarrow R(a, \ldots, h, (w_t \ggg 15) \bmod 32) \\
h &\leftarrow g & h &\leftarrow g + w_{t+7} \\
g &\leftarrow f \ggg ((w_t \ggg 5) \bmod 32) & g &\leftarrow f \ggg ((w_t \ggg 20) \bmod 32) \\
f &\leftarrow e + w_{t+3} & f &\leftarrow e + w_{t+6} \\
e &\leftarrow d \ggg ((w_t \ggg 10) \bmod 32) & e &\leftarrow d \ggg ((w_t \ggg 25) \bmod 32) \\
d &\leftarrow G_{w_t \ggg 30}(a, b, c) + w_{t+2} & d &\leftarrow G_{t \bmod 4}(a, b, c) + w_{t+5} \\
c &\leftarrow b & c &\leftarrow b + w_t \\
b &\leftarrow a & b &\leftarrow a \\
a &\leftarrow T + w_{t+1} & a &\leftarrow T + w_{t+4}
\end{aligned}
$$

2.2 Compression Functions

Given a chaining value $h_0, \ldots, h_7$ and a message block $w_0, \ldots, w_{15}$, the compression function of Dynamic SHA (Dynamic SHA2, respectively) produces a new chaining value, as described in Algorithm 1 (Algorithm 2, resp.).

The compression function of Dynamic SHA is composed of an initialization, an iterative part of 48 rounds, and a feedforward of the initial chaining value. It uses three constants TT_0, TT_1, TT_2.

The compression function of Dynamic SHA2 is composed of an initialization followed by three iterative parts, and finally by a feedforward. Note that, when calling COMP with the message words $w_8, \ldots, w_{15}$ and an integer t, w_t stands for w_8, w_{t+1} stands for w_9, etc. Dynamic SHA2 surprisingly enough, uses no constants.

3 Collision Attack on Dynamic SHA

This section describes a practical collision attack on Dynamic SHA. It builds on a 9-step local collision that exploits an important differential property of the function $R1$, which we will introduce first. The same local collision pattern is repeated three times to find collisions for the entire compression function. Furthermore, these three instances of the local collision pattern can be decoupled, which drastically reduces the attack complexity. We present the attack on Dynamic SHA-256 here. We could adapt it to Dynamic SHA-512 with only minimal changes, as detailed in Appendix C.

3.1 A Differential Property of the Function $R1$

To overcome the obstacle of data-dependent rotation, our attack ensures that no difference occurs in any of the data-dependent rotation amounts. This section clarifies how to achieve this.

The data-dependent rotations are located in the 8-input function $R1$. For Dynamic SHA-256, consider the difference $\Delta = \texttt{80004000}$, i.e., only bits 31

Algorithm 1 Compression function of Dynamic SHA.

Initialization

$$a = h_0 \quad b = h_1 \quad c = h_2 \quad d = h_3 \quad e = h_4 \quad f = h_5 \quad g = h_6 \quad h = h_7$$

Iterative part

for $t = 0, 1 \ldots, 47$

$$T \leftarrow R1(a, b, c, d, e, f, g, h)$$
$$U \leftarrow G(a, b, c, t \bmod 4) + w_{t \bmod 16} + TT_{t \gg 4}$$
$$(a, b, c, d, e, f, g, h) \leftarrow (T, a, b, U, d, e, f, g)$$

Feedforward

$$h_0 \leftarrow h_0 + a \quad h_1 \leftarrow h_1 + b \quad h_2 \leftarrow h_2 + c \quad h_3 \leftarrow h_3 + d$$
$$h_4 \leftarrow h_4 + e \quad h_5 \leftarrow h_5 + f \quad h_6 \leftarrow h_6 + g \quad h_7 \leftarrow h_7 + h$$

Algorithm 2 Compression function of Dynamic SHA2.

Initialization

$$a = h_0 \quad b = h_1 \quad c = h_2 \quad d = h_3 \quad e = h_4 \quad f = h_5 \quad g = h_6 \quad h = h_7$$

First iterative part

$$\mathsf{COMP}\,(a, b, c, d, e, f, g, h, w_0, w_1, \ldots, \ w_7, 0)$$
$$\mathsf{COMP}\,(a, b, c, d, e, f, g, h, w_8, w_9, \ldots, w_{15}, 0)$$

Second iterative part

for $t = 0, 1 \ldots, 8$

$$T \leftarrow R1(a, b, c, d, e, f, g, h)$$
$$(a, b, c, d, e, f, g, h) \leftarrow (T, a, b, c, d, e, f, g)$$

Third iterative part

for $t = 1, 2 \ldots, 7$

$$\mathsf{COMP}\,(a, b, c, d, e, f, g, h, w_0, w_1, \ldots, \ w_7, t)$$
$$\mathsf{COMP}\,(a, b, c, d, e, f, g, h, w_8, w_9, \ldots, w_{15}, t)$$

Feedforward

$$h_0 \leftarrow h_0 + a \quad h_1 \leftarrow h_1 + b \quad h_2 \leftarrow h_2 + c \quad h_3 \leftarrow h_3 + d$$
$$h_4 \leftarrow h_4 + e \quad h_5 \leftarrow h_5 + f \quad h_6 \leftarrow h_6 + g \quad h_7 \leftarrow h_7 + h$$

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and 14 are set. Let one of the first seven inputs to the function $R1$ have this difference, i.e., one of $x_1, \ldots, x_7$. In the first step of $R1$, an intermediary word t_0 is computed as follows:

$$t_0 \leftarrow ((((((x_1 + x_2) \oplus x_3) + x_4) \oplus x_5) + x_6) \oplus x_7 .$$

The difference in the MSB always propagates to t_0. Assuming that no carry occurs for bit 14, the intermediary t_0 also has the difference Δ. If t_0 has a difference Δ, this difference is then absorbed by the rest of the function $R1$. Indeed, the next step computes the intermediary word t_1 as

$$t_1 \leftarrow ((t_0 \ggg 17) \oplus t_0) \wedge \texttt{0001FFFF} .$$

Note that $(\Delta \ggg 17) \oplus \Delta = \texttt{80000000}$, which is absorbed by the logical AND operation.

We now estimate the probability that a single Δ-difference in one of the first seven inputs of the function $R1$ is absorbed. As a Δ-difference in t_0 is absorbed with certainty, it suffices that a Δ-difference in one of the seven first inputs propagates to t_0. This happens when no carry difference occur for bit 14 in any of the modular additions. The probability that a one-bit difference in one of the summands in an addition does not cause a carry difference is $1/2$. Thus, the probability that a Δ-difference is absorbed by the function $R1$ can be estimated to 2^{-k}, where k is the number of modular additions the difference propagates through. For instance, a difference in x_3 activates two modular additions, so $k = 2$.

However, the actual probability is higher, as the undesirable effects of a carry difference in one modular addition can be reverted by another carry difference in a subsequent addition. The combination of modular additions and XOR can be represented compactly in a trellis, and a variant of the Viterbi algorithm can be used to efficiently count the probability that a Δ-difference is passed to t_0 unchanged. Our computer aided research revealed that this is indeed an important effect: For a difference in x_3 or x_4, the actual probability is $2^{-1.58}$ rather than 2^{-2}, and for a difference in x_1 or x_2, the actual probability is $2^{-2.07}$ rather than 2^{-3}. For differences in the other words, only one modular addition is affected, so no carry differences can be canceled. Hence, in those cases, the simple estimation is correct.

3.2 A 9-step Local Collision

We present a simple 9-step local collision for Dynamic SHA in Table 1. A difference of $\Delta = \texttt{80004000}$ is introduced, then, all further diffusion of this difference is avoided. After seven more steps, the difference has rotated through the internal state of Dynamic SHA once, and can be canceled via an appropriate difference in the message word. The characteristic has probability $2^{-20.3}$.

In step 0, a Δ-difference is introduced via the message word. Note that the message word itself can contain any additive difference that can cause a Δ-difference in the state. In steps 1 to 4, the Δ-difference in one of the state

Table 1. A 9-step local collision for Dynamic SHA. The difference at step t is the difference in the state *before* computing step t.

t	a	b	c	d	e	f	g	h	w	Pr
0	0	0	0	0	0	0	0	0	Δ	2^{-1}
1	0	0	0	Δ	0	0	0	0	0	$2^{-1.58}$
2	0	0	0	0	Δ	0	0	0	0	2^{-1}
3	0	0	0	0	0	Δ	0	0	0	2^{-1}
4	0	0	0	0	0	0	Δ	0	0	1
5	0	0	0	0	0	0	0	Δ	0	2^{-5}
6	Δ	0	0	0	0	0	0	0	0	$2^{-2.07} \cdot 2^{-2}$
7	0	Δ	0	0	0	0	0	0	0	$2^{-2.07} \cdot 2^{-2}$
8	0	0	Δ	0	0	0	0	0	Δ	$2^{-1.58} \cdot 2^{-1}$
	0	0	0	0	0	0	0	0		

variables is absorbed by the function $R1$, as described in Section 3.1. Then, at the beginning of step 5, there is a Δ-difference in the internal state word h. This word is rotated by a data-dependent amount, and thus we can require that it is rotated by zero bits, i.e., not rotated at all. In steps 6 and 7, the Δ-difference should be absorbed by the G-functions. Any G-function except XOR absorbs differences in its first two inputs with probability 1/2 per bit. Also, $R1$ should absorb the differences in these steps. Finally, in step 8, the difference in the state variable c is canceled by another Δ-difference coming from the message word.

The probability that the local collision pattern is followed is estimated by simply multiplying the probabilities of all the events discussed above. The probabilities of each step are indicated in Table 1. This yields an overall probability of $2^{-20.3}$ for the entire 9-step local collision.

3.3 The Attack

Our attack repeats the 9-step collision three times. This made possible by the simple message schedule, which consists of a simple repetition of the 16 words in a message block. Thus, the only message words that have a difference are w_0, which introduces the differences, and w_8, which cancels them.

A straightforward attack would consist of choosing an arbitrary message block, and applying a difference of $\Delta = \text{80004000}$ to w_0 and w_8. As the local collision is repeated three times, the complexity of this attack would be approximately $(2^{20.3})^3 = 2^{61}$. This can be improved tremendously by making the three local collisions independent. Then, the three local collision complexities can be added rather than multiplied.

The first two local collisions can be decoupled in a straightforward way as only the message words w_0 to w_8 influence the first local collision. Therefore, once suitable values for these message words have been found, there is still enough freedom remaining in the other message words. The words w_0 to w_8 can thus

be kept constant, while values for w_9 to w_{15} are searched such that the second local collision is also achieved.

Controlling Internal State Values. In each step of Dynamic SHA, the new value of the internal state word d is found as the modular addition of a message word and an intermediate depending on the internal state words a, b and c. Full control over message words allows an adversary to give the internal state word d any desired value. Indeed, it holds that

$$w_{t \bmod 16} = d_{\text{new}} - G(a, b, c, t \bmod 4) - TT_{t \ggg 4} \ .$$

Applying this to eight consecutive steps allows one to almost fully control the final internal state. In every step, the new value of d is fixed to some desired value. These values then shift through the internal state words a number of times, to end up as one of the internal state words after the eighth step. However, a complication arises with the first three steps, which ends up in the state words a, b and c. Before a controlled value from d ends up in one of these three state words, it is be rotated by a data-dependent amount. An obvious way to sidestep this issue is to choose a rotation-invariant value for these three words, i.e., `00000000` or `FFFFFFFF`. Then, the data-dependent rotations have no influence.

Decoupling All Three Local Collisions. Our attack consists of three phases, each dealing with one local collision. The first phase satisfies the first local collision, using the message words w_0 to w_8. It would be possible to use message modification techniques here to find a conforming message pair quicker, but as the later phases of the attack dominate the overall complexity anyway, no significant gains can be made in this way.

To satisfy the second local collision, we use the freedom in the remaining message words. However, we do not choose the remaining message words directly, but rather choose the internal state after step 15. We then use the words w_8 to w_{15} to connect to this state, using the technique outlined earlier. We fix the values of a, b and c to zero, to make them rotation-invariant, and choose the remaining five words arbitrarily. Note that w_8 was already determined in phase 1, so it should not be modified again, but w_8 is used here to force a zero value, which ends up in the internal state word d after step 15. This issue is solved by shifting this condition on w_8 to phase 1. Instead of arbitrarily choosing w_8 there, it is computed such that the required zero is generated. This does not change the complexity of the first phase.

Finally, to satisfy the third local collision, we modify w_7. Then, only d changes after step seven. As the value in w_8, which should force d to zero after step eight, depends only on the internal state words a, b and c before step eight, modifying w_7 does not require a correction in w_8. Thus, such modifications do not change the fact that the first local collision pattern is followed. The values of w_9 to w_{15} are then updated such that the internal state after step 15 is unchanged, and so the start of the second local collision will be unaltered. For

the same reasons as before, the change in w_7 also does not affect the end of the second local collision pattern.

Hence, we dispose of a modification algorithm that leaves the first two local collisions unaffected, but changes the internal state values before the third local collision randomly. This provides the required freedom to also satisfy this third and final local collision. Hence, the overall attack complexity can be estimated at about 2^{21} Dynamic SHA compression function computations. Appendix A reports on our implementation of the attack, with an example of collision.

4 Preimage Attack on Dynamic SHA

This section describes (first and second) preimage attacks on Dynamic SHA. We first describe how to find preimages for the compression function of Dynamic SHA, and then explain how to extend this to first and second preimage attacks. on the Dynamic SHA hash function. We describe how to attack Dynamic SHA-256 here, and refer to Appendix C for details on how to adapt the attack to Dynamic SHA-512.

Conceptually, our preimage attack bears some similarity to the work on SHA-0 and SHA-1 by De Cannière and Rechberger [2], for it finds a preimage bit slice per bit slice. If all data-dependent rotation amounts in Dynamic SHA are assumed to be zero, then a bit of any intermediate word cannot be influenced by any other bit of higher position. This is because, besides rotations, all operations are either bit-wise or modular additions.

4.1 Preimage Attack on the Compression Function

Assume that the rotations in a block of Dynamic SHA are all zero. Then, all words in Dynamic SHA can be divided into bit slices, as all computations are now T-functions [5]. As noted above, bit i of each word can only be influenced by bits 0 to i of other words. When bits 0 to $(i-1)$ of each word are known, bit i of all words can be determined.

In a preimage attack on the Dynamic SHA compression function, the internal state is given before step 0 and after step 47. Our attack starts by determining the LSB of each word. To determine this bit of all of the internal state words in every step, only the LSBs of the 16 message words need to be known. There are 2^{16} choices for these 16 bits. Then, it can be verified whether the LSBs of the eight internal state words after step 47 are correct. This occurs with probability 2^{-8}, so 2^8 choices are expected to survive.

We then proceed to the next bit slice. Keeping the choice for the LSB slice fixed, the same procedure can be repeated. For each choice of the LSB slice again 2^8 choices for the second LSB are expected to survive. For Dynamic SHA-256, this procedure is repeated until the 28 LSBs (bits 0–27) have been determined. At that point, one of the bits of each of the 48 rotation constants can be determined, as it does not depend on the higher bits of any word. Now, it can be verified if the initial assumption that all rotation constants are zero indeed holds. This

corresponds to a 48-bit condition, i.e., for all rotation constants to be zero, surely this single bit of each rotation constant has to be zero. Any choices that do not satisfy this condition are eliminated. Then, the next bit is determined as before, after which another bit of each rotation constant can be verified. This is repeated until all bits have been determined.

4.2 Complexity Evaluation

The attack can be described as a simple tree search, where a tree level corresponds to a bit slice, and a node represents an assignment for all bits in the slice under consideration, and all LSB slices. To expand a node in the tree, one guesses the 16 message bits of the next slice, and checks that the conditions on the state words after step 47 are satisfied. As explained above, on average about 2^8 choices are expected to survive, i.e., the tree has a branching factor of 2^8. When the 28 LSB slices are known, however, the average number of child nodes drops by 2^{-48} due to the additional filtering. The cost of expanding one node is about 2^{16} Dynamic SHA compression function evaluations, as 2^{16} choices have to be investigated. The expected number of solutions is equal to the expected number of nodes at the deepest level of the tree, which is $2^{8\cdot32} \cdot 2^{-48\cdot5} = 2^{16}$. This agrees with the observation that for a given input/output chaining values of the compression function, there are expected to be 2^{256} message blocks that conform to this combination. For each of these, the probability that all the rotations are by 0 positions is 2^{-240}, so about 2^{16} remain.

As we aim to find just one solution, i.e., any node on the deepest level of the tree, a depth-first search is well suited to our application. It requires only negligible memory and can easily be parallelised. Since, for Dynamic SHA-256, 2^{16} solutions are expected, the depth-first search needs to search only about a fraction 2^{-16} of the entire tree before encountering the first solution. Due to the large branching factor, the total number of nodes in the tree is well approximated by the number of nodes on the widest level of the tree, which has $2^{8\cdot27} = 2^{216}$ nodes for Dynamic SHA-256. The search is thus expected to expand about 2^{200} nodes, each of which costs 2^{16} Dynamic SHA-256 compression function evaluations, resulting in a total attack complexity of 2^{216} Dynamic SHA-256 compression function evaluations.

4.3 Application to the Hash Function

Our preimage attack on the compression function directly gives a second preimage attack on the Dynamic SHA hash function with the same complexity, provided that there is at least one message block that does not contain any padding in the challenge message.

For a first preimage, the padding bits limit the control an attacker has over the message bits. It is not possible to simply copy the padding as in a second preimage attack. Thus, we use the following approach instead. First, choose a message length such that the last padded message block only contains 65 bits of padding, which is the minimum. Then, choose an arbitrary message for all but

the last message block. Finally, a modified version of the attack in Section 4.1 is used to determine the last message block.

The main difference is that the last 65 bits of the message block can not be chosen by the adversary, as they are padding bits. Their contents are fixed by the choice of the message length. However, the same approach as in Section 4.1 can still be applied, except that fewer bits can be chosen in each bit slice. For Dynamic SHA-256, the expected number of solutions in the search tree now becomes $2^{6 \cdot 27} \cdot 2^{-42 \cdot 4} \cdot 2^{-43 \cdot 1} = 2^{-49}$. A solution is thus only expected to exist with probability 2^{-49}, thus the attack is repeated sufficiently many times with a different message length. The number of nodes at the widest level of the tree is $2^{6 \cdot 27}$, and the cost for expanding a single node at this level is 2^{14} Dynamic SHA compression function calls. Thus, the total attack complexity becomes approximately $2^{49} \cdot 2^{6 \cdot 27} \cdot 2^{14} = 2^{225}$ Dynamic SHA compression function evaluations.

5 Collision Attack on Dynamic SHA2

To attack Dynamic SHA2, we use similar ideas as for Dynamic SHA. Specifically, we use the control of the message to ensure that as many rotations as possible are by the amounts that we need. Moreover, as many of the rotations amounts are directly determined by the message, our task becomes easier. Our attack is based on introducing a difference in the most significant bit of two message words, w_8 and w_{14}. As a 32-bit condition is imposed on the chaining value, a two-block collision finding technique is used, where the first block is searched until a suitable chaining value is encountered. We describe our attack on Dynamic SHA2-256 here. It can be adapted to Dynamic SHA2-512, as Appendix C shows.

5.1 First Iterative Part

Given an initial value $a, \ldots, h$, the first iterative part of the compression function of Dynamic SHA2 updates the chaining value words $a, \ldots, h$ by computing

$$\mathsf{COMP}(a, b, \ldots, h, w_0, w_1, \ldots, w_7) \ ,$$

Since there is no difference in the message words $w_0, \ldots, w_7$ nor in the initial value, we have no difference at this stage.

Then, Dynamic SHA2 computes

$$\mathsf{COMP}(a, b, \ldots, h, w_8, w_9, \ldots, w_{15}) \ .$$

To follow our characteristic, the difference in w_8 and in w_{14} should lead to a difference $\Delta = \mathtt{80000000}$ in c and in f. Below, we show that, to obtain these differences, it suffices to set $w_8^{30} = 1$ and to ensure that b equals $\mathtt{FFFFFFFF}$ after the first COMP. These conditions are easily satisfied, and do not increase the complexity of our attack.

We note that w_{14} is used only once in the first iterative part. Thus the difference Δ in w_{14} only propagates to f, when COMP sets $f \leftarrow e + w_{14}$. The

word w_8, however, is used eight times, but as only the MSB has a difference, only two of these require our attention: first, when setting $c \leftarrow b + w_8$ (which gives the difference Δ in c with probability one), and second when setting

$$d \leftarrow G_{w_8 \ggg 30}(a, b, c) + w_{10} \ .$$

Here, the two MSBs of w_8 encode the index of the function used in G. Since we have a difference in the MSB of w_8, different functions are applied to (a, b, c). To obtain the same output, we require that the functions G_1 and G_3 are used, that is, we set the bit $w_8^{30} = 1$. The reason for this is that, when b equals FFFFFFFF, it is ensured that the outputs of both functions are equal, as can readily be seen from the definition of the G-functions in Section 2.1

To summarize, a difference Δ in w_8 and w_{14} yields a difference Δ in c and f after the first iterative part. To have $b = \text{FFFFFFFF}$, it is sufficient to start from a chaining values that gives at the very first COMP a T such that $T + w_1 = \text{FFFFFFFF}$. Such a chaining value can be reached in about 2^{32} trials, and needs to be precomputed only once. That is, one first needs to find a message block leading to a chaining value that satisfies $T + w_1 = \text{FFFFFFFF}$, before starting the actual differential attack with a second block. Actually, by using the freedom in w_0 and w_1 rather than fixing them a priori, this step can be accelerated further. However, as the other parts of the attack dominate the overall complexity, no significant gains can be made in this way.

5.2 Second Iterative Part

Table 2 describes our differential characteristic for the second iterative part of Dynamic SHA2. Note that no message word is input in this part. A set of conditions that ensure that this characteristic is followed, is relatively simple. Indeed, except when $t = 2$ and $t = 5$, the two differences Δ vanish in the first step of the computation of $R1$, namely when computing

$$(((((a + b) \oplus c) + d) \oplus e) + f) \oplus g \ .$$

Therefore, particular conditions are only required for $t = 2$ and $t = 5$.

When $t = 2$, the difference in e gives a difference of 16 in the rotation amounts, and so the function $R1$ returns $h \ggg r$ and $(h \oplus \Delta) \ggg (r + 16 \bmod 32)$, respectively. In order to obtain, as required by our differential characteristic, the relation

$$(h \ggg r) \oplus \Delta = (h \oplus \Delta) \ggg (r + 16 \bmod 32) \ ,$$

a sufficient condition is to have $r = 16$, and h invariant under 16-bit rotation, i.e., $(h \ggg 16) = h$. This means that h should be of the form XYZTXYZT, which we call *symmetric*. When $t = 5$, we require similar conditions.

Now, observe that the words that should be symmetric are c and f obtained after the first iterative part. The values of c and f then directly depend on w_8 and w_{14} (see description of COMP in Section 2). We now have to find values of w_8 and of w_{14} that give symmetric c and f.

Table 2. Differential characteristic for the second iterative part of Dynamic SHA2. The difference at step t is the difference in the state *before* computing step t.

t	a	b	c	d	e	f	g	h
0	0	0	Δ	0	0	Δ	0	0
1	0	0	0	Δ	0	0	Δ	0
2	0	0	0	0	Δ	0	0	Δ
3	Δ	0	0	0	0	Δ	0	0
4	0	Δ	0	0	0	0	Δ	0
5	0	0	Δ	0	0	0	0	Δ
6	Δ	0	0	Δ	0	0	0	0
7	0	Δ	0	0	Δ	0	0	0
8	0	0	Δ	0	0	Δ	0	0

Such w_8 and w_{14} can be found as follows: first fix w_{14} to some arbitrary value, and search for a w_8 that gives a symmetric c, in 2^{16} trials. Then, fix w_8 to the value found, and search for a pair (w_5, w_{14}) that gives a symmetric f after the first iterative part. Here we need w_5 to have enough freedom, since for certain choices of w_5, there does not exist a suitable w_{14}. Again, 2^{16} trials are expected. Then we are enough degrees of freedom in the message words that do not affect c and f to find rotation $r = 16$.

Assuming symmetric c and f after the first iterative part, the characteristic is followed with probability 2^{-10}, since the condition $r = 16$ is satisfied for both $t = 2$ and $t = 5$ with probability $2^{-5} \times 2^{-5}$. By trying several values of, for example, w_9, and leaving the other message words fixed, one can thus find a conforming message pair for the first two iterative parts in about 2^{10} trials.

5.3 Third Iterative Part

Given the final difference of the second iterative part, we found a characteristic for the second round that yields no difference in the final state, thus given a collision. Table 5 in Appendix B describes our differential characteristic. Appendix B also explains in detail why the characteristic can be followed with probability 2^{-42}, given some conditions on the input.

Combining our differential characteristics with their respective conditions on the message, we obtain a method for finding a 2-block collision in about $2^{42+10} = 2^{52}$ trials. The attack succeeds with probability close to one.

6 Conclusion

In this paper we have discussed the security of the two SHA-3 candidates Dynamic SHA and Dynamic SHA2. We have analyzed their security, and found out that, despite their reliance on data-dependent rotations and in the case of Dynamic SHA2 even data-dependent functions, their security is subverted by the vast control and knowledge the adversary has while attacking a hash function.

Table 3. Summary of our results.

Hash Function	Attack	Complexity	Section
Dynamic SHA-256	Collision	2^{21}	3
Dynamic SHA-512	Collision	2^{22}	3,C
Dynamic SHA-256	Second preimage	2^{216}	4
Dynamic SHA-512	Second preimage	2^{256}	4,C
Dynamic SHA-256	First preimage	2^{225}	4
Dynamic SHA-512	First preimage	2^{262}	4,C
Dynamic SHA2-256	Collision	2^{52}	5
Dynamic SHA2-512	Collision	2^{85}	5,C

We also showed that neither Dynamic SHA nor Dynamic SHA2 are suitable to be selected as SHA-3, following their lack of security. Table 3 summarizes our results.

Acknowledgements

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Table 4. Collision example for Dynamic SHA-256: two messages and their common digest.

```
34BC5378 1150D86C 3085EB92 7538ECEE    199FB91A 5A9614EC 4D21FB88 728FF21E
22FBFA2E 08CE50DF 95CDE61F 71E5F222    3D30C361 EB7676B8 F1AE9728 758B70AF
00000000 00000000 00000000 00000000    00000000 00000000 00000000 00000000
00000000 00000000 00000000 00000000    00000000 00000000 00000000 00000000

B4BC9378 1150D86C 3085EB92 7538ECEE    199FB91A 5A9614EC 4D21FB88 728FF21E
A2FBBA2E 08CE50DF 95CDE61F 71E5F222    3D30C361 EB7676B8 F1AE9728 758B70AF
00000000 00000000 00000000 00000000    00000000 00000000 00000000 00000000
00000000 00000000 00000000 00000000    00000000 00000000 00000000 00000000

703C40F7 9DDFE2C6 8298F6D0 8D2B45B6    664CBB71 8BAB1BE3 DD563F77 0D0901E6
```

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A Practical Results

We have implemented our collision attack on Dynamic SHA. Collisions for Dynamic SHA-256 and Dynamic SHA-512 are found in a matter of seconds on an average desktop PC. A collision example for Dynamic SHA-256 is given in Table 4. An all-zero block was appended to both messages to circumvent an error in the padding routine of the Dynamic SHA reference implementation, which causes part of the last message block to be reused in the padding block.

B Differential Characteristic for Dynamic SHA2

This appendix describes the differential characteristic for the third iterative part of Dynamic SHA2, used in our collision attack presented in Section 5.

A transition in Table 5 has probability $1/2$ when there is a difference in a or b and G_1, G_2 or G_3 is used. In this case, the difference does (not) propagate

with probability $1/2$. When there is a difference only in c, it always propagates to the output of the G function, independent of the function used. We also note that a difference Δ in one operand of R is always transferred to T, and thus to a, except when w_{t+1} or w_{t+4} are w_8 or w_{14}, in which case the differences vanish. When two operands of T have a difference Δ, they cancel out and yield no difference in T.

The probabilities for each step assume some conditions on the message. We will take as example the first COMP when $t = 2$: we start with a difference

$$0 \quad \Delta \quad 0 \quad \Delta \quad 0 \quad 0 \quad 0 \quad 0$$

in the chaining value $a, b, \ldots, h$. In the computation of COMP (first half), there is no difference in T, because the Δ difference in b cancels that of d. The assignment of the new values of f, g, h requires no condition on the message, for it only involves words with no difference. To obtain a difference Δ in e, we need that d is rotated by zero bit positions, that is, we need the bits 10 to 14 of w_2 to be zero. This is easy as we have direct control over w_2. Then, to obtain no difference in d, we require that the difference in b does not propagate in G. This is only possible if the Boolean function in G is not $x_1 \oplus x_2 \oplus x_3$ (see Section 2.1). Since the Boolean function is determined by the last two bits of w_2, we require $w_2^{30} \vee w_2^{31} = 1$, i.e., these bits should not be both zero. Now, the difference will not propagate in G with probability $1/2$. Finally, we get a difference Δ in c with probability 1.

By applying a similar reasoning to all the steps of our differential characteristic, we obtain conditions on the message $w_0, \ldots, w_{15}$ that are sufficient to conform to the characteristic with probability 2^{-42}. Table 6 summarizes these conditions, along with the conditions for the other iterative parts.

Conditions on $w_0, \ldots, w_7$ ensure that in the first COMP of each step the rotations are by bit zero positions, and thus the difference remains in the MSB. The probabilities smaller than one are the probabilities that the function G absorbs or passes a difference in a or b. In the second COMP, we need some rotations to be zero in order the difference to stay in the MSB. This is achieved by setting conditions on the message, for example at $t = 1$, the first ten bits of w_9 should be zero. Table 6 summarizes these conditions. After satisfying all these conditions, about 200 bits of freedom remain; indeed, besides w_8 and w_{14}, the message words w_1 to w_4 have to be fixed to let the symmetric c and f unchanged after the first iterative part.

At step $t = 6$, the difference in the MSB of w_{14} implies that G will apply different functions to (a, b, c). Similarly to Section 5.1, we will require $w_{14}^{30} = 1$ and $b = \texttt{EFFFFFFF}$, which will occur with probability 2^{-32}. The MSB of b should be zero in order the difference to propagate, which will happen with probability $1/2$, thus the total probability for this step $1/2 \times 2^{-32} = 2^{-33}$

Table 5. Differential characteristic for the third iterative part of Dynamic SHA2. The difference at step t is the difference in the state *before* computing step t. The column T indicates the difference in the temporary variable T. The probability on a line is the probability to reach the *next* difference, when conditions on the message are satisfied.

t	(message input)	a	b	c	d	e	f	g	h	T	prob.
1	$(w_1, \ldots, w_0)$	0	0	0	Δ	0	0	Δ	0	0	1
		0	0	0	0	Δ	0	0	Δ	0	1
1	$(w_9, \ldots, w_8)$	0	0	0	0	0	Δ	0	0	Δ	1
		Δ	0	0	0	0	0	Δ	0	0	2^{-1}
2	$(w_2, \ldots, w_1)$	0	Δ	0	Δ	0	0	0	0	0	2^{-1}
		0	0	Δ	0	Δ	0	0	0	0	1
2	$(w_{10}, \ldots, w_9)$	0	0	0	Δ	0	Δ	0	0	0	1
		0	0	0	0	Δ	0	Δ	0	0	1
3	$(w_3, \ldots, w_2)$	Δ	0	0	0	0	0	0	Δ	0	2^{-1}
		0	Δ	0	0	0	0	0	0	Δ	2^{-1}
3	$(w_{11}, \ldots, w_{10})$	Δ	0	Δ	0	0	0	0	0	0	2^{-1}
		0	Δ	0	Δ	0	Δ	0	0	Δ	2^{-1}
4	$(w_4, \ldots, w_3)$	Δ	0	Δ	0	Δ	0	Δ	0	0	2^{-1}
		0	Δ	0	Δ	0	Δ	0	Δ	0	1
4	$(w_{12}, \ldots, w_{11})$	0	0	Δ	Δ	Δ	0	Δ	0	0	1
		0	0	0	0	Δ	Δ	0	Δ	Δ	1
5	$(w_5, \ldots, w_4)$	0	0	0	0	0	Δ	Δ	0	0	1
		0	0	0	0	0	0	Δ	Δ	0	1
5	$(w_{13}, \ldots, w_{12})$	0	0	0	0	0	0	0	Δ	Δ	1
		0	0	0	0	0	Δ	0	0	Δ	1
6	$(w_6, \ldots, w_5)$	Δ	0	0	0	0	0	Δ	0	0	1
		0	Δ	0	Δ	0	0	0	Δ	Δ	2^{-1}
6	$(w_{14}, \ldots, w_{13})$	Δ	0	Δ	Δ	Δ	0	0	0	Δ	2^{-33}
		0	Δ	0	Δ	Δ	Δ	0	0	0	2^{-1}
7	$(w_7, \ldots, w_6)$	0	0	0	Δ	Δ	Δ	Δ	0	0	1
		0	0	0	0	Δ	Δ	Δ	Δ	0	1
7	$(w_{15}, \ldots, w_{14})$	0	0	0	0	0	Δ	Δ	Δ	Δ	1
		0	0	0	0	0	0	Δ	Δ	0	1
		0	0	0	0	0	0	0	0		

Table 6. Conditions on the message words $w_0, \ldots, w_{15}$ sufficient to follow our differential characteristic.

Word	Condition
w_0	$-$
w_1	$w_1 = 0$
w_2	$w_2^{10} = \cdots = w_2^{14} = 0$, $w_2^{25} = \cdots = w_2^{29} = 0$, $w_2^{30} \vee w_2^{31} = 1$
w_3	$w_3^{30} \vee w_3^{31} = 1$
w_4	$w_4^{20} = \cdots = w_4^{29} = 0$, $w_4^{30} \vee w_4^{31} = 1$
w_5	$w_5^5 = \cdots = w_5^9 = 0$
w_6	$w_6^0 = \cdots = w_6^4 = 0$, $w_6^{15} = \cdots = w_6^{19} = 0$, $w_6^{20} = \cdots = w_6^{29} = 0$
w_7	$w_7^5 = \cdots = w_7^{14} = 0$, $w_7^{20} = \cdots = w_7^{24} = 0$
w_8	difference in w_8^{31}, $w_8^{30} = 1$
w_9	$w_9^0 = \cdots = w_9^9 = 0$
w_{10}	$w_{10}^5 = \cdots = w_{10}^{14} = 0$
w_{11}	$w_{11}^{15} = \cdots = w_{11}^{29} = 0$, $w_{11}^{30} \vee w_{11}^{31} = 1$
w_{12}	$w_{12}^{10} = \cdots = w_{12}^{24} = 0$
w_{13}	$w_{13}^0 = \cdots = w_{13}^4 = 0$, $w_{13}^{15} = \cdots = w_{13}^{24} = 0$
w_{14}	difference in w_{14}^{31}, $w_{14}^{10} = \cdots = w_{14}^{14} = 0$, $w_{14}^{20} = \cdots = w_{14}^{29} = 0$, $w_{14}^{30} = 1$
w_{15}	$w_{15}^0 = \cdots = w_{15}^9 = 0$

C Extensions to the 512-bit Versions

The attacks presented in this paper can be extended to the 512-bit versions of Dynamic SHA and Dynamic SHA2 in a straightforward way. This appendix details how the attacks can be adapted.

Collision Attack on Dynamic SHA. The attack on Dynamic SHA-256 can be adapted to Dynamic SHA-512 with almost no change. Due to the different $R1$ function, the difference word is $\Delta = 8000000080000000$. Also, the probability of the local collision is lowered by about 2^{-1} compared to Dynamic SHA-256, as in the fifth step six rotation bits have to be fixed to zero instead of only five.

Preimage Attack on Dynamic SHA. The preimage attack on Dynamic SHA-512 is similar to that on Dynamic SHA-256, except that the 59 LSBs are determined, instead of the 28 LSBs. Then, when building the tree, 2^{224} solutions are expected, leading to an attack complexity of 2^{256} on the compression function. Calculations for preimages on the full hash function (with correct padding bits) give a cost of of 2^{262} compression function evaluations.

Collision Attack on Dynamic SHA2. To attack Dynamic SHA2-512 we use a similar differential path. The changes are that the condition on the first block is on 64 bits (starting from a chaining value with $b = \texttt{FFFFFFFFFFFFFFFF}$), the fact that in the second iterative part the probability is 2^{-6} for each of the two transitions, the decrease in the probability only of the sixth COMP from 2^{-33} to 2^{-65}, and the different set of conditions on the message described in Table 7. Hence, the total time complexity of this attack is 2^{85}. We note that in this approach the attack fixes w_i^{60} and w_i^{61} to $i \bmod 4$ (which causes the same function to be used in this case as in the attack on Dynamic SHA2-256).

Table 7. Conditions on the message words $w_0, \ldots, w_{15}$ sufficient to follow our differential characteristic in Dynamic SHA2-512

Word	Condition
w_0	$-$
w_1	$w_1 = 0$, $w_1^{18} = \cdots = w_1^{23} = 0$, $w_1^{42} = \cdots = w_1^{47} = 0$, $w_1^{60} = 1, w_1^{61} = 0$
w_2	$w_2^{18} = \cdots = w_2^{29} = 0$, $w_2^{42} = \cdots = w_2^{47} = 0$, $w_2^{60} = 0, w_2^{61} = 1, w_2^{62} \vee w_2^{63} = 1$
w_3	$w_3^{54} = \cdots w_3^{59} = 0$, $w_3^{60} = w_3^{61} = 1$, $w_3^{62} \vee w_3^{63} = 1$
w_4	$w_4^{6} = \cdots = w_4^{11} = 0$, $w_4^{18} = \cdots = w_4^{23} = 0$, $w_4^{42} = \cdots = w_4^{47} = 0$, $w_4^{60} = w_4^{61} = 0$, $w_4^{62} \vee w_4^{63} = 1$
w_5	$w_5^{6} = \cdots = w_5^{11} = 0$, $w_5^{60} = 1, w_5^{61} = 1$
w_6	$w_6^{48} = \cdots = w_6^{53} = 0$, $w_6^{60} = 0, w_6^{61} = 1$
w_7	$w_7^{6} = \cdots = w_7^{23} = 0$, $w_7^{36} = \cdots = w_7^{53} = 0$, $w_7^{60} = w_7^{61} = 1$
w_8	difference in w_8^{63}, $w_8^{62} = 1$
w_9	$w_9^{12} = \cdots = w_9^{17} = 0$, $w_9^{36} = \cdot = w_9^{41} = 0$, $w_9^{60} = 1, w_9^{61} = 0$
w_{10}	$w_{10}^{6} = \cdots = w_{10}^{11} = 0$, $w_{10}^{18} = \cdots = w_{10}^{23} = 0$, $w_{10}^{42} = \cdots = w_{10}^{47} = 0$, $w_{10}^{60} = 0$, $w_{10}^{61} = 1$
w_{11}	$w_{11}^{36} = \cdots = w_{11}^{41} = 0$, $w_{11}^{48} = \cdots = w_{11}^{59} = 0$, $w_{11}^{60} = w_{11}^{61} = 1, w_{11}^{62} \vee w_{11}^{63} = 1$
w_{12}	$w_{12}^{12} = \cdots = w_{12}^{23} = 0$, $w_{12}^{36} = \cdots = w_{12}^{47} = 0$, $w_{12}^{60} = w_{12}^{61} = 0$
w_{13}	$w_{13}^{36} = \cdots = w_{13}^{41} = 0$, $w_{13}^{60} = 1, w_{13}^{61} = 0$
w_{14}	difference in w_{14}^{63}, $w_{14}^{12} = \cdots = w_{14}^{23} = 0$, $w_{14}^{36} = \cdots = w_{14}^{53} = 0$, $w_{14}^{60} = 0, w_{14}^{61} = 1$
w_{15}	$w_{15}^{6} = \cdots = w_{15}^{11} = 0$, $w_{15}^{36} = \cdots = w_{15}^{41} = 0$, $w_{15}^{60} = w_{15}^{61} = 1$