

An Efficient Password Security of Key Exchange Protocol based on ECDLP

Jayaprakash Kar
Department of Information Technology
Al Musanna College of Technology
Sultanate of Oman*

Banshidhar Majhi
Department of Computer Science & Engineering
National Institute of Technology
Rourkela,INDIA[†]

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Abstract

In this paper we have proposed an efficient password security of Three-Party and Multiparty Key Exchange Protocol based on Elliptic Curve Discrete Logarithm Problem. In three party Key exchange protocols allow two clients with a trusted Server where they registered their password and communicate over a public network to establish a common secret key called session key. Similarly multiparty key exchange protocol allow a group of parties communicating over a public network to establish the session key. Due to their significance by in building a secure communication channel, a number of key exchange protocols have been suggested over the years for a variety of settings. Here we have taken two one-way hash functions to built the level of security high.

Key word : Key exchange protocol, Password based, secure communication, off-line dictionary attack,ECDLP.

1 Introduction

Three-party authenticated key exchange protocol is an important cryptographic technique in the secure communication areas, by which two clients, each shares a human-memorable password with a trusted server, can agree a secure session key. Where as group key exchange protocol is the cryptographic technique in public network, by which a group shares a human-memorable password with a trusted

*e-mail: jayaprakashkar@yahoo.com

[†]e-mail: bmajhi@nitrkl.ac.in

server. Over the past years, many three-party and multiparty authenticated key exchange protocols have been proposed. However, to our best knowledge, not all of them can meet the requirements of security and efficiency simultaneously. Therefore, in this paper, we would like to propose two protocols for the three party and multiparty exchange protocol. Compared with other existing protocols, our proposed protocol does not require any server's public key, but can resist against various known attacks. Therefore, we believe it is suitable for some practical scenarios.

With the proliferation of the hand-held wireless information appliances, the ability to perform security functions with limited computing resources has become increasingly important. In mobile devices such as personal digital assistants (PDAs) and multimedia cell phones, the processing resources, memory and power are all very limited, but the need for secure transmission of information may increase due to the vulnerability to attackers of the publicly accessible wireless transmission channel [1]. New smaller and faster security algorithms provide part of the solution, the elliptic curve cryptography ECC provide a faster alternative for public key cryptography. Much smaller key lengths are required with ECC to provide a desired level of security, which means faster key exchange, user authentication, signature generation and verification, in addition to smaller key storage needs. The terms elliptic curve cipher and elliptic curve cryptography refers to an existing generic cryptosystem which use numbers generated from an elliptic curve. Empirical evidence suggests that cryptosystems that utilize number derived from elliptic curve can be more secure [2]. As with all cryptosystems and especially with public-key cryptosystems, it takes years of public evaluation before a reasonable level of confidence in a new system is established. ECC seem to have reached that level now. In the last couple of years, the first commercial implementations are appearing, as toolkits but also in real-world applications, such as email security, web security, smart cards, etc. The security of ECC has not been proven but it is based on the difficulty of computing elliptic curve discrete logarithm in the elliptic curve group [3].

2 Background

In this section we brief overview of Elliptic Curve over finite field, Elliptic Curve Discrete Logarithm Problem, Key exchange, Elliptic Curve Diffie-Helman(ECDH) and about three-party key exchange protocol.

2.1 The finite field F_p

Let p be a prime number. The finite field F_p is comprised of the set of integers $0, 1, 2, \dots, p - 1$ with the following arithmetic operations [5] [6] [7]:

- Addition: If $a, b \in F_p$, then $a + b = r$, where r is the remainder when $a + b$ is divided by p and $0 \leq r \leq p - 1$. This is known as addition modulo p .

- **Multiplication:** If $a, b \in F_p$, then $a \cdot b = s$, where s is the remainder when $a \cdot b$ is divided by p and $0 \leq s \leq p - 1$. This is known as multiplication modulo p .
- **Inversion:** If a is a non-zero element in F_p , the inverse of a modulo p , denoted a^{-1} , is the unique integer $c \in F_p$ for which $a \cdot c = 1$.

2.2 Elliptic Curve over F_p

Let $p \geq 3$ be a prime number. Let $a, b \in F_p$ be such that $4a^3 + 27b^2 \neq 0$ in F_p . An elliptic curve E over F_p defined by the parameters a and b is the set of all solutions (x, y) , $x, y \in F_p$, to the equation $y^2 = x^3 + ax + b$, together with an extra point \mathcal{O} , the point at infinity. The set of points $E(F_p)$ forms an abelian group with the following addition rules [9]:

1. **Identity :** $P + \mathcal{O} = \mathcal{O} + P = P$, for all $P \in E(F_p)$
2. **Negative :** if $P(x, y) \in E(F_p)$ then $(x, y) + (x, -y) = \mathcal{O}$, The point $(x, -y)$ is denoted as $-P$ called negative of P .
3. **Point addition:** Let $P((x_1, y_1), Q(x_2, y_2) \in E(F_p)$, then $P + Q = R \in E(F_p)$ and coordinate (x_3, y_3) of R is given by $x_3 = \lambda^2 - x_1 - x_2$ and $y_3 = \lambda(x_1 - x_3) - y_1$ where $\lambda = \frac{y_2 - y_1}{x_2 - x_1}$
4. **Point doubling :** Let $P(x_1, y_1) \in E(K)$ where $P \neq -P$ then $2P = (x_3, y_3)$ where $x_3 = (\frac{3x_1^2 + a}{2y_1})^2 - 2x_1$ and $y_3 = (\frac{3x_1^2 + a}{2y_1})(x_1 - x_3) - y_1$

2.3 Elliptic Curve Discrete Logarithm Problem (ECDLP)

Given an elliptic curve E defined over a finite field F_p , a point $P \in E(F_p)$ of order n , and a point $Q \in \langle P \rangle$, find the integer $l \in [0, n - 1]$ such that $Q = lP$. The integer l is called discrete logarithm of Q to base P , denoted $l = \log_p Q$ [9].

2.4 Key exchange

Key exchange protocols allow two parties to agree on a secret shared secret key that they can use to do further encryption for a long message. One of these protocols is the Diffie-Hellman, which is the most used one. The Elliptic curve Diffie-Hellman is considered as an extension to the standard Diffie-Hellman.

2.5 Elliptic Curve Diffie-Hellman

Elliptic curve Diffie-Hellman protocol (ECDH) is one of the key exchange protocols used to establish a shared key between two parties. ECDH protocol is based on the additive elliptic curve group. ECDH begins by selecting the underlying field $F(P)$ or $GF(2^k)$, the curve E with parameters a, b and the base point P . The

order of the base point P is equal to n . The standards often suggest that we select an elliptic curve with prime order and therefore any element of the group would be selected and their order will be the prime number n [5]. At the end of the protocol, the communicating parties end up with the same value K which is a point on the curve.

2.6 Three-Party Key Exchange Protocol

Recently, Chang proposed a practical three-party key exchange (C-3PEKE) protocol with round efficiency [4]. It allows two parties A and B to share an easy-to-remember password with a trusted server S . S acts as a coordinator between two communication parties to complete mutual authentication. Once authentication is achieved, two parties can share a session key to encrypt and decrypt their communication. A practical 3PEKE protocol should comply with the following requirements [4]:

1. The session key should be agreed by the communication parties instead of being assigned by the server directly.
2. Except the password, no extra secret information should be needed - the public key for example.
3. The server has to authenticate both communication parties.
4. Computation and round efficiencies should be provided at the same time.

2.7 Multiparty Key Exchange Protocol

Consider the dynamic scenario where participants may join or leave a multi-cast group at any given time. As a result of the increased popularity of group oriented applications, the design of an efficient authenticated group key agreement protocol has recently received much attention in the literature. A comprehensive treatment have been made to extend the two party (and three party) key agreement protocols to multi party setting. Notable solutions have been suggested by Ingemerson et al. [13], Burmester and Desmedt [10], Steiner et al. [12] and Becker and Willie [11]. All these works assume a passive (eavesdropping) adversary, and the last three provide rigorous proofs of security. For practical applications, efficiency is a critical concern in designing group key agreement in addition to provable security. In particular, number of rounds may be crucial in an environment where number of group members are quite large and the group is dynamic. Handling dynamic membership changes get much attention to the current research community. A group key agreement scheme in a dynamic group must ensure that the session key is updated upon every membership change so that subsequent communication sessions are protected from leaving members and previous communication sessions are protected from joining members. Although this can be achieved by running any

authenticated group key agreement protocol from scratch whenever group membership changes, alternative approaches to handle this dynamic membership more effectively would be clearly preferable in order to minimize cost of the rekeying operations associated with group updates. The problem of key agreement in Dynamic Peer Groups (DPG) were studied by Steiner et al. [12]. They proposed a class of generic n -party Diffie-Hellman protocols". Atenise et al. [14] [15] introduced authentication into the class of protocols and heuristically analyze their security against active adversary. Steiner et al. [16] consider a number of different scenario of group membership changes and introduced a complete key management suite CLIQUES studied specially for DPGs which enable addition and exclusion of group members as well as refreshing of the keys. The security analysis of these schemes are heuristic against active adversaries. However, Pereira and Quisquater [20] have described a number of potential attacks, highlighting the need for ways to obtain greater assurance in the security of these protocols. Bresson et al. [17] [18] have recently given a formal security model for group authenticated key agreement. They provided the first provably secure protocols based on the protocols of Steiner et al. [12] for this setting which requires $O(n)$ rounds to establish a key among a group of n users. The initial works [18] respectively consider the static and dynamic case, the security of both of which are in random oracle model following the formalized security model introduced by themselves under the computational Diffie-Hellman (CDH) assumption. They further refine in [18] the existing security model to incorporate major missing details, (e.g. strong corruption and concurrent sessions) and proposed an authenticated dynamic group Diffie-Hellman key agreement proven secure under the DDH assumption within this model. Their security result holds in the standard model instead of random oracle model.

3 Proposed Protocol for Three Party

Let A and B be two clients who wish to establish a session key, and S be a trusted authentication server with which client A has registered a password pw_A . Then the 3PAKE protocol runs among the three parties, S , A and B , with the following parameters established:

- Let the elliptic curve E defined over a finite field F_p two field elements $a, b \in F_p$, which defined the equation of the elliptic curve E over F_p i.e $y^2 = x^3 + ax + b$ in the case $p \geq 3$, where $4a^3 + 27b^2 \neq 0$.
- Let M and N be any two group elements in $E(F_p)$.
- Two one-way hash functions \mathcal{G} and \mathcal{H} , where the output are the elements of F_p
- Iteration Count is the number to be randomly choosed and both the hash function will be executed that nos of time. Let the number be $c \in [1, n - 1]$. So we have to compute the hash \mathcal{G} and \mathcal{H} c times.

The proposed protocol follows the following steps.

- Step -I : A chooses a random number t from the interval $[1, n-1]$, computes the point $P = t \cdot Q$ and $P' = P + M \cdot pw_A$ and sends $A||P'$ to B .
- Step -II : B selects a random number s from the interval $[1, n-1]$, computes $R = s \cdot Q$ and $R' = R + N \cdot pw_B$ and sends $A||P'||B||R'$ to S .
- Step-III : Upon receiving $A||P'||B||R'$, S first recovers P and R by computing $P = P' - M \cdot pw_A$ and $R = R' - N \cdot pw_B$. Next S select random number u from $[1, n-1]$ and computes $\tilde{P} = u \cdot P$ and $\tilde{R} = u \cdot R$, then compute the following

$$pw'_A(1) = pw_A \cdot \mathcal{G}(A||S||P)$$

$$pw'_A(2) = \mathcal{G}(pw'_A(1))$$

\vdots

$$pw'_A(c) = \mathcal{G}(pw'_A(c-1))$$

Finally we get $pw'_A = \mathcal{G}(pw'_A(c))$

Similarly $pw'_B(1) = pw_B \cdot \mathcal{G}(B||S||P)$

$$pw'_B(2) = \mathcal{G}(pw'_B(1))$$

\vdots

$$pw'_B(c) = \mathcal{G}(pw'_B(c-1))$$

Finally we get $pw'_B = \mathcal{G}(pw'_B(c))$

$$\tilde{P}' = pw'_B \cdot \tilde{P}$$

$$\tilde{R}' = pw'_A \cdot \tilde{R}$$

and sends $\tilde{P}'||\tilde{R}'$ to B .

- Step -IV : After having received $\tilde{P}'||\tilde{R}'$, B computes

$$pw'_B(1) = pw_B \cdot \mathcal{G}(B||S||R)$$

$$pw'_B(2) = \mathcal{G}(pw'_B(1))$$

\vdots

$$pw'_B(c) = \mathcal{G}(pw'_B(c-1))$$

$$pw'_B = \mathcal{G}(pw'_B(c))$$

$$K = s \cdot pw_B^{-1} \cdot (\tilde{P}')$$

$$L_1(1) = \mathcal{G}(A||B||K)$$

$$L_1(2) = \mathcal{G}(L_1(1))$$

\vdots

$$L_1(c) = \mathcal{G}(L_1(c-1))$$

$$L_1 = L_1(c) \text{ and sends } \tilde{R}'||L_1 \text{ to } A.$$

- Step-V : With $\tilde{R}'||L_1$ from B , A computes $pw'_A = pw_A \cdot \mathcal{G}(A||S||P)$, $K = t \cdot pw_A^{-1} \cdot (\tilde{R}')$ and verifies that L_1 is equal to $L_1(c)$ by computing $L_1(1) = \mathcal{G}(A||B||K)$, $L_1(2) = \mathcal{G}(L_1(1)) \dots L_1(c) = \mathcal{G}(L_1(c-1))$. If the verification fails, then A aborts the protocol. Otherwise, A computes the session key SK as

$$\begin{aligned}
SK(1) &= \mathcal{H}(A\|B\|K) \\
SK(2) &= \mathcal{H}(SK(1)) \\
&\vdots \\
SK(c) &= \mathcal{H}(SK(c-1)) \\
SK &= SK(c)
\end{aligned}$$

and sends $L_2 = \mathcal{G}(A\|B\|K)$.

- Step-VI : B verifies the correctness of L_2 is equal to $L_2(c)$ by checking the equation $L_2(1) = \mathcal{G}(B\|A\|K), L_2(2) = \mathcal{G}(L_2(1)) \dots L_2(c) = \mathcal{G}(L_2(c-1))$. If it holds, then B computes the session key $SK = \mathcal{H}(A\|B\|K)$. Otherwise, B abort the protocol.

3.1 Verification of Correctness of 3PAKE

The correctness of 3PAKE can be verified as

$$\begin{aligned}
K &= s.pw'_B{}^{-1}.\tilde{P}' = s.pw'_B{}^{-1}.pw'_B\tilde{P} = s.\tilde{P} = s.u.P \\
&= sut.Q \\
\text{and } K &= t.pw'_A{}^{-1}.\tilde{R}' = t.pw'_A{}^{-1}.pw'_A\tilde{R} \\
t.\tilde{R} &= tu.R = tus.Q
\end{aligned}$$

4 Proposed Protocol for Multiparty

Our protocol is designed for use in multi cast network. The protocol participants consists of a single server S and multi clients $C_1, \dots C_m$. The protocol assumes that each client C_i has shared a password pw_i with server S via a secure channel. The followings are the public system parameters used in the protocol.

- Let the elliptic curve E defined over a finite field F_p two field elements $a, b \in F_p$, which defined the equation of the elliptic curve E over F_p i.e $y^2 = x^3 + ax + b$ in the case $p \geq 3$, where $4a^3 + 27b^2 \neq 0$.
- Let $M_1, M_2 \dots M_m$ be m number of group elements in $E(F_p)$.
- Two one-way hash functions \mathcal{G} and \mathcal{H} , where the output are the elements of F_p
- Iteration Count is the number to be randomly selected and both the hash function will be executed that nos of time. Let the number be $c \in [1, n-1]$. So we have to compute the hash \mathcal{H} and \mathcal{G} for c times.

The proposed protocol follows the follows the following steps.

- Step -I : Let each client C_i for $i = 1 \dots m$ selects random numbers $t_i \in [1, n-1]$ and computes the point $P_i = t_i.Q$ and $P'_i = P_i + M_i.pw_i$ and broadcasts P'_i to rest of group.

- Step -II : Clients sends $(C_1\|P'_1)\|(C_2\|P'_2) \dots (C_n\|P'_m)$ to S .
- Step-III : Upon receiving , S first recovers P_i by computing $P_i = P'_i - M_i \cdot pw_i$. Next S select random number u from $[1, n - 1]$ and computes $\tilde{P}_i = u \cdot P_i$ for all $i = 1, \dots m$ then compute the following

$$pw'_i(1) = pw_i \cdot \mathcal{G}(C_i\|S\|P_i) \text{ for all } i = 1, \dots m$$

$$pw'_i(2) = \mathcal{G}(pw'_i(1))$$

$$\vdots$$

$$pw'_i(c) = \mathcal{G}(pw'_i(c-1))$$
 Finally gets $pw'_i = \mathcal{G}(pw'_i(c))$
 Then computes $\tilde{P}'_i = pw'_j \cdot \tilde{P}_i, j = 1 \dots m$ and $i \neq j$ and sends $\tilde{P}'_1\|\tilde{P}'_2\| \dots \tilde{P}'_m$ to rest of the group.
- Step -IV : After having received $\tilde{P}'_1\|\tilde{P}'_2\| \dots \tilde{P}'_m$, C_i computes the pairwise key as

$$K_j = t_j \cdot pw'_j{}^{-1} \cdot (\tilde{P}'_i) \text{ where } i, j = 1 \dots m \text{ and } i \neq j$$

$$\alpha_1 = \mathcal{G}(C_1\|C_2\| \dots \|C_m\|K), \text{ Where } K = K_i = K_j \text{ for}$$
 all $i, j = 1 \dots m$ and $i \neq j$.

$$\alpha_2 = \mathcal{G}(\alpha_1)$$

$$\vdots$$

$$\alpha_c = \mathcal{G}(\alpha_{c-1})$$

$$\alpha = \alpha_c .$$
 Client C_j send $\tilde{P}'_i\|\alpha$ to C_i for $i, j = 1 \dots m$ and $i \neq j$
- Step-V : With $\tilde{P}'_i\|\alpha$ from C_j , C_i computes

$$pw'_i = pw_i \cdot \mathcal{G}(C_i\|S\|P_i), K_i = t_i \cdot (pw'_i)^{-1} \cdot \tilde{P}'_i$$
 and verifies that $\alpha = \alpha_c$ by computing $\alpha_1, \alpha_2 \dots \alpha_c$ and $\alpha_1 = \mathcal{G}(C_1\|C_2\| \dots \|C_m\|K)$
 If the verification fails, then C_i aborts the protocol. Otherwise, C_i computes the session key SK as

$$SK(1) = \mathcal{H}(C_1\|C_2\| \dots \|C_m\|K)$$

$$SK(2) = \mathcal{H}(SK(1))$$

$$\vdots$$

$$SK(c) = \mathcal{H}(SK(c-1))$$

$$SK = SK(c)$$
 and sends $\beta = \beta_c$, where $\beta_1 = \mathcal{G}(C_1\|C_2\| \dots \|C_m\|K)$ and $\mathcal{G}(\beta_{c-1}) = \beta_c$.
- Step-VI : Each client C_i verifies the correctness of β is equal to β_c by checking the equation $\beta_1 = \mathcal{G}(C_1\|C_2\| \dots \|C_m\|K)$,
 $\beta_2 = \mathcal{G}(\beta_1) \dots \beta_c = \mathcal{G}(\beta_{c-1})$. If it holds, then each C_i computes the session key $SK = \mathcal{H}(C_1\|C_2\| \dots \|C_m\| \dots \|K)$. Otherwise, C_i abort the protocol.

4.1 Verification of Correctness of the Protocol

The correctness of the protocol can be verified for each clients $C_1, C_2 \dots C_m$

Let for the client C_1 , the key $K_1 = \tilde{P}'_2 \cdot (pw'_1)^{-1} \cdot t_1$ can be verified with the client C_2 having the key $K_2 = \tilde{P}'_1 \cdot (pw'_2)^{-1} \cdot t_2$ by computing as

$$K_1 = \tilde{P}'_2 \cdot (pw'_1)^{-1} \cdot t_1 = (pw'_1)^{-1} \cdot (pw'_1) \cdot \tilde{P}_2 \cdot t_1 = u \cdot P_2 \cdot t_1 = u \cdot t_1 \cdot t_2 \cdot Q$$

$$K_2 = \tilde{P}'_1 \cdot (pw'_2)^{-1} \cdot t_2 = (pw'_2)^{-1} \cdot (pw'_2) \cdot \tilde{P}_1 \cdot t_2 = u \cdot P_1 \cdot t_2 = u \cdot t_2 \cdot t_1 \cdot Q$$

Similarly for each other clients $C_3 \dots C_m$ the correctness of the protocol can be verified.

5 Security discussions

Theorem 1 *The Protocol does not leak any information that allows to verify the correctness of password guesses.*

Proof : Since \mathcal{G} is a one-way hash function is executed c times and s, u and t are all random numbers, so the protocol does not leak any information that allow the adversary to verify the correctness of password guesses.

Theorem 2 *The Protocol is secure against off-line password guessing attacks.*

Proof : If the hacker intends to tract out the password, first he has to find out the iteration count c which is a random number and process that number of times. Further he has to solve Elliptic Curve Discrete Logarithm problem(ECDLP) which is computationally infeasible takes fully exponential time . So we can say it is secured against off-line password guessing attacks.

6 Conclusion

In this research a new protocol for exchanging key between two parties and group of parties with a trusted Server has been defined. This new protocol has two major advantages over all previous key exchange protocol, first this protocol does not leak any information that allow the adversary to verify the correctness of password guesses. The second one is that this protocol does not leak any information that allows to verify the correctness of password guesses. The proposed protocol is also easy to implement. The security of our system is based on Elliptic Curve Discrete Logarithm Problem (ECDLP). The primary reason for the attractiveness of ECC over systems such as RSA and DSA is that the best algorithm known for solving the underlying mathematical problem (namely, the ECDLP) takes fully exponential time. In contrast, sub-exponential time algorithms are known for underlying mathematical problems on which RSA and DSA are based, namely the integer factorization (IFP) and the discrete logarithm (DLP) problems. This means that the algorithms for solving the ECDLP become infeasible much more rapidly as the problem size increases than those algorithms for the IFP and DLP. For this

reason, ECC offers security equivalent to RSA and DSA while using far smaller key sizes. The attractiveness of ECC will increase relative to other public-key cryptosystems as computing power improvements force a general increase in the key size. The benefits of this higher-strength per-bit include higher speeds, lower power consumption, bandwidth savings, storage efficiencies, and smaller certificates.

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