Tweakable Enciphering Schemes From Stream Ciphers With IV

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Abstract. We present the first construction of a tweakable enciphering scheme from a stream cipher supporting an initialization vector. This construction can take advantage of the recent advances in hardware efficient stream ciphers to yield disk encryption systems with a very small hardware footprint. Such systems will be attractive for resource constrained devices.

Keywords: stream cipher with IV, tweakable encryption, disk encryption.

1 Introduction

A length preserving permutation is a bijective function from binary strings to binary strings such that the lengths of the input and the output strings are equal. A length preserving encryption function is a family $\{\mathbf{E}_K\}_{K\in\mathcal{K}}$ of length preserving permuations which is indexed by a secret key K. Both $\mathbf{E}_K$ and its inverse $\mathbf{E}_K^{-1}$ has to be efficiently computable for every value of K. The security requirement is that for a uniform random K, the pair of functions $\mathbf{E}_K, \mathbf{E}_K^{-1}$ is computationally indistinguishable from $\mathbf{\Gamma}, \mathbf{\Gamma}^{-1}$, where $\mathbf{\Gamma}$ is a uniform random length preserving permutation of the set of all binary strings, i.e., for each possible i, Γ is a uniform random permutation of $\{0,1\}^i$.

The above extends the notion of strong pseudorandom permutation (SPRP) that was introduced by Luby and Rackoff [10] to handle variable length strings. This extension was introduced by Halevi and Rogaway [7]. They also incorporated the idea of tweak from [9] into the above definition. A tweak is an additional input to $\mathbf{E}_K$ and the idea of the tweak is to provide for flexibility in applications. A tweakable length preserving encryption function satisfying the above notion of security is called a tweakable enciphering scheme (TES). An important practical application of a TES is that of disk encryption. In this application, the disk is encrypted sector-wise and the sector address corresponds to the tweak. Till date all known constructions of TES have used a block cipher in a particular mode of operation. There are several such constructions [7, 8, 11, 18, 4, 12, 16, 14, 17].
Our Contributions. We present the first construction of a TES using a stream cipher with an initialization vector (IV). This has important consequences to the practical problem of disk encryption. In this application, the implementation is typically in hardware and the module resides just above the disk controller in the overall architecture.

Stream ciphers are nowadays designed for two widely different environments – for extremely fast software implementation and for very small hardware foot print. In the recently concluded estream [1] selection these are called Profile 1 and Profile 2 respectively. There are some hardware efficient stream ciphers in Profile 2. Our construction makes it possible to use such stream ciphers to build a TES for disk encryption system. Overall this would lead to a hardware which is significantly smaller than any hardware implementing a block cipher.

The construction makes three calls to the stream cipher with three IVs. For the first two calls, n bits of the generated keystream are used, where n is the length of the IV supported by the stream cipher. The third call to the stream cipher generates the main keystream which is used to encrypt most of the message (whole of the message except for the first $2n$ bits). Typically, a stream cipher with IV requires an initialization phase which can take much more time compared to the key generation phase. Consequently, it is useful to keep the number of initializations as small as possible. The three calls to the stream cipher that the TES construction makes require three initializations. This does not impact the performance too much.

Apart from the stream cipher, the TES construction uses an almost XOR universal (AXU) hash function. The role of this hash function is similar to that played in the block cipher based construction [17]. The differences are in the way the hashing keys are derived and the manner in which the tweak and the length are handled. When working with a block cipher, it is easy to apply the block cipher encryption to a string and consider the output to be a hash key. Doing the same with an IV based stream cipher can be time consuming, since it will involve a stream cipher initialization. This necessitated some changes to the actual hash function design compared to [17].

An AXU hash function can be instantiated by usual polynomial hashing or the more efficient hashing introduced by Bernstein [3] based on earlier work by Rabin and Winograd [13] which is called BRW hashing. BRW hashing requires about half the number of multiplications compared to usual polynomial hashing. Another design of AXU hash function [15] uses a tower field representation of $GF(2^n)$ and does not require any multiplication at all. The trade-off, however, is that the size of the hashing key is as long as the message. Later we show how to tackle this in the context of disk encryption. Combining a hardware efficient stream cipher with the AXU hash function from [15] gives rise to an efficient TES with a very small hardware footprint. This will be attractive for implementing disk encryption on resource constrained systems such as portable and handheld devices.

1.1 Previous and Related Works

2 Construction

We follow the notation used in [16, 17]. $\mathsf{First}_r(Z)$ denotes the most significant r bits of the n-bit binary string Z and $\mathsf{pad}_i(Z)$ denotes the string obtained from Z by appending i zero bits; for $0 \leq \ell \leq 2^n - 1$, $\mathsf{bin}_n(\ell)$ denotes the n-bit binary representation of ℓ.

Let $\mathbb{F} = GF(2^n)$ be the finite field of 2^n elements. The addition operation over $\mathbb{F}$ will be denoted by $\oplus$. Elements of $\mathbb{F}$ can also be considered to be n-bit strings. Let $h : \mathcal{K} \times \mathbb{F}^m \to \mathbb{F}$ be a function such that for $\mathbf{X}, \mathbf{X}' \in \mathbb{F}^m$, $\mathbf{X} \neq \mathbf{X}'$ and for any $\gamma \in \mathbb{F}$, $\Pr_\tau[h_\tau(\mathbf{X}) \oplus h_\tau(\mathbf{X}') = \gamma] \leq \epsilon$, where $h_\tau(\mathbf{X}) \stackrel{\Delta}{=} h(\tau, \mathbf{X})$ and the probability is over uniform random choice of τ. Such an h is called an ϵ-almost XOR universal (AXU) hash function. Note that ϵ could depend on m (correspondingly, we write ϵ_m). Usual polynomial hashing requires m multiplications and has $\epsilon_m = m/2^n$. If the BRW [3] polynomials are used to instantiate h then $\max(1, \lfloor m/2 \rfloor)$ multiplications are required (assuming that the terms $\tau^2, \tau^4, \ldots$ are pre-computed and available) and $\epsilon_m = m/2^{n-1}$. Another possibility for instantiating h is from [15] which has $\epsilon_m = 1/2^n$ for all m and does not require any multiplications; but, the length of the key τ is as long as the message. Later we discuss how to tackle this issue in the context of TES.

We also require h to satisfy an ϵ-uniformity property: for any $\mathbf{X} \in \mathbb{F}^m$ and $\gamma \in \mathbb{F}$, $\Pr_\tau[h_\tau(\mathbf{X}) = \gamma] \leq \epsilon$. For an ϵ-AXU function, this property is satisfied if h is linear which is the case for the above mentioned instantiations.

A hash function with double block output [17]. Let $h_\tau : \cup_{i\geq 1}\mathbb{F}^i \to \mathbb{F}$ be an AXU function, such that for $\mathbf{X}, \mathbf{X}' \in \mathbb{F}^m$ with $\mathbf{X} \neq \mathbf{X}'$, $\Pr_\tau[h_\tau(\mathbf{X}) = h_\tau(\mathbf{X}')] \leq \epsilon_m$. Define $\Upsilon_\tau : \cup_{i\geq 3}\mathbb{F}^i \to \mathbb{F}^2$ as follows

$$\Upsilon_\tau(X_1, X_2, X_3, \ldots, X_m) = (X_1 \oplus Z, X_2 \oplus Z) \tag{1}$$

where $Z = h_\tau(X_3, \ldots, X_m)$.

It is not difficult to show that for $\mathbf{X} \neq \mathbf{X}'$, $\Pr_\tau[\Upsilon_\tau(\mathbf{X}) = \Upsilon_\tau(\mathbf{X}')] \leq \epsilon_{m-2}$. If $(A_1, A_2) = \Upsilon_\tau(X_1, X_2, X_3, \ldots, X_m)$, then $(X_1, X_2) = \Upsilon_\tau(A_1, A_2, X_3, \ldots, X_m)$, a property which is required for decryption.

Stream cipher with IV. Let $\mathrm{SC}_K : \{0,1\}^n \rightarrow \{0,1\}^L$ be a stream cipher with IV, i.e., for every choice of K, SC_K maps an IV of length n bits to a string of length L bits. The length L is assumed to be long enough for practical sized messages to be encrypted. Actual encryption of a plaintext P of ℓ bits with an IV V is done by XORing the first ℓ bits of $\mathrm{SC}_K(V)$ to the message. By a slight abuse of notation, we will write this as $P \oplus \mathrm{SC}_K(V)$. The key K is from a suitable key space and there is no restriction on this key space. The security proof will assume $\{\mathrm{SC}_K\}$ to be a family of pseudo-random functions, so that the output $\mathrm{SC}_K(V)$ can be assumed to be indistinguishable from a uniform random string of length ℓ. This is the design goal of practical stream ciphers with IV. See [2] for more discussion on this point.

Parsing. Let the length of the plaintext or the ciphertext be ℓ bits. Write $\ell = (m-1)n + r$, where $1 \leq r \leq n$. There are a total of m blocks $X_1, \ldots, X_m$, with $X_1, \ldots, X_{m-1}$ being full blocks and the length of the last block is r which is a possible partial block. *We require $\ell > 2n$ so that $m \geq 3$.*

SCTES*.* The details of the encryption and the decryption algorithms for the tweakable enciphering scheme are shown in Table 1. The required sub-routine is shown in Table 2. We denote the new construction by SCTES.

There are three invocations to SC_K. Two of these are within the Feistel structure. For each of these two invocations, the input IV to SC_K is an n-bit string. Formally, the output is an L-bit string. But, only the first n bits are used and hence only these bits are generated. The third invocation of SC_K is for the encryption of the bulk of the message. Again the input IV is an n-bit string and formally the output is and L-bit string of which only the first $(m-2)n$ bits are generated and used. The key for all the three invocations is the same key K.

There are 4 invocations of the AXU hash function h. Two of these are in the Feistel structure and the other two are as part of the two invocations of Υ. The key for the first two is τ' while the key for the other two is τ. The reason for using independent keys is that otherwise certain probability calculations do not go through.

Informally, Υ ensures that with high probability the inputs to the Feistel structure are distinct. The Feistel structure itself realizes an SPRP and the input and the output of the Feistel structure are XORed to form the input to the third invocation of SC_K. Assuming the Feistel structure to be an SPRP ensures that with high probability the inputs to this third invocation of SC_K are all distinct and so the outputs can be assumed to be independent and uniform random strings. Actual encryption (or decryption) is done by XORing with these strings which ensures that the adversary is unable to distinguish the plaintext (or ciphertext) from true random strings. This is the intuition behind the construction and is made precise in the security proof below.

Note that the invertibility of the Feistel structure does not depend on the nature of SC_K. This property has also been used in [17] to construct a TES which does not require the decryption function of a block cipher. In fact, the construction in [17] can be considered to be a motivation for the current construction. A lot of the details, however, are different necessitating a separate security proof.

Table 1. Encryption and decryption algorithms. The stream cipher key is K and the hash key is (τ, τ'). Let $\mathbf{K} = (K, \tau, \tau')$.

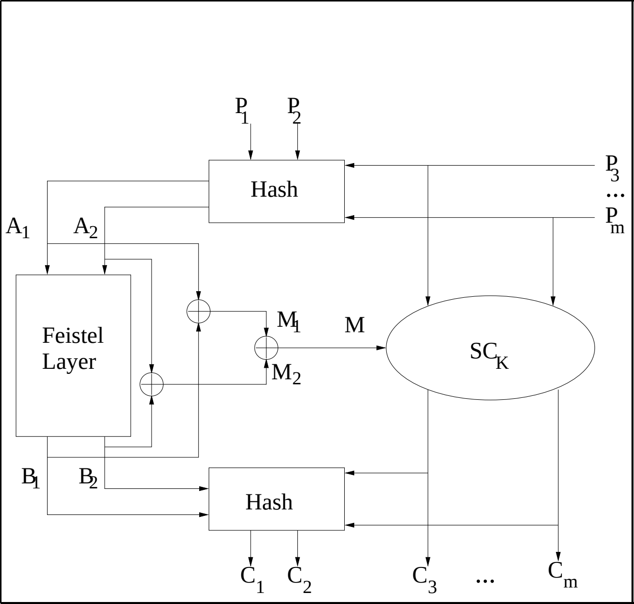
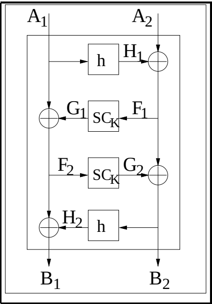
 P1 P2 Hash P3 ... Pm A1 A2 Feistel Layer M1 M M2 SCK B1 B2 Hash C1 C2 C3 ... Cm	**Algorithm** $\mathsf{Encrypt}^T_{\mathbf{K}}(P_1, \ldots, P_m)$
	1. $M_m = \mathsf{pad}_{n-r}(P_m)$;
	2. $(A_1, A_2) = \Upsilon_\tau(P_1, \ldots, P_{m-1}, M_m, T, \mathsf{bin}_n(\ell))$;
	3. $(B_1, B_2) = \mathsf{Feistel}_{K,\tau'}(A_1, A_2)$;
	4. $M_1 = A_1 \oplus B_1$; $M_2 = A_2 \oplus B_2$; $M = M_1 \oplus M_2$;
	5. $(C_3, \ldots, C_{m-1}, C_m)$
	$= (P_3, \ldots, P_{m-1}, P_m) \oplus \mathrm{SC}_K(M)$;
	6. $U_m = \mathsf{pad}_{n-r}(C_m)$;
	7. $(C_1, C_2) = \Upsilon_\tau(B_1, B_2, C_3, \ldots, C_{m-1}, U_m, T, \mathsf{bin}_n(\ell))$;
	return $(C_1, \ldots, C_m)$.
	Algorithm $\mathsf{Decrypt}^T_{\mathbf{K}}(P_1, \ldots, P_m)$
	1. $U_m = \mathsf{pad}_{n-r}(C_m)$;
	2. $(B_1, B_2) = \Upsilon_\tau(C_1, \ldots, C_{m-1}, U_m, T, \mathsf{bin}_n(\ell))$;
	3. $(A_1, A_2) = \mathsf{Feistel}^{-1}_{K,\tau'}(B_1, B_2)$;
	4. $M_1 = A_1 \oplus B_1$; $M_2 = A_2 \oplus B_2$; $M = M_1 \oplus M_2$;
	5. $(P_3, \ldots, P_{m-1}, P_m)$
	$= (C_3, \ldots, C_{m-1}, C_m) \oplus \mathrm{SC}_K(M)$;
	6. $M_m = \mathsf{pad}_{n-r}(P_m)$;
	7. $(P_1, P_2) = \Upsilon_\tau(A_1, A_2, P_3, \ldots, P_{m-1}, M_m, T, \mathsf{bin}_n(\ell))$;
	return $(P_1, \ldots, P_m)$.

Table 2. A four-round Feistel construction. (A similar construction was used in [17].)

 A1 A2 h H1 G1 SCK F1 F2 SCK G2 H2 h B1 B2	$\mathsf{Feistel}_{K,\tau'}(A_1, A_2)$	$\mathsf{Feistel}^{-1}_{K,\tau'}(B_1, B_2)$
	1. $H_1 = h_{\tau'}(A_1)$;	1. $H_2 = h_{\tau'}(B_2)$;
	2. $F_1 = H_1 \oplus A_2$;	2. $F_2 = B_1 \oplus H_2$;
	3. $F_2 = A_1 \oplus \mathrm{SC}_K(F_1)$;	3. $F_1 = B_2 \oplus \mathrm{SC}_K(F_2)$;
	4. $B_2 = F_1 \oplus \mathrm{SC}_K(F_2)$;	4. $A_1 = F_2 \oplus \mathrm{SC}_K(F_1)$;
	5. $H_2 = h_{\tau'}(B_2)$;	5. $H_1 = h_{\tau'}(A_1)$;
	6. $B_1 = H_2 \oplus F_2$;	6. $A_2 = H_1 \oplus F_1$;
	return (B_1, B_2).	return (A_1, A_2).

Fixed length inputs. If the length of plaintexts and ciphertexts are fixed, then there is no need to provide $\mathsf{bin}_n(\ell)$ as input to the hash function. This will improve the efficiency of the construction (by a small amount) without affecting the security.

2.1 Efficiency

The key for SCTES consists of the key for the stream cipher and the keys τ, τ' for the hash functions. For polynomial based hashing or hashing using the BRW polynomials, both τ and τ' are n-bit blocks.

Let $[H_m]$ denote the time for hashing m blocks using h; [ISC] the time for initializing the stream cipher SC; and $[\mathrm{SC}_i]$ the time for generating i bits using SC *after* it has been initialized. The time complexity of SCTES for encrypting an m-block message is $2([H_m]+[H_1])+3[\mathrm{ISC}]+[\mathrm{SC}_{\ell-2n}]$. The derivation of this cost is explained below.

For some stream ciphers, the initialization time [ISC] can be quite high. This is the reason for separately accounting for this. The first two initializations are incurred due to the application of the two SC_K calls in the Feistel structure. The third one is to generate the key stream for the actual encryption of the $\ell - 2n$ bits of the message which correspond to the blocks $P_3, \ldots, P_m$. The time for this generation is accounted for by the term $[\mathrm{SC}_{\ell-2n}]$. The call $\Upsilon_\tau(P_1, \ldots, P_{m-1}, M_m, T, \mathsf{bin}_n(\ell))$ in turn invokes $h_\tau(P_3, \ldots, P_{m-1}, M_m, T, \mathsf{bin}_n(\ell))$. So, the two calls to Υ requires time $2[H_m]$. The two calls to h within the Feistel structure take $[H_1]$ time each. Thus, the total time for hashing is $2([H_m]+[H_1])$.

If polynomial hashing is used, then the time for hashing is $2(m+1)$ multiplications over $GF(2^n)$, whereas for BRW-based hashing, the time for hashing is approximately $m + 2$ multiplications over $GF(2^n)$.

There is a restriction that the message length must be greater than $2n$ bits. But, there is no upper bound on the length and varying length messages can be handled. For disk encryption and other practical applications, the restriction of more than $2n$ bits on the message length is not a concern.

2.2 Disk Encryption

A disk is encrypted sector-wise and the sector address is the tweak. The length of each sector is fixed (typical value is 512 bytes, which correspond to 32 128-bit blocks). In this case, the input $\mathsf{bin}_n(\ell)$ to Υ need not be provided. For polynomial or BRW-based hashing, this reduces the number of multiplications by two.

It has been mentioned in [7] that the hardware for disk encryption resides just above the disk controller. For such a hardware implementation, it might be worthwhile to use the hashing scheme from [15]. This scheme does not require any multiplications and being parallelizable is quite efficient to implement in hardware using registers and XOR gates. The downside is that this scheme requires a key which is as long as the message. For disk encryption, this means that the key τ' used by h in Υ is also required to be 512 bytes, whereas the key τ' used by h in the Feistel structure is 16 bytes (if $n = 128$). Note, however, that the same 512 byte key will be used for all the sectors and it is not the case that each sector requires a different key.

The maintenance of a 512-byte key can be a problem. A way out is to generate this key using the stream cipher itself. Let κ be an n-bit string and then generate τ as $\tau = \mathrm{SC}_K(\kappa)$. This τ is used for the actual hashing. In this case, κ is now the secret key from which the actual hashing key is derived. The time for generating τ from κ is $[\mathrm{ISC}]+[\mathrm{SC}_{2^{12}}]$. In practice, τ will be generated once during a read/write session and hence, the time for generating τ will amortized over the number of sectors which are processed in one session. Effectively, the effect of the time for generating τ from κ on the overall time will be negligible. Also, the effect of this strategy on the security bound is negligible.

3 Security

The basic encryption primitive that is used in the construction of SCTES is a stream cipher with IV. This is defined to be a family of functions $\{\mathsf{SC}_K\}_{K\in\mathcal{K}}$, where $\mathsf{SC}_K : \{0,1\}^n \to \{0,1\}^L$. The proper formal model for this primitive is a PRF [2]. This means that a computationally bounded adversary is unable to distinguish the output of SC_K for a uniform random K from the output of a uniform random function $\rho : \{0,1\}^n \to \{0,1\}^L$.

The security analysis of SCTES that we perform in the paper is information theoretic with SC_K being replaced by a uniform random function ρ. Hence, without loss of generality, the adversary $\mathcal{A}$ can be considered to be a deterministic algorithm. Passing from information theoretic security to computational security for SCTES is based on the computational security of SC_K. This task is quite standard and follows along lines similar to that taken in moving from information theoretic to computational security for other constructions. See [7, 5] for details.

The security model for a tweakable enciphering scheme (which can handle inputs of length greater than $2n$ bits) is described below. An adversary $\mathcal{A}$ interacts with the encryption and the decryption oracles of the tweakable enciphering scheme and finally outputs either 0 or 1. Oracles are written as superscripts. The encryption oracle $\mathbf{\Pi}$ takes as input (T, P), where T is an n-bit string and P is a string of length greater than $2n$ and returns C which is of length equal to that of P. Similarly, the decryption oracle $\mathbf{\Pi}^{-1}$ takes as input (T, C) and returns P. The notation $\mathcal{A}^{\mathbf{\Pi},\mathbf{\Pi}^{-1}} \Rightarrow 1$ denotes the event that the adversary $\mathcal{A}$, interacts with the oracles $\mathbf{\Pi}$ and $\mathbf{\Pi}^{-1}$, and finally outputs the bit 1.

We consider an adversary's advantage in distinguishing a tweakable enciphering scheme from an oracle which simply returns random bit strings. This advantage is defined in the following manner.

$$\mathbf{Adv}^{\pm\mathrm{rnd}}_{\mathbf{\Pi}}(\mathcal{A}) = \Pr\left[\mathcal{A}^{\mathbf{\Pi},\mathbf{\Pi}^{-1}} \Rightarrow 1\right] - \Pr\left[\mathcal{A}^{\$(.,.),\$(.,.)} \Rightarrow 1\right]$$

where $\$(., P)$ returns random bits of length $|P|$. See [7, 6] for other definitions of advantages and the relation to the above definition of advantage.

The query complexity σ_n of an adversary is defined to be the total number of n-bit blocks it provides in all its encryption and decryption queries. This includes the plaintext and ciphertext blocks as well as the n-bit tweak. By $\mathbf{Adv}(q, \sigma_n)$ (with suitable sub and super-scripts) we denote the maximum advantage of any adversary which makes a total of q queries and has query complexity σ_n.

Theorem 1. *Fix n and σ_n to be positive integers. Suppose that an adversary uses a total of σ_n blocks in all its queries, where each block is an n-bit string.*

1. *If $\epsilon_m \le cm/2^n$ for some constant c, then*

$$\mathbf{Adv}^{\pm\mathrm{rnd}}_{\mathsf{SCTES}}(\sigma_n) \le \frac{13q^2 + 2cq\sigma_n}{2^{n-1}}. \tag{2}$$

2. *If $\epsilon_m = 1/2^n$ for all m, then*

$$\mathbf{Adv}^{\pm\mathrm{rnd}}_{\mathsf{SCTES}}(\sigma_n) \le \frac{15q^2}{2^{n-1}}. \tag{3}$$

For the usual polynomial based instantiation of h, the constant c in the above statement is 1 while for BRW based hashing the constant c is 2. In both cases, we get a $q\sigma_n/2^n$ type bound which is significantly better than what has been previously obtained for other tweakable enciphering schemes. If we use an AXU function for which $\epsilon_m = 1/2^n$ (as given in [15]), then the bound improves to a constant multiple of $q^2/2^n$. A few words will be in order to place these bounds in the proper perspective.

Informally, the reason why we obtain such improved bounds is that we work with a stream cipher where we can assume the "long" output to be indistinguishable from a uniform random string. For block cipher based constructions, for any invocation we can only assume that we obtain a uniform random string of length n bits. Thus, for long strings, the bound invariably is of the type $\sigma_n^2/2^n$. This may seem to suggest that using a stream cipher gives a better bound. But, the correct view is that the improvement in the bound on information theoretic security will probably be compensated by the greater computational advantage of an adversary in attacking the PRF-property of the stream cipher with IV. So, on the whole, we would say that the computational bounds for both block and stream cipher based construction will be similar.

The other point is the difference in the bound when ϵ_m is $1/2^n$ versus when it is bounded above by $cm/2^n$. In this case, the bound with $\epsilon_m = 1/2^n$ is actually better. But, the downside is that for such hash functions the key is equal to the length of the message. In the context of disk encryption, the way to tackle such long key length has been discussed in Section 2.2. This method requires one invocation of the stream cipher with IV and has a miniscule effect on the security which is accounted for in the constant 15 given in the theorem statement.

3.1 Proof of Theorem 1

We need to consider collisions among internal variables and so we define the internal variables.

Internal variables. There are two invocations of ρ in the Feistel structure and one invocation of ρ for encryption (or decryption) from the third block onwards. Denote the inputs and outputs of the Feistel calls by F_1, F_2 and G_1, G_2 respectively and denote the input and output of the other call by F_3 and G_3. The quantities F_1, F_2, F_3 and G_1, G_2, G_3 can be expressed in terms of the plaintext and ciphertext blocks.

$$\begin{array}{ll} F_1 = h_{\tau'}(A_1) \oplus A_2; & F_2 = B_1 \oplus h_{\tau'}(B_2); \\ G_1 = A_1 \oplus F_2 = A_1 \oplus B_1 \oplus h_{\tau'}(B_2); & G_2 = B_2 \oplus F_1 = B_2 \oplus A_2 \oplus h_{\tau'}(A_1); \end{array}$$

where $M_m = \mathsf{pad}_{n-r}(P_m)$, $U_m = \mathsf{pad}_{n-r}(C_m)$ and

$$\begin{array}{ll} A_1 = P_1 \oplus h_\tau(P_3, \ldots, P_{m-1}, M_m, T, \mathsf{bin}_n(\ell)); & A_2 = P_2 \oplus h_\tau(P_3, \ldots, P_{m-1}, M_m, T, \mathsf{bin}_n(\ell)); \\ B_1 = C_1 \oplus h_\tau(C_3, \ldots, C_{m-1}, U_m, T, \mathsf{bin}_n(\ell)); & B_2 = C_2 \oplus h_\tau(C_3, \ldots, C_{m-1}, U_m, T, \mathsf{bin}_n(\ell)). \end{array}$$

$$F_3 = M = A_1 \oplus A_2 \oplus B_1 \oplus B_2 = P_1 \oplus P_2 \oplus C_1 \oplus C_2;$$
$$G_3 = \begin{cases} (P_3, \ldots, P_{m-1}, P_m||0^{n-r}) \oplus (C_3, \ldots, C_{m-1}, D_m) & \text{if } \mathsf{ty} = \mathsf{enc}; \\ (P_3, \ldots, P_{m-1}, V_m) \oplus (C_3, \ldots, C_{m-1}, C_m||0^{n-r}) & \text{if } \mathsf{ty} = \mathsf{dec}. \end{cases}$$

Notation. The adversary makes a total of q queries of possibly different lengths. We use the superscript ${}^{(s)}$ to denote quantities corresponding to the s-th query. For example, the length is $\ell^{(s)}$; the number of blocks is $m^{(s)}$; the tweak is $T^{(s)}$; the plaintext blocks are $P_1^{(s)}, \ldots, P_{m^{(s)}}^{(s)}$ and the ciphertext blocks are $C_1^{(s)}, \ldots, C_{m^{(s)}}^{(s)}$. Similarly, we denote the internal variables. A query can be either an encryption or a decryption query. The variable $\mathsf{ty}^{(s)}$ denotes the type of the query, i.e., if the query is an encryption query, then $\mathsf{ty}^{(s)} = \mathsf{enc}$, and if the query is a decryption query, then $\mathsf{ty}^{(s)} = \mathsf{dec}$.

The oracles $\mathbf{\Pi}$ and $\mathbf{\Pi}^{-1}$ are built using the uniform random function ρ. Suppose all queries made by the adversary $\mathcal{A}$ are answered in the manner shown in Table 3. For both encrypt and decrypt queries, the adversary obtains independent and uniform random strings. Then clearly $\mathcal{A}$ cannot distinguish between $\mathbf{\Pi}, \mathbf{\Pi}^{-1}$ and $\$(\cdot,\cdot), \$(\cdot,\cdot)$. But, answering queries in this manner may not always be consistent with the fact that ρ is a uniform random permutation.

Table 3. Response to queries from an adversary.

$\mathsf{ty}^{(s)} = \mathsf{enc}$
1. choose $C_1^{(s)}, C_2^{(s)}$ to be independent and uniform random n-bit strings;
2. choose $G_3^{(s)}$ to be an independent and uniform random $((m^{(s)} - 2)n)$-bit string;
3. set $C_3^{(s)}, \ldots, C_{m^{(s)}-1}^{(s)}, D_{m^{(s)}}^{(s)}$ to be $(P_3, \ldots, P_{m-1}, P_m \vert\vert 0^{n-r^{(s)}}) \oplus G_3^{(s)}$;
4. set $C_{m^{(s)}}^{(s)} = \mathsf{First}_{r^{(s)}}(D_{m^{(s)}}^{(s)})$;
5. return $C_1^{(s)}, \ldots, C_{m^{(s)}-1}^{(s)}, C_{m^{(s)}}^{(s)}$ to the adversary.

$\mathsf{ty}^{(s)} = \mathsf{dec}$
1. choose $P_1^{(s)}, P_2^{(s)}$ to be independent and uniform random n-bit strings;
2. choose $G_3^{(s)}$ to be independent and uniform random $((m^{(s)} - 2)n)$-bit string;
3. set $P_3, \ldots, P_{m^{(s)}-1}^{(s)}, V_{m^{(s)}}^{(s)}$ to be $(C_3^{(s)}, \ldots, C_{m-1}, C_m \vert\vert 0^{n-r^{(s)}}) \oplus G_3^{(s)}$;
4. set $P_{m^{(s)}}^{(s)} = \mathsf{First}_{r^{(s)}}(V_{m^{(s)}}^{(s)})$;
5. return $P_1^{(s)}, \ldots, P_{m^{(s)}-1}^{(s)}, P_{m^{(s)}}^{(s)}$ to the adversary.

Let $\mathbf{E}$ be the event that the random variables $F_3^{(1)}, \ldots, F_3^{(q)}$ take distinct values and further that these values are distinct from the values taken by the variables $F_1^{(1)}, F_2^{(2)}, \ldots, F_1^{(q)}, F_2^{(q)}$. Conditioned on the event $\mathbf{E}$, the uniform random function ρ is applied to the "new" and distinct values $F_3^{(1)}, \ldots, F_3^{(q)}$. Consequently, the outputs $G_3^{(1)}, \ldots, G_3^{(q)}$ are independent and uniformly distributed. (The output of the sth invocation of ρ is an L-bit string and we assume that for any s, $G_3^{(s)}$ is the first $(m^{(s)} - 2)n$ bits of this string.) As a result, using the definition of G_3, if $\mathsf{ty}^{(s)} = \mathsf{enc}$, then $C_3^{(s)}, \ldots, C_{m^{(s)}-1}^{(s)}, C_{m^{(s)}}^{(s)}$ is independent of the other random variables and is distributed uniformly over $\{0,1\}^{\ell^{(s)}-2n}$; if $\mathsf{ty}^{(s)} = \mathsf{dec}$, then $P_3^{(s)}, \ldots, P_{m^{(s)}-1}^{(s)}, P_{m^{(s)}}^{(s)}$ is independent of the other random variables and is distributed uniformly over $\{0,1\}^{\ell^{(s)}-2n}$.

Let $\mathcal{D}$ be the set of random variables $\{F_1^{(1)}, F_2^{(1)}, F_3^{(1)}, \ldots, F_1^{(q)}, F_2^{(q)}, F_3^{(q)}\}$ and $\mathcal{R}$ be the set of random variables $\{G_1^{(1)}, G_2^{(2)}, \ldots, G_1^{(q)}, G_2^{(q)}\}$. Let $\mathsf{Coll}(\mathcal{D})$ be the event that two random variables in $\mathcal{D}$ take the same value and similarly define $\mathsf{Coll}(\mathcal{R})$. Note that $\overline{\mathsf{Coll}(\mathcal{D})}$ implies $\mathbf{E}$. Let Coll be the event $\mathsf{Coll}(\mathcal{D}) \vee \mathsf{Coll}(\mathcal{R})$. For any adversary $\mathcal{A}$, we clearly have

$$\Pr[\mathcal{A}^{\mathbf{\Pi},\mathbf{\Pi}^{-1}} \Rightarrow 1 | \overline{\mathsf{Coll}}] = \Pr[\mathcal{A}^{\$,\$} \Rightarrow 1].$$

In other words, if Coll does not occur, then in the real game, i.e. during the interaction with $\mathbf{\Pi}, \mathbf{\Pi}^{-1}$, the adversary gets independent and uniform random strings as responses which is the same as it would get while interacting with oracles that return independent and uniform random strings.

$$\begin{aligned}
\Pr[\mathcal{A}^{\mathbf{\Pi},\mathbf{\Pi}^{-1}} \Rightarrow 1] &= \Pr[(\mathcal{A}^{\mathbf{\Pi},\mathbf{\Pi}^{-1}} \Rightarrow 1) \wedge (\mathsf{Coll} \vee \overline{\mathsf{Coll}})] \\
&= \Pr[(\mathcal{A}^{\mathbf{\Pi},\mathbf{\Pi}^{-1}} \Rightarrow 1)|\mathsf{Coll}]\Pr[\mathsf{Coll}] + \Pr[(\mathcal{A}^{\mathbf{\Pi},\mathbf{\Pi}^{-1}} \Rightarrow 1)|\overline{\mathsf{Coll}}]\Pr[\overline{\mathsf{Coll}}] \\
&\leq \Pr[\mathsf{Coll}] + \Pr[\mathcal{A}^{\mathbf{\Pi},\mathbf{\Pi}^{-1}} \Rightarrow 1|\overline{\mathsf{Coll}}] \\
&= \Pr[\mathsf{Coll}] + \Pr[\mathcal{A}^{\$,\$} \Rightarrow 1].
\end{aligned}$$

This gives $\mathsf{Adv}_{\mathbf{\Pi}}(\mathcal{A}) = \Pr[\mathcal{A}^{\mathbf{\Pi},\mathbf{\Pi}^{-1}} \Rightarrow 1] - \Pr[\mathcal{A}^{\$,\$} \Rightarrow 1] \leq \Pr[\mathsf{Coll}]$. We next obtain an upper bound for $\Pr[\mathsf{Coll}]$ which would then complete the proof.

Collision analysis. Below we prove some results on the probability of certain kinds of collisions.

Claim. Let τ' be chosen uniformly at random from $\mathbb{F}$. For $1 \leq s, t \leq q$, $\Pr_{\tau}[F_1^{(s)} = F_2^{(s)}] = 1/2^{n-1}$.

Proof of claim. $A_1^{(s)} = P_1^{(s)} \oplus \mathsf{rest}^{(s)}$ and $B_2^{(t)} = C_2^{(t)} \oplus \mathsf{rest}_1^{(t)}$, where $\mathsf{rest}^{(s)}$ and $\mathsf{rest}_1^{(t)}$ are independent of both $P_1^{(s)}$ and $C_2^{(t)}$. So, $A_1 = B_2$ holds if and only if $P_1^{(s)} \oplus C_2^{(t)} = \mathsf{rest}^{(s)} \oplus \mathsf{rest}_1^{(t)}$. If the t-th query is an encrypt query, then $C_2^{(t)}$ is an independent and uniform random string and if the s-th query is a decrypt query then $P_1^{(s)}$ is an independent and uniform random string. So, irrespective of whether s equals t or not, one of $P_1^{(s)}$ and $C_2^{(t)}$ is a uniform random string which is independent of the other as also independent of $\mathsf{rest}^{(s)} \oplus \mathsf{rest}_1^{(t)}$. This shows $\Pr[A_1^{(s)} = B_2^{(t)}] \leq 1/2^n$.

$$\begin{aligned}
\Pr[F_1^{(s)} = F_2^{(t)}] &= \Pr_{\tau'}[h_{\tau'}(A_1^{(s)}) \oplus A_2^{(s)} = h_{\tau'}(B_2^{(t)}) \oplus B_1^{(t)}] \\
&= \Pr_{\tau'}\left[\left(h_{\tau'}(A_1^{(s)}) \oplus A_2^{(s)} = h_{\tau'}(B_2^{(t)}) \oplus B_1^{(t)}\right) \wedge ((A_1^{(s)} = B_2^{(t)}) \vee (A_1^{(s)} \neq B_2^{(t)}))\right] \\
&= \Pr_{\tau'}\left[\left(h_{\tau'}(A_1^{(s)}) \oplus A_2^{(s)} = h_{\tau'}(B_2^{(t)}) \oplus B_1^{(t)}\right) |(A_1^{(s)} = B_2^{(t)})\right] \Pr_{\beta_1}[A_1^{(s)} = B_2^{(t)}] \\
&\quad + \Pr_{\tau'}\left[\left(h_{\tau'}(A_1^{(s)}) \oplus A_2^{(s)} = h_{\tau'}(B_2^{(t)}) \oplus B_1^{(t)}\right) |(A_1^{(s)} \neq B_2^{(t)})\right] \Pr[A_1^{(s)} \neq B_2^{(t)}] \\
&\leq \Pr[A_1^{(s)} = B_2^{(t)}] + \Pr_{\tau'}\left[\left(h_{\tau'}(A_1^{(s)}) \oplus A_2^{(s)} = h_{\tau'}(B_2^{(t)}) \oplus B_1^{(t)}\right) |(A_1^{(s)} \neq B_2^{(t)}))\right] \\
&\leq \frac{2}{2^n}.
\end{aligned}$$

The last inequality follows from the XOR-universal property of h. □

Claim. Suppose that τ and τ' are chosen independently and uniformly at random from $\mathbb{F}$; $1 \leq s < t \leq q$ and suppose without loss of generality that $\ell^{(s)} \leq \ell^{(t)}$.

1. If $(P_1^{(s)}, \ldots, P_{m-1}^{(s)}, M_m^{(s)}, T^{(s)}, \mathsf{bin}_n(\ell^{(s)})) \neq (P_1^{(t)}, \ldots, P_{m-1}^{(t)}, M_m^{(t)}, T^{(t)}, \mathsf{bin}_n(\ell^{(t)}))$, then $\Pr_{\tau,\tau'}[F_1^{(s)} = F_1^{(t)}] \leq \epsilon_{m^{(t)}} + 1/2^n$.
2. If $(C_1^{(s)}, \ldots, C_{m-1}^{(s)}, U_m^{(s)}, T^{(s)}, \mathsf{bin}_n(\ell^{(s)})) \neq (C_1^{(t)}, \ldots, C_{m-1}^{(t)}, U_m^{(t)}, T^{(t)}, \mathsf{bin}_n(\ell^{(t)}))$, then $\Pr_{\tau,\tau'}[F_2^{(s)} = F_2^{(t)}] \leq \epsilon_{m^{(t)}} + 1/2^n$.

Proof of claim. We prove (1), the proof of (2) being similar. Note that $F_1^{(s)} = h_{\tau'}(A_1^{(s)}) \oplus A_2^{(s)}$ and $F_1^{(t)} = h_{\tau'}(A_1^{(t)}) \oplus A_2^{(t)}$.

If at least one of $\mathsf{ty}^{(s)}$ or $\mathsf{ty}^{(t)}$ is equal to dec, then one of $P_2^{(s)}$ and $P_2^{(t)}$ is a uniform random string which is independent of all other random variables. So, in this case, it follows from the definition of F_1 that the probability that $F_1^{(s)}$ equals $F_1^{(t)}$ is $1/2^n$.

Now suppose that both $\mathsf{ty}^{(s)}$ and $\mathsf{ty}^{(t)}$ are equal to enc.

Case $(P_3^{(s)}, \ldots, P_{m-1}^{(s)}, M_m^{(s)}, T^{(s)}, \mathsf{bin}_n(\ell^{(s)})) = (P_3^{(t)}, \ldots, P_{m-1}^{(t)}, M_m^{(t)}, T^{(t)}, \mathsf{bin}_n(\ell^{(t)}))$. Then necessarily $(P_1^{(s)}, P_2^{(s)}) \neq (P_1^{(t)}, P_2^{(t)})$ (as otherwise the two queries would be same). If $P_1^{(s)} = P_1^{(t)}$ (and so, $P_2^{(s)} \neq P_2^{(t)}$), then $A_1^{(s)} = A_1^{(t)}$; $P_2^{(s)} \neq P_2^{(t)}$ implies $A_2^{(s)} \neq A_2^{(t)}$ whereby we have $F_1^{(s)} \neq F_1^{(t)}$. If $P_2^{(s)} = P_2^{(t)}$ (and so, $P_1^{(s)} \neq P_1^{(t)}$), then $A_2^{(s)} = A_2^{(t)}$; $P_1^{(s)} \neq P_1^{(t)}$ implies $A_1^{(s)} \neq A_1^{(t)}$ whereby $F_1^{(s)} = F_1^{(t)}$ implies $h_\tau(A_1^{(s)}) = h_\tau(A_1^{(t)})$. Since $A_1^{(s)} \neq A_1^{(t)}$, by the XOR-universality of h, the last condition holds with probability $1/2^n$.

Case $(P_3^{(s)}, \ldots, P_{m-1}^{(s)}, M_m^{(s)}, T^{(s)}, \mathsf{bin}_n(\ell^{(s)})) \neq (P_3^{(t)}, \ldots, P_{m-1}^{(t)}, M_m^{(t)}, T^{(t)}, \mathsf{bin}_n(\ell^{(t)}))$. By the XOR-universality of h, $\Pr_\tau[A_1^{(s)} = A_1^{(t)}] \leq \epsilon_m$. Also, note that $A_1^{(s)}, A_2^{(s)}, A_1^{(t)}$ and $A_2^{(t)}$ are independent of τ'.

$$\begin{aligned}
\Pr_{\tau,\tau'}[F_1^{(s)} = F_1^{(t)}] &= \Pr_{\tau,\tau'}[h_{\tau'}(A_1^{(s)}) \oplus A_2^{(s)} = h_{\tau'}(A_1^{(t)}) \oplus A_2^{(t)}] \\
&\leq \Pr_{\tau}[A_1^{(s)} = A_1^{(t)}] + \Pr_{\tau'}[h_{\tau'}(A_1^{(s)}) \oplus A_2^{(s)} = h_{\tau'}(A_1^{(t)}) \oplus A_2^{(t)} | (A_1^{(s)} \neq A_1^{(t)})] \\
&\leq \epsilon_m + \frac{1}{2^n}.
\end{aligned}$$

The last relation holds due to the XOR-universality of h. □

Claim. Let $1 \leq s, t \leq q$.

1. $\Pr_{\tau'}[F_3^{(s)} = F_1^{(t)}] = \Pr_{\tau'}[F_3^{(s)} = F_2^{(t)}] = \frac{1}{2^n}$.
2. If $s \neq t$, then $\Pr[F_3^{(s)} = F_3^{(t)}] = \frac{1}{2^n}$.

Proof of claim.

(1). We prove the result for $\Pr_{\tau'}[F_3^{(s)} = F_1^{(t)}]$, the other case being similar. If $s \neq t$, then $P_1^{(s)}$ is independent of $F_1^{(t)}$ and since $F_3^{(s)} = P_1^{(s)} \oplus P_2^{(s)} \oplus C_1^{(s)} \oplus C_2^{(s)}$, $\Pr[F_3^{(s)} = F_1^{(t)}] = 1/2^n$ without involving τ'. So suppose that $s = t$. Then $F_3^{(s)} \oplus F_1^{(t)} = A_1^{(s)} \oplus B_1^{(s)} \oplus B_2^{(s)} \oplus h_{\tau'}(A_1^{(s)})$. The result now follows from the uniformity property of h.

(2). Since $s \neq t$, $P_1^{(s)}, P_2^{(s)}, C_1^{(s)}, C_2^{(s)}$ and $P_1^{(t)}, P_2^{(t)}, C_1^{(t)}, C_2^{(t)}$ are independent and uniformly distributed and so $F_3^{(s)} = P_1^{(s)} \oplus P_2^{(s)} \oplus C_1^{(s)} \oplus C_2^{(s)}$ and $F_3^{(t)} = P_1^{(t)} \oplus P_2^{(t)} \oplus C_1^{(t)} \oplus C_2^{(t)}$ are independent and uniformly distributed. □

Claim. Let $1 \leq s, t \leq q$.

1. $\Pr[G_1^{(s)} = G_2^{(t)}] = \frac{1}{2^n}$.
2. If $s \neq t$, then $\Pr[G_1^{(s)} = G_1^{(t)}] = \Pr[G_2^{(s)} = G_2^{(t)}] = \frac{1}{2^n}$.

Proof of claim. Recall that $G_1 = A_1 \oplus B_1 \oplus h_{\tau'}(B_2)$ and $G_2 = A_2 \oplus B_2 \oplus h_{\tau'}(A_1)$. So, we can write $G_1 = P_1 \oplus C_1 \oplus \mathsf{rest}$ and $G_2 = P_2 \oplus C_2 \oplus \mathsf{rest}_1$ where P_1, C_1, P_2, C_2 is independent of rest and rest_1. For any query, one of P_1 and C_1 is a uniform random string which is independent of all other strings and similarly for P_2 and C_2. Consequently, for any query $P_1 \oplus C_1$ and $P_2 \oplus C_2$ are uniformly distributed and independent of each other as well as rest and rest_1. Using this, it is easy to obtain the stated probabilities. □

Claim. 1. $\Pr[\mathsf{Coll}(\mathcal{R})] \leq q^2/2^{n-2}$.
2. (a) If $\epsilon_m \leq cm/2^n$ for some constant c, then $\Pr[\mathsf{Coll}(\mathcal{D})] \leq 11q^2/2^n + 2cq\sigma/2^{n-1}$.
   (b) If $\epsilon_m = 1/2^n$ for all m, then $\Pr[\mathsf{Coll}(\mathcal{D})] \leq 13q^2/2^n$.

Proof of claim. There are a total of $2q$ random variables in $\mathcal{R}$ and the probability that any two of these take the same value is $1/2^n$. This gives the bound on the probability of $\mathsf{Coll}(\mathcal{R})$.

$\mathcal{D}$ contains $3q$ random variables. Apart from the $q(q-1)$ pairs of random variables of the form $(F_1^{(s)}, F_1^{(t)})$ and $(F_2^{(s)}, F_2^{(t)})$, the probability that any other pair of random variables take the same value is $1/2^n$. The probability that a pair of the form $(F_j^{(s)}, F_j^{(t)})$, $j = 1, 2$ take the same value is $\epsilon_{m^{(t)}} + 1/2^n$. If $\epsilon_m = 1/2^n$ for all m, then the result easily holds.

Suppose that $\epsilon_m \leq cm/2^n$. Let $m^{(1)} \geq m^{(2)} \geq \cdots \geq m^{(q)}$. Then the probability that any pair of the form $(F_j^{(s)}, F_j^{(t)})$, $j = 1, 2$ take the same value is at most

$$\begin{aligned}\frac{q^2}{2^n} + \frac{2c}{2^n} \times \sum_{s=0}^{q-1} (q-s) m_{s+1} &\leq \frac{q^2}{2^n} + \frac{2qc}{2^n} \times \sum_{s=0}^{q-1} m_{s+1} \\ &\leq \frac{q^2}{2^n} + \frac{cq\sigma}{2^{n-1}}.\end{aligned}$$

Now the statement of Theorem 1 follows. □

References

1. eSTREAM, the ECRYPT Stream Cipher Project. `http://www.ecrypt.eu.org/stream/`.
2. Côme Berbain and Henri Gilbert. On the security of IV dependent stream ciphers. In Alex Biryukov, editor, *FSE*, volume 4593 of *Lecture Notes in Computer Science*, pages 254–273. Springer, 2007.
3. Daniel J. Bernstein. Polynomial evaluation and message authentication, 2007. `http://cr.yp.to/papers.html#pema`.
4. Debrup Chakraborty and Palash Sarkar. HCH: A new tweakable enciphering scheme using the hash-encrypt-hash approach. In Rana Barua and Tanja Lange, editors, *INDOCRYPT*, volume 4329 of *Lecture Notes in Computer Science*, pages 287–302. Springer, 2006. full version available at `http://eprint.iacr.org/2007/028`.
5. Debrup Chakraborty and Palash Sarkar. A general construction of tweakable block ciphers and different modes of operations. *IEEE Transactions on Information Theory*, 54(5):1991–2006, 2008.
6. Shai Halevi. Invertible universal hashing and the TET encryption mode. In Alfred Menezes, editor, *CRYPTO*, volume 4622 of *Lecture Notes in Computer Science*, pages 412–429. Springer, 2007.
7. Shai Halevi and Phillip Rogaway. A tweakable enciphering mode. In Dan Boneh, editor, *CRYPTO*, volume 2729 of *Lecture Notes in Computer Science*, pages 482–499. Springer, 2003.
8. Shai Halevi and Phillip Rogaway. A parallelizable enciphering mode. In Tatsuaki Okamoto, editor, *CT-RSA*, volume 2964 of *Lecture Notes in Computer Science*, pages 292–304. Springer, 2004.
9. Moses Liskov, Ronald L. Rivest, and David Wagner. Tweakable block ciphers. In Moti Yung, editor, *CRYPTO*, volume 2442 of *Lecture Notes in Computer Science*, pages 31–46. Springer, 2002.
10. Michael Luby and Charles Rackoff. How to construct pseudorandom permutations from pseudorandom functions. *SIAM J. Comput.*, 17(2):373–386, 1988.
11. David A. McGrew and Scott R. Fluhrer. The extended codebook (XCB) mode of operation. Cryptology ePrint Archive, Report 2004/278, 2004. `http://eprint.iacr.org/`.

12. Kazuhiko Minematsu and Toshiyasu Matsushima. Tweakable enciphering schemes from hash-sum-expansion. In K. Srinathan, C. Pandu Rangan, and Moti Yung, editors, *INDOCRYPT*, volume 4859 of *Lecture Notes in Computer Science*, pages 252–267. Springer, 2007.
13. Michael O. Rabin and Shmuel Winograd. Fast evaluation of polynomials by rational preparation. *Communications on Pure and Applied Mathematics*, 25:433–458, 1972.
14. Palash Sarkar. A general mixing strategy for the ECB-Mix-ECB mode of operation. *Inf. Process. Lett.*, 109(2):121–123, 2008.
15. Palash Sarkar. A new universal hash function and other cryptographic algorithms suitable for resource constrained devices. Cryptology ePrint Archive, Report 2008/216, 2008. `http://eprint.iacr.org/`.
16. Palash Sarkar. Efficient tweakable enciphering schemes from (block-wise) universal hash functions. *IEEE Transactions on Information Theory*, 2009. To appear.
17. Palash Sarkar. Tweakable enciphering schemes using only the encryption function of a block cipher. Cryptology ePrint Archive, Report 2009/216, 2009. `http://eprint.iacr.org/`.
18. Peng Wang, Dengguo Feng, and Wenling Wu. HCTR: A variable-input-length enciphering mode. In Dengguo Feng, Dongdai Lin, and Moti Yung, editors, *CISC*, volume 3822 of *Lecture Notes in Computer Science*, pages 175–188. Springer, 2005.