Data-Depend Hash Algorithm

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Abstract: We study some technologys that people had developed to analyse and attack hash algorithm. We find a way that use data-depend function to resist differential attack. Then we design a hash algorithm that called Data-Depend Hash Algorit(DDHA). And DDHA is simple and strong under differential attack.

Key Word: Hash algorithm, data-depend function

1. Introduction

Hash algorithm is the algorithm that computes a fixed size message digest from arbitrary size messages. After SHA-0 was published, some technologys that analyse and attack hash algorithm is developed. The major technologys is Differential attack. Papers[Wy05, Dau05] has explain the attack.

Differential attack is the best technique to attack hash function. To attack hash function, it need do the work as follow:

1. Constitute a feasible difference path that has good possibility.
2. Constitute the adequate conditions for the difference path.
3. Find some technique to raise the possibility of the difference path.

From mentioned above description of differential attack, it is easy to know that constituting a feasible difference path is the hinge. If it can make it hard to constitute a feasible difference path, it will be hard to attack the hash function. In appenix 1, we know that the data-depend circular shift has good defence feature. And we find a message expension function that make any difference path will has at least eight data-depend circular shift difference [appenix 2]. This make it hard to constitute a feasible difference path.

At the same time, we study some technologys[1,2] that used to attack hash algorithm, DDHA use some ways to resist these attack ways.

The following operations are applied to 32-bit or 64-bit words in DDHA:

1. ← variable assignment

2. Bitwise logical word operations: ‘∧’ –AND , ‘∨’ –OR, ‘⊕’ – XOR and ‘¬’ –Negation.

3. Addition ‘+’ modulo 2^{32} or modulo 2^{64} .

4. The shift right operation, $SHR^n(x)$, where x is a 32-bit or 64-bit word and n is an integer with 0⩽n<32 (resp. 0⩽n<64).

5.The shift left operation, $SHL^n(x)$, where x is a 32-bit or 64-bit word and n is an integer with 0⩽n<32 (resp. 0⩽n<64).

6. The rotate right (circular right shift) operation, $ROTR^n(x)$, where x is a 32-bit or 64-bit word and n is an integer with $0 \leqslant n < 32$ (resp. $0 \leqslant n < 64$).

7. The rotate left (circular left shift) operation, $ROTL^n(x)$, where x is a 32-bit or 64-bit word and n is an integer with $0 \leqslant n < 32$ (resp. $0 \leqslant n < 64$).

2. Data-Depend Hash Algorithm (DDHA)

DDHA has two hash functions: DDHA-256(32-bitversion), DDHA-512 (64–bitversion). DDHA-256 is used for message no bigger than 2^{64}, DDHA-512 is used for message no bigger than 2^{64}, The properties as follow:

	word	Message size	Block size	Hash value size
DDHA-256	32	$< 2^{64}$	512	256
DDHA-512	64	$< 2^{64}$	1024	512

Properties of DDHA hash functions(size in bits)

In DDHA, the message will be preprocessed. After message is preprocessed, the message will prased in N message blocks, these blocks will be processed with a compression function in order.

2.1 Preprocessing

Preprocessing in DDHA include steps:

a. padding the message M, parsing the padded message into message blocks,

b. setting the initial hash value,

2.1.1 Padding and parsing

Suppose that the length of the message M is L bits. Append the bit “1”

to the end of the message, followed by k zero bits, where k is the smallest, non-negative solution to the equation L+1+k $\equiv$ 448 mod 512 (resp. L+1+k $\equiv$ 960 mod 1024). Then append the 64-bit block that is equal to the number L expressed using a binary representation.

After message is padded, the message will be parsed into N 512-bits(resp. 1024-bits) message blocks.

2.1.2 Initial Hash Value and constants

DDHA use the same initial hash value as that of SHA-2 (given as follow):

DDHA-256	DDHA-512
$H_0^0 = 0x6a09e667,$	$H_0^0 = 0x6a09e667f3bcc908,$
$H_1^0 = 0xbb67ae85,$	$H_1^0 = 0xbb67ae8584caa73b,$
$H_2^0 = 0x3c6ef372,$	$H_2^0 = 0x3c6ef372fe94f82b,$
$H_3^0 = 0xa54ff53a,$	$H_3^0 = 0xa54ff53a5f1d36f1,$
$H_4^0 = 0x510e527f,$	$H_4^0 = 0x510e527fade682d1f,$
$H_5^0 = 0x9b05688c,$	$H_5^0 = 0x9b05688c2b3e6c1f,$
$H_6^0 = 0x1f83d9ab,$	$H_6^0 = 0x1f83d9abfb41bd6b,$
$H_7^0 = 0x5be0cd19,$	$H_7^0 = 0x5be0cd19137e2179,$

The initial hash value for DDHA

DDHA use 32 contant words, these words separate into two parts C1 and C2, C1 as follow:

DDHA-256	DDHA-512
$C1_0 = 0xd76aa478,$	$C1_0 = 0xd76aa478fffa3942,$
$C1_1 = 0xe8c7b756,$	$C1_1 = 0xe8c7b7568771f681,$
$C1_2 = 0x242070db,$	$C1_2 = 0x242070db699d6122,$
$C1_3 = 0xc1bdceee,$	$C1_3 = 0xc1bdceeefde5380c,$
$C1_4 = 0xf57c0faf,$	$C1_4 = 0xf57c0fafa4beea44,$
$C1_5 = 0x4787c62a,$	$C1_5 = 0x4787c62a4bdecfa9,$
$C1_6 = 0xa8304613,$	$C1_6 = 0xa8304613f6bb4b60,$
$C1_7 = 0xfd469501,$	$C1_7 = 0xfd469501bebfbc70,$
$C1_8 = 0x698098d8,$	$C1_8 = 0x698098d8289b7ec6,$
$C1_9 = 0x8b44f7af,$	$C1_9 = 0x8b44f7afeaa127fa,$

$C1_{10} = 0xffff5bb1,$	$C1_{10} = 0xffff5bb1d4ef3085,$
$C1_{11} = 0x895cd7be,$	$C1_{11} = 0x895cd7be04881d05,$
$C1_{12} = 0x6b901122,$	$C1_{12} = 0x6b901122d9d4d039,$
$C1_{13} = 0xfd987193,$	$C1_{13} = 0xfd987193e6db99e5,$
$C1_{14} = 0xa679438e,$	$C1_{14} = 0xa679438e1fa27cf8,$
$C1_{15} = 0x49b40821,$	$C1_{15} = 0x49b40821c4ac5665,$

Constants C1 of DDHA

C2 as follow:

DDHA-256	DDHA-512
$C2_0 = 0xf61e2562,$	$C2_0 = 0xf61e2562f4292244,$
$C2_1 = 0xc040b340,$	$C2_1 = 0xc040b340432aff97,$
$C2_2 = 0x265e5a51,$	$C2_2 = 0x265e5a51ab9423a7,$
$C2_3 = 0xe9b6c7aa,$	$C2_3 = 0xe9b6c7aafc93a039,$
$C2_4 = 0xd62f105d,$	$C2_4 = 0xd62f105d655b59c3,$
$C2_5 = 0x02441453,$	$C2_5 = 0x024414538f0ccc92,$
$C2_6 = 0xd8a1e681,$	$C2_6 = 0xd8a1e681ffeff47d,$
$C2_7 = 0xe7d3fbc8,$	$C2_7 = 0xe7d3fbc885845dd1,$
$C2_8 = 0x21e1cde6,$	$C2_8 = 0x21e1cde66fa87e4f,$
$C2_9 = 0xc33707d6,$	$C2_9 = 0xc33707d6fe2ce6e0,$
$C2_{10} = 0xf4d50d87,$	$C2_{10} = 0xf4d50d87a3014314,$
$C2_{11} = 0x455a14ed,$	$C2_{11} = 0x455a14ed4e0811a1,$
$C2_{12} = 0xa9e3e905,$	$C2_{12} = 0xa9e3e905f7537e82,$
$C2_{13} = 0xfcefa3f8,$	$C2_{13} = 0xfcefa3f8bd3af235,$
$C2_{14} = 0x676f02d9,$	$C2_{14} = 0x676f02d92ad7d2bb,$
$C2_{15} = 0x8d2a4c8a,$	$C2_{15} = 0x8d2a4c8aeb86d391,$

Constants C2 of DDHA

2.2 processing.

If there are N message blocks $M_0,...,M_{N-1}$.

The DDHA has a compression function. The input of compression function include chaining variable(8 words, $H_0^i,...,H_7^i$), message block(16 words, $m_0^i,...,m_{15}^i$), constants(32 words, $C1_0,...,C1_{15}, C2_0,...,C2_{15}$).

Then the processing as foollow:

$$for \quad j = 0 \quad to \quad 15$$
$$tm_j \leftarrow 0$$
$$next \quad j$$

$$
\begin{aligned}
&for \quad i=1 \quad to \quad N-1\\
&\quad for \quad j=0 \quad to \quad 15\\
&\qquad j1 \leftarrow j+1\\
&\qquad tm_{j1-1} \leftarrow tm_{j1-1}+m_{j1-1}^{i-1}\\
&\qquad if \quad (tm_{j1-1} < m_{j1-1}^{i-1}) \quad then\\
&\qquad\quad while \quad j1<16\\
&\qquad\qquad tm_{j1} \leftarrow tm_{j1}+1\\
&\qquad\qquad if \quad tm_{j1}<1 \quad then\\
&\qquad\qquad\quad j1 \leftarrow j1+1\\
&\qquad\qquad else\\
&\qquad\qquad\quad j1 \leftarrow 16\\
&\qquad\qquad end \quad if\\
&\qquad\quad end \quad while\\
&\qquad end \quad if\\
&\quad next \quad j\\
&\quad h^i \leftarrow compression(repeat\_times, m^{i-1}, h^{i-1}, c1, c2)\\
&next \quad i\\
&h^{N-1} \leftarrow \neg h^{N-1}\\
&c2 \leftarrow \neg c2\\
&c1 \leftarrow c1 \oplus tm\\
&h^N \leftarrow compression(repeat\_times, m^{N-1}, h^{N-1}, c1, c2)\\
&return \quad h^N
\end{aligned}
$$

Processing of DDHA

2.3 compresion fuction

The function compression(repeat_times,m,h,ct1,ct2) takes as input five values:

* an integer value *repeat_times*. User can set *repeat_times* to get higher intensity. The default value of *repeat_times* is 1.

* a chain value $h = h_0,...,h_7$

* a message block $m = m_0,...,m_{32}$

* a Constant $ct1 = ct1_0,...,ct1_{15}$

* a Constant $ct2 = ct2_0,...,ct2_{15}$

The compression function use two functions: sub_round(m, h,ct1,ct2),

message_expension(m). In DDHA, the word is carved up to sixteen parts, every part is used as parameter of data-depend circular shift once. And in function message_expension(m), the circular shift operation is based on part not bit.

2.3.1 sub_round(m,h,ct1,ct2)

The function sub_round(m,h,ct1,ct2) takes as input four values:

* a chain value $h = h_0,...,h_7$
* a message block $m = m_0,...,m_{32}$
* a Constant $ct1 = ct1_0,...,ct1_{15}$
* a Constant $ct2 = ct2_0,...,ct2_{15}$

And sub_round(m,h,ct1,ct2) as follow:

$for \quad i = 0 \quad to \quad 15$

$im \leftarrow i >> 1 + (i \bmod 2) \times 8$

$h_0 \leftarrow (h_0 + m_{im})$

$for \quad j = 1 \quad to \quad 7$

$im1 \leftarrow (i >> 1 + j) \bmod 8 + (i \bmod 2) \times 8$

$h_j \leftarrow ROTR^{(m_{im} >> ((j-1) \times rl)) \wedge rv}(h_j + m_{im1})$

$next \quad j$

$for \quad j = 1 \quad to \quad 7$

$h_j \leftarrow ROTR^{(m_{im} >> ((j+6) \times rl)) \wedge rv}(h_j + h_{j-1})$

$next \quad j$

$t \leftarrow ROTR^{(m_{im} >> (14 \times rl)) \wedge rv}(((h_4 + h_5) \oplus h_6) + h_7) + ct1_i$

$h_3 \leftarrow ROTR^{(m_{im} >> (15 \times rl)) \wedge rv}(((h_0 + h_1) \oplus h_2) + h_3) + ct2_i$

$for \quad j = 1 \quad to \quad 7$

$h_j \leftarrow h_{j-1}$

$next \quad j$

$h_0 \leftarrow t$

$next \quad i$

sub_round function of DDHA

In DDHA-256, the word length is 32, rl is 2, rv is 3. In DDHA- 512, the word length is 64, rl is 4, rv is 15.

2.3.2 message_expension(m)

The function message_expension(m) takes as input one value:

* a message block $m = m_0,...,m_{15}$

And message_expension1(m) as follow:

$t = \bigoplus_{i=0}^{15}(m_i)$

for $i = 0$ *to* 15

$m_i \leftarrow (t \oplus m_i)$

next i

for $i = 0$ *to* 15

$m_i \leftarrow ROTR^{i \times rl}(m_i)$

next i

$t = \bigoplus_{i=0}^{15}(m_i)$

for $i = 0$ *to* 15

$m_i \leftarrow (m_i \oplus t)$

next i

return m

function message_expension1 of DDHA

In DDHA-256, the word length is 32, rl is 2. In DDHA- 512, the word length is 64, rl is 4.

With function sub_round(m,h,ct1,ct2), message_expension(m), the compression function as follows:

```
h ← h^i
c3 ← ct1
c4 ← ct2
ma ← m
mb ← message_expension(m)
for j = 1 to repeat_times
    sub_round(ma, h, c3, c4)
    sub_round(mb, h, c4, c3)
    if repeat_times > 1 then
        for j1 = 0 to 15
            c3_j1 ← ROTR^1(c3_j1)
            c4_j1 ← ROTR^1(c4_j1)
        next j1
    end if
next j
h ← h + h^i
return h
```

compression function of DDHA

In DDHA-256, the word bit-length is 32. In DDHA- 512, the word bit-length is 64.

3 Security of DDHA

In this section, we discuss the resistance of DDHA to Differential attack, Length extension, Multicollisions.

3.1 Differential attack

From appendix 2, we will know that if there is any difference in the message, the difference path for DDHA will has at least eight data-depend circular shift difference that $\Delta r \neq 0$, r is the parameter of data-depend circular shift.

By theorem A.1, it is known that if a data-depend circular shift difference that $(r_1 - r_2) \neq 0$, the possibility of a data-depend circular shift difference is $2^{\mathrm{gcd}-n}$, gcd is the **greatest common divisor of** $(r1 - r2)$ **and** n.

In DDHA, the bit-length of the parameter of data-depend circular shift less than 4(resp.16). then there has:

$0 \le r_1, r_2 < 4$	$0 \le r_1, r_2 < 16$
$-4 < (r1 - r2) < 4$	$-16 < (r1 - r2) < 16$
$\mathrm{gcd} = GCD(r1 - r2, 32) \le 2$	$\mathrm{gcd} = GCD(r1 - r2, 64) \le 8$
$DDHA - 256$	$DDHA - 512$

So the possibility of a difference path for DDHA will be:

$$p = 2^{(\mathrm{gcd}-n)\times 8} \le 2^{16-8\times n} (resp. 2^{64-8\times n})$$

At the same time, in a difference path for DDHA if a chain value that $\Delta r = 0$ has defences, some bits in the parameter r will be fixed, this depend on the defences that the chain value has. Here we suppose attacker can find the needed defences.

3.2 Length extension

Let $\sum$ is the sum of several members, and the addition operator is modulo 2^{512} or modulo 2^{1024}.

Length extension is the attaca against keyed hash of form $h = H_k(m)$ or $h = H(k \| m)$. The attack as: given $h = H_k(m)$, the padding data is p, then find m' that make $h = H(k \| m \| p \| m')$. The $(m \| p \| m')$ is the fabricated message.

In DDHA, if $(... \| m^{N-2} \| m^{N-1})$ is padded message data, and m^{N-1} is the

last message block, the final chain values is $h^N = compression(\neg h^{N-1}, m^{N-1}, c1 \oplus (\sum_{i=0}^{N-2} m^i), \neg c2)$. If a block m^N is extended, then the chain value between m^{N-1}, m^N will be $h^N = compression(h^{N-1}, m^{N-1}, c1, c2)$ from $h^N = compression(\neg h^{N-1}, m^{N-1}, c1 \oplus (\sum_{i=0}^{N-2} m^i), \neg c2)$. The knowledge of $DDHA(... \| m^{N-2} \| m^{N-1})$ can not be used to compute the hash of $(... \| m^{N-2} \| m^{N-1} \| m^N)$.

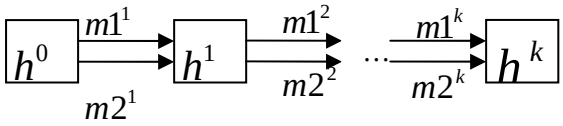
3.3 Multicollisions

Let $\sum$ is the sum of several members, and the addition operator is modulo 2^{512} or modulo 2^{1024}.

Many technique is developed to find Multicollisions of hash function, Joux's technique[1] and Kelsey/Schneier's technique[2] is representative technique.

3.3.1 Joux's technique

Joux [1] has proposed a technique to find a 2^k-collision for hash functions with n-bit hash values in $k \times 2^{n/2}$ as follow:



In Joux's technique, we can find that replace $m1^i$ with $m2^i$ will not change the final chain value h^k. In DDHA, if a message $m := (... \| m1^i \| ...)$, replace $m1^i$ with $m2^i$ will change the final chain value h^k from $h^k = compression(\neg h^{k-1}, m^k, c1 \oplus (\sum_{i=0}^{N-2} m^i), \neg c2)$ to $h^k = compression(\neg h^{k-1}, m^k, c1 \oplus ((\sum_{i=0}^{N-2} m^i) - m1^i + m2^i), \neg c2)$.

And it can alter Joux's technique to apply it on DDHA. To pair (h^i, h^{i+1}), if find message blocks $(m3^0, ..., m3^{i3-1}, m4^0, ..., m4^{i4-1})$ that satisfy:

$$\begin{cases} \sum_{ii=0}^{i3-1} m3^{ii} = \sum_{ii=0}^{i4-1} m4^{ii} \\ h3^1 = compression(h^i, m3^0, c1, c2) \\ h3^{ii+1} = compression(h3^{ii}, m3^{ii}, c1, c2) \quad ii = 1, ..., (i3-2) \\ h^{i+1} = compression(h3^{i3-1}, m3^{i3-1}, c1, c2) \\ h4^1 = compression(h^i, m4^0, c1, c2) \\ h4^{ii+1} = compression(h4^{ii}, m4^{ii}, c1, c2) \quad ii = 1, ..., (i4-2) \\ h^{i+1} = compression(h4^{i4-1}, m4^{i4-1}, c1, c2) \end{cases} \tag{3.1}$$

So to find Multicollisions of DDHA, to every pair chain value (h^i, h^{i+1}), it need find message blocks that satisfy (3.1). Then to 2^k-collision for DDHA, the message blocks must satify k systems that like (3.1).

3.3.2 Kelsey/Schneier's technique

Kelsey/Schneier's technique bases on fixed-points of hash function. But in DDHA, when admit a fixed-points mfp for DDHA, the the final chain value will be $h^k = compression(\neg h^{k-1}, m^k, c1 \oplus ((\sum_{i=0}^{k-1} m^i) + mfp), \neg c2)$ from $h^k = compression(\neg h^{k-1}, m^k, c1 \oplus (\sum_{i=0}^{k-1} m^i), \neg c2)$.

4. Improvement

In compression function, there are 512 data-depend circular shift operations, this will increase the calculation. There exists a way to improve it. The function sub_round(m,h,ct1,ct2) and compression(repeat_times,m,h, ct1,ct2) will be changed to sub_round1(m1,m2,h,ct1,ct2,b) and compression1(repeat_ times, m, h, ct1,ct2) as follow:

4.1 sub_round1(m1,m2,h,ct1,ct2,b)

The function sub_round1(m1,m2,h,ct1,ct2,b) takes as input six values:

* a chain value $h = h_0,...,h_7$
* a message block $m1 = m1_0,...,m1_{32}$
* a message block $m2 = m2_0,...,m2_{32}$
* a Constant $ct1 = ct1_0,...,ct1_{15}$
* a Constant $ct2 = ct2_0,...,ct2_{15}$
* an integer b

And sub_round1(m1,m2,h,ct1,ct2,b) as follow:

$$
\begin{aligned}
&for \quad i = b \quad to \quad 7 + b \\
&\quad im \leftarrow i >> 1 + (i \bmod 2) \times 8 \\
&\quad h_0 \leftarrow (h_0 + m2_{im}) \\
&\quad for \quad j = 1 \quad to \quad 7 \\
&\qquad im1 \leftarrow (i >> 1 + j) \bmod 8 + (i \bmod 2) \times 8 \\
&\qquad h_j \leftarrow ROTR^{(m1_{im} >> ((j-1) \times rl)) \wedge rv}(h_j + m2_{im1}) \\
&\quad next \quad j \\
&\quad for \quad j = 1 \quad to \quad 7 \\
&\qquad h_j \leftarrow ROTR^{(m1_{im} >> ((j+6) \times rl)) \wedge rv}(h_j + h_{j-1}) \\
&\quad next \quad j
\end{aligned}
$$

$t \leftarrow ROTR^{(m1_{im} >> (14 \times rl)) \wedge rv}(((h_4 + h_5) \oplus h_6) + h_7) + ct1_i$

$h_3 \leftarrow ROTR^{(m1_{im} >> (15 \times rl)) \wedge rv}(((h_0 + h_1) \oplus h_2) + h_3) + ct2_i$

for $j = 1$ *to* 7

$h_j \leftarrow h_{j-1}$

next j

$h_0 \leftarrow t$

next i

function sub_round1 of DDHA

In DDHA-256, the word length is 32, rl is 2, rv is 3. In DDHA- 512, the word length is 64, rl is 4, rv is 15.

compression(repeat_times,m,h,ct1,ct2) will be as follow:

4.2 compression1(repeat_times,m,h,ct1,ct2)

The function compression1(repeat_times,m,h,ct1,ct2) takes as input five values:

* an integer value *repeat_times*. User can set *repeat_times* to get higher intensity. The default value of *repeat_times* is 1.

* a chain value $h = h_0,...,h_7$

* a message block $m = m_0,...,m_{32}$

* a Constant $ct1 = ct1_0,...,ct1_{15}$

* a Constant $ct2 = ct2_0,...,ct2_{15}$

Function compression1(repeat_times,m,h,ct1,ct2) as follow:

```
h ← h^i
c3 ← ct1
c4 ← ct2
ma ← m
mb ← message_expension(m)
for j = 1 to repeat_times
    sub_round1(ma, ma, h, c3, c4, 0)
    sub_round1(ma, mb, h, c4, c3, 8)
    if repeat_times > 1 then
        for j1 = 0 to 15
            c3_j1 ← ROTR^1(c3_j1)
            c4_j1 ← ROTR^1(c4_j1)
        next j1
    end if
next j
h ← h + h^i
```

return *h*

function compression1 of DDHA

In DDHA-256, the word bit-length is 32. In DDHA- 512, the word bit-length is 64.

After sub_round(m,h,ct1,ct2) and compression(repeat_times,m,h, ct1, ct2) is changed to sub_round1(m1,m2,h,ct1,ct2,b) and compression1(repeat_times, m,h,ct1,ct2). There are 256 data-depend circular shift perations, this will reduce calculation, And at the same time, function message_expension(m) will make there are many defferences when there are few defferences in message. Here we will not discuss it detailedly.

5. Conclusions

After study the technologys[wy05, Dau05] and the defence feature of data-depend function, and we find a message expension function that will make every defence path for DDHA will has at least eight data-depend circular shift defence, this make it hard to constitute a feasible difference path that has good possibility. Base on data-depende function and the message expension function, we design the hash function DDHA.

At the same time, we study other attack technologys[1,2] and length extension, and we use some measures in view of these technologys, these measures wreck the condition that applying the technologys need, this make it harder to apply these technologys on DDHA.

DDHA uses a value that user can set the value to change rounds to change the strength.

So DDHA adopts various measures in view of the techniques that use to attack hash function, this will make DDHA resists these attacks.

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Appendix 1: Difference of data-depend circular shift

Here we just discuss circular right shift. And we just discuss XOR differences[Dau05].

If $x \in F_2^n$, x has n-bits as:

$$x \mapsto (x_{n-1},...,x_0)$$

If y,x is n-bits word, n is 32(resp. 64), $0 \le r < 32(resp.64)$ is an integer that has 5(resp. 6) bits, then the circular right shift is:

$$y = ROTR^r(x) = ((x << (n-r)) \vee (x >> r))$$

If there is (x1,y1,r1) and (x2,y2,r3) meet:

$$y1 = ROTR^{r1}(x1) \qquad y2 = ROTR^{r2}(x2)$$

At fisrt, there is:($wbl = \log_2^n - 1$)

$$\begin{cases} r1 - r2 = \sum_{i=0}^{wbl} (\Delta^{\pm}(r1_i, r2_i) \times 2^i) \\ y1 := (x1_{r1-1},...,x1_0, x1_{n-1},...,x1_{r1}) \\ y2 := (x2_{r2-1},...,x2_0, x2_{n-1},...,x2_{r2}) \\ \Delta^{\oplus}(x1,x2) := (\Delta^{\oplus}(x1_{n-1}, x2_{n-1}),...,\Delta^{\oplus}(x1_0, x2_0)) \\ \Delta^{\oplus}(y1,y2) := (\Delta^{\oplus}(x1_{r1-1}, x2_{r2-1}),...,\Delta^{\oplus}(x1_{r1}, x2_{r2})) \end{cases}$$

Let the **greatest common divisor of** $(r1-r2)$ **and** n is $\gcd = GCD(r1-r2, n)$. Then there exists:

1. if r1=r2, there has:

$$\begin{aligned} \Delta^{\oplus}(y1,y2) &= (\Delta^{\oplus}(x1_{r1-1}, x2_{r1-1}),...,\Delta^{\oplus}(x1_0, x2_0), \Delta^{\oplus}(x1_{n-1}, x2_{n-1}),...,\Delta^{\oplus}(x1_{r1}, x2_{r1})) \\ &= ROTR^{r1}(\Delta^{\oplus}(x1,x2)) \end{aligned}$$

So, if r1=r2, the difference of y1 and y2 will just depend on the difference of x1 and x2. of course it also depend on r1.

To given $\Delta^{\oplus}$, there are 2^n $x1$. To given $(x1, \Delta^{\oplus})$, there is a $x2$ that meet $x2 = x1 \oplus \Delta^{\oplus}(x1,x2)$. So there are 2^n pair $(x1,x2)$ has same $\Delta^{\oplus}$.

2. if $r1 \neq r2$.

Divide $\Delta^{\oplus}(y1,y2)$ **and** $\Delta^{\oplus}(x1,x2)$ into $\gcd$ parts as follow:

$$\begin{cases} px_j := (\Delta^{\oplus}(x1_{(j+i\times\gcd) \mod n}, x2_{(j+i\times\gcd) \mod n}) \\ \quad |i = 0,...,(n/\gcd-1)) \qquad j = 0,...,\gcd-1 \\ py_j := (\Delta^{\oplus}(x1_{(j+i\times\gcd) \mod n}, x2_{(n+r2-r1+j+i\times\gcd) \mod n}) \\ \quad |i = 0,...,(n/\gcd-1)) \qquad j = 0,...,\gcd-1 \end{cases} \tag{A.1}$$

To gven defence of patr px_j, py_j:

$$dx := (\Delta^{\oplus}(x1_{(j+i\times\gcd) \mod n}, x2_{(j+i\times\gcd) \mod n})$$
$$|\, i = 0,...,(n/\gcd-1))$$

There are $2^{n/\gcd-1}$ diffence as follow:

$$dy1 := (\Delta^{\oplus}(x1_{(j+i\times\gcd) \mod n}, x2_{(n+r2-r1+j+i\times\gcd) \mod n})$$
$$|\, i = 0,...,(n/\gcd-2))$$

To given pair (dx, dy1), there exists:

$$\bigoplus_{i=0}^{n/\gcd-1} \Delta^{\oplus}(x1_{(j+i\times\gcd) \mod n}, x2_{(j+i\times\gcd) \mod n})$$
$$\oplus\left(\bigoplus_{i=0}^{n/\gcd-2} \Delta^{\oplus}(x1_{(j+i\times\gcd) \mod n}, x2_{(n+r2-r1+j+i\times\gcd) \mod n})\right)$$
$$= \Delta^{\oplus}(x1_{(j+n-\gcd) \mod n}, x2_{(2\times n+r2-r1+j-\gcd) \mod n}) \tag{A.2}$$

Theorem A.1: if $r1 \neq r2$, the possibility of a difference pair $(\Delta^{\oplus}(y1, y2), \Delta^{\oplus}(x1, x2))$ is $2^{\gcd-n}$.

Proof:

At first, Divide $\Delta^{\oplus}(y1, y2)$ **and** $\Delta^{\oplus}(x1, x2)$ into $\gcd$ parts as (A.1), and every part satisfy (A.2). To given pair (dx, dy1), it will has the system:

$$\begin{cases} dx_{j,i} = x1_{(j+i\times\gcd) \mod n} \oplus x2_{(j+i\times\gcd) \mod n} \\ \qquad\qquad i = 0,...,(n/\gcd-1) \\ dy1_{j,i} = x1_{(j+i\times\gcd) \mod n} \oplus x2_{(n+r2-r1+j+i\times\gcd) \mod n} \\ \qquad\qquad i = 0,...,(n/\gcd-2) \end{cases} \tag{A.3}$$

The system has $2\times(n/\gcd-1)$ variables and $2\times n/\gcd-3$ equations. The

system has two roots on GF(2).

The difference pair $(\Delta^{\oplus}(y1, y2), \Delta^{\oplus}(x1, x2))$ include $\gcd$ parts that satisfy (A.2) (A.3). So there are $2^{\gcd}$ pair $(x1, x2)$ satify these systems.

So there are $2^{\gcd}$ pair $(x1, x2)$ have the given defference $(\Delta^{\oplus}(y1, y2), \Delta^{\oplus}(x1, x2))$. Of course these pairs $(x1, x2)$ satisfy $x2 = \Delta^{\oplus}(x1, x2) \oplus x1$.

To given defference $(\Delta^{\oplus}(x1, x2))$, there are 2^n $x1$, And to given pair $(\Delta^{\oplus}(x1, x2), x1)$, there is a $x2$ that satisfy $x2 = \Delta^{\oplus}(x1, x2) \oplus x1$, so there are 2^n pair $(x1, x2)$ have the given defference $(\Delta^{\oplus}(x1, x2))$.

So the possibility of a difference pair $(\Delta^{\oplus}(y1, y2), \Delta^{\oplus}(x1, x2))$ is $2^{\gcd - n}$. □

Appendix 2: Message_expension(m)

In DDHA, the message m is expand from 16 words to 32 words. It can use a $512\times1024(resp.1024\times2048)$ generator matrix to describe it. It a little hard to find out the minimum deffences in expand message words with the big matrix. We will find out the minimum deffences in expand message words with other way.

At first, the follow facts is used to simplify the discussion:

1. Because the degree of the Algebraic Normal Form (ANF) that describe function Message_expension(m) is 1. Finding out the minimum deffences in expand message words is be equal finding out the minimum hamming weight of the expand message words if the Hamming weight of message bigger than 0.
2. The words in DDHA is carved up to sixteen parts. So it can describe a word as follow:

$$W := (w_{15},...,w_0)$$

Where $w_i := (b_J,...,b_0) \quad 0 \le i < 16$, every part w_i has *J* bits. Then the message words *m* and expand message words *em* as follow:

$$\begin{cases} m := (m_{0,15}, m_{0,14},..., m_{0,0},.., m_{15,0}) \\ em := (em_{0,15}, em_{0,14},..., em_{0,0},.., em_{31,0}) \\ em_{i,j} = m_{i,j} \qquad 0 \le i, j \le 15 \end{cases}$$

Then function Message_expension(m) can be described with steps as follow, let *m1* and *m2* include 16 words.

$$\begin{cases} m1_i \leftarrow (\bigoplus_{j=0}^{15} m_j) \oplus m_i \qquad 0 \le i \le 15 \\ m2_i \leftarrow ROTR^i(m1_i) \qquad 0 \le i \le 15 \\ em_{i+16} \leftarrow (\bigoplus_{j=0}^{15} m2_j) \oplus m2_i \qquad 0 \le i \le 15 \end{cases}$$

Then there exists:

$$m2_{i,j} = m1_{i,(16-i+j) \mod 16}$$

Let HW(w) is Hamming weight of w. Then there exists:

Theorem B.1: If

$$\begin{cases} x := (x_0,..., x_{15}) \qquad 0 \le i \le 15 \\ y := (y_0,..., x_{15}) \qquad 0 \le i \le 15 \\ y_i = x_i \oplus (\bigoplus_{j=0}^{15} x_j) \quad 0 \le i \le 15 \end{cases}$$

There exists:

1. If $\bigoplus_{j=0}^{15} x_j = 0$, then HW(y)=HW(x).
2. If $\bigoplus_{j=0}^{15} x_j = 1$, then HW(y)=16-HW(x).
3. If $HW(x) \geq 1$, then $HW(y) \geq 1$

*proof:*I

There exists:

$$HW(x) = \sum_{j=0}^{15} x_j \leq 16 \qquad (B.1.1)$$

When $x_j = 1 \quad 0 \leq j < 15$, the equality sign hold water.

1. If $\bigoplus_{j=0}^{15} x_j = 0$ Then

$$y_i = x_i \oplus (\bigoplus_{j=0}^{15} x_j) = x_i \qquad 0 \leq i \leq 15$$

Then

HW(y)=HW(x)

2. If $\bigoplus_{j=0}^{15} x_j = 1$ Then

$$y_i = x_i \oplus (\bigoplus_{j=0}^{15} x_j) = x_i \oplus 1 = \neg x_i \qquad 0 \leq i \leq 15$$

Then

$$HW(y) = \sum_{i=0}^{15} y_i = \sum_{i=0}^{15} (1 - x_i) = 16 - \sum_{i=0}^{15} (x_i) = 16 - HW(x)$$

3. If $HW(x) \geq 1$ and $\bigoplus_{j=0}^{15} x_j = 0$, then $HW(y) = HW(x) \geq 1$.

If $HW(x) \geq 1$ and $\bigoplus_{j=0}^{15} x_j = 1$, then by (B.1.1), there exists:

16> HW(x)>0

HW(y)=16- $HW(x) \geq 1$. □

Theorem B.2: In message words of DDHA, if there exists $0 \leq j1 \leq 15$ make $\bigoplus_{i=0}^{15} m_{i,j1} = 1$, Then there exist $HW(em) \geq 16$.

Proof:

There has:

$$em_i = m_i \qquad 0 \leq i \leq 15$$
$$\bigoplus_{i=0}^{15} em_{i,j1} = \bigoplus_{i=0}^{15} m_{i,j1} = 1$$

Suppose $I = \{i \mid m_{j1} = 1\}$ and $HW((em_{0,j1},...,em_{15,j1})) = H0$

There has: $m1_i = em_i \oplus (\bigoplus_{j=0}^{15} em_{i,j})$

By theorem B.1, thus

$$HW((m1_{0,j1},...,m1_{15,j1})) = 16 - H0$$
$$m1_{i,j1} = 1 \qquad i \notin I$$

Let $J = \{(i + j1) \mod 16 \mid i \notin I\}$, there are *16-H0* members in *J*.

Because $m2_{i,j} = m1_{i,(16-i+j) \mod 16}$

Then

$$\begin{aligned} m2_{i,j} &= m1_{i,(16-i+j) \mod 16} \qquad i \notin I \quad j \in J \\ &= m1_{i,(16-i+i+j1) \mod 16} \\ &= m1_{i,j1} \\ &= 1 \end{aligned}$$

So there exists: $16 \geq HW((m2_{0,j},...,m2_{15,j})) \geq 1 \qquad j \in J$

By theorem B.1, there exits:

$$HW((em_{16,j},...,m2_{31,j})) \geq 1 \qquad j \in J$$

$$\begin{aligned} HW((em_{16},...,m2_{31})) &= (\sum_{j \in J} \sum_{i=16}^{31} em_{i,j}) + \sum_{j \notin J} \sum_{i=16}^{31} em_{i,j} \\ &\geq \sum_{j \in J} \sum_{i=16}^{31} em_{i,j} \\ &\geq 16 - H0 \end{aligned}$$

Then

$$\begin{aligned} HW(em) &= HW((em_0,...,em_{15})) + HW((em_{16},...,em_{31})) \\ &\geq H0 + (16 - H0) \\ &= 16 \end{aligned}$$

□

Theorem B.3: In message words of DDHA, if $HW(m) \geq 0$ there exist $HW(em) \geq 8$.

Proof:

There exists:

$$em_i = m_i \qquad 0 \leq i \leq 15$$

$$HW((em_0,...,em_{15})) = HW((m_0,...,m_{15}))$$

1.if there exists *j1* make $\bigoplus_{i=0}^{15} em_{i,j1} = 1$, by theorem B.2, there has exists:

$$HW(em) \geq 16 > 8 \qquad (B.3.a).$$

2. if there exists

$$\bigoplus_{i=0}^{15} em_{i,j} = 0 \qquad 0 \leq j \leq 15$$

Then there exists:

$$m1_{i,j} = em_{i,j} \oplus (\bigoplus_{i1=0}^{15} em_{i1,j}) = em_{i,j} \qquad 0 \le i, j \le 15$$
$$m2_{i,j} = m1_{i,(i+j) \mod 16}$$

Let $I0_j = \{i \mid em_{i,j} = 1 \quad 0 \le i \le 15 \quad 0 \le j \le 15\}$, $I1_j = \{i \mid m1_{i,j} = 1 \quad 0 \le i \le 15 \quad 0 \le j \le 15\}$ then:

$$\begin{aligned} I1_j &= \{i \mid m1_{i,j} = 1 \quad 0 \le i \le 15 \quad 0 \le j \le 15\} \\ &= \{i \mid em_{i,j} = 1 \quad 0 \le i \le 15 \quad 0 \le j \le 15\} \\ &= I0_j \end{aligned}$$
$$HW(m1) = HW((em_0, ..., em_{15}))$$

Let $JB0 = \{jb_0, ...\} = \{j \mid (\sum_{i=0}^{15} I1_{i,j}) > 0 \quad 15 \ge j \ge 0\}$, and because

$$\bigoplus_{i=0}^{15} em_{i,j} = 0 \qquad 0 \le j \le 15$$

Then:

$$(\sum_{i \in I1_j} m1_{i,j}) = (\sum_{i \in I1_j} em_{i,j}) \ge 2 \qquad j \in JB0 \qquad (B.3.1)$$

Let $J2_j = \{i \mid m2_{i,j} = 1 \quad 0 \le i \le 15 \quad 0 \le j \le 15\}$ then there has:

$$J2_j = \{i \mid i \in I1_{(j+i) \mod 16}\} \qquad 0 \le j \le 15$$
$$I1_j = \{i \mid i \in J1_{(16-i+j) \mod 16}\} \qquad 0 \le j \le 15$$

This means that every bit of m2 is equal to a bit of m1, and every bit of m1 is equal to a bit of m2. So there exist:

$$HW((m2_0, ..., m2_{15})) = HW((m1_0, ..., m1_{15})) = HW((em_0, ..., em_{15})) \qquad (B.3.2)$$

2.1 If there exists *i1c* make $(\bigoplus_{i=0}^{15} m2_{i,i1c}) = 1$, by theroem B.1 and (B.3.2), there exists:

$$\begin{aligned} HW((em_{16}, ..., em_{31})) &\ge HW((em_{16,i1c}, ..., em_{31,i1c})) \\ &= 16 - HW((m2_{0,i1c}, ..., m2_{15,i1c})) \\ HW((em_0, ..., em_{31})) &= HW((em_0, ..., em_{15})) + HW((em_{16}, ..., em_{31})) \\ &\ge HW((m2_0, ..., m2_{15})) + 16 - HW((m2_{0,i1c}, ..., m2_{15,i1c})) \\ &\ge HW((m2_{0,i1c}, ..., m2_{15,i1c})) + 16 - HW((m2_{0,i1c}, ..., m2_{15,i1c})) \\ &= 16 > 8 \qquad (B.3.b) \end{aligned}$$

2.2 If $(\bigoplus_{i=0}^{15} m2_i) = 0$. Then there exist:

$$em_{i,j} = m2_{i-16,j} \oplus (\bigoplus_{i1=0}^{15} m2_{i1,j}) = m2_{i-16,j} \qquad 16 \le i \le 31$$
$$HW((em_{16},...,em_{31})) = HW(m2) \qquad (B.3.3)$$

Because $HW(m) \ge 0$, there are at least one member in *JB0*. Let $jb0 = jb_0$ and there are at least two members in $I1_{jb0}$. Suppose $i1a \ne i1b \in I1_{jb0}$, then:

$$m1_{i1a,jb0} = 1 \quad and \quad m1_{i1b,jb0} = 1$$

Then there has:

$$m1_{i1a,jb0} = m2_{i1a,\ (16+jb0-i1a)\ \bmod\ 16} = 1$$
$$m1_{i1b,jb0} = m2_{i1b,\ (16+jb0-i1b)\ \bmod\ 16} = 1$$

Because $(\bigoplus_{i=0}^{15} m2_i) = 0$, there are at least two members in $\{m2_{i,\ (16+jb0-i1a)\ \bmod\ 16} \quad 15 \ge i \ge 0\}$ and $\{m2_{i,\ (16+jb0-i1b)\ \bmod\ 16} \quad 15 \ge i \ge 0\}$ is 1. Then:

$$(\sum_{i=0}^{15} m2_{i,\ (16+jb0-i1a)\ \bmod\ 16}) \ge 2$$
$$(\sum_{i=0}^{15} m2_{i,\ (16+jb0-i1a)\ \bmod\ 16}) \ge 2$$
$$HW((em_{16},...,em_{31})) = HW(m2)$$
$$\ge HW((m2_{0,\ (16+jb0-i1a)\ \bmod\ 16},....,m2_{15,\ (16+jb0-i1a)\ \bmod\ 16})) +$$
$$HW((m2_{0,\ (16+jb0-i1b)\ \bmod\ 16},....,m2_{15,\ (16+jb0-i1b)\ \bmod\ 16}))$$
$$\ge 4 \qquad (B.3.4)$$

By (B.3.2) (B.3.3) (B.3.4), there exist:

$$HW((em_0,...,em_{31})) = HW((em_0,...,em_{15})) + HW((em_{16},...,em_{31}))$$
$$= 2 \times HW((em_{16},...,em_{31}))$$
$$\ge 2 \times 4 = 8 \qquad | \qquad (B.3.c)$$

So by (B.3.a) (B.3.b) (B.3.c), if $HW(m) \ge 0$ there exist $HW(em) \ge 8$. □

Because every part is as parameter of data-depend circular shift once. Theroem B.3 means in any difference path for DDHA, there will be at least eight data-depend circular shift defference.