Identity Based Online/Offline Encryption Scheme

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Abstract. Consider the situation where a low power device with limited computational power has to perform cryptographic operation in order to do secure communication to the base station where the computational power is not limited. The most obvious way is to split each and every cryptographic operations into resource consuming, heavy operations (which are performed when the device is idle) and the fast light weight operations (which are executed on the fly). This concept is called online/offline cryptography. In this paper, we show the security weakness of an identity based online offline encryption scheme proposed in ACNS 09 by Liu et al. [7]. The scheme in [7] is the first identity based online offline encryption scheme in the random oracle model, in which the message and recipient are not known during the offline phase. We show that this scheme is not CCA secure. We show the weakness in the security proof of CCA secure online/offline encryption system proposed by Chow et al. in [2]. We propose a new provably secure identity based online offline encryption scheme in which the message and receiver are not known during the offline phase. Since all the CCA secure identity based online/offline encryption schemes are shown to have weakness, ours is the first provably secure scheme with the aforementioned properties.

Keywords: Identity Based, encryption, online/offline, cryptanalysis.

1 Introduction

Separating the process of signing or encrypting into two phases namely, online phase and offline phase is the concept of "Online/Offline" cryptography. This notion was first introduced in the context of digital signatures by Even, Goldreich and Micali [4]. Their construction is inefficient as it increases the size of each signature by a quadratic factor. Shamir and Tauman [9] proposed an improved version which makes use of a new paradigm called "hash-sign-switch" to design more efficient online/offline signature schemes. During the offline phase most of the heavy computations like exponentiation and bilinear pairing are done and in the online phase in-order to make the execution faster, only light weight integer operations (multiplication and addition) and hashing are performed. In an online/offline signature scheme the message is not known in the offline phase and in an online/offline encryption scheme both the message and receiver are not known in the offline phase. Thus, online/offline schemes find use in low power devices such as PDA's, sensor networks, hand held devices including mobile phones and smart-cards.

Adi Shamir [8] introduced the concept of identity based cryptography and proposed the first identity based signature scheme. The idea of identity based cryptography is to enable

a user to use any arbitrary string that uniquely identifies him as his public key. Identity based cryptography serves as an efficient alternative to Public Key Infrastructure (PKI) based systems. Most of the identity based encryption (IBE) schemes use the costly bilinear pairing operation and the concept of online/offline computation is an important area of research with respect to IBE. The first identity based online/offline encryption scheme was proposed by Guo et al.[6]. It should be noted that, the major difference between online/offline signature and encryption schemes is that, the receiver is not known during the offline phase in encryption schemes. This makes it subtle and interesting to explore for new directions in constructing efficient and elegant online/offline encryption schemes. Few motivating examples for online/offline encryption schemes can be found in [6] and [7].

Guo et al. [6] have shown natural extension of the IBE of Boneh and Boyen [1] and Gentry [5]. They have also given constructions which efficiently divide the IBE schemes in [1] and [5]. All the schemes reported in [6] are secure in the standard model. In 2009, Joseph. K. Liu et al. [7] proposed the first identity based online/offline encryption scheme in the random oracle model. It was claimed to be chosen ciphertext (CCA) secure. It is more efficient than the scheme in [6] (due to random oracle assumption). In [2], Chow et al. proposed a generic way of constructing an CCA secure online/offline encryption scheme from any online/offline KEM.

Our Contribution: In this paper, we show that the scheme in [7] is not CCA secure, i.e. an adversary can distinguish the challenge ciphertext using the decryption oracle. We provide a fix for the bug in the scheme, further more, we show the weakness in the security proof of another result reported in [2]. Finally, we propose a new efficient construction for identity based online/offline encryption. We prove the new scheme in the random oracle model and ours is the only existing concrete identity based online/offline encryption scheme that is secure in the random oracle model.

2 Preliminaries

2.1 Bilinear Pairing

Let $\mathbb{G}_1$ be an additive cyclic group generated by P, with prime order q, and $\mathbb{G}_2$ be a multiplicative cyclic group of the same order q. Let $\hat{e}$ be a bilinear pairing $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \rightarrow \mathbb{G}_2$.

2.2 Computational Assumptions

In this section, we recall the computational assumptions related to bilinear maps[3] that are relevant to the security of our scheme.

k-BDHP assumption : This is the bilinear version of k-CAA problem. Given $(P, Q = aP) \in \mathbb{G}_1^2$, $((x_1, x_1 + a)^{-1}P), ..., (x_k, (x_k + a)^{-1}P)) \in (\mathbb{Z}_q^{*k}, \mathbb{G}_1^k)$ for unknown $a \in Z_q^*$ and known $x_1, ..., x_k \in Z_q^*$, the bilinear k-CAA problem is to compute $\hat{e}(P, P)^{(a+x^*)^{-1}} \in \mathbb{G}_2$ for some $x^* \notin \{x_1, ..., x_k\}$.

Definition 1. *The advantage of any probabilistic polynomial time algorithm $\mathcal{A}$ in solving the bilinear k-BDHP problem in $\mathbb{G}_1$ is defined as*

$$Adv_{\mathcal{A}}{}^{k-BDHP} = Pr[\mathcal{A}(P, aP, (x_1, (x_1+a)^{-1}P), ..., (x_k, (x_k+a)^{-1}P)|x_1, ..., x^k \in \mathbb{Z}_q^*) = \hat{e}(P,P)^{(a+x^*)^{-1}}|a, x^* \in_R Z_q{}^*,\ x^* \notin \{x_1, ..., x_k\}].$$

We say that the k-BDHP problem is (t, ϵ) hard if for any t time probabilistic algorithm $\mathcal{A}$, the advantage $Adv_{\mathcal{A}}{}^{k-BDHP} < \epsilon$.

3 Identity Based Online/Offline Encryption Schemes (IBOOE)

3.1 Generic Model

An identity based online/offline encryption scheme consists of the following algorithms.

- ***Setup***(1^κ) ***:*** Given a security parameter κ, the Private Key Generator (PKG) generates a master private key msk and public parameters $Params$. $Params$ is made public while msk is kept secret by the PKG.
- ***Extract*** (ID) ***:*** Given an identity ID, the PKG executes this algorithm to generate the private key D_{ID} corresponding to ID and transmits D_{ID} to the user with identity ID via. secure channel.
- ***Off-Encrypt*** $(Params)$ ***:*** To generate the offline share of the encryption, this algorithm is executed without the knowledge of message to be encrypted and the receiver of the encryption. The offline ciphertext is represented as ϕ.
- ***On-Encrypt*** (m, ID_A, ϕ) ***:*** For encrypting a message m to user with identity ID_A, any sender can run this algorithm to generate the encryption σ of message m. This algorithm uses a fresh offline ciphertext ϕ every time and generates the full encryption σ.
- ***Decrypt***(σ, ID_A, D_A) ***:*** For decryption of σ, the receiver ID_A uses his private key D_A and run this algorithm to get back the message m.

3.2 Security Model

Definition 2. *An ID-Based online/offline encryption scheme is said to be indistinguishable against adaptive chosen ciphertext attacks (IND-IBOOE-CCA2) if no polynomially bounded adversary has a non-negligible advantage in the following game.*

1. ***Setup :*** The challenger $\mathcal{C}$ runs the *Setup* algorithm with a security parameter κ and obtains public parameters $Params$ and the master private key msk. $\mathcal{C}$ sends $Params$ to the adversary $\mathcal{A}$ and keeps msk secret.
2. ***Phase I :*** The adversary $\mathcal{A}$ performs a polynomially bounded number of queries. These queries may be adaptive, i.e. current query may depend on the answers to the previous queries.

- ***Key extraction queries(Oracle*** $\mathcal{O}_{Extract}(ID)$***) :*** $\mathcal{A}$ produces an identity ID and receives the private key D_{ID}.
- ***Decryption queries(Oracle*** $\mathcal{O}_{Decrypt}(\sigma, ID_A)$***) :*** $\mathcal{A}$ produces the receiver identity $ID_{\mathbb{A}}$ and the ciphertext σ. $\mathcal{C}$ generates the private key D_A and sends the result of $Decrypt(\sigma, ID_A, D_A)$ to $\mathcal{A}$. This result will be "$Invalid$" if σ is not a valid ciphertext or the message m if σ is a valid encryption of message m to ID_A.

3. ***Challenge :*** $\mathcal{A}$ chooses two plaintexts, m_0 and m_1 and the receiver identity $ID_{\mathbb{R}}$, on which $\mathcal{A}$ wishes to be challenged. $\mathcal{A}$ should not have queried for the private key corresponding to $ID_{\mathbb{R}}$ in Phase I. $\mathcal{C}$ chooses randomly a bit $b \in \{0,1\}$, computes $\sigma = Encrypt(m_b, ID_{\mathbb{R}})$ and sends it to $\mathcal{A}$.
4. ***Phase II :*** $\mathcal{A}$ is now allowed to get training as in $Phase-I$. During this interaction, $\mathcal{A}$ is not allowed to extract the private key corresponding to $ID_{\mathbb{R}}$. Also, $\mathcal{A}$ cannot query the decryption oracle with $\sigma, ID_{\mathbb{R}}$ as input, i.e. $\mathcal{O}_{Decrypt}(\sigma, ID_{\mathbb{R}})$.
5. ***Guess :*** Finally, $\mathcal{A}$ produces a bit $b^{'}$ and wins the game if $b^{'} = b$.

$\mathcal{A}^{'}$ s advantage is defined as $\text{Adv}(\mathcal{A}) = 2\left|Pr\left[b^{'} = b\right] - 1\right|$ $where\ Pr\left[b^{'} = b\right]$ denotes the probability that $b^{'} = b$.

4 Review and Attack of Liu et al. Identity Based Online/Offline Encryption Scheme (L-IBOOE)[7]

4.1 Review of L-IBOOE Scheme [7]

Let $\mathbb{G}$ and $\mathbb{G}_T$ be groups of prime order q , and let $\hat{e} : \mathbb{G} \times \mathbb{G}_T \rightarrow \mathbb{G}_T$ be the bilinear pairing. We use a multiplicative notation for the operation in $\mathbb{G}$ and $\mathbb{G}_T$.

Setup: The PKG selects a generator $P \in \mathbb{G}$ and randomly chooses s, $w \in \mathbb{Z}_q^*$. It sets $P_{pub} = sP$, $P_{pub}^{'} = s^2P$ and $W = (w+s)^{-1}P$. Define $\mathcal{M}$ to be the message space. Let n_M be the number of bits used to represent a message. Let H_2:$\{0,1\}^* \times \mathbb{G}_T \rightarrow \mathbb{Z}_q^*$ and H_3:$\{0,1\}^* \rightarrow \{0,1\}^{n_M}$ be two hash functions. The public parameters $Params$ and master private key msk are given by,

$$Params = \langle \mathbb{G}, \mathbb{G}_T, q, P_{pub}, P_{pub}^{'}, W, w, \mathcal{M}, H_1, H_2, H_3 \rangle \text{ and } msk = s.$$

Extract(ID) :
- $q_{ID} = H_1(ID)$
- $D_{ID} = \dfrac{1}{q_{ID} + s} P.$

Off-Encrypt($Params$) **:**
- u, x, α, β, γ, $\delta \in_R \mathbb{Z}_q^*$
- $U = W\text{-}\ uP$
- $R = \hat{e}(wP + P_{pub}, P)^x$
- $T_0 = x(w\ \alpha P + (w+\gamma)P_{pub} + P_{pub}^{'})$
- $T_1 = xw\beta P.$
- $T_2 = x\delta P_{pub}.$
- Output the offline ciphertext $\phi = \langle u, x, \alpha, \beta, \gamma, \delta, U, R, T_0, T_1, T_2 \rangle$.

On-Encrypt(m, ID_A, ϕ) **:**
- $t_1 = \beta^{-1}(H_1(ID_A)\text{-}\ \alpha) \bmod q$
- $t_2 = \beta^{-1}(H_1(ID_A)\text{-}\ \gamma) \bmod q$

– $t = H_2(m, R)x+u \bmod q$
– $c = H_3(R)\oplus m$
– Output the ciphertext $\sigma = \langle U, T_0, T_1, T_2, t, t_1, t_2, c\rangle$

Decrypt(σ,ID_A, D_A) :

– $R = \hat{e}(T_0+ t_1 T_1 + t_2 T_2, D_A)$
– $m = c \oplus H_3(R)$
– and checks for $R^{H_2(m,R)} \stackrel{?}{=} \hat{e}(tP + U, wP + P_{pub}) \hat{e}(P,P)^{-1}$
– outputs m if equal. Otherwise outputs $\perp$

4.2 Attack on confidentiality

During the confidentiality game, after the completion of Phase-1 of training, the adversary $\mathcal{A}$ picks two messages, (m_0, m_1) of equal length and an identity $ID_{\mathbb{R}}$($D_{\mathbb{R}}$ is not known to $\mathcal{A}$), and submits to $\mathcal{C}$. $\mathcal{C}$ chooses a bit $b \in_R \{0, 1\}$ generates the challenge ciphertext σ of the message m_b and gives σ to $\mathcal{A}$.

– Let the challenge ciphertext σ of message m_b is,

$$\sigma = \langle U, T_0, T_1, T_2, t_1, t_2, t, c\rangle$$

– $\mathcal{A}$ cooks up the ciphertext σ^*= (U^*, T_0^*, T_1^*, T_2^*, t_1^*, t_2^*, t^*, c^*) from the challenge ciphertext σ as given below:
  • Chooses r^*, t_1^*, $t_2^* \in_R \mathbb{Z}_q^*$.
  • Computes $U^* = U$ - $r^*P = W$- $(u+ r^*)$P.
  • Chooses T_1^*, $T_2^* \in_R G$.
  • Computes $T_0^* = T_0$ - $(t_1^* T_1^*+ t_2^* T_2^*)$ +$(t_1 T_1+ t_2 T_2) = x(w+ s)(q_A+ s)$P-$(t_1^* T_1^*+t_2^* T_2^*)$
    (since $T_0+ t_1 T_1+ t_2 T_2 = x(w+ s)(q_A+ s)$P).
  • Sets $c^*= c$.
  • Computes $t^*= t + r^* \bmod q$.
– First, we show that $R^* = R$ i.e., R^* used for generation of σ^* and R used for the generation of σ are the same.
  • Now,

$$\begin{aligned} R^* &= \hat{e}(T_0^* + t_1^* T_1^* + t_2^* T_2^*, D_R) \\ &= \hat{e}(x(w+s)(q_R+s)P - (t_1^* T_1^* + t_2^* T_2^*) + t_1^* T_1^* + t_2^* T_2^*, D_R) \\ &= \hat{e}(x(w+s)(q_R+s)P, D_R) \\ &= \hat{e}(x(w+s)(q_R+s)P, \frac{1}{q_R+s}P) \\ &= \hat{e}(x(w+s)P, P) \\ &= \hat{e}((w+s)P, xP) \\ &= \hat{e}(wP + P_{pub}, P)^x \\ &= R \end{aligned}$$

  • From this it is clear that R^* of σ^* and R of the challenge ciphertext σ are equal.
– Now, we show that σ^* will pass the verification test of the decryption algorithm,

$$\begin{aligned}
\hat{e}(t^*P + U^*, wP + P_{pub})\hat{e}(P,P)^{-1} &= \hat{e}((t+r^*)P + W - (u+r^*)P, wP + P_{pub})\hat{e}(P,P)^{-1} \\
&= \hat{e}((xH_2(m_b,R^*) + u + r^*)P, wP + P_{pub}) \\
&\qquad \hat{e}(W - (u+r^*)P, wP + P_{pub})\hat{e}(P,P)^{-1} \\
&= \hat{e}(xH_2(m_b,R)P + W, wP + P_{pub})\hat{e}(P,P)^{-1} \\
&\qquad \text{(Since } R^* = R \text{)} \\
&= \hat{e}(xH_2(m_b,R)P, wP + P_{pub})\hat{e}(W, wP + P_{pub})\hat{e}(P,P)^{-1} \\
&= \hat{e}(xH_2(m_b,R)P, wP + P_{pub})\hat{e}((1/(w+s))P, wP + P_{pub})\hat{e}(P,P)^{-1} \\
&= \hat{e}(xH_2(m_b,R)P, wP + P_{pub})\hat{e}(P,P)\hat{e}(P,P)^{-1} \\
&= \hat{e}(wP + P_{pub}, P)^{xH_2(m_b,R)} \\
&= R^{H_2(m_b,R)} \\
&= R^{*H_2(m_b,R^*)}
\end{aligned}$$

- Thus σ^* is a valid encryption of the same message m_b that was encrypted in the challenge ciphertext σ. However, $\sigma \neq \sigma^*$,
  - $c^* = c$, $R^* = R$
  - $U^* \neq U$, $T_0^* \neq T_0$, $T_1^* \neq T_1$, $T_2^* \neq T_2$, $t^* \neq t$, $t_1^* \neq t_1$ and $t_2^* \neq t_2$,
- Hence, σ^* can be legally given to the decryption oracle during **Phase-II** of training. The response from the decryption oracle will be m_b and thus $\mathcal{A}$ will know exactly m_b during **Phase-II**.
- Since $R^* = R$ and $c^* = c$, we obtain

$$c^* \oplus H_3(R^*) = c \oplus H_3(R) = m_b$$

.

Another way of attacking the confidentiality :

- Let the challenge ciphertext σ of message m_b is,

$$\sigma = \langle U, T_0, T_1, T_2, t_1, t_2, t, c \rangle$$

- $\mathcal{A}$ cooks up the ciphertext σ^*= ($U - r^*, T_0, T_1, T_2, t_1, t_2, t + r^*, c$) from the challenge ciphertext σ. Since $\sigma^* \neq \sigma$ and both are valid encryption of the same message m_b, $\mathcal{A}$ can make use of the decrypt oracle to get back the message m_b.

4.3 Fixing the Weakness in [7]

The security weakness of [7] can be fixed by providing the modifications to the $On-Encrypt$ algorithm and the definition of the hash function H_2 and all the other algorithms remain the same. The improved On-Encrypt protocol:
On-Encrypt(m, ID_A, ϕ) **:**

- $t_1 = \beta^{-1}(H_1(ID_A)\text{- }\alpha) \bmod q$
- $t_2 = \beta^{-1}(H_1(ID_A)\text{- }\gamma) \bmod q$
- $t = H_2(m,\ R,\ \mathrm{U},\ T_0, T_1, T_2, t_1, t_2)x + u \bmod q$

– $c = H_3(R) \oplus m$
– Output the ciphertext $\sigma = \langle U, T_0, T_1, T_2, t, t_1, t_2, c \rangle$

The definition of H_2 should be modified to $H_2 : \{0,1\}^* \times \mathbb{G}_T \times \mathbb{G}^4 \times \mathbb{Z}_q^* \times \mathbb{Z}_q^* \rightarrow \mathbb{Z}_q^*$. With these changes, the system in [7] can be made CCA secure.

5 Weakness in the security proof of Chow et al.

In this we give the weakness in the security proof for Theorem 2 of the CCA secure scheme proposed by Chow et al. [2]. We are not reviewing the scheme in [2] since the scheme is available in a open source archive. Let $\mathcal{C}_1, \mathcal{C}_2, \mathcal{C}_3$ be the challenge ciphertext given to the adversary and let ID^* be the identity chosen by adversary during challenge phase and given to challenger for the generation of challenge ciphertext. Let $\mathcal{C}_1^*, \mathcal{C}_2^*, \mathcal{C}_3^*$ be the ciphertext generated by adversary in the following way.

– Randomly pick the elements of C_1^* according to the definition of the scheme.
– Choose r^* randomly and generate a valid $K^* = KEM^{Off}(r^*)$ (Note : r^* is not bound to C_1^*.
– Compute $\mathcal{C}_2^* = H(K^*, \mathcal{C}_1^*, m) \oplus r^*$
– Compute $\mathcal{C}_3^* = H'(K^*, \mathcal{C}_1^*)$.

It should be noted that $\mathcal{C}_1^*, \mathcal{C}_2^*, \mathcal{C}_3^*$ is not a valid encryption of m since $\mathcal{C}_1$ is not the component that is generated from $KEM^{Off}(r^*)$. But, if adversary submits this ciphertext to the decryption oracle, the challenger will respond with m. Hence the adversary will come to know that the decryption oracle is wrong and will abort. In the proof, the binding of key K' obtained from list is checked by $(K', \mathcal{C}_1') == KEM^{off}(r')$ (r' obtained from ciphertext). The components in $\mathcal{C}_1^*, b$ (output together with K^* during $KEM^{off}(r^*)$ are not verified during simulation of decryption oracle. Also it should be noted that even if $r^* = r'$, when the KEM^{off} is invoked with r^* two times the key K will be the same but not the rest of the components as different randomness are involved during each invocation. Therefore, $\mathcal{C}_1^* \neq \mathcal{C}_1'$. Hence, some explicit tests should be done with respect to the binding of K' obtained during decryption process.

6 New Identity Based Online/Offline Encryption Scheme (New-IBOOE)

Let $\mathbb{G}_1$ and $\mathbb{G}_2$ be groups of prime order q , and let $\hat{e} : \mathbb{G}_1 \times \mathbb{G}_1 \rightarrow \mathbb{G}_2$ be the bilinear pairing.

Setup: The PKG selects a generator $P \in \mathbb{G}_1$ and randomly chooses $s \in \mathbb{Z}_q^*$. It sets $P_{pub} = sP$ and $\alpha = \hat{e}(P, P)$. Let $\mathcal{M}$ denotes the message space Let n_M be the number of bits used to represent a message. Let $H_1 : \{0,1\}^* \rightarrow \mathbb{Z}_q^*$, H_2: $\mathbb{G}_2 \times \{0,1\}^{n_m} \times \mathbb{Z}_q^* \times \mathbb{G}_1 \times \mathbb{G}_1 \times \mathbb{Z}_q^* \rightarrow \{0,1\}^{n_2}$ and $H_3 : \mathbb{G}_2 \rightarrow \{0,1\}^{n_M+q+n_2}$ be the hash functions. The public parameters $Params$ and master private key msk are given by,

$$Params = \langle \mathbb{G}_1, \mathbb{G}_2, q, P_{pub}, \alpha, \mathcal{M}, H_1, H_2, H_3 \rangle, msk = s.$$

Extract(ID) :
- $q_{ID} = H_1(ID)$
- $D_{ID} = \frac{1}{q_{ID}+s}P.$

Off-Encrypt($Params$) :
- x, $\hat{a}$, $\hat{b} \in_R \mathbb{Z}_q^*$
- $R = \alpha^x$
- $\beta = H_3(R)$
- $T_1 = a^{-1}xP$
- $T_2 = x\hat{b}P + xsP = x(\hat{b}+s)P.$
- Outputs the offline ciphertext $\phi = \langle x, \hat{a}, \hat{b}, R, T_1, T_2, \beta\rangle$.

On-Encrypt(m, ID_A, ϕ) :
- $t_1 = \hat{a}(q_A - \hat{b}) \, mod \, q$
- $h = H_2(R, m, x, T_1, T_2, t_1)$
- $c = \beta \oplus m\|x\|h$
- Outputs the ciphertext $\sigma = \langle T_1, T_2, t_1, c\rangle$

Decrypt(σ, ID_A, D_A) :
- Let $Z = T_2 + t_1T_1$ and $Q_A = q_AP + P_{Pub}$
- $R' = \hat{e}(Z, D_A)$
- $m'\|x'\|h' = c \oplus H_3(R')$
- $h'' = H_2(R', m', x', T_1, T_2, t_1)$
- Checks whether $Z \stackrel{?}{=} x'Q_A$ and $h'' \stackrel{?}{=} h'$.
- Outputs m, if equal. Otherwise, outputs "$Invalid$"

Correctness : It is sufficient to show that $R = R'$, In-fact,

$$\begin{aligned} R' &= \hat{e}(Z, D_A) = \hat{e}(t_1T_1 + T_2, D_A) \\ &= \hat{e}(\hat{a}(q_A - \hat{b})a^{-1}xP + x(\hat{b}+s)P, D_A) \\ &= \hat{e}((q_A - \hat{b})xP + x(\hat{b}+s)P, D_A) \\ &= \hat{e}(q_AxP + xsP, 1/(q_A+s)P) \\ &= \hat{e}(x(q_A+s)P, 1/(q_A+s)P) \\ &= \hat{e}(P,P)^x \\ &= R \end{aligned}$$

7 Security Results

7.1 Proof of Confidentiality of New-IBOOE (IND-IBOOE-CCA2)

Theorem 1. *Assume that an IND-IBOOE-CCA2 adversary $\mathcal{A}$ making q_e key extraction queries, q_{H_i} queries to hash oracles H_i (i=1, 2, 3), and q_d decryption queries, has an advantage ϵ against New-IBOOE scheme. Then, there exists an algorithm $\mathcal{C}$ to solve the k-Bilinear Diffie Hellman Problem (k-BDHP) for $k = q_{H_1}$ with an advantage $\epsilon' \geq \epsilon\left(\frac{1}{q_{H_1}(q_{H_3}+q_{H_2})}\right)$.*

Proof: Algorithm $\mathcal{C}$ takes the k-BDHP instance $(P, Q = aP, l^*, \frac{1}{x_1+a}P, ..., \frac{1}{x_k+a}P)$, and aims to find $\hat{e}(P,P)^{\frac{1}{x^*+a}}$, for a random $x^* \notin \{x_1, ..., x_k\}$. $\mathcal{C}$ simulates the system with the various oracles $\mathcal{O}_{H_1}$, $\mathcal{O}_{H_2}$, $\mathcal{O}_{H_3}$, $\mathcal{O}_{Decrypt}$. $\mathcal{A}$ is allowed to make polynomially bounded number of queries, adaptively to the oracles provided by $\mathcal{C}$. The game between $\mathcal{C}$ and $\mathcal{A}$ can be demonstrated by:

- ***Setup*** *: $\mathcal{C}$ sets $P_{pub} = aP$, $\alpha = \hat{e}(P,P)$ and sends the system parameters $\langle \mathbb{G}_1, \mathbb{G}_2, P,, P_{pub}, \alpha, \hat{e}(.,.)\rangle$ to $\mathcal{A}$.*

– ***Phase-I*** *: For maintaining the consistency of the oracle query responses, $\mathcal{C}$ maintains three lists $\mathcal{L}_{H_i}$, (i = 1, 2, 3) which keeps track of the responses given by $\mathcal{C}$ to the corresponding oracles ($\mathcal{O}_{H_1}$, $\mathcal{O}_{H_2}$, $\mathcal{O}_{H_3}$) queries. $\mathcal{C}$ simulates $\mathcal{A}$'s queries as below.*

- $\mathcal{O}_{H_1}$ ***oracle query*** *: Without loss of generality, we assume that queries to $\mathcal{O}_{H_1}$ are distinct and that any query involving an identifier ID is preceded by the random oracle query $H_1(ID)$. $\mathcal{C}$ selects a random index γ, where $1 \leq \gamma \geq q_{H_1}$. When $\mathcal{A}$ generates the γ^{th} query on ID_γ , $\mathcal{C}$ decides to fix ID_γ as target identity for the challenge phase. Moreover, $\mathcal{C}$ responds to $\mathcal{A}$ as follows:*
  - *If it is the γ^{th} query, then $\mathcal{C}$ sets $q_\gamma = x^*$, returns q_γ as the response to the query and stores (ID_γ, q_γ) in the list $\mathcal{L}_{H_1}$*
  - *For all other queries, $\mathcal{C}$ sets $q_i = x_i$ where x_i is the value given in the instance of k-DBHP. The tuple (ID_i, q_i) is stored in the list $\mathcal{L}_{H_1}$.*
- $\mathcal{O}_{H_2}$ ***oracle query*** *: When $\mathcal{A}$ makes a query to this oracle with (R, m, x, T_1, T_2, t_1) as input, $\mathcal{C}$ does the following:*
  - *Searches for the tuple $(R, m, x, T_1, T_2, t_1, h)$ in the list $\mathcal{L}_{H_2}$ and if found, $\mathcal{C}$ responds with h.*
  - *Otherwise, $\mathcal{C}$ responds to $\mathcal{A}$ by choosing a random $h \leftarrow {\mathbb{Z}_q}^*$ such that no entry h does not exist in $\mathcal{L}_{H_2}$ and adds the tuple $(R, m, x, U, T_1, T_2, t_1, h)$ into $\mathcal{L}_{H_2}$.*
- $\mathcal{O}_{H_3}$ ***oracle query*** *: When $\mathcal{A}$ makes a query to this oracle with R as input, $\mathcal{C}$ does the following:*
  - *Searches for the tuple (R, β) in the list $\mathcal{L}_{H_3}$ and if found, $\mathcal{C}$ responds with β.*
  - *Otherwise, $\mathcal{C}$ responds to $\mathcal{A}$ by choosing a random $\beta \leftarrow {\mathbb{Z}_q}^*$ such that no entry $(.,\beta)$ exists in $\mathcal{L}_{H_3}$ and adds the tuple (R, β) in to the list $\mathcal{L}_{H_3}$.*
- $\mathcal{O}_{Extract}$ *query : On getting a request for private key of user $\mathcal{U}_i$ with identity ID_i, $\mathcal{C}$ aborts if $ID_i = ID_\gamma$. Else, $\mathcal{C}$ recovers the corresponding pair (ID_i, x_i) from the list $\mathcal{L}_{H_1}$ and responds to $\mathcal{A}$ with $\frac{1}{x_i + s}P$ which is given in the instance of k-BDHP.*
- $\mathcal{O}_{\textbf{Decrypt}}$ ***query*** *: Upon receiving an decryption query of ciphertext $\sigma =$ with ID_A as receiver, $\mathcal{C}$ proceeds as follows:*
  When $ID_A \neq ID_\gamma$, then $\mathcal{C}$ can directly decrypt the ciphertext, since $\mathcal{C}$ knows the private key D_A corresponding to ID_A. When the receiver identity $ID_A = ID_\gamma$ (i.e. $\mathcal{C}$ does not know the private key corresponding to ID_A), $\mathcal{C}$ generates the response as explained below:
  1. *For each $(R, \beta) \in \mathcal{L}_{H_3}$, perform the following :*
     (a) Compute $m \| x \| h = c \oplus \beta$
     (b) Check $R \stackrel{?}{=} \alpha^x$ and $h \stackrel{?}{=} \mathcal{O}_{H_2}(R, m, x, T_1, T_2, t_1)$
     (c) If not true, proceed with the next tuple in $\mathcal{L}_{H_3}$.
     (d) Else, check $(t_1 T_1 + T_2) \stackrel{?}{=} x(q_A P + P_{pub})$.
     (e) If true, output m
  2. *If none of the tuples obtained in step(1) passes the checks, then return "Invalid".*

- ***Challenge Phase*** *: In the challenge phase $\mathcal{A}$ chooses two equal length plain texts $m_0, m_1 \in \mathcal{M}$, a receiver identity $ID_{\mathbb{R}}$ on which $\mathcal{A}$ wishes to be challenged. $\mathcal{A}$ sends (m_0, m_1), $ID_{\mathbb{R}}$ to $\mathcal{C}$. It should be noted that $\mathcal{A}$ should not have queried the private key corresponding to $ID_{\mathbb{R}}$ in Phase-I. $\mathcal{C}$ aborts, if $ID_{\mathbb{R}} \neq ID_\gamma$; else, $\mathcal{C}$ chooses a bit $b \in \{0,1\}$ and computes the challenge ciphertext σ^* of m_b as follows :*
  - Picks $\eta \in_R \mathbb{Z}_q^*$
  - Chooses $t_1 \in_R \mathbb{Z}_q^*$ and $T_1 \in_R \mathbb{G}_1$
  - Computes $T_2 = \eta P - t_1 T_1$.
  - *Randomly picks a c of the size defined in the scheme.*
  - *$\mathcal{C}$ Outputs the challenge ciphertext $\sigma^* = \langle T_1, T_2, t_1, c \rangle$*

- ***Phase-II*** *: After receiving the challenge ciphertext, $\mathcal{A}$ gets training as in Phase-I, except that $\mathcal{A}$ is not allowed to ask decryption query on σ^* and extract query for $ID_{\mathbb{R}}$.*
  Here, as per the decryption algorithm $R = \hat{e}(T_2 + t_1 T_1, D_{\mathcal{R}}) = \hat{e}(\eta P, D_{\mathbb{R}})$. Hence, $R^{\eta^{-1}}$ will be equal to $\hat{e}(P, D_{\mathbb{R}}) = \hat{e}(P, D_\gamma)$. For $\mathcal{A}$ to know that σ^* is invalid, it should have queried the $\mathcal{H}_3$ oracle or $\mathcal{H}_2$ oracle with R as input. Hence, one of the entries in list $\mathcal{L}_{H_3}$ or $\mathcal{L}_{H_2}$ will contain the value R, the solution to the k-BDHP problem.

- ***Guess*** *: At the end of the Phase-II, $\mathcal{A}$ returns a bit b'. $\mathcal{C}$ ignores the response by $\mathcal{A}$. $\mathcal{C}$ fetches a (R) randomly from the list $\mathcal{L}_{H_3}$ or $\mathcal{L}_{H_2}$ and computes $\eta^* = R^{\eta^{-1}}$. With probability $\frac{1}{(q_{H_3} + q_{H_2})}$, $\eta^* = \hat{e}(P, (q_\gamma + s)^{-1} P)$ should have been queried by $\mathcal{A}$, since the simulation given by $\mathcal{C}$ is indistinguishable from the real protocol.*

$$\hat{e}(P,P)^{(q_\gamma + s)^{-1}} = \hat{e}(P,P)^{\frac{1}{x^* + s}}.$$

  Now, $\mathcal{C}$ returns (x^*, $\hat{e}(P,P)^{(x^*+s)^{-1}}$) as the output.
  The probability of success of $\mathcal{C}$ can be measured by analyzing the various events that happen during the simulation :
  The events in which $\mathcal{C}$ aborts the IND-IBOOE-CCA2 game are,
  1. *E_1 - when $\mathcal{A}$ queries the private key of the target identity ID_γ and $Pr[E_1] = \frac{q_e}{q_{H_1}}$.*
  2. *E_2 - when $\mathcal{A}$ does not choose the target identity ID_γ as the receiver during the challenge and $Pr[E_2]$ = 1 - $\frac{1}{q_{H_1} - q_e}$.*

The probability that $\mathcal{C}$ does not abort in the IND-IBOOE-CCA2 game is given by,

$$Pr[\neg E_1 \wedge \neg E_2] = \left(1 - \frac{q_e}{q_{H_1}}\right)\left(\frac{1}{q_{H_1} - q_e}\right) = \frac{1}{q_{H_1}}.$$

The probability that the random entry chosen by $\mathcal{C}$ from the list $\mathcal{L}_{H_3}$ becoming the solution to the k-BDHP is $\left(\frac{1}{(q_{H_3} + q_{H_2})}\right)$. Therefore the probability of $\mathcal{C}$ solving the k-BDHP is given by,

$\Pr[\mathcal{A}(P, aP, (x_1+a)^{-1}P, ..., (x_k+a)^{-1}P, x_1, ..., x^k) = \hat{e}(P,P)^{(a+x^*)^{-1}} | a, x^* \in_R \mathrm{Z}_q{}^*, x^* \notin \{x_1, ..., x_k\}] = \epsilon\left(\frac{1}{q_{H_1}(q_{H_3}+q_{H_2})}\right)$

As ϵ is non-negligible, the probability of $\mathcal{C}$ solving k-BDHP is also non-negligible. This clearly shows that no adversary exists who can solve the IND-IBOOE-CCA2 security of New-IBOOE scheme □

Conclusion

Identity based encryption schemes wherein the encryption is carried out in two phases namely, offline and online phase according to the complexity of the operations performed is known to be identity based online/offline encryption scheme. The subtle issue in designing an identity based online/offline encryption scheme is to split the operations into heavy weight (for offline phase) and light weight (for online phase) without knowing the message and receiver. [7] gives a solution for this problem in the random oracle model. In this paper, we pointed out that the scheme in [7] is not CCA secure. We proposed a possible fix for the same. We pointed out the weakness in the security proof of the online/offline system in [2]. Also, we proposed an efficient identity based online/offline encryption scheme. We formally proved the security of the new scheme in the random oracle model.

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