

The World is Not Enough: Another Look on Second-Order DPA

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Abstract. In a recent work, Mangard *et al.* showed that under certain assumptions, the (so-called) standard univariate side-channel attacks using a distance-of-means test, correlation analysis and Gaussian templates are essentially equivalent. In this paper, we show that in the context of multivariate attacks against masked implementations, this conclusion does not hold anymore. In other words, while a single distinguisher can be used to compare the susceptibility of different unprotected devices to first-order DPA, understanding second-order attacks requires to carefully investigate the information leakages and the adversaries exploiting these leakages, separately. Using a framework put forward by Standaert *et al.* at Eurocrypt 2009, we provide the first analysis that considers these two questions in the case of a masked device exhibiting a Hamming weight leakage model. Our results lead to new intuitions regarding the efficiency of various practically-relevant distinguishers. Further, we also investigate the case of second- and third-order masking (*i.e.* using three and four shares to represent one value). It turns out that moving to higher-order masking only leads to significant security improvements if the secret sharing is combined with a sufficient amount of noise. Eventually, we show that an information theoretic analysis allows determining this necessary noise level, for different masking schemes and target security levels, with high accuracy and smaller data complexity than previous methods.

1 Introduction

Masking (as described, *e.g.* in [2, 6, 17]) is a very frequently considered solution to thwart side-channel attacks. The basic idea is to randomize all the sensitive variables during a cryptographic computation by splitting them into d shares. The value $d - 1$ is usually denoted as the order of the masking scheme. As most countermeasures against side-channel attacks, masking does not totally prevent the leakages but it is expected to increase the difficulty of performing a successful key-recovery. For example, masking can be defeated because of technological issues such as glitches [7]. Alternatively, an adversary can always perform a higher-order DPA (*e.g.* [8, 11, 22]) in which he “combines” the leakages corresponding to the d shares in order to extract key-dependent information. From

a performance point of view, masking a block cipher implies significant performance overheads, because it requires to compute the encryption of the different shares separately. As a result, an important problem is to determine the exact security level that it provides in function of the order of the scheme $d - 1$.

In order to solve this problem, Prouff *et al.* proposed a comprehensive study of first-order masking (*i.e.* second-order power analysis) in [15]. In their paper, the two leakage samples corresponding to the different shares are first mingled with a combination function. Next, a (key-dependent) leakage model is used to predict the output of this function. Eventually, the physical leakages are compared with the key-dependent predictions, thanks to Pearson’s correlation coefficient. Different combination functions are analyzed in function of the efficiency of the resulting attacks, leading to the following conclusions:

1. For every device and combination function, there exists an optimal prediction function (or model) that leads to the best attack efficiency.
2. Following an analysis based on Pearson’s coefficient and assuming a “Hamming weight leakage model”, the “normalized product combining function” (both to be detailed in this paper) is the best available in the literature.

The first observation is in fact quite natural. Since every device is characterized by its leakage function, there is one optimal model to predict these leakages that perfectly captures their probability density function (pdf for short). And for every optimal model, there is one way to combine the leakage samples that leads to the best possible correlation. But the idea of *optimal combination function* also leads to a number of issues. On the one hand, as acknowledged by the authors of [15], their analysis is carried out for a fixed (Hamming weight) leakage function. Therefore, how the observations made in this context would be affected by a different leakage function is an open question. On the other hand, their analysis is also performed for a given statistical tool, *i.e.* Pearson’s correlation coefficient. Hence, one can wonder about the extent to which this statistical tool is generic enough for evaluating second-order DPA.

This second question is particularly interesting in view of the recent results of [10]. This reference shows that in the context of (so-called standard) first-order DPA and when provided with the same leakage model, the most popular distinguishers such as using distance-of-means tests, correlation analysis and Gaussian templates require approximately the same number of traces to extract keys. Differences observed in practice are only due to statistical artifacts. In addition, it is shown that the correlation coefficient can be related to the concept of conditional entropy which has been established as a measure for side-channel leakage in [18]. Therefore, a natural question is to know if these observations still hold in the second-order case. For example, can the correlation coefficient be used to evaluate the information leakage of a masked implementation?

In this paper, we answer this question negatively. We show that second-order DPA attacks are a typical context in which the two parts of the framework for the analysis of side-channel key-recovery of Eurocrypt 2009 lead to different intuitions. First, an information theoretic analysis measures the amount of leakage

provided by the masked device. It quantifies the security limits of the implementation and relates to the success rates of an adversary who can perfectly profile the leakage pdf. Second, a security analysis measures the effectiveness of one particular distinguisher. By applying this framework, we exhibit new intuitions regarding the behavior of different second-order DPA attacks and combination functions. We then discuss the impact of these observations in profiled and non-profiled attack scenarios and confirm our theoretical investigations with practical experiments. We note that our results do not contradict [15] but rather emphasize that a single distinguisher cannot capture all the specificities of a leakage function. Eventually, we extend our analysis towards higher-order masking. This allows us to confirm that, from an information theoretic point of view, increasing the number of shares in a masking scheme only improves the security of a chip if a sufficient amount of noise is limiting the quality of the adversary’s measurements [2]. Higher-order masking also provides a case for the information theoretic metric introduced in [18]. We show that this metric can be used to determine the exact amount of shares and noise required to reach a certain security level (against worst-case template attacks, exploiting intensively profiled leakage models), with smaller data complexity than previous methods. Hence, it constitutes a useful tool for evaluation laboratories in which the amount of measurements to evaluate the security of a protected chip is usually limited.

Summarizing, first-order side-channel attacks are a quite simple context in which (under certain conditions) most popular distinguishers behave similarly, if they are feeded with the same leakage models. As a consequence, it can be sound to use “one distinguisher for all” in this context. By contrast, second-order (or higher-order) DPA can benefit from probability distributions of physical leakages that take very different forms (*e.g.* mixtures, typically). Hence, given a certain amount of information leaked by a masked device, and even if feeded with the same leakage models, different statistical tools will take advantage of the key-dependencies in very different manners. In other words, depending on the devices, one or another attack may perform better, hence suggesting our title “the world is not enough”. Interestingly, this conclusion goes nicely together with the known intuition that the construction (or profiling) of a leakage model is highly dependent on the target device. Also in this context, an information theoretic analysis provides efficient ways to compare the quality of the profiling [20].

2 Boolean masking and second-order attacks

Many different masking schemes have been proposed in the literature. Although they can result in significantly different performances, the application of second-order attacks generally relies on the same principles, independent of the type of masking. In the following, we decided to focus on the Generalized Look Up Table (GLUT for short) that has been proposed in [14]. Such a scheme is represented in the lower left part of Figure 1, using the key addition and S-box layer of a block cipher as a concrete example. It can be explained as follows. For an input plaintext x_i , a random mask a_i is first generated within the device. The

value $x_i \oplus a_i$ is generally denoted as the masked variable. Then, the encryption algorithm (here, the key addition and S-box) are applied to the masked variables, where s denotes a secret key byte (we will use the term subkey in the following). Concurrently, some correction terms are also computed such that anytime during the cryptographic computation, the XOR between a masked variable and its corresponding mask produces the original variable. In the case of the GLUT proposal, a precomputed function Sbox' is used for this purpose. For example in Figure 1, the masked S-box output $\text{Sbox}(x_i \oplus a_i \oplus s)$ can be written as $\text{Sbox}(x_i \oplus s) \oplus b_i$, where b_i denotes an output mask produced by Sbox' .

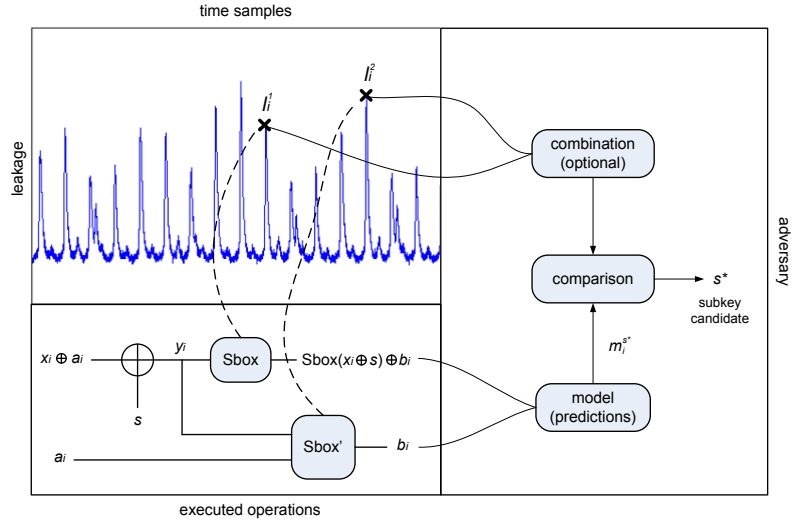


Fig. 1: Illustrative second-order DPA.

In practice, the GLUT countermeasure can be implemented in different manners. Mainly, the two S-box computations can be performed sequentially (as in a software device, typically) or in parallel (as in a hardware device, typically). In order to describe the second-order DPA that we investigate in this paper, we first use the sequential approach (the parallel approach will be discussed in the next section). Also, we rely on the terminology introduced in [18]. Essentially, the idea of a second-order DPA is to take advantage of the joint leakage of two intermediate computations during the encryption process (*i.e.* the masked value and its mask). In the software approach, the computation of these intermediate variables will typically be performed in two different clock cycles. Hence, two leakage samples l_i^1 and l_i^2 corresponding to these computations can be found in the leakage traces, as in the top of Figure 1. Following the standard DPA described in [10], the adversary will then work in three (plus one optional) steps:

1. For different plaintexts x_i and subkey candidates s^* , the adversary predicts some intermediate values in the target implementation. For example, one could predict the S-box outputs $\text{Sbox}(x_i \oplus s)$ in Figure 1.

2. For each of these predicted values, the adversary models the leakages. Because of the presence of a mask in the implementation, this prediction can use a pdf (where the probability is taken over the masks) or some simpler function *e.g.* capturing only certain moments of this pdf.
3. Optionally, the adversary combines the leakage samples into a single variable.
4. For each subkey candidate s^* , the adversary finally compares the modeled leakages with actual measurements, produced with the same plaintexts x_i and a secret subkey s . In a second-order DPA, each model is compared with two samples in the traces. This comparison is independent of all other points. Consequently, these attacks are referred to as *bivariate*. In practice, this comparison is applied to many pairs of points in the leakage traces and the subkey candidate that performs best is selected by the adversary.

As for the analysis of first-order attacks, comparing different distinguishers requires to provide them with the same leakage samples. However, contrary to the first-order case and as will be discussed in the following sections, the best pair of leakage samples is not necessarily the same for all distinguishers. This is because different distinguishers can take advantage of different leakage pdf with different efficiencies in this case. In practice, the best solution is to test all pairs of samples in the traces (but this means N^2 statistical tests to perform if the traces have N samples). In this paper, we will generally assume that this best pair of samples is provided to the attacks we perform (which can be done easily when simulating experiments and requires significant - but tractable - computational power when performing attacks based on real measurements).

Finally, we will use the following notations:

- $\mathbf{x}_q = [x_1, x_2, \dots, x_q]$: a vector of plaintext bytes.
- $\mathbf{a}_q = [a_1, a_2, \dots, a_q]$: a vector of random input mask bytes.
- $\mathbf{b}_q = [b_1, b_2, \dots, b_q]$: a vector of random output mask bytes.
- $v_i^1 = \text{Sbox}(x_i \oplus s) \oplus b_i$: an intermediate value in the encryption of x_i .
- $v_i^2 = b_i$: another intermediate value in the encryption of x_i .
- $\mathbf{l}_q^1 = [l_1^1, l_2^1, \dots, l_q^1]$: a vector of leakage samples (hypothetically) corresponding to the first intermediate values v_i^1 during the encryption process.
- $\mathbf{l}_q^2 = [l_1^2, l_2^2, \dots, l_q^2]$: a vector of leakage samples (hypothetically) corresponding to the second intermediate values v_i^2 during the encryption process.
- $\mathbf{m}_q^{s^*} = [m_1^{s^*}, m_2^{s^*}, \dots, m_q^{s^*}]$: a vector containing leakage models (*i.e.* predictions) corresponding to a subkey candidate s^* and the plaintexts \mathbf{x}_q .

3 Second-order attacks with Pearson's correlation

In theory, second-order DPA is possible if the joint probability distributions $\Pr[\mathbf{L}_q^1, \mathbf{L}_q^2 | s]$ are different for different subkeys s . This can be illustrated, *e.g.* for a Hamming weight leakage function that is frequently considered in the practice of side-channel attacks [9] and has been the running example in [15]. It means assuming that the leakage samples l_1^1 and l_1^2 can be written as:

$$\begin{aligned} l_1^1 &= W_H(v_i^1) + n_i^1, \\ l_1^2 &= W_H(v_i^2) + n_i^2, \end{aligned}$$

where W_H is the Hamming weight function and n_i^1, n_i^2 are normally distributed noise values with mean 0 and standard deviation σ_n . In this particular setting, we can observe 9 possible distributions, corresponding to the 9 Hamming weight values of a secret state $\Sigma_i = \mathbf{Sbox}(x_i \oplus s)$, as observed in [19] (we also used the AES S-box in our experiments, with the first byte of the master key as a target s). The left parts of Figures 10, 11 and 12 in Appendix A show the joint leakage distributions in this setting and clearly illustrate that they are key-dependent. As detailed in the previous section, taking advantage of these dependencies requires a comparison tool. In their statistical evaluation of second-order DPA, Prouff *et al.* use Pearson’s correlation coefficient [1]. In the context of first-order attacks exploiting a single leakage sample l_i , it implies computing:

$$\hat{\rho}(\mathbf{M}_q^{s^*}, \mathbf{L}_q) = \frac{\hat{\mathbf{E}}\left((l_i - \hat{\mathbf{E}}(\mathbf{L}_q)) \cdot (m_i^{s^*} - \hat{\mathbf{E}}(\mathbf{M}_q^{s^*}))\right)}{\hat{\sigma}(\mathbf{L}_q) \cdot \hat{\sigma}(\mathbf{M}_q^{s^*})},$$

where $\hat{\mathbf{E}}$ and $\hat{\sigma}$ denote the sample means and standard deviations of a random variable, respectively. In order to extend this tool towards the second-order case, the usual approach is to first combine the two leakage samples l_i^1 and l_i^2 with a *combination function* C . For example, Chari *et al.* proposed to take the (normalized) product of the two samples [2]: $C(l_i^1, l_i^2) = (l_i^1 - \hat{\mathbf{E}}(\mathbf{L}_q^1)) \cdot (l_i^2 - \hat{\mathbf{E}}(\mathbf{L}_q^2))$ and Messergers used the absolute difference between them [11]: $C(l_i^1, l_i^2) = |l_i^1 - l_i^2|$. As illustrated in the right parts of Figures 10, 11 and 12, those combining functions also lead to key-dependencies. In addition to these standard examples, we finally plotted the distribution of the sum combining function $C(l_i^1, l_i^2) = l_i^1 + l_i^2$ because it can be used to analyze the behavior of the GLUT masking in a hardware setting, where the two S-boxes of Figure 1 are computed in parallel.

3.1 Choice of a model and leakage-dependency of C

Given the above descriptions and assuming that the adversary knows a good leakage model for the samples l_i^1 and l_i^2 (here, we use Hamming weights), it remains to determine which model to use when computing $\hat{\rho}(\mathbf{M}_q^{\Sigma_i^*}, C(\mathbf{L}_q^1, \mathbf{L}_q^2))$. That is, we do not need to predict the leakage samples separately, but their combination. In addition and contrary to the first-order case, there is an additional variable (*i.e.* the mask) that is unknown to the adversary. But assuming a model for the separate samples directly allows deriving one for their combination by simply computing the following mean values:

$$m_{\Sigma_i^*} = \hat{\mathbf{E}}_{b_i}\left(C(W_H(\Sigma_i^* \oplus b_i), W_H(b_i))\right),$$

taken over the masks b_i . For example, Figure 2 illustrates the leakage models corresponding to the absolute difference and normalized product combination functions. They are again clearly key-dependent, as opposed to the ones of a sum combining function for which the mean value (taken over the masks) is constant for all secret states. Hence, as already observed in [9], this sum combining function will not lead to successful second-order correlation attacks.

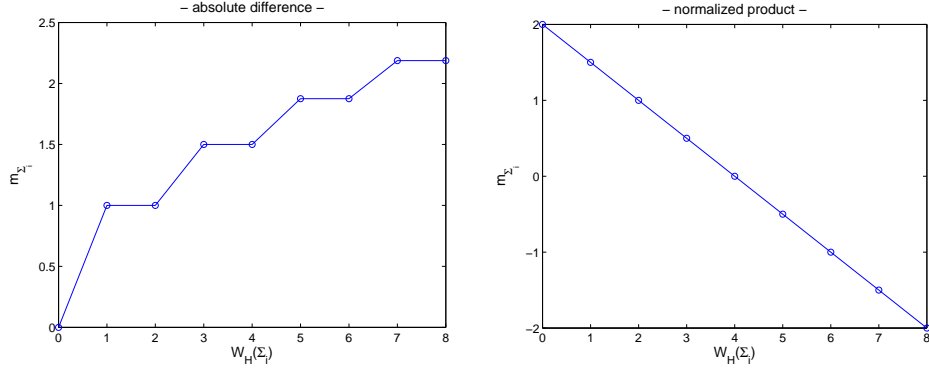


Fig. 2: Leakage models for second-order DPA using the correlation coefficient.

The figure also confirms the previous theoretical analysis of Prouff *et al.* where it is demonstrated that the normalized product combining function leads to the most efficient second-order side-channel attacks when using Pearson's coefficient and assuming a Hamming weight leakage model for the separate samples. Indeed, this particular setting gives rise to nicely linear dependencies of the models $m_{\Sigma_i^*}$ in the Hamming weight of the secret states $W_H(\Sigma_i)$. Also, and contrary to the absolute difference combining function, all the 9 possible Hamming weights correspond to a different model $m_{\Sigma_i^*}$ in this particular case.

Interestingly, the effectiveness of the normalized product combining function can be simply explained when looking at the equations since it computes:

$$\hat{\rho}(\mathbf{M}_q^{\Sigma^*}, \mathbf{C}(\mathbf{L}_q^1, \mathbf{L}_q^2)) = \frac{\hat{\mathbf{E}}\left(\left(\mathbf{C}(l_i^1, l_i^2) - \hat{\mathbf{E}}(\mathbf{C}(\mathbf{L}_q^1, \mathbf{L}_q^2))\right) \cdot \left(m_i^{\Sigma^*} - \hat{\mathbf{E}}(\mathbf{M}_q^{\Sigma^*})\right)\right)}{\hat{\sigma}(\mathbf{C}(\mathbf{L}_q^1, \mathbf{L}_q^2)) \cdot \hat{\sigma}(\mathbf{M}_q^{\Sigma^*})}.$$

As the product is normalized, we have that $\hat{\mathbf{E}}(\mathbf{C}(\mathbf{L}_q^1, \mathbf{L}_q^2)) = 0$, which leads to:

$$\hat{\rho}(\mathbf{M}_q^{\Sigma^*}, \mathbf{C}(\mathbf{L}_q^1, \mathbf{L}_q^2)) = \frac{\hat{\mathbf{E}}\left(\left(l_i^1 - \hat{\mathbf{E}}(\mathbf{L}_q^1)\right) \cdot \left(l_i^2 - \hat{\mathbf{E}}(\mathbf{L}_q^2)\right) \cdot \left(m_i^{\Sigma^*} - \hat{\mathbf{E}}(\mathbf{M}_q^{\Sigma^*})\right)\right)}{\hat{\sigma}(\mathbf{C}(\mathbf{L}_q^1, \mathbf{L}_q^2)) \cdot \hat{\sigma}(\mathbf{M}_q^{\Sigma^*})}. \quad (1)$$

And this formula is in fact very close to the straightforward generalization of Pearson’s correlation coefficient to the case of three random variables:

$$\hat{\rho}(\mathbf{M}_q^{\Sigma^*}, \mathbf{L}_q^1, \mathbf{L}_q^2) = \frac{\hat{\mathbf{E}}\left(\left(l_i^1 - \hat{\mathbf{E}}(\mathbf{L}_q^1)\right) \cdot \left(l_i^2 - \hat{\mathbf{E}}(\mathbf{L}_q^2)\right) \cdot \left(m_i^{\Sigma^*} - \hat{\mathbf{E}}(\mathbf{M}_q^{\Sigma^*})\right)\right)}{\hat{\sigma}(\mathbf{L}_q^1) \cdot \hat{\sigma}(\mathbf{L}_q^2) \cdot \hat{\sigma}(\mathbf{M}_q^{\Sigma^*})}. \quad (2)$$

The only difference between Equations (1) and (2) is in the denominator, *i.e.* in the standard deviation terms. But as detailed in [10] for first-order DPA, once properly estimated, these terms are all key-independent. Therefore, any difference of effectiveness between an attack using Equations (1) or (2) is only caused by statistical artifacts. Intuitively, these equations provide a very simple explanation of the normalized product combining function. That is, such a combining function will lead to key-dependent models (*i.e.* successful attacks) if the two leakage samples that they mingle are linearly correlated. As illustrated in Figures 10, 11 and 12, this condition is nicely achieved in the case of a Hamming weight leakage function for the two samples l_i^1 and l_i^2 .

4 Evaluating second-order leakage: IT analysis

The previous section highlights that the evaluation of higher-order side-channel attacks is not straightforward to capture. For example:

- Given Pearson’s correlation coefficient as a distinguisher and a Hamming weight leakage function, there exist combination functions for the samples (*e.g.* the sum) that do not lead to successful key recoveries.
- Given Pearson’s coefficient as a distinguisher and the normalized product combination function, there exist leakage functions (*e.g.* with no linear dependencies between the samples) that don’t lead to successful key recoveries.

These observations suggest that the simple situation in the first-order context where the correlation coefficient could (under certain physical assumptions detailed in [10]) be used both as a distinguisher and as a measure of side-channel leakage does not hold here. In the second-order case, this correlation is only a distinguisher (that can only exploit linear dependencies). Hence, it is a typical context in which the evaluation framework of Eurocrypt 2009 [18] is interesting to put into practice. This framework works in two steps:

1. First, an information theoretic analysis is performed, in order to evaluate the physical leakages independently of the adversary who exploits them. When applied to a countermeasure (*e.g.* masking), this step allows to quantify how much the asymptotic security of the device has been improved. In other words, it is an objective measure of the quality of the countermeasure.
2. Second, a security analysis is performed, in order to evaluate how efficiently a particular distinguisher (*e.g.* Pearson’s correlation coefficient with a given combining function) can exploit the available leakage. This step is useful to translate the information theoretic analysis into a “number of measurements required to extract the key” in a given scenario.

In this section, we tackle the first part of the analysis. For this purpose, and in order to compare our conclusions with previous works, we use exactly the same assumptions as [15], *i.e.* a Hamming weight leakage function for the two samples, just as described in Section 3. Following the definitions in [18], we compute:

$$H[S|\mathbf{L}_1^1, \mathbf{L}_1^2] = - \sum_s \Pr[s] \sum_{x_1} \Pr[x_1] \int_{l_1^1} \int_{l_1^2} \Pr[l_1^1, l_1^2 | s, x_1] \log_2 \Pr[s | l_1^1, l_1^2, x_1] dl_1^1 dl_1^2$$

Since the leakage samples are assumed to be normally distributed, this can be quite easily computed in function of the noise standard deviation σ_n . Some simplifications allow to speed up the computations, *e.g.* by observing that only nine distributions are possible, corresponding to the nine Hamming weights of the secret states Σ_i . Also, in order to evaluate the information loss caused by the different combination functions, we similarly evaluated $H[S|C(\mathbf{L}_1^1, \mathbf{L}_1^2)]$. This implies slightly more complex integrals since, *e.g.* the product combining gives rise to mixtures of normal product distributions. Figure 9 in Appendix A illustrates these distributions for two secret states and two σ_n 's. The mutual information values corresponding to these different information leakages (*i.e.* $I(S; (\mathbf{L}_1^1, \mathbf{L}_1^2)) = H[S] - H[S|\mathbf{L}_1^1, \mathbf{L}_1^2]$) are then plotted in Figure 3, in function of the noise variance σ_n^2 (in log scale). From this figure, we can observe:

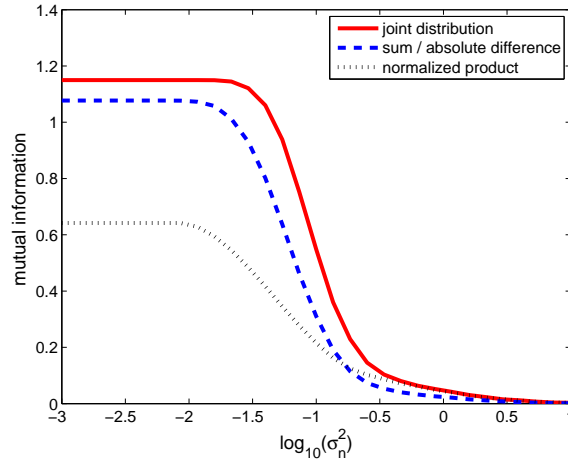


Fig. 3: Information leakage for different combination functions.

1. The best combination function is no combination: dealing directly with the 2-dimensional leakages allows avoiding any additional information loss.
2. The sum and absolute difference combining functions give rise to exactly the same information leakage. This can be understood from the shape of their distributions: the distribution of the absolute difference combining can be seen as the one of the sum combining that has been folded up.

3. For small σ_n^2 , the normalized product is the least informative combining.
4. The respective efficiency of different combining functions varies with the amount of noise. In particular, after a certain noise threshold, the product combining carries more information on S than the sum/absolute difference.

Note that the leakage of the sum combining exactly corresponds to the previous evaluation of [19] in which masking is analyzed in the hardware setting only.

5 Implications for profiled attacks: security analysis (I)

The previous information theoretic analysis is quite intriguing, since it contradicts parts of the prevailing intuitions regarding second-order DPA. For example, it shows that the sum combining function leads to information leakages (as can also be seen from the different pdf in appendix) although they cannot be directly exploited with Pearson’s correlation coefficient. Of course, there is no strong contradiction: Waddle and Wagner showed in [22] how to overcome this limitation by squaring the sum combining. But it raises the question whether these information leakages can be directly exploited (*i.e.* without squaring) by other distinguishers. In order to tackle this question, we apply the second part of the framework in [18], *i.e.* security analysis. In this section, we start with the evaluation of profiled side-channel attacks using templates, for which a strong relation with the previous information theoretic analysis should hold.

The results of various template attacks performed against the same masked AES S-box as in the previous sections are in Figure 4, given for two different noise standard deviations. We mention that these attacks do *not* use Gaussian templates as in [3] but the exact leakage distributions as in the previous information theoretic analysis (*e.g.* attacks using the joint distributions exploit Gaussian mixtures; attacks using the normalized product combining function exploit normal product distributions mixtures, ... as plotted in appendix). The

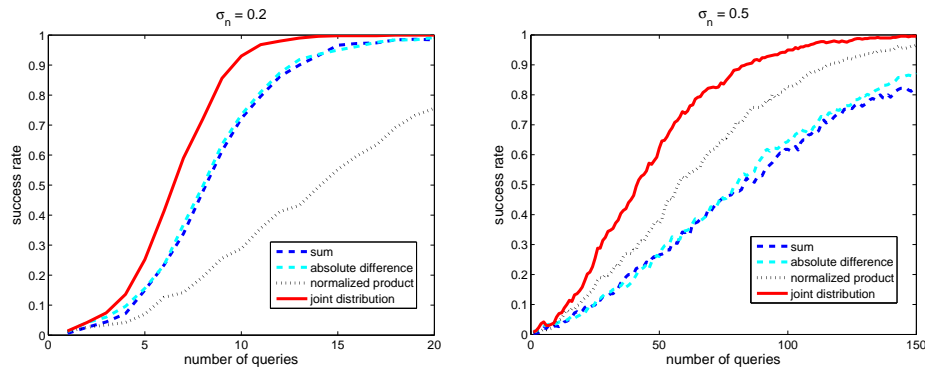


Fig. 4: Success rate of (simulated) profiled attacks against a masked AES S-box.

success rates are computed over 500 independent experiments and nicely confirm the theoretical predictions of Theorem 2 in [18]. First, we see that the sum and absolute difference combining functions lead to the same attack effectiveness in this profiled case (since they give rise to the same information leakages). Second, we see that for $\sigma_n = 0.2$ (*i.e.* left of the intersection in Figure 3, where $\log_{10}(0.2^2) \approx -1.39$), these two combining functions allow more efficient attacks than the normalized product one. By contrast, for $\sigma_n = 0.5$ (*i.e.* right of the intersection in Figure 3 where $\log_{10}(0.5^2) \approx -0.6$), the opposite conclusion holds.

Of course, these experiments are partially artificial since in practice, an adversary who can profile the leakages will generally use the templates based on the joint distribution only¹. However, together with [19], they show that an information theoretic analysis provides an objective evaluation of the quality of a countermeasure against the “best-available” template adversaries in the DPA setting. Consequently, these results emphasize that such an analysis is an important part in the evaluation of side-channel countermeasures. Eventually, our analysis also confirms the conclusions of [12], showing that resistance against sufficiently profiled template attacks cannot be achieved by masking only.

6 Implications for non-profiled attacks: security analysis (II)

The previous section showed that for carefully profiled template attacks, there is a strong connection between the information leakage of a device and the success rate of the adversary. By contrast, we know that in the non-profiled context of correlation attacks, this observation does not hold. Again, a natural question is to check whether this is a general fact, or if there exist non-profiled distinguishers that can be successful in this case. We answer this question positively, using the Mutual Information Analysis (MIA for short) first introduced in [4]. It can be seen as the counterpart of template attacks, in which the leakage distributions are estimated “on-the-fly” rather than prior to the attacks.

The results of correlation and MIA attacks using different combining functions are given in Figure 5, again using the (simulated) setting described in the previous section. In our experiments, MIA estimates the pdf using histograms and the number of bins is selected as proposed in [4]. That is, *e.g.* for MIA exploiting the joint distribution, we use 9 bins corresponding to the 9 Hamming weights of every leakage sample. This means a total of 81 bins for the bivariate distribution. The following observations can be emphasized:

1. In the low noise scenario, MIA with the sum and absolute difference combining functions works best, as similarly observed for template attacks.
2. By contrast, and contrary to template attacks, MIA without combining function (*i.e.* using the joint distribution directly, as in [5, 16]), is not the most

¹ At least, this is the best strategy if the adversary has enough data and time to profile the multivariate leakage pdf. But the complexity of profiling also increases with the number of dimensions which may become an issue in higher-order masking.

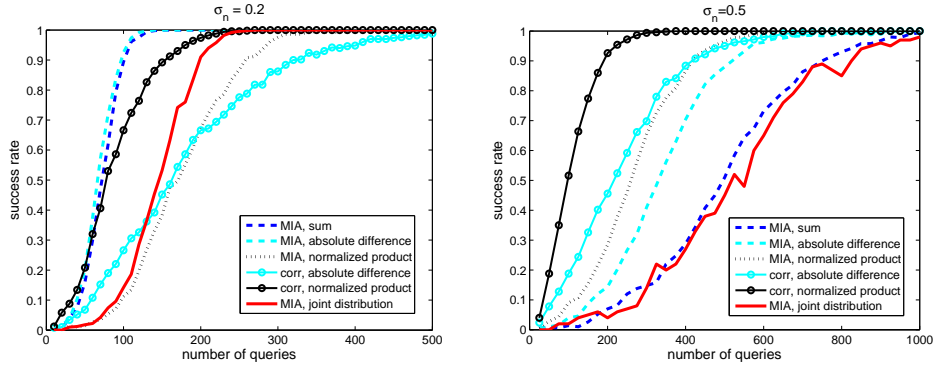


Fig. 5: Success rate of (simulated) non-profiled attacks - masked AES S-box.

efficient solution in our simulations. This is caused by the need to estimate two-dimensional distributions, which turns out to require more data.

3. For similar reasons (*i.e.* also related to the different effectiveness of the “on-the-fly” pdf estimation), when increasing the noise, MIA with the sum and absolute difference combining functions are not equivalent anymore.
4. Finally, attacks using Pearson’s correlation coefficient perform well, specially when combined with the normalized product (which is natural since our simulated leakages perfectly fulfill the requirements of Section 3.1).

Note that all these non-profiled distinguishers show significantly lower efficiencies than the profiled ones in the previous section. This suggests that the problem of estimating the leakage pdf is in fact more difficult than the one of extracting the keys once a good model is available. In other words, the complexity of profiling is higher than the one of online attacks in our example.

7 Experimental results

The previous sections evaluated the impact of masking a substitution box with respect to various side-channel distinguishers, based on simulations. But as for most investigations in the area of physically observable cryptography, it is important to verify that our conclusions reasonably match practical measurements performed against a real chip. For this purpose, we also carried out a set of attacks against a masked implementation of the DES in an 8-bit RISC microcontroller from the Atmel AVR family. Considering the DES (rather than the AES) was motivated by practical facilities. Since the output of the DES S-box is 4-bit wide, it allows easily considering two types of contexts: in a first scenario, the 4 remaining bits on the bus are kept constant; in a second scenario, these 4 bits can be chosen at random, in order to produce some additional algorithmic noise. This is an interesting opportunity since the noise level that was used as a parameter, *e.g.* in Figure 5, is typically fixed in a real experimental setting.

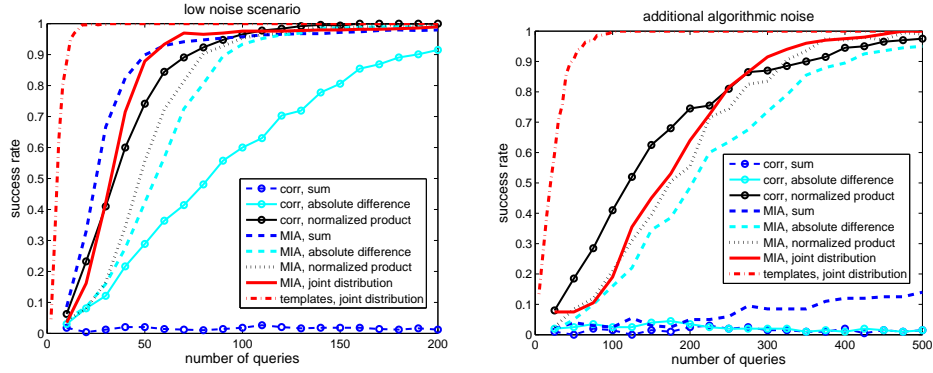


Fig. 6: Success rate of various experimental attacks against a masked DES.

The results in Figure 6 are nicely consistent with our previous simulations. For example, the excellent effectiveness of template attacks is again observed. Also, the good performance of correlation attacks using the normalized product is underlined. And in the high noise scenario, the success rate of MIA using the sum combining is very low. By contrast, we note two interesting differences:

1. MIA using the joint distribution is much more efficient than in the AES case. This is in fact related to the reduced number of bins that the 4-bit DES S-box allows in the pdf estimations (*i.e.* 25 rather than 81).
2. The absolute difference combining function provides very poor results in practice. In our setting, this observation can be explained by the weak resistance of this combination function against incorrect modeling (*i.e.* the fact that the device leakages do not perfectly correspond to Hamming weights).

Summarizing, these experiments confirm the “world is not enough” nature of second-order DPA. The only strong statement that can be made in this context is that an information theoretic metric estimated with perfect templates captures the security against the best possible profiled adversary. As for all the other distinguishers, they highly depend on the actual shape of the leakage pdf and the engineering knowledge that can be exploited when mounting an attack. And contrary to the first-order case discussed in [10], the Gaussian assumption that holds for the leakage samples does hold anymore from the adversary’s point of view (*e.g.* masks typically imply mixtures of Gaussians - or other - distributions).

8 Generalization to higher-orders

In order to improve the security of masking schemes further, the traditional approach is to increase their order. For this purpose, this final section analyzes the cost *vs.* security tradeoff that can be obtained by generalizing the GLUT countermeasure in such a way, and details the second- and third-order cases for

illustration. That is, rather than using one input mask per S-box, we now use two or three masks per S-box. In terms of cost, this implies using one or two additional tables Sbox'' and Sbox''' , as described, *e.g.* in [13]. Conveniently, all the tools used in second-order DPA can be easily generalized to this third- and fourth-order cases. In particular, the information theoretic analysis of Section 4 just requires to integrate over three or four leakage samples l_1^i, l_2^i, l_3^i and possibly l_4^i . Since it captures the worst-case scenario, it is the most revealing technique to evaluate the impact and interest of higher-order masking schemes.

The information leakage of these different masking schemes is represented in Figure 7, in function of the noise variance. On the same plot, we represented the average number of queries to the target device required for a perfectly profiled attack (similar to the ones in Section 5) to reach a success rate of 90%. These figures provide a quantitative insight to the observations in [2], where it is demonstrated that, given a large enough noise variance, the data complexity of a side-channel attack increases exponentially with the amount of shares in the masking scheme. That is, given a noise variance σ_n^2 in the leakage samples and k shares, the data complexity required to attack a masking scheme successfully is proportional to $(\sigma_n^2)^{k/2}$. The linear regions of the (log scale) curves that are observed in the right part of the figure suggest that this expectation is fulfilled in our experiments. Importantly, it also shows that the impact of (higher-order) masking can be extremely small in terms of security increases, for small σ_n^2 's.

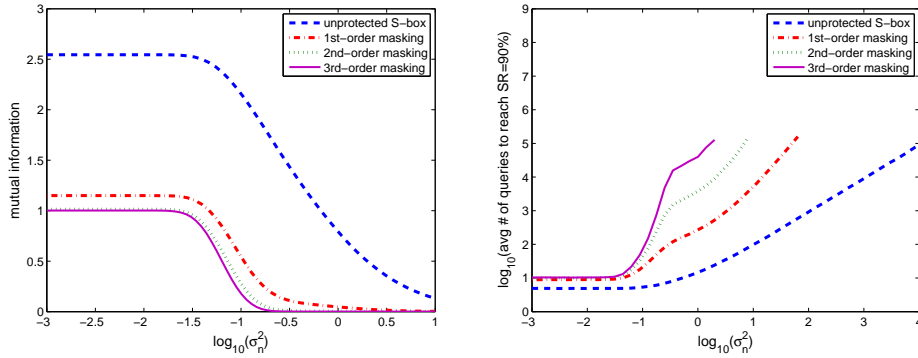


Fig. 7: Information leakage for 0^{th} , 1^{st} and 2^{nd} -order masking.

8.1 A case for the information theoretic metric

Looking at Figure 7, the main question for a designer (or evaluation laboratory) is to best trade the amount of shares and the amount of noise that he has to add to his implementation, in order to reach a certain security level. This is essential since increasing these parameters has a strong impact on the performances of the implementation. Unfortunately, for large security levels, the proper estimation of the number of traces required to reach a certain success rate becomes intensive

(because of statistical sampling issues). Already in simulations, running 1000 attacks, each of them using 10^5 queries is expensive. And when moving to the analysis of real traces (taking much more time to be generated and space to be stored), this limitation becomes even more critical. Interestingly, this is exactly the context where an information theoretic analysis becomes useful. That is, since the mutual information can be estimated from single query leakage vectors \mathbf{L}_1^i , it requires much less data to estimate properly. And following [18], Theorem 2, it should hold that the mutual information $I(S; \mathbf{L}_1^1, \mathbf{L}_1^2, \dots)$ is directly correlated with the number of traces required to reach a certain success rate. As a consequence, we also plotted an estimation of this number, based on the inverse of the mutual information multiplied with a constant factor c . As illustrated in Figure 8, this approximation holds nearly perfectly, with the same constant c for all attacks, essentially depending on the success rate to reach (here 90%). Summarizing, these simulated experiments confirm the relevance of an information theoretic analysis in the context of physical security evaluations.

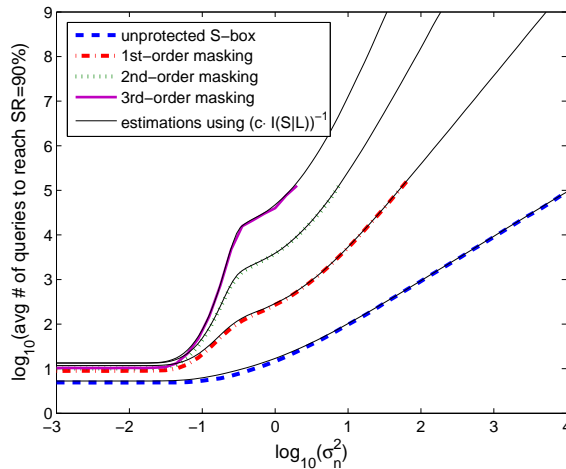


Fig. 8: Information leakage for different combination functions.

Before to conclude, we note again that such an information theoretic analysis only captures the most powerful adversaries for which the profiling of the leakage distributions is perfect. But in practice, the reduction of the information leakages is not the only effect that increases the security in masked implementations. Namely, the pdf estimation of multidimensional distributions may become too complex for allowing the exploitation of all the information in the traces. And the number of (pairs, triples, . . . of) samples to test in the attacks also increases their time complexity considerably (up to N^2 , N^3 , . . .). However, we believe that the formal analysis of a worst-case scenario as in this paper is an important step towards a better understanding of masked implementations.

9 Conclusions

The results in this paper provide a first complete and unifying treatment of higher-order power analysis. They allow putting forward the strengths and weaknesses of various approaches to second-order DPA and provide a sound explanation for them. Our analysis illustrates that in the context of cryptographic devices protected with masking, it is not sufficient to run a single arbitrary distinguisher to quantify the security of an implementation. Evaluations should hold in two steps. First, an information theoretic analysis determines the actual information leakage (*i.e.* the impact of the countermeasure, independently of the adversary). Second, a security analysis determines the efficiency of various distinguishers in exploiting this leakage. By applying such a methodology to simulated and practical experiments, we consequently obtain a fair and comprehensive evaluation of the security increases that a masking scheme can ensure.

While not in contradiction with previous results in the field, these investigations reshape the understanding of certain assumptions and bring new intuitions. First, both theoretical analysis and empirical attacks show a large gap between the efficiency of profiled attacks that best exploit the information from two or more leakage samples and the one of non-profiled attacks that are most frequently used in practice. This relates to the observation that the statistics in side-channel attacks are only used to discriminate secret data (while their natural objective is to allow a good estimation). Hence, the study of advanced pdf estimation techniques in the context of side-channel attacks is an interesting direction for further research, as initiated with the MIA distinguisher in [4].

Second, the security improvement obtained when increasing the order of a masking scheme beyond one is negligible if it is not combined with a sufficient amount of noise in the leakages. This observation relates to the general guideline that side-channel resistance requires the combination of several countermeasures in order to be effective. And the best combination of masking with other available countermeasures is another interesting scope for further research.

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A Probability distributions

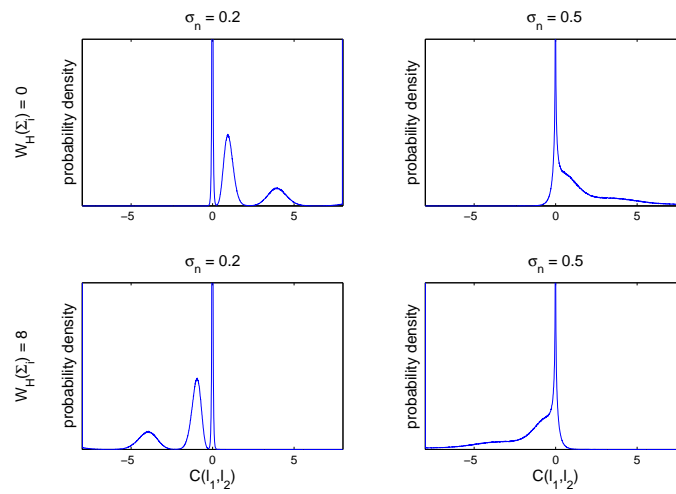


Fig. 9: Leakage probability distributions for the product combining function.

Fig. 10: Leakage probability distributions for three secret states ($W_H(\Sigma_i) = 0, 1, 2$).

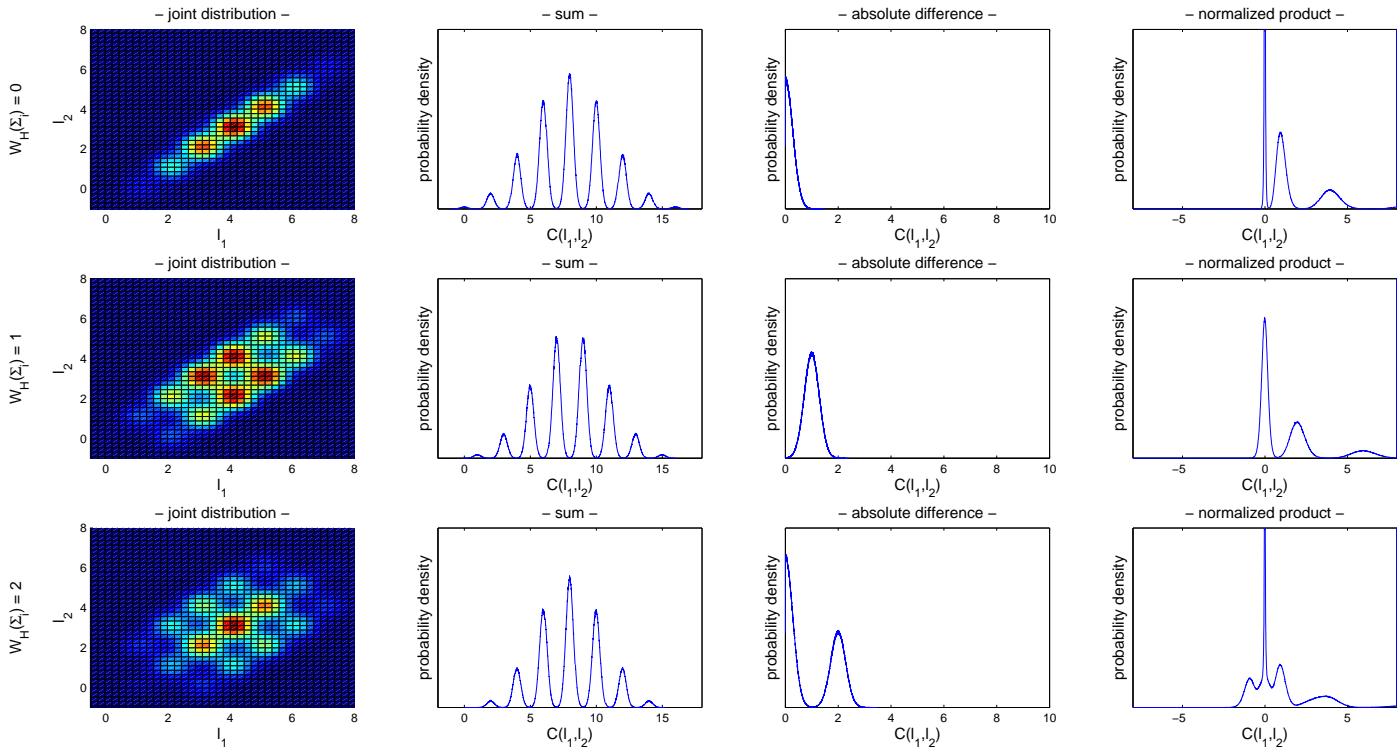


Fig. 11: Leakage probability distributions for three secret states ($W_H(\Sigma_i) = 3, 4, 5$).

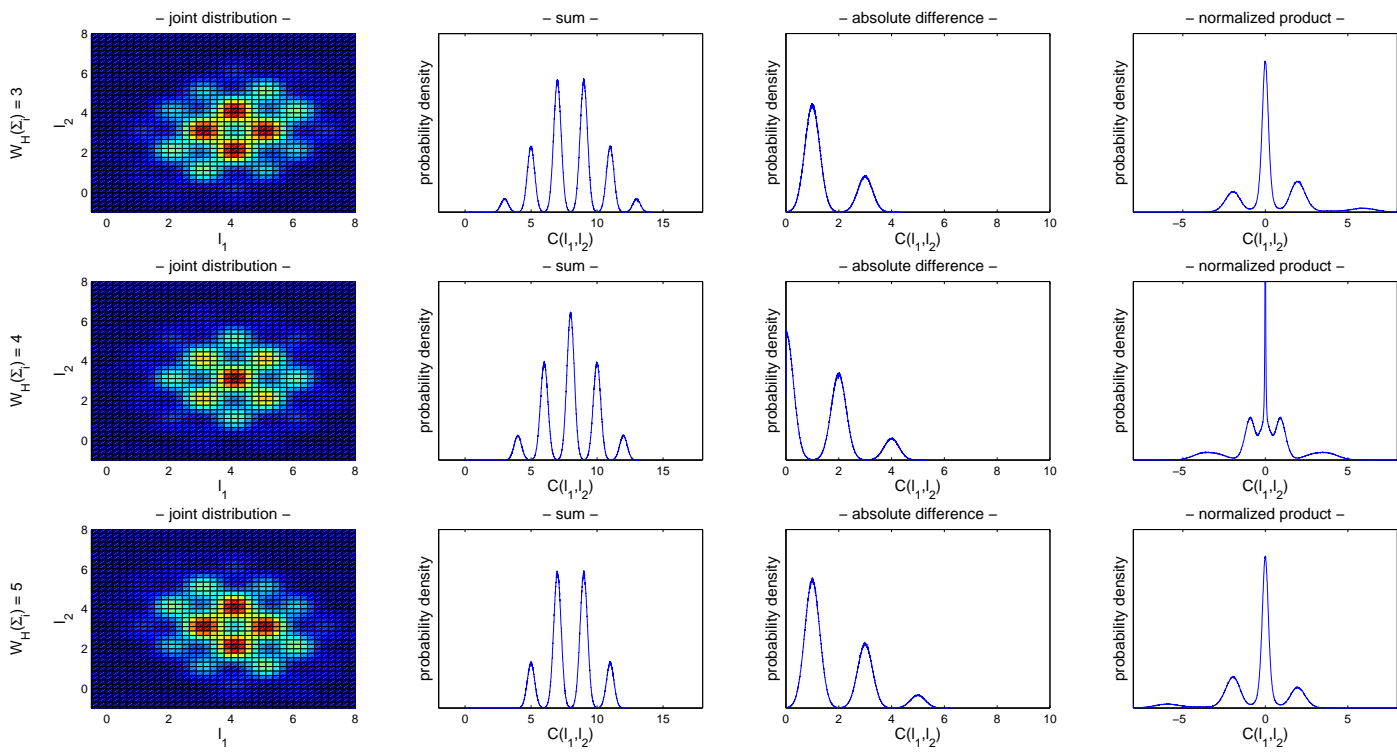


Fig. 12: Leakage probability distributions for three secret states ($W_H(\Sigma_i) = 6, 7, 8$).

