New Methods to Construct Golay Complementary Sequences Over the QAM Constellation

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Abstract. In this paper, based on binary Golay complementary sequences, we propose some methods to construct Golay complementary sequences of length 2^n for integer n, over the M^2-QAM constellation and $2M$-Q-PAM constellations ,where $M = 2^m$ for integer m . A method to judge whether a sequence constructed using the new general offset pairs over the QAM constellation is Golay complementary sequence is proposed. Base on this judging rule, we can construct many new Golay complementary sequences. In particular, we study Golay complementary sequences over 16-QAM constellation and 64-QAM constellation,many new Golay complementary sequences over these constellations have been found.

Key words: Golay Complementary Sequences, Quadrature Amplitude Modulation(QAM), Orthogonal Frequency Division Multiplexing($OFDM$), Quadrature Pulse Amplitude Modulation (Q-PAM).

1 Introduction

Complementary binary sequences were first introduced by Marcel Golay [1] to study problems in infrared multislit spectrometry.Both binary and polyphase Golay sequences have many applications in communications, including peak power control for orthogonal frequency division multiplexing signals, channel estimation, and complementary code code-division multiple access.

Despite many evident advantages of the orthogonal frequency division multiplexing ($OFDM$) modulation technique, a major drawback to $OFDM$ applications is the large peak to mean envelope power ratio ($PMEPR$)of the uncoded $OFDM$ signals. coding techniques are one of the main techniques to reduce $PMEPR$. In 1999, Davis and Jedwab[2] discovered an important link between Golay sequences and Reed-Muller codes, their method of generating binary and nonbinary Golay sequences is known as the GDJ construction. Davis and Jedwab made major progress in attacking the $PMEPR$ problem by coding techniques;

they proposed the coding scheme for $OFDM$ transmission for 2^h-ary PSK modulation to reduce the $PMEPR$. In 2000, Tarokh and Jafarkhani[6] introduced a geometric approach to the offset selection problem for PSK modulation. In 2000, Paterson and Tarokh[5] found the lower bound on the achievable rate of a code of a given length, the minimum Euclidean distance and the maximum peak-to-average power ratio ($PAPR$). 1n 2001, Roβing and Tarokh[7] made significant progress on the construction of complementary sequences for both amplitude and phase modulation. 1n 2003, Chong, Venkataramani, and Tarokh[8] explicitly constructed 16-QAM complementary sequences using cosets of second order Reed-Muller codes by setting up the two coordinates. 1n 2003, Tarokh and Sadjadpour[9] derived the upper bound for the PEP for square M-QAM Golay sequences under the assumption that all the symbols are equiprobable. 1n 2006, Heekwan Lee and Solomon W.Golomb extended the constructions of 16-QAM Golay sequences to 64-QAM constellation using the offsets discovered in [10]. 1n 2008, M. Anand and P. Vijay Kumar[11] studied the low correlation sequences over the QAM constellation. In 2008, Ying Li [15, 16] gave some corrections for the sequence pairing descriptions of 16-QAM and 64-QAM, he proposed two conjectures to describe the new offset pairs and enumerate all known first order offset pairs.

In this paper, we study how to construct M^2-QAM Golay complementary sequences and $2M$-Q-PAM Golay complementary sequences by binary Golay complementary sequences. we extend the techniques constructing 16-QAM Golay sequences and 64-QAM Golay sequences using the offsets discovered in [7, 10]to construct Golay complementary sequences over M^2-QAM constellations and $2M$-Q-PAM constellations. We propose a sufficient condition to judge whether the sequence constructed by binary complementary sequences is Golay complementary sequence. Because 16-QAM and 64-QAM are widely applied in the modern communications, we consider how to construct Golay complementary sequences over 16-QAM and 64-QAM in particular. Some sufficient conditions are given to judge whether a sequence using new offset pairs over 16-QAM constellation and 64-QAM constellation is Golay complementary, many new Golay complementary sequences over 16-QAM and 64-QAM have been found.

2 The Golay Complementary Sequences over M^2-QAM constellation

The M^2-QAM constellation is the set

$$\{a + bj| - (M-1) \leq a, b \leq M-1, a, b\ odd\}.$$

Where $M = 2^m$,and this constellation can alternately be described as

$$\{\sum_{k=0}^{m-1} 2^k(-1)^{a_k^1} + j\sum_{k=0}^{m-1} 2^k(-1)^{a_k^2} | a_k^1, a_k^2 \in Z_2, k = 0, 1, \cdots, m-1\}$$

These representations suggest that binary sequences can be used to construct the Golay complementary sequences over M^2-QAM constellation.

Theorem 1. *Let* $A(x)=\sum_{k=1}^{n-1}x_{\pi(k)}x_{\pi(k+1)}+\sum_{k=1}^{n}c_kx_k+c$,

$a_k^1(x)=A(x)+s_k^1(x)$, $a_k^2(x)=A(x)+s_k^2(x)$,$k=0,1,\cdots,m-1$.

Where $c_k\in Z_2$,$c\in Z_2$, π *is a permutation from* $\{1,2,\cdots,n\}$ *to* $\{1,2,\cdots,n\}$,

$s_k^1(x)$,$s_k^2(x)$ *and* $\mu(x)$ *satisfy the following cases.*

Case I: $s_k^1(x)=d_{k,0}^{(1)}+d_{k,1}^{(1)}x_{\pi(1)}$, $s_k^2(x)=d_{k,0}^{(2)}+d_{k,1}^{(2)}x_{\pi(1)}$,

$\mu(x)=x_{\pi(n)}$.

Case II: $s_k^1(x)=d_{k,0}^{(1)}+d_{k,1}^{(1)}x_{\pi(n)}$, $s_k^2(x)=d_{k,0}^{(2)}+d_{k,1}^{(2)}x_{\pi(n)}$,

$\mu(x)=x_{\pi(1)}$.

Case III: $s_k^1(x)=d_{k,0}^{(1)}+x_{\pi(\omega)}+x_{\pi(\omega+1)}$, $s_k^2(x)=d_{k,0}^{(2)}+x_{\pi(\omega)}+x_{\pi(\omega+1)}$,

with $1\le\omega\le n-1$,

$\mu(x)=x_{\pi(1)}$ *or* $\mu(x)=x_{\pi(n)}$.

Where $d_{k,0}^{(1)},d_{k,0}^{(2)}\in Z_2$, $k=0,1,\cdots,m-1$.

Case IV :

$s_k^1(x)=d_{k,0}^{(1)}+(x_{\pi(\omega_1)}+x_{\pi(\omega_1+1)})+(x_{\pi(\omega_2)}+x_{\pi(\omega_2+1)})+\cdots+(x_{\pi(\omega_l)}+x_{\pi(\omega_l+1)})$,

$s_k^2(x)=d_{k,0}^{(2)}+(x_{\pi(\omega_1)}+x_{\pi(\omega_1+1)})+(x_{\pi(\omega_2)}+x_{\pi(\omega_2+1)})+\cdots+(x_{\pi(\omega_l)}+x_{\pi(\omega_l+1)})$,

with $1\le\omega_1<\omega_1+1<\omega_1+2<\omega_2<\omega_2+1<\omega_2+2<\cdots<\omega_l\le n-1$,

$\mu(x)=x_{\pi(1)}$ *or* $\mu(x)=x_{\pi(n)}$,

where $d_{k,0}^{(1)},d_{k,0}^{(2)}\in Z_2$,$k=0,1$.

Then the M^2*-QAM sequences*

$c_i=\sum_{k=0}^{m-1}2^k(-1)^{a_k^1(i)}+j\sum_{k=0}^{m-1}2^k(-1)^{a_k^2(i)}$,

$d_i=\sum_{k=0}^{m-1}2^k(-1)^{b_k^1(i)}+j\sum_{k=0}^{m-1}2^k(-1)^{b_k^2(i)}$, *with* $M=2^m$,

are Golay complementary sequences.

Proof:

Case I:

$$C_c(\tau) = \sum_{i=0}^{2^n-1-\tau} c_i c_{i+\tau}^*$$

$$= \sum_{i=0}^{2^n-1-\tau}[\sum_{k=0}^{m-1} 2^k(-1)^{a_k^1(i)} + j\sum_{k=0}^{m-1} 2^k(-1)^{a_k^2(i)}]\times$$

$$[\sum_{k=0}^{m-1} 2^k(-1)^{a_k^1(i+\tau)} - j\sum_{k=0}^{m-1} 2^k(-1)^{a_k^2(i+\tau)}]$$

$$= \sum_{k=0}^{m-1} 2^{2k}(C_{a_k^1}(\tau) + C_{a_k^2}(\tau)) + \sum_{k,f,k\neq f} 2^{k+f}(C_{a_k^1,a_f^1}(\tau) + C_{a_k^2,a_f^2}(\tau))$$

$$+j\sum_{k,f} 2^{k+f}(C_{a_k^2,a_f^1}(\tau) - C_{a_k^1,a_f^2}(\tau))$$

Similarly,

$$C_d(\tau) = \sum_{i=0}^{2^n-1-\tau} d_i d_{i+\tau}^*$$

$$= \sum_{k=0}^{m-1} 2^{2k}(C_{b_k^1}(\tau) + C_{b_k^2}(\tau)) + \sum_{k,f,k\neq f} 2^{k+f}(C_{b_k^1,b_f^1}(\tau) + C_{b_k^2,b_f^2}(\tau))$$

$$+j\sum_{k,f} 2^{k+f}(C_{b_k^2,b_f^1}(\tau) - C_{b_k^1,b_f^2}(\tau))$$

For $\tau > 0$,

$$C_c(\tau) + C_d(\tau) = \sum_{k,f,k\neq f} 2^{k+f}(C_{a_k^1,a_f^1}(\tau) + C_{a_k^2,a_f^2}(\tau) + C_{b_k^1,b_f^1}(\tau) + C_{b_k^2,b_f^2}(\tau))$$

$$+j\sum_{k,f} 2^{k+f}(C_{a_k^2,a_f^1}(\tau) + C_{b_k^2,b_f^1}(\tau) - C_{a_k^1,a_f^2}(\tau) - C_{b_k^1,b_f^2}(\tau))$$

$$C_{a_k^1,a_f^1}(\tau) + C_{a_f^1,a_k^1}(\tau) + C_{b_k^1,b_f^1}(\tau) + C_{b_f^1,b_k^1}(\tau)$$

$$= \sum_{i=0}^{2^n-1-\tau}(-1)^{A(i)+d_{k,0}^{(1)}+d_{k,1}^{(1)}(i)_{\pi(1)}+A(i+\tau)+d_{f,0}^{(1)}+d_{f,1}^{(1)}(i+\tau)_{\pi(1)}}$$

$$+\sum_{i=0}^{2^n-1-\tau}(-1)^{A(i)+d_{f,0}^{(1)}+d_{f,1}^{(1)}(i)_{\pi(1)}+A(i+\tau)+d_{k,0}^{(1)}+d_{k,0}^{(1)}(i+\tau)_{\pi(1)}}$$

$$+\sum_{i=0}^{2^n-1-\tau}(-1)^{A(i)+d_{k,0}^{(1)}+d_{k,1}^{(1)}(i)_{\pi(1)}+(i)_{\pi(n)}+A(i+\tau)+d_{f,0}^{(1)}+d_{f,1}^{(1)}(i+\tau)_{\pi(1)}+(i+\tau)_{\pi(n)}}$$

$$+\sum_{i=0}^{2^n-1-\tau}(-1)^{A(i)+d_{f,0}^{(1)}+d_{f,1}^{(1)}(i)_{\pi(1)}+(i)_{\pi(n)}+A(i+\tau)+d_{k,0}^{(1)}+d_{k,1}^{(1)}(i+\tau)_{\pi(1)}+(i+\tau)_{\pi(n)}}$$

$$= \sum_{i=0}^{2^n-1-\tau}(-1)^{A(i)+A(i+\tau)}[(-1)^{d_{k,0}^{(1)}+d_{k,1}^{(1)}(i)_{\pi(1)}+d_{f,0}^{(1)}+d_{f,1}^{(1)}(i+\tau)_{\pi(1)}}$$

$$+(-1)^{d_{f,0}^{(1)}+d_{f,1}^{(1)}(i)_{\pi(1)}+d_{k,0}^{(1)}+d_{k,1}^{(1)}(i+\tau)_{\pi(1)}}]\times[1+(-1)^{(i)_{\pi(n)}+(i+\tau)_{\pi(n)}}]$$

If $(i)_{\pi(n)} \neq (i+\tau)_{\pi(n)}$,*then* $1+(-1)^{(i)_{\pi(n)}+(i+\tau)_{\pi(n)}} = 0$.

If $(i)_{\pi(n)} = (i+\tau)_{\pi(n)}$,*Let* ν *denote the largest index for which* $(i)_{\pi(\nu)} \neq (i+$

$\tau)_{\pi(\nu)}$,then $(i)_{\pi(k)} = (i+\tau)_{\pi(k)}$,$\nu < k \le n$. Let i' denote indexes whose binary representations differ from those of i only at position $\pi(\nu+1)$. Similar to the Proof in [1], we obtain $(-1)^{A(i)+A(i+\tau)} = -(-1)^{A(i')+A(i'+\tau)}$. Obviously,$\nu+1 \ne 1$, then

$$(-1)^{d_{k,0}^{(1)}+d_{k,1}^{(1)}(i)_{\pi(1)}+d_{f,0}^{(1)}+d_{f,1}^{(1)}(i+\tau)_{\pi(1)}} + (-1)^{d_{f,0}^{(1)}+d_{f,1}^{(1)}(i)_{\pi(1)}+d_{k,0}^{(1)}+d_{k,1}^{(1)}(i+\tau)_{\pi(1)}}$$
$$= (-1)^{d_{k,0}^{(1)}+d_{k,1}^{(1)}(i')_{\pi(1)}+d_{f,0}^{(1)}+d_{f,1}^{(1)}(i'+\tau)_{\pi(1)}} + (-1)^{d_{f,0}^{(1)}+d_{f,1}^{(1)}(i')_{\pi(1)}+d_{k,0}^{(1)}+d_{k,1}^{(1)}(i'+\tau)_{\pi(1)}}$$

Thus,

$$C_{a_k^1,a_f^1}(\tau) + C_{a_f^1,a_k^1}(\tau) + C_{b_k^1,b_f^1}(\tau) + C_{b_f^1,b_k^1}(\tau) = 0$$

Similarly,we have,

$$C_{a_k^2,a_f^1}(\tau) + C_{b_k^2,b_f^1}(\tau) - C_{a_k^1,a_f^2}(\tau) - C_{b_k^1,b_f^2}(\tau)$$
$$= \sum_{i=0}^{2^n-1-\tau}(-1)^{A(i)+d_{k,0}^{(2)}+d_{k,1}^{(2)}(i)_{\pi(1)}+A(i+\tau)+d_{f,0}^{(1)}+d_{f,1}^{(1)}(i+\tau)_{\pi(1)}}$$
$$- \sum_{i=0}^{2^n-1-\tau}(-1)^{A(i)+d_{k,0}^{(1)}+d_{k,1}^{(1)}(i)_{\pi(1)}+A(i+\tau)+d_{f,0}^{(2)}+d_{f,0}^{(2)}(i+\tau)_{\pi(1)}}$$
$$+ \sum_{i=0}^{2^n-1-\tau}(-1)^{A(i)+d_{k,0}^{(2)}+d_{k,1}^{(2)}(i)_{\pi(1)}+(i)_{\pi(n)}+A(i+\tau)+d_{f,0}^{(1)}+d_{f,1}^{(1)}(i+\tau)_{\pi(1)}+(i+\tau)_{\pi(n)}}$$
$$- \sum_{i=0}^{2^n-1-\tau}(-1)^{A(i)+d_{k,0}^{(1)}+d_{k,1}^{(1)}(i)_{\pi(1)}+(i)_{\pi(n)}+A(i+\tau)+d_{f,0}^{(2)}+d_{f,1}^{(2)}(i+\tau)_{\pi(1)}+(i+\tau)_{\pi(n)}}$$
$$= \sum_{i=0}^{2^n-1-\tau}(-1)^{A(i)+A(i+\tau)}[(-1)^{d_{k,0}^{(2)}+d_{k,1}^{(2)}(i)_{\pi(1)}+d_{f,0}^{(1)}+d_{f,1}^{(1)}(i+\tau)_{\pi(1)}}$$
$$-(-1)^{d_{k,0}^{(1)}+d_{k,1}^{(1)}(i)_{\pi(1)}+d_{f,0}^{(2)}+d_{f,1}^{(2)}(i+\tau)_{\pi(1)}}] \times [1+(-1)^{(i)_{\pi(n)}+(i+\tau)_{\pi(n)}}]$$
$$= 0$$

Thus,

$$C_c(\tau) + C_d(\tau) = 0$$

Case II:*The proof is similar to the proof in the case I.*

Case III:*In the Case III, we have $(i)_{\pi(\nu)} + (i+\tau)_{\pi(\nu)} = 1$,*

$(i)_{\pi(\nu+1)} = (i+\tau)_{\pi(\nu+1)} = 1 + (i')_{\pi(\nu+1)} = 1 + (i'+\tau)_{\pi(\nu+1)}$

It is easy to verify the following three equations.

$$
\begin{aligned}
&(-1)^{d_{k,0}^{(1)}+(i_{\pi(\omega)}+i_{\pi(\omega+1)})+d_{f,0}^{(1)}+((i+\tau)_{\pi(\omega)}+(i+\tau)_{\pi(\omega+1)})} \\
&+(-1)^{d_{f,0}^{(1)}+(i_{\pi(\omega)}+i_{\pi(\omega+1)})+d_{k,0}^{(1)}+((i+\tau)_{\pi(\omega)}+(i+\tau)_{\pi(\omega+1)})} \\
= \quad &(-1)^{d_{k,0}^{(1)}+(i'_{\pi(\omega)}+i'_{\pi(\omega+1)})+d_{f,0}^{(1)}+((i'+\tau)_{\pi(\omega)}+(i'+\tau)_{\pi(\omega+1)})} \\
&+(-1)^{d_{f,0}^{(1)}+(i'_{\pi(\omega)}+i'_{\pi(\omega+1)})+d_{k,0}^{(1)}+((i'+\tau)_{\pi(\omega)}+(i'+\tau)_{\pi(\omega+1)})}
\end{aligned}
$$

$$
\begin{aligned}
&(-1)^{d_{k,0}^{(2)}+(i_{\pi(\omega)}+i_{\pi(\omega+1)})+d_{k,0}^{(1)}+((i+\tau)_{\pi(\omega)}+(i+\tau)_{\pi(\omega+1)})} \\
&-(-1)^{d_{k,0}^{(1)}+(i_{\pi(\omega)}+i_{\pi(\omega+1)})+d_{k,0}^{(2)}+((i+\tau)_{\pi(\omega)}+(i+\tau)_{\pi(\omega+1)})} \\
= 0 &
\end{aligned}
$$

$$
\begin{aligned}
&(-1)^{d_{k,0}^{(2)}+(i_{\pi(\omega)}+i_{\pi(\omega+1)})+d_{f,0}^{(1)}+((i+\tau)_{\pi(\omega)}+(i+\tau)_{\pi(\omega+1)})} \\
&-(-1)^{d_{k,0}^{(1)}+(i_{\pi(\omega)}+i_{\pi(\omega+1)})+d_{f,0}^{(2)}+((i+\tau)_{\pi(\omega)}+(i+\tau)_{\pi(\omega+1)})} \\
&+(-1)^{d_{f,0}^{(2)}+(i_{\pi(\omega)}+i_{\pi(\omega+1)})+d_{k,0}^{(1)}+((i+\tau)_{\pi(\omega)}+(i+\tau)_{\pi(\omega+1)})} \\
&-(-1)^{d_{f,0}^{(1)}+(i_{\pi(\omega)}+i_{\pi(\omega+1)})+d_{k,0}^{(2)}+((i+\tau)_{\pi(\omega)}+(i+\tau)_{\pi(\omega+1)})} \\
&= 0
\end{aligned}
$$

$$
\begin{aligned}
&C_{a_k^1,a_f^1}(\tau) + C_{a_f^1,a_k^1}(\tau) + C_{b_k^1,b_f^1}(\tau) + C_{b_f^1,b_k^1}(\tau) = 0 \\
&C_{a_k^2,a_k^1}(\tau) + C_{b_k^2,b_k^1}(\tau) - C_{a_k^1,a_k^2}(\tau) - C_{b_k^1,b_k^2}(\tau) = 0 \\
&C_{a_k^2,a_f^1}(\tau) + C_{b_k^2,b_f^1}(\tau) - C_{a_k^1,a_f^2}(\tau) - C_{b_k^1,b_f^2}(\tau) \\
&\quad + C_{a_f^2,a_k^1}(\tau) + C_{b_f^2,b_k^1}(\tau) - C_{a_f^1,a_k^2}(\tau) - C_{b_f^1,b_k^2}(\tau) = 0, \\
&k \neq f.
\end{aligned}
$$

Thus,

$$C_c(\tau) + C_d(\tau) = 0$$

Case IV: *the proof is similar to the proof in the case III.*

3 The Golay Complementary Sequences over *Q-PAM* constellation

The class of Q-PAM constellation is the subset of the M^2-QAM constellation of size $2M = 2^{m+1}$ having representation

$\{[(-1)^{a_0^1} + j(-1)^{a_0^2}](1 + \sum_{k=1}^{m-1} 2^k(-1)^{a_k}) | a_0^1, a_0^2 \in Z_2, a_k \in Z_2, k = 1, 2, \cdots, m - 1\}$

These representations suggest that binary sequences be used to construct the Golay complementary sequences over $2M$-Q-PAM constellations.

Theorem 2. *Let* $A(x) = \sum_{k=1}^{n-1} x_{\pi(k)} x_{\pi(k+1)} + \sum_{k=1}^{n} c_k x_k + c,$

$$a_0^1(x) = A(x) + s_0^1(x), a_0^2(x) = A(x) + s_0^2(x),$$

$$b_0^1(x) = A(x) + s_0^1(x) + \mu(x), b_0^2(x) = A(x) + s_0^2(x) + \mu(x)$$

Where $c_k \in Z_2$, $c \in Z_2, \pi$ *is a permutation from* $\{1, 2, \cdots, n\}$ *to* $\{1, 2, \cdots, n\}$,

$s_k^1(x), s_k^2(x)$ *and* $\mu(x)$ *satisfy the following cases.*

Case I: $s_0^1(x) = d_{1,0}^{(0)} + d_{1,1}^{(0)} x_{\pi(1)}, s_0^2(x) = d_{2,0}^{(0)} + d_{2,1}^{(0)} x_{\pi(1)},$

$$s^{(k)}(x) = d_0^{(k)} + d_1^{(k)} x_{\pi(1)}, k = 1, \cdots, m - 1,$$

$$\mu(x) = x_{\pi(n)}.$$

Case II: $s_0^1(x) = d_{1,0}^{(0)} + d_{1,1}^{(0)} x_{\pi(n)}, s_0^2(x) = d_{2,0}^{(0)} + d_{2,1}^{(0)} x_{\pi(n)},$

$$s^{(k)}(x) = d_0^{(k)} + d_1^{(k)} x_{\pi(n)}, k = 1, \cdots, m - 1,$$

$$\mu(x) = x_{\pi(1)}.$$

Case III: $s_0^0(x) = x_{\pi(\omega_1)} + x_{\pi(\omega_1+1)}, s_0^1(x) = 1 + x_{\pi(\omega_1)} + x_{\pi(\omega_1+1)},$

$s^{(k)}(x) = d_0^{(k)} + (x_{\pi(\omega_2)} + x_{\pi(\omega_2+1)}) + \cdots + (x_{\pi(\omega_l)} + x_{\pi(\omega_l+1)}),$

$k = 1, \cdots, m-1, 1 \le \omega_1 < \omega_1 + 1 < \omega_1 + 2 < \cdots < \omega_l < \omega_l + 1 < \omega_l + 2 \le n - 1,$

$\mu(x) = x_{\pi(1)}$ *or* $\mu(x) = x_{\pi(n)}$.

Where $d_0^{(k)}, d_1^{(k)} \in Z_2$ $, k = 1, \cdots, m - 1.$

Then the $2M$-Q-PAM *sequences*

$c(x)=\sqrt{2j}[(-1)^{a_0^1(x)}+j(-1)^{a_0^2(x)}](1+\sum_{k=1}^{m-1}2^k(-1)^{s^{(k)}(x)})$,

$d(x)=\sqrt{2j}[(-1)^{b_0^1(x)}+j(-1)^{b_0^2(x)}](1+\sum_{k=1}^{m-1}2^k(-1)^{s^{(k)}(x)})$ *,where* $M=2^m$

are Golay complementary sequences.

The proof of the theorem 2 is similar to the proof of the theorem 1.

4 Golay Complementary Sequences over QAM constellation Constructed by New Offset Pairs

In the following, we consider the problem how to construct Golay complementary sequences over QAM constellation by general offset pairs. A sufficient condition is proposed to judge whether a sequence constructed by general offset pairs over QAM constellation is Golay complementary sequence.

Let $A(x)=\sum_{k=1}^{n-1}x_{\pi(k)}x_{\pi(k+1)}+\sum_{k=1}^{n}c_kx_k+c$,

$a_k^1(x)=A(x)+s_k^1(x)$,

$a_k^2(x)=A(x)+s_k^2(x)$,

$b_k^1(x)=A(x)+a_k^1(x)+\mu(x)$,

$b_k^2(x)=A(x)+a_k^2(x)+\mu(x)$,

Where $c_l\in Z_2, l=1,2,\cdots,n$, $c\in Z_2$,π is a permutation from $\{1,2,\cdots,n\}$

to $\{1,2,\cdots,n\}$,$s_k^1(x), s_k^2(x)$ and $\mu(x)$ satisfy the following cases.

$s_k^1(x)=d_{k,0}^{(1)}+d_{k,1}^{(1)}x_{\pi(\omega_1)}+d_{k,2}^{(1)}x_{\pi(\omega_2)}+\cdots+d_{k,l}^{(1)}x_{\pi(\omega_l)}$,

$s_k^2(x)=d_{k,0}^{(2)}+d_{k,1}^{(2)}x_{\pi(\omega_1)}+d_{k,2}^{(2)}x_{\pi(\omega_2)}+\cdots+d_{k,l}^{(2)}x_{\pi(\omega_l)}$,

$\mu(x)=x_{\pi(1)}$ or $\mu(x)=x_{\pi(n)}$,

where $d_{k,j}^{(1)}, d_{k,j}^{(2)}\in Z_2, k=0,1,\cdots,m-1,\ j=0,1,\cdots,l.$

We construct M^2-QAM sequences

$c_i=\sum_{k=0}^{m-1}2^k(-1)^{a_k^1(i)}+j\sum_{k=0}^{m-1}2^k(-1)^{a_k^2(i)}$,

$d_i=\sum_{k=0}^{m-1}2^k(-1)^{b_k^1(i)}+j\sum_{k=0}^{m-1}2^k(-1)^{b_k^2(i)}$, where $M=2^m$.

The following equations can be checked easily.

$$C_{a_k^1,a_f^1}(\tau)+C_{a_f^1,a_k^1}(\tau)+C_{b_k^1,b_f^1}(\tau)+C_{b_f^1,b_k^1}(\tau)$$
$$=\sum_{i=0}^{2^n-1-\tau}(-1)^{A(i)+A(i+\tau)}[(-1)^{s_k^1(i)+s_f^1(i+\tau)}+(-1)^{s_f^1(i)+s_k^1(i+\tau)}]$$
$$\times[1+(-1)^{i_{\pi(n)}+(i+\tau)_{\pi(n)}}]$$

$$\sum_{k,f,k\neq f}2^{k+f}(C_{a_k^1,a_f^1}(\tau)+C_{a_k^2,a_f^2}(\tau)+C_{b_k^1,b_f^1}(\tau)+C_{b_k^2,b_f^2}(\tau))$$
$$=\sum_{i=0}^{2^n-1-\tau}\sum_{k,f,k\neq f}2^{k+f}(-1)^{A(i)+A(i+\tau)}[(-1)^{s_k^1(i)+s_f^1(i+\tau)}+(-1)^{s_k^2(i)+s_f^2(i+\tau)}]$$
$$\times[1+(-1)^{i_{\pi(n)}+(i+\tau)_{\pi(n)}}]$$
$$=\sum_{i=0}^{2^n-1-\tau}(-1)^{A(i)+A(i+\tau)}\sum_{k,f,k\neq f}2^{k+f}[(-1)^{s_k^1(i)+s_f^1(i+\tau)}+(-1)^{s_k^2(i)+s_f^2(i+\tau)}]$$
$$\times[1+(-1)^{i_{\pi(n)}+(i+\tau)_{\pi(n)}}]$$

$$C_{a_k^2,a_f^1}(\tau)+C_{b_k^2,b_f^1}(\tau)-C_{a_k^1,a_f^2}(\tau)-C_{b_k^1,b_f^2}(\tau)$$
$$=\sum_{i=0}^{2^n-1-\tau}(-1)^{A(i)+A(i+\tau)}[(-1)^{s_k^2(i)+s_f^1(i+\tau)}-(-1)^{s_k^1(i)+s_f^2(i+\tau)}]$$
$$\times[1+(-1)^{i_{\pi(n)}+(i+\tau)_{\pi(n)}}]$$

$$\sum_{k,f}2^{k+f}(C_{a_k^2,a_f^1}(\tau)+C_{b_k^2,b_f^1}(\tau)-C_{a_k^1,a_f^2}(\tau)-C_{b_k^1,b_f^2}(\tau))$$
$$=\sum_{i=0}^{2^n-1-\tau}(-1)^{A(i)+A(i+\tau)}\sum_{k,f}2^{k+f}[(-1)^{s_k^2(i)+s_f^1(i+\tau)}-(-1)^{s_k^1(i)+s_f^2(i+\tau)}]$$
$$\times[1+(-1)^{i_{\pi(n)}+(i+\tau)_{\pi(n)}}]$$

Similar to proof of theorem 1,we obtain.
If $(i)_{\pi(n)}=(i+\tau)_{\pi(n)}$,Let ν denote the largest index for which $(i)_{\pi(\nu)}\neq(i+\tau)_{\pi(\nu)}$,then $(i)_{\pi(k)}=(i+\tau)_{\pi(k)}$,$\nu<k\leq n$. Let i' denote indexes whose binary representations differ from those of i only at position $\pi(\nu+1)$. Similar to the Proof in[1], we obtain

$$(-1)^{A(i)+A(i+\tau)}=-(-1)^{A(i')+A(i'+\tau)}$$

Thus,if

$$\sum_{k,f,k\neq f}2^{k+f}[(-1)^{s_k^1(i)+s_f^1(i+\tau)}+(-1)^{s_k^2(i)+s_f^2(i+\tau)}]$$

$= \sum_{k,f,k\neq f} 2^{k+f}[(-1)^{s_k^1(i')+s_f^1(i'+\tau)} + (-1)^{s_k^2(i')+s_f^2(i'+\tau)}]$

$\sum_{k,f} 2^{k+f}[(-1)^{s_k^2(i)+s_f^1(i+\tau)} - (-1)^{s_k^1(i)+s_f^2(i+\tau)}]$

$= \sum_{k,f} 2^{k+f}[(-1)^{s_k^2(i')+s_f^1(i'+\tau)} - (-1)^{s_k^1(i')+s_f^2(i'+\tau)}]$

Then the above constructed sequence pairs are Golay complementary pairs over M^2-QAM constellation.

Because sequences over 16-QAM constellation and 64-QAM constellation have very important practical value, in the following, we will study how to construct new Golay complementary sequences over 16-QAM and 64-QAM constellation.

5 New Golay Complementary Sequences over 16-QAM constellation

In the following, we will find some new Golay complementary sequences with two cases of new proposed offset pairs called case V and case VI respectively over 16-QAM constellation.

Let $A(x) = \sum_{l=1}^{n-1} x_{\pi(l)}x_{\pi(l+1)} + \sum_{l=1}^{n} c_l x_l + c$,

$a_k^1(x) = A(x) + s_k^1(x), a_k^2(x) = A(x) + s_k^2(x)$,

$b_k^1(x) = A(x) + s_k^1(x) + \mu(x), b_k^2(x) = A(x) + s_k^2(x) + \mu(x)$,

Where $c_l \in Z_2$,$l = 1, 2, \cdots, n$,$c \in Z_2$,π is a permutation from $\{1, 2, \cdots, n\}$

to $\{1, 2, \cdots, n\}$,$s_k^1(x)$, $s_k^2(x)$ and $\mu(x)$ satisfy the following cases.

Case V:

$s_k^1(x) = d_{k,0}^{(1)} + d_{k,1}^{(1)}x_{\pi(\omega_1)} + d_{k,2}^{(1)}x_{\pi(\omega_2)} + d_{k,3}^{(1)}x_{\pi(\omega_3)} + d_{k,4}^{(1)}x_{\pi(\omega_4)}$

$s_k^2(x) = d_{k,0}^{(2)} + d_{k,1}^{(2)}x_{\pi(\omega_1)} + d_{k,2}^{(2)}x_{\pi(\omega_2)} + d_{k,3}^{(2)}x_{\pi(\omega_3)} + d_{k,4}^{(2)}x_{\pi(\omega_4)}$,$k = 0, 1$.

Where $1 \leq \omega_1 < \omega_1 + 1 < \omega_2 < \omega_2 + 1 < \omega_3 < \omega_3 + 1 < \omega_4 \leq n$,

$\mu(x) = x_{\pi(1)}$ or $\mu(x) = x_{\pi(n)}$.

Case VI:

$s_k^1(x) = d_{k,0}^{(1)} + d_{k,1}^{(1)}x_{\pi(\omega)} + d_{k,2}^{(1)}x_{\pi(\omega+1)}$,

$s_k^2(x) = d_{k,0}^{(2)} + d_{k,1}^{(2)} x_{\pi(\omega)} + d_{k,2}^{(2)} x_{\pi(\omega+1)}$.

$\mu(x) = x_{\pi(1)}$ or $\mu(x) = x_{\pi(n)}$.

Where $d_{k,i}^{(j)} \in Z_2$,$k = 0, 1$,$i = 0, 1, 2$,$j = 1, 2$.

In the following, we will research what conditions the coefficients

$\{(d_{k,0}^{(1)}, d_{k,1}^{(1)}, d_{k,2}^{(1)}, d_{k,3}^{(1)}, d_{k,4}^{(1)}, d_{k,0}^{(2)}, d_{k,1}^{(2)}, d_{k,2}^{(2)}, d_{k,3}^{(2)}, d_{k,4}^{(2)}), k = 0, 1\}$

or $\{(d_{k,0}^{(1)}, d_{k,1}^{(1)}, d_{k,2}^{(1)}, d_{k,0}^{(2)}, d_{k,1}^{(2)}, d_{k,2}^{(2)}), k = 0, 1\}$ must satisfy ,so that the sequences

$c_i = \sum_{k=0}^{1} 2^k (-1)^{a_k^1(i)} + j \sum_{k=0}^{1} 2^k (-1)^{a_k^2(i)}$,

$d_i = \sum_{k=0}^{1} 2^k (-1)^{b_k^1(i)} + j \sum_{k=0}^{1} 2^k (-1)^{b_k^2(i)}$

are Golay complementary sequences.

Firstly,we consider **case V**.

The following condition can be proved easily.

$$\begin{aligned}
&\sum_{k,f,k\neq f} 2^{k+f}[(-1)^{s_k^1(i)+s_f^1(i+\tau)} + (-1)^{s_k^2(i)+s_f^2(i+\tau)}] \\
&= 2\times\{(-1)^{s_0^1(i)+s_1^1(i+\tau)} + (-1)^{s_1^1(i)+s_0^1(i+\tau)} + (-1)^{s_0^2(i)+s_1^2(i+\tau)} + (-1)^{s_1^2(i)+s_0^2(i+\tau)}\} \\
&= 2\times\{[(-1)^{d_{0,0}^{(1)}+d_{0,1}^{(1)} i_{\pi_{\omega_1}} + d_{0,2}^{(1)} i_{\pi_{\omega_2}} + d_{0,3}^{(1)} i_{\pi_{\omega_3}} + d_{0,4}^{(1)} i_{\pi_{\omega_4}}} \\
&\qquad \times(-1)^{d_{1,0}^{(1)}+d_{1,1}^{(1)}(i+\tau)_{\pi_{\omega_1}} + d_{1,2}^{(1)}(i+\tau)_{\pi_{\omega_2}} + d_{1,3}^{(1)}(i+\tau)_{\pi_{\omega_3}} + d_{1,4}^{(1)}(i+\tau)_{\pi_{\omega_4}}}] \\
&\quad +[(-1)^{d_{1,0}^{(1)}+d_{1,1}^{(1)} i_{\pi_{\omega_1}} + d_{1,2}^{(1)} i_{\pi_{\omega_2}} + d_{1,3}^{(1)} i_{\pi_{\omega_3}} + d_{1,4}^{(1)} i_{\pi_{\omega_4}}} \\
&\qquad \times(-1)^{d_{0,0}^{(1)}+d_{0,1}^{(1)}(i+\tau)_{\pi_{\omega_1}} + d_{0,2}^{(1)}(i+\tau)_{\pi_{\omega_2}} + d_{0,3}^{(1)}(i+\tau)_{\pi_{\omega_3}} + d_{0,4}^{(1)}(i+\tau)_{\pi_{\omega_4}}}] \\
&\quad +[(-1)^{d_{0,0}^{(2)}+d_{0,1}^{(2)} i_{\pi_{\omega_1}} + d_{0,2}^{(2)} i_{\pi_{\omega_2}} + d_{0,3}^{(2)} i_{\pi_{\omega_3}} + d_{0,4}^{(2)} i_{\pi_{\omega_4}}} \\
&\qquad \times(-1)^{d_{1,0}^{(2)}+d_{1,1}^{(2)}(i+\tau)_{\pi_{\omega_1}} + d_{1,2}^{(2)}(i+\tau)_{\pi_{\omega_2}} + d_{1,3}^{(2)}(i+\tau)_{\pi_{\omega_3}} + d_{1,4}^{(2)}(i+\tau)_{\pi_{\omega_4}}}] \\
&\quad +[(-1)^{d_{1,0}^{(2)}+d_{1,1}^{(2)} i_{\pi_{\omega_1}} + d_{1,2}^{(2)} i_{\pi_{\omega_2}} + d_{1,3}^{(2)} i_{\pi_{\omega_3}} + d_{1,4}^{(2)} i_{\pi_{\omega_4}}} \\
&\qquad \times(-1)^{d_{0,0}^{(2)}+d_{0,1}^{(2)}(i+\tau)_{\pi_{\omega_1}} + d_{0,2}^{(2)}(i+\tau)_{\pi_{\omega_2}} + d_{0,3}^{(2)}(i+\tau)_{\pi_{\omega_3}} + d_{0,4}^{(2)}(i+\tau)_{\pi_{\omega_4}}}]\ \}
\end{aligned}$$

$$
\begin{aligned}
&\sum_{k,f} 2^{k+f}[(-1)^{s_k^2(i)+s_f^1(i+\tau)} - (-1)^{s_k^1(i)+s_f^2(i+\tau)}] \\
&= \{(-1)^{s_0^2(i)+s_0^1(i+\tau)} - (-1)^{s_0^1(i)+s_0^2(i+\tau)}\} \\
&\quad +2\times\{(-1)^{s_0^2(i)+s_1^1(i+\tau)}+(-1)^{s_1^2(i)+s_0^1(i+\tau)}-(-1)^{s_0^1(i)+s_1^2(i+\tau)}-(-1)^{s_1^1(i)+s_0^2(i+\tau)}\} \\
&\quad +4\times\{(-1)^{s_1^2(i)+s_1^1(i+\tau))} - (-1)^{s_1^1(i)+s_1^2(i+\tau)}\} \\
&= \{\ [(-1)^{d_{0,0}^{(2)}+d_{0,1}^{(2)}i_{\pi_{\omega_1}}+d_{0,2}^{(2)}i_{\pi_{\omega_2}}+d_{0,3}^{(2)}i_{\pi_{\omega_3}}+d_{0,4}^{(2)}i_{\pi_{\omega_4}}} \\
&\qquad \times(-1)^{d_{0,0}^{(1)}+d_{0,1}^{(1)}(i+\tau)_{\pi_{\omega_1}}+d_{0,2}^{(1)}(i+\tau)_{\pi_{\omega_2}}+d_{0,3}^{(1)}(i+\tau)_{\pi_{\omega_3}}+d_{0,4}^{(1)}(i+\tau)_{\pi_{\omega_4}}}] \\
&\quad -\ [(-1)^{d_{0,0}^{(1)}+d_{0,1}^{(1)}i_{\pi_{\omega_1}}+d_{0,2}^{(1)}i_{\pi_{\omega_2}}+d_{0,3}^{(1)}i_{\pi_{\omega_3}}+d_{0,4}^{(1)}i_{\pi_{\omega_4}}} \\
&\qquad \times(-1)^{d_{0,0}^{(2)}+d_{0,1}^{(2)}(i+\tau)_{\pi_{\omega_1}}+d_{0,2}^{(2)}(i+\tau)_{\pi_{\omega_2}}+d_{0,3}^{(2)}(i+\tau)_{\pi_{\omega_3}}+d_{0,4}^{(2)}(i+\tau)_{\pi_{\omega_4}}}]\ \}+ \\
&2\times\{\ [(-1)^{d_{0,0}^{(2)}+d_{0,1}^{(2)}i_{\pi_{\omega_1}}+d_{0,2}^{(2)}i_{\pi_{\omega_2}}+d_{0,3}^{(2)}i_{\pi_{\omega_3}}+d_{0,4}^{(2)}i_{\pi_{\omega_4}}} \\
&\qquad \times(-1)^{d_{1,0}^{(1)}+d_{1,1}^{(1)}(i+\tau)_{\pi_{\omega_1}}+d_{1,2}^{(1)}(i+\tau)_{\pi_{\omega_2}}+d_{1,3}^{(1)}(i+\tau)_{\pi_{\omega_3}}+d_{1,4}^{(1)}(i+\tau)_{\pi_{\omega_4}}}] \\
&\quad +\ [(-1)^{d_{1,0}^{(2)}+d_{1,1}^{(2)}i_{\pi_{\omega_1}}+d_{1,2}^{(2)}i_{\pi_{\omega_2}}+d_{1,3}^{(2)}i_{\pi_{\omega_3}}+d_{1,4}^{(2)}i_{\pi_{\omega_4}}} \\
&\qquad \times(-1)^{d_{0,0}^{(1)}+d_{0,1}^{(1)}(i+\tau)_{\pi_{\omega_1}}+d_{0,2}^{(1)}(i+\tau)_{\pi_{\omega_2}}+d_{0,3}^{(1)}(i+\tau)_{\pi_{\omega_3}}+d_{0,4}^{(1)}(i+\tau)_{\pi_{\omega_4}}}] \\
&\quad -\ [(-1)^{d_{0,0}^{(1)}+d_{0,1}^{(1)}i_{\pi_{\omega_1}}+d_{0,2}^{(1)}i_{\pi_{\omega_2}}+d_{0,3}^{(1)}i_{\pi_{\omega_3}}+d_{0,4}^{(1)}i_{\pi_{\omega_4}}} \\
&\qquad \times(-1)^{d_{1,0}^{(2)}+d_{1,1}^{(2)}(i+\tau)_{\pi_{\omega_1}}+d_{1,2}^{(2)}(i+\tau)_{\pi_{\omega_2}}+d_{1,3}^{(2)}(i+\tau)_{\pi_{\omega_3}}+d_{1,4}^{(2)}(i+\tau)_{\pi_{\omega_4}}}] \\
&\quad -\ [(-1)^{d_{1,0}^{(1)}+d_{1,1}^{(1)}i_{\pi_{\omega_1}}+d_{1,2}^{(1)}i_{\pi_{\omega_2}}+d_{1,3}^{(1)}i_{\pi_{\omega_3}}+d_{1,4}^{(1)}i_{\pi_{\omega_4}}} \\
&\qquad \times(-1)^{d_{0,0}^{(2)}+d_{0,1}^{(2)}(i+\tau)_{\pi_{\omega_1}}+d_{0,2}^{(2)}(i+\tau)_{\pi_{\omega_2}}+d_{0,3}^{(2)}(i+\tau)_{\pi_{\omega_3}}+d_{0,4}^{(2)}(i+\tau)_{\pi_{\omega_4}}}]\ \}+ \\
&4\times\ \{\ [(-1)^{d_{1,0}^{(2)}+d_{1,1}^{(2)}i_{\pi_{\omega_1}}+d_{1,2}^{(2)}i_{\pi_{\omega_2}}+d_{1,3}^{(2)}i_{\pi_{\omega_3}}+d_{1,4}^{(2)}i_{\pi_{\omega_4}}} \\
&\qquad \times(-1)^{d_{1,0}^{(1)}+d_{1,1}^{(1)}(i+\tau)_{\pi_{\omega_1}}+d_{1,2}^{(1)}(i+\tau)_{\pi_{\omega_2}}+d_{1,3}^{(1)}(i+\tau)_{\pi_{\omega_3}}+d_{1,4}^{(1)}(i+\tau)_{\pi_{\omega_4}}}] \\
&\quad -\ [(-1)^{d_{1,0}^{(1)}+d_{1,1}^{(1)}i_{\pi_{\omega_1}}+d_{1,2}^{(1)}i_{\pi_{\omega_2}}+d_{1,3}^{(1)}i_{\pi_{\omega_3}}+d_{1,4}^{(1)}i_{\pi_{\omega_4}}} \\
&\qquad \times(-1)^{d_{1,0}^{(2)}+d_{1,1}^{(2)}(i+\tau)_{\pi_{\omega_1}}+d_{1,2}^{(2)}(i+\tau)_{\pi_{\omega_2}}+d_{1,3}^{(2)}(i+\tau)_{\pi_{\omega_3}}+d_{1,4}^{(2)}(i+\tau)_{\pi_{\omega_4}}}]\ \}
\end{aligned}
$$

Let $i_{\pi_{\omega_1}} = a, i_{\pi_{\omega_2}} = b, i_{\pi_{\omega_3}} = c, i_{\pi_{\omega_4}} = d, (i+\tau)_{\pi_{\omega_1}} = e, (i+\tau)_{\pi_{\omega_2}} = f$, $(i+\tau)_{\pi_{\omega_3}} = g$, $(i+\tau)_{\pi_{\omega_4}} = h$.

We represent

$$\{(-1)^{s_0^2(i)+s_0^1(i+\tau)} - (-1)^{s_0^1(i)+s_0^2(i+\tau)}\}$$

$$+2\times\{(-1)^{s_0^1(i)+s_1^1(i+\tau)}+(-1)^{s_1^1(i)+s_0^1(i+\tau)}+(-1)^{s_0^2(i)+s_1^2(i+\tau)}+(-1)^{s_1^2(i)+s_0^2(i+\tau)}\}$$

$$+4\times\{(-1)^{s_1^2(i)+s_1^1(i+\tau))} - (-1)^{s_1^1(i)+s_1^2(i+\tau)}\}$$

as $F(a,b,c,d,e,f,g,h)$,and

Represent

$$\{(-1)^{s_0^2(i)+s_0^1(i+\tau)} - (-1)^{s_0^1(i)+s_0^2(i+\tau)}\}$$

$$+2\times\{(-1)^{s_0^2(i)+s_1^1(i+\tau)}+(-1)^{s_1^2(i)+s_0^1(i+\tau)}-(-1)^{s_0^1(i)+s_1^2(i+\tau)}-(-1)^{s_1^1(i)+s_0^2(i+\tau)}\}$$

$$+4\times\{(-1)^{s_1^2(i)+s_1^1(i+\tau))} - (-1)^{s_1^1(i)+s_1^2(i+\tau)}\}$$

as $G(a,b,c,d,e,f,g,h)$.

Thus,if the coefficients $d_{0,0}^{(1)}, d_{0,1}^{(1)}, d_{0,2}^{(1)}, d_{0,3}^{(1)}, d_{0,4}^{(1)},\quad d_{1,0}^{(1)}, d_{1,1}^{(1)}, d_{1,2}^{(1)}, d_{1,3}^{(1)}, d_{1,4}^{(1)},$ $d_{0,0}^{(2)}, d_{0,1}^{(2)}, d_{0,2}^{(2)}, d_{0,3}^{(2)}, d_{0,4}^{(2)},\quad d_{1,0}^{(2)}, d_{1,1}^{(2)}, d_{1,2}^{(2)}, d_{1,3}^{(2)}, d_{1,4}^{(2)}$ satisfy the following eight equations, the corresponding sequences are Golay complementary sequences.

$F(0,b,c,d,0,f,g,h) = F(1,b,c,d,1,f,g,h)$,for every $b,c,d,f,g,h \in Z_2$.

$G(0,b,c,d,0,f,g,h) = G(1,b,c,d,1,f,g,h)$,for every $b,c,d,f,g,h \in Z_2$.

$F(a,0,c,d,e,0,g,h) = F(a,1,c,d,e,1,g,h)$,for every $a,c,d,e,g,h \in Z_2$.

$G(a,0,c,d,e,0,g,h) = G(a,1,c,d,e,1,g,h)$,for every $a,c,d,e,g,h \in Z_2$.

$F(a,b,0,d,e,f,0,h) = F(a,b,1,d,e,f,1,h)$,for every $a,b,d,e,f,h \in Z_2$

$G(a,b,0,d,e,f,0,h) = G(a,b,1,d,e,f,1,h)$,for every $a,b,d,e,f,h \in Z_2$

$F(a,b,c,0,e,f,g,0) = F(a,b,c,1,e,f,g,1)$,for every $a,b,c,e,f,g \in Z_2$

$G(a,b,c,0,e,f,g,0) = G(a,b,c,1,e,f,g,1)$,for every $a,b,c,e,f,g \in Z_2$

By exhaust search, there are 3840 sets of$\{ d_{0,0}^{(1)}, d_{0,1}^{(1)}, d_{0,2}^{(1)}, d_{0,3}^{(1)}, d_{0,4}^{(1)},$ $d_{1,0}^{(1)}, d_{1,1}^{(1)}, d_{1,2}^{(1)}, d_{1,3}^{(1)}, d_{1,4}^{(1)},\quad d_{0,0}^{(2)}, d_{0,1}^{(2)}, d_{0,2}^{(2)}, d_{0,3}^{(2)}, d_{0,4}^{(2)},\quad d_{1,0}^{(2)}, d_{1,1}^{(2)}, d_{1,2}^{(2)}, d_{1,3}^{(2)}, d_{1,4}^{(2)}\}$

satisfying the above equations.

Due to the limited space, we just list a few of the coefficients we found.

0 0 0 1 1	0 1 1 1 1	0 0 1 1 1	1 1 0 1 1
0 1 0 0 1	0 1 1 1 1	0 1 0 1 1	1 1 1 0 1
0 1 0 1 1	0 0 1 1 1	0 1 1 1 1	1 0 0 1 1
0 1 1 0 0	0 1 1 1 1	0 1 1 0 1	1 1 1 1 0
0 1 1 0 1	0 0 1 1 1	0 1 1 1 1	1 0 1 0 1
0 1 1 1 0	0 0 1 1 1	0 1 1 1 1	1 0 1 1 0
1 0 1 1 0	0 1 1 1 1	0 0 1 1 1	0 1 1 1 0
1 1 0 0 1	0 1 1 1 1	0 1 1 0 1	0 1 0 1 1
1 1 0 1 1	0 1 1 1 0	1 1 1 1 1	1 1 0 1 0
1 1 1 1 1	0 0 0 1 1	1 0 1 1 1	1 1 0 1 1

Second,we consider **case VI**. In this case, we have

$$2\times\{\ (-1)^{s_0^1(i)+s_1^1(i+\tau)}+(-1)^{s_1^1(i)+s_0^1(i+\tau)}+(-1)^{s_0^2(i)+s_1^2(i+\tau)}+(-1)^{s_1^2(i)+s_0^2(i+\tau)}\}$$

$$=2\times\{\ (-1)^{d_{0,0}^{(1)}+d_{0,1}^{(1)}i_{\pi(\omega)}+d_{0,2}^{(1)}i_{\pi(\omega+1)}+d_{1,0}^{(1)}+d_{1,1}^{(1)}i_{\pi(\omega)}+d_{1,2}^{(1)}i_{\pi(\omega+1)}}$$

$$+(-1)^{d_{1,0}^{(1)}+d_{1,1}^{(1)}i_{\pi(\omega)}+d_{1,2}^{(1)}i_{\pi(\omega+1)}+d_{0,0}^{(1)}+d_{0,1}^{(1)}i_{\pi(\omega)}+d_{0,2}^{(1)}i_{\pi(\omega+1)}}$$

$$+(-1)^{d_{0,0}^{(2)}+d_{0,1}^{(2)}i_{\pi(\omega)}+d_{0,2}^{(2)}i_{\pi(\omega+1)}+d_{1,0}^{(2)}+d_{1,1}^{(2)}i_{\pi(\omega)}+d_{1,2}^{(2)}i_{\pi(\omega+1)}}$$

$$+(-1)^{d_{1,0}^{(2)}+d_{1,1}^{(2)}i_{\pi(\omega)}+d_{1,2}^{(2)}i_{\pi(\omega+1)}+d_{0,0}^{(2)}+d_{0,1}^{(2)}i_{\pi(\omega)}+d_{0,2}^{(2)}i_{\pi(\omega+1)}}\}$$

$$\{(-1)^{s_0^2(i)+s_0^1(i+\tau)}-(-1)^{s_0^1(i)+s_0^2(i+\tau)}\}$$

$$+2\times\{(-1)^{s_0^2(i)+s_1^1(i+\tau)}+(-1)^{s_1^2(i)+s_0^1(i+\tau)}-(-1)^{s_0^1(i)+s_1^2(i+\tau)}-(-1)^{s_1^1(i)+s_0^2(i+\tau)}\}$$

$$+4\times\{(-1)^{s_1^2(i)+s_1^1(i+\tau))}-(-1)^{s_1^1(i)+s_1^2(i+\tau)}\}$$

$$=\{\ (-1)^{d_{0,0}^{(2)}+d_{0,1}^{(2)}i_{\pi(\omega)}+d_{0,2}^{(2)}i_{\pi(\omega+1)}+d_{0,0}^{(1)}+d_{0,1}^{(1)}(i+\tau)_{\pi(\omega)}+d_{0,2}^{(1)}(i+\tau)_{\pi(\omega+1)}}$$

$$-\ (-1)^{d_{0,0}^{(1)}+d_{0,1}^{(1)}i_{\pi(\omega)}+d_{0,2}^{(1)}i_{\pi(\omega+1)}+d_{0,0}^{(2)}+d_{0,1}^{(2)}(i+\tau)_{\pi(\omega)}+d_{0,2}^{(2)}(i+\tau)_{\pi(\omega+1)}}\ \}+$$

$$2\times\{\ (-1)^{d_{0,0}^{(2)}+d_{0,1}^{(2)}i_{\pi(\omega)}+d_{0,2}^{(2)}i_{\pi(\omega+1)}+d_{1,0}^{(1)}+d_{1,1}^{(1)}(i+\tau)_{\pi(\omega)}+d_{1,2}^{(1)}(i+\tau)_{\pi(\omega+1)}}\ +$$

$$(-1)^{d_{1,0}^{(2)}+d_{1,1}^{(2)}i_{\pi(\omega)}+d_{1,2}^{(2)}i_{\pi(\omega+1)}+d_{0,0}^{(1)}+d_{0,1}^{(1)}(i+\tau)_{\pi(\omega)}+d_{0,2}^{(1)}(i+\tau)_{\pi(\omega+1)}}$$

$$-\ (-1)^{d_{0,0}^{(1)}+d_{0,1}^{(1)}i_{\pi(\omega)}+d_{0,2}^{(1)}i_{\pi(\omega+1)}+d_{1,0}^{(2)}+d_{1,1}^{(2)}(i+\tau)_{\pi(\omega)}+d_{1,2}^{(2)}(i+\tau)_{\pi(\omega+1)}}\ -$$

$$(-1)^{d_{1,0}^{(1)}+d_{1,1}^{(1)}i_{\pi(\omega)}+d_{1,2}^{(1)}i_{\pi(\omega+1)}+d_{0,0}^{(2)}+d_{0,1}^{(2)}(i+\tau)_{\pi(\omega)}+d_{0,2}^{(2)}(i+\tau)_{\pi(\omega+1)}}\ \}+$$

$$4\times\ \{\ (-1)^{d_{1,0}^{(2)}+d_{1,1}^{(2)}i_{\pi(\omega)}+d_{1,2}^{(2)}i_{\pi(\omega+1)}+d_{1,0}^{(1)}+d_{1,1}^{(1)}(i+\tau)_{\pi(\omega)}+d_{1,2}^{(1)}(i+\tau)_{\pi(\omega+1)}}$$

$$-\ (-1)^{d_{1,0}^{(1)}+d_{1,1}^{(1)}i_{\pi(\omega)}+d_{1,2}^{(1)}i_{\pi(\omega+1)}+d_{1,0}^{(2)}+d_{1,1}^{(2)}(i+\tau)_{\pi(\omega)}+d_{1,2}^{(2)}(i+\tau)_{\pi(\omega+1)}}\ \}$$

Let $i_{\pi(\omega)}=a$,$i_{\pi(\omega+1)}=b$,$(i+\tau)_{\pi(\omega)}=c$,$(i+\tau)_{\pi(\omega+1)}=d$.

We represent

$$\{(-1)^{s_0^2(i)+s_0^1(i+\tau)}-(-1)^{s_0^1(i)+s_0^2(i+\tau)}\}$$

$$+2\times\{(-1)^{s_0^1(i)+s_1^1(i+\tau)}+(-1)^{s_1^1(i)+s_0^1(i+\tau)}+(-1)^{s_0^2(i)+s_1^2(i+\tau)}+(-1)^{s_1^2(i)+s_0^2(i+\tau)}\}$$

$$+4\times\{(-1)^{s_1^2(i)+s_1^1(i+\tau))}-(-1)^{s_1^1(i)+s_1^2(i+\tau)}\}$$

as $F(a,b,c,d)$,and

Represent

$$\{(-1)^{s_0^2(i)+s_0^1(i+\tau)}-(-1)^{s_0^1(i)+s_0^2(i+\tau)}\}$$

$$+2\times\{(-1)^{s_0^2(i)+s_1^1(i+\tau)}+(-1)^{s_1^2(i)+s_0^1(i+\tau)}-(-1)^{s_0^1(i)+s_1^2(i+\tau)}-(-1)^{s_1^1(i)+s_0^2(i+\tau)}\}$$

$$+4\times\{(-1)^{s_1^2(i)+s_1^1(i+\tau))}-(-1)^{s_1^1(i)+s_1^2(i+\tau)}\}$$

as $G(a,b,c,d)$.

As discussed above, if the coefficients $d_{0,0}^{(1)},d_{0,1}^{(1)},d_{0,2}^{(1)},\quad d_{1,0}^{(1)},d_{1,1}^{(1)},d_{1,2}^{(1)}$, $d_{0,0}^{(2)},d_{0,1}^{(2)},d_{0,2}^{(2)},\quad d_{1,0}^{(2)},d_{1,1}^{(2)},d_{1,2}^{(2)}$ satisfy the following eight equations

$F(0,1,0,0)=F(1,1,1,0)$

$F(0,0,0,1)=F(1,0,1,1)$

$G(0,0,0,1)=G(1,0,1,1)$

$G(0,1,0,0)=G(1,1,1,0)$

$F(0,0,1,0)=F(0,1,1,1)$

$F(1,0,0,0)=F(1,1,0,1)$

$G(0,0,1,0)=G(0,1,1,1)$

$G(1,0,0,0)=G(1,1,0,1)$

the corresponding sequences are Golay complementary sequences.

By exhaust search, there are 736 sets of$\{$ $d_{0,0}^{(1)}, d_{0,1}^{(1)}, d_{0,2}^{(1)}, \quad d_{1,0}^{(1)}, d_{1,1}^{(1)}, d_{1,2}^{(1)},$

$d_{0,0}^{(2)}, d_{0,1}^{(2)}, d_{0,2}^{(2)}, \quad d_{1,0}^{(2)}, d_{1,1}^{(2)}, d_{1,2}^{(2)}\}$ satisfying the above equations.

Here , we list some of the coefficients.

0 0 0 1 0 0 0 1 0 1 0 1, 0 0 0 1 0 0 1 0 1 0 0 1,
0 0 1 0 0 1 1 0 0 1 1 1, 0 0 1 0 1 0 1 0 0 1 0 0,
0 1 0 0 0 1 1 0 0 1 0 0, 0 1 1 0 0 0 0 0 1 1 1 0,
0 1 1 0 1 1 0 0 1 1 0 1, 0 1 1 0 1 1 1 1 0 0 1 0,
0 1 1 1 1 1 0 0 1 1 1 0, 1 0 1 0 1 0 0 0 0 1 1 1.

6 New Golay Complementary Sequences over 64-QAM constellation

In this part, we will construct new Golay complementary sequences over 64-QAM constellation.

Let $A(x)=\sum_{l=1}^{n-1} x_{\pi(l)}x_{\pi(l+1)}+\sum_{l=1}^{n} c_l x_l + c,$

$$a_k^1(x)=A(x)+s_k^1(x),$$

$$a_k^2(x)=A(x)+s_k^2(x),$$

$$b_k^1(x)=A(x)+s_k^1(x)+\mu(x),$$

$$b_k^2(x)=A(x)+s_k^2(x)+\mu(x),$$

Where $c_l\in Z_2$,$l=1,2,\cdots,n, c\in Z_2$,π is a permutation from $\{1,2,\cdots,n\}$

to $\{1,2,\cdots,n\}$,$s_k^1(x), s_k^2(x)$ and $\mu(x)$ satisfy the following cases.

$$s_k^1(x)=d_{k,0}^{(1)}+d_{k,1}^{(1)}x_{\pi(\omega_1)}+d_{k,2}^{(1)}x_{\pi(\omega_2)}+d_{k,3}^{(1)}x_{\pi(\omega_3)}+d_{k,4}^{(1)}x_{\pi(\omega_4)},$$

$$s_k^2(x)=d_{k,0}^{(2)}+d_{k,1}^{(2)}x_{\pi(\omega_1)}+d_{k,2}^{(2)}x_{\pi(\omega_2)}+d_{k,3}^{(2)}x_{\pi(\omega_3)}+d_{k,4}^{(2)}x_{\pi(\omega_4)}\ ,$$

$$1\le\omega_1<\omega_1+1<\omega_2<\omega_2+1<\omega_3<\omega_3+1<\omega_4\le n,$$

$$\mu(x)=x_{\pi(1)} \text{ or } \mu(x)=x_{\pi(n)},$$

Where $d_{k,i}^{(1)}, d_{k,i}^{(2)} \in Z_2, k = 0, 1, 2,\ i = 0, 1, 2, 3, 4.$

We construct the following sequences over M^2-QAM constellation.

$c_i = \sum_{k=0}^{2} 2^k (-1)^{a_k^1(i)} + j \sum_{k=0}^{2} 2^k (-1)^{a_k^2(i)}$,

$d_i = \sum_{k=0}^{2} 2^k (-1)^{b_k^1(i)} + j \sum_{k=0}^{2} 2^k (-1)^{b_k^2(i)}$,

The following equations can be checked easily.

$$
\begin{aligned}
&8[(-1)^{s_2^1(i)+s_1^1(i+\tau)} + (-1)^{s_2^2(i)+s_1^2(i+\tau)}] + 8[(-1)^{s_1^1(i)+s_2^1(i+\tau)} + (-1)^{s_1^2(i)+s_2^2(i+\tau)}] \\
&+4[(-1)^{s_2^1(i)+s_0^1(i+\tau)} + (-1)^{s_2^2(i)+s_0^2(i+\tau)}] + 4[(-1)^{s_0^1(i)+s_2^1(i+\tau)} + (-1)^{s_0^2(i)+s_2^2(i+\tau)}] \\
&+2[(-1)^{s_1^1(i)+s_0^1(i+\tau)} + (-1)^{s_1^2(i)+s_0^2(i+\tau)}] + 2[(-1)^{s_0^1(i)+s_1^1(i+\tau)} + (-1)^{s_0^2(i)+s_1^2(i+\tau)}] \\
&= 8\{[(-1)^{d_{2,0}^{(1)}+d_{2,1}^{(1)}i_{\pi(\omega_1)}+d_{2,2}^{(1)}i_{\pi(\omega_2)}+d_{2,3}^{(1)}i_{\pi(\omega_3)}+d_{2,4}^{(1)}i_{\pi(\omega_4)}} \\
&\quad\times(-1)^{d_{1,0}^{(1)}+d_{1,1}^{(1)}(i+\tau)_{\pi(\omega_1)}+d_{1,2}^{(1)}(i+\tau)_{\pi(\omega_2)}+d_{1,3}^{(1)}(i+\tau)_{\pi(\omega_3)}+d_{1,4}^{(1)}(i+\tau)_{\pi(\omega_4)}}] \\
&\quad+[(-1)^{d_{2,0}^{(2)}+d_{2,1}^{(2)}i_{\pi(\omega_1)}+d_{2,2}^{(2)}i_{\pi(\omega_2)}+d_{2,3}^{(2)}i_{\pi(\omega_3)}+d_{2,4}^{(2)}i_{\pi(\omega_4)}} \\
&\quad\times(-1)^{d_{1,0}^{(2)}+d_{1,1}^{(2)}(i+\tau)_{\pi(\omega_1)}+d_{1,2}^{(2)}(i+\tau)_{\pi(\omega_2)}+d_{1,3}^{(2)}(i+\tau)_{\pi(\omega_3)}+d_{1,4}^{(2)}(i+\tau)_{\pi(\omega_4)}}] \\
&\quad+[(-1)^{d_{1,0}^{(1)}+d_{1,1}^{(1)}i_{\pi(\omega_1)}+d_{1,2}^{(1)}i_{\pi(\omega_2)}+d_{1,3}^{(1)}i_{\pi(\omega_3)}+d_{1,4}^{(1)}i_{\pi(\omega_4)}} \\
&\quad\times(-1)^{d_{2,0}^{(1)}+d_{2,1}^{(1)}(i+\tau)_{\pi(\omega_1)}+d_{2,2}^{(1)}(i+\tau)_{\pi(\omega_2)}+d_{2,3}^{(1)}(i+\tau)_{\pi(\omega_3)}+d_{2,4}^{(1)}(i+\tau)_{\pi(\omega_4)}}] \\
&\quad+[(-1)^{d_{1,0}^{(2)}+d_{1,1}^{(2)}i_{\pi(\omega_1)}+d_{1,2}^{(2)}i_{\pi(\omega_2)}+d_{1,3}^{(2)}i_{\pi(\omega_3)}+d_{1,4}^{(2)}i_{\pi(\omega_4)}} \\
&\quad\times(-1)^{d_{2,0}^{(2)}+d_{2,1}^{(2)}(i+\tau)_{\pi(\omega_1)}+d_{2,2}^{(2)}(i+\tau)_{\pi(\omega_2)}+d_{2,3}^{(2)}(i+\tau)_{\pi(\omega_3)}+d_{2,4}^{(2)}(i+\tau)_{\pi(\omega_4)}}]\} \\
&+4\{[(-1)^{d_{2,0}^{(1)}+d_{2,1}^{(1)}i_{\pi(\omega_1)}+d_{2,2}^{(1)}i_{\pi(\omega_2)}+d_{2,3}^{(1)}i_{\pi(\omega_3)}+d_{2,4}^{(1)}i_{\pi(\omega_4)}} \\
&\quad\times(-1)^{d_{0,0}^{(1)}+d_{0,1}^{(1)}(i+\tau)_{\pi(\omega_1)}+d_{0,2}^{(1)}(i+\tau)_{\pi(\omega_2)}+d_{0,3}^{(1)}(i+\tau)_{\pi(\omega_3)}+d_{0,4}^{(1)}(i+\tau)_{\pi(\omega_4)}}] \\
&\quad+[(-1)^{d_{2,0}^{(2)}+d_{2,1}^{(2)}i_{\pi(\omega_1)}+d_{2,2}^{(2)}i_{\pi(\omega_2)}+d_{2,3}^{(2)}i_{\pi(\omega_3)}+d_{2,4}^{(2)}i_{\pi(\omega_4)}} \\
&\quad\times(-1)^{d_{0,0}^{(2)}+d_{0,1}^{(2)}(i+\tau)_{\pi(\omega_1)}+d_{0,2}^{(2)}(i+\tau)_{\pi(\omega_2)}+d_{0,3}^{(2)}(i+\tau)_{\pi(\omega_3)}+d_{0,4}^{(2)}(i+\tau)_{\pi(\omega_4)}}] \\
&\quad+[(-1)^{d_{0,0}^{(1)}+d_{0,1}^{(1)}i_{\pi(\omega_1)}+d_{0,2}^{(1)}i_{\pi(\omega_2)}+d_{0,3}^{(1)}i_{\pi(\omega_3)}+d_{0,4}^{(1)}i_{\pi(\omega_4)}} \\
&\quad\times(-1)^{d_{2,0}^{(1)}+d_{2,1}^{(1)}(i+\tau)_{\pi(\omega_1)}+d_{2,2}^{(1)}(i+\tau)_{\pi(\omega_2)}+d_{2,3}^{(1)}(i+\tau)_{\pi(\omega_3)}+d_{2,4}^{(1)}(i+\tau)_{\pi(\omega_4)}}]
\end{aligned}
$$

$$
\begin{aligned}
&+[(-1)^{d_{0,0}^{(2)}+d_{0,1}^{(2)}i_{\pi(\omega_1)}+d_{0,2}^{(2)}i_{\pi(\omega_2)}+d_{0,3}^{(2)}i_{\pi(\omega_3)}+d_{0,4}^{(2)}i_{\pi(\omega_4)}}\\
&\quad\times(-1)^{d_{2,0}^{(2)}+d_{2,1}^{(2)}(i+\tau)_{\pi(\omega_1)}+d_{2,2}^{(2)}(i+\tau)_{\pi(\omega_2)}+d_{2,3}^{(2)}(i+\tau)_{\pi(\omega_3)}+d_{2,4}^{(2)}(i+\tau)_{\pi(\omega_4)}}]\}\\
&+2\{[(-1)^{d_{1,0}^{(1)}+d_{1,1}^{(1)}i_{\pi(\omega_1)}+d_{1,2}^{(1)}i_{\pi(\omega_2)}+d_{1,3}^{(1)}i_{\pi(\omega_3)}+d_{1,4}^{(1)}i_{\pi(\omega_4)}}\\
&\quad\times(-1)^{d_{0,0}^{(1)}+d_{0,1}^{(1)}(i+\tau)_{\pi(\omega_1)}+d_{0,2}^{(1)}(i+\tau)_{\pi(\omega_2)}+d_{0,3}^{(1)}(i+\tau)_{\pi(\omega_3)}+d_{0,4}^{(1)}(i+\tau)_{\pi(\omega_4)}}]\\
&+[(-1)^{d_{1,0}^{(2)}+d_{1,1}^{(2)}i_{\pi(\omega_1)}+d_{1,2}^{(2)}i_{\pi(\omega_2)}+d_{1,3}^{(2)}i_{\pi(\omega_3)}+d_{1,4}^{(2)}i_{\pi(\omega_4)}}\\
&\quad\times(-1)^{d_{0,0}^{(2)}+d_{0,1}^{(2)}(i+\tau)_{\pi(\omega_1)}+d_{0,2}^{(2)}(i+\tau)_{\pi(\omega_2)}+d_{0,3}^{(2)}(i+\tau)_{\pi(\omega_3)}+d_{0,4}^{(2)}(i+\tau)_{\pi(\omega_4)}}]\\
&+[(-1)^{d_{0,0}^{(1)}+d_{0,1}^{(1)}i_{\pi(\omega_1)}+d_{0,2}^{(1)}i_{\pi(\omega_2)}+d_{0,3}^{(1)}i_{\pi(\omega_3)}+d_{0,4}^{(1)}i_{\pi(\omega_4)}}\\
&\quad\times(-1)^{d_{1,0}^{(1)}+d_{1,1}^{(1)}(i+\tau)_{\pi(\omega_1)}+d_{1,2}^{(1)}(i+\tau)_{\pi(\omega_2)}+d_{1,3}^{(1)}(i+\tau)_{\pi(\omega_3)}+d_{1,4}^{(1)}(i+\tau)_{\pi(\omega_4)}}]\\
&+[(-1)^{d_{0,0}^{(2)}+d_{0,1}^{(2)}i_{\pi(\omega_1)}+d_{0,2}^{(2)}i_{\pi(\omega_2)}+d_{0,3}^{(2)}i_{\pi(\omega_3)}+d_{0,4}^{(2)}i_{\pi(\omega_4)}}\\
&\quad\times(-1)^{d_{1,0}^{(2)}+d_{1,1}^{(2)}(i+\tau)_{\pi(\omega_1)}+d_{1,2}^{(2)}(i+\tau)_{\pi(\omega_2)}+d_{1,3}^{(2)}(i+\tau)_{\pi(\omega_3)}+d_{1,4}^{(2)}(i+\tau)_{\pi(\omega_4)}}]\}
\end{aligned}
$$

$$
\begin{aligned}
&16[(-1)^{s_2^2(i)+s_2^1(i+\tau)}-(-1)^{s_2^1(i)+s_2^2(i+\tau)}]\\
&+8[(-1)^{s_2^2(i)+s_1^1(i+\tau)}-(-1)^{s_2^1(i)+s_1^2(i+\tau)}+(-1)^{s_1^2(i)+s_2^1(i+\tau)}-(-1)^{s_1^1(i)+s_2^2(i+\tau)}]\\
&+4[(-1)^{s_2^2(i)+s_0^1(i+\tau)}-(-1)^{s_2^1(i)+s_0^2(i+\tau)}+(-1)^{s_0^2(i)+s_2^1(i+\tau)}-(-1)^{s_0^1(i)+s_2^2(i+\tau)}\\
&\quad+(-1)^{s_1^2(i)+s_1^1(i+\tau)}-(-1)^{s_1^1(i)+s_1^2(i+\tau)}]\\
&+2[(-1)^{s_1^2(i)+s_0^1(i+\tau)}-(-1)^{s_1^1(i)+s_0^2(i+\tau)}+(-1)^{s_0^2(i)+s_1^1(i+\tau)}-(-1)^{s_0^1(i)+s_1^2(i+\tau)}]\\
&+[(-1)^{s_0^2(i)+s_0^1(i+\tau)}-(-1)^{s_0^1(i)+s_0^2(i+\tau)}]\\
&=16\{[(-1)^{d_{2,0}^{(2)}+d_{2,1}^{(2)}i_{\pi(\omega_1)}+d_{2,2}^{(2)}i_{\pi(\omega_2)}+d_{2,3}^{(2)}i_{\pi(\omega_3)}+d_{2,4}^{(2)}i_{\pi(\omega_4)}}\\
&\quad\times(-1)^{d_{2,0}^{(1)}+d_{2,1}^{(1)}(i+\tau)_{\pi(\omega_1)}+d_{2,2}^{(1)}(i+\tau)_{\pi(\omega_2)}+d_{2,3}^{(1)}(i+\tau)_{\pi(\omega_3)}+d_{2,4}^{(1)}(i+\tau)_{\pi(\omega_4)}}]\\
&+[(-1)^{d_{2,0}^{(1)}+d_{2,1}^{(1)}i_{\pi(\omega_1)}+d_{2,2}^{(1)}i_{\pi(\omega_2)}+d_{2,3}^{(1)}i_{\pi(\omega_3)}+d_{2,4}^{(1)}i_{\pi(\omega_4)}}\\
&\quad\times(-1)^{d_{2,0}^{(2)}+d_{2,1}^{(2)}(i+\tau)_{\pi(\omega_1)}+d_{2,2}^{(2)}(i+\tau)_{\pi(\omega_2)}+d_{2,3}^{(2)}(i+\tau)_{\pi(\omega_3)}+d_{2,4}^{(2)}(i+\tau)_{\pi(\omega_4)}}]\}\\
&+8\{[(-1)^{d_{2,0}^{(2)}+d_{2,1}^{(2)}i_{\pi(\omega_1)}+d_{2,2}^{(2)}i_{\pi(\omega_2)}+d_{2,3}^{(2)}i_{\pi(\omega_3)}+d_{2,4}^{(2)}i_{\pi(\omega_4)}}\\
&\quad\times(-1)^{d_{1,0}^{(1)}+d_{1,1}^{(1)}(i+\tau)_{\pi(\omega_1)}+d_{1,2}^{(1)}(i+\tau)_{\pi(\omega_2)}+d_{1,3}^{(1)}(i+\tau)_{\pi(\omega_3)}+d_{1,4}^{(1)}(i+\tau)_{\pi(\omega_4)}}]
\end{aligned}
$$

$$
\begin{aligned}
&-[(-1)^{d_{2,0}^{(1)}+d_{2,1}^{(1)}i_{\pi(\omega_1)}+d_{2,2}^{(1)}i_{\pi(\omega_2)}+d_{2,3}^{(1)}i_{\pi(\omega_3)}+d_{2,4}^{(1)}i_{\pi(\omega_4)}} \\
&\quad\times(-1)^{d_{1,0}^{(2)}+d_{1,1}^{(2)}(i+\tau)_{\pi(\omega_1)}+d_{1,2}^{(2)}(i+\tau)_{\pi(\omega_2)}+d_{1,3}^{(2)}(i+\tau)_{\pi(\omega_3)}+d_{1,4}^{(2)}(i+\tau)_{\pi(\omega_4)}}] \\
&+[(-1)^{d_{1,0}^{(2)}+d_{1,1}^{(2)}i_{\pi(\omega_1)}+d_{1,2}^{(2)}i_{\pi(\omega_2)}+d_{1,3}^{(2)}i_{\pi(\omega_3)}+d_{1,4}^{(2)}i_{\pi(\omega_4)}} \\
&\quad\times(-1)^{d_{2,0}^{(1)}+d_{2,1}^{(1)}(i+\tau)_{\pi(\omega_1)}+d_{2,2}^{(1)}(i+\tau)_{\pi(\omega_2)}+d_{2,3}^{(1)}(i+\tau)_{\pi(\omega_3)}+d_{2,4}^{(1)}(i+\tau)_{\pi(\omega_4)}}] \\
&-[(-1)^{d_{1,0}^{(1)}+d_{1,1}^{(1)}i_{\pi(\omega_1)}+d_{1,2}^{(1)}i_{\pi(\omega_2)}+d_{1,3}^{(1)}i_{\pi(\omega_3)}+d_{1,4}^{(1)}i_{\pi(\omega_4)}} \\
&\quad\times(-1)^{d_{2,0}^{(2)}+d_{2,1}^{(2)}(i+\tau)_{\pi(\omega_1)}+d_{2,2}^{(2)}(i+\tau)_{\pi(\omega_2)}+d_{2,3}^{(2)}(i+\tau)_{\pi(\omega_3)}+d_{2,4}^{(2)}(i+\tau)_{\pi(\omega_4)}}]\} \\
+4\{&[(-1)^{d_{2,0}^{(2)}+d_{2,1}^{(2)}i_{\pi(\omega_1)}+d_{2,2}^{(2)}i_{\pi(\omega_2)}+d_{2,3}^{(2)}i_{\pi(\omega_3)}+d_{2,4}^{(2)}i_{\pi(\omega_4)}} \\
&\quad\times(-1)^{d_{0,0}^{(1)}+d_{0,1}^{(1)}(i+\tau)_{\pi(\omega_1)}+d_{0,2}^{(1)}(i+\tau)_{\pi(\omega_2)}+d_{0,3}^{(1)}(i+\tau)_{\pi(\omega_3)}+d_{0,4}^{(1)}(i+\tau)_{\pi(\omega_4)}}] \\
&-[(-1)^{d_{2,0}^{(1)}+d_{2,1}^{(1)}i_{\pi(\omega_1)}+d_{2,2}^{(1)}i_{\pi(\omega_2)}+d_{2,3}^{(1)}i_{\pi(\omega_3)}+d_{2,4}^{(1)}i_{\pi(\omega_4)}} \\
&\quad\times(-1)^{d_{0,0}^{(2)}+d_{0,1}^{(2)}(i+\tau)_{\pi(\omega_1)}+d_{0,2}^{(2)}(i+\tau)_{\pi(\omega_2)}+d_{0,3}^{(2)}(i+\tau)_{\pi(\omega_3)}+d_{0,4}^{(2)}(i+\tau)_{\pi(\omega_4)}}] \\
&+[(-1)^{d_{0,0}^{(2)}+d_{0,1}^{(2)}i_{\pi(\omega_1)}+d_{0,2}^{(2)}i_{\pi(\omega_2)}+d_{0,3}^{(2)}i_{\pi(\omega_3)}+d_{0,4}^{(2)}i_{\pi(\omega_4)}} \\
&\quad\times(-1)^{d_{2,0}^{(1)}+d_{2,1}^{(1)}(i+\tau)_{\pi(\omega_1)}+d_{2,2}^{(1)}(i+\tau)_{\pi(\omega_2)}+d_{2,3}^{(1)}(i+\tau)_{\pi(\omega_3)}+d_{2,4}^{(1)}(i+\tau)_{\pi(\omega_4)}}] \\
&-[(-1)^{d_{0,0}^{(1)}+d_{0,1}^{(1)}i_{\pi(\omega_1)}+d_{0,2}^{(1)}i_{\pi(\omega_2)}+d_{0,3}^{(1)}i_{\pi(\omega_3)}+d_{0,4}^{(1)}i_{\pi(\omega_4)}} \\
&\quad\times(-1)^{d_{2,0}^{(2)}+d_{2,1}^{(2)}(i+\tau)_{\pi(\omega_1)}+d_{2,2}^{(2)}(i+\tau)_{\pi(\omega_2)}+d_{2,3}^{(2)}(i+\tau)_{\pi(\omega_3)}+d_{2,4}^{(2)}(i+\tau)_{\pi(\omega_4)}}] \\
&+[(-1)^{d_{1,0}^{(2)}+d_{1,1}^{(2)}i_{\pi(\omega_1)}+d_{1,2}^{(2)}i_{\pi(\omega_2)}+d_{1,3}^{(2)}i_{\pi(\omega_3)}+d_{1,4}^{(2)}i_{\pi(\omega_4)}} \\
&\quad\times(-1)^{d_{1,0}^{(1)}+d_{1,1}^{(1)}(i+\tau)_{\pi(\omega_1)}+d_{1,2}^{(1)}(i+\tau)_{\pi(\omega_2)}+d_{1,3}^{(1)}(i+\tau)_{\pi(\omega_3)}+d_{1,4}^{(1)}(i+\tau)_{\pi(\omega_4)}}] \\
&-[(-1)^{d_{1,0}^{(1)}+d_{1,1}^{(1)}i_{\pi(\omega_1)}+d_{1,2}^{(1)}i_{\pi(\omega_2)}+d_{1,3}^{(1)}i_{\pi(\omega_3)}+d_{1,4}^{(1)}i_{\pi(\omega_4)}} \\
&\quad\times(-1)^{d_{1,0}^{(2)}+d_{1,1}^{(2)}(i+\tau)_{\pi(\omega_1)}+d_{1,2}^{(2)}(i+\tau)_{\pi(\omega_2)}+d_{1,3}^{(2)}(i+\tau)_{\pi(\omega_3)}+d_{1,4}^{(2)}(i+\tau)_{\pi(\omega_4)}}]\} \\
+2\{&[(-1)^{d_{1,0}^{(2)}+d_{1,1}^{(2)}i_{\pi(\omega_1)}+d_{1,2}^{(2)}i_{\pi(\omega_2)}+d_{1,3}^{(2)}i_{\pi(\omega_3)}+d_{1,4}^{(2)}i_{\pi(\omega_4)}} \\
&\quad\times(-1)^{d_{0,0}^{(1)}+d_{0,1}^{(1)}(i+\tau)_{\pi(\omega_1)}+d_{0,2}^{(1)}(i+\tau)_{\pi(\omega_2)}+d_{0,3}^{(1)}(i+\tau)_{\pi(\omega_3)}+d_{0,4}^{(1)}(i+\tau)_{\pi(\omega_4)}}] \\
&-[(-1)^{d_{1,0}^{(1)}+d_{1,1}^{(1)}i_{\pi(\omega_1)}+d_{1,2}^{(1)}i_{\pi(\omega_2)}+d_{1,3}^{(1)}i_{\pi(\omega_3)}+d_{1,4}^{(1)}i_{\pi(\omega_4)}} \\
&\quad\times(-1)^{d_{0,0}^{(2)}+d_{0,1}^{(2)}(i+\tau)_{\pi(\omega_1)}+d_{0,2}^{(2)}(i+\tau)_{\pi(\omega_2)}+d_{0,3}^{(2)}(i+\tau)_{\pi(\omega_3)}+d_{0,4}^{(2)}(i+\tau)_{\pi(\omega_4)}}]
\end{aligned}
$$

$$+[(-1)^{d_{0,0}^{(2)}+d_{0,1}^{(2)}i_{\pi(\omega_1)}+d_{0,2}^{(2)}i_{\pi(\omega_2)}+d_{0,3}^{(2)}i_{\pi(\omega_3)}+d_{0,4}^{(2)}i_{\pi(\omega_4)}}$$

$$\times(-1)^{d_{1,0}^{(1)}+d_{1,1}^{(1)}(i+\tau)_{\pi(\omega_1)}+d_{1,2}^{(1)}(i+\tau)_{\pi(\omega_2)}+d_{1,3}^{(1)}(i+\tau)_{\pi(\omega_3)}+d_{1,4}^{(1)}(i+\tau)_{\pi(\omega_4)}}]$$

$$+[(-1)^{d_{0,0}^{(1)}+d_{0,1}^{(1)}i_{\pi(\omega_1)}+d_{0,2}^{(1)}i_{\pi(\omega_2)}+d_{0,3}^{(1)}i_{\pi(\omega_3)}+d_{0,4}^{(1)}i_{\pi(\omega_4)}}$$

$$\times(-1)^{d_{1,0}^{(2)}+d_{1,1}^{(2)}(i+\tau)_{\pi(\omega_1)}+d_{1,2}^{(2)}(i+\tau)_{\pi(\omega_2)}+d_{1,3}^{(2)}(i+\tau)_{\pi(\omega_3)}+d_{1,4}^{(2)}(i+\tau)_{\pi(\omega_4)}}]\}$$

Let $i_{\pi_{\omega_1}} = a, i_{\pi_{\omega_2}} = b, i_{\pi_{\omega_3}} = c, i_{\pi_{\omega_4}} = d, (i+\tau)_{\pi_{\omega_1}} = e, (i+\tau)_{\pi_{\omega_2}} = f,$

$(i+\tau)_{\pi_{\omega_3}} = g, (i+\tau)_{\pi_{\omega_4}} = h.$

We represent

$$8[(-1)^{s_2^1(i)+s_1^1(i+\tau)} + (-1)^{s_2^2(i)+s_1^2(i+\tau)}] + 8[(-1)^{s_1^1(i)+s_2^1(i+\tau)} + (-1)^{s_1^2(i)+s_2^2(i+\tau)}]$$

$$+4[(-1)^{s_2^1(i)+s_0^1(i+\tau)}+(-1)^{s_2^2(i)+s_0^2(i+\tau)}]+4[(-1)^{s_0^1(i)+s_2^1(i+\tau)}+(-1)^{s_0^2(i)+s_2^2(i+\tau)}]$$

$$+2[(-1)^{s_1^1(i)+s_0^1(i+\tau)}+(-1)^{s_1^2(i)+s_0^2(i+\tau)}]+2[(-1)^{s_0^1(i)+s_1^1(i+\tau)}+(-1)^{s_0^2(i)+s_1^2(i+\tau)}]$$

as $F(a, b, c, d, e, f, g, h)$.

Represent

$$16[(-1)^{s_2^2(i)+s_2^1(i+\tau)} - (-1)^{s_2^1(i)+s_2^2(i+\tau)}]$$

$$+8[(-1)^{s_2^2(i)+s_1^1(i+\tau)} - (-1)^{s_2^1(i)+s_1^2(i+\tau)} + (-1)^{s_1^2(i)+s_2^1(i+\tau)} - (-1)^{s_1^1(i)+s_2^2(i+\tau)}]$$

$$+4[(-1)^{s_2^2(i)+s_0^1(i+\tau)} - (-1)^{s_2^1(i)+s_0^2(i+\tau)} + (-1)^{s_0^2(i)+s_2^1(i+\tau)} - (-1)^{s_0^1(i)+s_2^2(i+\tau)} + (-1)^{s_1^2(i)+s_1^1(i+\tau)} - (-1)^{s_1^1(i)+s_1^2(i+\tau)}]$$

$$+2[(-1)^{s_1^2(i)+s_0^1(i+\tau)} - (-1)^{s_1^1(i)+s_0^2(i+\tau)} + (-1)^{s_0^2(i)+s_1^1(i+\tau)} - (-1)^{s_0^1(i)+s_1^2(i+\tau)}]$$

as $G(a, b, c, d, e, f, g, h)$.

We know when the coefficients $d_{0,0}^{(1)}, d_{0,1}^{(1)}, d_{0,2}^{(1)}, d_{0,3}^{(1)}, d_{0,4}^{(1)},\quad d_{1,0}^{(1)}, d_{1,1}^{(1)}, d_{1,2}^{(1)}, d_{1,3}^{(1)}, d_{1,4}^{(1)},$ $d_{2,0}^{(1)}, d_{2,1}^{(1)}, d_{2,2}^{(1)}, d_{2,3}^{(1)}, d_{2,4}^{(1)},\quad d_{0,0}^{(2)}, d_{0,1}^{(2)}, d_{0,2}^{(2)}, d_{0,3}^{(2)}, d_{0,4}^{(2)},\quad d_{1,0}^{(2)}, d_{1,1}^{(2)}, d_{1,2}^{(2)}, d_{1,3}^{(2)}, d_{1,4}^{(2)},$ $d_{2,0}^{(2)}, d_{2,1}^{(2)}, d_{2,2}^{(2)}, d_{2,3}^{(2)}, d_{2,4}^{(2)}$ satisfy the following equations, the corresponding sequences are Golay complementary sequences.

$F(0, b, c, d, 0, f, g, h) = F(1, b, c, d, 1, f, g, h)$,for every $b, c, d, f, g, h \in Z_2$.

$G(0,b,c,d,0,f,g,h) = G(1,b,c,d,1,f,g,h)$,for every $b,c,d,f,g,h \in Z_2$.

$F(a,0,c,d,e,0,g,h) = F(a,1,c,d,e,1,g,h)$,for every $a,c,d,e,g,h \in Z_2$.

$G(a,0,c,d,e,0,g,h) = G(a,1,c,d,e,1,g,h)$,for every $a,c,d,e,g,h \in Z_2$.

$F(a,b,0,d,e,f,0,h) = F(a,b,1,d,e,f,1,h)$,for every $a,b,d,e,f,h \in Z_2$

$G(a,b,0,d,e,f,0,h) = G(a,b,1,d,e,f,1,h)$,for every $a,b,d,e,f,h \in Z_2$

$F(a,b,c,0,e,f,g,0) = F(a,b,c,1,e,f,g,1)$,for every $a,b,c,e,f,g \in Z_2$

$G(a,b,c,0,e,f,g,0) = G(a,b,c,1,e,f,g,1)$,for every $a,b,c,e,f,g \in Z_2$

By exhaust search, there are 25600 sets of{ $d^{(1)}_{0,0}, d^{(1)}_{0,1}, d^{(1)}_{0,2}, d^{(1)}_{0,3}, d^{(1)}_{0,4}$,

$d^{(1)}_{1,0}, d^{(1)}_{1,1}, d^{(1)}_{1,2}, d^{(1)}_{1,3}, d^{(1)}_{1,4}$, $d^{(1)}_{2,0}, d^{(1)}_{2,1}, d^{(1)}_{2,2}, d^{(1)}_{2,3}, d^{(1)}_{2,4}$, $d^{(2)}_{0,0}, d^{(2)}_{0,1}, d^{(2)}_{0,2}, d^{(2)}_{0,3}, d^{(2)}_{0,4}$,

$d^{(2)}_{1,0}, d^{(2)}_{1,1}, d^{(2)}_{1,2}, d^{(2)}_{1,3}, d^{(2)}_{1,4}$, $d^{(2)}_{2,0}, d^{(2)}_{2,1}, d^{(2)}_{2,2}, d^{(2)}_{2,3}, d^{(2)}_{2,4}$} satisfying the above equations.

Some of the coefficients we found are shown here.

0 0 0 1 1	0 0 0 1 1	0 1 1 1 1	0 1 0 1 1	0 1 0 1 1	1 0 1 1 1,
0 0 1 0 1	0 1 1 1 1	0 1 1 1 1	0 1 1 0 1	0 0 1 1 1	1 1 1 0 1,
0 1 0 0 1	0 1 1 1 1	0 1 1 1 1	0 1 1 0 1	1 1 0 1 1	1 1 0 1 1,
0 1 1 0 0	0 1 1 1 1	1 1 1 1 1	0 1 1 1 0	1 1 1 0 1	0 1 1 0 1,
0 1 1 0 1	1 1 1 1 1	1 1 1 1 1	1 1 1 0 1	0 1 1 1 1	0 1 1 0 1,
1 0 0 1 1	1 1 1 1 1	1 1 1 1 1	1 0 1 1 1	0 1 0 1 1	0 1 0 1 1,
1 0 1 0 1	1 1 1 1 1	0 0 1 0 1	1 0 1 1 1	1 1 1 0 1	1 1 1 0 1,
1 1 0 0 1	0 1 1 1 1	1 1 0 0 1	0 1 0 1 1	0 1 1 0 1	0 1 0 1 1,
1 1 1 1 0	0 0 1 1 1	0 1 1 1 0	0 0 1 1 0	0 1 1 1 1	1 0 1 1 0,
1 1 1 1 1	0 0 0 1 1	0 1 1 1 1	1 0 1 1 1	0 1 0 1 1	0 1 0 1 1.

For the 64-QAM, if offset pairs are the Case VI, by exhaust search, there are 10048 sets coefficients satisfying the related equations. In the following,we list some of the coefficients.

0 0 0	0 0 1	0 1 0	0 0 0	1 0 1	1 1 0,
0 0 0	0 1 1	1 1 1	1 0 1	0 0 1	0 0 1,
0 1 0	1 1 0	1 0 1	0 1 1	1 0 0	1 1 1,
0 1 1	0 0 1	1 1 1	0 0 1	0 1 1	0 1 1,
0 1 1	0 1 1	0 1 1	1 1 0	1 0 1	1 1 0,
1 0 0	0 1 1	1 0 0	1 0 1	0 0 1	0 1 0,
1 0 1	0 0 1	0 0 1	1 1 1	0 1 1	1 0 0,
1 1 0	0 0 1	0 0 1	1 1 1	0 1 1	1 0 0,
1 1 1	0 1 1	0 0 1	1 1 1	0 1 1	1 0 1,

1 1 1 0 1 1 0 1 1 0 1 0 1 0 1 0 1 0.

7 Conlusion

We have proposed a new method to judge whether the sequences constructed using new offset pairs over the QAM constellation are Golay complementary sequences. Based on this method and new offset pairs, we construct new Golay complementary sequences over 16-QAM constellation and 64-QAM constellation. By exhaust search,we also find 3840 offset pairs with discrete four terms and 736 offset pairs with continue two terms for constructing Golay complementary sequences over 16-QAM constellation, 25600 offset pairs with discrete four terms and 10048 offset pairs with continue two terms for constructing new Golay complementary sequences over 64-QAM constellation. Based on the method proposed in this paper, one can find many new Golay complementary sequences over M^2-QAM constellation and $2M$-Q-PAM constellation.

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