

# Strongly Secure Certificate-Based Encryption Scheme with Low Communication Bandwidth

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**Abstract:** Certificate-based encryption (CBE) is a new asymmetric encryption paradigm which was introduced to solve the certificate management problem in traditional public-key encryption (PKE). It combines PKE and identity-based encryption (IBE) while preserving some of their most attractive features. CBE provides an efficient implicit certificate mechanism which eliminates the third-party queries and simplifies the certificate revocation problem in the traditional PKI. It also solves the key escrow problem and the key distribution problem inherent in IBE. In this paper, we first present a new security model for CBE, which defines the public key replacement attack and strengthens the power of adversaries against CBE. Our model is more elaborated and stronger compared with other existing ones. We then propose an efficient CBE scheme which is proved to be secure in the proposed security model under the random oracle model. When compared with other existing CBE schemes, our scheme enjoys better performance, especially on the communication bandwidth.

**Keywords:** certificate-based encryption; security model; public key replacement attack; random oracle model

## 1 Introduction

In Eurocrypt 2003, Gentry [1] introduced the notion of certificate-based encryption (CBE) to solve the certificate management problem in traditional public-key encryption (PKE). This new asymmetric encryption paradigm combines identity-based encryption (IBE) and PKE while preserving some of their most attractive features. As in the traditional PKI, each user in CBE generates his own public/private key pair and requests a certificate from a trusted third party, which is called as the certifier. The certifier generates a certificate as in a traditional PKI and is responsible for pushing a fresh certificate only to the holder of the public key periodically. A certificate in CBE has all the functionalities of a traditional PKI certificate, and also acts as a partial decryption key. This additional functionality provides an implicit certificate mechanism so that the sender is not required to obtain fresh information on certificate status and the receiver can only decrypt the ciphertext using his private key along with an up-to-date certificate from its certifier. The feature of implicit certificate allows us to eliminate third-party queries for the certificate status and to simplify the public key revocation problem so that CBE does not need infrastructures like CRL and OCSP. Therefore, CBE can be used to construct a more efficient PKI requiring fewer infrastructures. Furthermore, there is no key escrow problem (since the certifier does not know the private keys of users) and key distribution problem (since the certificates need

not be kept secret) in CBE.

In the original work [1], Gentry constructed the first CBE scheme in the random oracle [2] from the BF-IBE scheme [3]. A subsequent work by Yum and Lee [4] provided a formal equivalence theorem among IBE, certificateless public key encryption (CL-PKE) [5] and CBE, and showed that IBE implies both CBE and CL-PKE by giving a generic construction from IBE to those primitives. However, Galindo et al. [6] pointed out that a dishonest authority could break the security of their generic constructions. Actually, these generic constructions were inherently flawed due to a naive use of double encryption without further treatments. In [7], Lu et al. solved this problem by using the Fujisaki-Okamoto conversions [8, 9] and gave methods to achieve generic CCA-secure CBE constructions from PKE and IBE in the random oracle model. Moreover, Lu et al. [10] also proposed generic techniques to build CBE schemes from PKE and IBE in the standard model. In 2005, Al-Riyami and Paterson [11] gave an analysis of Gentry's CBE concept and repaired a number of problems in the original definition and security model for CBE. They also presented a generic conversion from CL-PKE to CBE and claimed that a secure CBE scheme could be constructed from any secure CL-PKE scheme using this conversion. Kang and Park [12] pointed out that their conversion was incorrect due to the flaw in their security proof. This implies that the derived CBE scheme by Al-Riyami and Paterson [11] is therefore invalid. In [13], Yum and Lee proposed a separable implicit certificate revocation system called status CBE to relieve the certifier's burden of certificate revocation, in which the authenticity of a public key is guaranteed by a long-lived certificate and the certificate revocation problem is resolved by a short-lived certificate. However, their status CBE scheme is insecure under the public key replacement attack [14]. In 2006, Morillo and Ràfols [15] proposed the first concrete CBE scheme in the standard model from the Waters-IBE scheme [16] and the BB-IBE scheme [17]. Subsequently, Galindo et al. [18] revised this CBE scheme and gave an improved scheme. In 2008, Liu and Zhou [19] proposed another CBE scheme in the standard model from the Gentry-IBE scheme [20]. Recently, Lu et al. [21] proposed a quite efficient CBE scheme in the random oracle model from the SK-IBE scheme [22, 23], which requires computing only one pairing in the encryption algorithm.

*Our Contributions.* The main contributions of this paper are twofold. We first present a new security model for CBE. As we know, a reasonable and elaborated security model is necessary for constructing provably secure cryptographic schemes. In [11], Al-Riyami and Paterson refined the original security model of CBE and proposed a revised one, but their model is still not elaborated and strong enough. Inspired by the recent improvements in the definition of security notions for certificate-based signature [24], we repair those problems still in the security model by Al-Riyami and Paterson, and strengthen the power of the adversaries against CBE, then propose a more elaborated and stronger security model for CBE. Our definition will provide a systematic approach for constructing CBE schemes secure against the public key replacement attack and proving their security on a stronger security level. The second contribution is that we construct an efficient CBE scheme with low communication bandwidth and prove it to be secure against adaptive chosen-ciphertext attack in the proposed security model. The proposed CBE scheme requires computing only one bilinear pairing in the decryption algorithm and introduces no redundancies in ciphertexts. When compared with other existing ones, our CBE scheme enjoys better performances both on the computation efficiency and the communication bandwidth, while its security is proved in a stronger security model.

## 2 Preliminaries

In this section, we review some basic concepts, including bilinear map, underlying hard problems and chosen-ciphertext secure symmetric encryption.

### 2.1 Bilinear Map and Underlying Hard Problems

Throughout the paper,  $G_1$  denotes an additive cyclic group of prime order  $q$  and  $G_2$  denotes a multiplicative cyclic group of the same order. For a group  $G$ , we use  $G^*$  to denote the set  $G \setminus \{O\}$  where  $O$  is the identity element in the group  $G$ . We let  $P$  denote a generator of  $G_1$ . For us, a bilinear paring is a map  $e: G_1 \times G_1 \rightarrow G_2$  with following properties:

- Bilinearity:  $\forall P_1, P_2 \in G_1, \forall a, b \in \mathbb{Z}_q^*$ , we have  $e(aP_1, bP_2) = e(P_1, P_2)^{ab}$ .
- Non-degeneracy:  $e(P, P) \neq 1$ .
- Computability:  $\forall P_1, P_2 \in G_1, e(P_1, P_2)$  can be efficiently computed.

The security of the CBE scheme in this paper is based on the hardness of the computational Diffie-Hellman (CDH) problem and the gap bilinear Diffie-Hellman (Gap-BDH) problem.

**Definition 1.** The CDH problem in  $G_1$  is, given a tuple  $(P, aP, bP)$  where  $a, b \in \mathbb{Z}_q^*$ , to compute  $abP \in G_1$ .

Let  $\mathcal{B}$  be an adversary against the hardness of the CDH problem.  $\mathcal{B}$ 's advantage to solve the CDH problem is defined to be  $Adv(\mathcal{B}) = \Pr[\mathcal{B}(P, aP, bP) = abP]$ .

**Definition 2 [25].** The Gap-BDH problem in  $(G_1, G_2)$  is, given a tuple  $(P, aP, bP, cP)$  where  $a, b, c \in \mathbb{Z}_q^*$ , to compute  $e(P, P)^{abc}$  with the help of a DBDH oracle  $\mathcal{O}_{DBDH}(\cdot)$  that takes  $(P, aP, bP, cP, T)$  as input and outputs 1 if  $T = e(P, P)^{abc}$  and 0 otherwise.

Let  $\mathcal{B}$  be an adversary against the hardness of the Gap-BDH problem.  $\mathcal{B}$ 's advantage to solve the Gap-BDH problem is defined to be  $Adv(\mathcal{B}) = \Pr[\mathcal{B}(P, aP, bP, cP, \mathcal{O}_{DBDH}(\cdot)) = e(P, P)^{abc}]$ .

### 2.2 Chosen-Ciphertext Secure Symmetric Encryption

A symmetric encryption scheme is specified by an encryption algorithm  $E$  and a decryption algorithm  $D$ . Let  $\lambda$  be a security parameter. Algorithm  $E$  takes a symmetric key  $K \in \{0,1\}^\lambda$  and a message  $M$  as input, and returns a ciphertext  $C = E(K, M)$ . Algorithm  $D$  takes  $K \in \{0,1\}^\lambda$  and a ciphertext  $C$  as input, and returns either a message  $M = D(K, C)$  or a special symbol  $\perp$  denoting a decryption failure. Here, we restrict ourselves to the deterministic algorithms  $E$  and  $D$ .

Next, we briefly review the notion of chosen-ciphertext security for a symmetric encryption scheme described in [26].

**Definition 3.** A symmetric encryption scheme  $(E, D)$  is said to be IND-CCA secure if no PPT adversary  $\mathcal{A}$  has non-negligible advantage in the following game:

**Setup:** The challenger randomly chooses a key  $K \in \{0,1\}^\lambda$ .

**Phase 1:**  $\mathcal{A}$  issues a series of queries to the encryption oracle and the decryption oracle. The challenger responds these queries by using the key  $K$ .

**Challenge:**  $\mathcal{A}$  outputs two plaintexts  $M_0$  and  $M_1$  which were not submitted to the encryption oracle or obtained from the decryption oracle. The challenger chooses a random bit  $\beta \in \{0,1\}$  and encrypts  $M_\beta$  under the key  $K$ , then outputs the resulting ciphertext  $C^*$  to  $\mathcal{A}$ .

**Phase 2:**  $\mathcal{A}$  issues more queries as in Phase 1, but with the restrictions that  $C^*$  can not be submitted to the decryption oracle and  $M_0, M_1$  can not be submitted to the encryption oracle.

**Guess:**  $\mathcal{A}$  outputs a guess  $\beta' \in \{0,1\}$  for  $\beta$  and wins the game if  $\beta = \beta'$ .  $\mathcal{A}$ 's advantage in this game is defined to be  $Adv(\mathcal{A}) = 2|\Pr[\beta = \beta'] - \frac{1}{2}|$ .

### 3 Certificate-Based Encryption

In this section, we will first review the definition of CBE schemes. Then, we will redefine a new security model for CBE.

#### 3.1 Definition of CBE

The following definition of CBE is essentially modified from [11], where the original definition given in [1] was reconsidered.

**Definition 4.** A certificate-based encryption (CBE) scheme consists of five algorithms:

- **Setup** is a probabilistic algorithm run by a certifier that takes as input a security parameter  $k$  to output a master key  $msk$  and a list of public parameters  $params$  that include the descriptions of a finite plaintext space  $MSPC$  and a finite ciphertext space  $CSPC$ . We consider  $params$  to be an implicit input to the rest of the algorithms.
- **UserKeyGen** is a probabilistic algorithm run by a user that takes as input an identity  $id$  to output a public/private key pair  $(PK_{id}, SK_{id})$ .
- **Certify** is a deterministic or probabilistic algorithm run by a certifier that takes as input a master key  $msk$ , an index  $\tau$  of the current time period, an identity  $id$  and a public key  $PK_{id}$ . It outputs a certificate  $Cert_{id,\tau}$  which is sent to the user  $id$  through an open channel.
- **Encrypt** is a probabilistic algorithm that takes as input an index  $\tau$  of the current time period, an identity  $id$ , a public key  $PK_{id}$  and a plaintext  $M \in MSPC$  to output a ciphertext  $C \in CSPC$ .
- **Decrypt** is a deterministic algorithm that takes as input a private key  $SK_{id}$ , a certificate  $Cert_{id,\tau}$  and a ciphertext  $C$  to output either a message  $M \in MSPC$  or a special symbol  $\perp$  indicating a decryption failure.

#### 3.2 Security Model

In [1] and [11], the security models of CBE schemes are both defined by two different types of adversary: Type I and Type II. A Type I adversary models a malicious third party (i.e. anyone except the legitimate receiver or the certifier) who does not have access to the master key, while a Type II adversary models an honest-but-curious certifier who has access to the master key. Although Al-Riyami and Paterson [11] have revised Gentry's security model, their model is still not elaborated enough. For example, the Type I adversary in their model is defined to have access to the target user's public/private key pair, but is not allowed to obtain the target user's certificate. However, in practice such an adversary may attack a CBE scheme by the target user's public key and certificate without knowing the corresponding private key. Moreover, this type adversary is required providing a private key along with the corresponding public key in all of decryption oracle queries. It enables the challenger to handle these decryption queries. This restriction is unnecessary and also restricts the ability of the adversary. Actually, the challenger can handle decryption queries using special purpose knowledge extractors without requiring the adversary to provide the private key. For another example, their security model does not capture the public key replacement attack. It seems that the public key replacement attack does not exist in CBE due to the use of certificates. However, in CBE, only the owner needs to check the validity of its certificate and other users do not need. Therefore, such attack actually does exist. A concrete example is the status CBE scheme proposed by Yum and Lee [13]. In [14], this scheme is pointed out to be insecure under the public key replacement attack, although it was proved to be secure

without considering such attack.

In the following, we define a more elaborated and stronger security model for CBE. Our model also distinguishes two types of adversary. Below, we first define the oracles that an adversary against CBE may query and how each query should be responded by a challenger.

- **Public Key Request:** On input an identity  $id$ , the challenger responds with the public key  $PK_{id}$  for  $id$ . If the identity  $id$  has no associated public key, then the challenger generates a public key  $PK_{id}$  for  $id$  by running **UserKeyGen**.
- **Public Key Replacement:** In our security model, an adversary can repeatedly replace the public key of any entity with a key of its choice. On input an identity  $id$  and a key  $PK'_{id}$ , the challenger replaces the current public key  $PK_{id}$  for  $id$  with  $PK'_{id}$ . Note that the current value of an entity's public key is used by the challenger in any computations or responses to the adversary's requests. This oracle models the adversary's ability to convince a legitimate user to use an invalid public key, and then enables our security model to capture the public key replacement attack.
- **Certificate Request:** On input an index  $\tau$  of a time period and an identity  $id$ , the challenger responds with the certificate  $Cert_{id,\tau}$ . If the identity  $id$  has no associated certificate in the time period  $\tau$ , then the challenger generates a certificate  $Cert_{id,\tau}$  by running **Certify** (after running **UserKeyGen** if necessary).
- **Private Key Extraction:** On input an identity  $id$ , the challenger responds with the private key  $SK_{id}$ . If the identity  $id$  has no associated private key, then the challenger generates a private key  $SK_{id}$  by running **UserKeyGen**. However, it is unreasonable to expect the challenger to respond to such a query if the public key for  $id$  has already been replaced.
- **Decryption:** On input an index  $\tau$  of a time period, an identity  $id$ , and a ciphertext  $C$ , the challenger responds with the correct decryption of  $C$ , even if the public key for  $id$  has been replaced. This is a rather strong property for the security model of CBE. After all, the challenger may no longer know the correct corresponding private key. However, this capability may give the adversary more power in breaking the scheme. For further discussion of this feature (but in CL-PKE setting), see [5]. Note that the security models in [1, 11] only allow the adversaries to query a weak decryption oracle which additionally requires taking the public key and the private key for  $id$  as input.

Next, we provide a detailed description of two adversary types in our security model.

**Type I Adversary:** Such an adversary  $\mathcal{A}_I$  does not have access to the master key, but is allowed to query the following oracles: **Public Key Request**, **Certificate Request**, **Private Key Extraction**, **Public Key Replacement** and **Decryption**. The restrictions on this type of adversary are that:

- $\mathcal{A}_I$  can not request both the private key for the challenge identity  $id^*$  and the certificate for the challenge identity  $id^*$  in the challenge time period  $\tau^*$  at any point.
- $\mathcal{A}_I$  can not extract the private key for any identity if the corresponding public key has been replaced.
- $\mathcal{A}_I$  can not request the certificate for the challenge identity  $id^*$  in the challenge time period  $\tau^*$  if the public key for  $id^*$  has been replaced before the challenge was issued.
- $\mathcal{A}_I$  can not make a decryption query on the challenge ciphertext  $C^*$  for the combination  $(id^*, \tau^*)$ .

**Type II Adversary:** Such an adversary  $\mathcal{A}_{II}$  has access to the master key, and is also allowed to query the following oracles: **Public Key Request**, **Private Key Extraction**, **Public Key Replacement** and **Decryption**. The restrictions on this type of adversary are that:

- $\mathcal{A}_H$  cannot extract the private key for the challenge identity  $id^*$  at any point.
- $\mathcal{A}_H$  cannot extract the private key for any identity if the corresponding public key has been replaced.
- $\mathcal{A}_H$  cannot output a challenge identity  $id^*$  for which it has replaced the public key.
- $\mathcal{A}_H$  cannot make a decryption query on the challenge ciphertext  $C^*$  for the combination  $(id^*, \tau^*)$ .

**Definition 5.** A CBE scheme is said to be IND-CBE-CCA2 secure if no polynomially bounded adversary  $\mathcal{A}$  of Type I or Type II has a non-negligible advantage in the following game:

**Setup:** The challenger  $\mathcal{C}$  runs  $Setup(1^k)$  to generate a master key  $msk$  and a list of public system parameters  $params$ , then outputs  $params$  to  $\mathcal{A}$ . If  $\mathcal{A}$  is of Type II,  $\mathcal{C}$  also outputs  $msk$  to  $\mathcal{A}$ . Otherwise, it keeps  $msk$  to himself.

**Phase 1:** In this phase,  $\mathcal{A}$  may have access to certain oracles defined above. These oracle queries are asked adaptively, but are subject to the restrictions described above.

**Challenge:** Once  $\mathcal{A}$  decides that Phase 1 is over, it outputs an index  $\tau^*$  of a time period, an identity  $id^*$  and two equal length plaintexts  $M_0, M_1$ , on which it wants to be challenged. Again, the restrictions described above apply.  $\mathcal{C}$  randomly chooses a bit  $b \in \{0, 1\}$ , computes the challenge ciphertext  $C^* = \text{Encrypt}(params, \tau^*, id^*, PK_{id^*}, M_b)$ , and then outputs  $C^*$  to  $\mathcal{A}$ .

**Phase 2:** As in Phase 1, but with the restrictions described above.

**Guess:** Finally,  $\mathcal{A}$  outputs a guess  $b' \in \{0, 1\}$  and wins the game if  $b = b'$ . We define the advantage of the adversary  $\mathcal{A}$  in this game to be  $Adv(\mathcal{A}) := 2|\Pr[b = b'] - \frac{1}{2}|$ .

## 4 Our CBE Scheme

In this section, we propose a new CBE scheme and prove it to be IND-CBE-CCA2 secure under the hardness of the CDH problem and Gap-BDH problem in our proposed security model. We also make a detailed comparison of our scheme and other existing CBE schemes on the computation efficiency and the communication bandwidth.

### 4.1 Concrete Construction

Our scheme is constructed based on a hybrid variant of the BF-IBE scheme proposed by Libert and Quisquater [26]. It is described as follows:

**Setup:** On input a security parameter  $k$ , this algorithm performs as follows:

- (1) Generate a pair of groups  $(G_1, G_2)$  of prime order  $q$  and a bilinear map  $e: G_1 \times G_1 \rightarrow G_2$ .
- (2) Choose a generator  $P \in G_1$ , randomly select  $s \in \mathbb{Z}_q^*$  and compute  $P_{pub} = sP$ .
- (3) Choose two cryptographic hash functions  $H_1: \{0,1\}^* \rightarrow G_1^*$  and  $H_2: (G_1)^3 \times G_2 \rightarrow \{0,1\}^\lambda$ , as well as an IND-CCA secure symmetric encryption scheme  $(E, D)$  of keylength  $\lambda$ , where  $\lambda$  is polynomial in  $k$ .

(4) Output  $msk = s$  as the master key and  $params = \{G_1, G_2, e, q, P, P_{pub}, H_1, H_2, \lambda, E, D, n\}$  as the public parameters, where  $n$  denotes a bound on the size of plaintexts. The plaintext space is  $MSPC = \{0,1\}^n$  and the ciphertext space is  $CSPC = G_1 \times \{0,1\}^n$ .

**UserKeyGen:** On input  $\langle params, id \rangle$ , this algorithm randomly chooses  $x \in \mathbb{Z}_q^*$  and outputs  $(PK_{id}, SK_{id}) = (xP, x)$  as the public/private key pair for the identity  $id$ .

**Certify:** On input  $\langle params, msk, \tau, id, PK_{id} \rangle$ , this algorithm computes  $Q_{id} = H_1(\tau, id, PK_{id})$  and outputs  $Cert_{id,\tau} = sQ_{id}$  as the certificate for the identity  $id$  in the time period  $\tau$ .

**Encrypt:** On input  $\langle params, \tau, id, PK_{id}, M \rangle$ , this algorithm performs as follows:

- (1) Randomly choose  $r \in Z_q^*$  and compute  $U = rP$ .
- (2) Compute  $Q_{id} = H_1(\tau, id, PK_{id})$  and  $SK_{sym} = H_2(Q_{id}, U, rPK_{id}, e(P_{pub}, Q_{id})^r)$ .
- (3) Compute  $V = E(SK_{sym}, M)$  and output  $C = (U, V)$  as the ciphertext.

Note that encryption needs to compute one pairing for a receiver in each time period. However, once  $e(P_{pub}, Q_{id})$  has been pre-computed, it does not require any pairing computations to send messages for the identity  $id$  in the whole time period  $\tau$ .

**Decrypt:** On input  $\langle params, Cert_{id, \tau}, SK_{id}, C = (U, V) \rangle$ , this algorithm computes  $SK_{sym} = H_2(Q_{id}, U, SK_{id} \cdot U, e(U, Cert_{id, \tau}))$  and outputs  $M = D(SK_{sym}, V)$ .

The consistency of our scheme is easy to check as we have

$$\begin{aligned} SK_{sym} &= H_2(Q_{id}, U, SK_{id} \cdot U, e(U, Cert_{id, \tau})) \\ &= H_2(Q_{id}, U, xrP, e(rP, sQ_{id})) \\ &= H_2(Q_{id}, U, rPK_{id}, e(P_{pub}, Q_{id})^r). \end{aligned}$$

## 4.2 Security Analysis

**Theorem 1.** Suppose that  $H_1, H_2$  are random oracles and  $\mathcal{A}_I$  is a Type I adversary against the IND-CBE-CCA2 security of our CBE scheme with advantage  $\varepsilon$  when running in time  $t$ , making  $q_{pub}$  public key requests,  $q_{pri}$  private key requests,  $q_{cer}$  certificate requests,  $q_{dec}$  decryption queries and  $q_i$  random oracle queries to  $H_i$  ( $i = 1, 2$ ). Then, for any  $0 \leq \nu \leq \varepsilon$ , there exists

- either an algorithm  $\mathcal{B}$  to solve the Gap-BDH problem with advantage  $\varepsilon' \geq \frac{\varepsilon}{q_1} - \nu$  and

running in time  $t' \leq t + (q_1 + q_{pub} + q_{pri} + q_{cer})\tau_{mul} + q_2\phi + q_{dec}(\tau_{sym} + 2\tau_{pair})$ , where  $\tau_{mul}$  and  $\tau_{pair}$  respectively denote the time for computing a multiplication in  $G_1$  and the one for a pairing, while  $\tau_{sym}$  and  $\phi$  respectively denote the time for a symmetric decryption and the one for a call to the DBDH oracle.

- or an adversary that breaks the chosen-ciphertext security of the symmetric encryption scheme  $(E, D)$  with advantage  $\nu$  within time  $t$ .

**Proof.** We show how to construct an algorithm  $\mathcal{B}$  to solve the Gap-BDH problem from the Type I adversary  $\mathcal{A}_I$ . Assume that the algorithm  $\mathcal{B}$  is given a random Gap-BDH instance  $(P, aP, bP, cP, \mathcal{O}_{DBDH})$ .  $\mathcal{B}$  finds  $e(P, P)^{abc}$  by interacting with  $\mathcal{A}_I$  as follows:

**Setup:**  $\mathcal{B}$  randomly chooses an index  $I$  with  $1 \leq I \leq q_1$  and sets  $P_{pub} = aP$ . It then supplies  $\mathcal{A}_I$  with  $params = \{G_1, G_2, e, q, P, P_{pub}, H_1, H_2, \lambda, \mathcal{E}, \mathcal{D}, n\}$  as the public parameters, where  $H_1$  and  $H_2$  are random oracles controlled by  $\mathcal{B}$ . Note that the corresponding master key  $msk$  is  $a$  which is unknown to  $\mathcal{B}$ .

$\mathcal{A}_I$  may make queries to  $H_1$  and  $H_2$  at any time during its attack and  $\mathcal{B}$  responds as follows:

**$H_1$  queries:**  $\mathcal{B}$  maintains a list  $H_1List$  of tuples  $\langle id_i, \tau_i, PK_{id_i}, \delta_i, Q_{id_i} \rangle$  which is initially empty.

On receiving such a query on  $(id_i, \tau_i, PK_{id_i})$ ,  $\mathcal{B}$  does the following:

- If  $\langle id_i, \tau_i, PK_{id_i}, \delta_i, Q_{id_i} \rangle$  exists in  $H_1List$ , then  $\mathcal{B}$  returns  $Q_{id_i}$ .
- Otherwise, if  $i = I$ ,  $\mathcal{B}$  adds  $\langle id_i, \tau_i, PK_{id_i}, ?, cP \rangle$  to  $H_1List$  and returns  $cP$ ; else if  $i \neq I$ ,  $\mathcal{B}$  randomly chooses  $\delta_i \in Z_q^*$ , computes  $Q_{id_i} = \delta_i P$ , and then adds  $\langle id_i, \tau_i, PK_{id_i}, \delta_i, Q_{id_i} \rangle$  to  $H_1List$  and returns  $Q_{id_i}$ .

**$H_2$  queries:**  $\mathcal{B}$  maintains three following lists which are initially empty.

- $H_2List-A$  contains tuples  $\langle Q_{id_i}, U_i, A_i, T_i, SK_i \rangle$  for which  $\mathcal{B}$  has assigned a value  $SK_i \in \{0, 1\}^\lambda$  to  $H_2(Q_{id_i}, U_i, A_i, T_i)$ .

- H<sub>2</sub>List-B contains tuples  $\langle Q_{id_i}, U_i, A_i, T_i \rangle$  such that  $\langle Q_{id_i}, U_i, A_i, T_i, SK_i \rangle$  exists in H<sub>2</sub>List-A for  $SK_i \in \{0,1\}^\lambda$  and  $\mathcal{O}_{DBDH}(P, P_{pub}, U_i, Q_{id_i}, T_i) = 1$ .
- H<sub>2</sub>List-C contains tuples  $\langle Q_{id_i}, U_i, A_i, SK_i \rangle$  for which  $\mathcal{B}$  has implicitly assigned a value  $SK_i \in \{0,1\}^\lambda$  to  $H_2(Q_{id_i}, U_i, A_i, T_i)$  although the value  $T_i$  such that  $\mathcal{O}_{DBDH}(P, P_{pub}, U_i, Q_{id_i}, T_i) = 1$  is unknown.

More concretely, when receiving such a query on  $(Q_{id_i}, U_i, A_i, T_i)$ ,  $\mathcal{B}$  does the following:

- If  $\langle Q_{id_i}, U_i, A_i, T_i, SK_i \rangle$  exists in H<sub>2</sub>List-A for some value  $SK_i \in \{0,1\}^\lambda$ ,  $\mathcal{B}$  returns  $SK_i$ .
- Otherwise,  $\mathcal{B}$  submits  $(P, P_{pub}, U_i, Q_{id_i}, T_i)$  to the oracle  $\mathcal{O}_{DBDH}(\cdot)$  to decide whether it is a valid BDH tuple.
  - If it does,  $\mathcal{B}$  adds  $\langle Q_{id_i}, U_i, A_i, T_i \rangle$  to H<sub>2</sub>List-B and checks if  $\langle Q_{id_i}, U_i, A_i, SK_i \rangle$  exists in H<sub>2</sub>List-C. If it does,  $\mathcal{B}$  adds  $\langle Q_{id_i}, U_i, A_i, T_i, SK_i \rangle$  to H<sub>2</sub>List-A and returns  $SK_i$ .
  - Otherwise,  $\mathcal{B}$  randomly chooses  $SK_i \in \{0,1\}^\lambda$ , adds  $\langle Q_{id_i}, U_i, A_i, T_i, SK_i \rangle$  to H<sub>2</sub>List-A and returns  $SK_i$ .

**Phase 1:**  $A_I$  issues a series of oracle queries as defined in Definition 5, and  $\mathcal{B}$  responds these queries as follows:

**Public Key Request:**  $\mathcal{B}$  maintains a list **KeyList** of tuples  $\langle id_i, PK_{id_i}, SK_{id_i}, flag_i \rangle$  which is initially empty. On receiving such a query on  $id_i$ ,  $\mathcal{B}$  does the following:

- If  $\langle id_i, PK_{id_i}, SK_{id_i}, flag_i \rangle$  exists in **KeyList**,  $\mathcal{B}$  returns  $PK_{id_i}$ .
- Otherwise,  $\mathcal{B}$  randomly chooses  $x_i \in Z_q^*$  as the private key for  $id_i$  and computes  $x_i P$  as the corresponding public key. It then adds  $\langle id_i, x_i P, x_i, 0 \rangle$  to **KeyList** and returns  $x_i P$ .

**Private Key Extraction:** On receiving such a query on  $id_i$ ,  $\mathcal{B}$  does the following:

- If  $\langle id_i, PK_{id_i}, SK_{id_i}, flag_i \rangle$  exists in **KeyList** and  $flag_i = 1$ , namely that the public key for  $id_i$  has been replaced, then  $\mathcal{B}$  rejects this query; else if  $flag_i = 0$ ,  $\mathcal{B}$  returns  $SK_{id_i}$ .
- Otherwise,  $\mathcal{B}$  randomly chooses  $x_i \in Z_q^*$ , adds  $\langle id_i, x_i P, x_i, 0 \rangle$  to **KeyList** and returns  $x_i$ .

**Public Key Replacement:** On receiving such a query on  $(id_i, PK'_{id_i})$ ,  $\mathcal{B}$  searches  $id_i$  in the list **KeyList** and updates the resulting tuple  $\langle id_i, PK_{id_i}, SK_{id_i}, flag_i \rangle$  with  $\langle id_i, PK'_{id_i}, SK_{id_i}, 1 \rangle$ .

**Certificate Request:**  $\mathcal{B}$  maintains a list **CertList** of tuples  $\langle id_i, \tau_i, Cert_{id_i, \tau_i} \rangle$  which is initially empty. On receiving such a query on  $(id_i, \tau_i)$ ,  $\mathcal{B}$  does the following:

- If  $i = I$ ,  $\mathcal{B}$  aborts.
- Otherwise,  $\mathcal{B}$  checks whether  $\langle id_i, \tau_i, Cert_{id_i, \tau_i} \rangle$  exists in **CertList**.
  - If it does,  $\mathcal{B}$  returns  $Cert_{id_i, \tau_i}$ .
  - Otherwise,  $\mathcal{B}$  searches H<sub>1</sub>List for a tuple  $\langle id_i, \tau_i, PK_{id_i}, \delta_i, Q_{id_i} \rangle$  (after running the simulation algorithm for  $H_1$  query if necessary) and computes  $Cert_{id_i, \tau_i} = \delta_i P_{pub}$ . It then adds  $\langle id_i, \tau_i, Cert_{id_i, \tau_i} \rangle$  to **CertList** and returns  $Cert_{id_i, \tau_i}$ .

**Decryption:** On receiving such a query on  $(id_i, \tau_i, C = (U, V))$ ,  $\mathcal{B}$  does the following:

- If  $i \neq I$  and the public key for  $id_i$  has not been replaced, then  $\mathcal{B}$  decrypts  $C$  in the normal way by using the corresponding private key  $SK_{id_i}$  and the certificate  $Cert_{id_i, \tau_i}$ .
- Otherwise,  $\mathcal{B}$  checks if  $\langle Q_{id_i}, U, A_i, T_i \rangle$  exists in H<sub>2</sub>List-B for some value  $T_i \in G_2$  satisfying  $e(U, PK_{id_i}) = e(A_i, P)$ .



- If it does,  $\mathcal{B}$  retrieves  $SK_i$  from the tuple  $\langle Q_{id_i}, U, A_i, T_i, SK_i \rangle$  in  $H_2\text{List-A}$  and returns  $M = D(SK_i, V)$ .
- Otherwise,  $\mathcal{B}$  checks whether  $H_2\text{List-C}$  contains a tuple  $\langle Q_{id_i}, U, A_i, SK_i \rangle$  for some value  $SK_i \in \{0,1\}^\lambda$  satisfying  $e(U, PK_{id_i}) = e(A_i, P)$ . If it does,  $\mathcal{B}$  returns  $M = D(SK_i, V)$ ; Otherwise,  $\mathcal{B}$  randomly chooses  $SK_i \in \{0,1\}^\lambda$ , adds  $\langle Q_{id_i}, U, T_i, SK_i \rangle$  to  $H_2\text{List-C}$  and returns  $M = D(SK_i, V)$ .

**Challenge Phase:**  $\mathcal{A}_I$  outputs  $id^*, \tau^*$  and two messages  $M_0, M_1$  on which it wants to be challenged. If  $(id^*, \tau^*) \neq (id_I, \tau_I)$ ,  $\mathcal{B}$  aborts. Otherwise,  $\mathcal{B}$  does the following:

- Set  $U^* = bP$ , retrieve  $SK_{id^*}$  from the tuple  $\langle id^*, PK_{id^*}, SK_{id^*}, flag^* \rangle$  in  $\text{KeyList}$ , and compute  $A^* = SK_{id^*} \cdot U^*$ .
- Check whether  $\langle Q_{id^*}, U^*, A^*, SK^* \rangle$  exists in  $H_2\text{List-C}$  for some value  $SK^* \in \{0,1\}^\lambda$  (satisfying  $e(U^*, PK_{id^*}) = e(A^*, P)$  if the public key for  $id^*$  has been replaced).
  - If it does,  $\mathcal{B}$  computes  $V^* = E(SK^*, M_\beta)$  for a random bit  $\beta \in \{0,1\}$ .
  - Otherwise,  $\mathcal{B}$  randomly chooses a value  $SK^* \in \{0,1\}^\lambda$ , adds  $\langle Q_{id^*}, U^*, A^*, SK^* \rangle$  to  $H_2\text{List-C}$ , and computes  $V^* = E(SK^*, M_\beta)$  for a random bit  $\beta \in \{0,1\}$ .
- Output  $C^* = (U^*, V^*)$  as the challenge ciphertext.

**Phase 2:** As in Phase 1, with the restrictions specified in Definition 5.

**Guess:** Finally,  $\mathcal{A}_I$  outputs a guess  $\beta' \in \{0,1\}$  for  $\beta$ .  $\mathcal{B}$  ignores the result and checks whether  $\langle Q_{id^*}, U^*, A^*, T^* \rangle$  exists in  $H_2\text{List-B}$ . If it does,  $\mathcal{B}$  outputs  $T^*$  as the solution to the Gap-BDH problem; otherwise, it loses the game. From the above simulation, it is easy to deduce that  $T^* = e(P, P)^{abc}$  which is the right solution to the Gap-BDH problem.

Next, we estimate  $\mathcal{B}$ 's advantage in solving the Gap-BDH problem. Let  $\text{AskH}_2^*$  denote the event that  $\langle Q_{id^*}, U^*, A^*, T^* \rangle$  has been queried to the random oracle  $H_2$ ,  $\text{BreakV}^*$  the event that  $V^*$  is broken as the challenge ciphertext in the chosen-ciphertext attack against the symmetric encryption scheme  $(E, D)$ , and  $\neg\text{Abort}$  the event that  $\mathcal{B}$  does not abort during the simulation.

Now, we define an event  $\mathbf{E}$  to be  $(\text{AskH}_2^* \wedge \text{BreakV}^*) | \neg\text{Abort}$ . It is clear that if  $\mathbf{E}$  does not happen, then  $\mathcal{A}_I$  does not gain any advantage greater than  $1/2$  to guess  $\beta$ . Namely, we have the probability  $\Pr[\beta' = \beta | \neg\mathbf{E}] \leq 1/2$ . Hence, by splitting  $\Pr[\beta' = \beta]$ , we obtain

$$\begin{aligned} \Pr[\beta' = \beta] &= \Pr[\beta' = \beta | \neg\mathbf{E}] \cdot \Pr[\neg\mathbf{E}] + \Pr[\beta' = \beta | \mathbf{E}] \cdot \Pr[\mathbf{E}] \\ &\leq \frac{1}{2} \cdot \Pr[\neg\mathbf{E}] + \Pr[\mathbf{E}] = \frac{1}{2} + \frac{1}{2} \Pr[\mathbf{E}]. \end{aligned}$$

By the definition of  $\varepsilon$ , we have

$$\begin{aligned} \varepsilon &\leq 2|\Pr[\beta' = \beta] - \frac{1}{2}| \leq \Pr[\mathbf{E}] \\ &\leq \frac{1}{\Pr[\neg\text{Abort}]} \cdot (\Pr[\text{AskH}_2^*] + \Pr[\text{BreakV}^*]). \end{aligned}$$

Since  $\Pr[\text{BreakV}^*] \leq v$  and  $\Pr[\neg\text{Abort}] = \frac{1}{q_1}$ , we obtain

$$\Pr[\text{AskH}_2^*] \geq \frac{\varepsilon}{q_1} - v.$$

Hence, we have  $\varepsilon \geq \Pr[\text{AskH}_2^*] \geq \frac{\varepsilon}{q_1} - v$ .

This completes the proof of this theorem.  $\square$

**Theorem 2.** Suppose that  $H_1, H_2$  are random oracles and  $\mathcal{A}_II$  is a Type II adversary against

the IND-CBE-CCA2 security of our CBE scheme with advantage  $\varepsilon$  when running in time  $t$ , making  $q_{pub}$  public key requests,  $q_{pri}$  private key requests,  $q_{dec}$  strong decryption queries and  $q_i$  random oracle queries to  $H_i$  ( $i = 1, 2$ ). Then, for any  $0 \leq \nu \leq \varepsilon$ , there exists

- either an algorithm  $\mathcal{B}$  to solve the CDH problem with advantage  $\varepsilon' \geq \frac{\varepsilon}{q_1} - \nu$  and running in

time  $t' \leq t + (q_{pub} + q_{pri})\tau_{mul} + 2q_2\tau_{pair} + q_{dec}(\tau_{sym} + \tau_{pair})$ , where  $\tau_{mul}$  and  $\tau_{pair}$  respectively denote the time for computing a multiplication in  $G_1$  and the one for a pairing, while  $\tau_{sym}$  denotes the time for a symmetric decryption.

- or an adversary that breaks the chosen-ciphertext security of the symmetric encryption scheme  $(E, D)$  with advantage  $\nu$  within time  $t$ .

**Proof.** We show how to construct an algorithm  $\mathcal{B}$  to solve the CDH problem from the Type II adversary  $\mathcal{A}_{II}$ . Assume that  $\mathcal{B}$  is given a random CDH instance  $(P, aP, bP)$ . It finds  $abP$  by interacting with  $\mathcal{A}_{II}$  as follows:

**Setup:**  $\mathcal{B}$  first randomly chooses an index  $I$  with  $1 \leq I \leq q_1$ . It then randomly chooses  $s \in \mathbb{Z}_q^*$ , computes  $P_{pub} = sP$ , and supplies  $\mathcal{A}_{II}$  with the public parameters  $params = \{G_1, G_2, e, q, P, P_{pub}, H_1, H_2, \lambda, E, D, n\}$  together with the master key  $s$ , where  $H_1$  and  $H_2$  are random oracles controlled by  $\mathcal{B}$ .  $\mathcal{A}_{II}$  may make queries to  $H_1$  and  $H_2$  at any time during its attack and  $\mathcal{B}$  responds as follows:

**H<sub>1</sub> queries:**  $\mathcal{B}$  maintains a list  $H_1List$  of tuples  $\langle id_i, \tau_i, PK_{id_i}, Q_{id_i} \rangle$  which is initially empty.

On receiving such a query on  $(id_i, \tau_i, PK_{id_i})$ ,  $\mathcal{B}$  does the following:

- If  $\langle id_i, \tau_i, PK_{id_i}, Q_{id_i} \rangle$  exists in  $H_1List$ ,  $\mathcal{B}$  returns  $Q_{id_i}$ .
- Otherwise,  $\mathcal{B}$  randomly chooses  $Q_{id_i} \in G_1^*$ , adds  $\langle id_i, \tau_i, PK_{id_i}, Q_{id_i} \rangle$  to  $H_1List$  and returns  $Q_{id_i}$ .

**H<sub>2</sub> queries:**  $\mathcal{B}$  maintains three following lists which are initially empty.

- $H_2List-A$  contains tuples  $\langle Q_{id_i}, U_i, A_i, T_i, SK_i \rangle$  for which  $\mathcal{B}$  has assigned a value  $SK_i \in \{0, 1\}^\lambda$  to  $H_2(Q_{id_i}, U_i, A_i, T_i)$ .
- $H_2List-B$  contains tuples  $\langle Q_{id_i}, U_i, A_i, T_i \rangle$  such that  $\langle Q_{id_i}, U_i, A_i, T_i, SK_i \rangle$  exists in  $H_2List-A$  for  $SK_i \in \{0, 1\}^\lambda$  and  $e(U_i, PK_{id_i}) = e(A_i, P)$ .
- $H_2List-C$  contains tuples  $\langle Q_{id_i}, U_i, T_i, SK_i \rangle$  for which  $\mathcal{B}$  has implicitly assigned a value  $SK_i \in \{0, 1\}^\lambda$  to  $H_2(Q_{id_i}, U_i, A_i, T_i)$  although the value  $A_i$  such that  $e(U_i, PK_{id_i}) = e(A_i, P)$  is unknown.

More concretely, when receiving such a query on  $(Q_{id_i}, U_i, A_i, T_i)$ ,  $\mathcal{B}$  does the following:

- If  $\langle Q_{id_i}, U_i, A_i, T_i, SK_i \rangle$  exists in  $H_2List-A$  for some value  $SK_i \in \{0, 1\}^\lambda$ ,  $\mathcal{B}$  returns  $SK_i$ .
- Otherwise,  $\mathcal{B}$  checks whether  $e(U_i, PK_{id_i}) = e(A_i, P)$ .
  - If it does,  $\mathcal{B}$  adds  $\langle Q_{id_i}, U_i, A_i, T_i \rangle$  to  $H_2List-B$  and checks whether  $\langle Q_{id_i}, U_i, T_i, SK_i \rangle$  exists in  $H_2List-C$ . If it does,  $\mathcal{B}$  adds  $\langle Q_{id_i}, U_i, A_i, T_i, SK_i \rangle$  to  $H_2List-A$  and returns  $SK_i$ .
  - Otherwise,  $\mathcal{B}$  randomly chooses  $SK_i \in \{0, 1\}^\lambda$ , adds  $\langle Q_{id_i}, U_i, A_i, T_i, SK_i \rangle$  to  $H_2List-A$  and returns  $SK_i$ .

**Phase 1:**  $\mathcal{A}_{II}$  issues a series of oracle queries as defined in Definition 5, and  $\mathcal{B}$  responds these queries as follows:

**Public Key Request:**  $\mathcal{B}$  maintains a list  $KeyList$  of tuples  $\langle id_i, PK_{id_i}, SK_{id_i}, flag_i \rangle$  which is initially empty. On receiving such a query on  $id_i$ ,  $\mathcal{B}$  does the following:

- If  $\langle id_i, PK_{id_i}, SK_{id_i}, flag_i \rangle$  exists in  $KeyList$ ,  $\mathcal{B}$  returns  $PK_{id_i}$  to  $\mathcal{A}_{II}$ .

- Otherwise, if  $i \neq I$ ,  $\mathcal{B}$  randomly chooses  $x_i \in \mathbb{Z}_q^*$  as the private key for  $id_i$  and computes  $x_i P$  as the corresponding public key, then adds  $\langle id_i, x_i P, x_i, 0 \rangle$  to **KeyList** and returns  $PK_{id_i}$ ; else if  $i = I$ ,  $\mathcal{B}$  adds  $\langle id_i, aP, ?, 0 \rangle$  to **KeyList** and returns  $aP$ .

**Private Key Extraction:** On receiving such a query on  $id_i$ ,  $\mathcal{B}$  does the following:

- If  $i = I$ ,  $\mathcal{B}$  aborts.
- Otherwise,  $\mathcal{B}$  checks whether  $\langle id_i, PK_{id_i}, SK_{id_i}, flag_i \rangle$  exists in **KeyList**.
  - If it does,  $\mathcal{B}$  rejects this query if  $flag_i = 1$  or returns  $SK_{id_i}$  if  $flag_i = 0$ .
  - Otherwise,  $\mathcal{B}$  randomly chooses  $x_i \in \mathbb{Z}_q^*$ , adds  $\langle id_i, x_i P, x_i, 0 \rangle$  in **KeyList** and returns  $x_i$ .

**Public Key Replacement:** On receiving such a query on  $(id_i, PK'_{id_i})$ ,  $\mathcal{B}$  does the following:

- If  $i = I$ ,  $\mathcal{B}$  aborts.
- Otherwise,  $\mathcal{B}$  searches  $id_i$  in **KeyList** and updates the resulting tuple  $\langle id_i, PK_{id_i}, SK_{id_i}, flag_i \rangle$  with  $\langle id_i, PK'_{id_i}, SK_{id_i}, 1 \rangle$ .

**Decryption:** On receiving such a query on  $(id_i, \tau_i, C = (U, V))$ ,  $\mathcal{B}$  does the following:

- If  $i \neq I$  and the public key for  $id_i$  has not been replaced, then  $\mathcal{B}$  decrypts  $C$  in the normal way by using the corresponding private key and the certificate  $(SK_{id_i}, Cert_{id_i, \tau_i})$ .
- Otherwise,  $\mathcal{B}$  computes  $T_i = e(U, Cert_{id_i, \tau_i})$  and checks whether  $\langle Q_{id_i}, U, A_i, T_i \rangle$  exists in **H<sub>2</sub>List-B** for some value  $A_i \in G_1$ .
  - If it does,  $\mathcal{B}$  retrieves  $SK_i$  from the tuple  $\langle Q_{id_i}, U, A_i, T_i, SK_i \rangle$  in **H<sub>2</sub>List-A** and returns  $M = D(SK_i, V)$ .
  - Otherwise,  $\mathcal{B}$  checks whether  $\langle Q_{id_i}, U, A_i, SK_i \rangle$  exists in **H<sub>2</sub>List-C** for some value  $SK_i \in \{0,1\}^\lambda$ . If it does,  $\mathcal{B}$  returns  $M = D(SK_i, V)$ . Otherwise,  $\mathcal{B}$  randomly chooses  $SK_i \in \{0,1\}^\lambda$ , adds  $\langle Q_{id_i}, U, T_i, SK_i \rangle$  to **H<sub>2</sub>List-C** and returns  $M = D(SK_i, V)$ .

**Challenge Phase:**  $\mathcal{A}_{II}$  outputs  $id^*$ ,  $\tau^*$  and two messages  $M_0, M_1$  on which it wants to be challenged. If  $(id^*, \tau^*) \neq (id_I, \tau_I)$ ,  $\mathcal{B}$  aborts. Otherwise,  $\mathcal{B}$  does the following:

- Set  $U^* = bP$  and then check whether  $\langle Q_{id^*}, U^*, T^*, SK^* \rangle$  exists in **H<sub>2</sub>List-C** for some value  $SK^* \in \{0,1\}^\lambda$  satisfying  $T^* = e(U^*, Cert_{id^*, \tau^*})$ .
  - If it does,  $\mathcal{B}$  computes  $V^* = E(SK^*, M_\beta)$  for a random bit  $\beta \in \{0,1\}$ .
  - Otherwise,  $\mathcal{B}$  randomly chooses  $SK^* \in \{0,1\}^\lambda$ , adds  $\langle Q_{id^*}, U^*, T^*, SK^* \rangle$  to **H<sub>2</sub>List-C**, and then computes  $V^* = E(SK^*, M_\beta)$  for a random bit  $\beta \in \{0,1\}$ .
- Output  $C^* = (U^*, V^*)$  as the challenge ciphertext.

**Phase 2:** As in Phase 1, with the restrictions specified in Definition 5. Note that  $\mathcal{A}_{II}$  can make the public key replacement query on the challenge identity  $id^*$  in this phase.

**Guess:** Finally,  $\mathcal{A}_{II}$  outputs a guess  $\beta' \in \{0,1\}$  for  $\beta$ .  $\mathcal{B}$  ignores the result and checks whether  $\langle Q_{id^*}, U^*, A^*, T^* \rangle$  exists in the list **H<sub>2</sub>List-B**. If it does, it outputs  $A^*$  as the solution to the CDH problem. Otherwise, it will lose the game. From the above simulation, it is easy to deduce that  $A^* = abP$  which is the right solution to the CDH problem.

Using the same method in the proof of Theorem 1, it is not difficult to deduce that  $\mathcal{B}$ 's advantage in solving the CDH problem is  $\varepsilon' \geq \frac{\varepsilon}{q_1} - \nu$ , where  $\nu$  denotes the advantage of the adversary against the chosen-ciphertext security of the symmetric encryption scheme  $(E, D)$ .

This completes the proof of this theorem.  $\square$

By combining Theorem 1 and Theorem 2, we can deduce that:

**Corollary 1.** If the symmetric encryption scheme  $(E, D)$  is IND-CCA secure, then our CBE scheme is IND-CBE-CCA2 secure under the hardness of the CDH problem and the Gap-BDH problem in the random oracle model.

### 4.3 Efficiency Analysis and Comparison

In this section, we will make a comparison of our CBE scheme and other existing ones. In the computation cost comparison, we consider three major operations: Pairing ( $p$ ), Multiplication ( $m$ ), and Exponentiation ( $e$ ). As usual, all symmetric operations are ignored. Some CBE schemes need a one-time signature scheme or a combination of an encapsulation scheme and a message authentication code to guarantee the IND-CBE-CCA2 security. We denote the signing algorithm and the verification algorithm in the signature scheme by  $(Sign, Vfy)$ . In the communication cost comparison, ciphertext expansion represents the length difference between the ciphertext and the plaintext. The length of a string  $X$  is denoted by  $|X|$ . We denote the public commitment string and the de-commitment string by  $com$  and  $dec$  respectively of an encapsulation scheme, the message authentication code by  $mac$  of a message authentication code, and the verification key and the signature by  $vk$  and  $\sigma$  respectively of a one-time signature scheme. In [1] and [21],  $l$  should be at least 160 in order to obtain a reasonable security. Furthermore, “rom” means random oracle model while “standard” means standard model. Considering the pre-computation, the detailed performances of all the CBE schemes are listed in Table 1.

**Table 1. Efficiency Comparison of the CBE Schemes**

Scheme	Model	Computation cost		Communication cost		
		Encryption	Decryption	Ciphertext expansion	Public key	Certificate
[1]	rom	$2p+1m+1e$	$1p+1m$	$ G_1 +l$	$ G_1 $	$ G_1 $
[15]	standard	$4m+2e$	$3p+3m$	$3 G_1 + dec + com + mac $	$2 G_1 $	$2 G_1 $
[18]	standard	$5m+2e+Sign$	$3p+3m+Vfy$	$3 G_1 + vk + \sigma $	$2 G_1 $	$2 G_1 $
[19]	standard	$2m+7e$	$2p+1m+2e$	$2 G_2 + G_1 $	$6 G_1 $	$3 G_1 +3 Z_q $
[21]	rom	$2m+2e$	$1p+1m+1e$	$ G_1 +l$	$ G_2 $	$ G_1 $
Ours	rom	$2m+1e$	$1p+1m$	$ G_1 $	$ G_1 $	$ G_1 $

From the table, we can see that our scheme has better performances both on the computation efficiency and the communication bandwidth, while its security is proved in a stronger security model. What is worth mentioning is that our scheme introduces non redundancies in ciphertexts and has short public keys and short certificates. So, our scheme is more suitable for the bandwidth limited network.

## 5 Conclusion

In this paper, we define a new security model of CBE schemes which strengthens the power of adversaries against CBE and captures the public key replacement attack. Our model is more elaborated and stronger by comparison with other existing security models of CBE schemes. We also proposed a new CBE scheme and proved it to be IND-CBE-CCA2 secure in the proposed security model. When compared with the existing CBE schemes, our scheme enjoys better performances both on the computation efficiency and the communication bandwidth, while its security reaches a stronger level. However, the security of our scheme is only proved in the random oracle model. Therefore, how to design a strongly secure and efficient CBE scheme in the

standard model is an interesting open problem.

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