# Efficient and provably-secure certificateless signature scheme 

# without bilinear pairings 

He Debiao*, Chen Jianhua, Zhang Rui<br>School of Mathematics and Statistics, Wuhan University, Wuhan, Hubei, China, 430072


#### Abstract

Many certificateless signature schemes using bilinear pairings have been proposed. But the relative computation cost of the pairing is approximately twenty times higher than that of the scalar multiplication over elliptic curve group. In order to improve the performance we propose a certificateless signature scheme without bilinear pairings. With the running time being saved greatly, our scheme is more practical than the previous related schemes for practical application.


Key words: Certificateless public key cryptography; Certificateless signature; Bilinear pairings; Elliptic curve

## 1. Introduction

Public-key cryptography(PKC) has become one of the essential techniques in providing security services in modern communications. In traditional public-key cryptosystems, a pair of public/private keys should be computed by each user. Since the public key is a string of random bits, a digital certificate of the public key is required to provide public-key authentication. Anyone who wants to send messages to others must obtain their authorized certificates that contain the public key. However, this requirement brings lots of certificate management problems in practice.

In order to simplify the public-key authentication, Shamir [1] introduced the concept of identity-based (ID-based) cryptosystem problem. In this system, each user needs to register at a key generator centre (KGC) with identify of himself before joining the network. Once a user is accepted, the KGC will generate a private key for the user and the user's identity (e.g. user's name or email address) becomes the corresponding public key. In this way, in order to verify a digital signature or send an encrypted message, a user only needs to know the "identity" of his communication partner and the public key of the KGC. However, this cryptosystem involves a KGC, which is responsible for generating a user's private key based on his identity. As a result, the KGC can literally decrypt any ciphertext or forge any user's signature on any message. To avoid the inherent key escrow problem in ID-based public key cryptosystem, Al-Riyami and Paterson [2] introduced a new approach called certificateless public key cryptography (CLPKC). The CLPKC is intermediate between traditional PKC and ID-based cryptosystem. In a certificateless cryptosystem, a user's private key is not generated by the KGC alone. Instead, it consists of partial private key generated by the KGC and some secret value chosen by the user. So, the KGC is unable to obtain the user's private key. In such a way that the key escrow problem can be solved. Intuitionally, CLPKC has nice features borrowed from both ID-based cryptography and traditional PKC. It alleviates the key escrow problem in ID-based cryptography and at the same time reduces the cost and simplifies the use of the technology when compared with traditional

[^0]PKC.
Following the pioneering work due to Al-Riyami and Paterson [2], several certificateless signature (CLS) schemes [3-10] have been proposed. All the above CLS schemes may be practical, but they are from bilinear pairings and the pairing is regarded as the most expensive cryptography primitive. The relative computation cost of a pairing is approximately twenty times higher than that of the scalar multiplication over elliptic curve group [11]. Therefore, CLS schemes without bilinear pairings would be more appealing in terms of efficiency.

In this paper, we present a CLS scheme without pairings. The scheme rests on the elliptic curve discrete logarithm problem (ECDLP).With the pairing-free realization, the scheme's overhead is lower than that of previous schemes [3-10] in computation.

## 2. Preliminaries

### 2.1.Background of elliptic curve group

Let the symbol $E / F_{p}$ denote an elliptic curve $E$ over a prime finite field $F_{p}$, defined by an equation

$$
\begin{equation*}
y^{2}=x^{3}+a x+b, \quad a, b \in F_{p} \tag{1}
\end{equation*}
$$

and with the discriminant

$$
\begin{equation*}
\Delta=4 a^{3}+27 b^{2} \neq 0 . \tag{2}
\end{equation*}
$$

The points on $E / F_{p}$ together with an extra point $O$ called the point at infinity form a group

$$
\begin{equation*}
G=\left\{(x, y): x, y \in F_{p}, E(x, y)=0\right\} \cup\{O\} . \tag{3}
\end{equation*}
$$

Let the order of $G$ be $n$. $G$ is a cyclic additive group under the point addition "+" defined as follows: Let $P, Q \in G, l$ be the line containing $P$ and $Q$ (tangent line to $E / F_{p}$ if $P=Q$ ), and $R$, the third point of intersection of $l$ with $E / F_{p}$. Let $l^{\prime}$ be the line connecting $R$ and $O$. Then $P$ " + " $Q$ is the point such that $l$ ' intersects $E / F_{p}$ at $R$ and $O$ and P " + " Q . Scalar multiplication over $\mathrm{E} / \mathrm{Fp}$ can be computed as follows:

$$
\begin{equation*}
t P=P+P+\cdots+P(t \text { times }) \tag{4}
\end{equation*}
$$

The following problem defined over $G$ is assumed to be intractable within polynomial time.

Eliptic curve discrete logarithm problem(ECDLP): For $x \in_{R} Z_{n}^{*}$ and $P$ the generator of
$G$, given $Q=x \cdot P$ compute $x$.

### 2.2.Certificateless signatures

A CLS scheme consists of seven algorithms[2]: Setup, Partial-Private-Key-Extract, Set-Secret-Value, Set-Private-Key, Set-Public-Key, Sign and Verify.

Setup: Taking security parameter $k$ as input and returns the system parameters params and master key.

Partial-Private-Key-Extract: This algorithm takes params, master key and a user's identity $I D$ as inputs and returns a partial private key $d_{I D}$.

Set-Secret-Value: This algorithm params and a user's identity ID as input and generates a secret value $r$.

Set-Private-Key: This algorithm takes params, a user's partial private key $d_{I D}$ and his secret value $r$ as inputs and outputs the full private key $s k_{I D}$.

Set-Public-Key: This algorithm takes params and a user's secret value $r$ as inputs and generates a public key $p k_{I D}$ for the user.

Sign: This algorithm takes params, a message m, a user's identity $I D$, and the user's private key $s k_{I D}$ as inputs and outputs a signature $S$.

Verify: This algorithm takes params, a public key $p k_{I D}$, a message $m$, a user's identity $I D$, and a signature $S$ as inputs and returns 1 means that the signature is accepted. Otherwise, 0 means rejected.

### 2.3.Security model for certificateless signatures

In CLS, as defined in [2], there are two types of adversaries with different capabilities, we assume Type 1 Adversary, $\mathscr{\mathscr { H }} 1$ acts as a dishonest user while Type 2 Adversary, $\mathscr{\mathscr { t }} 2$ acts as a malicious KGC:

Type 1 Adversary: Adversary $\mathscr{\mathscr { H }} 1$ does not have access to the master key, but $\mathscr{\mathscr { H }} 1$ can replace the public keys of any entity with a value of his choice, since there is no certificate involved in CLS.

Type 2 Adversary: Adversary $\mathscr{\mathscr { C }} 2$ has access to the master key, but cannot replace any user's public key.

Definition 1. Let $\mathscr{\mathscr { A }} 1$ and $\mathscr{\mathscr { C }} 2$ be a Type1Adversaryanda Type2Adversary, respectively. We consider two games Game 1 and Game 2 where $\mathscr{\mathscr { H }} 1$ and $\mathscr{\mathscr { H }} 2$ interact with its challenger in these
two games, respectively. We say that a CLS scheme is existentially unforgeable against adaptive chosen message attacks, if the success probability of both $\mathscr{\mathscr { A }} 1$ and $\mathscr{\mathscr { C }} 2$ is negligible.

Game 1: This is the game where $\mathscr{\mathscr { t }} 1$ interacts with its challenger $\mathscr{G}$ :
Setup: The challenger $\mathcal{G}$ takes a security parameter $k$ and runs Setup to generate master key and params, then sends params to $\mathscr{\mathscr { t }} 1 . \mathscr{\mathscr { C }} 1$ acts as the following oracle queries:

Hash Queries: $\mathscr{\mathscr { t }} 1$ can request the hash values for any input.
Extract Partial Private Key: $\mathscr{\mathscr { C }} 1$ is able to ask for the partial private key $d_{I D}$ for any $I D$ except the challenged identity $I D$. G computes the partial private key $d_{I D}$ corresponding to the identity $I D$ and returns $d_{I D}$ to $\mathscr{\mathscr { C }} 1$.

Extract Private Key: For any $I D$ except the challenged identity $I D, ~ G ~ c o m p u t e s ~ t h e ~$ private key $s k_{I D}$ corresponding to the identity $I D$ and returns $s k_{I D}$ to $\mathscr{\mathscr { A }} 1$.

Request Public Key: Upon receiving a public key query for any identity ID, 飞 computes the corresponding public key $p k_{I D}$ and sends it to $\mathscr{\mathscr { C }} 1$.

Replace Public Key: For any identity $I D, \mathscr{C} 1$ can pick a new secret value $r^{\prime}$ and compute the new public $p k_{I D}^{\prime}$ corresponding to the value $r^{\prime}$, and then replace $p k_{I D}$ with $p k_{I D}^{\prime}$.

Signing Queries: When a signing query for an identity $I D$ on some message $m$ is coming, T uses the private key $s k_{I D}$ corresponding to the identity $I D$ to compute the signature $S$ and sends it to $\mathscr{\mathscr { H }} 1$. If the public key $p k_{I D}$ has been replaced by $\mathscr{\mathscr { A }} 1$, then $\mathbb{G}$ cannot find $s k_{I D}$ and thus the signing oracle's answer may be incorrect. In such case, we assume that $\mathscr{\mathscr { H }} 1$ additionally submits the secret value $r^{\prime}$ corresponding to the replaced public key $s k_{I D}$ to the signing oracle.

Finally, $\mathscr{\mathscr { H }} 1$ outputs a signature $S^{*}$ on a message $m^{*}$ corresponding to a public key $p k_{I D^{*}}$ for an identity $I D^{*}$ which is the challenged identity $I D . \mathscr{\mathscr { C }} 1$ wins the game if $\operatorname{Verify}\left(\right.$ params, $\left.I D^{*}, m^{*}, p k_{I D^{*}}, S^{*}\right)=1$ and the following conditions hold:

- Extract private key on identity $I D^{*}$ has never been queried.
- $I D^{*}$ can not be an identity for which both the public key has been replaced and the partial private key has been extracted.
- Signing query on message $m^{*}$ for identity $I D^{*}$ with respect to $p k_{I D^{*}}$ has never been queried.
Game 2: This is a game in which $\mathscr{\mathscr { C }} 2$ interacts with its challenger $\mathcal{G}$.

Setup: T runs Setup to generate a master key and params. T gives both params and the master key to $\mathscr{\mathscr { H }} 2$.

Extract Private Key: For any identity $I D$ except the challenged $I D, \mathfrak{G}$ computes the private key $s k_{I D}$ corresponding to the identity $I D$ and returns $s k_{I D}$ to $\mathscr{\mathscr { t }} 2$.

Request Public Key: Upon receiving a public key query for any $I D, \mathcal{G}$ computes the corresponding public key $p k_{I D}$ and sends it to $\mathscr{\mathscr { C }} 2$.

Signing Queries: On receiving such a query on a identity $I D, \mathcal{G}$ uses the private key $s k_{I D}$ corresponding to the identity $I D$ to compute the signature $S$ and sends it to $\mathscr{\mathscr { A }} 2$.

Finally, $\mathscr{\mathscr { C }} 2$ outputs a signature $S^{*}$ on a message $m^{*}$ corresponding to a public key $p k_{I D^{*}}$ for an identity $I D^{*}$ which is the challenged identity $I D . \mathscr{\mathscr { C }} 2$ wins the game if the following conditions hold:

- Verify (params, $\left.I D^{*}, m^{*}, p k_{I D^{*}}, S^{*}\right)=1$
- $\operatorname{Sign}\left(I D^{*}, m^{*}\right)$ with respect to $p k_{I D^{*}}$ has been never queried.
- Extract Private Key on $I D^{*}$ has never been queried.


## 3. Our scheme

### 3.1.Scheme Description

In this section, we present an ID-based signature scheme without pairing. Our scheme consists of four algorithms: Setup, Extract, Sign, and Verify.

Setup: This algorithm takes a security parameter $k$ as input, returns system parameters and a master key. Given $k$, KGC does as follows.

1) Choose a $k$-bit prime $p$ and determine the tuple $\left\{F_{p}, E / F_{p}, G, P\right\}$ as defined in Secttion 2.1.
2) Choose the master private key $x \in Z_{n}^{*}$ and compute the master public key

$$
P_{p u b}=x \cdot P .
$$

3) Choose two cryptographic secure hash functions $H_{1}:\{0,1\}^{*} \rightarrow Z_{n}^{*}$ and

$$
H_{2}:\{0,1\}^{*} \rightarrow Z_{n}^{*}
$$

4) Publish params $=\left\{F_{p}, E / F_{p}, G, P, P_{\text {pub }}, H_{1}, H_{2}\right\}$ as system parameters and keep
the master key $X$ secretly.
Partial-Private-Key-Extract: This algorithm takes system parameters, master key and a user's identifier as input, returns the user's ID-based private key. With this algorithm, KGC works as follows for each user with identifier $I D_{U}$.
5) Choose at random $r_{I D} \in Z_{n}^{*}$, compute $R_{I D}=r_{I D} \cdot P$ and $h_{I D}=H_{1}\left(I D, R_{I D}\right)$.
6) Compute $s_{I D}=r_{I D}+h_{U} x \bmod n$.

The user's s partial private key is the tuple $d_{I D}=\left\{s_{I D}, R_{I D}\right\}$ and he can validate her private key by checking whether the equation $S_{I D} \cdot P=R_{I D}+h_{I D} \cdot P_{p u b}$ holds. The private key is valid if the equation holds and vice versa.

Set-Secret-Value: The user with identity $I D$ picks randomly $s_{I D}^{\prime} \in Z_{n}^{*}$ sets $s_{I D}^{\prime}$ as his secret value.

Set-Private-Key: Given params, the user's partial private key $d_{I D}$ and his secret value $s_{I D}^{\prime}$ and output a pair ( $\left.d_{I D}, s_{I D}^{\prime}\right)$ as the user's private key.

Set-Public-Key: This algorithm takes params, the user's secret value $s_{I D}^{\prime}$ as inputs, and generates the user's public key as $p k_{I D}=s_{I D}^{\prime} \cdot P$.

Sign: This algorithm takes system parameters, user's partial private key $d_{I D}$, user's secret value $s_{I D}^{\prime}$, and a message $m$ as inputs, returns a signature of the message $m$. The user does as follows.

1) Choose at random $l \in Z_{n}^{*}$ to compute $R=l \cdot P$.
2) Compute $h=H_{2}\left(m, R, p k_{I D}\right)$.
3) Verify whether the equation $\operatorname{gcd}(l+h, n)=1$ holds. If the equation does not hold, return to step 1).
4) Compute $s=(l+h)^{-1}\left(s_{I D}+s_{I D}^{\prime}\right) \bmod n$.
5) The resulting signature is $\left(R_{I D}, R, s\right)$.

Verify: To verify the signature ( $R_{I D}, R, s$ ) for message $m$ and identity $I D$, the verifier first computes $h_{I D}=H_{1}\left(I D, R_{I D}\right), h=H_{2}\left(m, R, p k_{I D}\right)$ and then checks whether

$$
\begin{equation*}
s \cdot(R+h \cdot P)=p k_{I D}+R_{I D}+h_{I D} P_{p u b} \tag{5}
\end{equation*}
$$

Accept if it is equal. Otherwise reject.
Since $R=l \cdot P, s_{I D}=r_{I D}+h_{U} x \bmod n$ and $s=(l+h)^{-1}\left(s_{I D}+s_{I D}^{\prime}\right) \bmod n$, we have

$$
\begin{align*}
& s \cdot(R+h \cdot P)=(l+h)^{-1}\left(s_{I D}+s_{I D}^{\prime}\right) \cdot(l \cdot P+h \cdot P) \\
& =(l+h)^{-1}\left(s_{I D}+s_{I D}^{\prime}\right) \cdot(l+h) \cdot P=\left(s_{I D}+s_{I D}^{\prime}\right) \cdot P  \tag{6}\\
& =s_{I D}^{\prime} \cdot P+s_{I D} \cdot P=p k_{I D}+R_{I D}+h_{U} \cdot P_{p u b}
\end{align*}
$$

Then the correctness of our scheme is proved.

### 3.2.Security Analysis

We prove the security of our scheme $\Sigma$ in the random oracle model which treats $H_{1}$ and $H_{2}$ as two random oracles [9] using the signature security model defined in [2]. As for the security of $\Sigma$, the following theorem is provided.

Theorem 1. Our scheme is secure against existential forgery under adaptively chosen message attacks in the random oracle model with the ECDLP is intractable.

This theorem follows from the following Lemmas 1 and 2.
Lemma 1. Let $\mathscr{\mathscr { L }} 1$ be a type 1 Adversary in game 1 . Assume $\mathscr{\mathscr { H }} 1$ makes $q_{H_{i}}$ queries to random oracles $H_{i}(\mathrm{i}=1,2)$ and $q_{E}$ queries to the partial private-key extraction oracle and $q_{E}^{\prime}$ queries to the private-key extraction oracle, and $q_{p k}$ queries to the public-key request oracle, and $q_{S}$ queries to signing oracle. If $\mathscr{\mathscr { A }} 1$ can break $\Sigma$ with probability
$\varepsilon \geq 10\left(q_{H_{2}}+1\right)\left(q_{H_{2}}+q_{S}\right) / 2^{k}$, then the ECDLP can be solved within running time $t \leq 23 q_{H_{2}} t / \varepsilon$ and with probability $\varepsilon^{\prime} \geq 1 / 9$.

Proof: Suppose that there is a type 1 Adversar $\mathscr{\mathscr { t }} 1$ for an adaptively chosen message attack against $\Sigma$. Then, we show how to use the ability of $\mathscr{\mathscr { C }} 1$ to construct an algorithm $\not \subset$ solving the ECDLP.

Suppose $F$ is challenged with a ECDLP instance $(P, Q)$ and is tasked to compute $x \in Z_{n}^{*}$ satisfying $Q=x \cdot P$. To do so, Ficks an identity $I D_{I}$ at random as the challenged $I D$ in this game, and gives $\left\{F_{p}, E / F_{p}, G, P, P_{p u b}=Q, H_{1}, H_{2}\right\}$ to $\mathscr{\mathscr { H }} 1$ as the public parameters. Then $\mathscr{F}$ answers $\mathscr{\mathscr { C }}$ 1's queries as follows.

H1-Queries: Fmaintains a hash list $L_{H_{1}}$ of tuple ( $I D_{i}, R_{I D_{i}}, S_{I D_{i}}, h_{I D_{i}}$ ) as explained below. The list is initially empty. When $\mathscr{\mathscr { C }} 1$ makes a hash oracle query on $I D_{i}$, if the query $I D_{i}$ has already appeared on $L_{H_{1}}$, then the previously defined value is returned. Otherwise, $\mathcal{F}$ acts as described in the partial private key extraction queries.

Partial Private Key Extraction Queries: $\mathscr{\mathscr { H }} 1$ is allowed to query the extraction oracle for an identity $I D_{i}$. Fquery $H_{1}$ oracle, $I D_{i}$ is on $L_{H_{1}}$, then $\not \approx$ response with ( $I D_{i}, R_{I D_{i}}, S_{I D_{i}}, h_{I D_{i}}$ ). Otherwise, if simulates the oracle as follows. It chooses $a_{i}, b_{i} \in Z_{n}^{*}$ at random, sets $R_{I D_{i}}=a_{i} \cdot P_{p u b}+b_{i} \cdot P, S_{I D_{i}}=b_{i}, h_{I D_{i}}=H_{1}\left(I D_{i}, R_{I D_{i}}\right) \leftarrow-a_{i} \bmod n$, response with ( $\left.I D_{i}, R_{I D_{i}}, s_{I D_{i}}, h_{I D_{i}}\right)$, and inserts $\left(I D_{i}, R_{I D_{i}}, s_{I D_{i}}, h_{I D_{i}}\right)$ into $L_{H_{1}}$. Note that $\left(R_{I D_{i}}, s_{I D_{i}}, h_{I D_{i}}\right)$ generated in this way satisfies the equation $S_{I D} \cdot P=R_{I D}+h_{I D} \cdot P_{p u b}$ in the partial private key extraction algorithm. It is a valid secret key.

Public Key Extraction Queries: Fmaintains a list $L_{p k}$ of tuple $\left(I D_{i}, s_{I D_{i}}^{\prime}, p k_{I D_{i}}\right)$ which is initially empty. When $\mathscr{\mathscr { C }} 1$ queries on input $I D_{i}$, $\neq$ checks whether $L_{p k}$ contains a tuple for this input. If it does, the previously defined value is returned. Otherwise, Fpicks a random value $s_{I D}^{\prime} \in Z_{n}^{*}$, computes $p k_{I D_{i}}=s_{I D_{i}}^{\prime} \cdot P$ and returns $p k_{I D_{i}}$. Then, adds $\left(I D_{i}, s_{I D_{i}}^{\prime}, p k_{I D_{i}}\right)$ to the $L_{p k}$.

Private Key Extraction Queries: For query on input $I D_{i}$, If $I D_{i}=I D_{I}$, F stops and outputs "failure". Otherwise, Fperforms as follows:

If the $L_{H_{1}}$ and the $L_{p k}$ contain the corresponding tuple ( $I D_{i}, R_{I D_{i}}, s_{I D_{i}}, h_{I D_{i}}$ ) and the tuple $\left(I D_{i}, s_{I D_{i}}^{\prime}, p k_{I D_{i}}\right)$ respectively, $F$ sets $s k_{I D_{i}}=\left\{d_{I D_{i}}, s_{I D_{i}}^{\prime}\right\}$ and sends it to $\mathscr{\mathscr { C }} 1$, where $d_{I D}=\left\{s_{I D}, R_{I D}\right\}$. Otherwise, Fmakes a partial private key extraction query and a public key extraction query on $I D_{i}$, then simulates as the above process and sends $s k_{I D_{i}}=\left\{d_{I D_{i}}, s_{I D_{i}}^{\prime}\right\}$ to $\mathscr{\mathscr { t }} 1$.
 tuple ( $I D_{i}, s_{I D_{i}}^{\prime}, p k_{I D_{i}}$ ) is contained in the $L_{p k}$. If it does, $F$ sets $p k_{I D_{i}}=\widetilde{p k_{I D_{i}}}$ and adds the
tuple $\left(I D_{i}, \widetilde{s_{I D_{i}}^{\prime}}, p k_{I D_{i}}\right)$ to the $L_{p k}$. Here we assume that $\not \subset$ can obtain a replacing secret value $\widetilde{s_{I D_{i}}^{\prime}}$ corresponding to the replacing public key $\widetilde{p k_{I D_{i}}}$ from $\mathscr{\mathscr { C }}$ 1. Otherwise, Fexecutes public key extraction to generate $\left(I D_{i}, s_{I D_{i}}^{\prime}, p k_{I D_{i}}\right)$, then sets $p k_{I D_{i}}=\widetilde{p k_{I D_{i}}}$ and adds $\left(I D_{i}, \widetilde{s_{I D_{i}}^{\prime}}, p k_{I D_{i}}\right)$ to the $L_{p k}$.

H2-Queries: $\neq$ maintains a hash list $L_{H_{2}}$ of tuple ( $m_{j}, R_{j}, I D_{i}, p k_{I D_{i}}, h_{j}$ ). When $\mathscr{\mathscr { C }} 1$ makes $H 2$ queries for identity $I D_{i}$ on the message $m_{j}$, F chooses a random value $h_{j} \in Z_{n}^{*}$, sets $h_{j}=H_{2}\left(m_{j}, R_{j}, p k_{I D_{i}}\right)$ and adds $\left(m_{j}, R_{j}, I D_{i}, p k_{I D_{i}}, h_{j}\right)$ to $L_{H_{2}}$, and sends $h_{j}$ to $\mathscr{\mathscr { H }} 1$.

Signing Queries: When a signing query on $\left(I D_{i}, m_{j}\right)$ is coming, Facts as follows:

If $I D_{i}=I D_{I}$, $\mathscr{F}$ outputs "failure". Otherwise, $\mathscr{F}$ recovers $\left(I D_{i}, R_{I D_{i}}, S_{I D_{i}}, h_{I D_{i}}\right)$ from $L_{H_{1}}$ and $\left(I D_{i}, s_{I D_{i}}^{\prime}, p k_{I D_{i}}\right)$ from $L_{p k}$. Then Fgets the secret key $s k_{I D_{i}}=\left\{d_{I D_{i}}, s_{I D_{i}}^{\prime}\right\}$, he can execute the sign algorithm as described in section 3.1. At last, Fresponse with $\left(R_{I D_{i}}, R_{j}, s_{j}\right)$.

Finally, $\mathscr{\mathscr { H }} 1$ stops and outputs a signature $S^{*}=\left\{R_{I D^{*}}, R_{j}, S_{j}\right\}$ on the message $m^{*}$ with respect to the public key $p k_{I D^{*}}$ for the identity $I D^{*}$, which satisfies the following equation $\operatorname{Verify}\left(\right.$ params, $\left.I D^{*}, m^{*}, p k_{I D^{*}}, S^{*}\right)=1$.

If $I D_{i} \neq I D_{I}$, Foutputs "failure" and aborts. Otherwise, Frecovers the tuple $\left(I D_{i}, R_{I D_{i}}, s_{I D_{i}}, h_{I D_{i}}\right)$ from $L_{H_{1}}$, the tuple $\left(I D_{i}, s_{I D_{i}}^{\prime}, p k_{I D_{i}}\right)$ from $L_{p k}$ and the tuple $\left(m_{j}, R_{j}, I D_{i}, p k_{I D_{i}}, h_{j}\right)$ from $L_{H_{2}}$.

Then, we have

$$
\begin{equation*}
s_{j} \cdot\left(R_{j}+h_{j} \cdot P\right)=p k_{I D_{i}}+R_{I D_{i}}+h_{I D_{i}} P_{p u b} \tag{7}
\end{equation*}
$$

From the forgery lemma[12], if we have a replay of $F$ with the same random tape but different choice of $H_{2}$ will output another valid signatures $\left\{R_{I D_{i}}, R_{j}, s_{j}^{\prime}\right\}$. Then we have

$$
\begin{equation*}
s_{j}^{\prime} \cdot\left(R_{j}+h_{j}^{\prime} \cdot P\right)=p k_{I D_{i}}+R_{I D_{i}}+h_{I D_{i}} P_{p u b} \tag{8}
\end{equation*}
$$

When eliminating $R_{j}$ in the above two equation, we could have

$$
\begin{equation*}
h_{I D_{i}}\left(s_{j}^{\prime}-s_{j}\right) P_{p u b}=s_{j} s_{j}^{\prime}\left(h_{j}-h_{j}^{\prime}\right) P-\left(s_{j}-s_{j}^{\prime}\right)\left(p k_{I D_{i}}+R_{I D_{i}}\right) \tag{9}
\end{equation*}
$$

Let $p k_{I D_{i}}=s_{I D_{i}}^{\prime} \cdot P, R_{I D_{i}}=a_{i} \cdot P_{p u b}+b_{i} \cdot P, P_{p u b}=Q=x \cdot P$, then we have

$$
\begin{equation*}
h_{I D_{i}}\left(s_{j}^{\prime}-s_{j}\right) x=s_{j} s_{j}^{\prime}\left(h_{j}-h_{j}^{\prime}\right)-\left(s_{j}-s_{j}^{\prime}\right)\left(s_{I D_{i}}^{\prime}+a_{i} \cdot x+b_{i}\right) . \tag{10}
\end{equation*}
$$

Hence, we have

$$
\begin{equation*}
\left(h_{I D_{i}}-a_{i}\right)\left(s_{j}^{\prime}-s_{j}\right) x=s_{j} s_{j}^{\prime}\left(h_{j}-h_{j}^{\prime}\right)-\left(s_{j}-s_{j}^{\prime}\right)\left(s_{I D_{i}}^{\prime}+b_{i}\right) . \tag{11}
\end{equation*}
$$

Let $u=\left(\left(h_{I D_{i}}-a_{i}\right)\left(s_{j}^{\prime}-s_{j}\right)\right)^{-1} \bmod n$ and $v=s_{j} s_{j}^{\prime}\left(h_{j}-h_{j}^{\prime}\right)-\left(s_{j}-s_{j}^{\prime}\right)\left(s_{I D_{i}}^{\prime}+b_{i}\right)$, then, we get $x=u v \bmod n$. According to [12, Lemma 4], the ECDLP can be solved with probability $\varepsilon^{\prime} \geq 1 / 9$ and time $t^{\prime} \leq 23 q_{H_{2}} t / \varepsilon$.

Lemma 2. Let $\mathscr{\mathscr { C }} 2$ be a type 2 Adversary in game 2. Assume that, $\mathscr{\mathscr { C }} 2$ makes $q_{H_{i}}$ queries to random oracles $H_{i}(\mathrm{i}=1,2)$ and $q_{E}$ queries to the partial private-key extraction oracle and $q_{E}^{\prime}$ queries to the private-key extraction oracle, and $q_{p k}$ queries to the public-key request oracle, and $q_{S}$ queries to signing oracle. If $\mathscr{\mathscr { C }} 2$ can break $\Sigma$ with probability $\varepsilon \geq 10\left(q_{H_{2}}+1\right)\left(q_{H_{2}}+q_{S}\right) / 2^{k}$, then the ECDLP can be solved within running time $t \leq 23 q_{H_{2}} t / \varepsilon$ and with probability $\varepsilon^{\prime} \geq 1 / 9$.

Proof: Suppose that there is a type 2 Adversar $\mathscr{\mathscr { A }} 2$ for an adaptively chosen message attack against $\Sigma$. Then, we show how to use the ability of $\mathscr{\mathscr { H }} 2$ to construct an algorithm $\mathscr{F}$ solving the ECDLP.

Suppose $\nsim$ is challenged with a ECDLP instance $(P, Q)$ and is tasked to compute $y \in Z_{n}^{*}$ satisfying $Q=y \cdot P$. To do so, $\neq$ randomly picks a value $x \in Z_{n}^{*}$ as the system master key, sets $P_{p u b}=x \cdot P$, picks an an identity $I D_{I}$ at random as the challenged $I D$ in this game, and gives the public parameters $\left\{F_{p}, E / F_{p}, G, P, P_{p u b}, H_{1}, H_{2}\right\}$ and the system master key $x$ to $\mathscr{\mathscr { H }}$ 2. Then $\mathscr{F}$ answers $\mathscr{\mathscr { t }} 1$ 's queries as follows.

H1-Queries: Fmaintains a hash list $L_{H_{1}}$ of tuple ( $I D_{i}, R_{I D_{i}}, h_{I D_{i}}$ ). The list is initially empty. When $\mathscr{\mathscr { C }} 2$ makes a hash oracle query on $I D_{i}$, if the query $I D_{i}$ has already appeared on $L_{H_{1}}$, then the previously defined value is returned. Otherwise, $\mp$ chooses a random value $h_{I D_{i}} \in Z_{n}^{*}$,
sets $h_{I D_{i}}=H_{1}\left(I D_{i}, R_{I D_{i}}\right)$ and adds $\left(I D_{i}, R_{I D_{i}}, h_{I D_{i}}\right)$ to $L_{H_{1}}$, and sends $h_{I D_{i}}$ to $\mathscr{\mathscr { t }} 1$.

Public Key Extraction Queries: $F$ maintains a list $L_{p k}$ of tuple ( $I D_{i}, s_{I D_{i}}^{\prime}, p k_{I D_{i}}$ ) which is initially empty. When $\mathscr{\mathscr { C }} 2$ queries on input $I D_{i}$, F checks whether $L_{p k}$ contains a tuple for this input. If it does, the previously defined value is returned. Otherwise, if $I D_{i}=I D_{I}$, $\neq$ sets $s_{I D_{i}}^{\prime}=\perp, \quad p k_{I D_{i}}=Q$, adds $\left(I D_{i}, s_{I D_{i}}^{\prime}, p k_{I D_{i}}\right)$ to the $L_{p k}$. If $I D_{i} \neq I D_{I}$, Fpicks a random value $s_{I D}^{\prime} \in Z_{n}^{*}$, computes $p k_{I D_{i}}=s_{I D_{i}}^{\prime} \cdot P$ and returns $p k_{I D_{i}}$. Then, adds $\left(I D_{i}, s_{I D_{i}}^{\prime}, p k_{I D_{i}}\right)$ to the $L_{p k}$.

Private Key Extraction Queries: F maintains a list $L_{\text {sk }}$ of tuple $\left(I D_{i}, R_{I D_{i}}, S_{I D_{i}}, h_{I D_{i}}\right)$
which is initially empty. For query on input $I D_{i}$, Ferforms as follows:

If the query $I D_{i}$ has already appeared on $L_{s k}$, then the previously defined value is returned.

Otherwise, Fgenerates a random number $\quad r_{I D_{i}} \in Z_{n}^{*}$, compute $R_{I D_{i}}=r_{I D_{i}} \cdot P$, queries $H_{1}$ oracle to get the value $h_{I D_{i}}=H_{1}\left(I D_{i}, R_{I D_{i}}\right)$ and adds the tuple $\left(I D_{i}, R_{I D_{i}}, S_{I D_{i}}, h_{I D_{i}}\right)$ to $L_{s k}$. Then $F$ makes the public key extraction queries and gets the tuple ( $I D_{i}, s_{I D_{i}}^{\prime}, p k_{I D_{i}}$ ). If $I D_{i}=I D_{I}$, Foutputs "failure". Otherwise, Fsets $s k_{I D_{i}}=\left\{d_{I D_{i}}, s_{I D_{i}}^{\prime}\right\}$ and sends it to $\mathscr{\mathscr { C }} 2$, where $d_{I D}=\left\{\mathrm{s}_{I D}, R_{I D}\right\}$. Otherwise, makes a partial private key extraction query and a public key extraction query on $I D_{i}$, then simulates as the above process and sends $s k_{I D_{i}}=\left\{d_{I D_{i}}, s_{I D_{i}}^{\prime}\right\}$ to $\mathscr{\mathscr { C }}$ 2. and adds $\left(I D_{i}, r_{I D_{i}}, R_{I D_{i}}, S_{I D_{i}}, h_{I D_{i}}\right)$ to $L_{H_{1}}$, and sends $h_{I D_{i}}$ to $\mathscr{C} 2$.

H2-Queries: $\mathscr{F}$ maintains a hash list $L_{H_{2}}$ of tuple $\left(m_{j}, R_{j}, I D_{i}, p k_{I D_{i}}, h_{j}\right)$. When $\mathscr{\mathscr { O }} 2$ makes $H 2$ queries for identity $I D_{i}$ on the message $m_{j}$, $\not \subset$ chooses a random value $h_{j} \in Z_{n}^{*}$, sets $h_{j}=H_{2}\left(m_{j}, R_{j}, p k_{I D_{i}}\right)$ and adds $\left(m_{j}, R_{j}, I D_{i}, p k_{I D_{i}}, h_{j}\right)$ to $L_{H_{2}}$, and sends $h_{j}$ to $\mathscr{\mathscr { C }} 2$.

Signing Queries: When a signing query on $\left(I D_{i}, m_{j}\right)$ is coming, $F$ acts as follows:

If $I D_{i}=I D_{I}$, Foutputs "failure". Otherwise, $\mp$ recovers $\left(I D_{i}, R_{I D_{i}}, S_{I D_{i}}, h_{I D_{i}}\right)$ from $L_{H_{1}}$
and $\left(I D_{i}, s_{I D_{i}}^{\prime}, p k_{I D_{i}}\right)$ from $L_{p k}$. Then $F$ gets the secret key $s k_{I D_{i}}=\left\{d_{I D_{i}}, s_{I D_{i}}^{\prime}\right\}$, he can execute the sign algorithm as described in section 3.1. At last, Fresponse with ( $\left.R_{I D_{i}}, R_{j}, S_{j}\right)$.

Finally, $\mathscr{\mathscr { C }} 2$ stops and outputs a signature $S^{*}=\left\{R_{I D^{*}}, R_{j}, S_{j}\right\}$ on the message $m^{*}$ with respect to the public key $p k_{I D^{*}}$ for the identity $I D^{*}$, which satisfies the following equation $\operatorname{Verify}\left(\right.$ params $\left., I D^{*}, m^{*}, p k_{I D^{*}}, S^{*}\right)=1$.

If $I D_{i} \neq I D_{I}$, Foutputs "failure" and aborts. Otherwise, Frecovers the tuple $\left(I D_{i}, R_{I D_{i}}, s_{I D_{i}}, h_{I D_{i}}\right)$ from $L_{H_{1}}$, the tuple $\left(I D_{i}, s_{I D_{i}}^{\prime}, p k_{I D_{i}}\right)$ from $L_{p k}$ and the tuple $\left(m_{j}, R_{j}, I D_{i}, p k_{I D_{i}}, h_{j}\right)$ from $L_{H_{2}}$.

Then, we have

$$
\begin{equation*}
s_{j} \cdot\left(R_{j}+h_{j} \cdot P\right)=p k_{I D_{i}}+R_{I D_{i}}+h_{I D_{i}} P_{p u b} \tag{7}
\end{equation*}
$$

From the forgery lemma[12], if we have a replay of $\mp$ with the same random tape but different choice of $H_{2}$ will output another valid signatures $\left\{R_{I D_{i}}, R_{j}, s_{j}^{\prime}\right\}$. Then we have

$$
\begin{equation*}
s_{j}^{\prime} \cdot\left(R_{j}+h_{j}^{\prime} \cdot P\right)=p k_{I D_{i}}+R_{I D_{i}}+h_{I D_{i}} P_{p u b} \tag{8}
\end{equation*}
$$

When eliminating $R_{j}$ in the above two equation, we could have

$$
\begin{equation*}
\left(s_{j}-s_{j}^{\prime}\right) p k_{I D_{i}}=s_{j} s_{j}^{\prime}\left(h_{j}-h_{j}^{\prime}\right) P-\left(s_{j}-s_{j}^{\prime}\right) R_{I D_{i}}-h_{I D_{i}}\left(s_{j}^{\prime}-s_{j}\right) P_{p u b} \tag{9}
\end{equation*}
$$

Let $p k_{I D_{i}}=Q=y \cdot P, \quad R_{I D_{i}}=r_{I D_{i}} \cdot P, P_{p u b}=x \cdot P$, then we have

$$
\begin{equation*}
\left(s_{j}-s_{j}^{\prime}\right) y=s_{j} s_{j}^{\prime}\left(h_{j}-h_{j}^{\prime}\right)-\left(s_{j}-s_{j}^{\prime}\right) r_{I D_{i}}-h_{I D_{i}}\left(s_{j}^{\prime}-s_{j}\right) x \tag{10}
\end{equation*}
$$

Let $u=\left(s_{j}-s_{j}^{\prime}\right)^{-1} \bmod n$ and $v=s_{j} s_{j}^{\prime}\left(h_{j}-h_{j}^{\prime}\right)-\left(s_{j}-s_{j}^{\prime}\right) r_{I D_{i}}-h_{I D_{i}}\left(s_{j}^{\prime}-s_{j}\right) x$, then, we get $x=u v \bmod n$. According to [12, Lemma 4], the ECDLP can be solved with probability $\varepsilon^{\prime} \geq 1 / 9$ and time $t^{\prime} \leq 23 q_{H_{2}} t / \varepsilon$.

## 4. Comparison with previous scheme

In this section, we will compare the efficiency of our new scheme with three latest CLS schemes, i.e. Huang et al.'s scheme [8], Tso et al.'s scheme [9] and Du et al.'s scheme [10]. In the computation efficiency comparison, we obtain the running time for cryptographic operations using MIRACAL [13], a standard cryptographic library.

The hardware platform is a PIV 3-GHZ processor with 512-MB memory and a Windows XP operation system. For the pairing-based scheme, to achieve the 1024-bit RSA level security, we use the Tate pairing defined over the supersingular elliptic curve $E / F_{p}: y^{2}=x^{3}+x$ with embedding degree $2 \cdot q$ is a 160 -bit Solinas prime $q=2^{159}+2^{17}+1$ and $p$ a 512-bit prime satisfying $p+1=12 q r$. For the ECC-based schemes, to achieve the same security level, we employed the parameter secp160r1[14], recommended by the Certicom Corporation, where $p=2^{160}-2^{31}-1$. The running times are listed in Table 1 where sca.mul. stands for scalar multiplication.

Table 1. Cryptographic Operation Time(in milliseconds)

| Modular <br> exponentiation | Pairing | Pairing-based <br> sca.mul | ECC-based <br> sca.mul. | Map-to-point <br> hash |
| :---: | :---: | :---: | :---: | :---: |
| 5.31 | 20.04 | 6.38 | 2.21 | 3.04 |

To evaluate the computation efficiency of different schemes, we use the simple method from [15]. For example, the sign algorithm of our scheme requires one ECC-based scale multiplication; thus, the computation time of the sign algorithm is $2.21 \times 1=2.21 \mathrm{~ms}$; the verify algorithm has to carry out three ECC-based scalar multiplications, and the resulting running time is $2.21 \times 3=6.63$ ms. As another example, in Huang et al.'s scheme[8], the sign algorithm should carry out a pairing-based scalar multiplications and a map-to-point hash computation; thus, the computation time for a client is $6.38+3.04=9.42 \mathrm{~ms}$; the verify algorithm has to carry out three pairing, a map-to-point hash computation , then the resulting running time is $20.04 \times 3+3.04=63.16 \mathrm{~ms}$. Table 2 shows the results of the performance comparison.

Table 2. Performance comparison of different schemes

|  | Running time |  |
| :---: | :---: | :---: |
|  | Sign | Verify |
| Huang et al.'s <br> scheme [8] | 9.42 ms | 63.16 ms |
| Tso et al.'s <br> scheme [9] | 5.31 ms | 48.39 ms |
| Du et al.'s <br> scheme [10] | 6.38 ms | 26.40 ms |
| Our scheme | 2.21 ms | 6.63 ms |

According to Table 2, the running time of the sign algorithm of our scheme is $23.46 \%$ of Huang et al.'s schemes, $41.62 \%$ of Tso et al.'s scheme and $34.64 \%$ of Du et al.'s scheme, the running time of the verify algorithm of our scheme is $10.50 \%$ of Huang et al.'s schemes, $13.70 \%$ of Tso et al.'s scheme and $25.12 \%$ of Du et al.'s scheme. Thus our scheme is more useful and efficient than the previous schemes[3-10].

## 5. Conclusion

In this paper, we have proposed an efficient certificateless signature scheme without bilinear pairings. We also prove the security of the scheme under random oracle. Compared with previous scheme, the new scheme reduces both the running time. Therefore, our scheme is more practical than the previous related schemes for practical application.

## 6. References

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[^0]:    *Corresponding author.
    E-mail: hedebiao@163.com, Tel:+0086015307184927

