Universally Composable Key-Management

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Abstract. We present the first universally composable key-management functionality, formalized in the GNUC framework by Hofheinz and Shoup. It allows the enforcement of a wide range of security policies and can be extended by diverse key usage operations with no need to repeat the security proof. We illustrate its use by proving an implementation of a security token secure with respect to arbitrary key-usage operations and explore a proof technique that allows the storage of cryptographic keys externally, a novel development in simulation-based security frameworks.

Keywords: Key management, Security APIs, Universal Composability

1 Introduction

Security critical applications often store keys on dedicated hardware security modules (HSM) or key-management servers to separate highly sensitive cryptographic operations from more vulnerable parts of the network. Access to such devices is given to protocol parties by the means of *Security APIs*. Examples of such APIs are the RSA PKCS#11 standard [1], IBM's CCA [2] and the trusted platform module (TPM) [3] API. Building on the work of Longley and Rigby [4] and Bond and Anderson [5] on API attacks, several recent papers have investigated the security of APIs on the logical level adapting symbolic techniques for protocol analysis [6–8], finding many new attacks. More recent work has tried to define appropriate security notions for APIs in terms of cryptographic games [9, 10]. This approach has two major disadvantages: First, it is not clear how the security notion will compose with other protocols implemented by the API. Second, it is difficult to see whether a definition covers the attack model completely, since the game may be tailored to a specific API. Since security APIs are foremost used as building blocks in other protocols, composability is crucial. In this work, we adapt the more approach to API security of Kremer et al. [10] to a framework that allows for composition.

Composability can be proven in frameworks for simulation-based security, such as GNUC [11], a deviation of the Universal Composability (UC) framework [12]. The requirements of a protocol are formalized by abstraction: an *ideal functionality* computes the protocol's inputs and outputs securely, while a 'secure' protocol is one that emulates the ideal functionality. Simulation-based security naturally models the composition of the API with other protocols, so that proofs of security can be performed in a modular fashion. We decided to use the GNUC model because it avoids shortcomings of the original UC framework which have been pointed out over the years. Moreover, the GNUC framework is well structured and well documented resulting in more rigorous and readable security proofs.

Contributions. We present, to the best of our knowledge, the first composable definition of secure key-management in form of a key-management functionality $\mathcal{F}_{\text{KM}}$. It assures that keys are transferred correctly from one Security API to another, that the ity policy is respected and that operations which use keys are computed correctly. The latter is achieved by describing operations unrelated to key-management by so-called key-usage functionalities. $\mathcal{F}_{\text{KM}}$ is parametric in the policy and the set of key-usage functionalities, which can be arbitrary. This facilitates revision of security devices, because changes to operations that are not part of the key-management or the addition of new functions do not affect the emulation proof. In order to achieve this extensibility, we investigate what exactly a "key" means in simulation-based security. Common functionalities in such settings do not allow two parties to share the same key, in fact, they do not have a concept of keys, but a concept of "the owner of a functionality" instead. The actual key is kept in the internal state of a functionality, used for computation, but never output. Dealing with key-management, we need the capability to export and import keys and we propose an abstraction of the concept of keys, that we call *credentials*. The owner of a credential can not only compute a cryptographic operation, but he can also delegate this capacity by transmitting the credential. We think this concept is of independent interest, and as a further contribution, subsequently introduce a general proof method that allows the substitution of credentials by actual keys when instantiating a functionality. Some aspects of the ideal functionality $\mathcal{F}_{\text{crypto}}$ by Küsters et al. [13] are similar to our key-management functionality in that both provide cryptographic primitives to a number of users and enjoy composability. However, the $\mathcal{F}_{\text{crypto}}$ approach aims at abstracting a specified set of cryptographic operations on client machines to make the analysis of protocols in the simulation-based security models easier, and does not address key-management nor policies.

Limitations. Our key-management functionality is tightly coupled with the employment of deterministic, symmetric authenticated encryption scheme that is secure against key-dependant messages for key export and import. While deterministic, symmetric authenticated encryption is indeed typically used to transfer keys (see, e. g., RFC 3394), it restricts the analysis to security devices providing this kind of encryption. We have not yet covered asymmetric encryption of keys in $\mathcal{F}_{\text{KM}}$ (but we cover asymmetric encryption of user-supplied data), although $\mathcal{F}_{\text{KM}}$ could be extended to support this. Second, adaptive corruption of parties, or of keys that produce an encryption, provokes the well-known commitment problem [14], so we place limitations on the types of corruptions that the environment may produce.

2 Background: GNUC

The GNUC ("GNUC is Not UC") framework was recently proposed by Hofheinz and Shoup [11] as an attempt to address several known shortcomings in UC. In particular, in UC the notion of a poly-time protocol implies that the interface of a protocol has to contain enough input padding to give sub-protocols of the implementation enough running time, hence the definition of an interface that is supposed to be abstract depends on the complexity of its implementation. Moreover, the proof of the composition theorem is flawed due to an inadequate formulation of the composition operation [11], though here the authors remark that, "none of the objections we raise point to gaps in security proofs of existing protocols. Rather, they seem artifacts of the concrete technical formulation of the underlying framework". These shortcomings are also addressed to a greater or lesser extent by other altenative frameworks [15–17]: we chose GNUC because it is similar in spirit to the original UC yet rigorous and well documented. We now give a short introduction to GNUC and refer the reader to [11] for additional details.

2.1 Preliminaries

Definition 1 (probabilistic polynomial-time). *We say that a probabilistic program A runs in polynomial-time, if the probability that A's runtime on an input of length n is bounded by a polynomial in n is 1. If so, we say such a program is PPT.*

Definition 2 (Computationally indistinguishable). *Let $X := \{X_\eta\}_\eta$ and $Y := \{Y_\eta\}_\eta$ be two families of random variables, where each random variable takes values in the set $\Sigma^* \cup \{\bot\}$. We say that X and Y are* computationally indistinguishable*, written $X \approx Y$, if for every PPT program D that takes as input a string over Σ we have that $|\Pr[D(X_\eta) = 1] - \Pr[D(Y_\eta) = 1]|$ is negligible in η.*

2.2 Machines and interaction

In GNUC a protocol π is modeled as a library of programs, that is, a function from protocol names to code. This code will be executed by interactive Turing machines. There are two distinguished machines, the environment and the adversary, that π does not define code for. All other machines are called *protocol machines*. Protocol machines themselves can be divided into two subclasses: *regular* protocol machines and *ideal* protocol machines. They come to life when they are called by the environment and are addressed using machine ids. A machine id contains two parts: the party id, which is of the form `<reg,`*basePID*`>` for regular protocol machines and `<ideal>` for ideal protocol machines, and the session id. The session ids are of the form <α_1, . . . ,α_k>. The last component α_k must be of the particular form *protName*, *sp*. When the environment sends the first message to a protocol machine, a machine running the code defined by the protocol name *protName* is created. The code will often make decisions based on the session parameter *sp* and the party id. A machine M, identified by its machine id $< pid, < \alpha_1, \ldots, \alpha_k >>$, can call a subroutine, i.e., a machine with the machine id $< pid, < \alpha_1, \ldots, \alpha_k, \alpha_{k+1} >>$. M is called the *caller* with respect to this machine. Two protocol machines, regular or ideal, are *peers* if they have the same session id. Programs have to declare which other programs they will call as subroutines, defining a static call graph which must be acyclic and have a program r with in-degree 0 – then we say that the protocol is rooted at r.

GNUC imposes the following communication constraints on a regular protocol machine M: It can only send messages to the adversary, to its ideal peer (i. e., a machine with party id `<ideal>` and the same session id), its subroutines and its caller. If the caller is the environment, a sandbox mechanism translates its machine id, which is simply `<env>` to the machine id of the caller of M (which is uniquely defined). As a consequence, regular protocol machines cannot talk directly to regular peers. They can communicate via the adversary, which models an insecure network, or via the ideal peer. This ideal peer is a party that can communicate directly with all regular protocol parties and the adversary.

2.3 Defining security via ideal functionalities

As in other universal composability frameworks, the security of a protocol is specified by a so-called *ideal functionality*, which acts as a third party and is trusted by all participants. Formally, an ideal functionality is a protocol that defines just one protocol name, say r. The behavior defined for this protocol name depends on the type of machine: All regular protocol machines act as “dummy parties” and forward messages received by their caller (which might be the environment) to their ideal peer. The ideal protocol machine interacts with the regular parties and the adversary: Using the inputs of the parties, the ideal functionality defines a secure way of computing anything the protocol shall compute, explicitly computing the data that is allowed to leak to the attacker.

Example 1. For instance, a secret channel is specified as a functionality that takes a message from Alice and sends it to Bob, notifying the attacker of the length of the message, which would be leaked if this channel were to be realized using encryption.

Now we can define a second protocol, which is rooted at r, and does not necessarily define any behaviour for the ideal party, but for the regular protocol machines. The role of the environment Z is to distinguish whether it is interacting with the ideal system (dummy users interacting with an ideal functionality) or the real system (users executing a protocol). We say that a protocol π *emulates* a functionality $\mathcal{F}$ if for all attackers interacting with π, there exists an attacker, the simulator *Sim*, interacting with $\mathcal{F}$, such that no environment can distinguish between interacting with the attacker and the real protocol π, or the simulation of this attack (generated by *Sim*) and $\mathcal{F}$. It is actually not necessary to quantify over all possible adversaries: The most powerful adversary is the so called dummy attacker A_D that merely acts as a relay forwarding all messages between the environment and the protocol [11, Theorem 5].

Let Z be a program defining an environment, i. e., a program that satisfies the communication constraints that apply to the environment (e. g., it sends messages only to regular protocol machines or to the adversary). Let A be a program that satisfies the constraints that apply to the adversary (e. g., it sends messages only to protocol machines (ideal or regular) it previously received a message from). The protocol π together with A and Z defines a structured system of interactive Turing machines (formally defined in [11, § 4]) denoted $[\pi, A, Z]$. The execution of the system on external input 1^η is a randomized process that terminates if Z decides to stop running the protocol and output a string Σ^*. The random variable $\mathrm{Exec}[\pi, A, Z](\eta)$ describes the output of Z at the end of this process (or $\mathrm{Exec}[\pi, A, Z](\eta) = \bot$ if it does not terminate). Let $\mathrm{Exec}[\pi, A, Z]$ denote the family of random variables $\{\mathrm{Exec}[\pi, A, Z](\eta)\}_{\eta=1}^{\infty}$. An environment Z is well-behaved if the data-flow from Z to the regular protocol participants and the adversary is limited by a polynomial in the security parameter η. If Z is rooted at r, it may only invoke machines with the same session identifier referring to the protocol name r. Because of lack of space we do not define the notion of a *poly-time protocol* here and refer the reader to the definition in [11, § 6].

Definition 3 (emulation with respect to the dummy adversary). *Let π and π' be poly-time protocols rooted at r. Suppose that there exists an adversary Sim that is bounded for π, such that for every well-behaved environment Z rooted at r, we have* $\mathrm{Exec}[\pi, \mathit{Sim}, Z] \approx \mathrm{Exec}[\pi', A_D, Z]$ *Then* π' emulates π.

3 An ideal key management functionality and its implementation

The task of designing an ideal functionality is notoriously difficult. A famous example is the formulation of UC digital signatures: Obtaining a satisfactory formulation of this basic cryptographic operation took years because of repeated revisions caused by subtle flaws making the functionality unrealizable. The functionality we define will at some point need to preserve authenticity in a similar way to this signature functionality, but in a multi-session setting. So we must expect a key-management functionality to be *at least* as complex. Nonetheless we aim to keep it as simple as possible, and so justify the inclusion of each feature by discussing what minimum functionality we expect from a key-management system.

The goal of key-management is to preserve some kind of *policy* on a global level. We consider simple policies that consist of two kinds of requirements: usage policies of the form "key A can be only used for tasks X and Y", and dependency policies of the form "the security of key A may depend on the security of keys B and C". The need for the first is obvious, the need for the second arises because almost all non-trivial key management systems allow keys to encrypt other keys, or derive keys by, e. g., encrypting an identifier with a master key. Typically, the policy defines *roles* for keys, i. e., groups of tasks that can be performed by a key, and *security levels*, which define a hierarchy between keys. The difficulty lies in enforcing this policy globally when key-management involves a number of distributed security tokens that can communicate only via an untrusted network. Our ideal key-management functionality considers a distributed set of Security Tokens as a single trusted third party. It makes sure that every use of a key is compliant with the (global) policy. Therefore, if a set of well-designed security tokens with a sound local policy emulates the ideal key-management functionality, they can never reach a state where a key is used for an operation that is contrary to the policy. This implies that, in general, the key should be kept secret from the user, as the user cannot be forced to comply with the policy. Thus, keys are only accessed via an interface that executes only operations on the key permitted by the policy. The functionality associates some meta-data, an *attribute*, to each key. This attribute defines the key's role, and thus its uses. Existing industrial standards [1] and recent academic proposals [9, 10] are similar in this respect.

A key created on one security token is *a priori* only available to users that have access to this token (since it is hidden from the user). Many cryptographic protocols require that the participants share some key, so in order to be able to run a protocol between two users of different security tokens, we need to be able to "transfer" keys between devices without revealing them. There are several ways to do this but we will opt for the simplest, key-wrapping (the encryption of one key by another).While it is possible to define key-management with a more conceptual view of "transferring keys" and allow the implementation to decide for an option, we think that since key-wrapping is relevant in practice (it is defined in RFC 3394), the choice for this option allows us to define the key-management in a more comprehensible way. We leave the definition of a notion more general in this regard for future work.

The use of key-wrapping requires some initial shared secret values to be available before keys can be transferred. We model the setup in the following way: A subset of users, *Room*, is assumed to be in a secure environment during a limited setup-phase. Afterwards, the only secure channel is between a user U_i, and his security token ST_i. The intruder can access all other channels, and corrupt any party at any time, as well as corrupt keys, i. e., learn the value of the key stored inside the security token. This models the real world situation where tokens can be initialised securely but then may be lost or subject to, e. g., side channel attacks once deployed in the field.

Taken together, these requirements give a set of operations that key-management demands: `new` (create keys), `attr_change` (alter their attributes), `wrap` and `unwrap` (our chosen method of transferring keys), `corrupt` (corruption of keys) and `share/finish_setup` (modelling a setup phase in a secure environment). We argue that a reasonable definition of secure key-management has to provide at least those operations. Furthermore, the users need a way to access the keys stored in the security tokens, so there is a set of operations for each type of key. A signature key, for example, allows the operations *sign* and *verify*. This allows the following classification: The first group of operations defines *key-management*, the second *key-usage*. While key-management operations, for example `wrap` might operate on two keys of possibly different types, key-usage operations are restricted to calling an operation on a single key and user-supplied data. This is coherent with global policies as mentioned above: The form "key A can be used for task X" expresses key-usage, the form "the security of key A depends on keys B and C" expresses a constraint on the key-management.

In the following, we will introduce a concept that allows us to define the key-management functionality with respect to arbitrary key-usage operations. We will then formally define policies, before introducing our key-management functionality, which is called $\mathcal{F}_{\text{KM}}$, bit by bit.

3.1 Key-usage (KU) functionalities

In GNUC and similar frameworks, a cryptographic operation is specified by a functionality $\mathcal{F}$, a fact that we will exploit for the definition of $\mathcal{F}_{\text{KM}}$. For every KU operation, $\mathcal{F}_{\text{KM}}$ calls the corresponding KU functionality, receives the response and outputs it to the user. We define $\mathcal{F}_{\text{KM}}$ for arbitrary KU operations, and consider a security token secure, with respect to the implemented KU functionalities, if it emulates the ideal functionality $\mathcal{F}_{\text{KM}}$ parametrized by those KU functionalities. This allows us to provide an implementation for secure key-management independent of which KU functionalities are used. The security of the operation itself is part of the definition of $\mathcal{F}$.

This approach imposes assumptions on the KU functionalities, as they need to be implementable in a key-manageable way: *a*) There is an algorithm $impl_{\texttt{new}}$ that outputs a key (and possibly some public information) on input 1^η, and *b*) there is a set of commands $C \in \mathcal{C}^{priv}$ each of which can be implemented using an algorithm $impl_C$ which takes the key and some user data as input, and *c*) there is a set of public commands $C' \in \mathcal{C}^{priv}$ each of which can be implemented using an algorithm $impl'_C$ which takes the key and some user data as input. In other words, an implementation $\hat{I}$ emulating $\mathcal{F}$ is, once a key is created, stateless w.r.t. queries concerning this key.

Many functionalities bind the roles of the parties, e. g., signer and verifier, to machine ID encoded in the session parameters, e. g., [12]. But, an implementation of those functionalities usually employs keys, which means that these roles are not really bound to machines, but to the ownership of those keys. Those functionalities are not caller-independent and therefore fail to capture that a key allows to pass the *capacity* to, e. g., generate valid signatures for a certain verification key from one ST to another. While for most applications this is not really a restriction, it is for *key*-management. The privilege to perform an operation must be transferable as some piece of information, which cannot be the actual key: A signing functionality, for example, that exposes its keys to the environment is not realizable, since the environment could then generate dishonest signatures itself. The solution is to generate a key, but only send out a *credential*, which is a hard-to-guess pointer that refers to this key. Whoever knows the credential is allowed to sign. Keys not only allow a distinguished party, the owner of the key, to perform operation but also allow *delegation* of this capacity. Abstraction of 'keys' by 'credentials' is thus more powerful than abstraction of the 'owner of a key' by a 'fixed identity of party that is allowed to sign'. In our scenario, where keys are exported, it is necessary to abstract keys. The requirements on a KU functionality are formalized by the following definitions.

Definition 4 (key-manageable implementation). *A key-manageable implementation $\hat{I}$ is defined by a set of commands, $\mathcal{C}$ that can be partitioned into private and public commands, as well as key-generation, i. e., $\mathcal{C} = \mathcal{C}^{priv} \dot{\bigcup} \mathcal{C}^{pub} \dot{\bigcup} \{\texttt{new}\}$, and a set of PPT algorithms implementing those commands, $\{impl_C\}_{C \in \mathcal{C}}$, such that for the key-generation algorithm $impl_{\texttt{new}}$ the following holds: For all k, $\Pr[k' = k | (k', p) \leftarrow impl_{\texttt{new}}(1^\eta)]$ is negligible in η.*

$\hat{I}$ is a protocol in the sense of [11, §5], i. e., a run-time library that defines only one protocol name, `prot-f-i` from some i. The session parameter encodes a machine id P. When called on this machine, the following code is executed, otherwise no message is accepted:

```
new: accept <new> from parentId
  (key, public) ← $impl_{\texttt{new}}(1^\eta)$
  (credential, <ignore>) ← $impl_{\texttt{new}}(1^\eta)$
  L:= L ∪ { (credential, key) }
  send <new•, credential, public> to parentId
command: accept <C, cred, m> from parentId
  if (credential, key)∈L for some key
    send <C•, $impl_C$(key,m)> to parentId
public_command: accept <C, public, m> from parentId
  send <C•, $impl_C$(public,m)> to parentId
corrupt: accept <corrupt,cred> from parentId
  if (credential, key)∈L for some key
    send <corrupt•, key> to A
```

Definition 5 (key-manageable functionality). *A poly-time functionality $\mathcal{F}$ (to be precise, an ideal protocol, see [11, § 8.2]) is* key-manageable *iff there is a set of commands $\mathcal{C}$ and implementations of those, i. e., PPT algorithms $\texttt{Impl}_{\mathcal{F}} = \{impl_C\}_{C \in \mathcal{C}}$, defining a key-manageable implementation $\hat{I}$ (also poly-time) which emulates $\mathcal{F}$.*

3.2 Policies

Since all credentials on different security tokens in the network are abstracted to a central storage, $\mathcal{F}_{\mathrm{KM}}$ can implement a global policy. Every credential in $\mathcal{F}_{\mathrm{KM}}$ is associated to an attribute from a set of attributes A and to the KU functionality it belongs to (which we will call its type).

Definition 6 (Policy). *Given the KU functionalities $\mathcal{F}_i$, $i \in \{1, \ldots, l\}$ and corresponding sets of commands $\mathcal{C}_i$, a* policy *is a quaternary relation $\Pi \subset \{\mathcal{F}_1, \ldots, \mathcal{F}_l, \texttt{KW}\} \times \cup_{i \in \{1,\ldots,l\}} \mathcal{C}_i \cup \{\texttt{wrap}, \texttt{unwrap}, \texttt{attribute\_change}\} \times A \times A$.*

$\mathcal{F}_{\mathrm{KM}}$ is parametrized by a policy Π. If $(\mathcal{F}, C, a, a') \in \Pi$ and

- $C = \mathtt{new}$, then $\mathcal{F}_{\mathrm{KM}}$ allows the creation of a new key for the functionality $\mathcal{F}$ with attribute a.
- $\mathcal{F} = \mathcal{F}_i$ and $C \in \mathcal{C}_i$, then $\mathcal{F}_{\mathrm{KM}}$ will permit sending the command C to $\mathcal{F}$, if the key is of type $\mathcal{F}$ and has the attribute a.
- $\mathcal{F} = \mathtt{KW}$ and $C = \mathtt{wrap}$, then $\mathcal{F}_{\mathrm{KM}}$ allows the wrapping of a key with attribute a' using a wrapping key with attribute a.
- $\mathcal{F} = \mathtt{KW}$ and $C = \mathtt{unwrap}$, then $\mathcal{F}_{\mathrm{KM}}$ allows the unwrapping of a wrapping annotated with the attribute a' using a wrapping key with attribute a.
- if $C = \mathtt{attribute\_change}$, then $\mathcal{F}_{\mathrm{KM}}$ allows the changing of a key's attribute from a to a'.

Note that a' is only relevant for the `attribute_change` command, and that this command implies that a key can have different attributes set for different users of $\mathcal{F}_{\mathrm{KM}}$, corresponding to different security tokens in the real word.

3.3 The key-management functionality and reference implementation

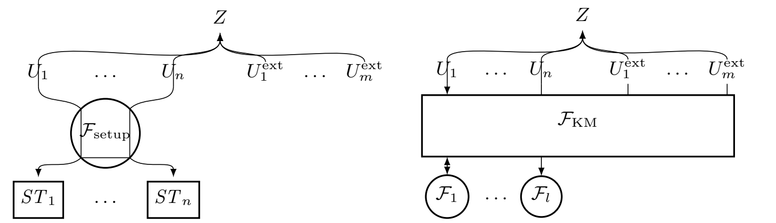
The principle structure of $\mathcal{F}_{\mathrm{KM}}$ is that of a proxy service to the KU functionalities. It is possible to create keys, which means that $\mathcal{F}_{\mathrm{KM}}$ asks the KU functionality for the credentials and stores them, but outputs only a *handle*, referring to the key. This handle can be the position of the key in memory, or a running number – we just assume that there is a way to draw them such that they are unique. When a command $C \in \mathcal{C}_i$ is called with a handle and a message, $\mathcal{F}_{\mathrm{KM}}$ substitutes the handle with the associated credential, and forwards the output to $\mathcal{F}_i$. The response from $\mathcal{F}_i$ is forwarded unaltered. All queries are checked against the policy. The environment may corrupt parties connected to security tokens, as well as individual keys.

Definition 7 (Parameters to a security token Network). *We summarize the parameters of a security token Network as a two tuples, ($\mathcal{U}$,$\mathcal{U}^{\mathrm{ext}}$,$\mathcal{ST}$, Room) and ($\overline{\mathcal{F}}$,$\overline{\mathcal{C}}$,$\Pi$). The first tuple defines* network parameters*:*

- *$\mathcal{U} = \{U_1, \ldots, U_n\}$ are the party IDs of the users connected to a security token*
- *$\mathcal{U}^{\mathrm{ext}} = \{U_1^{\mathrm{ext}}, \ldots, U_m^{\mathrm{ext}}\}$ are the party IDs of external users, i. e., users that do not have access to a security token.*
- *$\mathcal{ST} = \{ST_1, \ldots, ST_n\}$ are the party IDs of the Security Tokens accessed by $U_1, \ldots, U_n$.*
- *Room $\subset \mathcal{U}$.*

The second tuple defines key-usage parameters*:*

- *$\overline{\mathcal{F}} = \{\mathcal{F}_1, \ldots, \mathcal{F}_l\}$, and*
- *$\overline{\mathcal{C}} = \{\mathcal{C}_1, \ldots, \mathcal{C}_l\}$ are key-manageable functionalities with corresponding sets of commands. Note that* `KW` *$\notin \{\mathcal{F}_1, \ldots, \mathcal{F}_l\}$, and that each $\mathcal{C}_i \in \overline{\mathcal{C}}$ is partitioned into the private $\mathcal{C}_i^{priv}$ and public commands $\mathcal{C}_i^{pub}$.*


Z
U_1 … U_n U_1^{ext} … U_m^{ext}
$\mathcal{F}_{\text{setup}}$
ST_1 … ST_n
Z
U_1 … U_n U_1^{ext} … U_m^{ext}
$\mathcal{F}_{\text{KM}}$
$\mathcal{F}_1$ … $\mathcal{F}_l$

(a) Distributed security tokens in the network (b) An idealized functionality $\mathcal{F}_{\text{KM}}$ in the same network

Fig. 1: Distributed security tokens in the network (left-hand side) and idealized functionality $\mathcal{F}_{\text{KM}}$ in the same network (right-hand side).

- *Π is a policy for $\overline{\mathcal{F}}$ (cf. Definition 6) and a membership test on Π can be performed efficiently.*

Figure 1 shows the network of distributed users and security tokens on the left, and their abstraction $\mathcal{F}_{\text{KM}}$ on the right. There are two kinds of users: $U_1, \ldots, U_n =: \mathcal{U}$, each of whom has access to exactly one security token ST_i, and external users $U_1^{\text{ext}}, \ldots, U_m^{\text{ext}} =: \mathcal{U}^{\text{ext}}$, who cannot access any security token. The security token ST_i can only be controlled via the user U_i. The functionality $\mathcal{F}_{\text{setup}}$ in the real world captures our setup assumptions, which need to be achieved using physical means. Among other things, $\mathcal{F}_{\text{setup}}$ assures a secure channel between each pair (U_i, ST_i). The necessity of this channel follows from the fact that *a*) GNUC forbids direct communication between two regular protocol machines (indirect communication via A is used to model an insecure channel) and *b*) $U_1, \ldots, U_n$ can be corrupted by the environment, while $ST_1, \ldots, ST_n$ are incorruptible.

In the following, we will give a detailed description of $\mathcal{F}_{\text{KM}}$ on the left-hand side of Figures 2 to 8. At the same time, to illustrate our definition and demonstrate its use, we present the implementation of a Security API that allows the use of arbitrary key-manageable functionalities as KU functionalities on the right-hand side of the same figures. This implementation describes a way to implement a Security API for key-management that is independent of the KU functions it provides. In Section 4, we show that this implementation is a realization of $\mathcal{F}_{\text{KM}}$. We emphasize that extending $\mathcal{F}_{\text{KM}}$ and the implementation by a new KU functionality does not require a new proof.

In GNUC, functionalities are completely defined by the code that is run on the ideal peer (all regular peers run the so-called dummy party, described in [11, §8.2]). The session id `sid` is of the form `<`a_1`,`$\ldots$`,`a_{k-1}`,<prot-fkm,`sp`>>`, where the session parameter *sp* is some encoding of the network parameters $\mathcal{U}, \mathcal{U}^{\text{ext}}, \mathcal{ST}$, *Room*. The code itself is parametric in the KU parameters $\overline{\mathcal{F}}, \overline{\mathcal{C}}, \Pi$. When we refer to $\mathcal{F}_{\text{KM}}$ as a network identity, we mean the machine id `<ideal,sid >`.

Similarly, each security token $ST_i \in \{ST_1, \ldots, ST_n\}$ is addressed via the machine id `<`ST_i`,sid>`. We will abuse notation by identifying the machine id with

ST_i, whenever the session id is clear from the context. The session parameter within `sid` encodes the network parameters $\mathcal{U}, \mathcal{U}^{\text{ext}}, \mathcal{ST}, \mathit{Room}$. ST_i makes subroutine calls to the functionality $\mathcal{F}_{\text{setup}}$ which subsumes our setup assumptions. $\mathcal{F}_{\text{setup}}$ provides two things: 1. a secure channel between each pair U_i and ST_i, 2. a secure channel between some pairs ST_i and ST_j during the setup phase (we will come to this later). ST_i receives commands from a machine $U_i \in \mathcal{U}$, which is defined in Definition 9, and relays arbitrary commands sent by the environment via $\mathcal{F}_{\text{setup}}$. The environment cannot talk directly to ST_i, but the attacker can send queries on behalf of any corrupted user, given that the user has been corrupted previously (by the environment).

The implementation ST is inspired by [10] and is parametric on the KU parameters $\overline{\mathcal{F}}, \overline{\mathcal{C}}, \Pi$ and the implementation functions $\overline{\texttt{Impl}} := \{\texttt{Impl}_F\}_{F \in \overline{\mathcal{F}}}$. It is composable in the following sense: If a device performs the key-management according to our implementation, it does not matter how many, and which functionalities it enables access to, as long as those functionalities provide the amount of security the designer aims to achieve (cf. Corollary 1). In Section 5, we show how to instantiate those KU functionalities to fully instantiate a "secure" security token, and how $\mathcal{F}_{\text{KM}}$ facilitates analysis of this configuration.

To describe the code of the machines we use a similar notation to [11, § 12]. An input/output step is defined by a block of the form

```
name [conditions]: accept-clause; P
```

`name` is the label identifying the step. The logical expression `[conditions]` is a *guard* that must be satisfied to trigger a step, as well as the *accept* clause following it, which describes the form of the message that may trigger this step (if the guard expression is true). Note that a step may be triggered multiple times. This is different in [11, § 12], but their results hold independently of their pseudo-code program notation. A step name in the guard expression means that the corresponding step has been triggered at some previous point. The accept clause, too, might have logical conditions that must be satisfied in order to trigger the step. Any message not triggering any step is processed by sending an error message to A. The convention is that the response to a query $(\text{Command}, \texttt{sid}, \ldots)$ is always of the form $(\text{Command}^{\bullet}, \texttt{sid}, \ldots)$, or $\perp$. Furthermore, when we say that $\mathcal{F}_{\text{KM}}$ calls $\mathcal{F}$, we mean that it sends a message to a regular peer that calls $\mathcal{F}$ as a sub-protocol and relays the answers. Formally, $\mathcal{F}_{\text{KM}}$ sends a message to the machine id $F = \texttt{<<reg,}\mathcal{F}\texttt{ >,<sid>>}$, who in turn addresses $\texttt{<<reg,}\mathcal{F}\texttt{ >,<sid,<}\mathcal{F}\texttt{,<}F\texttt{>>>>}$ as a dummy party. This is necessary since Condition C6 in [11, §4.5] disallows ideal parties from making sub-routine calls. Note first that, for unambiguity, we use the code $\mathcal{F}$ as the party id for this user. Note secondly that $\mathcal{F}$ uses the session parameter F to identify F as the only machine id is accepts messages from.

A user can create keys of type $\mathcal{F}$ and attribute a using the command $\texttt{<new,}\mathcal{F}\texttt{,}\ a\texttt{>}$ (see Figure 2). In $\mathcal{F}_{\text{KM}}$, the functionality $\mathcal{F}$ is asked for a new credential and some public information. The credential is stored with the meta-data at a freshly chosen position h in the store. Similarly, ST stores an actual key, instead of a credential. Both output the handle h and the public information given by $\mathcal{F}$ or

```
new[ready]:
accept <new,F,a> from $U \in \mathcal{U}$
if <F,new,a,*> $\in \mathit{\Pi}$ and F$\neq$FKW then
  call F with <new>
  accept <new$^\bullet$,cred,public>
                    from F
  create $h$; Store[$U$,$h$] $\leftarrow$ <F,a,cred>
  send <new$^\bullet$,h,public> to $U$
if <F,new,a,*> $\in \mathit{\Pi}$ and F=KW then
  (k,public) $\leftarrow$ $impl^{KW}_{\texttt{new}}(1^\eta)$
  create $h$; Store[$U$,$h$] $\leftarrow$ <F,a,k>
  send <new$^\bullet$,h,public> to $U$
```

```
new[ready]:
accept <new,$F$,$a$> from $\mathcal{F}_{\text{setup}}$
  if <$F$,new,$a$,*> $\in \mathit{\Pi}$
    ($k$, $public$) $\leftarrow$ $impl^{F}_{\texttt{new}}(1^\eta)$
    create $h$; Store[$U_i$,$h$] $\leftarrow$ <$F$,$a$,$k$>
    send <new$^\bullet$,$h$,$public$> to $\mathcal{F}_{\text{setup}}$
```

Fig. 2: Creating keys of type $\mathcal{F}$ and attribute a using the command <new,$\mathcal{F}$,a>.

produced by the key-generation algorithm. $\mathcal{F}_{\text{KM}}$ treats wrapping keys differently: It calls the key-generation function for KW.

Once the setup phase is finished, the expression setup_finished will evaluate to true (the description of the setup phase follows later). If the policy $\mathit{\Pi}$ permits execution of a command $C \in \mathcal{C}_i$, $\mathcal{F}_{\text{KM}}$ calls the corresponding functionality as a sub-protocol, substituting the handle by the corresponding credential. Similarly, ST_i uses the corresponding key to compute the output of the implementation function $impl_C$ of the command C (Figure 3).

The commands that are important for key-management are handled by $\mathcal{F}_{\text{KM}}$ itself. So are wrap and unwrap. To transfer a key from one security token to another in the real world, the environment instructs, for instance, U_1 to ask for a key to be *wrapped* (see Figure 4). A wrapping of a key is the encryption of a key with another key, the wrapping key. The wrapping key must of course be on both security tokens prior to that. U_1 will receive the wrap from ST_1 and forward it to the environment, which in turn instructs U_2 to unwrap the data it just received from U_1. The implementation ST_i just verifies if the wrapping confirms the policy, and then produces a wrapping of c_2 under c_1, with additionally authenticated information: the type and the attribute of the key, plus a user-chosen identifier that is bound to a wrapping in order to identify which key was wrapped. This could, e. g., be a key digest provided by the KU functionality the key belongs to.

The definition of $\mathcal{F}_{\text{KM}}$ enforces that: 1. a wrapping created using an uncorrupted credential ($c_1 \notin \mathcal{K}_{\text{cor}}$) does not contain any information about c_2 besides its length and 2. that every wrapping created using on the device is saved in

```
command[finish_setup]:
accept <$C$,$h$,$m$> from $U \in \mathcal{U}$
if Store[U,$h$]=<$F$,$a$,$c$>
      and <$F$,$C$,$a$,*>$\in \mathit{\Pi}$
  call $F$ with <$C$,$c$,$m$>
  accept <$C^\bullet$,$r$> from $F$
  send <$C^\bullet$,$r$> to $U$
```

```
command[finish_setup]:
accept <$C$,$h$,$m$> from $\mathcal{F}_{\text{setup}}$
if Store[$U_i$,$h$]=<$F$,$a$,$c$>
      and <$F$,$C$,$a$,*>$\in \mathit{\Pi}$
  send <$C^\bullet$,$impl_C(c,m)$> to $\mathcal{F}_{\text{setup}}$
```

Fig. 3: Executing a command C on a handle h with data m.

```
wrap[finish_setup]:
accept <wrap,$h_1$,$h_2$,$id$> from $U \in \mathcal{U}$
 if Store[U,$h_1$]=<KW,$a_1$,$c_1$> and
    Store[U,$h_2$]=<$F_2$,$a_2$,$a_2$> and
    <KW,wrap,$a_1$,$a_2$>$\in \mathit{\Pi}$
   if <$c_2$,<$F_2$,$a_2$,$id$>,w>$\in$encs[$c_1$]
     send <wrap$^\bullet$,w> to $U$
   else
     $C \leftarrow C \cup \{(c_1, c_2)\}$
     if $c_1 \in \mathcal{K}_{\mathrm{cor}}$
       for all $c_3$ reachable from $c_2$ in $C$
         $\mathcal{K}_{\mathrm{cor}} \leftarrow \mathcal{K}_{\mathrm{cor}} \cup \{c_3\}$
         for any Store[$U'$,$h'$]=<$F'$,$a'$,$c_3$>
             call $F'$ with <corrupt,$c_3$>
       $w \leftarrow \mathsf{wrap}^{<F_2,a_2,id>}(c_1, c_2)$
     else
       $w \leftarrow \mathsf{wrap}^{<F_2,a_2,id>}(c_1, 0^{|c_2|})$
     encs[$c_1$] $\leftarrow$ encs[$c_1$]
                  $\cup${ <$c_2$,<$F_2$,$a_2$,$id$>,w>}
     send <wrap$^\bullet$,$w$> to $U$
```

```
wrap[finish_setup]:
  accept <wrap,$h_1$,$h_2$,$id$> from $\mathcal{F}_{\mathrm{setup}}$
  if Store[$U_i$,$h_1$]=<KW,$a_1$,$c_1$> and
      Store[$U_i$,$h_2$]=<$F_2$,$a_2$,$a_2$> and
      <KW,wrap,$a_1$,$a_2$>$\in \mathit{\Pi}$
    $w \leftarrow \mathsf{wrap}^{<F_2,a_2,id>}(c_1, c_2)$
    send <wrap$^\bullet$,$w$> to $\mathcal{F}_{\mathrm{setup}}$
```

Fig. 4: Wrapping the key h_2 under the key h_1, adding additional information id.

encs[c_1]. We will see that unwrap only allows to import a key with c_1 if it has an entry in encs[c_1] (unless c_1 is corrupted). This assures authentication. 3. Furthermore, $\mathcal{F}_{\mathrm{KM}}$ maintains a corruption graph C that stores which key has been wrapped with which. Should a key be wrapped under a corrupted key, it is regarded as corrupted itself.

When a wrapped key is unwrapped using an uncorrupted key, $\mathcal{F}_{\mathrm{KM}}$ checks if the wrapping was produced before, using the same identifier (see Figure 5). Furthermore, $\mathcal{F}_{\mathrm{KM}}$ checks if the given attribute and types are correct. If this is the case, it creates another entry in Store, i. e., a new handle h' for the user U pointing to the *correct* credentials, type and attribute type of the key. This way, $\mathcal{F}_{\mathrm{KM}}$ can guarantee the consistency of its database for uncorrupted keys, see the following Theorem 1. If the key used to unwrap is corrupted, this guarantee cannot be given, but the resulting entry in the store is marked corrupted.

There is an improvement that became apparent during the proof of emulation (Theorem 2 below). Namely, when unwrapping with a corrupted key, $\mathcal{F}_{\mathrm{KM}}$ checks the attribute that is going to be assigned to the (imported) key against the policy, instead of just accepting that a corrupted wrapping-key might just import any wrapping the attacker generated. This prevents, for example, a corrupted wrapping-key of low security from *creating* a high-security wrapping-key by unwrapping dishonestly produced wrappings. This detail in the definition of $\mathcal{F}_{\mathrm{KM}}$ enforces a stronger implementation than the one in [10]: ST validates the attribute given with a wrapping, enforcing that it is at least sound according to the policy, instead of blindly trusting the authenticity of the wrapping mechanism. Hence our implementation is more robust.

```
unwrap[finish_setup]:
accept <unwrap,$h_1$,$w$,$a_2$,$F_2$,$id$>
                from $U \in \mathcal{U}$
if Store[U,$h_1$]=<KW,$a_1$,$c_1$> and
  <KW,unwrap,$a_1$,$a_2$>$\in \Pi$,$F_2 \in \overline{\mathcal{F}}$
  if $c_1 \in \mathcal{K}_{\mathrm{cor}}$
    $c_2 = \mathsf{unwrap}^{<F_2,a_2,id>}(c_1, w)$
    if $c_2 \neq \bot$
      $\mathcal{K}_{\mathrm{cor}} \leftarrow \mathcal{K}_{\mathrm{cor}} \cup \{c_2\}$
      create $h_2$
      Store[$U$,$h_2$] $\leftarrow$ <$F_2$,$a_2$,$c_2$>
      send <unwrap$^\bullet$,h> to $U$
  else if ( $c_1 \notin \mathcal{K}_{\mathrm{cor}}$ and
     $\exists c_2$.<$c_2$,<$F_2$,$a_2$,$id$>,w>$\in$encs[$c_1$])
    create $h_2$
    Store[$U$,$h_2$] $\leftarrow$ <$F_2$,$a_2$,$c_2$>
    send <unwrap$^\bullet$,h> to $U$
```

```
unwrap[finish_setup]:
accept <unwrap,$h_1$,$w$,$a_2$,$F_2$,$id$>
                from $\mathcal{F}_{\mathrm{setup}}$
if Store[$U_i$,$h_1$]=<KW,$a_1$,$c_1$>
  and $F_2 \in \overline{\mathcal{F}}$
  and <KW,unwrap,$a_1$,$a_2$>$\in \Pi$
  and $c_2 = \mathsf{unwrap}^{<F_2,a_2,id>}(c_1, w) \neq \bot$
    create $h_2$
    Store[$U$,$h_2$] $\leftarrow$ <$F_2$,$a_2$,$c_2$>
    send <unwrap$^\bullet$,h> to $\mathcal{F}_{\mathrm{setup}}$
```

Fig. 5: Unwrapping w created with attribute a_2, F_2 and id using the key h_1.

It is possible to change the attributes of a key, if the policy permits (see Figure 6).

```
attr_change[finish_setup]:
accept <attr_change,$h$,$a'$>
                from $U \in \mathcal{U}$
if Store[U,$h$]=<$F$,$a$,$c$> and
  <F,attr_change,$a$,$a'$>$\in \Pi$
  Store[$U$,$h$]=<$F$,$a'$,$c$>
  send <attr_change$^\bullet$> to $U$
```

```
attr_change[finish_setup]:
accept <attr_change,$h$,$a'$>
                from $\mathcal{F}_{\mathrm{setup}}$
if Store[$U_i$,$h$]=<$F$,$a$,$c$> and
  <F,attr_change,$a$,$a'$>$\in \Pi$
  Store[$U_i$,$h$]=<$F$,$a'$,$c$>
  send <attr_change$^\bullet$> to $\mathcal{F}_{\mathrm{setup}}$
```

Fig. 6: Changing the attribute of h to a'.

Since keys might be used to wrap other keys, we would like to know how the loss of a key to the adversary affects the security of other keys. When an environment "corrupts a key" in $\mathcal{F}_{\mathrm{KM}}$, the adversary learns the credentials to access the functionalities (see Figure 7). These credentials will allow the environment to access the underlying functionality directly. ST implements this corruption by outputting the actual key to the adversary.

Some cryptographic operations (e. g., digital signatures) allow users without access to a security token to perform certain operations (e. g., signature verification). Those commands do not require knowledge of the credential (in $\mathcal{F}_{\mathrm{KM}}$), or the secret part of the key (in ST). They can be computed using publicly available information. Note that $\mathcal{F}_{\mathrm{KM}}$ does relay this call to the underlying KU functionality unaltered, and independent of its store and policy (see Figure 7).

```
corrupt[finish_setup]:
  accept <corrupt,$h$> from $U \in \mathcal{U}$
  if Store[U,$h$]=<$F$,$a$,$c$>
    if $F$=KW
      for all $c'$ reachable from $c$ in $C$
        $\mathcal{K}_{\text{cor}} \leftarrow \mathcal{K}_{\text{cor}} \cup \{c'\}$
        for any Store[$U, h$]=<$F$,$a$,$c'$>
          call $F$ with <corrupt,c'>
      send <corrupt$^\bullet$,$h$,$c$> to $A$
    else
      $\mathcal{K}_{\text{cor}} \leftarrow \mathcal{K}_{\text{cor}} \cup \{c\}$
      call $F$ with <corrupt,c>
```

```
corrupt[finish_setup]:
  accept <corrupt,$h$> from $\mathcal{F}_{\text{setup}}$
  if Store[$U_i$,$h$]=<$F$,$a$,$c$>
    send <corrupt$^\bullet$,$h$,$c$> to $A$
```

```
public_command: accept <$C$,$public$,$m$>
  from $U \in \mathcal{U} \cup \mathcal{U}^{\text{ext}}$
if $C \in \mathcal{C}_{i,pub}$
  call $F_i$ with <$C$,$public$,$m$>
  accept <$C^\bullet$,$r$> from F
  send <$C^\bullet$,$r$> to $U$
```

(U_i, U_i^{ext} compute $impl_C(public, m)$ themselves.)

Fig. 7: Corrupting h / Computing the public commands C using *public* and m.

We model the setup phase as follows: All users in *Room* are allowed to share keys during the setup phase, i. e., the implementation is allowed to use secure channels to transport keys during this phase, but not later (see Figure 8). On a physical device this would correspond to pre-installed keys at manufacturing time or installation of keys by an administrator using a trusted machine. Once the setup phase is finished the functionality enters the run phase, where users may create new keys, wrap and unwrap keys, change their attributes and send commands to the KU functionalities. We assume a secure environment for the setup phase, permitting all users in the set *Room* to generate keys and share them among themselves. This is implemented by the functionality $\mathcal{F}_{\text{setup}}$, defined in Listing A in Appendix A. This secure channel *between two security tokens* ST is only used during the setup phase. Afterwards, $\mathcal{F}_{\text{setup}}$ provides only a secure channel between a user U_i, which takes commands from the environment, and his security token ST_i. When we say that ST_i calls $\mathcal{F}_{\text{setup}}$, we mean that it sends a message to the machine id <ST_i,<sid,<prot-fsetup,<$\mathcal{U}$,$\mathcal{U}^{\text{ext}}$,$\mathcal{ST}$, $Room$>>>>.

Remark 1: Credentials for different KU functionalities are distinct. It is nonetheless possible to encrypt and decrypt arbitrary credentials using *Wrap* and *Unwrap*. Suppose a designer wants to prove a Secure API secure which uses shared keys for different operations . One way or another, she would need to prove that those roles do not interfere. For this case, we suggest providing a functionality that combines the two KU functionalities, and proving that the implementation of the two operations combined emulates the combined functionality. It is possible to assign different attributes to keys of the same KU functionality, and thus restrict their use to certain commands, effectively providing different roles for credentials to the same KU functionality. This can be

```
share[¬finish_setup∧ready]:
accept <share,$h_1$,$U_2$> from $U_1 \in \mathcal{U}$
if $U_i, U_2 \in Room$
  create $h_2$; Store[$U_2$,$h_2$]=Store[$U_1$,$h_1$]
  send <share$^\bullet$> to $U_2$
```

```
finish_setup[¬finish_setup∧ready]:
accept <finish_setup> from $U \in \mathcal{U}$
send <finish_setup$^\bullet$> to $A$
```

```
share[¬finish_setup∧ready]:
accept <share,$h_1$,$U_2$> from $\mathcal{F}_{\text{setup}}$
  if Store[$U$,$h_1$]=s
    call $\mathcal{F}_{\text{setup}}$ with <send,s,$U_2$>
import[¬finish_setup∧ready]:
  accept <deliver,$s$,$U_1$> from $\mathcal{F}_{\text{setup}}$
  if $U_1 \in Room$
    create $h_2$; Store[$U_i$,$h_2$]=$s$
    send <share$^\bullet$> to $\mathcal{F}_{\text{setup}}$
```

```
finish_setup[¬finish_setup∧ready]:
accept <close> from $\mathcal{F}_{\text{setup}}$
send <close$^\bullet$> to $\mathcal{F}_{\text{setup}}$
```

Fig. 8: The setup phase.

done by specifying two attributes for the two roles and defining a policy that restricts which operation is permitted for a key of each attribute.

Remark 2: Many commonly used functionalities are not *caller-independent*, often the access to critical functions is restricted to a network party that is encoded in the session identifier. However, it is possible to construct caller-independent functionalities for a large class of functionalities, if the implementation relies on keys but is otherwise stateless. See Section 6 for details.

Remark 3: Constraint C6 in [11, §8.2] requires each regular machine to send a message to $\mathcal{F}_{\text{setup}}$ before it can address it. The initialization procedure and the parts of the definition of $\mathcal{F}_{\text{KM}}$, ST and $\mathcal{F}_{\text{setup}}$ that perform this procedure are explained in detail in Appendix A.

Definition 8. *We define a family of* attribute policy graphs $\mathcal{A}_\Pi$ *for each KU functionality $\mathcal{F}$: a is a node in $\mathcal{A}_\Pi$ if $(\mathcal{F}, C, a, a') \in \Pi$ from some C and a'. Additionally, a is marked* `new` *if $C =$* `new`*. An edge is between a and a' whenever $(\mathcal{F}, \texttt{attribute\_change}, a, a') \in \Pi$.*

Theorem 1. *Every instance of $\mathcal{F}_{\text{KM}}$ with parameters $\overline{\mathcal{F}}, \overline{\mathcal{C}}, \Pi$ and session parameters $\mathcal{U}, \mathcal{U}^{\text{ext}}, \mathcal{ST}$, Room has the following properties:*

(1) At any step of an execution of $[\mathcal{F}_{\text{KM}}, A_D, Z]$, the following holds for $\mathcal{F}_{\text{KM}}$: for all $\texttt{Store}[U, h] =< \mathcal{F}, a, c >$ such that $c \notin \mathcal{K}_{\text{cor}}$, there is a `new` *node labelled a' in the attribute policy graph for $\mathcal{F}$ in Π such that the attribute a is reachable from that node and there was a step* `new` *where $\texttt{Store}[U', h'] =< \mathcal{F}, a', c >$ was added.*

(2) At any step of an execution of $[\mathcal{F}_{\text{KM}}, A_D, Z]$, the following holds for $\mathcal{F}_{\text{KM}}$: all $c \in \mathcal{K}_{\text{cor}}$, were either corrupted directly, i. e., there was a `corrupt` *triggered by a query $< \texttt{corrupt}, \texttt{h} >$ from U while* `Store[U,h]`*$=< \mathcal{F}, a, c >$, or indirectly,i. e., there is $c' \in \mathcal{K}_{\text{cor}}$ such that at some point the* `wrap` *step*

was triggered by a message <wrap,h',h,id> *from* U *while* Store[U,h']=< KW, a', c' >, Store[U,h]=< $\mathcal{F}, a, c$ >.

(3) *At any step of an execution of* $[\mathcal{F}_{\mathrm{KM}}, A_D, Z]$*, the following holds: Whenever an ideal machine* $\mathcal{F}_i$ = <ideal,<sid,<$\mathcal{F}_i$,F>>>, F =<<reg,$\mathcal{F}$ >,<sid>>, *accepts the message* <corrupt,c> *for some* c *such that* $\mathcal{F}_{\mathrm{KM}}$ *in session* sid *has an entry* Store[U, h]= <$\mathcal{F}_i$,a,c>, *then* $c \in \mathcal{K}_{\mathrm{cor}}$ *in* $\mathcal{F}_{\mathrm{KM}}$.

The formal proof of these claims can be found in Appendix B.

4 Proof overview

We show that, for arbitrary KU parameters $\overline{\mathcal{F}}, \overline{\mathcal{C}}, \Pi$, the network $\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\texttt{Impl}}}$, consisting of the set of users $\mathcal{U}$ connected to security tokens $\mathcal{ST}$, the set of external users $\mathcal{U}^{\mathrm{ext}}$ and the functionality $\mathcal{F}_{\mathrm{setup}}$, emulates the key-management functionality $\mathcal{F}_{\mathrm{KM}}$. We will only give a proof sketch here, the complete proof can be found in Appendix C

Let $\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\texttt{Impl}}}$ (in the following: π) denote the network consisting of the programs $\pi(\texttt{prot}-\texttt{fkm})$ and $\pi(\texttt{prot}-\texttt{fsetup})$. $\pi(\texttt{prot}-\texttt{fkm})$ defines the behaviours for users in $\mathcal{U}$, $\mathcal{U}^{\mathrm{ext}}$ and $\mathcal{ST}$. Parties in $\mathcal{U} \cup \mathcal{U}^{\mathrm{ext}}$ will act according to the convention on machine corruption defined in [11, § 8.1], while parties in $\mathcal{ST}$ will ignore corruption requests (security tokens are assumed to be incorruptible). $\pi(\texttt{prot}-\texttt{fkm})$ is *totally regular*, that is, for other machines, in particular ideal machines, it responds to any message with an error message to the adversary. The protocol π is a $\mathcal{F}_{\mathrm{setup}}$-hybrid protocol.

The proof that π implements $\mathcal{F}_{\mathrm{KM}}$ proceeds in several steps: Making use of the composition theorem, the last functionality $\mathcal{F}_l$ in $\mathcal{F}_{\mathrm{KM}}$ can be substituted by its key-manageable implementation $\hat{I}_L$. Then, $\mathcal{F}_{\mathrm{KM}}$ can simulate $\hat{I}$ instead of calling it. Let $\mathcal{F}_{\mathrm{KM}}^{\{\mathcal{F}_l/\hat{I}_l\}}$ be the resulting functionality. In the next step, calls to this simulation are substituted by calls to the functions used in $\hat{I}$, $impl_C$ for each $C \in \mathcal{C}_l$. The resulting, partially implemented functionality $\mathcal{F}_{\mathrm{KM}}^{\{\mathcal{F}_l/\texttt{Impl}_{\mathcal{F}_l}\}}$ saves keys rather than credentials (for $\mathcal{F}_l$). We repeat the previous steps until $\mathcal{F}_{\mathrm{KM}}$ does not call any KU functionalities anymore, i. e., we have $\mathcal{F}_{\mathrm{KM}}^{\{\mathcal{F}_1/\texttt{Impl}_{\mathcal{F}_1},\ldots,\mathcal{F}_n/\texttt{Impl}_{\mathcal{F}_n}\}}$. Then we show that the network of distributed token π emulates the monolithic block $\mathcal{F}_{\mathrm{KM}}^{\{\mathcal{F}_1/\texttt{Impl}_{\mathcal{F}_1},\ldots,\mathcal{F}_n/\texttt{Impl}_{\mathcal{F}_n}\}}$ that does not call KU functionalities anymore, using a reduction to the security of the key-wrapping scheme.

The first four steps are the subject of Lemma 1, the last step is Lemma 2:

Lemma 1. *Let* $\overline{\mathcal{F}}, \overline{\mathcal{C}}, \Pi$ *be KU parameters such that all* $F \in \overline{\mathcal{F}}$ *are key-manageable. Let* $\texttt{Impl}_{\mathcal{F}_i}$ *be the functions defining the key-manageable implementation* $\hat{I}_i$ *of* $\mathcal{F}_i$*. Then* $\mathcal{F}_{\mathrm{KM}}^{\mathcal{F}_1/\texttt{Impl}_{\mathcal{F}_1},\ldots,\mathcal{F}_l/\texttt{Impl}_{\mathcal{F}_l}}$ *emulates* $\mathcal{F}_{\mathrm{KM}}$*. Furthermore, it is poly-time.*

Lemma 2. *For any KU parameter* $\overline{\mathcal{F}}, \overline{\mathcal{C}}, \Pi$ *and set of sets of ppt algorithms* $\overline{\texttt{Impl}}$*, let* $\mathcal{F}_{\mathrm{KM}}^{\texttt{impl}} := \mathcal{F}_{\mathrm{KM}}^{\mathcal{F}_1/\texttt{Impl}_{\mathcal{F}_1},\ldots,\mathcal{F}_l/\texttt{Impl}_{\mathcal{F}_l}}$ *be the partial implementation of* $\mathcal{F}_{\mathrm{KM}}$

with respect to all KU functionalities in $\overline{\mathcal{F}}$. *If* $KW = (\mathsf{KG}, \mathsf{wrap}, \mathsf{unwrap})$ *is a secure and correct key-wrapping scheme (Definition 11) then* $\mathcal{F}_{\mathrm{KM}}^{\mathtt{impl}}$ *emulates* $\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\mathtt{Impl}}}$ *for environments that guarantee* corrupt-before-wrap.

The main result follows from the transitivity of emulation and Lemmas 1 and 2:

Corollary 1. *Let* $\overline{\mathcal{F}}, \overline{\mathcal{C}}, \Pi$ *be KU parameters such that all* $F \in \overline{\mathcal{F}}$ *are keymanageable. Let* $\mathtt{Impl}_{\mathcal{F}_i}$ *be the functions defining the keymanageable implementation* $\hat{I}_i$ *of* $\mathcal{F}_i$. *Then* $\mathcal{F}_{\mathrm{KM}}$ *emulates* $\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\mathtt{Impl}}}$ *for environments that guarantee* corrupt-before-wrap.

5 Realizing key-usage functionalities for a static key-hierarchy

To demonstrate the use of Corollary 1, we equip the security token with the functionalities $\mathcal{F}_1 = \mathcal{F}^{\mathrm{Nonce}}$ and $\mathcal{F}_2 = \mathcal{F}^{\mathrm{SIG}}$ defined below. The resulting security token $ST^{\mathcal{F}^{\mathrm{Nonce}},\mathcal{F}^{\mathrm{SIG}}}$ is able to encrypt keys and nonces and sign user-supplied data. It is not able to sign keys, as this task is part of the key-management. The first functionality, $\mathcal{F}^{\mathrm{Nonce}}$, is an unusual functionality, but demonstrates what can be done within the design of $\mathcal{F}_{\mathrm{KM}}$. It models the creation of nonces, tests of equality and corruption, which means here that the adversary learns the value of the nonce. The definition of $\mathcal{F}^{\mathrm{Nonce}}$ follows on the left-hand side, and the definition of two functions implementing it, $impl_{\mathtt{new}}$ and $impl_{\mathtt{equal}}$ on the right-hand side:

```
new: accept <new> from parentId (=:$p$)
credential ← $\{0,1\}^\eta$
L:= L ∪ { (credential, 0) }
send <new$^\bullet$,credential,> to $p$
command:
accept <equal,cred,$n$> from $p$
if (credential , key)∈L for some key
  if key$\notin \mathcal{K}_{\mathrm{cor}}$
    send <equal$^\bullet$,false> to $p$
  else if $n$=key
    send <equal$^\bullet$,true> to $p$
corrupt:
accept <corrupt,cred> from $p$
if ($c$, 0)∈L for some key
  key ← $\{0,1\}^\eta$
  L:= $(L \setminus \{(c,0)\}) \cup \{$(c, key) }
  $\mathcal{K}_{\mathrm{cor}} = \mathcal{K}_{\mathrm{cor}} \cup \{$ key }
  send <corrupt$^\bullet$, key> to $A$
```

```
new: $impl_{\mathtt{new}}$ on input $1^\eta$
  $n \leftarrow \{0,1\}^\eta$
  output $(n,\_)$
command: $impl_{\mathtt{equal}}$ on input $n, n'$
  output $n = n'$
```

Due to space restrictions, the signature functionality $\mathcal{F}^{\mathrm{SIG}}$ is presented in Appendix D. In the following, we will consider $\mathcal{F}_{\mathrm{KM}}$ for the parameters $\overline{\mathcal{F}} = \{\mathcal{F}^{\mathrm{Nonce}}, \mathcal{F}^{\mathrm{SIG}}\}$, $\overline{\mathcal{C}} = \{\{\mathtt{equal}\}, \{\mathtt{sign}, \mathtt{verify}\}\}$ and a static key-hierarchy Π, which is defined as the relation that consists of all 4-tuples ($\mathcal{F}$,Cmd,attr$_1$,attr$_2$) such that the conditions in one of the lines in the following table hold. Note that we omit the "=" sign when we mean equality and "*" denotes that no condition has to hold for the variable.

$\mathcal{F}$	Cmd	attr$_1$	attr$_2$
KW	new	> 0	*
$\neq$ KW	new	0	*
*	attribute_change	a	a
KW	wrap	> 0	attr$_1$ $>$ attr$_2$
KW	unwrap	> 0	attr$_1$ $>$ attr$_2$
$\mathcal{F}_i$	$C \in \mathcal{C}_i$	0	*

(where $a \in \mathbb{N}$)

Theorem 1 allows immediately to conclude some useful properties on this instantiation of $\mathcal{F}_{\mathrm{KM}}$: From *(1)* we conclude that all keys with $c \notin \mathcal{K}_{\mathrm{cor}}$ have the attribute they were created with. This also means that the same credential has the same attribute, no matter which user accesses it. From *(2)*, we can see that for each corrupted credential $c \in \mathcal{K}_{\mathrm{cor}}$, there was either a query $<$ corrupt, h $>$, where Store[U,h]=$< \mathcal{F}, a, c >$, or there exists Store[U,h']=$<$ KW, $a', c' >$, Store[U,h]=$< \mathcal{F}, a, c >$ and a query <wrap,h',h,id> was emitted, for $c' \in \mathcal{K}_{\mathrm{cor}}$. By the definition of the strict key-hierarchy policy, the last case implies that $a' > a$. It follows that, for any credential c for $\mathcal{F}$, such that Store[U,h]=$< \mathcal{F}, a, c >$ for some U, h and a, $c \notin \mathcal{K}_{\mathrm{cor}}$, as long as every corruption query $<$ corrupt, h$^* >$ at U was addressed to a different key of lower or equal rank key, i. e., Store[U,h^*]=$<$ KW, $a^*, c^* >$, $c^* \neq c$ and $a^* \leq a$. By *(3)*, those credentials have not been corrupted in their respective functionality, i. e., it has never received a message <corrupt,c>.

6 Conclusions and outlook

We have presented a provably secure framework for key management in the GNUC model. In further work, we are currently developing a technique for transforming functionalities that use keys but are not key-manageable into key-manageable functionalities in the sense of Definition 4. This way, existing proofs could be used to develop a secure implementation of cryptographic primitives in a plug-and-play manner. Investigating the restrictions of this approach could teach us more about the modelling of keys in simulation-based security.

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A Initialisation procedure and definition of user and setup functionality

All regular protocol machines that shall accept messages from $\mathcal{F}_{\mathrm{KM}}$ need to send a message to $\mathcal{F}_{\mathrm{KM}}$ first [11, § 4.5]. A similar behaviour needs to be emulated by $\mathcal{F}_{\mathrm{setup}}$ in the network with the actual tokens. The involved protocol machines are $\mathcal{M} := \mathcal{U} \cup \mathcal{ST} \cup \{\mathcal{F}_1, \ldots, \mathcal{F}_l\}$, where $\mathcal{F}_i$ denotes the regular protocol machine that makes subroutine calls to $\mathcal{F}_i$, identified with the machine id `<<reg,`$\mathcal{F}_i$`>, sid >`.

We add the following parts to the definition of $\mathcal{F}_{\mathrm{KM}}$ and ST:

```
ready$-P$: accept <ready> from $P \in \mathcal{M}$
  send <ready$^\bullet$,$P$> to $A$
ready [ ready$-P$ $\forall P \in \mathcal{M}$ ]
```

```
ready [$\neg$ ready]:
accept <ready> from parentId
   call $\mathcal{F}_{\mathrm{setup}}$ with <ready>
```

The definition of $\mathcal{F}_{\mathrm{setup}}$.

```
ready$-U_i$: accept <ready,␣$ST_i$> from $U_i \in \mathcal{U}$
    send <ready$^\bullet$,$U_i$> to $A$
ready$-P$: accept <ready> from $P \in \mathcal{M} \setminus \mathcal{U}$
    send <ready$^\bullet$,$ST_i$> to $A$
ready [ready$-P$ $\forall P \in \mathcal{M}$]

share[ready$\wedge$ $\neg$finish_setup]: accept <send,$x$,$ST_j$> from $ST_i$
  if $U_i, U_j \in Room$
    send <deliver,$x$,$ST_i$> to $ST_j$
  else
    send <$\perp$,$ST_i$> to $U_i$
finish_setup[ready $\wedge$ $\neg$finish_setup]:
  accept <finish_setup> from $U \in \mathcal{U}$
  from $i$:=1 to n
    send <close> to $ST_i$
    accept <close$^\bullet$> from $ST_i$
  send <finish_setup$^\bullet$> to $A$
  else
    send <$\perp$,$ST_i$> to $U_i$
relay_receive [ready]: accept <$x$,$ST_i$> from $U_i$
  send <$x$> to $ST_i$
relay_send [ready]: accept <$x$> from $ST_i$
  send <$x$,$ST_i$> to $U_i$
```

B Proof for Theorem 1

Theorem 1. *Every instance of $\mathcal{F}_{\mathrm{KM}}$ with parameters $\overline{\mathcal{F}}, \overline{\mathcal{C}}, \Pi$ and session parameters $\mathcal{U}, \mathcal{U}^{\mathrm{ext}}, \mathcal{ST}, Room$ has the following properties:*

(1) At any step of an execution of $[\mathcal{F}_{\mathrm{KM}}, A_D, Z]$, the following holds for $\mathcal{F}_{\mathrm{KM}}$: for all `Store`$[U,h]$ $=< \mathcal{F}, a, c >$ *such that $c \notin \mathcal{K}_{\mathrm{cor}}$, there is a* `new` *node*

labelled a' in the attribute policy graph for $\mathcal{F}$ in Π such that the attribute a is reachable from that node and there was a step `new` *where* $\mathtt{Store}[U', h'] =< \mathcal{F}, a', c >$ *was added.*

(2) *At any step of an execution of $[\mathcal{F}_{\mathrm{KM}}, A_D, Z]$, the following holds for $\mathcal{F}_{\mathrm{KM}}$: all $c \in \mathcal{K}_{\mathrm{cor}}$, were either corrupted directly, i. e., there was a* `corrupt` *triggered by a query $< \mathtt{corrupt}, \mathtt{h} >$ from U while $\mathtt{Store[}U\mathtt{,}h\mathtt{]}=< \mathcal{F}, a, c >$, or indirectly,i. e., there is $c' \in \mathcal{K}_{\mathrm{cor}}$ such that at some point the* `wrap` *step was triggered by a message $\mathtt{<wrap,}h'\mathtt{,}h\mathtt{,}id\mathtt{>}$ from U while $\mathtt{Store[}U\mathtt{,}h'\mathtt{]}=< \mathtt{KW}, a', c' >$, $\mathtt{Store[}U\mathtt{,}h\mathtt{]}=< \mathcal{F}, a, c >$.*

(3) *At any step of an execution of $[\mathcal{F}_{\mathrm{KM}}, A_D, Z]$, the following holds: Whenever an ideal machine $\mathcal{F}_i$ = $\mathtt{<ideal,<sid,<}\mathcal{F}_i\mathtt{,F>>>}$, $F = \mathtt{<<reg,}\mathcal{F}\ \mathtt{>,<sid>>}$, accepts the message $\mathtt{<corrupt,}c\mathtt{>}$ for some c such that $\mathcal{F}_{\mathrm{KM}}$ in session* `sid` *has an entry $\mathtt{Store[}U\mathtt{,}\ \ h\mathtt{]=}\ \mathtt{<}\mathcal{F}_i\mathtt{,}a\mathtt{,}c\mathtt{>}$, then $c \in \mathcal{K}_{\mathrm{cor}}$ in $\mathcal{F}_{\mathrm{KM}}$.*

Proof. (1) Proof by induction over the number of epochs since $\mathcal{F}_{\mathrm{KM}}$'s first activation, t: If $t = 0$, `Store` is empty. $t > 0$: Since the property was true in the previous step, there are only three steps we need to look at: If a key $< \mathcal{F}, \mathtt{a}, \mathtt{c} >$ is added to `Store` at step `new`, then it is created only if the policy contains an entry ($\mathcal{F}$, `new`,a), i. e., a itself is a `new` node. If a key $< \mathcal{F}, \mathtt{a}, \mathtt{c} >$ is added to `Store` at step `unwrap`, let $\mathtt{<unwrap,}h_1\mathtt{,}w\mathtt{,}\mathcal{F}\mathtt{,}id\mathtt{>}$ be the arguments sent by a user U, and $\mathtt{Store[}U\mathtt{,}h_1\mathtt{]=<KW,}a_w\mathtt{,}c_w\mathtt{>}$. If $c \notin \mathcal{K}_{\mathrm{cor}}$, then $c_w \notin \mathcal{K}_{\mathrm{cor}}$, too, and thus there is an entry $< c, < \mathcal{F}, a, id >, w >\in \mathtt{encs}[c_w]$. `encs` is only written in `wrap`, therefore there was a position $[U', h']$ in the store, such that $\mathtt{Store[}U'\mathtt{,}h'\mathtt{]=<}\mathcal{F}\mathtt{,}a\mathtt{,}c\mathtt{>}$. Using the induction hypothesis, we see that a is reachable in the attribute policy graph. The third and last step where the store is written to is `AttributeChange`. If this step alters the attribute from a' to a, there must have been an entry $(\mathcal{F}, \mathtt{attribute\_change}, a', a) \in \Pi$. By induction hypothesis, a' is reachable from a `new` node, therefore a' is, too.

(2) A credential c is only added to the set $\mathcal{K}_{\mathrm{cor}}$ in two steps: If it is added in `corrupt`, then a message $\mathtt{<corrupt,}h\mathtt{>}$ was received from $U \in \mathcal{U}$ and $\mathtt{Store[}U\mathtt{,}h\mathtt{]}=< F, a, c >$. If it was added in `wrap`, then a message $\mathtt{<wrap,}h_1\mathtt{,}$ $h_2\mathtt{>}$ must have been received from $U \in \mathcal{U}$ while $\mathtt{Store[}U\mathtt{,}h_1\mathtt{]}=< \mathtt{KW}, a_1, c_1 >$, and c was reachable from c_1. Let c' be the last node on the path to c. $c' \in \mathcal{K}_{\mathrm{cor}}$ because it is reachable from c_1, too. Since $(c', c) \in C$, there was another wrapping query $\mathtt{<wrap,}h'\mathtt{,}h\mathtt{,}id\mathtt{>}$ with $\mathtt{Store[}U\mathtt{,}h'\mathtt{]}=< \mathtt{KW}, a', c' >$ and $\mathtt{Store[}U\mathtt{,}h\mathtt{]}=< \mathcal{F}, a, c >$. Since entries in the store are never deleted (only the attribute can be altered), and credentials are never removed from $\mathcal{K}_{\mathrm{cor}}$, the property holds.

(3) $\hat{I}_i$ accepts only messages coming from the party F, and F in turn only accepts messages coming from $\mathcal{F}_{\mathrm{KM}}$. Therefore, we can conclude from the definition of step `corrupt` in $\mathcal{F}_{\mathrm{KM}}$ that $c \in \mathcal{K}_{\mathrm{cor}}$.

C Proofs for Lemma 1 and Lemma 2

Definition 9. *For KU parameters $\overline{\mathcal{F}}, \overline{\mathcal{C}}, \Pi$ and implementation functions $\overline{\mathtt{Impl}} := \{\mathtt{Impl}_F\}_{F \in \overline{\mathcal{F}}}$, define the protocol $\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\mathtt{Impl}}}$ as follows: $\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\mathtt{Impl}}}$ defines only $\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\mathtt{Impl}}}(\mathtt{prot-fkm})$ and $\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\mathtt{Impl}}}(\mathtt{prot-fsetup})$. The session parameter is expected to be an encoding of the network parameters $\mathcal{U}, \mathcal{U}^{\text{ext}}, \mathcal{ST}, Room$. The code executed depends on the party running $\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\mathtt{Impl}}}(\mathtt{prot-fkm})$: If the party has identity `<<reg,u-i>,sid>`, the following code U_i is executed:*

```
relay_send: accept <m> from parentId
  call $\mathcal{F}_{\text{setup}}$ with <m,$ST_i$>
relay_receive : accept <m,$ST_i$> or <$m = \bot$> from $\mathcal{F}_{\text{setup}}$
  send <m> to parentId
public_command:
  accept <C,public,m> from parentId
  if $C \in \mathcal{C}_{i,pub}$
    send <$C^\bullet$,$impl_C(public, m)$> to parentId
```

If the party has identity `ext-u-i`, the following code U_i^{ext} is executed:

```
public_command:
  accept <C,public,m> from parentId
  if $C \in \mathcal{C}_{i,pub}$
    send <$C^\bullet$,$impl_C(public, m)$> to parentId
```

A regular protocol machine with machine id `<reg,`$\mathcal{F}_i$`>,sid` for $\mathcal{F}_i \in \{\mathcal{F}_1, \ldots, \mathcal{F}_l\}$ runs the following code:

```
relay: accept <m> from parentId
  call $\mathcal{F}_{\text{setup}}$ with <m>
```

If the party has identity `<<reg,st-i>,sid>`, then ST_i is executed. The code for ST_i is given in Section 3.3 and in Appendix A. For parties with the identities `<<reg,u-i>,sid>` or `<<reg,ext-u-i>,sid>`, $\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\mathtt{Impl}}}$ will act according to the convention on machine corruption defined in [11, § 8.1], while for `<<reg,st-i>,sid>`, it will ignore corruption request (security tokens are assumed to be incorruptible). For other machines, including ideal machines, it responds to any message with an error message to the adversary, i. e., $\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\mathtt{Impl}}}(\mathtt{prot-fkm})$ is totally regular. *$\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\mathtt{Impl}}}(\mathtt{prot-fkm})$ declares the use of `prot-fsetup` as a subroutine. $\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\mathtt{Impl}}}(\mathtt{prot-fsetup})$ runs $\mathcal{F}_{\text{setup}}$, i. e., $\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\mathtt{Impl}}}$ is a $\mathcal{F}_{\text{setup}}$-hybrid protocol.*

The static call graph has only an edge from `prot-fkm` to `prot-fsetup`, $\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\mathtt{Impl}}}$ is thus rooted at `prot-fkm`.

Lemma 3. *For all KU parameters $\overline{\mathcal{F}}, \overline{\mathcal{C}}, \Pi$, $\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\mathtt{Impl}}}$ is a poly-time protocol.*

Proof. By Definition 2 in [11, § 6], we need to show that there exists a polynomial p such that for every well-behaved environment Z that is rooted at `prot-fkm`, we have:

$$\begin{aligned}\Pr[\mathrm{Time}_{\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\texttt{Impl}}}}[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\texttt{Impl}}}, A_D, Z](\eta) \\ &> p(\mathrm{Flow}_{Z\rightarrow\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\texttt{Impl}}},A_D}[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\texttt{Impl}}}, A_D, Z](\eta))] \\ &= \mathrm{negl}(\eta).\end{aligned}$$

Let $p_{\max}$ be a polynomial such that for all $\mathcal{F}_i$ and $C \in \mathcal{C}$, the algorithm impl_C terminates in a running time smaller than $p_{\max}(n)$, where n is the length of the input. $\mathit{impl}^F_{\texttt{new}}$ is always called on input of length η, thus all keys have a length smaller $p_{\max}(\eta)$. In step `new`, F and a are provided by the environment (as input to U_i, which then asks $\mathcal{F}_{\mathrm{setup}}$ to relay the request to ST_i). Since messages have at least length η, we can overapproximate by saying that `Store` grows at most by some polynomial $p_{\mathrm{growth-new}}$ in the length of the environment's input. Similarly, an `<unwrap,...>` query cannot grow the Store by more than $p_{\mathrm{growth-unwrap}}$. Therefore, at any point in time t (we simply count the number of epochs, i. e., activations of the environment), the store is smaller than $p'(\mathrm{Flow}_{Z\rightarrow\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\texttt{Impl}}},A_D}[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\texttt{Impl}}}, A_D, Z](\eta))$ for a polynomial p'.

We observe that there is not a single activation of a machine in $\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\texttt{Impl}}}$, neither a U_i, an ST_i nor $\mathcal{F}_{\mathrm{setup}}$, where the running time is not polynomial in the environment's input and the length of the `Store`. A_D might corrupt user $U \in \mathcal{U} \cup \mathcal{U}^{\mathrm{ext}}$, but they do not have any state. Thus, we have for the running time of $\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\texttt{Impl}}}$ at point t, i. e., $\mathrm{Time}_{\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\texttt{Impl}}},t}[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\texttt{Impl}}}, A_D, Z](\eta)$,

$$\begin{aligned}\mathrm{Time}_{\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\texttt{Impl}}},t}&[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\texttt{Impl}}}, A_D, Z](\eta) \\ &= \mathrm{Time}_{\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\texttt{Impl}}},t-1}[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\texttt{Impl}}}, A_D, Z](\eta)+ \\ &\quad p'(\mathrm{Flow}_{Z\rightarrow\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\texttt{Impl}}},A_D}[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\texttt{Impl}}}, A_D, Z](\eta)) \\ &\leq t \cdot p'(\mathrm{Flow}_{Z\rightarrow\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\texttt{Impl}}},A_D}[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\texttt{Impl}}}, A_D, Z](\eta)) \\ &\leq p''(\mathrm{Flow}_{Z\rightarrow\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\texttt{Impl}}},A_D}[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\texttt{Impl}}}, A_D, Z](\eta))\end{aligned}$$

for another polynomial p'', because

$$t < \mathrm{Flow}_{Z\rightarrow\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\texttt{Impl}}},A_D}[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\texttt{Impl}}}, A_D, Z](\eta)).$$

The proof that π implements $\mathcal{F}_{\mathrm{KM}}$ proceeds in several steps: Making use of the composition theorem, the last functionality $\mathcal{F}_l$ in $\mathcal{F}_{\mathrm{KM}}$ can be substituted by its key-manageable implementation $\hat{I}_L$. Then, $\mathcal{F}_{\mathrm{KM}}$ can simulate $\hat{I}$ instead of calling it. Let $\mathcal{F}_{\mathrm{KM}}^{\{\mathcal{F}_l/\hat{I}_l\}}$ be the resulting functionality. In the next step, calls to this simulation are substituted by calls to the functions used in $\hat{I}$, impl_C for each $C \in \mathcal{C}_l$. The resulting, partially implemented functionality $\mathcal{F}_{\mathrm{KM}}^{\{\mathcal{F}_l/\texttt{Impl}_{\mathcal{F}_l}\}}$ saves keys rather than credentials (for $\mathcal{F}_l$). We repeat the previous steps until $\mathcal{F}_{\mathrm{KM}}$ does not call any KU functionalities anymore, i. e., we have $\mathcal{F}_{\mathrm{KM}}^{\{\mathcal{F}_1/\texttt{Impl}_{\mathcal{F}_1},\ldots,\mathcal{F}_n/\texttt{Impl}_{\mathcal{F}_n}\}}$.

Then we show that the network of distributed token π emulates the monolithic block $\mathcal{F}_{\mathrm{KM}}^{\{\mathcal{F}_1/\mathtt{Impl}_{\mathcal{F}_1},\ldots,\mathcal{F}_n/\mathtt{Impl}_{\mathcal{F}_n}\}}$ that does not call KU functionalities anymore, using a reduction to the security of the key-wrapping scheme.

The first four steps will be the subject of Lemma 1, the last step is Lemma 2. But before we come to this, the following definition expresses partial implementations of $\mathcal{F}_{\mathrm{KM}}$. In fact, the formal definition of $\mathcal{F}_{\mathrm{KM}}$ is the special case in which the set of substituted functionalities is empty:

Definition 10 **($\mathcal{F}_{\mathrm{KM}}$ with partial implementation).** *Given the KU parameters $\overline{\mathcal{F}}, \overline{\mathcal{C}}, \Pi$, and functions ($impl^{\mathtt{KW}}_{\mathtt{new}}$, wrap, unwrap), let $\mathtt{Impl}_{\mathcal{F}_i}$ be the algorithms defining the keymanageable implementation $\hat{I}_i$ of $\mathcal{F}_i \in \{\mathcal{F}_1, \ldots, \mathcal{F}_p\} \subset \overline{\mathcal{F}}$. We will define the partial implementation of $\mathcal{F}_{\mathrm{KM}}$ with respect to the KU functionalities $\mathcal{F}_1, \ldots, \mathcal{F}_p$, denoted $\mathcal{F}_{\mathrm{KM}}^{\{\mathcal{F}_1/\mathtt{Impl}_{\mathcal{F}_1},\ldots,\mathcal{F}_p/\mathtt{Impl}_{\mathcal{F}_p}\}}$,*

Furthermore, the protocol defines the ideal protocols `prot-f-`$(p+1)$,…, `prot-f-`l, running the logic defined by $\mathcal{F}_{p+1}, \ldots, \mathcal{F}_l$, and `prot-fkm`. For `prot-fkm`, the protocol defines the following behaviour: A regular protocol machine with machine id `<<reg,`$\mathcal{F}_i$`>,sid>` for $\mathcal{F}_i \in \{\mathcal{F}_1, \ldots, \mathcal{F}_l\}$ runs the following code:

```
dummy_to: accept <m> from parentId
  send <m> to <ideal,sid> $(= \mathcal{F}_{\mathrm{KM}})$
dummy_from: accept <m> from <ideal,sid> $(= \mathcal{F}_{\mathrm{KM}})$
  send <m> to parentId
relay_to: accept <m> from <ideal,sid> $(= \mathcal{F}_{\mathrm{KM}})$
  send <m> to <<reg,0>,<sid,<prot-f-$i$,<>>> $(= \mathcal{F}_i)$
relay_from: accept <m> from <<reg,0>,<sid,<prot-f-$i$,<>>> $(= \mathcal{F}_i)$
  send <m> to <ideal,sid> $(= \mathcal{F}_{\mathrm{KM}})$
```

The ideal party runs the logic for $\mathcal{F}_{\mathrm{KM}}$ described in Section-3.3, with the following alteration in the `new` and `command` step:

```
new[ready]: accept <new,F,a> from $U \in \mathcal{U}$
if <F,new,a,*> $\in \Pi$ then
  if $F \in \{\mathcal{F}_1, \ldots, \mathcal{F}_p, \mathtt{KW}\}$
    (k, public) $\leftarrow impl^{F}_{\mathtt{new}}(1^\eta)$
    create h; Store[U,h] $\leftarrow$ <F,a,k>
    send <new$^\bullet$,h,public> to U
  else
    call F with <new>
    accept <new$^\bullet$,cred,public> from F
    create h; Store[U,h] $\leftarrow$ <F,a,cred>
    send <new$^\bullet$,h,public> to U
```

```
command[finish_setup]:
  accept <C,h,m> from $U \in \mathcal{U}$
  if Store[U,h]=<F,a,c> and <F,C,a,*>$\in \Pi$
    if $F \in \{\mathcal{F}_1, \ldots, \mathcal{F}_p\}$
      send <C$^\bullet$,$impl_C(c,m)$> to U
    else
```

```
        call $F$ with <$C$,$c$,$m$>
        accept <$C^\bullet$,$r$> from $F$
        send <$C^\bullet$,$r$> to $U$
```

```
public_command:
  accept <$C$,$public$,$m$> from $U \in \mathcal{U}$
  if $C \in \mathcal{C}_{i,pub}$
    if $F \in \{\mathcal{F}_1, \dots, \mathcal{F}_p\}$
      send <$C^\bullet$,$impl_C(public, m)$> to $U$
    else
      call $F_i$ with <$C$,$public$,$m$>
      accept <$C^\bullet$,$r$> from $F$
      send <$C^\bullet$,$r$> to $U$
```

```
corrupt[finish_setup]:
  accept <corrupt,$h$> from $U \in \mathcal{U}$
  if Store[$U$,$h$]=<$F$,$a$,$c$>
    if $F \in \{\mathcal{F}_1, \dots, \mathcal{F}_p\}$
      send <corrupt$^\bullet$,$h$,$c$> to $A$
    else if $F \in \{\mathcal{F}_{p+1}, \dots, \mathcal{F}_l\}$
      call $F$ with <corrupt,c>
    else if $F$=KW
      for all $c'$ reachable from $c$ in $C$
        $\mathcal{K}_{\text{cor}} \leftarrow \mathcal{K}_{\text{cor}} \cup \{c'\}$
      send <corrupt$^\bullet$,$h$,$c$> to $A$
```

Note that the partial implementation of $\mathcal{F}_{\text{KM}}$ is not an ideal protocol in the sense of [11, § 8.2], since not every regular protocol machine runs the dummy party protocol – the party <reg,$\mathcal{F}_i$> relays the communication with the KU functionalities.

Lemma 1. *Let $\overline{\mathcal{F}}, \overline{\mathcal{C}}, \Pi$ be KU parameters such that all $F \in \overline{\mathcal{F}}$ are key-manageable. Let $\texttt{Impl}_{\mathcal{F}_i}$ be the functions defining the key-manageable implementation $\hat{I}_i$ of $\mathcal{F}_i$. Then $\mathcal{F}_{\text{KM}}^{\mathcal{F}_1/\texttt{Impl}_{\mathcal{F}_1}, \dots, \mathcal{F}_l/\texttt{Impl}_{\mathcal{F}_l}}$ emulates $\mathcal{F}_{\text{KM}}$. Furthermore, it is poly-time.*

Proof. Induction on the number of substituted KU functionalities.

Base case: $\mathcal{F}_{\text{KM}}^{\{\}}$ actually equals $\mathcal{F}_{\text{KM}}$. Since emulation is reflexive, $\mathcal{F}_{\text{KM}}^{\{\}}$ emulates $\mathcal{F}_{\text{KM}}$. It is left to show that $\mathcal{F}_{\text{KM}}$ is poly-time: The argument is actually the same as for $\pi_{\overline{\mathcal{F}}, \overline{\mathcal{C}}, \Pi, \overline{\texttt{Impl}}}$ (see proof to Lemma 3), after we have established five things: 1. The implementation functions for all $\mathcal{F} \neq \texttt{KW}$ are running on the same values. 2. The implementation function for wrapping might run a different value, but it has the same length, i. e., the same upper bound holds for its running time. 3. Graph reachability is linear in the number of credentials, which in turn is polynomial, because the Flow from the environment is polynomial, and thus the number of new queries. 4. The relaying of messages from U_i via $\mathcal{F}_{\text{setup}}$ does not add more than linearly in η to the running time, 5. simiarily, for the distribution of the <finish_setup> message.

Induction Step: Assume $i \geq 1$ and that $\mathcal{F}_{\mathrm{KM}}^{\mathcal{F}_1/\mathtt{Impl}_{\mathcal{F}_1},\ldots,\mathcal{F}_{i-1}/\mathtt{Impl}_{\mathcal{F}_{i-1}}}$ emulates $\mathcal{F}_{\mathrm{KM}}$. Since emulation is transitive, it suffices to show that $\mathcal{F}_{\mathrm{KM}}^{\mathcal{F}_1/\mathtt{Impl}_{\mathcal{F}_1},\ldots,\mathcal{F}_i/\mathtt{Impl}_{\mathcal{F}_i}}$ emulates $\mathcal{F}_{\mathrm{KM}}^{\mathcal{F}_1/\mathtt{Impl}_{\mathcal{F}_1},\ldots,\mathcal{F}_{i-1}/\mathtt{Impl}_{\mathcal{F}_{i-1}}}$. We will proceed in three step: First, we will substitute $\mathcal{F}_i$ by its key-manageable implementation $\hat{I}_i$. Then, we will alter $\mathcal{F}_{\mathrm{KM}}$ to simulate $\hat{I}_i$ inside. The main part of the proof is showing that $\hat{I}_i$ can be emulated by calling $\mathtt{Impl}_{F_i}$ inside $\mathcal{F}_{\mathrm{KM}}$, storing keys instead of credentials.

The first step is a consequence of composition theorem [11, Theorem 7]. The induction hypothesis give us that $\mathcal{F}_{\mathrm{KM}}^{\mathcal{F}_1/\mathtt{Impl}_{\mathcal{F}_1},\ldots,\mathcal{F}_{i-1}/\mathtt{Impl}_{\mathcal{F}_{i-1}}}$ (in the following: $\mathcal{F}_{\mathrm{KM}}^{i-1}$) is a poly-time protocol, and it is rooted in `fkm`. Since $\mathcal{F}_i$ is key-manageable, we know that $\hat{I}_i$ is a polytime protocol that emulates $\hat{I}_i$. $\hat{I}_i$ defines only `F-`i, therefore $\hat{I}_F$ is substitutable for $\mathcal{F}_i$ in $\mathcal{F}_{\mathrm{KM}}^{i-1}$. Hence, $F^{i-1}[\mathcal{F}_i/\hat{I}_i]$ is poly-time and emulates $\mathcal{F}_{\mathrm{KM}}^{i-1}$.

In the second step, we alter $F^{i-1}[\mathcal{F}_i/\hat{I}_i]$ (in the following: $\mathcal{F}_{\mathrm{KM}}^{i-1'}$) such that the ideal functionality defined in $\mathcal{F}_{\mathrm{KM}}^{i-1'}(\mathtt{prot}-\mathtt{fkm})$ simulates $\hat{I}_i$ locally, and calls this simulation whenever `<<reg,`$\mathcal{F}_i$`>,sid>` would be addressed in $\mathcal{F}_{\mathrm{KM}}$. $\hat{I}_i$ might send a message to A, in which case this message is indeed relayed to A. Since the simulation will only be called by $\mathcal{F}_{\mathrm{KM}}$, it will only respond to $\mathcal{F}_{\mathrm{KM}}$. We will call this protocol $\mathcal{F}_{\mathrm{KM}}^{i-1''}$. To show that $\mathcal{F}_{\mathrm{KM}}^{i-1''}$ emulates $\mathcal{F}_{\mathrm{KM}}^{i-1'}$, we have to make sure that $\hat{I}_i$ can only be addressed by $\mathcal{F}_{\mathrm{KM}}$, since then the observable output to the environment is exactly the same. By definition, `<<reg,`$\mathcal{F}_i$`>,sid>` ignores messages from the environment (except for the initial `<ready>` message). It can be adressed by any other regular parties. It might be adressed by `<adv>`, but it ignores those messages. This behaviour can easily be simulated: whenever the environment instructs the simulator in $\mathcal{F}_{\mathrm{KM}}^{i-1'}$ to address $\hat{I}_i$, the Simulator behaves as if this machine would not exist (e. g.by sending a message to a non-existent machine). Therefore, $\mathcal{F}_{\mathrm{KM}}^{i-1''}$ emulates $\mathcal{F}_{\mathrm{KM}}^{i-1'}$. Since $\hat{I}_i$ is poly-time, $\mathcal{F}_{\mathrm{KM}}^{i-1''}$ can simulate it and is still poly-time.

In the third step, we show that F_i emulates $\mathcal{F}_{\mathrm{KM}}^{i-1''}$. We claim that in fact, with overwhelming probability, F_i provides a perfect simulation of $\mathcal{F}_{\mathrm{KM}}^{i-1''}$, namely, when the list L maintained in $\mathcal{F}_{\mathrm{KM}}^{i-1}$ describes a bijection between credentials and keys. Since for all k, $\Pr[k' = k | k' \leftarrow \mathit{impl}_{\mathtt{new}}^{F_i}(1^\eta)]$ is negligible by assumption (see Definition 4), this is the case. So we can assume without loss of generality that this list describes a bijection. Then, as careful inspection of the steps `new, command, wrap, unwrap` and `corrupt` shows, keys and credentials are interchangeable, and $\mathtt{Impl}_{\mathcal{F}_i}$ can be inlined, resulting in F_i.

By transitivity of emulation, we have that F_i emulates $\mathcal{F}_{\mathrm{KM}}$. By the fact that F_i actually computes less than $\mathcal{F}_{\mathrm{KM}}^{i-1''}$, we know it is poly-time.

For the next step, we need to define what we understand under a key-wrapping scheme. We took the definition from [10] as a basis and will repeat it here. It is based on the notion of deterministic, authenticated encryption from [19], but it additionally supports key-dependant messages. We changed the definition, so it allows to wrap the same key with the same wrapping key but under different attributes, just like in the DAE definition from [19].

Definition 11 (Multi-user setting for key wrapping). *We define experiments $\mathbf{Exp}^{\mathsf{wrap,real}}_{\mathcal{A},\mathsf{KW}}(\eta)$ and $\mathbf{Exp}^{\mathsf{wrap,fake}}_{\mathcal{A},\mathsf{KW}}(\eta)$. In both experiments the adversary can access a number of keys $k_1, k_2, \ldots, k_n \ldots$ (which he can ask to be created via a query NEW). In his other queries, the adversary refers to these keys via symbols $K_1, K_2, \ldots, K_n$ (where the implicit mapping should be obvious). By abusing notation we often use K_i as a placeholder for k_i so, for example, $\mathsf{Wrap}^a_{K_i}(K_j)$ means $\mathsf{Wrap}^a_{k_i}(k_j)$. We now explain the queries that the adversary is allowed to make, and how they are answered in the two experiments.*

- $\mathsf{NEW}(K_i)$*: a new key* k_i *is generated via* $k_i \leftarrow \mathsf{KG}(\eta)$
- $\mathsf{ENC}(K_i, a, m)$ *where* $m \in \mathcal{K} \cup \{K_i \mid i \in \mathbb{N}\}$ *and* $h \in \mathcal{H}$*. The experiment returns* $\mathsf{Wrap}^a_{k_i}(m)$*.*
- $\mathsf{TENC}(K_i, a, m)$ *where* $m \in \mathcal{K} \cup \{K_i \mid i \in \mathbb{N}\}$ *and* $a \in \mathcal{H}$*. The real experiment returns* $\mathsf{Wrap}^a_{k_i}(m)$*, whereas the fake experiment returns* $\$^{|\mathsf{Wrap}^a_{k_i}(m)|}$
- $\mathsf{DEC}(K_i, a, c)$*: the real experiment returns* $\mathsf{UnWrap}^a_{k_i}(c)$*, the fake experiment returns* $\perp$*.*
- $\mathsf{CORR}(K_i)$*: the experiment returns* k_i

Correctness of the wrapping scheme requires that for any $k_1, k_2 \in \mathcal{K}$ and any $a \in \mathcal{H}$, if $c \leftarrow \mathit{Wrap}^a_{k_1}(k_2)$ then $\mathit{Unwrap}^a_{k_1}(c) = k_1$.

Consider the directed graph whose nodes are the symbolic keys K_i and in which there is an edge from K_i to K_j if the adversary issues a query $\mathsf{ENC}(K_i, a, K_j)$. We say that a key K_i is corrupt if either the adversary corrupted the key from the start, or if the key is reachable in the above graph from a corrupt key. If a handle, respectively pointer, points to a corrupted key, we call the pointer corrupted as well.

We make the following assumptions on the behaviour of the adversary.

- For all i the query $\mathsf{NEW}(K_i)$ is issued at most once.
- All the queries issued by the adversary contain keys that have already been generated by the experiment.
- The adversary never makes a test query $\mathsf{TENC}(K_i, a, K_j)$ if K_i is corrupted at the end of the experiment.
- If A issues a test query $\mathsf{TENC}(K_i, a, m)$ then A does not issue $\mathsf{TENC}(K_j, a', m')$ or $\mathsf{ENC}(K_j, a', m')$ with $(K_i, a, m) = (K'_j, a', m')$
- The adversary never queries $\mathsf{DEC}(K_i, a, c)$ if c was the result of a query $\mathsf{TENC}(K_i, a, m)$ or of a query $\mathsf{ENC}(K_i, a, m)$ or K_i is corrupted.

At the end of the execution the adversary has to output a bit b which is also the result of the experiment. The advantage of adversary A in breaking the key-wrapping scheme KW is defined by:

$$\begin{aligned} A^{\mathsf{wrap}}_{\mathsf{KW},A}(\eta) = \Big| &\Pr\left[b \leftarrow \mathbf{Exp}^{\mathsf{wrap,real}}_{\mathsf{KW},A}(\eta) \ : b = 1\right] - \\ &\Pr\left[b \leftarrow \mathbf{Exp}^{\mathsf{wrap,fake}}_{\mathsf{KW},A}(\eta) \ : b = 1\right]\Big| \end{aligned}$$

and KW is secure if the advantage of any probabilistic polynomial time algorithm is negligible.

Definition 12 (guaranteeing environment). *Suppose Z is an environment that is rooted at r, and p is a predicate on sequences of (id_0, id_1, m). Let $S_p(Z)$ be a sandbox that runs Z but checks at the end of each activation if the predicate holds true on the list of messages sent and received by the environment (including the message about to be send). If the predicate does not hold true, S_p aborts Z_p and outputs some error symbol fail $\in \Sigma$. We say that Z* guarantees *a predicate p, if there exists such a sandbox $S_p(Z)$, and for every protocol Π rooted at r, for every adversary A, we have that:*

$$\Pr[\mathrm{Exec}[\Pi, A, Z] = \mathit{fail}]$$

is negligible in η.

Let us denote a list of messages m_i from a_i to b_i, as $M^t = ((a_0, b_0, m_0), \ldots, (a_t, b_t, m_t))$. We will denote the i-prefix of this list by M^i. We can filter messages by their session id: $M^i_{|\mathrm{SP}}$ denotes a messages (a_i, b_i, m_i) where either $a_i = <\texttt{env}>$ and b_i is of the form `<<reg,`*basePID*`>,<`$\alpha_1, \ldots, \alpha_{k-1}$`,<prot-fkm,<`SP`>>>>`, or vice versa. We say (a_j, b_j, m_j) is a response to (a_i, b_i, m_i) if (a_j, b_j, m_j) is the earliest message such that $i < j$, $a_i = b_j$, $b_j = a_i$, and that no other message at an epoch $k < i$ exists such that (a_i, b_i, m_i) is a response to (a_k, b_k, m_k). This assumes that there is a response to every query. (In case of an error, $\mathcal{F}_{\mathrm{KM}}$ responds with $\perp$ rather than ignoring the query.) In order to tell which handles are corrupted, we need to define which handles point to the same key a given moment t.

Given $M_{|NP} = M_{|\mathcal{U},\mathcal{U}^{\mathrm{ext}},\mathcal{ST},Room} = ((a_0, b_0, m_0), \ldots, (a_n, b_n, m_n))$, we define $\equiv^0$ to be the empty relation and for all $1 \leq t \leq n$, we define $\equiv^t$ as the least symmetric transitive relation such that

1. $\equiv^t \subset \equiv^{t-1} \cup \{(U, h), (U, h)\}$, if $m_t = <\texttt{new}^\bullet, \texttt{h}, \mathit{public}) >$, $a_t = U$ and $\exists s < t, F, a : m_s = <\texttt{new}, \texttt{F}, \texttt{a}>$ and (a_t, b_t, m_t) is a response to (a_s, b_s, m_s)
2. $\equiv^t \subset \equiv^{t-1} \cup \{(U_1, h_1), (U_2, h_2)\}$, if $m_t = <\texttt{share}^\bullet>$, $a_t = U_1$ and $\exists s < t :\ m_s = <\texttt{share}, (\texttt{U}_1, \texttt{h}_1), (\texttt{U}_2, \texttt{h}_2)>$ and (a_t, b_t, m_t) is a response to (a_s, b_s, m_s)
3. $\equiv^t \subset \equiv^{t-1} \cup \{(U_1, h_1), (U_2, h_2)\}$, if $m_t = <\texttt{unwrap}^\bullet, \texttt{h}_2>$, $a_t = U_2$ and $\exists q, r, s :$ such that (a_t, b_t, m_t) is a response to (a_s, b_s, m_s), and (a_r, b_r, m_r) is a response to (a_q, b_q, m_q), and $r < s$. Furthermore:
   $m_q = <\texttt{wrap}, \texttt{h}_1, \texttt{h}_2, \texttt{id}>, b_q = U_1, m_r = <\texttt{wrap}^\bullet, \texttt{w}>, a_r = U_1$ and
   $m_s = \texttt{unwrap}, \texttt{h}'_1, \texttt{w}, \texttt{a}, \texttt{F}, \texttt{id}>, b_s = U_1$ and $(U_2, h'_1) \equiv^{t-1} (U_1, h_1)$.
4. $\equiv^t = \equiv^{t-1}$, otherwise.

Using this relation, we define following predicate: $\mathit{corrupted}_{M_{|NP}}(U, h)$ holds iff either some (U^*, h^*), $((U^*, h^*) \equiv^t (U, h))$ were corrupted directly, or via wrapping with a corrupted key, formally:

- $m_j = (\mathtt{adv}, \mathtt{env}, < \mathtt{corrupt}^\bullet, \mathtt{h}, \mathtt{c} >), m_i = (\mathtt{env}, U, < \mathtt{corrupt}, \mathtt{h} >) \in M_{|NP}$ and m_j is a response to m_i (for some c), or
- there are $m_j = (U, \mathtt{env}, < \mathtt{wrap}^\bullet, \mathtt{w} >), m_i = (\mathtt{env}, U, < \mathtt{wrap}, \mathtt{h}_1, \mathtt{h}_2, \mathtt{id} >) \in M_{|NP}$ and m_j is a response to m_i (for some c), while $(U_1, h_1) \equiv^t (U^*, h^*)$ and $(U_1, h_2) \equiv^t (U, h)$ for some (U^*, h^*) that were corrupted directly.

Finally, let *corrupt-before-wrap* be the following predicate on a list of messages $M^t = ((a_0, b_0, m_0), \dots, (a_t, b_t, m_t))$: For all $i \leq t$ and network parameters $NP = \mathcal{U}, \mathcal{U}^{\mathrm{ext}}, \mathcal{ST}, \mathit{Room}$, we have

$$\mathit{corrupted}_{M_{|NP}}(U, h) \wedge (\mathtt{env}, U, < \mathtt{wrap}, \mathtt{h}, \mathtt{h'} >) \in M^i_{|NP} \Rightarrow \mathit{corrupted}_{M^i_{|NP}}(U, h).$$

Lemma 2. *For any KU parameter $\overline{\mathcal{F}}, \overline{\mathcal{C}}, \Pi$ and set of sets of ppt algorithms $\overline{\mathtt{Impl}}$, let $\mathcal{F}^{\mathtt{impl}}_{\mathrm{KM}} := \mathcal{F}^{\mathcal{F}_1/\mathtt{Impl}_{\mathcal{F}_1}, \dots, \mathcal{F}_l/\mathtt{Impl}_{\mathcal{F}_l}}_{\mathrm{KM}}$ be the partial implementation of $\mathcal{F}_{\mathrm{KM}}$ with respect to all KU functionalities in $\overline{\mathcal{F}}$. If $KW = (\mathsf{KG}, \mathsf{wrap}, \mathsf{unwrap})$ is a secure and correct key-wrapping scheme (Definition 11) then $\mathcal{F}^{\mathtt{impl}}_{\mathrm{KM}}$ emulates $\pi_{\overline{\mathcal{F}}, \overline{\mathcal{C}}, \Pi, \overline{\mathtt{Impl}}}$ for environments that guarantee* corrupt-before-wrap.

Proof. Proof by contradiction: Assuming that there is no adversary *Sim* such that for all well-behaved environments Z that are rooted at `prot-fkm` and guarantee *corrupt-before-wrap* $\mathrm{Exec}[\pi_{\overline{\mathcal{F}}, \overline{\mathcal{C}}, \Pi, \overline{\mathtt{Impl}}}, A_D, Z] \approx \mathrm{Exec}[\mathcal{F}^{\mathtt{impl}}_{\mathrm{KM}}, \mathit{Sim}, Z]$ holds, we chose a *Sim* that basically simulates $\mathcal{F}_{\mathrm{setup}}$ for corrupted users in $\mathcal{F}^{\mathtt{impl}}_{\mathrm{KM}}$, and a Z that is indeed able to distinguish $\mathrm{Exec}[\pi_{\overline{\mathcal{F}}, \overline{\mathcal{C}}, \Pi, \overline{\mathtt{Impl}}}, A_D, Z]$ and $\mathrm{Exec}[\mathcal{F}^{\mathtt{impl}}_{\mathrm{KM}}, \mathit{Sim}, Z]$. Then, we use it to construct an attacker B_Z against the key-wrapping challenger. B_Z will be carefully crafted, such that *a*) it is a valid adversary *b*) it has the same output distribution in the fake key-wrapping experiment as Z has when interacting with $\mathcal{F}^{\mathtt{impl}}_{\mathrm{KM}}$ and *Sim* *c*) it has the same output distribution in the real key-wrapping experiment as Z has when interacting with $\pi_{\overline{\mathcal{F}}, \overline{\mathcal{C}}, \Pi, \overline{\mathtt{Impl}}}$ and A_D.

Sim defines the same code as the dummy adversary (see [11, §4.7]), but when instructed by the environment to instruct a corrupted party to call $\mathcal{F}_{\mathrm{setup}}$, it simulates $\mathcal{F}_{\mathrm{setup}}$ (because $\mathcal{F}^{\mathtt{impl}}_{\mathrm{KM}}$ does not define `prot-fsetup`). This means: *Sim* waits for `<ready`$^\bullet$`,P>` from $\mathcal{F}^{\mathtt{impl}}_{\mathrm{KM}}$ from all parties $P \in \mathcal{U} \cup \mathcal{ST}$ before operating - for corrupted parties $U \in \mathcal{U}$ (security tokens are incorruptible, $U^{\mathrm{ext}} \in \mathcal{U}^{\mathrm{ext}}$ are ignored), it waits be instructed to send ready and simulates the reception of `<ready`$^\bullet$`,P>` itself. Afterwards, it accepts instructions to send `<send,`m`,`ST_i`>` as U_i to $\mathcal{F}_{\mathrm{setup}}$– in this case, *Sim* instructs U_i to send m to $\mathcal{F}^{\mathtt{impl}}_{\mathrm{KM}}$. Similarly, the response from $\mathcal{F}^{\mathtt{impl}}_{\mathrm{KM}}$ is simulated to be transmitted via $\mathcal{F}_{\mathrm{setup}}$. When U_i is instructed to send `finish_setup` to $\mathcal{F}_{\mathrm{setup}}$, *Sim* sends `finish_setup` to $\mathcal{F}^{\mathtt{impl}}_{\mathrm{KM}}$ instead (and relays the answer). It is only before having received a message `<finish_setup`$^\bullet$`>` that *Sim* simulates the sets `share` and `finish_setup`.

Given Z, we will now construct the attacker $\mathcal{B}_Z$ against the key-wrapping game in Definition 11. Recall that Z is rooted at `prot-fkm`. This means that Z only calls machines with the same SID and the protocol name `prot-fkm`. In particular, the session parameters $(\overline{\mathcal{F}}, \overline{\mathcal{C}}, \Pi)$ are the same (see [11, §5.3]), so from now on, we will assume them to be arbitrary, but fixed. The construction of $\mathcal{B}_Z$ aims at simulating Z in communication with the simulator Sim given above and the key-management functionality $\mathcal{F}_{\mathrm{KM}}^{\mathtt{impl}}$, but instead of performing wrapping and unwrapping in $\mathcal{F}_{\mathrm{KM}}^{\mathtt{impl}}$ itself, $\mathcal{B}_Z$ queries the challenger in the wrapping experiment. In case of the fake experiment, the simulation is very close to the network $[\mathcal{F}_{\mathrm{KM}}^{\mathtt{impl}}, Sim, Z]$, for the case of the real experiment, we have to show that the output is indistinguishable from a network of distributed security tokens and a dummy adversary $[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\mathtt{Impl}}}, A_D, Z]$. This will be the largest part of the proof. $\mathcal{B}_Z$ is defined as follows: $\mathcal{B}_Z$ simulates the network $[\mathcal{F}_{\mathrm{KM}}^{\mathtt{impl},\mathsf{KW}}, Sim, Z]$, where $\mathcal{F}_{\mathrm{KM}}^{\mathtt{impl},\mathsf{KW}}$ is defined just as $\mathcal{F}_{\mathrm{KM}}^{\mathtt{impl}}$, but `new`,`wrap`,`unwrap` and `corrupt` are altered such that send queries to the experiment instead. Note that for this reason, $\mathcal{F}_{\mathrm{KM}}^{\mathtt{impl},\mathsf{KW}}$ is not a valid machine in the GNUC model - we just use it as a convenient way to describe the simulation that $\mathcal{B}_Z$ runs.

```
new[ready]: accept <new,F,a> from $U \in \mathcal{U}$
if <F,new,a,*> $\in \Pi$ then
  if $F \in \{\mathcal{F}_1, \ldots, \mathcal{F}_p\}$
    $(k, public) \leftarrow impl^F_{\mathtt{new}}(1^\eta)$
    create $h$; Store[$U$,$h$] $\leftarrow$ <$F$,$a$,$k$>
    send <new$^\bullet$,$h$,$public$> to $U$
  else if $F$ =KW
    create $K_i$, $h$
    query $\mathsf{NEW}(K_i)$
    Store[$U$,$h$] $\leftarrow$ <KW,$a$,$K_i$>
    send <new$^\bullet$,$h$,> to $U$
```

```
wrap[finish_setup]:
  accept <wrap,$h_1$,$h_2$,$id$> from $U \in \mathcal{U}$
  if Store[U,$h_1$]=<KW,$a_1$,$c_1$> and
     Store[U,$h_2$]=<$F_2$,$a_2$,$a_2$> and
     <KW,wrap,$a_1$,$a_2$>$\in \Pi$
    if <$c_2$,<$F_2$,$a_2$,$id$>,w>$\in$encs[$c_1$]
      send <wrap$^\bullet$,w> to $U$
    else
      $C \leftarrow C \cup \{(c_1, c_2)\}$
      if $c_1 \in \mathcal{K}_{\mathrm{cor}}$
        for all $c_3 \notin \mathcal{K}_{\mathrm{cor}}$ reachable from $c_2$ in $C$
          $\mathcal{K}_{\mathrm{cor}} \leftarrow \mathcal{K}_{\mathrm{cor}} \cup \{c_3\}$
        if $F_2$ =KW
          $w \leftarrow \mathsf{wrap}^{<F_2,a_2,id>}$(key[$c_1$],key[$c_2$])
        else
          $w \leftarrow \mathsf{wrap}^{<F_2,a_2,id>}$(key[$c_1$],$c_2$)
      else
        query $w = \mathsf{TENC}(c_1, < F_2, a_2, id >, c_2)$
      encs[$c_1$] $\leftarrow$ encs[$c_1$] $\cup$\{ <$c_2$,<$F_2$,$a_2$,$id$>,w> \}
      send <wrap$^\bullet$,$w$> to $U$
```

```
unwrap[finish_setup]:
  accept <unwrap,$h_1$,$w$,$a_2$,$F_2$,$id$> from $U \in \mathcal{U}$
  if Store[U,$h_1$]=<KW,$a_1$,$c_1$> and
     <KW,unwrap,$a_1$,$a_2$>$\in \Pi$ and $F_2 \in \overline{\mathcal{F}}$
    if ( $c_1 \in \mathcal{K}_{\text{cor}}$ and
      $c_2 = \mathsf{unwrap}^{<F_2,a_2,id>}(\text{key}[c_1], w) \neq \bot$)
      create $h_2$, $K_2$; Store[$U$,$h_2$]$\leftarrow$ <$F_2$,$a_2$,$K_2$>
      $\mathcal{K}_{\text{cor}} \leftarrow \mathcal{K}_{\text{cor}} \cup \{k_2\}$
      keys[$K_2$] $\leftarrow c_2$
      send <unwrap$^\bullet$,h> to $U$
    else if ( $c_1 \notin \mathcal{K}_{\text{cor}}$ and
       $\exists c_2$.<$c_2$,<$F_2$,$a_2$,$id$>,w>$\in$encs[$c_1$])
      create $h_2$; Store[$U$,$h_2$]$\leftarrow$ <$F_2$,$a_2$,$c_2$>
      send <unwrap$^\bullet$,h> to $U$
    else if $c_1 \notin \mathcal{K}_{\text{cor}}$ and
       $\neg\exists c_2$.<$c_2$,<$F_2$,$a_2$,$id$>,w>$\in$encs[$c_1$])
      query $c_2 = \mathsf{DEC}(c_1, < F_2, a_2, id >, w)$;
       if $c_2 \neq \bot$ //bad event
         halt and output 0.
```

```
corrupt[finish_setup]:
  accept <corrupt,$h$> from $U \in \mathcal{U}$
  if Store[U,$h$]=<$F$,$a$,$c$>
    if $F$=KW
      for all $c'$ reachable from $c$ in $C$
        $\mathcal{K}_{\text{cor}} \leftarrow \mathcal{K}_{\text{cor}} \cup \{c'\}$
      query $k = \mathsf{CORR}(c)$
      key[$c$] $\leftarrow k$
      send <corrupt$^\bullet$,$h$,$k$> to $A$
    else
      send <corrupt$^\bullet$,$h$,$c$> to $A$
```

$\mathcal{B}_Z$ is a valid adversary We will argue about each assumption on the behaviour of the adversary, one after another:

1. For all i, the query $\mathsf{NEW}(K_i)$ is issued at most once, because we specified $\mathcal{F}_{\text{KM}}^{\texttt{impl},\mathsf{KW}}$ to select a new K_i for each such query
2. All queries issued by $\mathcal{B}_Z$ contain keys that have already been generated by the experiment. We claim that there are only keys that have either been generated using NEW or that are in $\mathcal{K}_{\text{cor}}$ the third position of the store (regarding only the elements in the store that start with KW, of course). If this claim holds, then the fact that all queries are proceeded by a conditional that checks if the argument to the query is in the store makes sure that this assumption hold. Now, assume that our claim does not hold true, and there exist a key in the store that has not previously been generated. Assume we are at the first point of the execution where such a key is added to the store. The store is only written in the new and unwrap step. In new, a new K_i is

created. In `unwrap`, there are three cases in which the store is written to: *a*) If $c_1 \in \mathcal{K}_{\text{cor}}$, then $c_2 \in \mathcal{K}_{\text{cor}}$. Once something is marked as corrupted, it stays corrupted. *b*) If $c_1 \notin \mathcal{K}_{\text{cor}}$, but $\exists \mathtt{c_2}. < \mathtt{c_2}, < \mathtt{F_2}, \mathtt{a_2}, id >, \mathtt{w} > \in \mathtt{encs}[\mathtt{c_1}])$. Only `wrap` can write to `encs`, so c_2 must have been in the store before.

3. The adversary never makes a test query $\mathsf{TENC}(K_i, a, K_j)$ if K_i is corrupted at the end of the experiment, because a TENC query is only output in the step `wrap` if $c_1 \notin \mathcal{K}_{\text{cor}}$. The condition *corrupt-before-wrap* enforces that if c_1 is not corrupted at that point, it will never be corrupted. (A detailed analysis about how *corrupt-before-wrap* is correct with respect to the definition if $\mathcal{F}_{\text{KM}}^{\texttt{impl},\mathsf{KW}}$ is left to the reader.)
4. If $\mathcal{B}_Z$ issues a test query $\mathsf{TENC}(K_i, a, m)$ then $\mathcal{B}_Z$ does not issue $\mathsf{TENC}(K_j, a',$ $m')$ or $\mathsf{ENC}(K_j, a', m')$ with $(K_i, a, m) = (K'_j, a, m')$, since $\mathcal{B}_Z$ never issues ENC queries at all and only issues TENC queries if the same combination of (K_i, a, m) was not stored in `encs` before. Every time TENC is called, `encs` is updated with those parameters.
5. $\mathcal{B}_Z$ never queries $\mathsf{DEC}(K_i, a, c)$ if c was the result of a query $\mathsf{TENC}(K_i, a, m)$ or of a query $\mathsf{ENC}(K_i, a, m)$ or K_i is corrupted, because *a*) TENC queries are store in `encs` and the step `unwrap` checks this variable before querying DEC, *b*) `enc` queries are never issued, *c*) is a credential c_1 inside the `unwrap` step is corrupted, the query DEC is not issued.

We conclude that $\mathcal{B}_Z$ fulfills the assumptions on the behaviour of the adversary expressed in Definition 11.

$\mathcal{B}_Z$ simulates $\mathcal{F}_{\text{KM}}^{\texttt{impl},\mathsf{KW}}$ in the fake experiment $\mathcal{B}_Z$ is defined to be a simulation of the network $\left[\mathcal{F}_{\text{KM}}^{\texttt{impl},\mathsf{KW}}, Sim, Z\right]$, where $\mathcal{F}_{\text{KM}}^{\texttt{impl},\mathsf{KW}}$ is $\mathcal{F}_{\text{KM}}^{\texttt{impl}}$, except for the altered steps `new,wrap,unwrap` and `corrupt`. We claim that, in the fake experiment, those alterations do not change the input/output behaviour.

- `new`: The handle and the entry to the store are created in the same way, except for the handle stored in the third position in case of $\mathtt{F} = \mathtt{KW}$. Given such a handle c will refer to the key that is generated upon the NEW call by $k(c)$. The functions `wrap,unwrap` and `corrupt` depend on this value, so we will show in the following that $k(c)$ is used for those operations. Not that, by definition of `wrap` and `corrupt`, whenever a credential becomes corrupted, a CORR query is issued to determine the actual value of this key, and the key is stored in the lookup table called `key`. Thus, `key` is defined for each wrapping credential $c \in \mathcal{K}_{\text{cor}}$ and points to $k(c)$.
- `wrap`: If $c_1 \in \mathcal{K}_{\text{cor}}$ is called, then the functions wrap are used with the key stored upon corruption of c_1. By definition of the experiment, this is the key that was generated when $\mathsf{NEW}(c_1)$ was called, so the same output is created as in $\mathcal{F}_{\text{KM}}^{\texttt{impl}}$. If $c_1 \notin \mathcal{K}_{\text{cor}}$, TENC is called, which by definition outputs $\mathsf{wrap}^{<F_2, a_2, id>}(k(c_1), 0^{k(c_2)})$. If $c_1 \in \mathcal{K}_{\text{cor}}$, then (since we established that $\mathsf{enc}[c_1] = k(c_1)$), the output is $\mathsf{wrap}^{<F_2, a_2, id>}(k(c_1), k(c_2))$
- `unwrap`: If $c_1 \in \mathcal{K}_{\text{cor}}$, $\mathsf{unwrap}^{<F_2, a_2, id>}(k(c_1), w)$ is output, just as in $\mathcal{F}_{\text{KM}}^{\texttt{impl}}$. Similarly, if $c_1 \notin \mathcal{K}_{\text{cor}}$, $\mathcal{B}_Z$ outputs the wrapping recorded before, or (by definition of the fake experiment) $\perp$, just as $\mathcal{F}_{\text{KM}}^{\texttt{impl}}$ does.

– `corrupt:` correctly outputs $k(c)$ for wrapping keys.

We can conclude that

$$\mathbf{Exp}^{\mathsf{wrap},\mathsf{fake}}_{\mathsf{KW},\mathcal{B}_Z}(\eta) = \mathrm{Exec}[\mathcal{F}^{\mathtt{impl}}_{\mathrm{KM}}, \mathit{Sim}, Z](\eta).$$

$\mathcal{B}_Z$ simulates $\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\mathtt{Impl}}}$ in the real experiment In the fake experiment, it is not possible that $\mathcal{B}_Z$ halts at the end of the `unwrap` step (marked "bad event"), since DEC always outputs $\perp$. Thus, the probability that $\mathcal{B}_Z$ halts at the "bad event" mark whilst in the real experiment must be negligible, as this would contradict the assumption that KW is a secure wrapping scheme right here. The representation of the this part of the proof benefits from altering $\mathcal{B}_Z$ such that instead of halting, $\mathcal{B}_Z$ continues to run $\mathcal{F}^{\mathtt{impl},\mathsf{KW}}_{\mathrm{KM}}$, by running the following code:

```
create $h_2$; Store[$U$,$h_2$]$\leftarrow$ <$F_2$,$a_2$,$c_2$>
send <unwrap$^\bullet$,h> to $U$
```

We call this sightly different attacker $\mathcal{B}'_Z$. Since this part of the code is only executed with negligible probability, we have that

$$\Pr[b \leftarrow \mathbf{Exp}^{\mathsf{wrap},\mathsf{real}}_{\mathsf{KW},\mathcal{B}_Z}(\eta) : b = 1] - \Pr[b \leftarrow \mathbf{Exp}^{\mathsf{wrap},\mathsf{real}}_{\mathsf{KW},\mathcal{B}'_Z}(\eta) : b = 1]$$

is negligible in η.

Fix an arbitrary security parameter η. Then, let $\mathit{View}_{\mathcal{B}'_Z}(t)$ be the view of Z, i.e. the distribution of messages it is simulated to send to the protocol machine or the adversary, in the tth step of the simulation of $\mathcal{B}'_Z$ in the real experiment. Furthermore, let $\mathit{Store}_{\mathcal{B}_Z;}(t)$ be the distribution of the variable `Store` within the simulated machine `ideal,sid`,i. e.,$\mathcal{F}^{\mathtt{impl},\mathsf{KW}}_{\mathrm{KM}}$, but with the following substitution that affects the wrapping keys: Every entry `<KW,`a`,`K_i`>` in the variable `Store[`U`,`h`]` for some U and h is substituted by an entry `<KW,`a`,`k_i`>`, where k_i is the key that the key-wrapping experiment associates to K_i, denoted in the following by $k(K_i)$. We denote the view of Z in the execution of the network $[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\mathtt{Impl}}}, A_D, Z]$ by View_π and the distribution of the union of all `Store` variables of all security tokens $ST_1, \ldots, ST_n$ in the network as Store_π. The union of those variables is well defined, because the first element of each key-value of this table is different for all ST. A step t is an epoch [11, §5.3], i. e., it ends begins with and activation of Z and ends with the next activation.

We define the following invariant, which will allow us to conclude that $\mathcal{B}'_Z$ has the same output distribution as $[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\mathtt{Impl}}}, A_D, Z]$. For each number of steps t, the following three conditions hold:

– *state consistency (s.c.)* $\mathit{Store}_{\mathcal{B}_Z;}$ (t) and Store_π are equally distributed
– *output consistency (o.c.)* $\mathit{View}_{\mathcal{B}'_Z}$ (t) and View_π are equally distributed
– *phase consistency (p.c.)* The probability that the flag `ready` is set in $\mathcal{F}^{\mathtt{impl},\mathsf{KW}}_{\mathrm{KM}}$ in $\mathbf{Exp}^{\mathsf{wrap},\mathsf{real}}_{\mathsf{KW},\mathcal{B}'_Z}$ equals the probability that `ready` is set in $\mathcal{F}_{\mathrm{setup}}$ in $[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\mathit{\Pi},\overline{\mathtt{Impl}}}$, $A_D, Z]$. Furthermore, the probability that the flag `setup_finished` is set in

$\mathcal{F}_{\text{KM}}^{\texttt{impl},\mathsf{KW}}$ in $\mathbf{Exp}_{\mathsf{KW},\mathcal{B}'_Z}^{\mathsf{wrap,real}}$ equals the probability that `setup_finished` is set in all $ST \in \mathcal{ST}$. .

If $t = 0$, the protocol has not been activated, thus there was no output, and not state changes. The invariant holds trivially. If $t > 0$, we can assume that s.c., o.c. and p.c. were true at the end of the preceding epoch. Note that Z is restricted to addressing top-level parties with the same `sid`. In particular, it cannot address $\mathcal{F}_{\text{setup}}$ directly (but it can corrupt a user to do this). Since the `sid` has to have a specific format that is expected by all machines in both networks, we assume `sid` to encode $\mathcal{U}, \mathcal{U}^{\text{ext}}, \mathcal{ST}, \mathit{Room}$. Case distinction over the recipient and content of the message that Z sends at the beginning of the next epoch:

1. Z sends a message to $ST_i \in \mathcal{ST}$, and
   (a) the message is `<ready>`: In $\mathbf{Exp}_{\mathsf{KW},\mathcal{B}'_Z}^{\mathsf{wrap,real}}$, $\mathcal{F}_{\text{KM}}^{\texttt{impl},\mathsf{KW}}$ records `ready-`ST_i and sends $< \texttt{ready}^\bullet, ST_\texttt{i} >$ to Sim, if the message is recorded for the first time. Sim behaves just like A_D in this case. If all other $ST_j \in \mathcal{ST}$ and all $U \in \mathcal{U}$ have sent this message before, the flag `ready` is set to true.
   In $[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\texttt{Impl}}}, A_D, Z]$, ST_i accepts the message and forwards it (as `<ready,` ST_i`>` to $\mathcal{F}_{\text{setup}}$. Then, $\mathcal{F}_{\text{setup}}$ records `ready-`ST_i and sends $< \texttt{ready}^\bullet, ST_\texttt{i} >$ to A_D, if the message is recorded for the first time. If all other $ST_j \in \mathcal{ST}$ and all $U \in \mathcal{U}$ have sent this message before, the flag `ready` is set to true. We see that p.c. and o.c. hold. S.c. holds trivially, because the Store did change in neither execution.
   (b) the message is of any other form: In $[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\texttt{Impl}}}, A_D, Z]$, ST accepts no other message from the environment. In $\mathbf{Exp}_{\mathsf{KW},\mathcal{B}'_Z}^{\mathsf{wrap,real}}$, $\mathcal{F}_{\text{KM}}^{\texttt{impl},\mathsf{KW}}$ ignores any other message coming from ST_i, too. So p.c., o.c. and s.c. hold.
2. Z sends a message to $U_i \in \mathcal{U}$:
   In $\mathbf{Exp}_{\mathsf{KW},\mathcal{B}'_Z}^{\mathsf{wrap,real}}$, $\mathcal{F}_{\text{KM}}^{\texttt{impl},\mathsf{KW}}$ will receive this message, and treat it depending on its form (if its flag `ready` is set). In $[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\texttt{Impl}}}, A_D, Z]$, will relay this message m in the form `<`m`,`ST_i`>` to $\mathcal{F}_{\text{setup}}$, who in turn will send m to ST_i, (if its flag `ready` is set).
   (a) Let m be `<ready>`: If the `ready` flag has not been set before, and U_i is the last party in $\mathcal{U} \cup \mathcal{ST}$ that has not sent this message yet, $\mathcal{F}_{\text{KM}}^{\texttt{impl},\mathsf{KW}}$ in $\mathbf{Exp}_{\mathsf{KW},\mathcal{B}'_Z}^{\mathsf{wrap,real}}$ will set the `ready` flag, otherwise it will not. The same holds for $\mathcal{F}_{\text{setup}}$ in $[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\texttt{Impl}}}, A_D, Z]$. Therefore, we have p.c. In both cases, Sim, respectively A_D, forward the acknowledgement to the environment (after recording the state change). Thus, s.c. and o.c. hold trivially.
   For the following cases, assume `ready` to be set in both $\mathbf{Exp}_{\mathsf{KW},\mathcal{B}'_Z}^{\mathsf{wrap,real}}$ and $[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\texttt{Impl}}}, A_D, Z]$. If it is unset in one of them, by induction hypothesis, it is unset in both. If it is not set, any other message will not be accepted by neither $\mathcal{F}_{\text{KM}}^{\texttt{impl},\mathsf{KW}}$, nor $\mathcal{F}_{\text{setup}}$ (thus never reach ST_i). Therefore, in the following cases we will assume `ready` to be set in $\mathcal{F}_{\text{KM}}^{\texttt{impl},\mathsf{KW}}$ and $\mathcal{F}_{\text{setup}}$, i. e., $\mathcal{F}_{\text{setup}}$ delivering commands that U_i receives from the environment to ST_i.

(b) Let m= <new,F,a> : If $F \neq \mathtt{KW}$, the same code is executed, p.c.,s.c. and o.c. hold trivially. Assume $F = \mathtt{KW}$ and <KW,new,a,*>$\in \Pi$ (otherwise, $\perp$ is output in both executions). $\mathcal{F}_{\mathrm{KM}}^{\mathtt{impl},\mathsf{KW}}$ draws a new K_i and call NEW to create the key. K_i is created, just like handles, in a way that makes sure it is unique. Therefore, since throughout $\mathbf{Exp}^{\mathsf{wrap},\mathsf{real}}_{\mathsf{KW},\mathcal{B}'_Z}$, K_i is always substituted for the same key, there is a function mapping K_i to the key k_i created by the experiment, and this function is injective. Note that, by the definition of $Store_{\mathcal{B}_Z;}$, $Store_{\mathcal{B}_Z;}(t)$ is $Store_{\mathcal{B}_Z;}(t-1)$ with an additional entry KW,a,k_i at $[U,h]$, where k_i is distributed according to KG. In $[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\mathtt{Impl}}}, A_D, Z]$, ST_i calls the key-generation directly ($impl^{KW}_{\mathtt{new}}$ calls KG, adding nothing but an empty public part). The output in both cases is <new$^\bullet$,h,> for an equally distributed h. Thus, o.c. holds. $Store_\pi$ is $Store_\pi(t-1)$ with an additional entry KW,a,k_i at $[U,h]$ where k_i is distributed according to the same KG as above. Therefore, s.c. holds. (P.c. holds trivially.)

(c) Let m= <share,h_i,U_j> : In $\mathbf{Exp}^{\mathsf{wrap},\mathsf{real}}_{\mathsf{KW},\mathcal{B}'_Z}$, assuming $U_i, U_j \in Room$, $\mathcal{F}_{\mathrm{KM}}^{\mathtt{impl},\mathsf{KW}}$ outputs <share$^\bullet$>, and $Store_{\mathcal{B}_Z;}(t)$ is $Store_{\mathcal{B}_Z;}(t-1)$ extended by a copy of its entry $[U_i, h_i]$ at $[U_j, h_j]$. In $[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\mathtt{Impl}}}, A_D, Z]$, ST_i checks the same conditions, which by p.c. have an equal probability of success, implicitly: It sends the content of it's store at $[U_i, h_i]$ to $\mathcal{F}_{\mathrm{setup}}$, which verifies $U_i, U_j \in Room$. If this is not the case, $\mathcal{F}_{\mathrm{setup}}$ sends $\perp$ to ST_i, which sends this to the environment (via $\mathcal{F}_{\mathrm{setup}}$), behaving just like $\mathbf{Exp}^{\mathsf{wrap},\mathsf{real}}_{\mathsf{KW},\mathcal{B}'_Z}$. If the condition is met, ST_i sends the content of the store at $[U_i, h_i]$ to $\mathcal{F}_{\mathrm{setup}}$, who delivers this information to ST_j, which in the next step extends $Store_\pi(t-1)$ by a copy of its entry $[U_i, h_i]$ at $[U_j, h_j]$. Thus, s.c. holds. Both output <share$^\bullet$> upon success, so o.c. holds, too. p.c. holds trivially.

(d) Let m= <finish_setup> : By p.c., we have that $\mathbf{Exp}^{\mathsf{wrap},\mathsf{real}}_{\mathsf{KW},\mathcal{B}'_Z}$ and $[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\mathtt{Impl}}}, A_D, Z]$ either both have finish_setup set, or none has. If both have it set, both output $\perp$ and do nothing. Assume none has finish_setup set and both ready.

    In $\mathbf{Exp}$, $\mathcal{F}_{\mathrm{KM}}^{\mathtt{impl},\mathsf{KW}}$ sets the flag finish_setup and responds. In $[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\mathtt{Impl}}}, A_D, Z]$, U_i sends <finish_setup> to $\mathcal{F}_{\mathrm{setup}}$, which in turn, instead of forwarding it to ST_i like for the majority of commands, sends <close> to every single $ST_j \in \mathcal{ST}$, accepting the response (and thus taking control) after each of those have set the finish_setup flag. By the time $\mathcal{F}_{\mathrm{setup}}$ finishes this step, and hands communication over to U_i, which forwards finish_setup to the environment, every ST_i has left the setup phase. We see that p.c. and o.c. are preserved.

(e) Let m= <C,h,m> : $\mathcal{F}_{\mathrm{KM}}^{\mathtt{impl},\mathsf{KW}}$ and ST_i execute the same code on their outputs, so by s.c., the invariant is preserved.

(f) Let m= <corrupt,h> : ST_i outputs the credential. $\mathcal{F}_{\mathrm{KM}}^{\mathtt{impl},\mathsf{KW}}$ does the same, except for wrapping keys. It substitutes the credential by the output of CORR, i. e., $k = k(c)$. By definition of $Store_\pi$ as s.c., $< \mathtt{KW}, \mathtt{a}, \mathtt{k} > \in$

$\mathit{Store}_\pi[U_i,h]$ with the same probability as $< \mathtt{KW}, \mathsf{a}, \mathsf{c} > \in \mathit{Store}_{\mathcal{B}_Z;}[U_i,h]$, thus the output is equally distributed. S.c. and p.c. hold trivially.

(g) Let m= <wrap, h_1, h_2, id> : For this case and the following case, observe that $\mathcal{F}_{\mathrm{KM}}^{\mathtt{impl},\mathsf{KW}}$ initialises $\mathtt{key}[c]$ only at steps wrap and corrupt. It always contains the output of a query CORR, thus its value is always $k(c)$ if it is defined. It is defined whenever $c \in \mathcal{K}_{\mathrm{cor}}$, because if c is added to $\mathcal{K}_{\mathrm{cor}}$ at step corrupt, the response is written to $\mathtt{key}[c]$, and if a $c_3 \notin \mathcal{K}_{\mathrm{cor}}$ is found during step wrap, the condition *corrupt-before-wrap* must have been violated by Z: If such a c_3 is reachable from c_1, with out lost of generality, assume it to have minimal distance from c_1 in C. Then, second-before last node on this path is in $\mathcal{K}_{\mathrm{cor}}$, as the distance would not be minimal otherwise. By definition of the step wrap, this node could not have been wrapped without adding it to $\mathcal{K}_{\mathrm{cor}}$, therefore this node was corrupted after it was used to create this wrapping.

Assume now ST_i and $\mathcal{F}_{\mathrm{KM}}^{\mathtt{impl},\mathsf{KW}}$ both have finish_setup set, as otherwise either p.c. was violated in the previous step, or both would output $\perp$ and trivially satisfy the invariant. (This argument is valid for each of the following sub-cases, but the last one).

Both machines check the same conditions on the Store and the policy. ST_i computes $w = \mathsf{wrap}^{<F_2,a_2,id>}(c_1,c_2)$ on the values $< \mathtt{KW}, \mathsf{a}_1, \mathsf{c}_1 >$ and $< \mathtt{F}_2, \mathsf{a}_2, \mathsf{c}_2 >$ at $[U_i,h_1]$ and $[U_i,h_2]$ in Store_π.

$\mathcal{F}_{\mathrm{KM}}^{\mathtt{impl},\mathsf{KW}}$ performs a case distinction, but we will show that in each cases, it outputs the same value. If $< \mathsf{c}_2, < \mathtt{F}_2, \mathsf{a}_2, id >, \mathtt{w} > \in \mathtt{encs}[\mathsf{c}_1]$, then by observing that encs is only written at the end of this function, we see that p.c. would have been violated in an earlier step, if the output now was differently distributed then the output in $[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\mathtt{Impl}}}, A_D, Z]$.

If $c_1 \in \mathcal{K}_{\mathrm{cor}}$, then $c_2 \in \mathcal{K}_{\mathrm{cor}}$, too. Then, since $\mathtt{key}(c_1) = k(c_1)$ (and $\mathtt{key}(c_2) = k(c_2)$ in case $F_2 = \mathtt{KW}$), the output is $w = \mathsf{wrap}^{<F_2,a_2,id>}(k(c_1), c_2))$ (or $w = \mathsf{wrap}^{<F_2,a_2,id>}(k(c_1), k(c_2)))$, which preserves o.c., since s.c. from the last step guarantees that $< \mathtt{KW}, \mathsf{a}, \mathsf{c} > \in \mathit{Store}_\pi[U_i,h] = < \mathtt{KW}, \mathsf{a}, \mathsf{k}(\mathsf{c}) > \in \mathit{Store}_{\mathcal{B}_Z;}[U_i,h]$ for $c = c_1$ and $c = c_2$, in case c_2 is a wrapping key. By definition of the real experiment, it performs the same substitutions in case $c_1 \notin \mathcal{K}_{\mathrm{cor}}$, so the same argument can be applied. Therefore, the output is equally distributed in all three cases, assuming that s.c. was true for the previous step. s.c. and p.c. hold trivially.

(h) Let m= <unwrap, h_1, w, a_2, F_2, id> : In $[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\mathtt{Impl}}}, A_D, Z]$, if policy and store allow, i.e., $< F_1, a_1, c_1 > \in \mathit{Store}_\pi(U_i,h_1)$, ST_i writes $< F_2, a_2, \mathsf{unwrap}^{<F_2,a_2,id>}(c_1,w)$ at a fresh place $[U_i,h]$ in Store_π, unless unwrap returned $\perp$.

$\mathcal{F}_{\mathrm{KM}}^{\mathtt{impl},\mathsf{KW}}$ chooses U_i and a new h exactly the same way.

- If $c_1 \in \mathcal{K}_{\mathrm{cor}}$, it writes $< F_2, a_2, \mathsf{unwrap}^{<F_2,a_2,id>}(k(c_1),w)$. By s.c., the probability that $< F_1, a_1, c_1 > \in \mathit{Store}_\pi(U_i,h_1)$ is equal to the probability that $< F_1, a_1, f(c_1) > \in \mathit{Store}_\pi(U_i,h_1)$, since $F_1 = \mathtt{KW}$
- If $c_1 \notin \mathcal{K}_{\mathrm{cor}}$ and w was recorded earlier, inspection of $\mathcal{F}_{\mathrm{KM}}^{\mathtt{impl},\mathsf{KW}}$ shows that encs is written to only at the wrap step, which implies that

$w = \mathsf{wrap}^{<F_2,a_2,id>}(A,B)$. From the correctness of the scheme, we conclude that $< F_2, a_2, \mathsf{unwrap}^{<F_2,a_2,id>}(c_1, w) >$ is written to this position.

  - If $c_2 \notin \mathcal{K}_{\mathrm{cor}}$, by definition of DEC, $< F_2, a_2, \mathsf{unwrap}^{<F_2,a_2,id>}(k(c_1), w)$ is written. (Same argument as in the first case, follows from s.c.)

(i) Let m= `<attr_change,`h`,`a'`>` : The same code is executed in $\mathbf{Exp}^{\mathsf{wrap,real}}_{\mathsf{KW},\mathcal{B}'_Z}$ and $[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\mathtt{Impl}}}, A_D, Z]$, thus p.c., s.c., and o.c. hold trivially.

(j) Let m= `<`C`,`$public$`,`m`>` : In $[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\mathtt{Impl}}}, A_D, Z]$, U_i and $\mathcal{F}^{\mathtt{impl},\mathsf{KW}}_{\mathrm{KM}}$ perform the same computations, thus p.c., s.c., and o.c. hold trivially.

3. Z sends a message to $U^{\mathrm{ext}} \in \mathcal{U}^{\mathrm{ext}}$
$\mathcal{F}^{\mathtt{impl},\mathsf{KW}}_{\mathrm{KM}}$, as well as U^{ext} only accept messages of the form `<`C`,`$public$`,`m`>` for $C \in \mathcal{C}_{i,pub}$. Both perform the same computations, thus p.c., s.c., and o.c. hold trivially.

4. Z sends a message to `<adv>`.
Both A_D and Sim ignore messages that are no instruction. So we can assume that Z instructs the adversary to send a message to some party.

(a) Assume Z instructs `<adv>` to send a message to a corrupted party, namely:

i. $U_i \in \mathcal{U}$: U_i can only be addressed by the adversary if it was corrupted before, as otherwise it has never sent a message to the adversary. Note that the code run by U_i in $[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\mathtt{Impl}}}, A_D, Z]$, as well as in $\mathbf{Exp}$, does not depend on any internal state. U_i can only talk to the environment, the adversary (Sim acts like A_D in this case) and it can call $\mathcal{F}_{\mathrm{setup}}$, which Sim has to simulate in $\mathbf{Exp}^{\mathsf{wrap,real}}_{\mathsf{KW},\mathcal{B}'_Z}$. Sim is described above and receives all the information necessary to simulate it, that is: `<ready`$^\bullet$`,`P`>`, when a protocol party in $\mathcal{U} \cup ST$ receives `<ready>` from the environment, `<finish_setup`$^\bullet$`>`, when a protocol party in $\mathcal{U}$ receives `<ready>` from the environment, and all messages that $\mathcal{F}^{\mathtt{impl},\mathsf{KW}}_{\mathrm{KM}}$ sends to the corrupted U_U (and $\mathcal{F}_{\mathrm{setup}}$ would need to relay). Thus, if p.c. holds in the previous step, the invariant is preserved in case that the message is `<ready,`U_i`>` or`<finish_setup>`. A message of form `<send,...>` is ignored by $\mathcal{F}_{\mathrm{setup}}$ and Sim, so the invariant is trivially preserved here. The communication relayed in the steps `relay_receive` and `relay_send` in $\mathcal{F}_{\mathrm{setup}}$ is simulated as described above and thus falls back to case 2.

ii. $U^{\mathrm{ext}}_i \in \mathcal{U}^{\mathrm{ext}}$: Like in the previous case, only that $\mathcal{F}_{\mathrm{setup}}$ ignores messages from U^{ext}_i, which Sim simulates correctly.

iii. other parties cannot become corrupted

(b) Assume Z instructs `<adv>` to send a message to a party that cannot be corrupted, but that addressed `<adv>` before, i. e., $ST \in \mathcal{ST}$ or $\mathcal{F}_{\mathrm{setup}}$. Since both ST and $\mathcal{F}_{\mathrm{setup}}$ are specified to ignore messages in this case, Sim can simply mask their presence by reacting like ST or $\mathcal{F}_{\mathrm{setup}}$ react upon reception of an unexpected message: answer with $\perp$.

We conclude that the invariant is preserved for an arbitrary number of steps. Since output consistency implies that Z has an identical view, the distribution of Z's output is the same in both games. Thus:

$$\Pr[b \leftarrow \mathrm{Exec}[\pi_{\mathcal{F},\overline{\mathcal{C}},\Pi,\overline{\texttt{Impl}}}, A_D, Z](\eta) : b = 1] = \Pr[b \leftarrow \mathbf{Exp}^{\mathsf{wrap,real}}_{\mathsf{KW},\mathcal{B}'_Z}(\eta) : b = 1]$$

and therefore:

$$\begin{aligned}
&|\Pr[b \leftarrow \mathrm{Exec}[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\texttt{Impl}}}, A_D, Z](\eta) : b = 1] \\
&\qquad\quad - \Pr[b \leftarrow \mathrm{Exec}[\mathcal{F}^{\texttt{impl}}_{\mathrm{KM}}, \mathit{Sim}, Z](\eta) : b = 1]| \\
&= |\Pr[b \leftarrow \mathbf{Exp}^{\mathsf{wrap,fake}}_{\mathsf{KW},\mathcal{B}_Z}(\eta) : b = 1] - \Pr[b \leftarrow \mathbf{Exp}^{\mathsf{wrap,real}}_{\mathsf{KW},\mathcal{B}'_Z}(\eta) : b = 1]| \\
&> |\Pr[b \leftarrow \mathbf{Exp}^{\mathsf{wrap,fake}}_{\mathsf{KW},\mathcal{B}_Z}(\eta) : b = 1] - \Pr[b \leftarrow \mathbf{Exp}^{\mathsf{wrap,real}}_{\mathsf{KW},\mathcal{B}_Z}(\eta) : b = 1]| - \epsilon(\eta),
\end{aligned}$$

where ϵ is negligible in η. This contradicts the indistinguishability of $\mathrm{Exec}[\mathcal{F}^{\texttt{impl}}_{\mathrm{KM}}, \mathit{Sim}, Z]$ and $\mathrm{Exec}[\pi_{\overline{\mathcal{F}},\overline{\mathcal{C}},\Pi,\overline{\texttt{Impl}}}, A_D, Z]$ and thus concludes the proof.

D The signature functionality

The digital signature functionality is designed after the one described in [20] and detailed in Listing 1.1. It is parametrized by three algorithms KG, sign and verify. It expects the session parameter to encode a machine id P, and implements $\mathcal{C}^{priv} = \{\texttt{sign}\}$ and $\mathcal{C}^{pub} = \{\texttt{verify}\}$.

```
new: accept <new> from $P$
  $(sk, vk) \leftarrow \mathsf{KG}(1^\eta)$
  ( credential ,<ignore>) $\leftarrow \mathsf{KG}(1^\eta)$
  L:= L $\cup$ { (credential,$sk$,$vk$) }
  send <new$^\bullet$,credential,$vk$> to $P$
sign: accept <sign,credential,$m$> from $P$
  if ( credential , $sk$,$vk$)$\in$L for some key
    $\sigma \leftarrow \mathsf{sign}(k, m)$
    if $\mathsf{verify}(vk, m, \sigma) \neq \bot$
      signs [$vk$]= signs[$vk$] $\cup\{(m, \sigma)\}$
    else $\sigma \leftarrow \bot$
    send <sign$^\bullet$,$\sigma$> to $P$
verify: accept <verify,$vk$,$m$> from $P$
  $b \leftarrow \mathsf{verify}(vk, m, \sigma)$
  if $\exists c, sk.$ ($c$,$sk$,$vk$)$\in$L and
    $k \notin \mathcal{K}_{\mathrm{cor}}$ and $b = 1$ and $\nexists\sigma' : (m, \sigma') \in$signs[$vk$] or $b \notin \{0, 1\}$
    $b \leftarrow \bot$
  send <verify$^\bullet$,$b$> to $P$
corrupt: accept <corrupt,cred> from $P$
  if ( credential , $sk$,$vk$)$\in$L for some $sk$, $vk$
    $\mathcal{K}_{\mathrm{cor}} \leftarrow \mathcal{K}_{\mathrm{cor}} \cup \{sk\}$
    send <corrupt$^\bullet$,$sk$> to $A$
```

Listing 1.1: A signature functionality $\mathcal{F}^{\mathrm{SIG}}$

In Section 6, we mention that future work could enable us to produce an implementation $\hat{I}^{\mathrm{SIG}}$ such that $\hat{I}^{\mathrm{SIG}}$ emulates $\mathcal{F}^{\mathrm{SIG}}$ from the proof presented in [20] for a non-key-manageable signature functionality. Since this is out of the scope of this work, we leave this as an assumption.