A Cryptanalysis of HummingBird-2: The Differential Sequence Analysis

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Abstract. Hummingbird-2 is one recent design of lightweight block ciphers targeting constraint devices, which not only enables a compact hardware implementation and ultra-low power consumption but also meets the stringent response time as specified in ISO18000-6C.
In this paper, we present the first cryptanalytic result on the full version of this cipher using two pairs of related keys. We discover that the differential sequences for the last invocation of the round function can be computed by running the full cipher, due to which the search space for the key can be reduced. Base upon this observation, we propose a probabilistic attack encompassing two phases, preparation phase and key recovery phase. The preparation phase, requiring 2^{80} effort in time, aims to reach an internal state, with 0.5 success probability, that satisfies particular conditions. In the key recovery phase, by attacking the last invocation of the round function of the encryption (decryption resp.) using the proposed differential sequence analysis (DSA), we are able to recover 36 bits (another 44 bits resp.) of the 128-bit key. In addition, the rest 48 bits of the key can be exhaustively searched and the overall time complexity of the key recovery phase is $2^{48.14}$.
Note that the proposed attack, though exhibiting an interesting tradeoff between the success probability and time complexity, is only of a theoretical interest at the moment and does not affect the security of the Hummingbird-2 in practice.

Keywords: lightweight cryptography, differential cryptanalysis, Hummingbird encryption

1 Introduction

Passive RFID tags and other constraint computing devices are usually characterized by extremely tight cost and power consumption requirements. The needs of cryptographic primitives on such devices have been increasing with the growing pervasiveness and mass deployment of these devices. To this end, considerable lightweight stream/block ciphers are proposed in recent years, targeting very small hardware footprint and reduced power consumption. Typical examples are listed in Table 1. Meanwhile, cryptanalysis of these lightweight primitives has received considerable attention due to a widely-accept concern – the pursue of efficiency at the cost of reducing the security margin or applying innovative but less well understood technologies lead lightweight candidates to be less durable relative to regular symmetric ciphers. This concern has been further confirmed by the successful cases of attacking KeeLoq [33], Crypto-I [32], Atmel Cipher [22, 9], PRESENT [13, 11], KTANTAN [7, 3], PRINTCipher [2, 29], reduced KLEIN [1], A2U2 [12] and so on.

Table 1. Recent Design/Implementation of Lightweight Ciphers (ordered by gate equivalent (GE))

	Key size[bits]	Block size[bits]	Area[GE]	Throughput[Kb/s]	Logic process[μm]
PRINTCipher-48 [26]	80	48	402	6.25	0.18
KTANTAN32 [14]	80	32	462	12.5	0.13
PRINTCipher-48 [26]	80	48	503	100	0.18
KTANTAN48 [14]	80	48	571	9.4	0.13
GOST [34]	256	64	651	24.24	0.18
Piccolo-80 [38]	80	64	683	14.8	0.13
KTANTAN64 [14]	80	64	684	8.4	0.13
LED-64 [21]	64	64	688	5.1	0.18
LED-128 [21]	128	64	700	3.4	0.18
PRINTCipher-96 [26]	160	96	726	3.13	0.18
Piccolo-128 [38]	128	64	758	12.1	0.13
KATAN32 [14]	80	32	802	12.5	0.13
KATAN48 [14]	80	48	916	9.4	0.13
PRINTCipher-96 [26]	160	96	967	100	0.18
KATAN64 [14]	80	64	1,027	8.4	0.13
PRESENT [35]	80	64	1,075	11.4	0.18
KLEIN-64 [20]	64	64	1,981	N/A	0.18
KLEIN-80 [20]	80	64	2,097	N/A	0.18
HummingBird-2 [17]	**128**	**16**	**2,159**	**N/A**	**0.13**
KLEIN-96 [20]	96	64	2,213	N/A	0.18
AES [19]	128	128	3,400	12.4	0.35

A Brief History of Hummingbird Cipher: Motivated by the design of the well-known Enigma machine, the first generation of Hummingbird (call it HB-1) was proposed by the engineers in Revere Security and was further analyzed and published in [16] as an ultra-lightweight cryptographic algorithm targeting low-cost RFID tags, smart cards, and wireless sensor nodes to meet the stringent response time and power consumption requirements. Although HB-1, with an innovative hybrid structure of block cipher and stream cipher, was designed to provide 256-bit security, Saarinen, in FSE'11, showed a chosen-IV and chosen-message attack [36] that can recover the full secret key with at most 2^{64} off-line computational effort under two related IVs. Recently, Revere Security published the second generation of Hummingbird (call it Hummingbird-2 or HB-2) in [17], which inherits the design philosophy from HB-1, e.g., it has a small block size of 16-bit to adapt the needs of encrypting short messages in RFID applications and it retains the hybrid structure as a security compensation for the small block size. High level differences between HB-1 and HB-2 are: (1) key size has been reduced to 128 bits to satisfy the actual need for constrained devices; (2) size of the internal state has been increased from 80 bits to 128 bits; (3) the nonlinear keyed transformation in HB-2 has four invocations of the S-boxes, compared to five in HB-1, to further increase the throughput.

In addition, it is claimed in the same paper that HB-2 can withstand differential, linear and algebraic attacks and the four 4-bit S-Boxes in HB-2 belong to the optimal classes as discussed in [31]. Its resistance to the side-channel cube attack is recently investigated in [18], where the author applied cube attack to recover 48 bits of the secret key providing the attacker could access the internal states of HB-2 during an early stage in the initialization. However, this attack is marginal since it only threats HB-2 before the finishing of its initialization.

Our Contribution: By refining/improving our preliminary results in [10], we present, in this paper, the first cryptanalytic result on the full version of this cipher using two pairs of related keys. Our attack makes use of the internal state of such a cipher and our philosophy is general: (1) the outputs of the encryption/decryption may leak information of the subkeys (under the differential cryptanalysis) as long as the internal states of the cipher satisfy particular conditions; (2) due to the birthday paradox, such a condition always happens with 1/2 probability providing $2^{L/2}$ attempts are made, where L (in bit) is the size of the internal state. To be specific, we propose the following attack encompassing two phases, a probabilistic preparation phase and a key recovery phase.

- To realize the two particular conditions regarding the internal states, the preparation phase spends 2^{80} effort in time to achieve the succeed probability 0.5 (due to the birthday paradox). If succeeds, one could proceed to the key recovery phase.
- The key recovery phase is basically an instance of a novel cryptanalytic technique – we call it differential sequence analysis (DSA) – which can be seen as a hybrid of the conventional differential cryptanalysis and saturation attack. After exhibiting DSA's definitions and properties, we present its applications in the attacking scenarios, i.e.,
  - by using the encryption of HB-2, DSA recovers 36-bit (out of 128-bit) of the key, if condition (A) (regarding HB-2's secret key, input and internal state) holds.
  - by using the decryption of HB-2, DSA recovers another recovers another 44-bit of the key, if condition (B) (regarding HB-2's secret key, input and internal state) holds.
  - the rest 48-bit of the key can be exhaustively searched and the overall time complexity is $2^{48.14}$.

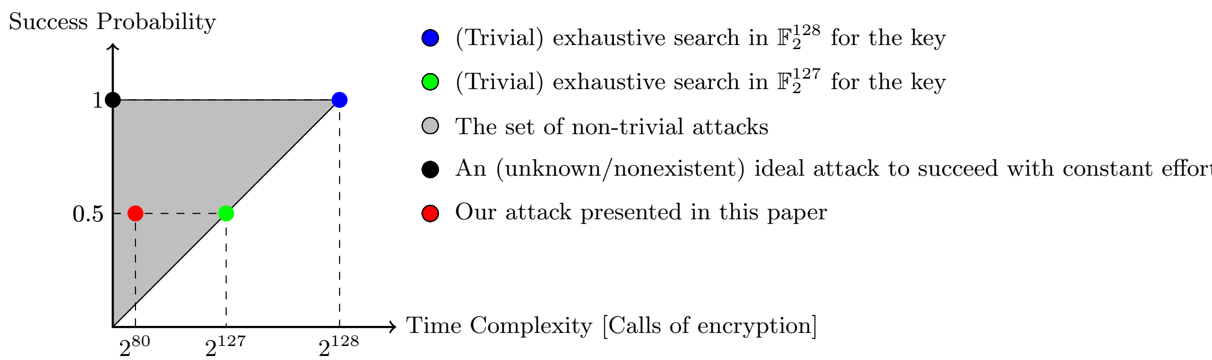

Fig. 1. Tradeoff between Success Probability and Time Complexity when attacking HB-2 (In fact, since one encryption only provides 16-bit entropy of the key, the exhaustive search needs a bit more than 2^{128} calls of the encryption function following the "key testing" procedure as desired in [4], i.e., $2^{128}+2^{112}+2^{96}+2^{80}+2^{64}+2^{48}+2^{32}+2^{16} \approx 2^{128.000022}$ and 8 plaintext-ciphertext pairs to uniquely determine the key with probability 1.)

Note that our results in this paper exhibit an interesting tradeoff between the success probability and time complexity for HB-2, as shown in Fig. 1, which is analog to the collision attack in the hash function due to the birthday paradox. Stated in another way, to be successful with probability 0.5, our attack is faster than the exhaustive search (which is the best known) by a factor of 2^{50}. Unfortunately, to succeed with probability 1, our preparation phase requiring more effort in time than the exhaustive search, which makes the proposed method only of theoretical interests at the moment, i.e., the attack presented in this paper does not affect the security of the Hummingbird-2 in practice.

Organization: In Section 2, the specification of HB-2 is presented. Section 3 describes the principle of our attack at a high level. In Section 4, we devise the DSA technique, discuss its properties and how to use it to attack parts of HB-2. In Section 5, we show how to achieve the desired conditions. We conclude the paper in Section 6.

Notations: Throughout the rest of this paper, we make use of the following notation for illustration.

- An hexadecimal number is indicated by a prefix "0x", e.g., 0x10 = 16.
- Unless otherwise stated, "+" denotes the addition in $\mathbb{F}_2$, which can also be vector-wise, e.g., $(a, b) + (c, d) = (a + c, b + d)$, where $a, b, c, d \in \mathbb{F}_2^m$.
- "$\boxplus$" or "$\boxminus$" operator denote addition or subtraction modulo 2^{16}.
- The high-bit XOR differential is defined as H =0x8000, a nice property of which is, given $x, x', y \in \mathbb{F}_2^{16}$ and $x + x' = H$, the following holds with probability 1,

$$(x \boxplus y) + (x' \boxplus y) = H, \quad (x \boxminus y) + (x' \boxminus y) = H, \quad (y \boxminus x) + (y \boxminus x') = H.$$

  That is to say, as pointed out in [36], the differential H behaves the same under "+" and "$\boxplus/\boxminus$".

2 Specification of Hummingbird-2

Hummingbird-2 is a 16-bit block cipher with a 128-bit secret key $K = (K_1, ..., K_8) \in (\mathbb{F}_2^{16}, ..., \mathbb{F}_2^{16}) = \mathbb{F}_2^{128}$ and a 64-bit public initialization vector $IV = (IV_1, ..., IV_4) \in (\mathbb{F}_2^{16}, ..., \mathbb{F}_2^{16}) = \mathbb{F}_2^{64}$. As opposed to conventional block ciphers, it has an 128-bit internal state $R = (R_1, ..., R_8) \in (\mathbb{F}_2^{16}, ..., \mathbb{F}_2^{16}) = \mathbb{F}_2^{128}$, which participates in each encryption/decryption and is updated after that.

Building Block: $WD16 : \{0, 1\}^{16} \mapsto \{0, 1\}^{16}$ is the fundamental block or round function of HB-2 encryption, which is defined as

$$WD16(x, K_a, K_b, K_c, K_d) = f(f(f(f(x + K_a) + K_b) + K_c) + K_d),$$

where x is the varying input, e.g., plaintext, intermediate state, K_a, K_b, K_c, K_d are four 16-bit secret keys and the nonlinear function f is specified as

$$\begin{aligned}
&S(x) = S_1(x_1)||S_2(x_2)||S_3(x_3)||S_4(x_4), x = (x_1, x_2, x_3, x_4)\\
&L(x) = x + (x <<< 6) + (x <<< 10)\\
&f(x) = L(S(x)).
\end{aligned}$$

Note that the four S-boxes, i.e., $S_1(x_i)$ to $S_4(x_i)$, are given in Table 2.

Besides, the inverse of $WD16$ is employed in the decryption, which is defined as

$$WD16^{-1}(y, K_d, K_c, K_b, K_a) = f^{-1}(f^{-1}(f^{-1}(f^{-1}(y) + K_d) + K_c) + K_b) + K_a,$$

Table 2. S-boxes in HummingBird-2

x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$S_1(x)$	7	12	14	9	2	1	5	15	11	6	13	0	4	8	10	3	$S_1^{-1}(x)$	11	5	4	15	12	6	9	0	13	3	14	8	1	10	2	7
$S_2(x)$	4	10	1	6	8	15	7	12	3	0	14	13	5	9	11	2	$S_2^{-1}(x)$	9	2	15	8	0	12	3	6	4	13	1	14	7	11	10	5
$S_3(x)$	2	15	12	1	5	6	10	13	14	8	3	4	0	11	9	7	$S_3^{-1}(x)$	12	3	0	10	11	4	5	15	9	14	6	13	2	7	8	1
$S_4(x)$	15	4	5	8	9	7	2	1	10	3	0	14	6	12	13	1	$S_4^{-1}(x)$	10	7	6	9	1	2	12	5	3	4	8	15	13	14	11	0

where $y = WD16(x, K_a, K_b, K_c, K_d)$ and f^{-1} is the inverse of f. The four S-boxes used in f^{-1} are also listed in Table 2.

Initialization: Hummingbird-2 is initialized before use. Let $(R_1^{(r)}, ...R_8^{(r)}) \in \{0,1\}^{128}$ denote the internal state at the rth iteration in the initialization. The initialization can thus be formulated as, for $r = 0, 1, 2, 3$,

$$
\begin{aligned}
t_1 &= WD16(R_1^{(r)} \boxplus < r >, K_1, K_2, K_3, K_4) && (1)\\
t_2 &= WD16(R_2^{(r)} \boxplus t_1, K_5, K_6, K_7, K_8) && (2)\\
t_3 &= WD16(R_3^{(r)} \boxplus t_2, K_1, K_2, K_3, K_4) && (3)\\
t_4 &= WD16(R_4^{(r)} \boxplus t_3, K_5, K_6, K_7, K_8) && (4)\\
R_1^{(r+1)} &= (R_1^{(r)} \boxplus t_4) \lll 3 && (5)\\
R_2^{(r+1)} &= (R_2^{(r)} \boxplus t_1) \lll 1 && (6)\\
R_3^{(r+1)} &= (R_3^{(r)} \boxplus t_2) \lll 8 && (7)\\
R_4^{(r+1)} &= (R_4^{(r)} \boxplus t_3) \lll 1 && (8)\\
R_5^{(r+1)} &= R_5^{(r)} + R_1^{(r+1)} && (9)\\
R_6^{(r+1)} &= R_6^{(r)} + R_2^{(r+1)} && (10)\\
R_7^{(r+1)} &= R_7^{(r)} + R_3^{(r+1)} && (11)\\
R_8^{(r+1)} &= R_8^{(r)} + R_4^{(r+1)}, && (12)
\end{aligned}
$$

where $< r >$ represents a counter and $(R_1^{(0)}, ..., R_8^{(0)}) = (IV_1, IV_2, IV_3, IV_4, IV_1, IV_2, IV_3, IV_4)$.

Note that R_5, R_6, R_7, R_8 do not participate in the randomization, i.e., Eq. (6)-(9), but simply XOR the historical statuses of R_1, R_2, R_3, R_4 respectively (behaving like XOR-MAC). This fact may nullify their contribution to the overall cryptanalytic strength of HB-2 under a side-channel injection attack – 64 injections and 64 invocations of HB-2 encryption are needed to recover (R_5, R_6, R_7, R_8). Details are provided in Appendix A.

Encryption: After the initialization, each encryption, by invoking the round function for four times, transforms a single plaintext word $P_i \in \mathbb{F}_2^{16}, i = 1, 2, ...,$ to a corresponding ciphertext word C_i, i.e.,

$$t_1 = WD16(R_1^{(i)} \boxplus P_i, K_1, K_2, K_3, K_4) \tag{13}$$
$$t_2 = WD16(R_2^{(i)} \boxplus t_1, K_5 + R_5^{(i)}, K_6 + R_6^{(i)}, K_7 + R_7^{(i)}, K_8 + R_8^{(i)}) \tag{14}$$
$$t_3 = WD16(R_3^{(i)} \boxplus t_2, K_1 + R_5^{(i)}, K_2 + R_6^{(i)}, K_3 + R_7^{(i)}, K_4 + R_8^{(i)}) \tag{15}$$
$$C_i = WD16(R_4^{(i)} \boxplus t_3, K_5, K_6, K_7, K_8) \boxplus R_1^{(i)}, \tag{16}$$

where $(R_1^{(i)}, ..., R_8^{(i)}) \in \mathbb{F}_2^{128}$ is the internal state during the ith encryption and it is updated, at the end of the encryption, as follows:

$$R_1^{(i+1)} = R_1^{(i)} \boxplus t_3 \tag{17}$$
$$R_2^{(i+1)} = R_2^{(i)} \boxplus t_1 \tag{18}$$
$$R_3^{(i+1)} = R_3^{(i)} \boxplus t_2 \tag{19}$$
$$R_4^{(i+1)} = R_4^{(i)} \boxplus t_1 \boxplus R_1^{(i+1)} \tag{20}$$
$$R_5^{(i+1)} = R_5^{(i)} + R_1^{(i+1)} \tag{21}$$
$$R_6^{(i+1)} = R_6^{(i)} + R_2^{(i+1)} \tag{22}$$
$$R_7^{(i+1)} = R_7^{(i)} + R_3^{(i+1)} \tag{23}$$
$$R_8^{(i+1)} = R_8^{(i)} + R_4^{(i+1)} \tag{24}$$

A shorthand of Eq. (13)-(24) is $C_i = E(P_i, K) = E(P_i, (K_1, ..., K_8))$.

Decryption: Decryption of a single word $C_i \in \mathbb{F}_2^{16}, i = 1, 2, ...,$ followed by the same initialization, is

$$u_3 = WD16^{-1}(C_i \boxminus R_1^{(i)}, K_8, K_7, K_6, K_5) \tag{25}$$
$$u_2 = WD16^{-1}(u_3 \boxminus R_4^{(i)}, K_4 + R_8^{(i)}, K_3 + R_7^{(i)}, K_2 + R_6^{(i)}, K_1 + R_5^{(i)}) \tag{26}$$
$$u_1 = WD16^{-1}(u_2 \boxminus R_3^{(i)}, K_8 + R_8^{(i)}, K_7 + R_7^{(i)}, K_6 + R_6^{(i)}, K_5 + R_5^{(i)}) \tag{27}$$
$$P_i = WD16^{-1}(u_1 \boxminus R_2^{(i)}, K_4, K_3, K_2, K_1) \boxminus R_1^{(i)}. \tag{28}$$

After this, the internal states are updated as in the encryption, i.e., using Eq. (17)-(24), where $t_3 = u_3 \boxminus R_4^{(i)}$, $t_2 = u_2 \boxminus R_3^{(i)}$ and $t_1 = u_1 \boxminus R_2^{(i)}$.

3 Overview of Our Cryptanalytic Method on the Full HB-2

Adversary Model: We consider a scenario that two paralleled executions of encryptions are $C_i = E(P_i, K)$ and $C'_{i'} = E(P'_{i'}, K')$, where the internal states are $(R_1^{(i)}, ..., R_8^{(i)})$ and $({R'}_1^{(i')}, ..., {R'}_8^{(i')})$ respectively, the intermediate values are (t_1, t_2, t_3) and (t'_1, t'_2, t'_3) respectively, and K and K' are related. (Similar for the decryption). The attacker follows the chosen plaintext/ciphertext model such that the attacker is free to choose plaintext $P_i \in \mathbb{F}_2^{16}$ and $P'_{i'} \in \mathbb{F}_2^{16}$, launch encryption without knowing the related keys, and observe the corresponding $C_i \in \mathbb{F}_2^{16}$ and $C'_{i'} \in \mathbb{F}_2^{16}$; or chooses $C_i \in \mathbb{F}_2^{16}$ and $C'_{i'} \in \mathbb{F}_2^{16}$, launches decryption without knowing the related keys, and observes the corresponding $P_i \in \mathbb{F}_2^{16}$ and $P'_{i'} \in F_2^{16}$.

Attack In A Nutshell: Block ciphers are usually based on iterating a cryptographically weak function sufficient number of times without disturbing, e.g., modifying, the outputs of intermediate rounds except whitening them with round-keys. Our attack on the full HB-2 exploits the fact that the internal states, which, instead of enhancing the overall cryptanalytic strength, give the attacker an opportunity to create an input differential for the last invocation of $WD16$ ($WD16^{-1}$ resp.) in the encryption (decryption resp.) and to retrieve the corresponding distribution of the output differences (call the collection of them a *differential sequence*) caused by the last invocation of the round function, which is information-rich in (a subset of) $(K_5, ..., K_8)$ ($(K_1, ..., K_4)$ resp.). Henceforth, after obtaining such a *template sequence*, the attacker, in an off-line environment, could search for the key bits associated, which usually constitute a subset of entire key bits. In all, our full attack can be divided into two phases: **preparation phase** as described in Section 5 and **key recovery phase** as described in Section 4.

Key Recovery Phase: In the key recovery phase, to remove the undesired interference introduced by the varying internal states when consecutive words are encrypted/decrypted, our attack here targets a specific encryption/decryption after the *preparation*, i.e., ith encryption/decryption for one HB-2 instance and i'th encryption/decryption for the other one. This is because given the key, IV, and the plaintext chain fed are fixed, the ith internal state and the i'th internal state are fixed as well. Henceforth, we omit the superscript/subscript i and i' of HB-2 variables for convenience when describing operations in the key recovery phase.

Providing the preparation phase succeeds, the attacker accomplishes the following utilizing the properties of the differential sequence analysis:

- Step 1. 36 bits of $(K_5, ..., K_8) \in \mathbb{F}_2^{64}$ are recovered using the differential sequence obtained from the last invocation of $WD16$ in the encryption if a particular condition meets, as shown in Fig. 2.
- Step 2. 28 bits of $(K_4, ..., K_1) \in \mathbb{F}_2^{64}$ are recovered using the differential sequence obtained from the last invocation of $WD16$ in the decryption if another particular condition meets.
- Step 3. the rest 64-bit key are exhaustively searched using either encryption or decryption.

To be specific, the condition needed to launch Step 1 in *key recovery phase* is:

$$\text{Condition (A)}: \quad \begin{aligned} \Delta K &= (K_1, ..., K_8) + (K'_1, ..., K'_8) = (H, 0, 0, 0, H, 0, 0, 0) \\ \Delta P &= P + P' = H \\ \Delta R &= (R_1, ..., R_8) + (R'_1, ..., R'_8) = (0, 0, 0, 0, H, 0, 0, 0). \end{aligned}$$

The condition needed to launch Step 2 in *key recovery phase* is:

$$\text{Condition (B)}: \quad \begin{aligned} \Delta K &= (K_1, ..., K_8) + (K'_1, ..., K'_8) = (0, 0, 0, H, 0, 0, 0, H) \\ \Delta C &= C + C' = H \\ \Delta R &= (R_1, ..., R_8) + (R'_1, ..., R'_8) = (0, 0, 0, 0, 0, 0, 0, H). \end{aligned}$$

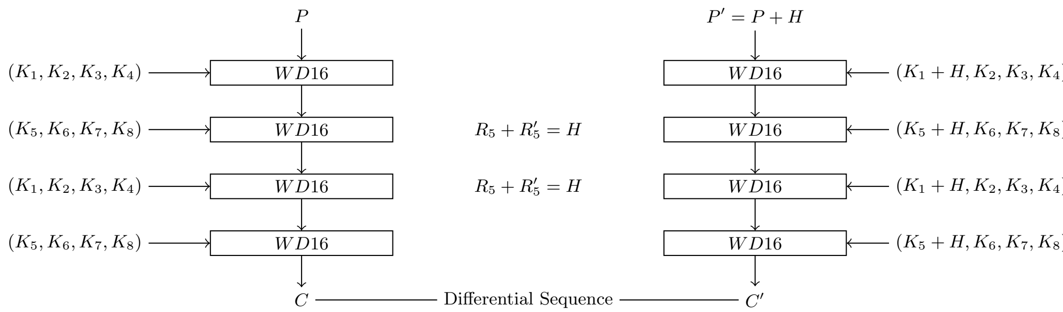

Fig. 2. Constructing Differential Sequence from Encryption with Condition (A)

To reach ΔP or ΔC in the two conditions above, the adversary model already allows the plaintext/ciphertext to be freely chosen; to reach ΔK, two pair of related-keys have to be used in our attack; and to reach ΔR, an extra phase, called preparation phase, has to be introduced.

Preparation Phase: As one may expected, preparation phase of our attack copes with the realization of ΔRs one at a time. To this end, one obvious way is to mount side-channel injection attack as shown in Appendix D, which gives the attacker no time/memory penalty, i.e., the overall time/memory complexity of the attack is dominated by that of the key recovery phase.

However, side-channel injection attack is not considered much in this work. Instead, we realize both conditions in a probabilistic manner, i.e.,

- $(R_1^{(i)}, ..., R_8^{(i)})$ and $(R_1'^{(i')}, ..., R_8'^{(i')})$ can be "randomized" by feeding both HB-2 instances with either different IVs and/or chains of random plaintext words. According to the birthday paradox, there is at least 0.5 chance that the randomized $(R_1^{(i)}, ..., R_8^{(i)}) \in \mathbb{F}_2^{128}$ and the randomized $(R_1'^{(i')}, ..., R_8'^{(i')}) \in \mathbb{F}_2^{128}$ satisfies ΔR in condition (A) (condition (B) resp.) providing 2^{64} attempts are made.
- Note that, in the previous step, even if ΔR happens, the attacker is usually unaware. To determine, we improve the mechanism above in light of another characteristic of HB-2, i.e., if condition (A) (condition (B) resp.) holds at the current round, it also holds for the next round. Hence, the differential sequences produced at the current round by $((R_1^{(i)}, ..., R_8^{(i)}), (R_1'^{(i')}, ..., R_8'^{(i')}))$ is exactly the same as that produced at the next round by $((R_1^{(i+1)}, ..., R_8^{(i+1)}), (R_1'^{(i'+1)}, ..., R_8'^{(i'+1)}))$.
- If the above step succeeds, the attacker proceeds to the key recovery phase to attack.

In what follows, we detail each of the above phases and steps.

4 Differentials Sequence Analysis (DSA)

In this section, we present a novel technique called differential sequence analysis (DSA) rooted in the differential cryptanalysis and the saturation attack. To be specific, we exhibiting its definitions, properties, and applications to attack one round of the HB-2, that constitutes the *key recovery phase* in our whole attack.

4.1 Differential Cryptanalysis and Saturation Attack

Differential cryptanalysis is a method analyzing the effect of particular differences in plaintext pairs on the differences of the resultant ciphertext pairs, which is based on a crucial observation that for any particular input differential, not all the output differential are possible, and the possible ones may not appear uniformly. In the original version of differential cryptanalysis [37], a unique differential is exploited to recover the subkey used in the last round of a block cipher. This idea has been extended in several ways: Biham and Shamir themselves further considered in [37] to use a trail of differentials to attack; Lai in [27] connected differential cryptanalysis with derivative of polynomials and presented a fine definition of higher order differentials; Knudsen [25] considered to use part of the input and output that have differential characteristics for the analysis; Biham, Biryukov and Shamir proposed in [5] to use differentials that happens with probability 0 as distinguishers; and recently, Blondeau and Gérard demonstrated the multiple differential cryptanalysis in [6], where a set of input/output differentials are considered together.

Saturation attack [23, 28, 8] exploits the fact that the output set is saturated, i.e., the outputs forms the whole space of $\mathbb{F}_2^m$, if the input set for the m-bit core injective function is saturated. Since the saturation of the outputs is observable, this technique usually serves as a distinguisher for the attacker.

At a high level, our differential sequence analysis in this paper can be understood as a hybrid of the conventional differential cryptanalysis and saturation attack, i.e., the set of the output differentials (instead of the outputs themselves) with respect to a particular/fixed input differential and a saturated set of inputs is considered. From another angle, due to the use of output differentials caused by a saturated set of inputs, our attack is also a special case of multiple differential cryptanalysis [6].

4.2 (First-order) Differential Sequence

Assume we have a keyed permutation $h(w, K)$ mapping $w \in \mathbb{F}_2^m$ to $h(w, K) \in \mathbb{F}_2^m$ with respect to the secret key K, where m is a positive integer. Given a fixed $\theta \in \mathbb{F}_2^m$, the first-order differential is known as

$$\Delta_{\theta,K}(w) = h(w, K) + h(w + \theta, K).$$

The *(first-order) differentials sequence* of h at θ is basically one row in the differential distribution table of h with respect to the input differential θ. To discuss its properties, we define it in a more formal way.

Definition 1. *The first-order differential sequence (DS) of h at θ is a non-binary sequence of 2^m entries, i.e.,*

$$\Delta_{\theta,K} = (z_0, z_1, ..., z_{2^m-1}),$$

where z_i denotes the multiplicity (that is, number of occurrences) of i in the set $\{w \in \mathbb{F}_2^m | \Delta_{\theta,K}(w)\}$, i.e.,

$$z_i = |\{w \in \mathbb{F}_2^m |\ \Delta_{\theta,K}(w) = i\}|.$$

Note that this definition can be extended to higher orders. In this paper, we constrained ourself to the first-order case.

For example, the differential sequence is $\{0, 0, 0, 2, 0, 0, 2, 0, 0, 4, 2, 0, 4, 0, 0, 2\}$ providing $\{w = \mathbb{F}_2^4 | \Delta_{\theta,K}(w), \} = \{12, 10, 3, 9, 6, 9, 15, 12, 12, 10, 3, 9, 6, 9, 15, 12\}$ and $\theta = 0x08$. The length of the differential sequence is the sum of all its multiplicities (16 in this example).

4.3 Properties of the Differential Sequence

The saturated set of inputs brings quite a lot interesting properties to the conventional differential cryptanalysis. We list the core properties to attack HB-2 here.

Property 1. For a fixed $\theta \in \mathbb{F}_2^m$, $\Delta_{\theta,K}$ is constructed by evaluating and counting $(h(w,K)+h(w+\theta,K))$ for every w in $\mathbb{F}_2^m$ regardless of the order of $w \in \mathbb{F}_2^m$ been accessed.

This property follows immediately from Definition 1 and is useful in the sense that even though $h(w,K)$ is an intermediate round in a cipher (thus, w is an intermediate value), we are able to capture $\Delta_{\theta,K}$ given that θ can be fixed in a particular way and w traverses the whole space of $\mathbb{F}_2^m$. Stated in another way, we have the property below.

Property 2. Let $perm(w)$ be a permutation of w in $\mathbb{F}_2^m$, i.e., $perm(w) : \mathbb{F}_2^m \mapsto \mathbb{F}_2^m$. For a fixed $\theta \in \mathbb{F}_2^m$ and every $w \in \mathbb{F}_2^m$, $\Delta_{\theta,K}$ can be obtained either by evaluating and counting $(h(w,K) + h(w+\theta,K))$, or by evaluating and counting $(h(perm(w),K) + h(perm(w)+\theta,K))$.

In what follows, we use

$$[h(w,K) + h(w+\theta,K)|w \in \mathbb{F}_2^m] = [h(perm(w),K) + h(perm(w)+\theta,K)|w \in \mathbb{F}_2^m]$$

as a symbolic expression for Property 2, where [...] actually defines a multiset and θ is always a fixed value in $\mathbb{F}_2^m$ for the rest of the paper. Henceforth, a straightforward extension of Property 2 can be derived below.

Property 3. Let $perm_i$, $i = 1, ..., n$, be permutations in $\mathbb{F}_2^m$. We have

$$\begin{aligned}&[h(w,K) + h(w+\theta,K)|w \in \mathbb{F}_2^m] \\ = &[h(perm_n(...(perm_1(w))),K) + h(perm_n(...(perm_1(w))) + \theta,K)|w \in \mathbb{F}_2^m].\end{aligned}$$

Proof. $perm_n(...(perm_1(w)))$ can be written as $perm(w)$ in $\mathbb{F}_2^m$. □

As aforementioned, the obtained differential sequence is primarily used to search for the key bits associated. Henceforth, we are especially interested in the correspondences between the differential sequence and the K in the underlying function $h(w,K)$, e.g., is the mapping from K to the differential sequence injective or not? To this end, we start with a special case of Property 2.

Property 4. Providing $K = K_a \bigcup K_b$, $K_a \bigcap K_b = \emptyset$ and $h(w,K) = h(w+K_a,K_b)$, we have

$$[h(w,K) + h(w+\theta,K)|w \in \mathbb{F}_2^m] = [h((w+K_a),K_b) + h((w+K_a)+\theta,K_b)|w \in \mathbb{F}_2^m].$$

Proof. By applying Property 2 and set $perm(w) = w + K_a$, this property follows immediately. □

From the property above, it is clear that all $K_a \in \mathbb{F}_2^{|K_a|}$ produces the same sequence while different K_bs may produce different sequences. Therefore, this property in fact implies that the obtained differential sequence of h at θ can be used to search for (a subset of) the key nonlinearly associated. Besides, there exists a more complicated correspondence between the key and the differential sequence. To discuss, we need to investigate the properties of sub-differential sequences.

Property 5. Let Γ be a subset of $\mathbb{F}_2^m$ and $perm$ is a permutation in Γ, we have

$$[h(w,K)+h(w+\theta,K)|w\in\Gamma]=[h(perm(w),K)+h(perm(w)+\theta,K)|w\in\Gamma].$$

Proof. This property follows from Definition 1, if and only if $perm$ is a permutation in Γ, i.e., $perm(w):\Gamma\mapsto\Gamma$. We call $[h(w,K)+h(w+\theta,K)|w\in\Gamma]$ or $[h(perm(w),K)+h(perm(w)+\theta,K)|w\in\Gamma]$ a *sub-differential sequence* of $\Delta_{\theta,K}$. □

Due to this, we can actually view a differential sequence obtained in $\mathbb{F}_2^m$ as a summation of several sub-differential sequences obtained in the disjoint subspaces of $\mathbb{F}_2^m$. This intuition can be written as below.

Property 6. Let Γ_i, $i=1,...,q$, be q disjoint partitions of $\mathbb{F}_2^m$, i.e.,

$$\Gamma_i\cap\Gamma_j=\emptyset,\qquad 1\leq i\neq j\leq q \tag{29}$$

$$\cup_{i=1}^q\Gamma_i=\mathbb{F}_2^m \tag{30}$$

and let the differential sequence obtained by $[h(w,K)+h(w+\theta,K)|w\in\Gamma_i]$ be $\Delta_{\theta,K}^{\{\Gamma_i\}}$, we thus have,

$$\Delta_{\theta,K}=\sum_{i=1}^{q}\Delta_{\theta,K}^{\{\Gamma_i\}}.$$

Following this reasoning, Property 4 can also be extended as below, which tells us that a differential sequence in Γ only corresponds to the key nonlinearly (in Γ) associated.

Property 7. Providing $K=K_a\bigcup K_b$, $K_a\bigcap K_b=\emptyset$ and $h(w,K)=h(w+K_a,K_b)$, we have, if Γ is a subset of $\mathbb{F}_2^m$ and $(w+K_a)$ is a permutation in Γ with respect to K_a,

$$[h(w,K)+h(w+\theta,K)|w\in\Gamma]=[h((w+K_a),K_b)+h((w+K_a)+\theta,K_b)|w\in\Gamma].$$

Therefore, if each of the sub-differential sequence stays the same with respect to the keys belonging to a particular set, denoted as Φ_0, the overall differential sequence remain the same under Φ_0. We formalize this correspondence as below.

Property 8. Let $\Phi_0=\cap_{i=1}^q\{k|w+k:\Gamma_i\mapsto\Gamma_i,k,w\in\mathbb{F}_2^m\}$, $K=K_a\bigcup K_b$, $K_a\bigcap K_b=\emptyset$ and $h(w,K)=h(w+K_a,K_b)$, we have

$$\Delta_{\theta,K}=\Delta_{\theta,\kappa}, \tag{31}$$

where $\kappa=\kappa_a\bigcup\kappa_b$, $\kappa_a\bigcap\kappa_b=\emptyset$, $K_a,\kappa_a\in\Phi_0$ and $\kappa_b=K_b$.

Proof. Let $\Delta_{\theta,K}^{\{\Gamma_i\}}$ be the sub-differential sequence obtained by Property 7. Thanks to Property 6, we have $\Delta_{\theta,K}=\sum_{i=1}^q\Delta_{\theta,K}^{\{\Gamma_i\}}$ and $\Delta_{\theta,\kappa}=\sum_{i=1}^q\Delta_{\theta,\kappa}^{\{\Gamma_i\}}$. Thanks to Property 7, for each i , $\Delta_{\theta,K}^{\{\Gamma_i\}}=\Delta_{\theta,\kappa}^{\{\Gamma_i\}}$ providing $K_a,\kappa_a\in\Phi_0$ and $\kappa_b=K_b$.

As opposed, providing $\kappa_b\neq K_b$ while $K_a,\kappa_a\in\Phi_0$, it is quite likely that $\Delta_{\theta,K}\neq\Delta_{\theta,\kappa}$ since $\Delta_{\theta,K}^{\{\Gamma_i\}}\neq\Delta_{\theta,\kappa}^{\{\Gamma_i\}}$ for each i.

□

4.4 Differential Sequence Analysis against HB-2

In this subsection, we attack the last invocation of $WD16$ ($WD16^{-1}$ resp.) in the encryption (decryption resp.) of HB-2 by exploiting the DSA as presented. To be specific, our Theorem 1 and Theorem 3 give answers to the question "how to obtain the differential sequences" while our Theorem 2 and Theorem 4 exhibit "how to use the differentials sequences". Since the HB-2 has a 16-bit block size, we have $m = 16$ for the rest.

Attacking $WD16$ in Encryption: To show our idea in a concise way, we assume that R_1 and R_1' are known (in fact, as they are identified by our algorithms in the preparation phase). In addition, let h in Definition 1 be the last invocation of $WD16$, i.e., Eq. (16), in the encryption. We thus have the following theorems.

Theorem 1. *When condition (A) meets, the differential sequence of the last $WD16$ in the encryption at $\theta = H$ can be extracted from executing the entire encryption.*

Proof: First of all, when condition (A) holds, we have,

$$\begin{aligned}
t_1' &= WD16(R_1' \boxplus P', K_1', K_2', K_3', K_4') \\
&= WD16(R_1 \boxplus (P + H), (K_1 + H), K_2, K_3, K_4) = t_1 \\
t_2' &= WD16(R_2' \boxplus t_1', K_5' + R_5', K_6' + R_6', K_7' + R_7', K_8' + R_8') \\
&= WD16(R_2 \boxplus t_1, (K_5 + H) + (R_5 + H), K_6 + R_6, K_7 + R_7, \\
&\quad K_8 + R_8) = t_2 \\
t_3' &= WD16(R_3' \boxplus t_2', K_1' + R_5', K_2' + R_6', K_3' + R_7', K_4' + R_8') \\
&= WD16(R_3 \boxplus t_2, (K_1 + H) + (R_5 + H), K_2 + R_6, K_3 + R_7, \\
&\quad K_4 + R_8) = t_3
\end{aligned}$$

Next, $\Delta_{H,(K_5,K_6,K_7,K_8)} = [z_0, z_1, ..., z_{2^{16}-1}]$ can be extracted, where

$$\begin{aligned}
z_i &= |\{t_3 \in \mathbb{F}_2^{16} |\ (WD16(R_4 \boxplus t_3, K_5, K_6, K_7, K_8) + WD16(R_4' \boxplus t_3', K_5', K_6', K_7', K_8')) = i\}| \\
&= |\{t_3 \in \mathbb{F}_2^{16} |\ (WD16(R_4 \boxplus t_3, K_5, K_6, K_7, K_8) + WD16(R_4 \boxplus t_3, (K_5 + H), K_6, K_7, K_8)) = i\}| \\
&= |\{t_3 \in \mathbb{F}_2^{16} |\ (WD16(R_4 \boxplus t_3, K_5, K_6, K_7, K_8) + WD16((R_4 + H) \boxplus t_3, K_5, K_6, K_7, K_8)) = i\}| \\
&= |\{P \in \mathbb{F}_2^{16}, P' = P + H |\ (C \boxminus R_1) + (C' \boxminus R_1) = i\}|.
\end{aligned}$$

The second last equality comes from the fact

$$(R_4 \boxplus t_3) + (K_5 + H) = ((R_4 + H) \boxplus t_3) + K_5,$$

which can be easily verified by the computer simulation.

Note that condition (A) is essentially a necessary condition for the following condition:

$$\begin{aligned}
\Delta K &= (K_1, ..., K_8) + (K_1', ..., K_8') = (0, 0, 0, 0, 0, 0, 0, 0) \\
\Delta P &= P_1 + P_{i'}' = 0 \\
\Delta R &= (R_1, ..., R_8) + (R_1', ..., R_8') = (0, 0, 0, H, 0, 0, 0, 0),
\end{aligned}$$

such that both of them produce the same differential sequence of $WD16$. However, we use condition (A) through the rest of the paper because it has an additional property that keeps the attacker informed once $\varDelta R$ happens (see Section 5.2). □

This theorem suggests that, after querying the encryption with every $P \in \mathbb{F}_2^{16}$ and obtaining the resultant output differentials, the attacker could have a *template sequence* $\varDelta_{H,(K_5,K_6,K_7,K_8)}$ to search for parts of (K_5, K_6, K_7, K_8). The next theorem discloses the correspondence between $\varDelta_{H,(K_5,K_6,K_7,K_8)}$ and (K_5, K_6, K_7, K_8).

Theorem 2. *Let $\varDelta_{H,(K_5,K_6,K_7,K_8)}$ be obtained from Theorem 1. For $\kappa_5 \in \mathbb{F}_2^{16}$ and $\kappa_6 \in \mathbb{F}_2^{16}$, we have*

$$\varDelta_{H,(K_5,K_6,K_7,K_8)} = \varDelta_{H,(\kappa_5,\kappa_6,K_7,K_8)},$$

where K_6 and κ_6 belong to the same set $\varPhi_i = \varPhi_0 + i$, $0 \le i \le 15$, and $\varPhi_0$, of cardinality 2^{12}, is tabulated in Appendix C.

Proof: To prove, we discuss the correspondence between K_5, K_6, K_7, K_8 and the template sequence in a respective way.

Correspondence Between K_5 and DS: For the time being, let us consider $h(w, K) = f(f(w+K_5)+K_6)$ (a simplified $WD16$), where $f : \mathbb{F}_2^{16} \mapsto \mathbb{F}_2^{16}$ (as described in Section 2) is an injective function, we thus have, by letting $w = R_4 \boxplus t_3$ and $\theta = H$,

$$\begin{aligned}
&[h(w,K) + h(w+\theta,K) | w \in \mathbb{F}_2^{16}] \\
&= [f(f(w+K_5)+K_6) + f(f(w+K_5+\theta)+K_6) | w \in \mathbb{F}_2^{16}] \\
&= [f(f(perm_1(w))+K_6) + f(f(perm_1(w)+\theta)+K_6) | w \in \mathbb{F}_2^{16}]
\end{aligned}$$

It is clear from the context that $perm_1(w) = w + K_5$ is a permutation in $\mathbb{F}_2^m$, and, due to Property 4, $\varDelta_{H,(K_5,K_6,K_7,K_8)}$ does not dependent on K_5.

Correspondence Between K_6 and DS: First of all, we define the following auxiliary variables for convenience:

- λ_i, $i = 1, ..., q$, are q possible output differences of f, given the input difference is θ.
- $\varGamma_i = \{f(w) | f(w) + f(w+\theta) = \lambda_i, w \in \mathbb{F}_2^{16}\}$, $i = 1, ..., q$, are q disjoint partitions of $\mathbb{F}_2^{16}$ such that: (1) Eq. (29) holds, otherwise there is a $w \in (\varGamma_i \cap \varGamma_j)$, $1 \le i \ne j \le q$, such that $f(w) + f(w+\theta)$ produces output differences λ_i and λ_j, $\lambda_i \ne \lambda_j$, which is impossible; (2) Eq. (30) holds, otherwise there is a $w \in (\mathbb{F}_2^{16} - \cup_{i=1}^{q} \varGamma_i)$, that produces an output difference $\notin \{\lambda_1, ..., \lambda_q\}$, which contradicts our definition.
- $\varPhi_0 = \cap_{i=1}^{q} \{k | f(w) + k : \varGamma_i \mapsto \varGamma_i, k \in \mathbb{F}_2^{16}\}$. Intuitively, $\varPhi_0$ encompasses all possible keys, which make $f(w) + k$ a permutation in $\varGamma_i$, $i = 1, ..., q$.

Furthermore, let us consider two cases: (1) $K_6 \in \varPhi_0$; and (2) $K_6 \in$ a coset of $\varPhi_0$.

For case (1), i.e., $K_6 \in \varPhi_0$, the above equations can be further written as, by setting $perm_2(w) = f(perm_1(w)) + K_6$,

$$\begin{aligned}
&[f(f(perm_1(w))+K_6) + f(f(perm_1(w)+\theta)+K_6) | w \in \mathbb{F}_2^{16}] \\
&= [f(f(perm_1(w))+K_6) + f(f(perm_1(w))+\lambda_i+K_6) | w \in \varGamma_i] && \text{for } i = 1, ..., q \\
&= [f(f(perm_1(w))+K_6) + f(f(perm_1(w))+K_6+\lambda_i) | w \in \varGamma_i] && \text{for } i = 1, ..., q \\
&= [f(perm_2(w)) + f(perm_2(w)+\lambda_i) | w \in \varGamma_i] && \text{for } i = 1, ..., q \\
&= [f(w) + f(w+\lambda_i) | w \in \varGamma_i] && \text{for } i = 1, ..., q
\end{aligned}$$

The above equation holds because of Property 8, e.g., for $K_6 \in \Phi_0$, every $[f(w)+f(w+\lambda_i)|w \in \Gamma_i]$ produces the same sub-differential sequence. Therefore, the overall differential sequence stays the same for every $K_6 \in \Phi_0$. Stating in another way, providing K_6 and κ_6 are both in Φ_0, $\Delta_{H,(K_5,K_6)} = \Delta_{H,(\kappa_5,\kappa_6)}$.

The above derivation is further confirmed through extensive experiments, where we found

$$(\lambda_1, ..., \lambda_6) = (\text{0x30cc}, \text{0x6198}, \text{0x9264}, \text{0xa2a8}, \text{0xc330}, \text{0xf3fc}),$$

$(\Gamma_1, ...\Gamma_6)$, and Φ_0 as tabulated in Appendix C, which is of cardinality 2^{12}.

In what follows, we prove case (2), i.e., the above equations are true for $K_6 \in \Phi_i = \Phi_0 + i$. This is because, by letting $K_6 = \triangleright K_6 + \triangleleft K_6$ such that $\triangleright K_6 \in \Phi_0$,

$$\begin{aligned}
&[f(f(perm_1(w)) + K_6) + f(f(perm_1(w)) + K_6 + \lambda_i)|w \in \Gamma_i] && \text{for } i = 1, ..., q\\
=\;&[f(f(perm_1(w)) + \triangleright K_6 + \triangleleft K_6) + f(f(perm_1(w)) + \triangleright K_6 + \triangleleft K_6 + \lambda_i)|w \in \Gamma_i] && \text{for } i = 1, ..., q\\
=\;&[f(perm_3(w) + \triangleleft K_6) + f(perm_3(w) + \triangleleft K_6 + \lambda_i)|w \in \Gamma_i] && \text{for } i = 1, ..., q\\
=\;&[f(w + \triangleleft K_6) + f(w + \triangleleft K_6 + \lambda_i)|w \in \Gamma_i] && \text{for } i = 1, ..., q
\end{aligned}$$

The second last equation holds because of our proof of case (1) (by letting $perm_3(w) = f(perm_1(w)) + \triangleright K_6$).

In addition, it is clear that:

- the sub-differential sequence $[f(w + \triangleleft K_6) + f(w + \triangleleft K_6 + \lambda_i)|w \in \Gamma_i]$, is different from $[f(w + f(w + \lambda_i)|w \in \Gamma_i]$ as long as $\triangleleft K_6 \neq 0$. So is the overall differential sequence with overwhelming probability.
- for $K_6 = \triangleright K_6 + \triangleleft K_6$, $\kappa_6 = \triangleright \kappa_6 + \triangleleft \kappa_6$, $K_6 \neq \kappa_6$, the sub-differential sequence $[f(w + \triangleleft K_6) + f(w + \triangleleft K_6 + \lambda_i)|w \in \Gamma_i]$, is the same as $[f(w + \triangleleft \kappa_6) + f(w + \triangleleft \kappa_6 + \lambda_i)|w \in \Gamma_i]$ as long as $\triangleleft K_6 = \triangleleft \kappa_6$. This is due to the possibility that $\triangleright K_6, \triangleright \kappa_6 \in \Phi_0$, $\triangleright K_6 \neq \triangleright \kappa_6$ could yield $\triangleleft K_6 = \triangleleft \kappa_6$.

From the accusation above and our extensive experiments, it can be concluded that the key space of $K_6 \in \mathbb{F}_2^{16}$ has been divided into 16 cosets, i.e., $\Phi_0, ..., \Phi_{15}$, and each is of cardinality 2^{12}.

Correspondence Between (K_7, K_8) **and DS**: We carry on all the notations above for K_7 except setting $h(w, K) = f(f(f(f(w + K_5) + K_6) + K_7) + K_8)$. We found that, for K_7, Φ_0 is always a empty set because too many λ_i divides $\mathbb{F}_2^{16}$ into numerous tiny subspaces Γ_i, for which there is no K_7 could make $f(w) + K_7$ a permutation in every Γ_i, $i = 1, ..., q$. Same phenomenon happens to K_8. In all, each choice of (K_7, K_8) produces a different differential sequence, which is further confirmed empirically. □

Attacking $WD16^{-1}$ **in Decryption**: Similar attack can be performed against the decryption. By assuming R_1 and R_1' are known and letting h in Definition 1 be the last invocation of $WD16^{-1}$, i.e., Eq. (28), we have the following results for our attack.

Theorem 3. *With the condition (B), the differential sequence of the last* $WD16^{-1}$ *in the decryption at* $\theta = H$ *can be extracted from executing the entire decryption.*

Proof: First of all, when condition (B) holds, we have,

$$\begin{aligned}
u_3 &= WD16^{-1}(C \boxminus R_1, K_8, K_7, K_6, K_5)\\
&= WD16^{-1}((C+H) \boxminus R_1', (K_8+H), K_7', K_6', K_5') = u_3'\\
u_2 &= WD16^{-1}(u_3 \boxminus R_4, K_4 + R_8, K_3 + R_7, K_2 + R_6, K_1 + R_5)\\
&= WD16^{-1}(u_3' \boxminus R_4', (K_4+H) + (R_8+H), K_3' + R_7', K_2' + R_6', K_1' + R_5') = u_2'\\
u_1 &= WD16^{-1}(u_2 \boxminus R_3, K_8 + R_8, K_7 + R_7, K_6 + R_6, K_5 + R_5)\\
&= WD16^{-1}(u_2' \boxminus R_3', (K_8+H) + (R_8+H), K_7' + R_7', K_6' + R_6', K_5' + R_5') = u_1'
\end{aligned}$$

Next, $\Delta_{H,(K_4,K_3,K_2,K_1)} = [z_0, z_1, ..., z_{2^{16}-1}]$ can be extracted, where,

$$\begin{aligned}
z_i &= |\{u_1 \in \mathbb{F}_2^{16}|\ (WD16^{-1}(u_1 \boxminus R_2, K_4, K_3, K_2, K_1) + WD16^{-1}(u_1' \boxminus R_2', K_4', K_3', K_2', K_1')) = i\}|\\
&= |\{u_1 \in \mathbb{F}_2^{16}|\ (WD16^{-1}(u_1 \boxminus R_2, K_4, K_3, K_2, K_1) + WD16^{-1}(u_1' \boxminus R_2', K_4 + H, K_3, K_2, K_1)) = i\}|\\
&= |\{C \in \mathbb{F}_2^{16}, C' = C + H|\ (P \boxplus R_1) + (P' \boxplus R_1) = i\}|.
\end{aligned}$$

□

A similar theorem describes the correspondence between $\Delta_{H,(K_4,K_3,K_2,K_1)}$ and (K_4, K_3, K_2, K_1).

Theorem 4. *Let $\Delta_{H,(K_4,K_3,K_2,K_1)}$ be obtained from Theorem 3. For $\kappa_1 \in \mathbb{F}_2^{16}$ and $\kappa_4 \in \mathbb{F}_2^{16}$,*

$$\Delta_{H,(K_4,K_3,K_2,K_1)} = \Delta_{H,(\kappa_4,K_3,K_2,\kappa_1)},$$

where K_4 and κ_4 belong to the same set $\Phi_i = \Phi_0 + i$, $0 \le i \le 2^{12} - 1$, and $\Phi_0 = \{0x0000, 0x0010, ..., 0x00f0\}$.

Proof: Similar as Theorem 2, except that we could easily observe from the experimental data that $\Phi_0 = \{0x0000, 0x0010, 0x0020, ..., 0x00f0\}$. □

Visualization of Differential Sequences From HB-2: Here we provide several examples of the differential sequences used in our experiments. Fig. 4 to Fig. 6 in Appendix B are the ones obtained from the last invocation of $WD16$ in the encryption with $IV = (0, 0, 0, 0)$ and different keys randomly selected. Fig. 7 to Fig. 9 in Appendix B are the ones obtained from the last invocation of $WD16^{-1}$ in the decryption with $IV = (0, 0, 0, 0)$ and different keys randomly selected. All of the sequences are substantially different from each other, which exhibits their correlations to the underlying keys in an intuitive way.

4.5 Local Search in DSA

After the template sequence is captured, the attacker could, in an off-line environment, launches $h(w, K) = WD16(.)$ ($h(w, K) = WD16^{-1}(.)$ resp.) to search for parts of (K_5, K_6, K_7, K_8) ($(K_1, K_2, K_3, K_4$ resp.), which is called the *local search* in DSA. Through the local search, the attacker recovers 36-bit (44-bit resp.) information regarding the key.

A naive way to search locally is to produce a complete local differential sequence from $[h(w, K) + h(w + H, K)|w \in \mathbb{F}_2^{16}]$ with a random K at first, comparing each entry of which with the corresponding entry of the template sequence. The cost per key trial is 2^{16} executions of $h(w, K)$s and 2^{16} comparisons.

The efficiency of this method can be substantially improved if the early-abort strategy [30] is adapted, i.e., given the ith entry in the local differential sequence is greater than the ith entry in the template sequence, one could assert that the trial key is incorrect and terminate the search in advance. We present this improved local search algorithm below.

---

let TDS be the template sequence obtained
initiate the local differential sequence LDS as a list of 2^{16} "0"s
for w from 0 to $2^{16}-1$ **do**
    randomly choose K
    $diff \leftarrow h(w+K)+h(w+H+K)$
    $LDS[diff] \leftarrow LDS[diff]+1$
    if $LDS[diff] > TDS[diff]$ **then**
        return NULL
    end if
end for
if $LDS[w] = TDS[w]$ for $w = 0, 1, ..., 2^{16}-1$ **then**
    return K
end if

The theoretical derivation of the time complexity of the above algorithm could be quite cumbersome. Instead, we recorded the number of the for-loops that are actually executed, denoted as l, during the search. Through repeated testings, we found that, in average, $1.640 < l < 1.660$ for-loops are spent per key trial for both local searches using $WD16(.)$ and $WD16^{-1}(.)$. Thus, we conclude the cost per key trial of our local search algorithm is 1.65 executions of (a pair of) $h(w,K)$s .

4.6 Differential Sequence Analysis (DSA) Against HB-2 and Its Time Complexity

We are ready to list out the steps performed by the attacker during the key recovery phase, as below.

1. When condition (A) holds, the attacker extracts the template sequence $\Delta_{H,(K_5,K_6,K_7,K_8)}$ using $((C \boxminus R_1) + (C' \boxminus R_1))$, where C and C' can be obtained by querying the encryption with P and $P' = P + H$, and R_1 and R_1' are obtained in the preparation phase. Then, the attacker locally searches 36-bit of (K_5, K_6, K_7, K_8) using the proposed local search algorithm.
2. Similarly, utilizing the decryption, when condition (B) holds, the attacker extracts another template sequence $\Delta_{H,(K_4,K_3,K_2,K_1)}$ using $(P \boxplus R_1) + (P' \boxplus R_1)$, and guesses to determine 44-bit of (K_4, K_3, K_2, K_1) using the proposed local search algorithm.
3. After that, the attacker searches the remaining 48-bit of the key using 2^{48} trial encryptions[1].

The overall complexity of the above steps is

$$\underbrace{2^{36} \times 1.65}_{\text{determine 36-bit of } (K_5,...,K_8)} + \underbrace{2^{44} \times 1.65}_{\text{determine 44-bit of } (K_1,...,K_4)} + \underbrace{2^{48}}_{\text{determine the rest}} \approx 2^{48.14},$$

where negligible memory is required by each steps.

[1] In fact, $2^{48} + 2^{32} + 2^{16} = 2^{48.00002201}$ trial encryptions and three plaintext-ciphertext pairs are required.

5 A Probabilistic Realization of Conditions (A) and (B)

The attacks in the last section solely depends on the occurrences of conditions (A) and (B), to reach ΔRs in which sounds unpractical at the first glance as the initialization of HB-2 makes the internal states unpredictable. In this section, we show a probabilistic approach to realize these conditions – when the internal states of two HB-2 instances are respectively random, there is a certain chance that the attacker could get the desired differentials in the internal states. To this end, we study how to randomize the internal states of HB-2 at first, and, how to determine whether the desired ΔRs happen.

5.1 Randomize the Internal States

There are two ways for the adversary to affect the internal states of HB-2:

– Providing the key is fixed, it is suffice, from Eq. (1)-(12), that $(IV_1, ..., IV_4) \mapsto (R_1, ..., R_4)$ is an injective mapping and so is $(IV_1, ..., IV_4) \mapsto (R_5, ..., R_8)$. Therefore, the attacker could easily generate 2^{64} (out of 2^{128}) different internal states by choosing different IVs and launching the initialization.
– For a fixed key and a particular IV, the attacker could choose plaintext P_1 to feed HB-2 at first. If a state transition graph is drawn, we can see that the starting state, i.e., $R^{(1)}$, transits to 2^{16} neighboring states while each $P_1 \in \mathbb{F}_2^{16}$ is encrypted. Next, if another encryption is performed, e.g., encrypting P_2, each of these "neighboring states" again transits to another 2^{16} states providing P_2 takes every value in $\mathbb{F}_2^{16}$. By continuing this process, we would have all 2^{128} states covered in this graph. Therefore, to produce a set of random internal states, i.e., $\{R^{(1)}, R^{(2)}, ...\}$, we could, as shown in Fig. 3, feed the encryptions with a plaintext chain where P_i is selected uniformly at random in $\mathbb{F}_2^{16}$ for $i = 1, 2, ...$. Similarly, a ciphertext chain could be fed to the decryption oracle to generate a set of random internal states as well. Note that feeding HB-2 encryption with a chain of N random inputs is equivalent to perform an N-step 2^{16}-dimensional random walk in its state transition graph. Therefore, $|\{R^{(1)}, R^{(2)}, ...\}| \approx N$ if$N \ll 2^{128}$ [15].

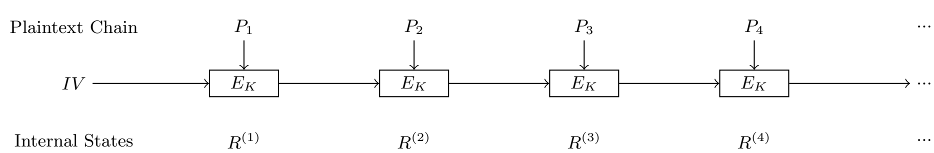

Fig. 3. Feeding HB-2 Encryption with a Plaintext Chain

Therefore, the algorithm below provides, to the later steps, the randomized internal states of two running HB-2 instances through an effort-saving way – one instance initializes a random IV and encrypts one random plaintext, while the other one, besides initializes a random IV, encrypts N random plaintexts consecutively. Since $\{R^{(1)}, R^{(2)}, ..., R^{(N)}\}$ is a set of random variables as analyzed, $\{R^{(1)} + R'^{(1)}, R^{(2)} + R'^{(1)}, ..., R^{(N)} + R'^{(1)}\}$ must also be a set of random variables.

Let $R^{(i)} \Leftarrow E(P_i, K)$ be the internal state $R^{(i)}$ after encrypting $P_1, ..., P_i$
Randomly choose IV' and P'_1, $R'^{(1)} \Leftarrow E(P'_1, K)$
Randomly choose IV
for i from 1 to N **do**
  Randomly choose P_i, $R^{(i)} \Leftarrow E(P_i, K)$
  if $R'^{(1)} + R^{(i)} = \Delta R$ **then**
    return "ΔR happens"
  end if
end for

Note that, currently, the given algorithm is only a skeleton for our attack, which is discussed in more detail in the next subsections and the full-fledged version is given at last. Nevertheless, we can already sense an interesting property from this skeleton algorithm.

Property 9. In the algorithm above, a certain ΔR happens with 0.5 probability when $N = 2^{64}$.

Proof. This property holds due to the birthday paradox. □

5.2 Determine while Guessing

To inform the attacker during the attempting, as long as condition (A) (condition (B) resp.) happens, we use one unusual differential characteristic in the encryption (decryption resp.), as first pointed out by HB-2's designers, such that the differentials in the internal states, secret keys and the inputs can be maintained and entered into the next round, i.e., for a positive integer i,

$$(\Delta P_i, \Delta K, \Delta R^{(i)}) = (\Delta P_{i+1}, \Delta K, \Delta R^{(i+1)}).$$

Therefore, the following theorem holds.

Theorem 5. *Let $\Delta^{(i')}_{H,(K_5,K_6,K_7,K_8)}$ ($\Delta^{(i')}_{H,(K_4,K_3,K_2,K_1)}$ resp.) be the differential sequence produced by the two encryption instances (two decryption instances resp.) with internal states $R^{(1)}$ and $R'^{(i')}$ and let $\Delta^{(i'+1)}_{H,(K_5,K_6,K_7,K_8)}$ ($\Delta^{(i'+1)}_{H,(K_4,K_3,K_2,K_1)}$ resp.) be the differential sequence produced by the two encryption instances (two decryption instances resp.) with internal states $R^{(2)}$ and $R'^{(i'+1)}$ (call $\Delta^{(i')}_{H,K}$ and $\Delta^{(i'+1)}_{H,K}$ neighboring template sequences). Therefore,*

- *If condition (A) happens during encryption, the adversary observes two identical neighboring template sequences, i.e.,*
$$\Delta^{(i')}_{H,(K_5,K_6,K_7,K_8)} = \Delta^{(i'+1)}_{H,(K_5,K_6,K_7,K_8)};$$
otherwise, the above equation holds with negligible probability.
- *If condition (B) happens during decryption, the adversary observes two identical neighboring template sequences, i.e.,*
$$\Delta^{(i')}_{H,(K_4,K_3,K_2,K_1)} = \Delta^{(i'+1)}_{H,(K_4,K_3,K_2,K_1)};$$
otherwise, the above equation hold with negligible probability.

Proof: It follows from Definition 1 and Property 1. □

Therefore, the above theorem can serve as an algorithm to determine the occurrences of condition (A) or condition (B), i.e., it returns either $(Success, \Delta_{H,K}^{(i')}, R_1^{(1)}, R_1^{(2)})$ or $(False, NULL, NULL, NULL)$ to the key recovery phase. Unfortunately, in this algorithm, the correct template sequences can only be extracted with the correct $R_1^{(1)}$ and $R_1^{(2)}$ due to Theorem 1 and Theorem 3. For instance, using the encryption, the two neighboring sequences are

$$\Delta^{(i')} = (z_0^{(i')}, z_1^{(i')}, ..., z_{65535}^{(i')}) \tag{32}$$
$$\Delta^{(i'+1)} = (z_0^{(i'+1)}, z_1^{(i'+1)}, ..., z_{65535}^{(i'+1)}) \tag{33}$$

where

$$z_j^{(i')} = |\{P_1 \in \mathbb{F}_2^{16}, P'_{i'} = P_1 + H|\ (C_1 \boxminus R_1^{(1)}) + (C'_{i'} \boxminus R'^{(i')}_1) = j\}| \qquad \text{and}$$
$$z_j^{(i'+1)} = |\{P_2 \in \mathbb{F}_2^{16}, P'_{i'+1} = P_2 + H|\ (C_2 \boxminus R_1^{(2)}) + (C'_{i'+1} \boxminus R'^{(i'+1)}_1) = j\}|$$

Henceforth, it is true that by guessing $R_1^{(1)}$ and $R_1^{(2)}$, the theorem/algorihtm above would cost 2^{32} encryptions/decryptions per execution.

To improve its efficiency, we make use of the following fact: as the modulo addition is only first-order correlation-immune, the two identical neighboring sequences obfuscated by modulo additions of different R_1s may have an apparent correlation, while two distinct neighboring sequences may not. This intuition is further verified by our extensive experiments. In parallel with Eq. (32) and Eq. (33), let us define the *raw neighboring sequences* as:

$$\underline{\Delta^{(i')}} = (\underline{z_0^{(i')}}, \underline{z_1^{(i')}}, ..., \underline{z_{65535}^{(i')}})$$
$$\underline{\Delta^{(i'+1)}} = (\underline{z_0^{(i'+1)}}, \underline{z_1^{(i'+1)}}, ..., \underline{z_{65535}^{(i'+1)}})$$

where

$$\underline{z_j^{(i')}} = |\{P_1 \in \mathbb{F}_2^{16}, P'_{i'} = P_1 + H|\ C_1 + C'_{i'} = j\}| \qquad \text{and}$$
$$\underline{z_j^{(i'+1)}} = |\{P_2 \in \mathbb{F}_2^{16}, P'_{i'+1} = P_2 + H|\ C_2 + C'_{i'+1} = j\}|.$$

We found that, for the identical neighboring sequences, the corresponding two raw neighboring sequences always have more than 30000 (out of 65536) identical entries, i.e.,

$$Corr(\underline{\Delta^{(i')}}, \underline{\Delta^{(i'+1)}}) = |\{\underline{z_j^{(i')}} = \underline{z_j^{(i'+1)}}, j = 0, 1, ..., 65535\}| > 30000, \text{ iff } \Delta^{(i')} = \Delta^{(i'+1)},$$

where $Corr(.,.)$ is the non-normalized correlation.

On the contrary, for the distinct neighboring sequences, the corresponding two raw neighboring sequences always have less than 19000 (out of 65536) identical entries, i.e.,

$$Corr(\underline{\Delta^{(i')}}, \underline{\Delta^{(i'+1)}}) = |\{\underline{z_j^{(i')}} = \underline{z_j^{(i'+1)}}, j = 0, 1, ..., 65535\}| < 19000, \text{ iff } \Delta^{(i')} \neq \Delta^{(i'+1)}.$$

By treating the correlation of the raw neighboring sequences as a criterion, Theorem 5 is now able to return whether $\Delta^{(i')}$ equals $\Delta^{(i'+1)}$ with 2^{16} time complexity. Once the identical neighboring sequences are identified, the adversary is able to guess to recover $R_1^{(1)}$ and $R_1^{(2)}$ with 2^{32} effort in time.

5.3 Preparation Phase and Its Time Complexity

We recap the whole process in the preparation phase for the encryption as shown below, which is an extension of the skeleton algorithm we shown before. Note that the preparation using the decryption is similar and omitted here.

---

randomly choose IV' and P'_1, $R'^{(1)} \Leftarrow E(P'_1, K)$
randomly choose IV
randomly choose a constant P'_2
for i from 1 to $N = 2^{64}$ **do**
  randomly choose P_i, $R^{(i)} \Leftarrow E(P_i, K)$
  generate $\Delta^{(i)}$ using $R'^{(1)}$ and $R^{(i)}$
  $R'^{(2)} \Leftarrow E(P'_2, K)$
  $R^{(i+1)} \Leftarrow E(P_{i+1}, K)$ where $P_{i+1} = P'_2 + H$
  generate $\Delta^{(i+1)}$ using $R'^{(2)}$ and $R^{(i+1)}$
  if $Corr(\Delta^{(i)}, \Delta^{(i+1)}) > 30000$ **then**
    guess to determine $R_1^{(1)}$ and $R_1^{(2)}$
    recover $\Delta_{H,K}^{(i')}$ from the raw neighboring sequences
    return $(Success, \Delta_{H,K}^{(i')}, R_1^{(1)}, R_1^{(2)})$, keep current states and enter the key recovery phase
  end if
  decrypt using C'_2 and C_{i+1} to roll back HB-2's states to $R'^{(1)}$ and $R^{(i)}$
end for
return $(False, NULL, NULL, NULL)$

Using the encryption (decryption resp.) only, the attacker has 0.5 probability to reach condition (A) (condition (B) resp.) with $2^{64} \times 2^{16} = 2^{80}$ time complexity. After that, he is able to guess to determine $R_1^{(1)}$ and $R_1^{(2)}$ with additional 2^{32} effort in time. In all, the time complexity of the preparation phase is

$$\underbrace{2^{64} \times 2^{16}}_{\text{test whether the condition happens}} + \underbrace{2^{32}}_{\text{guess to determine } R_1\text{s}} + \underbrace{2^{16}}_{\text{recover the template seuqnece}} \approx 2^{80}.$$

It is worthy to mention that to succeed with probability 1, the preparation phase requires $2^{128+16} = 2^{144}$ effort in time, which is slower than the exhaustive search.

6 Concluding Remarks

In this paper, we present a novel cryptanalytic technique called differential sequence analysis (DSA), which is especially effective if the differential sequence reflecting parts of a cipher associated with parts of the key can be obtained. In addition, we demonstrate the application of this technique, that constitutes the key recovery of the lightweight block cipher Hummingbird-2 with $2^{48.14}$ time complexity, given particular conditions hold in its internal states, secret keys and the inputs. Furthermore, we investigate how to reach these conditions in our preparation phase with 0.5 chance and 2^{80} effort in time. To the best of our knowledge, this is the first cryptanalytic result of the full Hummingbird-2.

The attack presented against Hummingbird-2 is a special case of the general DSA, to build the theoretic framework of which is part of our future work. In addition, it will be evaluated in the recent future: (1) whether the generalized DSA provides even better results against Hummingbird-2 and other potentially vulnerable ciphers, especially the ones with small block size and with internal states, e.g., stateful block ciphers [24]; (2) the possibility that the generalized DSA can work with other cryptanalysis technologies, e.g., meet-in-the-middle.

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A Side-channel Injection Attack to Recover (R_5, R_6, R_7, R_8)

As can be seen from Eq. (6)-(9), R_5, R_6, R_7, R_8 do not participate in the randomization process but simply record (by Xoring) the historical statuses of R_1, R_2, R_3, R_4 respectively. Therefore, following steps allow a side-channel attacker, who is able to inject "1" to a certain bit of the register storing R_j, $5 \le j \le 8$, to recover (R_5, R_6, R_7, R_8):

1. The attacker encrypts with a known IV and the target key to get a plaintext/cipher pair (P, C), where $P \in \mathbb{F}_2^{16}, C \in \mathbb{F}_2^{16}$;
2. He resets HB-2 and initializes HB-2 with the same IV and key. At any time during this initialization, he injects "1" to the qth bit, $0 \le q \le 15$, of the register which stores R_5. He then encrypts P and gets C'. If $C = C'$ (which implies the injection does not change the internal states of HB-2), the attacker in fact learns that the qth bit of R_5 is 1; otherwise it is 0. He repeats this step for every q in $\{0, 1, ..., 15\}$ to recover R_5;
3. Step (2) can be repeated to recover R_6, R_7 and R_8;

The cost of this injection attack to recover (R_5, R_6, R_7, R_8) is 64 injections and 64 invocations of HB-2 encryption. In addition, since the attacker has a large time window to perform the injection to the qth bit of R_j (any time during the rth iteration of the initialization), this side-channel attack seems quite practical.

B Visualization of Differential Sequences

Fig. 4 to Fig. 6 are the ones obtained from the last invocation of $WD16$ in the encryption with $IV = (0, 0, 0, 0)$ and different keys randomly selected. Fig. 7 to Fig. 9 are the ones obtained from the last invocation of $WD16^{-1}$ in the decryption with $IV = (0, 0, 0, 0)$ and different keys randomly selected. All of the sequences looks substantially different from each other, which exhibits their correlations to the underlying keys in an intuitive way.

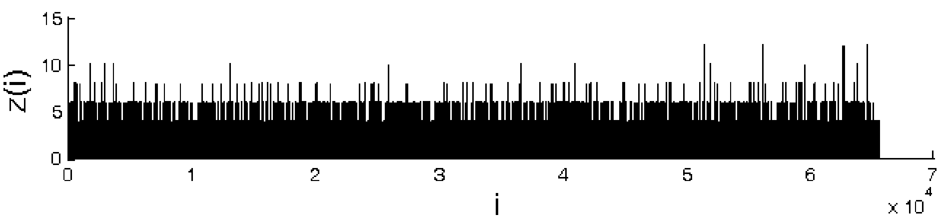

Fig. 4. DS from Enc. using (K_5, K_6, K_7, K_8) = (0xf1e3,0x524a,0xb28a,0xc987)

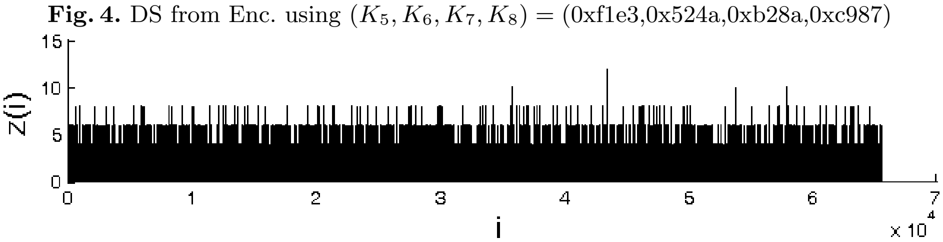

Fig. 5. DS from Enc. using (K_5, K_6, K_7, K_8) = (0x7c9f,0x0784,0x1c96,0xbcb4)

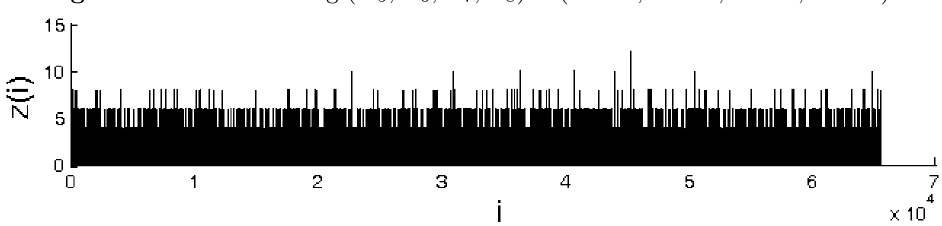

Fig. 6. DS from Enc. using (K_5, K_6, K_7, K_8) = (0x6b03,0xcf0c,0x1ba2,0xdc27)

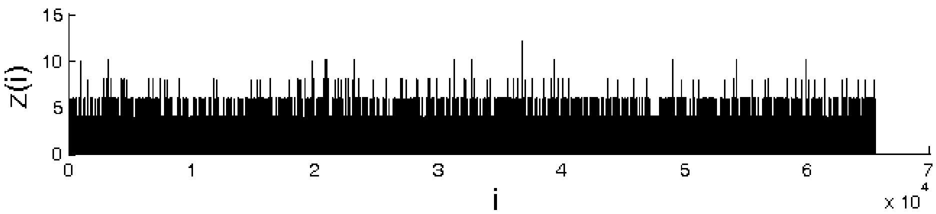

Fig. 7. DS from Dec. using (K_1, K_2, K_3, K_4) = (0x5d67,0xd0ef,0x8cec,0xa33a)

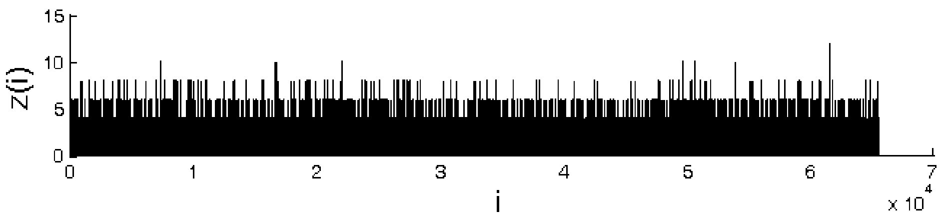

Fig. 8. DS from Dec. using (K_1, K_2, K_3, K_4) = (0x6601,0x0bd8,0xa6fa,0xcede)

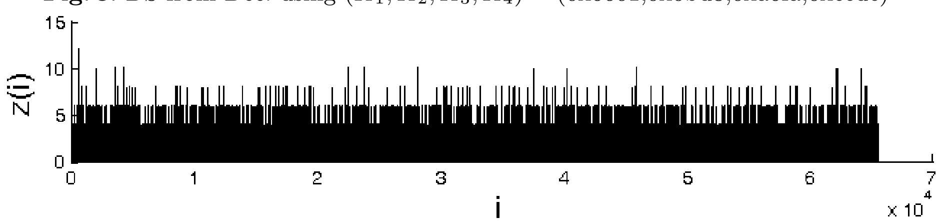

Fig. 9. DS from Dec. using (K_1, K_2, K_3, K_4) = (0x28dc,0xbde1,0x6e3d,0xa56d)

C The Set Φ_0

0x0	0x10	0x20	0x30	0x40	0x50	0x60	0x70	0x80	0x90	0xa0	0xb0	0xc0	0xd0	0xe0	0xf0	0x105	0x115	0x125	0x135	0x145
0x155	0x165	0x175	0x185	0x195	0x1a5	0x1b5	0x1c5	0x1d5	0x1e5	0x1f5	0x20a	0x21a	0x22a	0x23a	0x24a	0x25a	0x26a	0x27a	0x28a	0x29a
0x2aa	0x2ba	0x2ca	0x2da	0x2ea	0x2fa	0x30f	0x31f	0x32f	0x33f	0x34f	0x35f	0x36f	0x37f	0x38f	0x39f	0x3af	0x3bf	0x3cf	0x3df	0x3ef
0x3ff	0x401	0x411	0x421	0x431	0x441	0x451	0x461	0x471	0x481	0x491	0x4a1	0x4b1	0x4c1	0x4d1	0x4e1	0x4f1	0x504	0x514	0x524	0x534
0x544	0x554	0x564	0x574	0x584	0x594	0x5a4	0x5b4	0x5c4	0x5d4	0x5e4	0x5f4	0x60b	0x61b	0x62b	0x63b	0x64b	0x65b	0x66b	0x67b	0x68b
0x69b	0x6ab	0x6bb	0x6cb	0x6db	0x6eb	0x6fb	0x70e	0x71e	0x72e	0x73e	0x74e	0x75e	0x76e	0x77e	0x78e	0x79e	0x7ae	0x7be	0x7ce	0x7de
0x7ee	0x7fe	0x802	0x812	0x822	0x832	0x842	0x852	0x862	0x872	0x882	0x892	0x8a2	0x8b2	0x8c2	0x8d2	0x8e2	0x8f2	0x907	0x917	0x927
0x937	0x947	0x957	0x967	0x977	0x987	0x997	0x9a7	0x9b7	0x9c7	0x9d7	0x9e7	0x9f7	0xa08	0xa18	0xa28	0xa38	0xa48	0xa58	0xa68	0xa78
0xa88	0xa98	0xaa8	0xab8	0xac8	0xad8	0xae8	0xaf8	0xb0d	0xb1d	0xb2d	0xb3d	0xb4d	0xb5d	0xb6d	0xb7d	0xb8d	0xb9d	0xbad	0xbbd	0xbcd
0xbdd	0xbed	0xbfd	0xc03	0xc13	0xc23	0xc33	0xc43	0xc53	0xc63	0xc73	0xc83	0xc93	0xca3	0xcb3	0xcc3	0xcd3	0xce3	0xcf3	0xd06	0xd16
0xd26	0xd36	0xd46	0xd56	0xd66	0xd76	0xd86	0xd96	0xda6	0xdb6	0xdc6	0xdd6	0xde6	0xdf6	0xe09	0xe19	0xe29	0xe39	0xe49	0xe59	0xe69
0xe79	0xe89	0xe99	0xea9	0xeb9	0xec9	0xed9	0xee9	0xef9	0xf0c	0xf1c	0xf2c	0xf3c	0xf4c	0xf5c	0xf6c	0xf7c	0xf8c	0xf9c	0xfac	0xfbc
0xfcc	0xfdc	0xfec	0xffc	0x1001	0x1011	0x1021	0x1031	0x1041	0x1051	0x1061	0x1071	0x1081	0x1091	0x10a1	0x10b1	0x10c1	0x10d1	0x10e1	0x10f1	0x1104
0x1114	0x1124	0x1134	0x1144	0x1154	0x1164	0x1174	0x1184	0x1194	0x11a4	0x11b4	0x11c4	0x11d4	0x11e4	0x11f4	0x120b	0x121b	0x122b	0x123b	0x124b	0x125b
0x126b	0x127b	0x128b	0x129b	0x12ab	0x12bb	0x12cb	0x12db	0x12eb	0x12fb	0x130e	0x131e	0x132e	0x133e	0x134e	0x135e	0x136e	0x137e	0x138e	0x139e	0x13ae
0x13be	0x13ce	0x13de	0x13ee	0x13fe	0x1400	0x1410	0x1420	0x1430	0x1440	0x1450	0x1460	0x1470	0x1480	0x1490	0x14a0	0x14b0	0x14c0	0x14d0	0x14e0	0x14f0
0x1505	0x1515	0x1525	0x1535	0x1545	0x1555	0x1565	0x1575	0x1585	0x1595	0x15a5	0x15b5	0x15c5	0x15d5	0x15e5	0x15f5	0x160a	0x161a	0x162a	0x163a	0x164a
0x165a	0x166a	0x167a	0x168a	0x169a	0x16aa	0x16ba	0x16ca	0x16da	0x16ea	0x16fa	0x170f	0x171f	0x172f	0x173f	0x174f	0x175f	0x176f	0x177f	0x178f	0x179f
0x17af	0x17bf	0x17cf	0x17df	0x17ef	0x17ff	0x1803	0x1813	0x1823	0x1833	0x1843	0x1853	0x1863	0x1873	0x1883	0x1893	0x18a3	0x18b3	0x18c3	0x18d3	0x18e3
0x18f3	0x1906	0x1916	0x1926	0x1936	0x1946	0x1956	0x1966	0x1976	0x1986	0x1996	0x19a6	0x19b6	0x19c6	0x19d6	0x19e6	0x19f6	0x1a09	0x1a19	0x1a29	0x1a39
0x1a49	0x1a59	0x1a69	0x1a79	0x1a89	0x1a99	0x1aa9	0x1ab9	0x1ac9	0x1ad9	0x1ae9	0x1af9	0x1b0c	0x1b1c	0x1b2c	0x1b3c	0x1b4c	0x1b5c	0x1b6c	0x1b7c	0x1b8c
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0x2228	0x2238	0x2248	0x2258	0x2268	0x2278	0x2288	0x2298	0x22a8	0x22b8	0x22c8	0x22d8	0x22e8	0x22f8	0x230d	0x231d	0x232d	0x233d	0x234d	0x235d	0x236d
0x237d	0x238d	0x239d	0x23ad	0x23bd	0x23cd	0x23dd	0x23ed	0x23fd	0x2403	0x2413	0x2423	0x2433	0x2443	0x2453	0x2463	0x2473	0x2483	0x2493	0x24a3	0x24b3
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0x4051	0x4061	0x4071	0x4081	0x4091	0x40a1	0x40b1	0x40c1	0x40d1	0x40e1	0x40f1	0x4104	0x4114	0x4124	0x4134	0x4144	0x4154	0x4164	0x4174	0x4184	0x4194
0x41a4	0x41b4	0x41c4	0x41d4	0x41e4	0x41f4	0x420b	0x421b	0x422b	0x423b	0x424b	0x425b	0x426b	0x427b	0x428b	0x429b	0x42ab	0x42bb	0x42cb	0x42db	0x42eb
0x42fb	0x430e	0x431e	0x432e	0x433e	0x434e	0x435e	0x436e	0x437e	0x438e	0x439e	0x43ae	0x43be	0x43ce	0x43de	0x43ee	0x43fe	0x4400	0x4410	0x4420	0x4430
0x4440	0x4450	0x4460	0x4470	0x4480	0x4490	0x44a0	0x44b0	0x44c0	0x44d0	0x44e0	0x44f0	0x4505	0x4515	0x4525	0x4535	0x4545	0x4555	0x4565	0x4575	0x4585
0x4595	0x45a5	0x45b5	0x45c5	0x45d5	0x45e5	0x45f5	0x460a	0x461a	0x462a	0x463a	0x464a	0x465a	0x466a	0x467a	0x468a	0x469a	0x46aa	0x46ba	0x46ca	0x46da
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0x5554	0x5564	0x5574	0x5584	0x5594	0x55a4	0x55b4	0x55c4	0x55d4	0x55e4	0x55f4	0x560b	0x561b	0x562b	0x563b	0x564b	0x565b	0x566b	0x567b	0x568b	0x569b
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0x57fe	0x5802	0x5812	0x5822	0x5832	0x5842	0x5852	0x5862	0x5872	0x5882	0x5892	0x58a2	0x58b2	0x58c2	0x58d2	0x58e2	0x58f2	0x5907	0x5917	0x5927	0x5937
0x5947	0x5957	0x5967	0x5977	0x5987	0x5997	0x59a7	0x59b7	0x59c7	0x59d7	0x59e7	0x59f7	0x5a08	0x5a18	0x5a28	0x5a38	0x5a48	0x5a58	0x5a68	0x5a78	0x5a88
0x5a98	0x5aa8	0x5ab8	0x5ac8	0x5ad8	0x5ae8	0x5af8	0x5b0d	0x5b1d	0x5b2d	0x5b3d	0x5b4d	0x5b5d	0x5b6d	0x5b7d	0x5b8d	0x5b9d	0x5bad	0x5bbd	0x5bcd	0x5bdd
0x5bed	0x5bfd	0x5c03	0x5c13	0x5c23	0x5c33	0x5c43	0x5c53	0x5c63	0x5c73	0x5c83	0x5c93	0x5ca3	0x5cb3	0x5cc3	0x5cd3	0x5ce3	0x5cf3	0x5d06	0x5d16	0x5d26
0x5d36	0x5d46	0x5d56	0x5d66	0x5d76	0x5d86	0x5d96	0x5da6	0x5db6	0x5dc6	0x5dd6	0x5de6	0x5df6	0x5e09	0x5e19	0x5e29	0x5e39	0x5e49	0x5e59	0x5e69	0x5e79
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0x6126	0x6136	0x6146	0x6156	0x6166	0x6176	0x6186	0x6196	0x61a6	0x61b6	0x61c6	0x61d6	0x61e6	0x61f6	0x6209	0x6219	0x6229	0x6239	0x6249	0x6259	0x6269
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0xb28b	0xb29b	0xb2ab	0xb2bb	0xb2cb	0xb2db	0xb2eb	0xb2fb	0xb30e	0xb31e	0xb32e	0xb33e	0xb34e	0xb35e	0xb36e	0xb37e	0xb38e	0xb39e	0xb3ae	0xb3be	0xb3ce
0xb3de	0xb3ee	0xb3fe	0xb400	0xb410	0xb420	0xb430	0xb440	0xb450	0xb460	0xb470	0xb480	0xb490	0xb4a0	0xb4b0	0xb4c0	0xb4d0	0xb4e0	0xb4f0	0xb505	0xb515
0xb525	0xb535	0xb545	0xb555	0xb565	0xb575	0xb585	0xb595	0xb5a5	0xb5b5	0xb5c5	0xb5d5	0xb5e5	0xb5f5	0xb60a	0xb61a	0xb62a	0xb63a	0xb64a	0xb65a	0xb66a
0xb67a	0xb68a	0xb69a	0xb6aa	0xb6ba	0xb6ca	0xb6da	0xb6ea	0xb6fa	0xb70f	0xb71f	0xb72f	0xb73f	0xb74f	0xb75f	0xb76f	0xb77f	0xb78f	0xb79f	0xb7af	0xb7bf
0xb7cf	0xb7df	0xb7ef	0xb7ff	0xb803	0xb813	0xb823	0xb833	0xb843	0xb853	0xb863	0xb873	0xb883	0xb893	0xb8a3	0xb8b3	0xb8c3	0xb8d3	0xb8e3	0xb8f3	0xb906
0xb916	0xb926	0xb936	0xb946	0xb956	0xb966	0xb976	0xb986	0xb996	0xb9a6	0xb9b6	0xb9c6	0xb9d6	0xb9e6	0xb9f6	0xba09	0xba19	0xba29	0xba39	0xba49	0xba59
0xba69	0xba79	0xba89	0xba99	0xbaa9	0xbab9	0xbac9	0xbad9	0xbae9	0xbaf9	0xbb0c	0xbb1c	0xbb2c	0xbb3c	0xbb4c	0xbb5c	0xbb6c	0xbb7c	0xbb8c	0xbb9c	0xbbac
0xbbbc	0xbbcc	0xbbdc	0xbbec	0xbbfc	0xbc02	0xbc12	0xbc22	0xbc32	0xbc42	0xbc52	0xbc62	0xbc72	0xbc82	0xbc92	0xbca2	0xbcb2	0xbcc2	0xbcd2	0xbce2	0xbcf2
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0xbe58	0xbe68	0xbe78	0xbe88	0xbe98	0xbea8	0xbeb8	0xbec8	0xbed8	0xbee8	0xbef8	0xbf0d	0xbf1d	0xbf2d	0xbf3d	0xbf4d	0xbf5d	0xbf6d	0xbf7d	0xbf8d	0xbf9d
0xbfad	0xbfbd	0xbfcd	0xbfdd	0xbfed	0xbffd	0xc003	0xc013	0xc023	0xc033	0xc043	0xc053	0xc063	0xc073	0xc083	0xc093	0xc0a3	0xc0b3	0xc0c3	0xc0d3	0xc0e3
0xc0f3	0xc106	0xc116	0xc126	0xc136	0xc146	0xc156	0xc166	0xc176	0xc186	0xc196	0xc1a6	0xc1b6	0xc1c6	0xc1d6	0xc1e6	0xc1f6	0xc209	0xc219	0xc229	0xc239
0xc249	0xc259	0xc269	0xc279	0xc289	0xc299	0xc2a9	0xc2b9	0xc2c9	0xc2d9	0xc2e9	0xc2f9	0xc30c	0xc31c	0xc32c	0xc33c	0xc34c	0xc35c	0xc36c	0xc37c	0xc38c
0xc39c	0xc3ac	0xc3bc	0xc3cc	0xc3dc	0xc3ec	0xc3fc	0xc402	0xc412	0xc422	0xc432	0xc442	0xc452	0xc462	0xc472	0xc482	0xc492	0xc4a2	0xc4b2	0xc4c2	0xc4d2
0xc4e2	0xc4f2	0xc507	0xc517	0xc527	0xc537	0xc547	0xc557	0xc567	0xc577	0xc587	0xc597	0xc5a7	0xc5b7	0xc5c7	0xc5d7	0xc5e7	0xc5f7	0xc608	0xc618	0xc628
0xc638	0xc648	0xc658	0xc668	0xc678	0xc688	0xc698	0xc6a8	0xc6b8	0xc6c8	0xc6d8	0xc6e8	0xc6f8	0xc70d	0xc71d	0xc72d	0xc73d	0xc74d	0xc75d	0xc76d	0xc77d
0xc78d	0xc79d	0xc7ad	0xc7bd	0xc7cd	0xc7dd	0xc7ed	0xc7fd	0xc801	0xc811	0xc821	0xc831	0xc841	0xc851	0xc861	0xc871	0xc881	0xc891	0xc8a1	0xc8b1	0xc8c1
0xc8d1	0xc8e1	0xc8f1	0xc904	0xc914	0xc924	0xc934	0xc944	0xc954	0xc964	0xc974	0xc984	0xc994	0xc9a4	0xc9b4	0xc9c4	0xc9d4	0xc9e4	0xc9f4	0xca0b	0xca1b
0xca2b	0xca3b	0xca4b	0xca5b	0xca6b	0xca7b	0xca8b	0xca9b	0xcaab	0xcabb	0xcacb	0xcadb	0xcaeb	0xcafb	0xcb0e	0xcb1e	0xcb2e	0xcb3e	0xcb4e	0xcb5e	0xcb6e
0xcb7e	0xcb8e	0xcb9e	0xcbae	0xcbbe	0xcbce	0xcbde	0xcbee	0xcbfe	0xcc00	0xcc10	0xcc20	0xcc30	0xcc40	0xcc50	0xcc60	0xcc70	0xcc80	0xcc90	0xcca0	0xccb0
0xccc0	0xccd0	0xcce0	0xccf0	0xcd05	0xcd15	0xcd25	0xcd35	0xcd45	0xcd55	0xcd65	0xcd75	0xcd85	0xcd95	0xcda5	0xcdb5	0xcdc5	0xcdd5	0xcde5	0xcdf5	0xce0a
0xce1a	0xce2a	0xce3a	0xce4a	0xce5a	0xce6a	0xce7a	0xce8a	0xce9a	0xceaa	0xceba	0xceca	0xceda	0xceea	0xcefa	0xcf0f	0xcf1f	0xcf2f	0xcf3f	0xcf4f	0xcf5f
0xcf6f	0xcf7f	0xcf8f	0xcf9f	0xcfaf	0xcfbf	0xcfcf	0xcfdf	0xcfef	0xcfff	0xd002	0xd012	0xd022	0xd032	0xd042	0xd052	0xd062	0xd072	0xd082	0xd092	0xd0a2
0xd0b2	0xd0c2	0xd0d2	0xd0e2	0xd0f2	0xd107	0xd117	0xd127	0xd137	0xd147	0xd157	0xd167	0xd177	0xd187	0xd197	0xd1a7	0xd1b7	0xd1c7	0xd1d7	0xd1e7	0xd1f7
0xd208	0xd218	0xd228	0xd238	0xd248	0xd258	0xd268	0xd278	0xd288	0xd298	0xd2a8	0xd2b8	0xd2c8	0xd2d8	0xd2e8	0xd2f8	0xd30d	0xd31d	0xd32d	0xd33d	0xd34d
0xd35d	0xd36d	0xd37d	0xd38d	0xd39d	0xd3ad	0xd3bd	0xd3cd	0xd3dd	0xd3ed	0xd3fd	0xd403	0xd413	0xd423	0xd433	0xd443	0xd453	0xd463	0xd473	0xd483	0xd493
0xd4a3	0xd4b3	0xd4c3	0xd4d3	0xd4e3	0xd4f3	0xd506	0xd516	0xd526	0xd536	0xd546	0xd556	0xd566	0xd576	0xd586	0xd596	0xd5a6	0xd5b6	0xd5c6	0xd5d6	0xd5e6
0xd5f6	0xd609	0xd619	0xd629	0xd639	0xd649	0xd659	0xd669	0xd679	0xd689	0xd699	0xd6a9	0xd6b9	0xd6c9	0xd6d9	0xd6e9	0xd6f9	0xd70c	0xd71c	0xd72c	0xd73c
0xd74c	0xd75c	0xd76c	0xd77c	0xd78c	0xd79c	0xd7ac	0xd7bc	0xd7cc	0xd7dc	0xd7ec	0xd7fc	0xd800	0xd810	0xd820	0xd830	0xd840	0xd850	0xd860	0xd870	0xd880
0xd890	0xd8a0	0xd8b0	0xd8c0	0xd8d0	0xd8e0	0xd8f0	0xd905	0xd915	0xd925	0xd935	0xd945	0xd955	0xd965	0xd975	0xd985	0xd995	0xd9a5	0xd9b5	0xd9c5	0xd9d5
0xd9e5	0xd9f5	0xda0a	0xda1a	0xda2a	0xda3a	0xda4a	0xda5a	0xda6a	0xda7a	0xda8a	0xda9a	0xdaaa	0xdaba	0xdaca	0xdada	0xdaea	0xdafa	0xdb0f	0xdb1f	0xdb2f
0xdb3f	0xdb4f	0xdb5f	0xdb6f	0xdb7f	0xdb8f	0xdb9f	0xdbaf	0xdbbf	0xdbcf	0xdbdf	0xdbef	0xdbff	0xdc01	0xdc11	0xdc21	0xdc31	0xdc41	0xdc51	0xdc61	0xdc71
0xdc81	0xdc91	0xdca1	0xdcb1	0xdcc1	0xdcd1	0xdce1	0xdcf1	0xdd04	0xdd14	0xdd24	0xdd34	0xdd44	0xdd54	0xdd64	0xdd74	0xdd84	0xdd94	0xdda4	0xddb4	0xddc4
0xddd4	0xdde4	0xddf4	0xde0b	0xde1b	0xde2b	0xde3b	0xde4b	0xde5b	0xde6b	0xde7b	0xde8b	0xde9b	0xdeab	0xdebb	0xdecb	0xdedb	0xdeeb	0xdefb	0xdf0e	0xdf1e
0xdf2e	0xdf3e	0xdf4e	0xdf5e	0xdf6e	0xdf7e	0xdf8e	0xdf9e	0xdfae	0xdfbe	0xdfce	0xdfde	0xdfee	0xdffe	0xe001	0xe011	0xe021	0xe031	0xe041	0xe051	0xe061
0xe071	0xe081	0xe091	0xe0a1	0xe0b1	0xe0c1	0xe0d1	0xe0e1	0xe0f1	0xe104	0xe114	0xe124	0xe134	0xe144	0xe154	0xe164	0xe174	0xe184	0xe194	0xe1a4	0xe1b4
0xe1c4	0xe1d4	0xe1e4	0xe1f4	0xe20b	0xe21b	0xe22b	0xe23b	0xe24b	0xe25b	0xe26b	0xe27b	0xe28b	0xe29b	0xe2ab	0xe2bb	0xe2cb	0xe2db	0xe2eb	0xe2fb	0xe30e
0xe31e	0xe32e	0xe33e	0xe34e	0xe35e	0xe36e	0xe37e	0xe38e	0xe39e	0xe3ae	0xe3be	0xe3ce	0xe3de	0xe3ee	0xe3fe	0xe400	0xe410	0xe420	0xe430	0xe440	0xe450
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0xe5b5	0xe5c5	0xe5d5	0xe5e5	0xe5f5	0xe60a	0xe61a	0xe62a	0xe63a	0xe64a	0xe65a	0xe66a	0xe67a	0xe68a	0xe69a	0xe6aa	0xe6ba	0xe6ca	0xe6da	0xe6ea	0xe6fa
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0xe853	0xe863	0xe873	0xe883	0xe893	0xe8a3	0xe8b3	0xe8c3	0xe8d3	0xe8e3	0xe8f3	0xe906	0xe916	0xe926	0xe936	0xe946	0xe956	0xe966	0xe976	0xe986	0xe996
0xe9a6	0xe9b6	0xe9c6	0xe9d6	0xe9e6	0xe9f6	0xea09	0xea19	0xea29	0xea39	0xea49	0xea59	0xea69	0xea79	0xea89	0xea99	0xeaa9	0xeab9	0xeac9	0xead9	0xeae9
0xeaf9	0xeb0c	0xeb1c	0xeb2c	0xeb3c	0xeb4c	0xeb5c	0xeb6c	0xeb7c	0xeb8c	0xeb9c	0xebac	0xebbc	0xebcc	0xebdc	0xebec	0xebfc	0xec02	0xec12	0xec22	0xec32
0xec42	0xec52	0xec62	0xec72	0xec82	0xec92	0xeca2	0xecb2	0xecc2	0xecd2	0xece2	0xecf2	0xed07	0xed17	0xed27	0xed37	0xed47	0xed57	0xed67	0xed77	0xed87
0xed97	0xeda7	0xedb7	0xedc7	0xedd7	0xede7	0xedf7	0xee08	0xee18	0xee28	0xee38	0xee48	0xee58	0xee68	0xee78	0xee88	0xee98	0xeea8	0xeeb8	0xeec8	0xeed8
0xeee8	0xeef8	0xef0d	0xef1d	0xef2d	0xef3d	0xef4d	0xef5d	0xef6d	0xef7d	0xef8d	0xef9d	0xefad	0xefbd	0xefcd	0xefdd	0xefed	0xeffd	0xf000	0xf010	0xf020
0xf030	0xf040	0xf050	0xf060	0xf070	0xf080	0xf090	0xf0a0	0xf0b0	0xf0c0	0xf0d0	0xf0e0	0xf0f0	0xf105	0xf115	0xf125	0xf135	0xf145	0xf155	0xf165	0xf175
0xf185	0xf195	0xf1a5	0xf1b5	0xf1c5	0xf1d5	0xf1e5	0xf1f5	0xf20a	0xf21a	0xf22a	0xf23a	0xf24a	0xf25a	0xf26a	0xf27a	0xf28a	0xf29a	0xf2aa	0xf2ba	0xf2ca
0xf2da	0xf2ea	0xf2fa	0xf30f	0xf31f	0xf32f	0xf33f	0xf34f	0xf35f	0xf36f	0xf37f	0xf38f	0xf39f	0xf3af	0xf3bf	0xf3cf	0xf3df	0xf3ef	0xf3ff	0xf401	0xf411
0xf421	0xf431	0xf441	0xf451	0xf461	0xf471	0xf481	0xf491	0xf4a1	0xf4b1	0xf4c1	0xf4d1	0xf4e1	0xf4f1	0xf504	0xf514	0xf524	0xf534	0xf544	0xf554	0xf564
0xf574	0xf584	0xf594	0xf5a4	0xf5b4	0xf5c4	0xf5d4	0xf5e4	0xf5f4	0xf60b	0xf61b	0xf62b	0xf63b	0xf64b	0xf65b	0xf66b	0xf67b	0xf68b	0xf69b	0xf6ab	0xf6bb
0xf6cb	0xf6db	0xf6eb	0xf6fb	0xf70e	0xf71e	0xf72e	0xf73e	0xf74e	0xf75e	0xf76e	0xf77e	0xf78e	0xf79e	0xf7ae	0xf7be	0xf7ce	0xf7de	0xf7ee	0xf7fe	0xf802
0xf812	0xf822	0xf832	0xf842	0xf852	0xf862	0xf872	0xf882	0xf892	0xf8a2	0xf8b2	0xf8c2	0xf8d2	0xf8e2	0xf8f2	0xf907	0xf917	0xf927	0xf937	0xf947	0xf957
0xf967	0xf977	0xf987	0xf997	0xf9a7	0xf9b7	0xf9c7	0xf9d7	0xf9e7	0xf9f7	0xfa08	0xfa18	0xfa28	0xfa38	0xfa48	0xfa58	0xfa68	0xfa78	0xfa88	0xfa98	0xfaa8
0xfab8	0xfac8	0xfad8	0xfae8	0xfaf8	0xfb0d	0xfb1d	0xfb2d	0xfb3d	0xfb4d	0xfb5d	0xfb6d	0xfb7d	0xfb8d	0xfb9d	0xfbad	0xfbbd	0xfbcd	0xfbdd	0xfbed	0xfbfd
0xfc03	0xfc13	0xfc23	0xfc33	0xfc43	0xfc53	0xfc63	0xfc73	0xfc83	0xfc93	0xfca3	0xfcb3	0xfcc3	0xfcd3	0xfce3	0xfcf3	0xfd06	0xfd16	0xfd26	0xfd36	0xfd46
0xfd56	0xfd66	0xfd76	0xfd86	0xfd96	0xfda6	0xfdb6	0xfdc6	0xfdd6	0xfde6	0xfdf6	0xfe09	0xfe19	0xfe29	0xfe39	0xfe49	0xfe59	0xfe69	0xfe79	0xfe89	0xfe99
0xfea9	0xfeb9	0xfec9	0xfed9	0xfee9	0xfef9	0xff0c	0xff1c	0xff2c	0xff3c	0xff4c	0xff5c	0xff6c	0xff7c	0xff8c	0xff9c	0xffac	0xffbc	0xffcc	0xffdc	0xffec
0xfffc																				

D Side-channel Injection Attack to Realize $\boldsymbol{\Delta R}$

As mentioned in the Proof of Theorem 1, condition (A) gives us the same differential sequence as the following condition does,

$$\begin{aligned}\Delta K &= (K_1, ..., K_8) + (K'_1, ..., K'_8) = (0, 0, 0, 0, 0, 0, 0, 0)\\ \Delta P &= P_1 + P'_{i'} = 0\\ \Delta R &= (R_1, ..., R_8) + (R'_1, ..., R'_8) = (0, 0, 0, H, 0, 0, 0, 0),\end{aligned}$$

Therefore, to create the difference between R_4 and R'_4 (or between $f^{-1}(R_2 \boxminus u_1)$ and $f^{-1}(R'_2 \boxminus u'_1)$), one obvious way is to start with two instances initialized with the same IVs and keys and then mount side-channel injection attack, where the attacker simply injects H to the victim register, e.g., R_4 or $f^{-1}(R_2 \boxminus u_1)$, of one instance any time before the execution of the last round of encryption/decryption. Note that the *preparation through injection* gives the attacker no time/memory penalty, i.e., the overall time/memory complexity of the attack is dominated by that of the *key recovery phase*.