Anonymous Credentials Light

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Abstract. We define and propose an efficient and provably secure construction of blind signatures with attributes. Prior notions of blind signatures did not yield themselves to the construction of anonymous credential systems, not even if we drop the unlinkability requirement of anonymous credentials. Our new notion in contrast is a convenient building block for anonymous credential systems. The construction we propose is efficient: it requires just a few exponentiations in a prime-order group in which the decisional Diffie-Hellman problem is hard. Thus, for the first time, we give a provably secure construction of anonymous credentials that can work in the elliptic group setting without bilinear pairings. In contrast, prior provably secure constructions were based on the RSA group or on groups with pairings, which made them prohibitively inefficient for mobile devices, RFIDs and smartcards. The only prior efficient construction that could work in such elliptic curve groups, due to Brands, does not have a proof of security.

Keywords: Anonymous credentials, attributes, blind signatures, lightweight devices, private identity management.

1 Introduction

Anonymous credentials, envisioned by David Chaum [12], and first fully realized by Camenisch and Lysyanskaya [8], have proved to be a centrally important building block in privacy-minded identity management systems currently under development [34, 27, 10]. They allow users to prove possession of credentials without revealing any other information about themselves; when such a proof is carried out, it cannot be linked to previous uses of the same credential, or to any other identifying information about the user. Additionally, they give the users the ability to privately obtain credentials. The reason that they have become so popular is that they strictly adhere to data minimization principles [24]: no electronic transaction should require its participants to needlessly reveal private information. These principles are at the core of European privacy standards [28, 2] and also the National Strategy for Trusted Identities in Cyberspace (NSTIC) published by the U.S. government [14].

Efficiency considerations are important when deploying these algorithms in practice. While on a PC, the Camenisch-Lysyanskaya credentials [8] and follow-up work [7, 5, 11, 1] take only a fraction of a second of CPU time, in mobile devices, smartcards and RFID cards, they are not yet practical. The issue is that all existing anonymous credential systems use either the RSA group, or groups that admit bilinear pairings, and the security parameter choice needed to guarantee the required assumptions in these groups makes these systems too expensive for smartcards and mobile devices. Specifically, the original CL credentials require an RSA group with bitlength 2048, and exponentiation in such groups takes about 43 seconds on even the most advanced RFID cards [30]; while a bilinear pairing takes approximately 14 seconds (for a pairing with 128 bits of security) [21] (those results are a highly related to the specific microcontroller the RFID card uses as well as the frequency in which operates).

From the efficiency point of view, therefore, the UProve credential system [27] (based on Brands' work [6], acquired and implemented by Microsoft) seems attractive. UProve does not allow unlinkable reuse of credentials: in order to unlinkably use a credential again, a user must get it reissued. Yet, the credential issuing protocol, as well as the algorithm for verifying a credential, are practical enough for the more advanced end of RFID technology,

since they can work in any group where the discrete logarithm problem is hard, and require just a few exponentiations in this group. We currently have RFID chips that can compute exponentiations in elliptic curve groups in less than a second with security parameters that provide a good level of security [35]. So perhaps one could use these *linkable* anonymous credentials and just have them reissued as many times as needed. However, UProve is not provably secure! No proof of security has been given under any reasonable assumption; and in fact, recently, Baldimtsi and Lysyanskaya [4] showed that all known approaches for proving security in the random oracle model will fail for the Brands blind signature scheme on which UProve is based, under essentially any assumption.

One might ask what happens if, instead of the Brands blind signature, one tries to construct linkable anonymous credentials from a provably secure blind signature scheme. Unfortunately, blind signatures as traditionally defined [32, 31, 23, 3] do not give any consideration to users' attributes. In a blind signature, a user is free to choose whichever message he wants signed. In a credential scheme, a user should be able to get his particular attribute set signed by the signer. The signers can verify that what they are signing is the correct attribute set, even though they cannot see exactly what they are signing. For example, a signer might want to sign a user's secret key, his age, and his citizenship information, and wants to be able to verify that this is indeed what she is signing. Attributes are, in fact, essential: without them, it is impossible to tie a credential to a particular user's identity, and so users would be able to just pool each others' credentials as desired, instead of tying them to a particular secret key. Moreover, giving the user the ability to choose any message means that the content of the signed messages has not been vetted by the signer at all.

Thus, a better question is: how do we extend the notion of a blind signature so that it can efficiently accommodate anonymous credentials — if not full anonymous credentials a-la Camenisch and Lysyanskaya, then at least linkable lightweight anonymous credentials a-la Brands?

In this paper, we define and give an efficient construction of blind signatures *with attributes*. In such a blind signature scheme, the signer and the user both get as input a cryptographic commitment C to the user's attributes; this way, the user can prove that the commitment contains the correct attributes via a separate zero-knowledge proof. As output, the user obtains another, unlinkable, commitment $\tilde{C}$ to the same attributes, and a signature on this commitment and a message of the user's choice. Blindness ensures that, even upon seeing two signatures obtained this way on commitments of his own choice, the signer cannot link a signature to its issuing. Unforgeability ensures that a user cannot produce more signatures than he was issued, and also that the multiset of openings to the input commitments is the same as the multiset of openings to the output commitments. It is easy to see that this is the right building block for linkable anonymous credentials: a user with a particular set of attributes can form a commitment C to these attributes, prove in zero-knowledge that he has committed to the correct attributes, and then obtain a credential by running a blind signature with attributes on input this commitment. He can then prove that he has a credential with the desired attributes by revealing his signature and the output commitment $\tilde{C}$, and proving in zero knowledge that $\tilde{C}$ corresponds to the needed attributes.

For example, suppose that we allow users to obtain and anonymously show age credentials. Then Alice will form a commitment C to her secret key sk and her age age, prove to the signer who serves as an identity validator that she has committed to sk that corresponds to her pk, and to the correct age, and run a blind signature with attributes protocol to obtain a fresh commitment $\tilde{C}$ on the same sk and age, and a signature σ. Then, when she needs to prove her age, she can reveal $(\tilde{C}, \sigma)$ and prove that the age inside commitment $\tilde{C}$ allows her entry to some age-restricted resource, for example a video store that requires viewers to be over 13. If she wants to do it again, she needs to run the blind signature with attributes

protocol with the signer again. She can further anonymously obtain credentials that are connected to her identity: let's say that the video store wants to give her a credential for a free movie; she can obtain it by running a blind signature with attributes with the video store. She will form her input commitment C' by committing to the same sk and age as in $\tilde{C}$ and proving to the video store that she did so; once she runs the protocol with the video store, she receives an output $(\tilde{C}', \sigma)$, which is a credential from the video store (and not from the original signer) on her (sk, age) — even though the video store never saw the public key at all. More interestingly, we can require the free movie to be a single-use credential, by additional clever use of attributes (see Section 4), so that Alice can be traced if she tries to get two free movies using the same single-use credential more than once. Thus we see that blind signatures with attributes are the right building block for linkable anonymous credentials.

In addition to the definition, we give a lightweight construction of blind signatures with attributes; because of the connection to anonymous credentials we call our construction "anonymous credentials light" (ACL). In the random-oracle model, our construction is unlinkable under the decisional Diffie-Hellman assumption, and unforgeable under the discrete-logarithm assumption for sequential composition (we leave extending it to concurrent self-composition as an open problem). Our construction is inspired by the Abe blind signature [3]. It can be instantiated in elliptic-curve groups, in which exponentiation, for the security parameters that are currently considered reasonable, is, in general, almost a hundred times more efficient than in the RSA group; it also does not require expensive pairings (a pairing is also at least ten times the cost of exponentiation); thus it is much more usable on mobile devices than CL anonymous credentials. Specifically, the signing protocol in our ACL construction consists of three phases: registration, preparation and validation. Registration is the most expensive phase, but it need not be repeated for the same user/signer pair should the user need his credential reissued (as long as his attribute set is the same). Preparation and validation require the signer to perform 7 exponentiations, while the user performs 13; verification requires 8 exponentiations. These numbers are essentially the same as Abe's blind signature; UProve requires essentially the same number of exponentiations for signature issue, and 4 for verification. Thus, we get comparable efficiency to the most efficient examples of protocols that either lack essential features (such as Abe's blind signatures) or provable security (such as UProve). See Table 1 for a comparison of the above mentioned schemes.

	Brands [6]		**Abe [3]**		**ACL Scheme**		**CL credentials [7]**			
							Rsa		**Pairings**	
Efficiency	S[1]	U[2]	S	U	S	U	S	U	S	U
Signing[3]	2	13	6	12	7	13	10	8	15	14p
Reveal	7	0	11	1	11	1	NC[4]		5+6p	9+6p
Signature size	6 elem.		9 elem.		9 elem.		~ 0[5]		~ 0[5]	
Blindness	✓		✓		✓		✓		✓	
Provable Unforg.	✗		✓		✓		✓		✓	
Attributes	✓		✗		✓		✓		✓	

[1] Signer, [2] User, [3] In number of exponentiations
[4] Non-comparable, [5] it can be used multiple times

Table 1. Comparison of anonymous credentials schemes

2 Preliminaries

2.1 Basic Assumptions

We start by describing two standard cryptographic assumptions useful in the rest of the paper.

Definition 1 (Discrete Logarithm Assumption). *Let k be the security parameter. Let G_k be a set of groups such that G_k is a group of order q (k-bit prime) and generator g. Then, for every polynomial time algorithm $\mathcal{A}$ it holds that:*

$$Pr[h \leftarrow G; x \leftarrow A(h) : x = \log_g h] \leq \nu(k)$$

where $\nu(k)$ is a negligible function.

Definition 2 (Decisional Diffie-Hellman Assumption [26]). *Let DH be an instance generator that, on input the security parameter 1^k outputs (G, q, g, A, B, C) where G is a cyclic group of order q (k-bit prime) with generator g, and, for randomly chosen $a, b \in \mathbb{Z}_q$, $A = g^a$, $B = g^b$ and $C = g^{ab}$. Let Rand be an instance generator that, on input 1^k outputs (G, q, g, A, B, C) where G,q,g,A and B are as above, while $C = g^c$ for a random $c \in \mathbb{Z}_q$. We assume that DH and Rand are computationally indistinguishable.*

2.2 The OR-proof technique

Let $\mathcal{P}$ be a Σ-protocol for a relation R_L, (h_0, h_1) be a common input to (P, V) and P knows a x such that $h_b, x \in R_L$ for $b \in \{0, 1\}$ (for definitions of witness relations and Σ-protocols please refer to Appendices A, B). An OR-proof protocol $\mathcal{P}_{OR}$ is a Σ-protocol for proving that either $(h_0, x) \in R_L$ or $(h_1, x) \in R_L$ [16][1].

The main idea of an OR-proof is that P will complete two Σ protocols Σ_0, Σ_1, one for h_0 and one for h_1 in such a way that the verifier will not be able to tell for which of the two P knows the corresponding witness. For h_b the prover can actually follow the real Σ protocol while for h_{b-1} he will have to use a simulator M to create his answers. A $\mathcal{P}_{OR}$ protocol works as follows:

1. P computes the first message of Σ_b, a_b, using (h_b, x) as input. P randomly chooses c_{1-b} and runs the simulator M on input (h_{1-b}, c_{1-b}) and receives $(a_{1-b}, c_{1-b}, r_{1-b})$ as an output. Then, P sends a_0, a_1 to V.
2. V chooses a random string e and sends it to P.
3. P sets $c_b = e \oplus c_{b-1}$ and computes the answer r_b to challenge c_b using h_b, a_b, c_b, x as input. He sends c_0, c_1, r_0, r_1 to V.
4. V checks that $e = c_0 \oplus c_1$ and that both (a_0, c_0, r_0) and (a_1, c_1, r_1) are accepting conversations.

Let $R_{OR} = \{((h_0, h_1), x) | (h_0, x) \in R_L \text{ or } (h_1, x) \in R_L\}$. Then:

Theorem 1 ([16]). *The $\mathcal{P}_{OR}$ protocol is a Σ-protocol for R_{OR}. Moreover, for any verifier V^*, the probability distribution of conversations between P and V^*, where x is such that $(h_b, x) \in R_L$, is independent of b.*

[1] This is based on a more general result due to Cramer et. al. [13] where they present a protocol in which the prover demonstrates knowledge of the solution to some subset of n problem instances out of a collection of subsets.

2.3 Generalized Pedersen Commitment

A non-interactive commitment takes as input a message (or set of messages) m and randomness R and outputs a value that, on the one hand, reveals no information about the message but, on the other hand, it is hard to find a (m', R') such that $Commit(m; R) = Commit(m', R')$ but $m \neq m'$ (see[25] for a standard definition and treatment). The Pedersen commitment is a commitment scheme based on the discrete logarithm assumption [29]. Here we give a generalized version that allows a commitment to a set of messages $(L_1, \ldots, L_n)$ The scheme is defined as follows:

1. *Setup*: On input the security parameter 1^k, pick a group G of prime order $q = \Theta(2^k)$ with generators $h_0, h_1, \ldots, h_n$.
2. $Commit(L_1, \ldots, L_n; R) = h_0^R \prod_{i=1}^{n} h_i^{L_i}$; note that $L_i \in \mathbb{Z}_q$.

The Pedersen commitment scheme (constructed from a corresponding Σ-protocol) is information theoretically hiding and computationally binding. It is also important to note that there are efficient zero-knowledge proof protocols for proving that a commitment C is to a particular set of values; or to a set of values that satisfy a rich class of relations [6, 9, 15].

2.4 Generalized Commitment Scheme

Let $Commit_1$ be a commitment scheme that takes as input its parameters $params_1$, n attributes, $(L_1, \ldots, L_n)$, and randomness R_1 and outputs a commitment C_1. Let $Commit_2$ be a commitment scheme that takes as input its parameters $params_2$, an attribute L_0 and randomness R_2 and outputs a commitment C_2. Suppose that another commitment scheme, $Commit$, is a *combination* of these two commitments; i.e., on input C_1 and C_2 it produces a commitment to the combined attributes $(L_0, L_1, \ldots, L_n)$ with combined randomness $R = R_1 + R_2$.

For example, this can be instantiated by a generalized Pedersen commitment scheme: the parameters for the combined scheme are generators $(g_0, \ldots, g_n, h)$, $Commit_1(L_1, \ldots, L_n; R_1) = (\prod_{i=1}^{n} g_i^{L_i}) h^{R_1}$, $Commit_2(L_0; R_2) = g_0^{L_0} h^{R_2}$, and a combined commitment can be obtained either by multiplying together the two component commitments, or by computing it from scratch as $Commit(L_0, L_1, \ldots, L_n; R) = (\prod_{i=0}^{n} g_i^{L_i}) h^R$.

2.5 Blinded Pedersen Commitment Scheme

Here, we note that the Pedersen commitment scheme can be further extended. Let $(h, h_1, \ldots, h_n)$ be the parameters of the Pedersen commitment. Consider an additional parameter $z \in G$, where $z \neq 1$. Let $C = Commit(L_1, \ldots, L_n; R)$. Then the values (z^γ, C^γ) can also be viewed as a commitment to the same $(L_1, \ldots, L_n)$ with randomness (R, γ). Let us define a new commitment scheme, which we will call the *blinded* Pedersen commitment scheme: $Commit^B(L_1, \ldots, L_n; R, \gamma) = (z^\gamma, Commit(L_1, \ldots, L_n; R)^\gamma)$, where $Commit$ is the Pedersen commitment. It is easy to see that this commitment is unconditionally hiding, same as Pedersen. It is also easy to see that it is binding: given (z^γ, C^γ), γ is uniquely defined, and therefore so is C, which is binding. Finally, using well-known Σ-protocols, it is easy to see that the same set of relations that can be proven about values inside a Pedersen commitment can be proven about values inside a blinded Pedersen commitment.

3 Definition of Secure Blind Signatures with Attributes

In this section we are first going to define what a blind signature scheme with attributes is and then we will show how we can construct the first lightweight e-cash that allows for attributes and is still efficient and provable secure using Abe's [3] scheme as a building block. For definitions of standard (without attributes) blind signatures please refer to Appendix D.

Definition 3 (Blind Signatures with attributes). *Let $Commit(x; r)$ be a non-interactive commitment scheme. A blind signature scheme with n attributes, for this commitment scheme, consists of three algorithms: KeyGen, BlindSign, Verify where BlindSign is a protocol between the Signer(S) and the User(U).*

- *$KeyGen(1^k)$: is a probabilistic polynomial time key generation algorithm. It takes a security parameter k as input and outputs a pair (pk, sk) of public and secret keys for the system. sk is the secret key of the Signer.*
- *BlindSign: is an interactive, probabilistic polynomial time protocol between S and U. The public parameters are pk, the public parameters of the commitment scheme and $C = Commit(L_1, \ldots, L_n; R)$ where $(L_1, \ldots, L_n)$ is the set of attributes and R is some randomness. The Signer's private input is sk and User's private input is $(L_1, \ldots, L_n, R)$ and the message, m, that he wishes to have signed. The User's output in the protocol is a pair $(\tilde{R}, \sigma)$, where $\sigma = \sigma(m, \tilde{C})$ is the Signer's signature on $(m, \tilde{C})$, and $\tilde{C} = Commit(L_1, \ldots, L_n; \tilde{R})$; the Signer's output is "completed".*
- *$Verify(PK, m, \tilde{C}, \sigma)$: is the signature verification algorithm; i.e. a deterministic polynomial time algorithm that gets as input the public key, the message, a commitment to the attributes and the blind signature on the message and attributes and checks the validity of the signature σ. If it is valid the algorithm outputs "1", otherwise outputs "0".*

A blind signature scheme with attributes is secure if it is both blind and unforgeable. Blindness is defined in a similar way as in blind signature schemes without attributes: the Signer is unable to view the messages and the attributes he signs (protection for the User). A malicious Signer, $\mathcal{A}$, cannot link a $(m, \tilde{C}', \sigma)$ tuple to any particular execution of the protocol, even if $\mathcal{A}$ chooses $m, L_1, \ldots, L_n$.

Definition 4 (Blindness for blind signatures with attributes). *Let $\mathcal{A}$ be a malicious Signer and $b \in \{0, 1\}$ be a randomly chosen bit which is kept secret from $\mathcal{A}$. $\mathcal{A}$ will try to guess the value b by performing the following steps:*

1. *$(pk, sk) \leftarrow Gen(1^k)$*
2. *$\{m_0, m_1, \overrightarrow{L_0}, \overrightarrow{L_1}, R_0, R_1\} \leftarrow \mathcal{A}(1^k, pk, sk)$ (i.e. $\mathcal{A}$ produces two messages $\{m_0, m_1\}$, polynomial in 1^k, and two attribute vectors $\overrightarrow{L_0}, \overrightarrow{L_1}$ with the corresponding randomness).*
3. *$\mathcal{A}(1^k, pk,\ sk, m_0, m_1, \overrightarrow{L_0}, \overrightarrow{L_1}, R_0, R_1)$ engages in two parallel (and arbitrarily interleaved as desired by $\mathcal{A}$) interactive protocols, the first with $U(pk, \{m_b, \overrightarrow{L_0}, R_0\})$ and the second with $U(pk, \{m_{1-b}, \overrightarrow{L_1}, R_1\})$.*
4. *If neither of the User instances failed, then $\mathcal{A}$ gets two signatures and the corresponding blinded commitments: $\sigma(m_0, \tilde{C}_b), \tilde{C}_b$ and $\sigma(m_1, \tilde{C}_{1-b}), \tilde{C}_{1-b}$.*
5. *$\mathcal{A}$ outputs a bit b'.*

Then the probability, taken over the choice of b, over coin-flips of the key-generation algorithm, the coin-flips of $\mathcal{A}$, and (private) coin-flips of both users (from step 3), that $b' = b$ is at most $\frac{1}{2} + \nu(k)$, where $\nu(k)$ is a negligible function.

We give the definition of one-more unforgeability for the sequential composition case.

Definition 5 (Sequential one-more unforgeability for blind signatures with attributes). *(KeyGen, BlindSign, Verify) is a one-more unforgeable blind signature scheme with respect to Commit if $\forall$ ppt $\mathcal{A}$, the probability that $\mathcal{A}$ wins in the following game is negligible:*

1. *$(pk, sk) \leftarrow Gen(1^k)$*

2. *$\mathcal{A}(pk, C_i, m_i)$ engages in polynomially many (in k) adaptive, sequential interactive protocols with polynomially many copies of $S(pk, sk)$, where $\mathcal{A}$ decides in an adaptive fashion when to stop. Let ℓ be the number of executions, where the Signer output "completed" in the end of the protocol.*
3. *$\mathcal{A}$ outputs a collection $\{(\tilde{C}_1, m_1, \sigma_1), \ldots, (\tilde{C}_j, m_j, \sigma_j)\}$ where $(\tilde{C}_i, m_i, \sigma_i)$ for $1 \leq i \leq j$ are all accepted by $Verify(pk, \tilde{C}_i, m_i, \sigma_i)$, and all $(\tilde{C}_i, m_i)$'s are distinct.*

We say that $\mathcal{A}$ wins the game if either:

1. *$j > \ell$ (i.e $\mathcal{A}$ outputs more $(\tilde{C}, m, \sigma)$ tuples than he received).*
2. *$\mathcal{A}$ opens the sets of commitments $\{C_i\}$ and $\{\tilde{C}_i\}$ and the resulting multisets do not match.*

4 From Blind Signatures with Attributes to Single-Use Credentials

Single-use anonymous credentials ensure that a credential cannot be used more than once: if used more than once, the user's identity can be discovered, and the user can be penalized. This is a generalization of off-line electronic cash [12], in that such a credential can also contain attributes, while e-cash does not necessarily have any connection to a user's identity. Here we construct single-use anonymous credentials from blind signatures with attributes using standard techniques [12, 6, 7].

Recall our definition of generalized commitment schemes $(Commit_1, Commit_2, Commit)$ from Section 2.4. Consider the following construction of a single-use credential with n attributes from a blind signature scheme with $n+1$ attributes. The common inputs to the User and Signer are the Signer's public key for verifying signatures, and a commitment C_1 to the User's attributes $(L_1, \ldots, L_n)$. We assume that the attribute L_1 contains the user's identity, so that learning that attribute will allow one to trace the user. The Signer's private input is its signing key, while the User's private input is the message m that the user wants signed (m will serve as the serial number for this credential, so the user needs to have chosen it at random during the signing protocol) and the opening to C_1, $(L_1, \ldots, L_n; R_1)$. The commitment C_1 corresponds to this user's identity and never changes; each time this user obtains a signature, they input the same C_1.

First, the User forms a commitment $C_2 = Commit_2(L_0; R_2)$ for random (L_0, R_2); this commitment is specific to this credential, and the value L_0 will be used in the double-spending equation, below. Next, the user submits C_2 to the signer and provides a zero-knowledge proof of knowledge of its opening.

Next, the User and the Signer carry out the blind signature protocol relative to the combined commitment scheme $Commit$, whereby the User's output is a fresh commitment $\tilde{C}$ to the values $(L_0, \ldots, L_n)$ and randomness $\tilde{R}$, and the signature σ on $(m, \tilde{C})$.

To use this credential to a verifier V, the User reveals $(m, \tilde{C}, \sigma)$, obtains from V a challenge c, and reveals the double-spending equation $cL_1 + L_0$ (the multiplication and addition is in whatever ring the committed values belong to) and whatever other attributes are needed for the transaction; the User then provides a zero-knowledge proof that these values were revealed correctly, relative to commitment $\tilde{C}$.

To trace a user who spent a single-use credential twice, it is sufficient to examine two double-spending equations for the same credential. If double spending occurred, then the attribute L_1, which encodes the User's identity, will be revealed.

The following theorem is stated informally because we omitted a formal definition of single-use anonymous credentials.

Theorem 2. *(Informal) Let $KeyGen, BlindSign, Verify$ be a blind signature scheme with $n+1$ attributes, associated with the commitment scheme Commit which is a combination of $Commit_1$ and $Commit_2$ as explained above. Then the above construction is a single-use credential scheme with n attributes.*

5 Our Construction: ACL

Let's now describe our proposed construction of a blind signature scheme with attributes, called "Anonymous Credentials Light" (ACL). ACL is based on Abe's blind signature scheme [3] which has been modified appropriately so that it will also allow the users to be able to provably encode attributes in the signatures they get. Abe's scheme is a witness indistinguishable variant of the Schnorr signature scheme where the Signer owns a *real public key* $y = g^x$ and a *tag public key* $z = g^w$. The idea is that the signature can be issued only with the real secret key x but no one can distinguish which of the two (real or tag) secret keys was used (remember the OR-proof technique presented above).

Although the modification seems straightforward, proving the security of the resulting scheme turns out to be challenging as we will see later in the next section.

5.1 Setup

let G be a group of order q and g is a generator of this group. Let $\mathcal{H} : \{0,1\}^* \to \mathbb{Z}_q$ be a hash function. We assume the existence of a trusted party, TP, which chooses G, g and outputs $params = (q, G, g, z, h, h_0, \ldots, h_1)$ where $z, h, h_0, \ldots, h_n \in G$ and n is the maximum number of attributes a User can possibly have. We will use $(h_0, \ldots, h_n)$ as parameters of the generalized Pedersen commitment scheme *Commit*; we will use $(z, h_0, \ldots, h_n)$ as parameters for the blinded Pedersen commitment scheme $Commit^B$.

The Signer picks his secret key $x \in \mathbb{Z}_q$ and computes his public key $y = g^x \bmod q$. y is Signer's *real public key* while z (output by TP) can be used as his *tag public key.*

The Signer selects $h \in_R \langle g \rangle$ and $x \in_R \mathbb{Z}_q$ and computes his real public-key to be $y = g^x \bmod p$. The TP given (p, q, g, h, y) outputs z which is the tag public key of the Signer. The public key of the Signer is (p, q, g, h, y, z), and the private key is x.

5.2 Signature Issuing

The signature issuing protocol will be described in three phases: registration, preparation and validation. The registration phase actually only needs to happen once for each user/set of attributes and also, preparation and validation phases can happen simultaneously (we choose to present them separately in order to describe the construction in a more modular way).

The signature issuing protocol is basically a $\mathcal{P}_{OR}$-protocol for proving knowledge of one of the following:

- y-side: proof of knowledge x of $y = g^x$
- z-side: proof of knowledge (w_1, w_2) of $z_1 = g^{w_1},\ z_2 = h^{w_2}$ (where z_1, z_2 are the so called "one-time" tag keys that the signer creates).

Notice that the z-side witness in not known to the Signer, so the z-side proof will be done by simulation following the OR-proof technique paradigm described in Section 2.2.

Registration The User's input includes the system parameters *params*, the signer's real public key y, the message m to be signed and $(L_1, \ldots, L_n; R)$ where $L_1, \ldots, L_n$ is a set of attributes and R is some randomness. The Signer also gets as input the system parameters *params*, a commitment $C = Commit(L_1, \ldots, L_n; R)$ and his secret key x. During registration the User and the Signer carry out a standard interactive zero-knowledge proof of knowledge protocol where the User creates a proof π_1 to convince the Signer that he knows an opening of the commitment C.

Preparation The Signer prepares z_1 and z_2 (the z-side proof). He first picks $rnd \in \mathbb{Z}_q$ and creates the "one-time" tag keys: $z_1 = Cg^{rnd}$ and $z_2 = z/z_1$. The Signer sends rnd to the User in order to convince him that $\log_g z_1$ is not known to him. The User computes himself $z_1 = Cg^{rnd}$ and then picks $\gamma \in \mathbb{Z}_q^*$ and blinds z, z_1, z_2 into $\zeta = z^\gamma$, $\zeta = z^\gamma$, $\zeta_1 = z_1^\gamma$ so that $\log_z z_1 = \log_\zeta \zeta_1$ holds. Finally, the User picks $\tau \in \mathbb{Z}_q$ and computes $\eta = z^\tau$ (this will serve as an element of an additional Schnorr signature that proves knowledge of γ such that $\zeta = z^\gamma$).

Validation In this phase two Σ protocols are going to take place and combined according to the OR-proof. By (a, c, r) we are going to denote the transcript of Σ_y and by (a', c', r') we will denote the transcript of Σ_z. The Signer will compute Σ_z himself and Σ_y in interaction with the user.

1. The Signer begins by creating a for Σ_y by picking a random $u \in \mathbb{Z}_q$ and computing $a = g^u$. Then, for Σ_z, following the OR-proof he picks random $c' \in \mathbb{Z}_q$ and $r' = \{r'_1, r'_2 \in \mathbb{Z}_q\}$. and sets $a'_1 = g^{r'_1} z_1^{c'}$ and $a'_2 = h^{r'_2} z_2^{c'}$. Finally he sends to the User $a, a' = \{a'_1, a'_2\}$.
2. The User checks whether $a, a'_1, a'_2 \in G$ and then picks blinding factors $t_1, t_2, t_3, t_4, t_5 \in_R \mathbb{Z}_q$ and blinds a into $\alpha = ag^{t_1} y^{t_2}$ and a'_1, a'_2 into $\alpha'_1 = a_1^{'\gamma} g^{t_3} \zeta_1^{t_4}$ and $\alpha'_2 = a_2^{'\gamma} h^{t_5} \zeta_2^{t_4}$. We denote $\alpha' = \{\alpha'_1, \alpha'_2\}$. Then, computes $\varepsilon = \mathcal{H}(\zeta, \zeta_1, \alpha, \alpha', \eta, m)$ where m is the message to be signed. The User sends to the Signer $e = (\varepsilon - t_2 - t_4) \bmod q$.
3. The Signer, according to the OR-proof technique, computes $c = e - c' \bmod q$ and $r = u - cx \bmod q$. Then, sends (c, r, c', r') to the User.
4. Finally, the User "unblinds" the received values and gets: $\rho = r + t_1 \bmod q$, $\omega = c + t_2 \bmod q$, $\rho'_1 = \gamma r'_1 + t_3 \bmod q$, $\rho'_2 = \gamma r'_2 + t_5 \bmod q$, $\omega' = c' + t_4 \bmod q$, $\mu = \tau - \omega' \gamma \bmod q$.

A signature is a 8-tuple $\sigma = (m, (\zeta, \zeta_1, \rho, \omega, \rho' = \{\rho'_1, \rho'_2\}, \omega', \mu))$ where ζ_1 encodes the attributes of the User ((ζ, ζ_1) corresponds to $\tilde{C}$, which is a blinded Pedersen commitment to $(L_1, \ldots, L_n)$ with randomness (R, γ)).

The complete signature issuing protocol is also described in Appendix E.

Verification A signature tuple (m, ζ_1, σ) (where ζ_1 corresponds to $\tilde{C}$) verifies if $\zeta \neq 1$ and

$$\omega + \omega' = \mathcal{H}(\zeta, \zeta_1, g^\rho y^\omega, g^{\rho'_1} \zeta_1^{\omega'}, h^{\rho'_2} \zeta_2^{\omega'}, z^\mu \zeta^{\omega'}, m) \bmod q.$$

5.3 Discussion

As briefly mentioned in the description of our construction there are certain "tricks" we can do to improve its efficiency. First of all notice that the *Registration* phase only needs to happen once for each User and a corresponding set of attributes. The Signer stores the attribute commitment C together with some identification information asked from the User (e.g. passport, ID).Then, it is sufficient if every time in the beginning of the *Signature Issuing* protocol the User identifies himself to the Signer in order to prove ownership of his account. Moreover, notice that *Preparation* and *Validation* phases can be combined and executed simultaneously. As a result, the whole signing protocol consists of three rounds.

Let's now discuss the differences between our ACL construction and the blind signature protocol described by Abe [3]. The basic advantage of ACL is that allows for the encoding of users' attributes in the signatures they receive from the Signer. In order for this to happen we need the *Registration* phase during which the User commits to his set of attributes. Those attributes are then encoded in the signature in ζ_1. The crucial difference from Abe's approach is that z_1 can no longer be computed as the result of some hash function given a random input rnd. In ACL z_1 needs to include the attribute commitment C so $z_1 = Cg^{rnd}$ (we need the g^{rnd} factor so that two different signature issuings with the same user cannot be linked to

each other). The fact that z_1 is no longer the result of a hash function $\mathcal{H}'$ makes our security analysis challenging. We can no longer define $\mathcal{H}'$ so that it will return the output we need, instead we will have to make use of ZK extractors. The formal analysis follows in the next section.

6 Proof of Security

The correctness of the scheme is straightforward, see Appendix F.

6.1 Blindness

Theorem 3. *The proposed scheme satisfies blindness under the Decisional Diffie-Hellman assumption in the random oracle model.*

Proof. We wish to show that, when interacting with a challenger as described in the definition of blindness, the adversary cannot guess which signature is which (i.e. the adversary should not be able to tell which signature corresponds to each instance and each attribute set). We distinguish between two types of signatures that a challenger might output: a signature $\sigma = (m, (\zeta, \zeta_1, \rho, \omega, \rho'_1, \rho'_2, \omega', \mu))$ is *correct* for a particular interaction with the signer if there exists a value γ such that $\zeta = z^\gamma$ and $\zeta_1 = z_1^\gamma$ where z_1 is the value corresponding to this interaction with the signer, and the signature verifies. A signature is *fake* if no such γ exists. Note that it is easy for a challenger to generate both kinds of signatures: a correct signature can simply be output by correctly following the user's side of the protocol. With control of the random oracle, a fake signature is computed as follows: first, the challenger picks ζ and ζ_1 at random from G, and let $\zeta_2 = \zeta/\zeta_1$. Next, it picks random $\rho, \omega, \rho'_1, \rho'_2, \omega', \mu$ from $\mathbb{Z}_q$ and sets the random oracle such that $\omega + \omega' = \mathcal{H}(\zeta, \zeta_1, g^\rho y^\omega, g^{\rho'_1}\zeta_1^{\omega'}, h^{\rho'_2}\zeta_2^{\omega'}, z^\mu \zeta^{\omega'}, m)$, which ensures that the fake signature verifies.

We will prove the theorem by a hybrid argument. Consider the following three games in which the adversarial signer engages in two signature issuing instances with a challenger who outputs two signatures:

1. `Real`: The challenger outputs two correct signatures, σ_1, σ_2, in random order by honestly following the protocol. A correct signature is one that is honestly generated by interacting with the signer. Thus, in a correct signature the user blinds z, z_1 by the same γ so that $\log_z(z_1) = \log_\zeta(\zeta_1)$.
2. `Hybrid`: The challenger's output consists of a randomly picked correct signature and a fake signature, in random order. In a fake signature the challenger doesn't use the same random γ to blind z and z_1 so that $\log_z(z_1) \neq \log_\zeta(\zeta_1)$ except for negligible probability. Above, we explained how a fake signature can be generated, assuming that the challenger controls the random oracle.
3. `Fake`: The challenger outputs two fake signatures.

What we need to prove is that `Real`≈`Hybrid` and then that `Hybrid`≈`Fake` (we show this in Appendix G). Then, it holds that `Real` ≈ `Fake`. This proves the blindness of the ACL scheme since the view of the adversary is indistinguishable no matter if he receives two correct or two fake signatures. □

6.2 Unforgeability

Theorem 4. *The signature issuing protocol is $(\ell, \ell{+}1)-$unforgeable for polynomially bounded ℓ if the discrete logarithm problem is intractable and $\mathcal{H}$ is a random oracle.*

Outline of the proof Our proof follows Abe's results [3]. We first discuss the witness indistinguishability of the protocol (Lemma 1), which allows us to to simulate the Signer with either y-side or z-side witness(es) to extract the witness of the other side. Then, in Lemma 2 we prove that in order for the User to get a valid signature, he has to blind (z, z_1) into (ζ, ζ_1) only in such a way that $\log_z \zeta = \log_{z_1} \zeta_1$. In Lemma 3 we prove that is infeasible to create a valid signature without engaging in the issuing protocol with the legitimate Signer. From Lemmas 2 and 3 we see that if a User engages in the signature issuing protocol ℓ times and outputs $\ell + 1$ signatures, then, there exist at least two valid signatures linked to a particular run of the issuing protocol. Finally, it needs to be proven that a forger who manages to produce two signatures from a single protocol run can be used to solve the discrete logarithm problem.

We will denote the i-th execution of the issuing protocol by run_i. Recall that a transcript of run_i of the issuing protocol contains values $(z_{1,i}, z_{2,i})$; by the z-side witness of run_i we denote $(w_{1,i}, w_{2,i})$ such that $z_{1,i} = g^{w_{1,i}}$ and $z_{2,i} = h^{w_{2,i}}$.

Consider an alternative signing algorithm that, instead of using the y-side witness in the issuing protocol, uses the z-side witness. Let $params = (q, G, h, z, \{h_j\})$ be the public parameters, and y be the public key of the Signer. Suppose that the registration and the preparation phases of the signing protocol for run_i resulted in the signer and the user setting up the values $z_{1,i} = g^{w_{1,i}}$, $z_{2,i} = h^{w_{2,i}}$. The alternative signing algorithm takes as input the public parameters $params$, the public key y, the preparation phase output (z_1, z_2) and the values $(w_{1,i}, w_{2,i})$ instead of the secret key x, and works as follows:

1. Generate $c_i, r_i \in_U \mathbb{Z}_q$ and set $a_i := g^{r_i} y^{c_i}$.
2. Compute $a'_{1,i} := g^{u_{1,i}}$ and $a'_{2,i} := h^{u_{2,i}}$ with $u_{1,i}, u_{2,i} \in_U \mathbb{Z}_q$.
3. Sends $a_i, a'_{1,i}, a'_{2,i}$ to the user.
4. Given e_i from user, compute $c'_i := e_i - c_i \bmod q$, $r'_{1,i} := u_{1,i} - c'_i w_{1,i} \bmod q$ and $r'_{2,i} := u_{2,i} - c'_i w_{2,i} \bmod q$.
5. Send $r_i, c_i, r'_{1,i}, r'_{2,i}, c'_i$ to the user.

Definition 6. *The signing protocol described above is called* the z-side signer.

Lemma 1. *The signer's output is perfectly indistinguishable from the output of a z-side signer.*

Proof. Lemma 1 follows from the result of Cramer [13].

□

Note that, in order to run the z-side signer, it is necessary to somehow get the corresponding witness. We will see later that, with black-box access to an adversarial user in the registration phase, a reduction can use standard Σ-protocol extraction techniques in order to learn the representations of C; if the reduction is, in addition, given the discrete logarithms of all the public parameters to the base g, it will be able to compute the z-side witness, and so, inside a reduction, the z-side signer can be invoked.

Lemma 2. *(Restrictive Blinding [3]) Let $\mathcal{A}$ be a User that engages in the signature issuing protocol ℓ times, and outputs a valid signature $\sigma = (m, (\zeta, \zeta_1, \rho, \omega, \rho'_1, \rho'_2, \omega', \mu))$. Let run_i denote the i-th execution of the protocol and z_{1i} denote z_1 used by the signer $\mathcal{S}$ in run_i. For polynomially bounded ℓ and for all polynomial-time $\mathcal{A}$, the probability that $\log_z \zeta \neq \log_{z_{1i}} \zeta_1$ holds for all i, is negligible if the discrete logarithm problem is intractable and $\mathcal{H}$ is a random oracle.*

Proof. Following Abe [3], assume that, $\mathcal{A}$ having at most q_h accesses to $\mathcal{H}$ and asking at most ℓ signatures to $\mathcal{B}$, outputs a signature $\sigma = (m, (\zeta, \zeta_1, \rho, \omega, \rho'_1, \rho'_2, \omega', \mu))$ that satisfies

$\log_z \zeta \neq \log_{z_{1i}} \zeta_1$ for all i with probability ϵ_0 which is not negligible in n. Here, q_h and ℓ are bounded by a polynomial of security parameter n. We randomly fix an index $Q \in \{1, \ldots, q_h\}$ and regard $\mathcal{A}$ as successful only if the resulting signature corresponds to the Q-th query to $\mathcal{H}$. (If it does not correspond to any query, $\mathcal{A}$ is successful only with negligible probability due to the randomness of $\mathcal{H}$.) Accordingly, it is equivalent to assuming an adversary, we will call it $\mathcal{A}^*$, that asks $\mathcal{H}$ only once and succeeds with probability $\epsilon_1 \geq \epsilon_0/q_h$ [3]. Given this modified $\mathcal{A}^*$ we construct the reduction $\mathcal{B}$ that solves the discrete logarithm problem by simulating the interaction. Let $(\mathbf{p}; \mathbf{q}; \mathbf{g}; \mathbf{Y})$ be an instance of the DL problem.

Reduction Algorithm: $\mathcal{B}$ first sets $(p, q, g) := (\mathbf{p}, \mathbf{q}, \mathbf{g})$. Then, it flips a coin $\chi \in_U \{0, 1\}$ to either select $y := \mathbf{Y}$ (case $\chi = 0$), or $h := \mathbf{Y}$ (case $\chi = 1$).

Case $y = \mathbf{Y}$: (extracting y-side witness)

1. *Key Generation*: $\mathcal{B}$ selects $k, k', \{k_j\} \in_U \mathbb{Z}_q$ and sets $h := g^k$ and $h_j := g^{k_j}$, $z = g^{k'}$. As a result of setting the parameters this way, $\mathcal{B}$ will always be able to compute z-side witnesses, as long as $\mathcal{B}$ can successfully extract the user's attributes and randomness in the registration phase.
2. *Registration* $\mathcal{B}$ extracts values $(L_1, \ldots, L_n; R)$ using a knowledge extractor for proof π_1 (since we only worry about sequential composition the extractor may rewind) and defines $K = k_0 R + \sum_{i=1}^{n} k_i L_i$ (so that $C = g^K$).
3. *Preparation* $\mathcal{B}$ selects $w_{1,i} \in \mathbb{Z}_q$ and sets $rnd = w_{1,i} - K$. Then, computes $w_{2,i} = (k' - w_{1,i})/k \bmod q$ (so we have $z_{1,i} = g^{w_{1,i}}, z_{2,i} = h^{w_{2,i}}$). $\mathcal{B}$ sends rnd to $\mathcal{A}$.
4. *Validation* $\mathcal{B}$ runs $\mathcal{A}^*$ using the z-side signer described above. At the end of each run_i the z-side signer sends: $r_i, c_i, r'_{1,i}, r'_{2,i}, c'_i$ to $\mathcal{A}^*$. $\mathcal{B}$ simulates $\mathcal{H}$ by returning $\varepsilon \in_U \mathbb{Z}_q$ to the random oracle query issued by $\mathcal{A}^*$. $\mathcal{A}^*$ outputs a signature $\sigma = (\zeta, \zeta_1, \rho, \omega, \rho'_1, \rho'_2, \omega', \mu)$, that corresponds to ε.
5. *Rewinding*: Reset and restart $\mathcal{A}^*$ with the same setting. $\mathcal{B}$ simulates $\mathcal{H}$ with $\tilde{\varepsilon} \in_U \mathbb{Z}_q$. $\mathcal{A}^*$ outputs a signature, say $\tilde{\sigma} = (\zeta, \zeta_1, \tilde{\rho}, \tilde{\omega}, \tilde{\rho'_1}, \tilde{\rho'_2}, \tilde{\omega'}, \tilde{\mu})$, that corresponds to $\tilde{\varepsilon}$.
   If $\omega \neq \tilde{\omega}$, $\mathcal{B}$ outputs $x := (\rho - \tilde{\rho})/(\tilde{\omega} - \omega) \bmod q$. The simulation fails, otherwise.

Case $h = \mathbf{Y}$: (extracting z-side witness)

1. *Key generation*: $\mathcal{B}$ selects $x, \{k_j\} \in_U \mathbb{Z}_q$ and sets $y := g^x$ and $h_j := g^{k_j}$. It also selects $w_1, w_2 \in_U \mathbb{Z}_q$ and sets $z := g^{w_1} h^{w_2}$. $\mathcal{B}$ selects $I \in_U \{0, \ldots, \ell\}$ and $J \in_U \{1, \ldots, \ell\}$.
2. *Signature Issuing*: $\mathcal{B}$ runs $\mathcal{A}^*$ simulating the signer as follows.
   (a) For $i \neq J$, $\mathcal{B}$ follows the protocol with y-side witness, x.
   (b) For $i = J$, $\mathcal{B}$ engages in the issuing protocol using *both* y-side witness x and z-side witness (w_1, w_2) as follows.
      i. *Registration* Extract values $(L_1, \ldots, L_n; R)$ using a ZK extractor and define $K = k_0 R + \sum_{i=1}^{n} k_i L_i$ (so that $C = g^K$).
      ii. *Preparation* Set $rnd = w_1 - K$ (so we have $z_{1,J} = g^{w_1}, z_{2,J} = h^{w_2}$). $\mathcal{B}$ sends rnd to $\mathcal{A}$.
      iii. *Validation* Compute $a_J = g^{u_J}, a'_{1J} = g^{u_{1J}}, a'_{2J} = h^{u_{2J}}$ with $u_J, u_{1J}, u_{2J} \in_U \mathbb{Z}_{||}$.
      iv. Send (a_J, a'_{1J}, a'_{2J}) to $\mathcal{A}^*$.
      v. Given e_J from $\mathcal{A}^*$, choose $c'_J \in_U \mathbb{Z}_q$ and compute $c_J := e_J - c'_J \bmod q, r_J := u_J - c_J x \bmod q, r'_{1J} := u_{1J} - c'_J w_1 \bmod q$, and $r'_{2J} := u_{2J} - c'_J w_2 \bmod q$.
      vi. Send $(r_J, c_J, r'_{1J}, r'_{2J}, c'_J)$ to $\mathcal{A}^*$.
   $\mathcal{B}$ simulates $\mathcal{H}$ by returning $\varepsilon \in_U \mathbb{Z}_q$.
3. $\mathcal{A}^*$ outputs a signature, say $\sigma = (\zeta, \zeta_1, \rho, \omega, \rho'_1, \rho'_2, \omega', \mu)$, that corresponds to ε.
4. *Rewind*: Rewind and restart $\mathcal{A}^*$ with the same setting.
   – If $I = 0$, $\mathcal{B}$ simulates $\mathcal{H}$ by returning $\tilde{\varepsilon} \in_U \mathbb{Z}_q$. Otherwise, set $\tilde{\varepsilon} = \varepsilon$.

– If $I \neq 0$ and run_J has not yet been completed before the query to $\mathcal{H}_3$ is sent, $\mathcal{B}$ simulates the execution by using both y-side and z-side witnesses as above choosing $\tilde{c'}_J \in_U \mathbb{Z}_q$. Otherwise, $\mathcal{B}$ simulates only with y-side witness choosing $\tilde{c'}_J = c'_J$.

$\mathcal{A}^*$ outputs a signature, say $\tilde{\sigma} = (\zeta, \zeta_1, \tilde{\rho}, \tilde{\omega}, \tilde{\rho'_1}, \tilde{\rho'_2}, \tilde{\omega'}, \tilde{\mu})$, that corresponds to $\tilde{\varepsilon}$. If $\omega' \neq \tilde{omega'}$, $\mathcal{B}$ computes $w'_1 = (\rho'_1 - \tilde{\rho'}_1)/(\mu - \tilde{\mu}) \bmod q$, $w'_2 = (\rho'_2 - \tilde{\rho'_2})/(\mu - \tilde{\mu}) \bmod q$, and outputs $w = (w_1 - w'_1)/(w'_2 - w_2) \bmod q$. Simulation fails otherwise.

Evaluation of success probability Similarly to Abe [3], we need to make sure that the adversary can't defeat the reduction all the time, so that at least one of the following statements is true: (1) when $\chi = 0$, $\omega \neq \tilde{\omega}$ with non-negligible probability; (2) when $\chi = 1$, $\omega' \neq \tilde{\omega}'$ with non-negligible probability.

Consider fixing the adversary's view before the RO query. Consider, also, that the values (a_i, a'_i) that the adversary receives after the RO query are fixed; consider fixing the adversary's random tape as well. The forgery that the adversary outputs still depends on the output of the random oracle, and on the reduction's responses (c_i, r_i, c'_i, r'_i) that the adversary receives after the RO query.

Suppose that there exists some value δ such that with non-negligible probability, the adversary's signature $\sigma = (\zeta, \zeta_1, \rho, \omega, \rho'_1, \rho'_2, \omega', \mu)$ contains $\omega' = \delta$. In that case, (1) above happens with non-negligible probability: with non-negligible probability, we have $\delta = \omega' = \tilde{\omega}'$ which implies that $\omega \neq \tilde{\omega}$.

So, consider the case when no such δ exists and the reduction fails when $\chi = 0$. In this case, the value ω' output by the adversary is sensitive to the values (c_i, r_i, c'_i, r'_i) that the adversary receives after the RO query; thus when we run the adversary the second time around with access to the z-side witness, we send him the values $(c_i, r_i, \tilde{c}'_i, \tilde{r}'_i)$, and get two signatures from him where $\omega = \tilde{\omega}$, and so $\omega' \neq \tilde{\omega}'$.

What we would like to show is that this implies also that, when we run the adversary with the y-side witness, we will get $\omega' \neq \tilde{\omega}'$. However, with the y-side witness, the second time around the values we send will be $(\tilde{c}_i, \tilde{r}_i, c'_i, r'_i)$, which is not what will produce the desired effect of obtaining $\omega' \neq \tilde{\omega}'$: to get this effect, we need fresh $(\tilde{c}'_i, \tilde{r}'_i)$ instead of fresh $(\tilde{c}_i, \tilde{r}_i)$.

Luckily, by the Splitting Lemma due to Pointcheval and Stern [32, 31] in order to get $\omega' \neq \tilde{\omega}'$ with non-negligible probability, it is sufficient to produce fresh $(\tilde{c}'_J, \tilde{r}'_J)$ for just one randomly chosen run J, which is what the reduction does for $\chi = 1$.

The rest of the analysis is identical to Abe's.

□

Lemma 3. *Any poly-time adversary $\mathcal{A}^*$ outputs a valid signature without interacting with the Signer only with negligible probability if the discrete logarithm problem is intractable and $\mathcal{H}$ is a random oracle.*

Proof. We present the proof in Appendix H.

□

Proof. (Theorem 4). Suppose that there exists adversary $\mathcal{A}$ that outputs $\ell+1$ valid signatures with probability ϵ not negligible after interacting with the Signer at most ℓ times. The case $\ell = 0$ has been ruled out by Lemma 3. We consider $\ell \geq 1$.

By Lemmas 2 and 3, among the $\ell + 1$ signatures, there exist at least two signature-message pairs which contains (ζ, ζ_1) and $\bar{\zeta}, \bar{\zeta_1}$ such that $\log_\zeta \zeta_1 = \log_{\bar{\zeta}} \bar{\zeta_1} = \log_z z_{1I}$ holds for z_{1I} used in run_I for some I in $\{1, \ldots, \ell\}$. Now, there exist two queries to $\mathcal{H}$ that correspond to those signatures. In a similar way as in the proof of Lemma 2, we guess the indices of these queries and regard $\mathcal{A}$ as being successful only when the guess is correct. Accordingly this is equivalent to an adversary, say $\mathcal{A}^*$ that asks $\mathcal{H}$ only twice and succeeds with probability $\epsilon^* = \epsilon / \binom{q_h}{2}$ in producing two signatures in the expected relation.

We construct a reduction $\mathcal{B}$ that, given $(\mathbf{p}, \mathbf{q}, \mathbf{g}, \mathbf{Y})$, solves $\log_g \mathbf{Y}$ in $\mathbb{Z}_q$ by using $\mathcal{A}^*$.

Reduction Algorithm: $\mathcal{B}$ sets $(p, q, g) := (\mathbf{p}, \mathbf{q}, \mathbf{g})$. It then flips a coin, $\chi \in_U \{0, 1\}$, to select either $y := \mathbf{Y}$ (case $\chi = 0$), or $y := g^x$ with randomly chosen x (case $\chi = 1$). It also chooses a random $J \in \{1, 2\}$ which determines whether $\mathcal{A}^*$ is rewound to its first or second RO query.

1. *Setup* If $\chi = 0$, set up and use the z-side signer. Set h, $h_0, \ldots, h_n$ such that their discrete logarithm base g is known. Else, set up the y-side signer by letting $y = g^x$ for a randomly chosen x; set up h, $h_0, \ldots, h_n$ such that their discrete logarithm base Y is known.
2. *Registration* If $\chi = 0$, extract $(R, L_0, \ldots, L_n)$. Else follow the protocol.
3. *Preparation* Follow the protocol.
4. *Validation* If $\chi = 0$, use the z-side signer, else follow the protocol.
5. *Responding to the RO queries* $\mathcal{B}$ simulates $\mathcal{H}$ by returning random values, say ε_1 and ε_2, to the two RO queries.
6. *Rewinding* When $\mathcal{A}^*$ outputs two signatures $(\sigma^{(1)} = (\zeta^{(1)}, \zeta_1^{(1)}, \rho^{(1)}, \omega^{(1)}, \rho'^{(1)}_1, \rho'^{(1)}_2, \omega'^{(1)}, \mu^{(1)}))$ and $(\sigma^{(2)} = (\zeta^{(2)}, \zeta_1^{(2)}, \rho^{(2)}, \omega^{(2)}, \rho'^{(2)}_1, \rho'^{(2)}_2, \omega'^{(2)}, \mu^{(2)}))$ corresponding to ε_1 and ε_2, $\mathcal{B}$ resets it to its J'th random oracle query; this time it responds to the two RO queries with $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$ (although $\varepsilon_1 = \tilde{\varepsilon}_1$ if $J = 2$).
7. *Interacting with $\mathcal{A}^*$ after rewinding* is the same as before, according to the value of χ.
8. *Extracting* When $\mathcal{A}^*$ again outputs two signatures $(\tilde{\sigma}^{(1)} = (\zeta^{(1)}, \zeta_1^{(1)}, \tilde{\rho}^{(1)}, \tilde{\omega}^{(1)}, \tilde{\rho'}^{(1)}_1, \tilde{\rho'}^{(1)}_2, \tilde{\omega'}^{(1)}, \tilde{\mu}^{(1)})$ and $(\tilde{\sigma}^{(2)} = (\zeta^{(2)}, \zeta_1^{(2)}, \tilde{\rho}^{(2)}, \tilde{\omega}^{(2)}, \tilde{\rho'}^{(2)}_1, \tilde{\rho'}^{(2)}_2, \tilde{\omega'}^{(2)}, \tilde{\mu}^{(2)})$ corresponding to $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$, do: if $\chi = 0$ and $\omega_J \neq \tilde{\omega}_J$, then extract the y-side witness as described in the proof of Lemma 2; else if $\chi = 1$ and $\omega'_J \neq \tilde{\omega}'_J$, then extract the z-side witness as described in Lemma 2.

Evaluation of Success probability The analysis of the success probability is similar to Abe, except for several crucial differences. Abe's reduction uses the z-side witness for each run except run I independently of χ. This is possible because Abe's z_1 is determined by a random oracle query, and so he can embed the instance $\mathbf{Y}$ in z_1 specifically for run I, and choose it such that its discrete logarithm is known for every other run. As a result, when we look at the transcripts with the adversary, both the first time the adversary is run and after the adversary is rewound, only the transcripts corresponding to run I may potentially depend on χ.

Why doesn't our reduction do the same? The problem is that, in the case when $\chi = 1$, the setup is such that it is impossible for our reduction to know the z-side witness in any run. One could consider changing the setup somehow; the problem is that, if $\mathbf{Y}$ is embedded anywhere in the public parameters, then the reduction cannot know the z-side witness for any run at all: z_1's representation in h, $h_0 \ldots, h_n$, g is known to the reduction, so if in addition it also knows its discrete logarithm, then it can compute the discrete logarithm of the challenge $\mathbf{Y}$ from those.

Thus, we need pairs of transcripts (transcript before rewinding, transcript after rewinding) for runs other than I to be independent of χ without guaranteeing that the reduction always knows a z-side witness for runs other than I. If we limit our attention to sequential composition only (see Definition 5), we get this for free, just by witness indistinguishability (Lemma 1).

The rest of the analysis of the success of the reduction is identical to Abe's [3].

Thus the only thing left to prove is that an adversary cannot open the sets of commitments $\{C^{(i)}\}$ and $\{\zeta^{(i)}, \zeta_1^{(i)}\}$ to two different multisets. Note that $\zeta^{(i)}$ uniquely determines $\gamma^{(i)}$, and by Lemma 2, it follows that for every i, there exists some $z_1^{(i')}$ such that $\zeta_1^{(i)} = (z_1^{(i')})^{\gamma^{(i)}}$.

Thus, an alternative opening of the commitment $\{\zeta^{(i)}, \zeta_1^{(i)}\}$ gives rise to an alternative representation of $z_1^{(i')}$ in bases $h, h_1, \ldots, h_n, g$. In turn, by standard techniques, this breaks the discrete logarithm assumption. (The reduction will just honestly emulate the signer, and embed the DL instance into the parameters $h, h_1, \ldots, h_n, g$.)

□

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A Witness Relations and Interactive Proofs

A witness relation for a language $L \in \mathcal{NP}$ is defined as [18]:

Definition 7 (Witness relation). *A witness relation for a language $L \in \mathcal{NP}$ is a binary relation R_L that is polynomially bounded (i.e., $(h,x) \in R_L$ implies $|x| \leq poly(|h|)$), is polynomial-time-recognizable and characterizes L by*

$$L = \{h : \exists x \ s.t. \ (h,x) \in R_L\}$$

For $h \in L$, any x satisfying $(h,x) \in R_L$ is called a *witness* (for the membership $h \in L$). By $R_L(h)$ we denote the set of witnesses for the membership $h \in L$; that is, $R_L(h) = \{x : (h,x) \in R_L\}$.

A relation R_L is said to be "hard", if one can efficiently generate (h,x)'s such that, when given h it is hard to find the witness x.

Definition 8 (Interactive Proof System [19]). *An interactive proof system with soundness error $s \in [0,1]$ for a language L with witness relation R_L is a pair of algorithms (P,V) where v is probabilistic polynomial time and the following properties hold:*

1. *Completeness. For every $h \in L$ and every $x \in R_L(h)$,*

$$Pr[\langle P(x), V_L\rangle(h) = 1] = 1.$$

2. *s-Soundness. For every $h \notin L$, every $z \in \{0,1\}^*$ and every interactive algorithm P^**

$$Pr[\langle P^*(z), V\rangle(h) = 0] \geq 1 - s.$$

A useful property in the above setting would be if the verifier V wouldn't learn anything useful from P about the witness x besides the fact that P knows x. This property is called *zero knowledge*. If soundness error s is negligible, then this interactive proof system has *strong soundness*.

Definition 9 (Honest Verifiable Zero Knowledge (HVZK)). *An interactive proof system (P, V) for a language L is said to be* honest verifiable zero knowledge *if there exists a probabilistic polynomial time algorithm S (the Simulator) such that for all $h \in L$:*

$$view_V[P(h) \leftrightarrow V(h)] \approx S(h),$$

where view_V is the view of the honest verifier V of the interaction between V and P on input h.

B Σ-Protocols

Σ-protocols are a class of interactive proofs where (P, V) have a common input h and an x such that $(h, x) \in R_L$ is P's private input. Their main characteristic is that they have exactly 3 rounds of the following type: (1) P sends a message a to V, (2) V responds with a random challenge c chosen from a domain of size $\Theta(k)$ and (3) P resents a reply r. V decides whether to accept or not given the information he has seen: (h, a, c, r). Formally:

Definition 10 (Σ-Protocol). *A protocol $\mathcal{P}$ is said to be a Σ-protocol for a relation R_L if:*

- *$\mathcal{P}$ is of the above three rounds form, and if (P, V) follow the protocol, the verifier always accepts.*
- *From any h and any pair of accepting conversations on input h, (a, c, r), (a, c', r') where $c \neq c'$, one can efficiently compute x such that $(h, x) \in R_L$ (special soundness).*
- *There exists a polynomial-time simulator S, which on input h and a random c outputs an accepting conversation of the form (a, c, r), with the same probability distribution as conversations between the honest P, V on input h (special honest-verifier zero-knowledge).*

An example of a Σ-protocol is the Schnorr Identification scheme [33]. Let G be a group of prime order q with generator g, and let $\mathbb{Z}_q$ denote the field of integers modulo q. Schnorr's identification scheme works as follows:

$\text{Prover}(q, g, h = g^x)$		$\text{Verifier}(q, g, h)$
$y \leftarrow \mathbb{Z}_q$, $a = g^y$	$\xrightarrow{a}$	
	$\xleftarrow{c}$	$c \leftarrow \mathbb{Z}_q$
$r = y + cx \bmod q$	$\xrightarrow{r}$	$g^r \stackrel{?}{=} ah^c$

Σ-protocols are an essential building block for blind signatures and anonymous credentials. For example Brands' [6] and Abe's [3] schemes are based on a Σ-protocol, while CL anonymous credentials [8] uses ZK proofs which are based on Σ-protocols.

C Fiat-Shamir Heuristic

Fiat and Shamir [17] proposed a method to transform any three-round interactive proof system with negligible soundness error, such as Σ-protocols, into a digital signature scheme using a hash function, modeled as a random oracle.

To transform a three-round proof system into a signature scheme, one could, instead of a random c, compute $c = H(a, m)$, where $H \to \{0,1\}^*$ is a hash function. Then, the Fiat-Shamir transformation uses c to create a signature $\sigma(m) = (a, r)$. Specifically:

- $\text{Gen}(1^k) : (h, x) \leftarrow R_L, \text{SK} = x, \text{PK} = (h, H)$.
- $\text{Sig}(m)$: produce a honestly using x, set $c = H(m, a)$, produce r honestly for this c, output $\sigma(m) = (a, r)$.
- $\text{Ver}(m, (a, r))$: $c = H(a, m)$ and check if (a, c, r) is valid.

Famous digital signatures that have been constructed from Σ-protocols using the Fiat-Shamir heuristic include Schnorr's [33] and GQ signatures [22] and they have been proven secure in the RO model [31]. Note that, it has also been shown that the digital signatures produced by the Fiat-Shamir heuristic are not secure in the standard model [20].

D Blind Signatures Definitions

Blind signatures are a special case of digital signatures. They have a blind signature issuing protocol in which the signer doesn't learn anything about the message he is signing. We present the formal definitions for blind signatures as they were described in [23]. A blind signature scheme is a four-tuple consisting of two interactive Turing machines, the Signer and the User, (S, U) and two algorithms $(Gen, Verify)$.

- $Gen(1^k)$: is a probabilistic polynomial time key-generation algorithm which takes as an input a security parameter 1^k and outputs a pair (pk, sk) of public and secret keys.
- $S(pk, sk), U(pk, m)$: are polynomially- bounded probabilistic Interactive Turing machines who have the following (separate) tapes: read-only input tape, write-only output tape, a read/write work tape, a read-only random tape, and two communication tapes, a read-only and a write-only tape. They are both given (on their input tapes) as a common input a pk produced by a key generation algorithm. Additionally, S is given on her input tape a corresponding private key sk and U is given on her input tape a message m, where the length of all inputs must be polynomial in the security parameter 1^k of the key generation algorithm. Both S and U engage in an interactive protocol of some polynomial (in the security parameter) number of rounds. At the end of this protocol S outputs either "completed" or "not-completed" and U outputs either "fail" or $\sigma(m)$.
- $Verify(pk, m, \sigma(m))$: is a deterministic polynomial-time algorithm, which outputs "accept"/"reject" with the requirement that for any message m, and for all random choices of key generation algorithm, if both S and U follow the protocol then S always outputs "completed", and the output of U is always accepted by the verification algorithm.

A blind digital signature scheme is *secure* if for all the probabilistic polynomial time algorithms $\mathcal{A}$ there exists a security parameter $k_{\mathcal{A}}$ such that for all $k > k_{\mathcal{A}}$ the following two properties hold [23]:

- Blindness: the signer is unable to view the messages he signs (protection for the user). Furthermore, a malicious signer cannot link a $(m, \sigma(m))$ pair to any particular execution of the protocol. In order to define blindness formally consider the following experiment. Let $\mathcal{A}$ control the signer but not the user and $b \in \{0, 1\}$ is a randomly chosen bit which is kept secret from $\mathcal{A}$. $\mathcal{A}$ will try to guess the value of b by performing the following steps:

1. $(pk, sk) \leftarrow Gen(1^k)$
2. $\{m_0, m_1\} \leftarrow \mathcal{A}(1^k, pk, sk)$ (i.e. $\mathcal{A}$ produces two documents, polynomial in 1^k, where $\{m_0, m_1\}$ are by convention lexicographically ordered and may even depend on pk and sk).
3. We denote by $\{m_b, m_{1-b}\}$ the same two documents $\{m_0, m_1\}$, ordered according to the value of bit b, where the value of b is hidden from $\mathcal{A}$. $\mathcal{A}(1^k, pk,\ sk, m_0, m_1)$ engages in two parallel (and arbitrarily interleaved) interactive protocols, the first with $U(pk, m_b)$ and the second with $U(pk, m_{1-b})$.

4. If the fist User outputs on her private tape $\sigma(m_b)$ (i.e. does not output fail) and the second user outputs on her private tape $\sigma(m_{1-b})$ (i.e., also does not output fail) then $\mathcal{A}$ is given as an additional input $\{\sigma(m_0), \sigma(m_1)\}$.(We remark that we do not insist that this happens, and either one or both users may output fail).
5. $\mathcal{A}$ outputs a bit b' (given her view of steps 1 through 3, and if conditions are satisfied on step 4 as well).

Then the probability, taken over the choice of b, over coin-flips of the key-generation algorithm, the coin-flips of $\mathcal{A}$, and (private) coin-flips of both users (from step 3), that $b' = b$ is at most $\frac{1}{2} + \nu(k)$, where $\nu(k)$ is a negligible function.

-One-more Unforgeability: a user interacting with a signer S cannot output an additional, valid message/signature pair $(m, \sigma(m))$ no matter how many pairs of messages/ signatures of S he has seen (protection for the signer). To define that formally, consider an adversary $\mathcal{A}$ who controls the user but not the signer and executes the following experiment in order to get "one-more" signature (this is also called "one-more" forgery).

1. $(pk, sk) \leftarrow Gen(1^k)$
2. $\mathcal{A}(pk)$ engages in polynomially many (in k) adaptive, parallel and arbitrarily interleaved interactive protocols with polynomially many copies of $S(pk, sk)$, where $\mathcal{A}$ decides in an adaptive fashion when to stop. Let ℓ be the number of executions, where the Signer outputted "completed" in the end of Step 2.
3. $\mathcal{A}$ outputs a collection $\{(m_1, \sigma(m_1)), \ldots, (m_j, \sigma(m_j))\}$ subject to the constraint that $(m_i, \sigma(m_i))$ for $1 \leq i \leq j$ are all accepted by $Verify(pk, m_i, \sigma(m_i))$, and all m_i's are distinct.

Then the probability, taken over coin-flips of key-generation algorithm, the coin-flips of $\mathcal{A}$, and over the (private) coin-flips of the Signer, that $j > \ell$ is at most $\nu(k)$.

E The Complete ACL Scheme

In Figure 1 we describe all the phases of our ACL construction.

F Correctness

The correctness of the new scheme is easily verified:

$$\begin{aligned}
\omega + \omega' &= c + t_2 + c' + t_4 = e + t_2 + t_4 = \varepsilon \bmod q \\
g^{\rho} y^{\omega} &= g^{r+t_1} y^{c+t_2} = g^{r+cx} g^{t_1} y^{t_2} = \alpha \\
g^{\rho_1'} \zeta_1^{\omega'} &= g^{\gamma r_1' + t_3} \zeta_1^{c'+t_4} = (a_1' z_1^{-\omega'})^{\gamma} g^{t_3} \zeta_1^{c'+t_4} = a_1'^{\gamma} g^{t_3} \zeta_1^{t_4} = \alpha_1' \\
g^{\rho_2'} \zeta_2^{\omega'} &= h^{\gamma r_2' + t_5} \zeta_2^{c'+t_4} = (a_2' z_2^{-\omega'})^{\gamma} h^{t_5} \zeta_2^{c'+t_4} = a_2'^{\gamma} h^{t_5} \zeta_2^{t_4} = \alpha_2' \\
z^{\mu} \zeta^{\omega'} &= z^{\tau - \omega' \gamma} \zeta^{\omega'} = \zeta^{\gamma} = \eta.
\end{aligned}$$

G Blindness

In order to conclude the proof of Theorem 3 we will first show that `Real`≈`Hybrid` and then that `Hybrid`≈`Fake`.

Proof. (*contin.*) **Case 1:** `Real`≈`Hybrid`

Suppose that there exists a poly-time adversary $\mathcal{A}$ who is successful in distinguishing between `Real` and `Hybrid` with probability $1/2 + \epsilon$, where the advantage ϵ is not negligible. We then

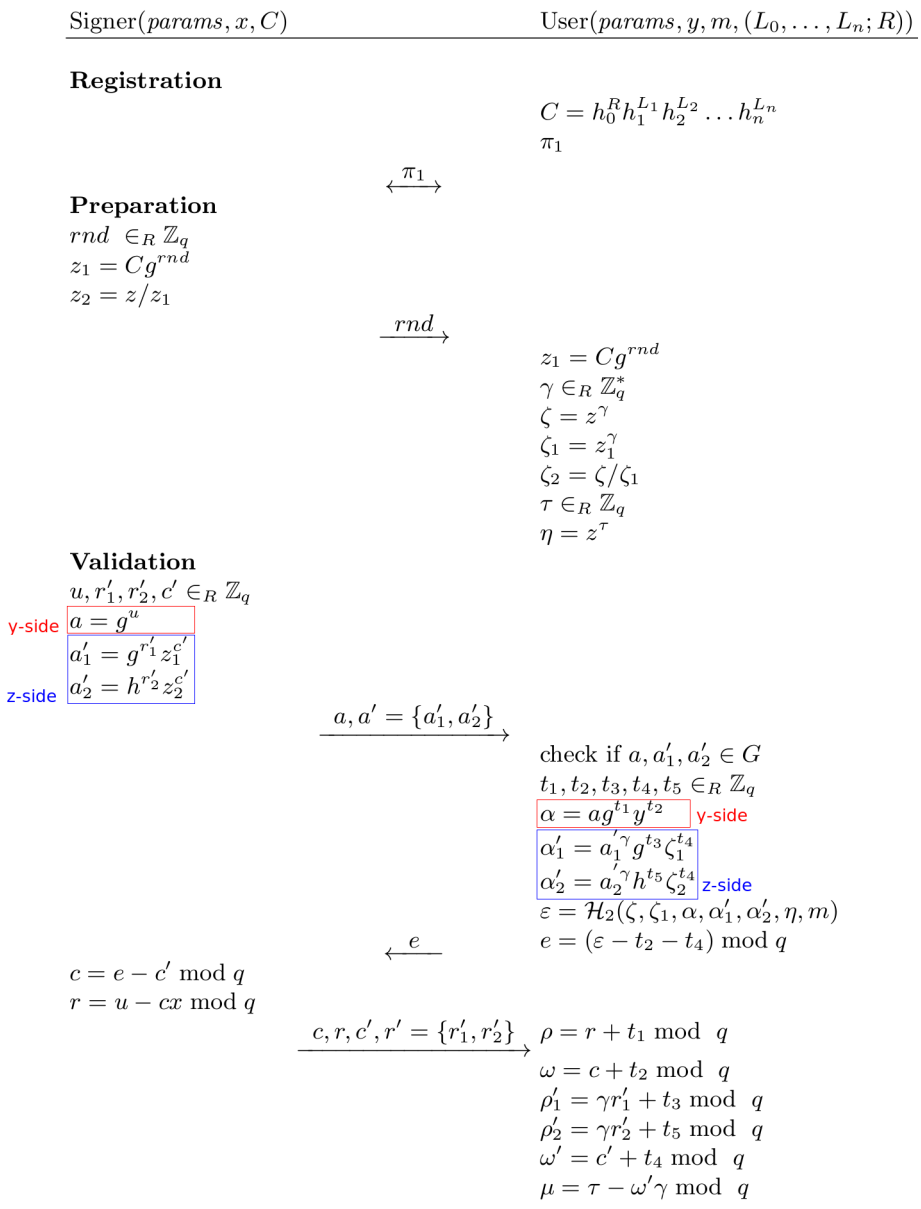

Fig. 1. Proposed ACL Construction

show that there exists a poly-time algorithm $\mathcal{B}$ that solves the DDH problem with non-negligible advantage.

The reduction $\mathcal{B}$ gets as input an instance (g, A, B, D) where (g, A, B, D) is a DDH instance. $\mathcal{B}$ first fixes $z = A$ and then sets $h_0 = g^{e_0}$, and $h_1, \ldots, h_n$ to $A^{e_0}, \ldots, A^{e_n}$ for randomly chosen $e_i \in \mathbb{Z}_q$. $\mathcal{B}$ sends the parameters to $\mathcal{A}$. The adversary $\mathcal{A}$ creates his public key y and sends y to $\mathcal{B}$ along with messages m_0, m_1, commitments C_0, C_1 and openings $(\overrightarrow{L_0}, \overrightarrow{L_1})$ which are two attributes vectors of size n. $\mathcal{A}$ and $\mathcal{B}$ engage in two interleaving registration and signature issuing instances. During the registration phase we have: $C_{(j)} = A^{k_1^{(j)}}$, $(k_1^{(j)} = e_0 R + e_1 L_{j,1} + \cdots + e_n L_{j,n})$ honestly created, for $j \in \{0, 1\}$. Note that the value $k_1^{(j)}$ is known to the reduction $\mathcal{B}$.

Then, during the two signature issuing instances, $\mathcal{A}$ computes $z_1^{(j)} = A^{k_1^{(j)}} g^{rnd^{(j)}}$ where $rnd^{(j)}$ was sent to $\mathcal{B}$. $\mathcal{B}$ responds with random $e^{(j)}$ and then computes two signatures σ_1, σ_2 in the following way:

$\mathcal{B}$ flips two random coins: $coin, b \in \{0, 1\}$. Then, lets σ_{2-coin} be the correct signature on $(m_0, \tilde{C}_b)$ (where $\tilde{C}$ is denoted as ζ_1 in our ACL scheme) for the corresponding instance, and σ_{coin+1} be a signature on $(m_1, \tilde{C}_{1-b})$ generated from the input to $\mathcal{B}$, as follows: set $\zeta^{(j)} = D$ and $\zeta_1^{(j)} = D^{k_1^{(j)}} B^{rnd^{(j)}}$; let $\zeta_2^{(j)} = \zeta^{(j)} / \zeta_1^{(j)}$, randomly choose $\rho, \omega, \rho_1', \rho_2', \omega', \mu$, and then define $\mathcal{H}$ so that the signature verifies.

After receiving the signatures, $\mathcal{A}$ outputs `Real` or `Hybrid`. If $\mathcal{A}$ outputs `Real` then $\mathcal{B}$ outputs "DH" or $\mathcal{B}$ outputs "random" otherwise.

Let us analyze the success of $\mathcal{B}$ in distinguishing Diffie-Hellman tuples from random tuples. If the input (g, A, B, D) is a DH tuple, then: $\zeta^{(j)} = D = g^{ab} = A^b = z^b$ and $\zeta_1^{(j)} = D^{k_1^{(j)}} B^{rnd^{(j)}} = g^{abk_1^{(j)}} g^{brnd^{(j)}} = (g^{ak_1^{(j)}} g^{rnd^{(j)}})^b = (A^{k_1^{(j)}} g^{rnd^{(j)}})^b = z_1^b$ so, the signature σ_{coin+1} is distributed identically to a correct signature, and this is precisely the `Real` game. Similarly, if the input is not a DH tuple then it is the `Hybrid` game. Therefore, $\mathcal{B}$ will be correct exactly when $\mathcal{A}$ is and will distinguish a DH tuple with probability $1/2 + \epsilon$, which contradicts to the DDH assumption.

Case 2: `Hybrid`$\approx$`Fake`
Similarly, we build a reduction $\mathcal{B}$ which given an adversary $\mathcal{A}$, who distinguishes between `Hybrid` and `Fake` with probability $1/2 + \epsilon$, solves the DDH problem.

Working in a similar way we did before, $\mathcal{B}$ will now create a fake signature and one using the input tuple. If $\mathcal{A}$ outputs `Hybrid` then $\mathcal{B}$ will output "DH" or "not DH" otherwise. If the input is a DH instance then the output will correspond to the `Hybrid` game and to the `Fake` otherwise. So, $\mathcal{B}$ will be successful whenever $\mathcal{A}$ is and will break the DDH assumption with non-negligible advantage ϵ which is a contradiction.

Finally, since `Real`$\approx$`Hybrid` and `Hybrid`$\approx$`Fake` it follows that `Real`$\approx$`Fake`.

□

H Unforgeability: Proof of Lemma 3

Proof. (sketch) In order to prove this we are going to use the *oracle replay attack* suggested by Pointcheval and Stern to prove unforgeability of blind signatures [32]. First, we consider as we did in the proof of Lemma 2 that $\mathcal{A}^*$ only makes one RO query. Given $\mathbf{Y} \in G$, we construct a reduction $\mathcal{B}$ that finds $\log_g \mathbf{Y}$ in $\mathbb{Z}_q$. During *key generation*, $\mathcal{B}$ first selects $\omega, \xi, \{k_i\}$ randomly and sets $y = \mathbf{Y}, h = g^w, h_i = g^{k_i} z = \mathbf{Y} g^{\xi}$. ($\mathcal{B}$ does not need to simulate the signer, so it can put $\mathbf{Y}$ in both y and z.) Then, $\mathcal{B}$ invokes $\mathcal{A}^*$ twice with the same initial settings and different ε and $\tilde{\varepsilon}$ as answers of $\mathcal{H}$. The resulting signatures are $\sigma = (\zeta, \zeta_1, \rho, \omega, \rho_1', \rho_2', \omega', \mu)$

and $\tilde{\sigma} = (\zeta, \zeta_1, \tilde{\rho}, \tilde{\omega}, \tilde{\rho_1'}, \tilde{\rho_2'}, \tilde{\omega'}, \tilde{\mu})$. Since $\omega + \omega' = \varepsilon \neq \tilde{\varepsilon} = \tilde{\omega} + \tilde{\omega'}$, at least one of $\omega \neq \tilde{\omega}$ or $\omega' \neq \tilde{\omega'}$ happens.

In the case that $\omega \neq \tilde{\omega}$, $\mathcal{B}$ computes $\log_g \mathbf{Y} = (\rho - \tilde{\rho})/(\tilde{\omega} - \omega) \bmod q$. For the case $\omega' \neq \tilde{\omega'}$, $\mathcal{B}$ computes $\gamma = \log_z \zeta = (\mu - \tilde{\mu})/(\omega' - \tilde{\omega'}) \bmod q$, $w_1 = \log_g \zeta_1 = (\rho_1' - \tilde{\rho_1'})/(\omega' - \tilde{\omega'}) \bmod q$, $w_2 = \log_g \zeta_2 = (\rho_2' - \tilde{\rho_2'})/(\omega' - \tilde{\omega'}) \bmod q$, and $\log_g \mathbf{Y} = \log_g z - \xi = (w_1 + w_2/w)/\gamma - \xi \bmod q$. □