

# Strongly Secure Authenticated Key Exchange Protocol from Bilinear Groups without Random Oracles

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May 9, 2013

## Abstract

Since the introducing of extended Canetti-Krawczyk (eCK) security model for two party key exchange, many protocols have been proposed to provide eCK security. However, most of those protocols are provably secure in the random oracle model or rely on special design technique well-known as the NAXOS trick. In contrast to previous schemes, we present an eCK secure protocol in the standard model, without NAXOS trick and without assuming impractical, strong, concurrent zero-knowledge proofs of knowledge of secret keys done to the CA at key registration. The security proof of our scheme is based on standard pairing assumption, collision resistant hash functions, bilinear decision Diffie-Hellman (BDDH) and decision linear Diffie-Hellman (DLIN) assumptions, and pseudo-random functions with pairwise independent random source  $\pi$ PRF [30]. Although our proposed protocol is based on bilinear groups, it doesn't need any pairing operations during protocol execution.

**Keywords:** one-round authenticated key exchange, pairing, insider security, standard model

## 1 Introduction

One-round authenticated key exchange protocols allow for low-latency key exchange that is executed to enable both parties to end up sharing a session key with assurance that the key is only known to them. These protocols typically combine the long-term and ephemeral Diffie-Hellman keys of the participants to compute the session key. Their security is usually proven in recent models like the Canetti-Krawczyk (CK) model [10] and the extended Canetti-Krawczyk (eCK) model [22], where the adversary is given partial access to session keys, ephemeral values, and intermediate states. Recently there are quite a few variants of (e)CK model used in literatures (e.g., [21, 12, 33, 14]). The CK+ model is recently used by Fujioka et al. [14], and the authors adopt `StateReveal` query in place of `EphemeralKeyReveal` query of eCK model to capture maximum exposure of states. Similar model has also been introduced before by Cremers [12] which was called eCK' model, where the `StateReveal` is used either. In this paper, we would like to call both CK+ and eCK' models as eCK model to avoid ambiguity since these models are based on the similar *freshness* restriction as the original eCK model that is distinct to the first CK model. The eCK model is known to be one of the strongest AKE models that covers the most desirable security attributes for AKE including resistance to key compromise impersonation (KCI) attacks, weak perfect forward secrecy (wPFS),

leakage of secret states and chosen identity and public key (CIDPK) attacks. Please note that both CK and eCK models leave out the definition of session state or ephemeral key to specific protocols. Because it is not easy to define session states for all protocols in a general way. On the other hand, to our best of knowledge, no AKE protocol is secure in either CK model or eCK model if ‘all’ session states can be revealed.

**PRACTICAL CONSIDERATIONS.** In order to protect those critical session states of AKE protocols, utilizing secure (tamper-proof) device might be a natural solution. A secure device (such as smart card) may usually be used to store long-term cryptographic keys and is able to implement cryptographic operations or primitives. An appropriate implementation approach with secure device for eCK model was introduced by Sarr et al. [33], that at each party an host machine (e.g. the personal computer) is used together with a secure device. Similar modeling technique involving secure hardware was previously used by Bresson et al. [7]. Basing the security models on specific implementation approach reduces the gap that often exists between formal models and practical security without loss of generality, and this also enables us to define the detailed content of `StateReveal` query. In this way it is possible to adopt a “All-and-Nothing” strategy to define the session states – namely we can assume that *all states* stored on untrusted host machine can be revealed via `StateReveal` query and *no state* would be exposed at secure device. As those secure devices might be short in both storage capacity and computational resource, the algorithm on secure device is often causing performance bottleneck of systems. This makes it necessary for us to optimize AKE protocols when they are realized involving secure device.

**Motivating Problems** So far most of CK and eCK protocols are only provably secure in the random oracle model [4] which assumes the existence of publicly available functions with truly random outputs. However, this model represents a very strong idealization of the real world. Canetti et al. [9] showed that there exist schemes which have a security proof in the random oracle model but are insecure when instantiated with any hash function. While investigating the AKE protocols without random oracles in either CK model or eCK model, there exist only few protocols. Jeong et al. [18, 19], proposed efficient one-round protocols, which are proven in the standard CK model. However their proposals do not provide resilience of key compromise impersonation (KCI) attacks. Boyd et al. [6], presented generic approaches (BCNP) exploiting KEMs to build one-round key exchange protocols that are secure in the standard model. However, neither of the above schemes [18, 19, 6] can resist with the leakage of ephemeral private keys from target session, which is modelled by eCK model. Most recently, Fujioka et al. [14] introduced a variant (FSXY) of BCNP construction which is proved secure without random oracles in the CK+ model. In FSXY construction, the twisted-PRF trick is overused to generate the de-facto ephemeral secret key, which is just a variant of NAXOS trick [22].<sup>1</sup> We note that only a small part of session states in FSXY scheme (i.e. the input randomness of twisted-PRF trick and the ephemeral key of IND-CPA KEM ) can be revealed via `StateReveal`. It is not hard to see if either the ephemeral keys generated by twisted-PRF trick or the encapsulation keys of IND-CCA secure KEM are defined as session states then the FSXY protocol is no longer secure in the eCK model. The problem of

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<sup>1</sup>The twisted-PRF (NAXOS) trick is first used by Okamoto [30] which is an implementation technique that hides the exponent of an ephemeral public key from an adversary using long-term key even if the adversary obtains the ephemeral secret keys. For instance, the exponent of public key is computed as  $x = \text{PRF}(\tilde{x}_1, a) \oplus \text{PRF}(a, \tilde{x}_2)$  where  $\tilde{x}_1, \tilde{x}_2$  are ephemeral keys that can be revealed via `StateReveal` query, the value  $a$  is the long-term private key and PRF is a secure pseudo-random function.

FSXY scheme is that to secure implement it with secure device one has to put all computations related to twisted-PRF trick (e.g. encapsulation and decapsulation algorithms of IND-CCA secure KEM) on secure device to avoid state leakage. This would lead to serious efficiency problem of the system. Another drawback of the BCNP-like protocols is that they require prior knowledge of the peer’s public key to run the KEM, namely those protocols can only run in *pre-specified peer* model [11]. This fact might result in those BCNP-like protocols being unable to execute efficiently in the *post-specified peer* model without additional message flows to exchange identifiers and static public keys. We stress that the specified peer model would dramatically affect the design of secure one-round AKE protocols in the standard model. Although the eCK secure protocol [28] is constructed without NAXOS trick which is proved in the standard model and are able to run under post-specified peer setting, it requires a rather strong assumption on public key registration, i.e. the knowledge of secret key assumption [24].<sup>2</sup> The protocol by Yang et al. [37] is proposed without KOSK assumption, but it needs one expensive pairing operation on secure device. In a nutshell, our major motivation of this paper is to seek efficient eCK secure construction without random oracles and KOSK assumption that can be efficiently implemented with secure device and under post-specified peer setting.

**Contributions** We present an eCK secure AKE protocol in the standard model, that is able to resist with chosen identity and public key attacks without KOSK assumption. Instead an alternative proof of public key and corresponding check procedure is given for key registration. The security of proposed protocol is based on standard pairing assumption, collision resistant hash function family, bilinear decision Diffie-Hellman (BDDH) and decision linear Diffie-Hellman (DLIN) assumptions, and pseudo-random function family with pairwise independent random source  $\pi$ PRF [30]. Moreover, our construction doesn’t rely on NAXOS like trick. Surprisingly, although our protocol is constructed based on bilinear groups, we avoid any pairing operations during protocol execution via pre-computation strategy. To implement the session key generation algorithm on secure device, only one exponentiation is required.

## 1.1 Related Work

In 1986, Matsumoto et al. [25] first studied the implicitly-authenticated DH protocols, which results in a line of research generated many protocols. Meanwhile, the most famous and efficient one is the MQV protocol by Law, Menezes, Qu, Solinas and Vanstone [26, 23]. On Crypto 2005 conference, Krawczyk [21] presented a hashed variant of MQV called HMQV, which is formally proven secure in a modified CK [10] model referred as  $\text{CH}_{\text{HMQV}}$ , relying on random oracles and strong assumptions (i.e. gap Diffie-Hellman [31] assumption and knowledge of exponent assumption [2]). Krawczyk also showed that HMQV protocol offers more security features, in particular for resilience of KCI attacks, UKS attacks and the leakage of secret state. Note that the chosen long-term public key attacks on HMQV are not formally studied in [21]. Besides, the application of secure module was also suggested by HMQV protocol [21] to prevent the leakage of intermediate secrets. But the detailed implementation mechanism was not elaborated.

In 2007 LaMacchia et al. [22] proposed an extended Canetti-Krawczyk (eCK) model, in which the adversary is equipped with an `EphemeralKeyReveal` (or `StateReveal` query in its variant) query

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<sup>2</sup>Basically the KOSK assumption assumes that there exists either efficient knowledge extractor (satisfying requirement in [1]) to be able to obtain the secret key from knowledge proof, or the adversary simply hands the challenger corresponding secret keys.

to access all ephemeral private input required to carry session key computations. Since then many AKE protocols [35, 33, 36, 27, 15, 30, 32, 28, 37, 14], are proposed to capture eCK security. However, most of those schemes are only provably secure in the random oracle model. When examining the eCK secure protocols in the standard model, the FSXY protocol [14] can run only in the pre-specified peer setting that means the protocol can not be executed within two pass. In addition, the NAXOS like trick is overused in the construction that may lead efficiency problem when it is realized with secure device. The MO protocol [28] requires KOSK assumption during key registration. Although the protocol [30] by Okamoto is eCK secure in the standard model without KOSK assumption, this protocol heavily relies on the NAXOS trick. Moreover, though the YS protocol [37] is also built without random oracles, it needs one pairing operation on secure device. The YS protocol can be seen as the predecessor of our proposal, but our scheme is more efficient than it.

With respect to the protection of session state involving secure device, two implementation scenarios (which are referred to as ImplM-I and ImplM-II in the following) and corresponding security models have been studied in literatures [33, 38]. Basically, the implementation of an AKE protocol is divided into two parts which are respectively run at insecure host machine (which is subject to attacks on session states) and at secure device. In corresponding security model, the adversary is able to reveal all possible states stored at host machine, but the secure device which is treated as a black-box and is resilience of leakage of intermediate values. We review those implementation models in the Appendix ???. However, Yoneyama et al., in their recent work [38], particularly showed errors in the security proofs of SMQV and FHMV [33] when they are realized by ImplM-II. They pointed out that the uniqueness of secret exponents interferes with the simulation to the query to reveal the result of intermediate computations. They thereafter formally clarified the technical limitations of Diffie-Hellman key exchange protocols for achieving provable security in the seCK model [33]. Unfortunately, their results showed that there is no scheme has been provably secure in the seCK model while involving the implementation scenario ImplM-II. So that in this work we mainly focus on the scenario ImplM-I without loss of generality.

## 2 Preliminaries

*Notations.* We let  $\kappa$  denote the security parameter and  $1^\kappa$  the string that consists of  $\kappa$  ones. Each party has a long-term authentication key which is used to prove the identity of the party in an AKE protocol. We let a ‘hat’ on top of a capital letter denotes an identifier of a participant, without the hat the letter denotes the public key of that party, and the same letter in lower case denotes a private key. For example, a party  $\hat{A}$  is supposed to register its public key  $A = g^a$  at certificate authority (CA) and keeps corresponding long-term secret key  $sk_A = a$  privately. Let  $[n] = \{1, \dots, n\} \subset \mathbb{N}$  be the set of integers between 1 and  $n$ . If  $S$  is a set, then  $a \xleftarrow{\$} S$  denotes the action of sampling a uniformly random element from  $S$ .

## 3 Collision-Resistant Hash Functions

Let  $\text{CRHF} : \mathcal{K}_{\text{CRHF}} \times \mathcal{M}_{\text{CRHF}} \rightarrow \mathcal{Y}_{\text{CRHF}}$  be a family of keyed-hash functions where  $\mathcal{K}_{\text{CRHF}}$  is the key space,  $\mathcal{M}_{\text{CRHF}}$  is the message space and  $\mathcal{Y}_{\text{CRHF}}$  is the hash value space. The public key  $hk_{\text{CRHF}} \in \mathcal{K}_{\text{CRHF}}$  defines a hash function, denoted by  $\text{CRHF}(hk_{\text{CRHF}}, \cdot)$ , which is generated by a PPT algorithm

CRHF.KG( $1^\kappa$ ) on input security parameter  $\kappa$ . On input a message  $m \in \mathcal{M}_{\text{CRHF}}$ , this function  $\text{CRHF}(hk_{\text{CRHF}}, m)$  generates a hash value  $y \in \mathcal{Y}_{\text{CRHF}}$ .

**Definition 1.** CRHF is called  $(t_{\text{CRHF}}, \epsilon_{\text{CRHF}})$ -secure if all  $t_{\text{CRHF}} = t_{\text{CRHF}}|(\kappa)$ -time adversaries  $\mathcal{A}$  have negligible advantage  $\epsilon_{\text{CRHF}} = \epsilon_{\text{CRHF}}(\kappa)$  with

$$\Pr \left[ \begin{array}{l} hk_{\text{CRHF}} \leftarrow \text{CRHF.KG}(1^\kappa), (m, m') \leftarrow \mathcal{A}(1^\kappa, hk_{\text{CRHF}}); \\ m \neq m', (m, m') \in \mathcal{M}_{\text{CRHF}}, \text{CRHF}(hk_{\text{CRHF}}, m) = \text{CRHF}(hk_{\text{CRHF}}, m') \end{array} \right] \leq \epsilon_{\text{CRHF}}$$

where the probability is over the random coins of the adversary and CRHF.KG.

If the hash key  $hk_{\text{CRHF}}$  is obvious from the context, we write  $\text{CRHF}(m)$  for  $\text{CRHF}(hk_{\text{CRHF}}, m)$ .

### 3.1 Pseudo-Random Functions

A *pseudo-random function* is an algorithm PRF that implements a deterministic function  $z = \text{PRF}(k, x)$ , taking as input a key (seed)  $k \in \mathcal{K}$  and some bit string  $x \in \mathcal{D}$ , and returning a string  $z \in \mathcal{R}$ , where  $\mathcal{K}$  is the key space,  $\mathcal{D}$  is the domain and  $\mathcal{R}$  is the range of PRF for security parameter  $\kappa$ . Let  $\mathcal{A}$  be an adversary that is given oracle access to either  $\text{PRF}(k, \cdot)$  for  $k \xleftarrow{\$} \mathcal{K}$  or a truly random function  $\text{RF}(\cdot)$  with the same domain and range as the pseudo-random function PRF.

**Definition 2.** We say that PRF is a  $(t, \epsilon_{\text{PRF}})$ -secure pseudo-random function, if any adversary  $\mathcal{A}$  running in probabilistic polynomial time  $t$  has at most an advantage of  $\epsilon_{\text{PRF}}$  to distinguish the pseudo-random function PRF from a truly random function RF, i.e.

$$\left| \Pr[\mathcal{A}^{\text{PRF}(k, \cdot)}(1^\kappa) = 1] - \Pr[\mathcal{A}^{\text{RF}(\cdot)}(1^\kappa) = 1] \right| \leq \epsilon_{\text{PRF}},$$

where  $\epsilon_{\text{PRF}} = \epsilon_{\text{PRF}}(\kappa)$  is a negligible function in  $\kappa$ .

### 3.2 Pseudo-Random Functions with Pairwise Independent Random Sources ( $\pi$ PRF)

This is a specific class of PRF introduced by Okamoto [30]. The  $\pi$ PRF family associated with key (seed) space  $\mathcal{K}$ , domain  $\mathcal{D}$  and range  $\mathcal{R}$  in the security parameter  $\kappa$ , states that if a specific variable  $k_{i_0} \xleftarrow{\$} \mathcal{K}$  is pairwise independent from other variable, then on input value  $m \in \mathcal{D}$  the output  $h \in \mathcal{R}$  of this function indexed by  $k_{i_0}$  is indistinguishable from random.

Suppose that function  $f_\Sigma : I_\Sigma \rightarrow X_\Sigma$  is a deterministic polynomial-time algorithm, where  $X_\Sigma$  is a set of random variables over  $\Sigma \xleftarrow{\$} \mathcal{K}$  and  $I_\Sigma$  is a set of indices regarding  $\Sigma$ , then this algorithm outputs  $k_i \in X_\Sigma$  from  $i \in I_\Sigma$ . Let  $(k_{i_0}, k_{i_1}, \dots, k_{i_{t(\kappa)}})$  ( $i_j \in I_\Sigma$ ) be pairwise independent random variables indexed by  $(I_\Sigma, f_\Sigma)$ , and each variable be uniformly distributed over  $\Sigma$ . That is, for any pair of  $(k_{i_0}, k_{i_j})$  ( $j = 1, \dots, t(\kappa)$ ), for any  $(x, y) \in \Sigma^2$ , we have  $\Pr[k_{i_0} \rightarrow x \wedge k_{i_j} \rightarrow y] = 1/|\Sigma|^2$ . Consider a PPT algorithm  $\mathcal{A}^{F, I_\Sigma}$  that can issue oracle queries. When  $\mathcal{A}$  sends  $q_j \in \mathcal{D}$  and  $i_j \in I_\Sigma$  to the query, the oracle replies with  $F_{k_j}^{\kappa, \Sigma, \mathcal{D}, \mathcal{R}}(q_j)$  for each  $j = 0, 1, \dots, t(\kappa)$ , where  $(\bar{k}_{i_0}, \bar{k}_{i_1}, \dots, \bar{k}_{i_{t(\kappa)}}) \xleftarrow{\$} (k_{i_0}, k_{i_1}, \dots, k_{i_{t(\kappa)}})$ .  $\mathcal{A}^{RF, I_\Sigma}$  is the same as  $\mathcal{A}^{F, I_\Sigma}$  except for  $F_{k_0}^{\kappa, \Sigma, \mathcal{D}, \mathcal{R}}(q_0)$  is replaced by a truly random function  $\text{RF}(q_0)$ .

**Definition 3.** We say that  $F$  is secure  $\pi$ PRF family if for any PPT adversary  $\mathcal{A}$  running in time  $t$  has at most an advantage of  $\epsilon_{\pi\text{PRF}} = \epsilon_{\pi\text{PRF}}(\kappa)$  in the security parameter  $\kappa$  to distinguish the  $\pi$ PRF from a truly random function, i.e.

$$|\Pr[\mathcal{A}^{F, I_\Sigma}(1^\kappa, \mathcal{D}, \mathcal{R}) = 1] - \Pr[\mathcal{A}^{\text{RF}, I_\Sigma}(1^\kappa, \mathcal{D}, \mathcal{R}) = 1]| \leq \epsilon_{\pi\text{PRF}},$$

where  $F$  is a  $\pi$ PRF family with index  $(I_\Sigma, f_\Sigma) \in \mathcal{K}$ .

### 3.3 Bilinear Groups

In the following, we briefly recall some of the basic properties of bilinear groups. Our AKE solution mainly consist of elements from a single group  $\mathbb{G}$ . We therefore concentrate on symmetric bilinear map (pairing).

**Definition 4** (Symmetric Bilinear groups). Let two cyclic groups  $\mathbb{G}$  and  $\mathbb{G}_T$  of prime order  $p$ . Let  $g$  be a generator of  $\mathbb{G}$ . The function

$$e : \mathbb{G} \times \mathbb{G} \rightarrow \mathbb{G}_T$$

is an (admissible) bilinear map if it holds that:

1. **Bilinear:** for all  $a, b \in \mathbb{G}$  and  $x, y \in \mathbb{Z}$ , we have  $e(a^x, b^y) = e(a, b)^{xy}$ .
2. **Non-degenerate:**  $e(g, g) \neq 1_{\mathbb{G}_T}$ , is a generator of group  $\mathbb{G}_T$ .
3. **Efficiency:**  $e$  is efficiently computable for all  $a, b \in \mathbb{G}$ .

We call  $(\mathbb{G}, g, \mathbb{G}_T, p, e)$  a symmetric bilinear groups.

### 3.4 The Decision Linear Diffie-Hellman Assumption

Let  $\mathbb{G}$  be a group of prime order  $p$ . Let  $g$  be a generator of  $\mathbb{G}$ , and along with arbitrary generators  $g_1$  and  $g_2$  of  $\mathbb{G}$ . Given tuple  $(g, g_1, g_2, g_1^a, g_2^b, g^c)$  for  $(a, b, c) \xleftarrow{\$} \mathbb{Z}_p^3$  the decisional linear Diffie-Hellman assumption says that it is hard to decide whether  $c = a + b \pmod{p}$ .

**Definition 5.** We say that the DLIN assumption holds if

$$\left| \Pr \left[ \mathcal{A}(g, g_1, g_2, g_1^a, g_2^b, g^{a+b}) = 1 \right] - \Pr \left[ \mathcal{A}(g, g_1, g_2, g_1^a, g_2^b, g^c) = 1 \right] \right| \leq \epsilon,$$

where  $(a, b, c) \xleftarrow{\$} \mathbb{Z}_p^3$ , for all probabilistic polynomial-time adversaries  $\mathcal{A}$ , where  $\epsilon_{\text{DLIN}} = \epsilon_{\text{DLIN}}(\kappa)$  is some negligible function in the security parameter.

### 3.5 The Bilinear Decisional Diffie-Hellman Assumption

Let  $\mathbb{G}$  and  $\mathbb{G}_T$  be groups of prime order  $p$ . Let  $e : \mathbb{G} \times \mathbb{G} \rightarrow \mathbb{G}_T$  be a bilinear map as definition 4. The Bilinear Decisional DH problem is stated as follows: given the tuple  $(g, g^a, g^b, g^c)$  for  $(a, b, c) \in \mathbb{Z}_p^3$  as input to distinguish the  $e(g, g)^{abc}$  from a random value.

**Definition 6.** We say that the  $(t, \epsilon)$ -BDDH assumption holds if

$$\left| \Pr \left[ \mathcal{A}(g, g^a, g^b, g^c, e(g, g)^{abc}) = 1 \right] - \Pr \left[ \mathcal{A}(g, g^a, g^b, g^c, e(g, g)^\gamma) = 1 \right] \right| \leq \epsilon,$$

where  $(a, b, c, \gamma) \xleftarrow{\$} \mathbb{Z}_p^4$ , for all probabilistic  $t$ -time adversaries  $\mathcal{A}$ , where  $\epsilon_{\text{BDDH}} = \epsilon_{\text{BDDH}}(\kappa)$  is some negligible function in the security parameter.

## 4 Security Model

In this section we present the formal security model for two party PKI-based authenticated key-exchange (AKE) protocol. In this model, while emulating the real-world capabilities of an active adversary, we provide an 'execution environment' for adversaries following an important line of research [5, 10, 22] which is initiated by Bellare and Rogaway [3]. In the sequel, we will use the similar framework as [34]. Let  $\mathcal{K}_{\text{KE}}$  be the key space of session key, and  $\{\mathcal{PK}, \mathcal{SK}\}$  be key spaces for long-term public/private key respectively. Let  $\mathcal{IDS}_{\text{KE}}$  be an identity space. Those spaces are associated with security parameter  $\kappa$  of considered protocol.

**Execution Environment.** In the execution environment, we fix a set of honest parties  $\{\text{ID}_1, \dots, \text{ID}_\ell\}$  for  $\ell \in \mathbb{N}$ , where  $\text{ID}$  is identity of a party which is chosen uniquely from space  $\mathcal{IDS}_{\text{KE}}$ . Each identity is associated with a long-term key pair  $(sk_i, pk_i) \in (\mathcal{SK}, \mathcal{PK})$  for entity authentication. Note that those identities are also lexicographically indexed via variable  $i \in [\ell]$ . For public key registration, each party  $\text{ID}_i$  might be required to provide extra information (denoted by  $\text{pf}$ ) to prove either the knowledge of the secret key or correctness of registered public key (via e.g. non-interactive proof of knowledge schemes). In practice, the concrete implementation of  $\text{pf}$  is up to the CA [8] and may be either interactive or non-interactive. Examples can be found in RFC 4210 [8] and PKCS#10. In this model we focus on non-interactive proof. Each honest party  $\text{ID}_i$  can sequentially and concurrently execute the protocol multiple times with different intended partners, this is characterized by a collection of oracles  $\{\pi_i^s : i \in [\ell], s \in [d]\}$  for  $d \in \mathbb{N}$ . Oracle  $\pi_i^s$  behaves as party  $\text{ID}_i$  carrying out a process to execute the  $s$ -th protocol instance, which has access to the long-term key pair  $(sk_i, pk_i)$  and to all other public keys. Moreover, we assume each oracle  $\pi_i^s$  maintains a list of independent internal state variables with semantics listed in Table 1.

Variable	Decryption
$\Psi_i^s$	recording the identity $\text{ID}_j$ of intended communication partner
$\Phi_i^s$	denoting the decision $\Phi_i^s \in \{\text{accept}, \text{reject}\}$
$K_i^s$	recording the session key $K_i^s \in \mathcal{K}_{\text{KE}}$ used for symmetric encryption
$st_i^s$	storing the maximum secret states that allows to be revealed by adversary
$sT_i^s$	recording the transcript of messages sent by oracle $\pi_i^s$
$rT_j^t$	recording the transcript of messages sent by oracle $\pi_i^s$

Table 1: Internal States of Oracles

All those variables of each oracle are initialized with empty string which is denoted by the symbol  $\emptyset$  in the following. At some point, each oracle  $\pi_i^s$  may complete the execution always with a decision state  $\Phi_i^s \in \{\text{accept}, \text{reject}\}$ . Furthermore, we assume that the session key is assigned to the variable  $K_i^s$  (such that  $K_i^s \neq \emptyset$ ) iff oracle  $\pi_i^s$  has reached an internal state  $\Phi_i^s = \text{accept}$ .

**Adversarial Model** An adversary  $\mathcal{A}$  in our model is a PPT Turing Machine taking as input the security parameter  $1^\kappa$  and the public information (e.g. generic description of above environment), which may interact with these oracles by issuing the following queries.

- $\text{Send}(\pi_i^s, m)$ : The adversary can use this query to send any message  $m$  of his own choice to oracle  $\pi_i^s$ . The oracle will respond the next message  $m^*$  (if any) to be sent according to the

protocol specification and its internal states. Oracle  $\pi_i^s$  would be initiated as *initiator* via sending the oracle the first message  $m = (\top, \widetilde{\text{ID}}_j)$  consisting of a special initialization symbol  $\top$  and a value  $\widetilde{\text{ID}}_j$ . The  $\widetilde{\text{ID}}_j$  is either the identity  $\text{ID}_j$  of intended partner or empty string  $\emptyset$ . After answering a `Send` query, the variables  $(\Psi_i^s, \Phi_i^s, K_i^s, st_i^s, T_i^s)$  will be updated depending on the specific protocol.<sup>3</sup>

- `RevealKey`( $\pi_i^s$ ): Oracle  $\pi_i^s$  responds with the contents of variable  $K_i^s$ .
- `StateReveal`( $\pi_i^s$ ): Oracle  $\pi_i^s$  responds with the secret state stored in variable  $st_i^s$ .
- `Corrupt`( $\text{ID}_i$ ): Oracle  $\pi_i^1$  responds with the long-term secret key  $sk_i$  of party  $\text{ID}_i$ . After this query, oracles  $\pi_i^s (s > 1)$  can still answer other queries.
- `EstablishParty`( $\text{ID}_m, pk_m, pf_m$ ): This query allows the adversary to register an identity  $\text{ID}_m (\ell < m < \mathbb{N})$  and a static public key  $pk_m$  on behalf of a party  $P_m$ , if either the adversary is ensured to know the secret key  $sk_m$  or the  $pk_m$  is ensured to be correctly formed corresponding to the public key  $pk_m$  by evaluating  $pf_m$  based on  $pk_m$ . Parties established by this query are called corrupted or dishonest.<sup>4</sup>
- `Test`( $\pi_i^s$ ): This query may only be asked once throughout the experiment. Oracle  $\pi_i^s$  handles this query as follows: If the oracle has state  $\Omega = \text{reject}$  or  $K_i^s = \emptyset$ , then it returns some failure symbol  $\perp$ . Otherwise it flips a fair coin  $b$ , samples a random element  $K_0$  from key space  $\mathcal{K}_{\text{KE}}$ , sets  $K_1 = K_i^s$  to the real session key, and returns  $K_b$ .

We stress that the exact meaning of the `StateReveal` must be defined by each protocol separately, and each protocol should be proven secure to resist with such kind of state leakage as its claimed. Namely a protocol should specify the content stored in the variable  $st$  during protocol execution.<sup>5</sup> The `EstablishParty` query is used to model the chosen public key attacks. In this query, the detail form of  $pf$  should be specified by each protocol. Please note that one could specify  $pf = \emptyset$  to model arbitrary public key registration without checking anything.

**Secure AKE Protocols.** To formalize the notion that two oracles are engaged in an on-line communication, we define the partnership via *matching sessions* which was first formulated by Krawczyk [21].

**Definition 7.** We say that an oracle  $\pi_i^s$  has a *matching session* to oracle  $\pi_j^t$ , if both oracles accept and

- $\Psi_i^s = \text{ID}_j$  and  $\Psi_j^t = \text{ID}_i$  and

<sup>3</sup>For example, the variable  $\Psi_i^s$  will be set as identity  $j$  at some point when the oracle receives a message containing the identity of its partner; the messages  $(m, m^*)$  will be orderly appended to transcript  $T_i^s$ . A protocol here might be either run in pre- or post-specified peer setting [11].

<sup>4</sup>We only require that the proof is non-interactive to simplify the model (if a common reference string is required we may assume that it is held by the execution environment and made publicly available). In practice, the concrete implementation of these proofs of possession is up to the CA and may also be interactive. We only require that it is secure under concurrent executions.

<sup>5</sup>To make our model simple, we only use a unified `StateReveal` query other than using different kinds of such queries with various ‘aliases’. Since no matter how many different kinds of ‘`StateReveal`’ are modelled, each protocol should specify which ‘`StateReveal`’ query it can resist with. We here just assume all states stored on untrusted host machine (e.g. the personal computer) are susceptible to `StateReveal` query.



- $sT_i^s = rT_j^t$  and  $rT_i^s = rT_j^s$

SECURITY GAME. This game is played between a challenger  $\mathcal{C}$  and an adversary  $\mathcal{A}$ , where the following steps are performed:

1. At the beginning of the game, the challenger  $\mathcal{C}$  implements the collection of oracles  $\{\pi_i^s : i \in [\ell], s \in [d]\}$ , and generates  $\ell$  long-term key pairs  $(pk_i, sk_i)$  and corresponding proof  $\text{pf}_i$  (if any) for all honest parties  $\text{ID}_i$  for  $i \in [\ell]$  where the identity  $\text{ID}_i \in \mathcal{IDS}_{\text{KE}}$  of each party is chosen uniquely.  $\mathcal{C}$  gives adversary  $\mathcal{A}$  all identities, public keys and corresponding proofs  $\{(\text{ID}_1, pk_1, \text{pf}_1), \dots, (\text{ID}_\ell, pk_\ell, \text{pf}_\ell)\}$  as input.
2.  $\mathcal{A}$  may issue polynomial number of queries as aforementioned, namely  $\mathcal{A}$  makes queries: `Send`, `StateReveal`, `Corrupt`, `EstablishParty` and `RevealKey`.
3. At some point,  $\mathcal{A}$  may issue a `Test`( $\pi_i^s$ ) query on an oracle  $\pi_i^s$  during the game with only once.
4. At the end of the game, the  $\mathcal{A}$  terminates with outputting a bit  $b'$  as its guess for  $b$  of `Test` query.

For the security definition, we need the notion of *freshness* of oracle.

**Definition 8** (Freshness). Let  $\pi_i^s$  be an accepted oracle held by an party  $\text{ID}_i$  with intended partner  $\text{ID}_j$ . Meanwhile, let  $\pi_j^t$  be an oracle (if it exists) held by a party  $\text{ID}_j$  with intended partner  $\text{ID}_i$ , such that  $\pi_i^s$  has a matching conversation to  $\pi_j^t$ . Then the oracle  $\pi_i^s$  is said to be *fresh* if none of the following conditions holds:

1. The party  $\text{ID}_j$  is established by adversary  $\mathcal{A}$  via `EstablishParty` query.
2.  $\mathcal{A}$  either makes query `RevealKey`( $\pi_i^s$ ), or `RevealKey`( $\pi_j^t$ ) (if such  $\pi_j^t$  exists).
3. If  $\pi_j^t$  exists,  $\mathcal{A}$  either makes queries:
  - (a) Both `Corrupt`( $\text{ID}_i$ ) and `StateReveal`( $\pi_i^s$ ), or
  - (b) Both `Corrupt`( $\text{ID}_j$ ) and `StateReveal`( $\pi_j^t$ ).
4. If  $\pi_j^t$  does not exist,  $\mathcal{A}$  either makes queries:
  - (a) Both `Corrupt`( $\text{ID}_i$ ) and `StateReveal`( $\pi_i^s$ ), or
  - (b) `Corrupt`( $\text{ID}_j$ ).

**Definition 9** (Session Key Security). A key exchange protocol  $\Sigma$  is called  $(t, \epsilon)$ -session-key-secure if for all adversaries  $\mathcal{A}$  running within time  $t$  in the above security game and for some negligible probability  $\epsilon = \epsilon(\kappa)$  in the security parameter  $\kappa$ , it holds that:

- If two oracles  $\pi_i^s$  and  $\pi_j^t$  accept with matching conversations, then both oracles hold the same session key.
- When  $\mathcal{A}$  returns  $b'$  such that
  - $\mathcal{A}$  has issued a `Test` query on an oracle  $\pi_i^s$  without failure,
  - $\pi_i^s$  is fresh throughout the security experiment,

then the probability that  $b'$  equals the bit  $b$  sampled by the `Test`-query is bounded by

$$|\Pr[b = b'] - 1/2| \leq \epsilon.$$

## 5 A One-round Two Party AKE Protocol from $\pi$ PRF

In this section we present a pairing-based strong AKE protocol without random oracles and NAXOS trick based on pseudo-Random function family with pairwise independent random sources ( $\pi$ PRF) as key derivation function.

### 5.1 Protocol Description

The AKE protocol takes as input the following building blocks:

- Symmetric bilinear groups  $(\mathbb{G}, g, \mathbb{G}_T, p, e)$ , where the generator of group  $\mathbb{G}_T$  is  $e(g, g)$  and along with another random generators  $g_1, g_2$  and  $h$  of  $\mathbb{G}$ .
- A collision resistant hash function  $\text{CRHF}(hk, \cdot) : \mathcal{K}_{\text{CRHF}} \times \{0, 1\}^* \rightarrow \mathbb{Z}_p$  where  $hk$  is chosen uniformly at random from space  $\mathcal{K}_{\text{CRHF}}$ .
- A pairwise independent pseudo-random function family ( $\pi$ PRF)  $F(\cdot, \cdot) : \mathbb{G}_T \times \{0, 1\}^* \rightarrow \mathcal{K}_{\text{KE}}$ , with index  $\{I_{\mathbb{G}_T}, f_{\mathbb{G}_T}\}$  where  $I_{\mathbb{G}_T} := \{(U, V, \alpha) | (U, V, \alpha) \in \mathbb{G}_T^2 \times \mathbb{Z}_p\}$  and  $f_{\mathbb{G}_T} : (U, V, \alpha) \rightarrow U^{r_1 + \alpha r_2} V$  with  $(r_1, r_2) \xleftarrow{\$} \mathbb{Z}_p^2$ .

The variable  $pms$  stores the public system parameters

$pms := (\mathcal{PG}, e(g, h), e(g_1, h), e(g_2, h), hk)$  where each element is initialized by corresponding building block as above.

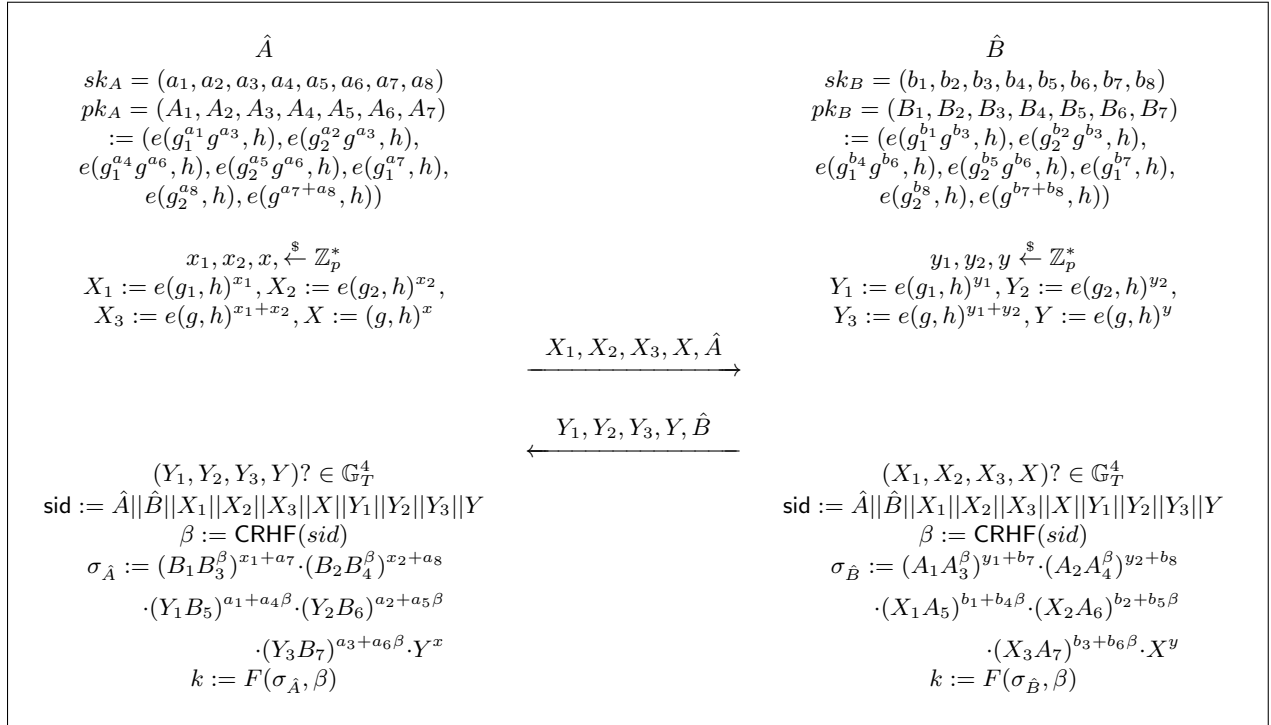


Figure 1: AKE Protocol from  $\pi$ PRF

**Long-term key generation and Registration:** On input the system parameter  $pms$ , the long-term keys of each party  $\hat{A}$  is generated as following:

- $\hat{A}$  selects long-term private keys :  
 $sk_A = (a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8) \xleftarrow{\$} \mathbb{Z}_p^8$ , and compute the long-term public keys:  
 $pk_A = (A_1, A_2, A_3, A_4, A_5, A_6, A_7) := (e(g_1^{a_1} g^{a_3}, h), e(g_2^{a_2} g^{a_3}, h), e(g_1^{a_4} g^{a_6}, h),$   
 $e(g_2^{a_5} g^{a_6}, h), e(g_1^{a_7}, h), e(g_2^{a_8}, h), e(g^{a_7+a_8}, h)).$

In order to register the public key  $pk_A$ , we require each party  $\hat{A}$  to provide the  $\text{pf}_{\hat{A}} := (g_1^{a_1}, g_2^{a_2}, g_1^{a_4}, g_2^{a_5}, g^{a_1}, g^{a_2}, g^{a_3}, g^{a_4}, g^{a_5}, g^{a_6})$ . Upon receiving a request on registering public key  $pk_{\hat{A}}$  with  $\text{pf}_{\hat{A}}$ , the CA does the following steps:

1. choose random values  $\theta_1, \theta_2, \theta_3, \theta_4 \xleftarrow{\$} \mathbb{Z}_p$ .
2. reject the registration if

$$e((g_1^{a_1})^{\theta_1} (g_2^{a_2})^{\theta_2} (g_1^{a_4})^{\theta_3} (g_2^{a_5})^{\theta_4}, g) \neq (e(g^{a_1}, g_1)^{\theta_1} (e(g^{a_2}, g_2))^{\theta_2} (e(g^{a_4}, g_1))^{\theta_3} (e(g^{a_5}, g_2))^{\theta_4}).$$

3. reject the registration if

$$A_1^{\theta_1} A_2^{\theta_2} A_3^{\theta_3} A_4^{\theta_4} \neq e((g_1^{a_1} g^{a_3})^{\theta_1} (g_2^{a_2} g^{a_3})^{\theta_2} (g_1^{a_4} g^{a_6})^{\theta_3} (g_2^{a_5} g^{a_6})^{\theta_4}, h).$$

4. accept the registration.

Those checks ensure that the public key  $pk_{\hat{A}}$  is consistent with the protocol specification (i.e. it is correctly formed). In particular the  $\text{pf}_{\hat{A}}$  is required while  $\mathcal{A}$  queries the  $\text{EstablishParty}(\hat{A}, pk_{\hat{A}}, \text{pf}_{\hat{A}})$  in which the above checks are performed. We will show in the proof that the  $\text{pf}_{\hat{A}}$  corresponding to registered public key  $pk_{\hat{A}}$  would make security proof go through.

**Protocol Execution :** On input  $pms$ , the protocol between two parties  $\hat{A}$  and  $\hat{B}$  is proceed as follows, which is also briefly illustrated in the Figure 1.

1. Upon activation a session at a party  $\hat{A}$ , it performs the steps: (a) choose three ephemeral private keys  $x_1, x_2, x, \xleftarrow{\$} \mathbb{Z}_p^3$ ;  
(b) compute  $X_1 := e(g_1, h)^{x_1}, X_2 := e(g_2, h)^{x_2}, X_3 := e(g, h)^{x_1+x_2}$  and  $X := (g, h)^x$ ; (c) send  $(X_1, X_2, X_3, X, \hat{A})$  to  $\hat{B}$ .
2. Upon activation a session at a party  $\hat{B}$ , it performs the steps: (a) choose three ephemeral private keys  $y_1, y_2, y, \xleftarrow{\$} \mathbb{Z}_p^3$ ;  
(b) compute  $Y_1 := e(g_1, h)^{y_1}, Y_2 := e(g_2, h)^{y_2}, Y_3 := e(g, h)^{y_1+y_2}$  and  $Y := e(g, h)^y$ ; (c) send  $(Y_1, Y_2, Y_3, Y, \hat{B})$  to  $\hat{A}$ .
3. Upon receiving  $(Y_1, Y_2, Y_3, Y, \hat{B})$  from  $\hat{B}$ ,  $\hat{A}$  does the following: (a) verify that  $(Y_1, Y_2, Y_3, Y) \in \mathbb{G}_T^4$ ; (b) set session identifier  
 $\text{sid} := \hat{A} \parallel \hat{B} \parallel X_1 \parallel X_2 \parallel X_3 \parallel X \parallel Y_1 \parallel Y_2 \parallel Y_3 \parallel Y$ , and compute  $\beta := \text{CRHF}(\text{sid})$ ; (c) compute  $\sigma_{\hat{A}} := (B_1 B_3^\beta)^{x_1+a_7} \cdot (B_2 B_4^\beta)^{x_2+a_8} \cdot (Y_1 B_5)^{a_1+a_4\beta} \cdot (Y_2 B_6)^{a_2+a_5\beta} \cdot (Y_3 B_7)^{a_3+a_6\beta} \cdot Y^x$ ; (d) compute session key as  $k := F(\sigma_{\hat{A}}, \beta)$  and erase all intermediate values and  $y_1, y_2, y$ .

4. Upon receiving  $(X_1, X_2, X_3, X, \hat{A})$  from  $\hat{A}, \hat{B}$  does the following: (a) verify that  $(X_1, X_2, X_3, X) \in \mathbb{G}_T^4$ ; (b) set session identifier  $\text{sid} := \hat{A} \parallel \hat{B} \parallel X_1 \parallel X_2 \parallel X_3 \parallel X \parallel Y_1 \parallel Y_2 \parallel Y_3 \parallel Y$ , and compute  $\beta := \text{CRHF}(\text{sid})$ ; (c) compute  $\sigma_{\hat{B}} := (A_1 A_3^\beta)^{y_1 + b_7} \cdot (A_2 A_4^\beta)^{y_2 + b_8} \cdot (X_1 A_5)^{b_1 + b_4 \beta} \cdot (X_2 A_6)^{b_2 + b_5 \beta} \cdot (X_3 A_7)^{b_3 + b_6 \beta} \cdot X^y$ ; (d) compute session key  $k := F(\sigma_{\hat{B}}, \beta)$  and erase all intermediate values and  $y_1, y_2, y$ .

**Session States:** We thus assume, only the ephemeral private keys, i.e.  $(x_1, x_2, x)$  would be stored as secret in the state variable  $st$ . This can be achieved by performing the computation in steps 2.(c) and 2.(d) on a smart card, where the long-term keys are stored. In this case, the intermediate values would not be exposed due to e.g. malware attacks on the PC, which we model with `StateReveal` query.

## 5.2 Comparisons

We summarize the comparisons of some existing well-known eCK (CK+) secure protocols without random oracles in the Table 2 from the following perspectives: (i) the knowledge of peer’s public key (peer setting), which would affect e.g. the round efficiency as aforementioned; (ii) the special design trick used to guarantee corresponding security; (iii) the security assumptions; (iv) number of long-term (LL) and ephemeral (Eph) keys which may affect the storage requirement, computation cost and communication cost; (v) overall computation cost and the computation cost on secure device (SD) of considered protocol in the order of regular exponentiation (E), multiple-exponentiation and pairing operation (P). In the table, ‘(T)CR’ denotes (target) collision-resistant hash, ‘DDH’ denotes the decisional Diffie-Hellman assumption, ‘DLIN’ denotes the decisional linear Diffie-Hellman assumption, ‘EXT’ denotes the strong randomness extractor, and ‘TP’ denotes the twisted-PRF trick.

	Peer setting	Design trick	Security assumptions	LL pk,sk	Eph pk,sk	Overall cost E,ME,P	SD cost E,ME,P
Okamoto [30]	post	TP	CR, DDH, $\pi$ PRF	2,4	3,2	3,1,0	2,1,0
MO [29]	post	-	CR, DDH, $\pi$ PRF	4,5	3,2	3,1,0	0,1,0
YS [37]	post	-	CR, BDDH DLIN, $\pi$ PRF	7,8	4,3	5,1,1	3,0,1
FSXY [14]	pre	TP	DDH, EXT, PRF	3,3	2,2	5,1,0	4,1,0
proposal	post	-	CR, BDDH DLIN, $\pi$ PRF	7,8	4,3	4,1,0	0,1,0

Table 2: Comparisons

As for the FSXY protocol, the situation is complicated since we have to instantiate the KEM used in their construction to make it comparable to other schemes. We select the DDH based KEM [16, 20] and the traditional Diffie-Hellman key exchange (DHKE) [13] for comparison, which are also suggested by FSXY protocol. But we note that such scheme can only run in pre-specified peer setting, as well as other efficient IND-CCA KEM with standard assumptions such as [17]. While instantiating FSXY protocol to be able to run in post-specified peer setting, the Okamoto KEM [30] might be a candidate but in which case the  $\pi$ PRF is required. Then the Okamoto protocol [30] can be seen as an ‘optimized’ version of FSXY scheme. In addition we assume without loss of generality the Okamoto [30], MO [29] and YS [37] protocols are realized with secure device to protect those critical session states. For simplicity and security, we just assume the whole KEM (both encapsulation/decapsulation algorithms) are realized at secure device as otherwise it

can not provide eCK security at all. Similarly the twisted-PRF trick used by both Okamoto and FSXY schemes might need to implement on secure device to avoid any leakage on its output.

### 5.3 Security Analysis

We show the security of proposed protocol in our strong security model. Assume each ephemeral key chosen during key exchange has bit-size  $\lambda \in \mathbb{N}$ .

**Theorem 1.** *Suppose that the  $(t, q, \epsilon_{\text{BDDH}})$ -Bilinear DDH assumption and  $(t, q, \epsilon_{\text{DLIN}})$ -Decision linear assumption hold in bilinear groups  $(\mathbb{G}, g, \mathbb{G}_T, p, e)$ , the hash function CRHF is  $(t, \epsilon_{\text{CRHF}})$ -secure, and a  $(t, \epsilon_{\pi\text{PRF}})$ -secure  $\pi\text{PRF}$  family with index  $\{I_{\mathbb{G}_T}, f_{\mathbb{G}_T}\}$  where  $I_{\mathbb{G}_T} := \{(U, V, \alpha) \mid (U, V, \alpha) \in \mathbb{G}^2 \times \mathbb{Z}_p\}$  and  $f_{\mathbb{G}_T} := (U, V, \alpha) \rightarrow U^{r_1 + \alpha r_2} V$  with  $(r_1, r_2) \xleftarrow{\$} \mathbb{Z}_p^2$ . Then the proposed protocol is a  $(t', \epsilon)$ -eCK secure AKE in the sense of Definition 9 with  $t' \approx t$  and  $\epsilon \leq \frac{d^2 \ell^2}{2^\lambda} + \epsilon_{\text{CRHF}} + 3 \cdot (d^2 \ell^2 \cdot \epsilon_{\text{BDDH}} + d\ell \cdot (\epsilon_{\text{DLIN}} + 6/2^\lambda)) + \epsilon_{\pi\text{PRF}}$ .*

We now verify that no polynomially bounded adversary can distinguish the real session key of a fresh oracle from a random key. We first introduce the notations might be used in the proof. When describing the communication between oracle  $\pi_{\hat{A}}^s$  and party  $\hat{C}^{(s)}$  ( $s \in [\ell - 1]$ ), we use the following notations:

- $(x_1^{(s)}, x_2^{(s)}, x^{(s)})$ : the ephemeral private keys of oracle  $\pi_{\hat{A}}^s$ .
- $(X_1^{(s)}, X_2^{(s)}, X_3^{(s)}, X^{(s)}) = (e(g_1, h)^{x_1^{(s)}}, e(g_2, h)^{x_2^{(s)}}, e(g, h)^{x_1^{(s)} + x_2^{(s)}}, e(g, h)^{x^{(s)}})$ : the ephemeral public keys of oracle  $\pi_{\hat{A}}^s$ .
- $(C_1^{(s)}, C_2^{(s)}, C_3^{(s)}, C_4^{(s)}, C_5^{(s)}, C_6^{(s)}, C_7^{(s)}) = (e(g_1^{c_1^{(s)}} g^{c_3^{(s)}}), h), e(g_2^{c_2^{(s)}} g^{c_3^{(s)}}), h), e(g_1^{c_4^{(s)}} g^{c_6^{(s)}}), h, e(g_2^{c_5^{(s)}} g^{c_6^{(s)}}), h, e(g_1, h)^{c_7^{(s)}}, e(g_2, h)^{c_8^{(s)}}, e(g, h)^{c_7^{(s)} + c_8^{(s)}})$ : the public keys of party  $\hat{C}^{(s)}$ .
- $\text{pk}_{\hat{C}^{(s)}} := (g_1^{c_1^{(s)}}, g_2^{c_2^{(s)}}, g_1^{c_4^{(s)}}, g_2^{c_5^{(s)}}, g^{c_1^{(s)}}, g^{c_2^{(s)}}, g^{c_3^{(s)}}, g^{c_4^{(s)}}, g^{c_5^{(s)}}, g^{c_6^{(s)}})$ : the valid consistent proof of  $\text{pk}_{\hat{C}^{(s)}}$ .
- $(W_1^{(s)}, W_2^{(s)}, W_3^{(s)}, W^{(s)}) = (e(g_1, h)^{w_1^{(s)}}, e(g_2, h)^{w_2^{(s)}}, e(g, h)^{w_1^{(s)} + w_2^{(s)}}, e(g, h)^{w^{(s)}})$ : the ephemeral public keys received by oracle  $\pi_{\hat{A}}^s$ .
- $\text{sid}_{\hat{A}}^{(s)} = (\hat{A}, \hat{C}^{(s)}, X_1^{(s)}, X_2^{(s)}, X_3^{(s)}, X^{(s)}, W_1^{(s)}, W_2^{(s)}, W_3^{(s)}, W^{(s)})$ .
- $\beta_{\hat{A}}^{(s)} = \text{CRHF}(\text{sid}_{\hat{A}}^{(s)})$ .
- $\sigma_{\hat{A}}^{(s)} = (C_1^{(s)} C_3^{(s) \beta_{\hat{A}}^{(s)}})^{x_1^{(s)} + a_7} \cdot (C_2^{(s)} C_4^{(s) \beta_{\hat{A}}^{(s)}})^{x_2^{(s)} + a_8} \cdot (W_1^{(s)} C_5^{(s)})^{a_1 + a_4 \beta_{\hat{A}}^{(s)}} \cdot (W_2^{(s)} C_6^{(s)})^{a_2 + a_5 \beta_{\hat{A}}^{(s)}} \cdot (W_3^{(s)} C_7^{(s)})^{a_3 + a_6 \beta_{\hat{A}}^{(s)}} \cdot W^{(s) x^{(s)}}$ .
- $k_{\hat{A}}^{(s)} = F(\sigma_{\hat{A}}^{(s)}, \beta_{\hat{A}}^{(s)})$ .

We describe the communication between oracle  $\pi_{\hat{B}}^t$  and party  $\hat{D}^{(t)}$  ( $t \in [\ell - 1]$ ), using the following notations:

- $(y_1^{(t)}, y_2^{(t)}, y^{(t)})$ : the ephemeral private keys of oracle  $\pi_{\hat{B}}^t$ .
- $(Y_1^{(t)}, Y_2^{(t)}, Y_3^{(t)}, Y^{(t)}) = (e(g_1, h)^{y_1^{(t)}}, e(g_2, h)^{y_2^{(t)}}, e(g, h)^{y_1^{(t)}+y_2^{(t)}}, e(g, h)^{y^{(t)}})$ : the ephemeral public keys of oracle  $\pi_{\hat{B}}^t$ .
- $(D_1^{(t)}, D_2^{(t)}, D_3^{(t)}, D_4^{(t)}, D_5^{(t)}, D_6^{(t)}, D_7^{(t)}) = (e(g_1^{d_1^{(t)}} g^{d_3^{(t)}}, h), e(g_2^{d_2^{(t)}} g^{d_3^{(t)}}, h), e(g_1^{d_4^{(t)}} g^{d_6^{(t)}}, h), e(g_2^{d_5^{(t)}} g^{d_6^{(t)}}, h), e(g_1, h)^{d_7^{(t)}}, e(g_2, h)^{d_8^{(t)}}, e(g, h)^{b_7+b_8})$ : the public keys of party  $\hat{D}^{(t)}$ .
- $\text{pk}_{\hat{D}^{(t)}} := (g_1^{d_1^{(t)}}, g_2^{d_2^{(t)}}, g_1^{d_4^{(t)}}, g_2^{d_5^{(t)}}, g^{d_1^{(t)}}, g^{d_2^{(t)}}, g^{d_3^{(t)}}, g^{d_4^{(t)}}, g^{d_5^{(t)}}, g^{d_6^{(t)}})$ : the valid consistent proof of public key  $\text{pk}_{\hat{D}^{(t)}}$ .
- $(N_1^{(t)}, N_2^{(t)}, N_3^{(t)}, N^{(t)}) = (e(g_1, h)^{n_1^{(t)}}, e(g_2, h)^{n_2^{(t)}}, e(g, h)^{n_1^{(t)}+n_2^{(t)}}, e(g, h)^{n^{(t)}})$ : the ephemeral public keys received by oracle  $\pi_{\hat{B}}^t$ .
- $\text{sid}_{\hat{A}}^{(s)} = (\hat{A}, \hat{C}^{(s)}, Y_1^{(t)}, Y_2^{(t)}, Y_3^{(t)}, Y^{(t)}, N_1^{(t)}, N_2^{(t)}, N_3^{(t)}, N^{(t)})$ .
- $\beta_{\hat{A}}^{(s)} = \text{CRHF}(\text{sid}_{\hat{A}}^{(s)})$ .
- $\sigma_{\hat{A}}^{(s)} = (D_1^{(t)} D_3^{(t) \beta_{\hat{A}}^{(s)}})^{y_1^{(t)}+a_7} \cdot (D_2^{(t)} D_4^{(t) \beta_{\hat{A}}^{(s)}})^{y_2^{(t)}+a_8} \cdot (N_1^{(t)} D_5^{(t)})^{a_1+a_4 \beta_{\hat{A}}^{(s)}} \cdot (N_2^{(t)} D_6^{(t)})^{a_2+a_5 \beta_{\hat{A}}^{(s)}} \cdot (N_3^{(t)} D_7^{(t)})^{a_3+a_6 \beta_{\hat{A}}^{(s)}} \cdot N^{(t) y^{(t)}}$ .
- $k_{\hat{B}}^{(t)} = F(\sigma_{\hat{B}}^{(t)}, \beta_{\hat{B}}^{(t)})$ .

We examine the probability of adversary in breaking the indistinguishability of session key of the test oracle, according to the following complementary events and all freshness related cases as Definition 8:

- **Event 1:** There is a oracle  $\pi_{\hat{B}}^{t^*}$  held by  $\hat{B}$ , such that  $\pi_{\hat{A}}^{s^*}$  and  $\pi_{\hat{B}}^{t^*}$  have matching conversations, and we have the following disjoint cases:
  - Case 1 ( $C1$ ): The adversary doesn't issue  $\text{Corrupt}(\hat{A})$  and  $\text{Corrupt}(\hat{B})$ .
  - Case 2 ( $C2$ ): The adversary doesn't issue  $\text{StateReveal}(\pi_{\hat{A}}^{s^*})$  and  $\text{StateReveal}(\pi_{\hat{B}}^{t^*})$ .
  - Case 3 ( $C3$ ): The adversary doesn't issue  $\text{StateReveal}(\pi_{\hat{A}}^{s^*})$  and  $\text{Corrupt}(\hat{B})$ .
  - Case 4 ( $C4$ ): The adversary doesn't issue  $\text{Corrupt}(\hat{A})$  and  $\text{StateReveal}(\pi_{\hat{B}}^{t^*})$ .
- **Event 2:** There is no oracle  $\pi_{\hat{B}}^{t^*}$  held by  $\hat{B}$ , such that  $\pi_{\hat{A}}^{s^*}$  and  $\pi_{\hat{B}}^{t^*}$  have matching conversations, and we have the following disjoint events:
  - Case 5 ( $C5$ ): The adversary doesn't issue  $\text{Corrupt}(\hat{A})$  and  $\text{Corrupt}(\hat{B})$ .
  - Case 6 ( $C6$ ): The adversary doesn't issue  $\text{StateReveal}(\pi_{\hat{A}}^{s^*})$  and  $\text{Corrupt}(\hat{B})$ .

In order to complete the proof of Theorem 1, we must provide the security proofs for above six cases. However, due to the Propositions 1 and 2 from [28], the security proofs for Cases  $C1, C3, C4$  can be reduced to the security proof for Case  $C5$ . Therefore, we prove the advantage of the adversary is negligible in security parameter  $\kappa$ , for cases  $C2, C5$  and  $C6$  respectively.

Let  $S_\delta$  be the event that the adversary wins the security experiment under the Game  $\delta$  and freshness cases in the set  $\{C2, C5, C6\}$ . Let  $\text{Adv}_\delta := \Pr[S_\delta] - 1/2$  denote the advantage of  $\mathcal{A}$  in Game  $\delta$ .

**Game 0.** This is the original eCK game with adversary  $\mathcal{A}$  under freshness cases  $C2, C5$  and  $C6$ . Meanwhile, the challenger chooses an uniform random value  $r \xleftarrow{\$} \mathbb{Z}_p$ , and sets  $h := g^r$  as public parameter. Thus we have that

$$\Pr[S_0] = 1/2 + \epsilon = \text{Adv}_0 + 1/2.$$

**Game 1.** In this game, the challenger proceeds exactly like previous game, except that we add an abort rule. The challenger raises event  $\text{abort}_{\text{eph}}$  and aborts, if during the simulation an ephemeral key  $X$  or  $Y$  replied by an oracle  $\pi_i^s$  but it has been sample by another oracle  $\pi_j^t$  or sent by adversary before. Since there are  $d\ell$  such ephemeral keys would be sampled uniform randomly from  $\mathbb{Z}_p$ . Thus, the event  $\text{abort}_{\text{eph}}$  occurs with probability  $\Pr[\text{abort}_{\text{eph}}] \leq \frac{d^2\ell^2}{2\lambda}$ . We therefore have that

$$|\text{Adv}_0 - \text{Adv}_1| \leq \frac{d^2\ell^2}{2\lambda}.$$

Note that the ephemeral key chosen by each oracle is unique in this game, so that the adversary can't replay any ephemeral key to result in two oracles generating the same session key but without matching sessions.

**Game 2.** This game proceeds as the previous game, except that the simulator aborts if the adversary completes two oracles  $\pi_A^s$  and  $\pi_B^t$  such that  $\text{CRHF}(\text{sid}_A^{(s)}) = \text{CRHF}(\text{sid}_B^{(t)})$  but those two oracles have no matching sessions. Hence we have for any  $\text{sid}_A^{(s)} \neq \text{sid}_B^{(t)}$  ( $s, t \in [d]^2$ ). Then, if the collision event does not occur, the simulation is equivalent to Game 1. When the event does occur, we can easily construct algorithm that breaks the collision-resistant hash function CRHF by outputting  $(\text{sid}_A^{(s)}, \text{sid}_B^{(t)})$ . Hence we have that

$$|\text{Adv}_1 - \text{Adv}_2| \leq \epsilon_{\text{CRHF}}.$$

**Game 3.** In this game proceeds as the previous one, but we modify the game according to guessed freshness cases as: (i) case  $C2$ : replace the key  $k_A^{(s^*)}$  of test oracle  $\pi_A^{s^*}$  and its partner oracle  $\pi_B^{t^*}$  with random value  $\widetilde{k_A^{(s^*)}}$ ; (ii) case  $C5$ : replace the DH tuple  $(g, g_1, g_2, A_5, A_6, A_7)$  with a random

tuple; and (iii) case  $C6$ : replace the DH tuple  $(g, g_1, g_2, X_1^{(s)}, X_2^{(s)}, X_3^{(s)})$  with a random tuple. First note that the probability that the chosen tuple for instance  $(g, g_1, g_2, A_5, A_6, A_7)$ , such that  $g_1 \neq g_2 \neq g \neq 1_G$  and  $\log_{g_1} A_5 + \log_{g_2} A_6 \neq \log_g A_7$ , is at least  $1 - 6/2^\lambda$ . Since the elements in DH tuple  $(g, g_1, g_2, A_5, A_6, A_7)$  are uniformly selected at random. If there exists an adversary  $\mathcal{A}$  can distinguish between the Game 3 from Game 2 then we can use it to construct a distinguisher  $\mathcal{D}$  to solve either the BDDH problem in case  $C2$  or DLIN in cases  $C5, C6$ . Basically,  $\mathcal{D}$  first guesses: (i) the fresh cases in set  $\{C2, C5, C6\}$  with probability at least  $1/3$ . (ii) the test oracle and its partner oracle for the case  $C2$  with probability at least  $1/(d^2\ell^2)$ ; (iii) the test oracle for the cases  $C5$  and  $C6$  with probability at least  $1/(d\ell)$ . Since the challenger activates  $d$  oracles for each  $\ell$  parties. Meanwhile, if  $\mathcal{D}$  guesses incorrectly in either case, then it aborts the game. Next  $\mathcal{D}$  simulates game for  $\mathcal{A}$  as the simulator in Game 2 except for the following modifications:

1. **Case  $C2$ .** Given a BDDH challenge instance  $(\bar{g}, u, v, w, \Gamma)$ ,  $\mathcal{B}$  does modifications:

- set  $g = \bar{g}$ ,  $h = w$  and  $X^{(s^*)} = e(u, h)$  and  $Y^{(s^*)} = e(v, h)$ ;
- set  $g_1 = g^{\mu_1}$  and  $g_2 = g^{\mu_2}$ , where  $\mu_1, \mu_2 \xleftarrow{\$} \mathcal{Z}_p^2$ ;
- compute the session key of test oracle as:
  - generate secret exponent  $\gamma = \mu_1(x_1^{(s^*)} + a_7)(b_1 + b_4\beta_{\hat{A}}^{(s^*)}) + \mu_2(x_2^{(s^*)} + a_8)(b_2 + b_5\beta_{\hat{A}}^{(s^*)}) + (x_1^{(s^*)} + x_2^{(s^*)} + a_7 + a_8)(b_3 + b_6\beta_{\hat{A}}^{(s^*)}) + \mu_1(y_1^{(s^*)} + b_7)(a_1 + a_4\beta_{\hat{A}}^{(s^*)}) + \mu_2(y_2^{(s^*)} + b_8)(a_2 + b_5\beta_{\hat{A}}^{(s^*)}) + ((y_1^{(s^*)} + y_2^{(s^*)})\beta_{\hat{A}}^{(s^*)} + b_7 + b_8)(a_3 + a_6\beta_{\hat{A}}^{(s^*)})$ , such that  $e(g, h)^\gamma = (B_1 B_3^{\beta_{\hat{A}}^{(s^*)}})^{x_1^{(s^*)} + a_7} \cdot (B_2 B_4^{\beta_{\hat{A}}^{(s^*)}})^{x_2^{(s^*)} + a_8} \cdot (Y_1^{(s^*)} B_5)^{a_1 + a_4\beta_{\hat{A}}^{(s^*)}} \cdot (Y_2^{(s^*)} B_6)^{a_2 + a_5\beta_{\hat{A}}^{(s^*)}} \cdot (Y_3^{(s^*)} B_7)^{a_3 + a_6\beta_{\hat{A}}^{(s^*)}}$ .
  - compute secret key material  $\sigma_{\hat{A}}^{(s^*)} = e(g, h)^\gamma \cdot \Gamma$ .
  - compute  $k^{(s^*)} = F(\sigma_{\hat{A}}^{(s^*)}, \beta_{\hat{A}}^{(s^*)})$ .

2. **Case  $C5$ .** Given a DLIN challenge instance  $(u, u_1, u_2, v, w, \Gamma)$ ,  $\mathcal{B}$  does modifications:

- set  $g = u$ ,  $g_1 = u_1$  and  $g_2 = u_2$ , and simulates all public/private keys as previous game except for the long-term public keys of  $\hat{A}$ :  $A_5, A_6$  and  $A_7$ ;
- set  $A_5 = e(v, h)$ ,  $A_6 = e(w, h)$  and  $A_7 = e(\Gamma, h)$ , and the values  $(v, w, \Gamma)$  would be included in the proof  $\text{pf}_{\hat{A}}$  of  $\hat{A}$ 's public key.
- change the computation of  $\sigma_{\hat{A}}^{(s)}$  for any oracle  $\pi_{\hat{A}}^s$  (including the test oracle) to

$$\begin{aligned} & (e(g^{x_1^{(s)}} v, g^{c_1^{(s)}} g^{c_4^{(s)}\beta_{\hat{A}}^{(s)}}) \cdot e(g^{x_2^{(s)}} w, g^{c_3^{(s)}} g^{c_5^{(s)}\beta_{\hat{A}}^{(s)}}) \cdot e(g^{x_1^{(s)} + x_2^{(s)}} \Gamma, g^{c_3^{(s)}} g^{c_6^{(s)}\beta_{\hat{A}}^{(s)}}))^r \\ & \quad \cdot (W_1^{(s)} C_5^{(s)})^{a_1 + a_4\beta_{\hat{A}}^{(s)}} \cdot (W_2^{(s)} C_6^{(s)})^{a_2 + a_5\beta_{\hat{A}}^{(s)}} \\ & \quad \cdot (W_3^{(s)} C_7^{(s)})^{a_3 + a_6\beta_{\hat{A}}^{(s)}} \cdot (W^{(s)})^{x^{(s)}}, \end{aligned}$$

where the values  $(g^{c_1^{(s)}}, g^{c_2^{(s)}}, g^{c_3^{(s)}}, g^{c_4^{(s)}}, g^{c_5^{(s)}}, g^{c_6^{(s)}})$  are included in  $\text{pf}_{\hat{C}^{(s)}}$ .

3. **Case  $C6$ .** Given a DLIN challenge instance  $(u, u_1, u_2, v, w, c)$ ,  $\mathcal{B}$  does modifications:



- set  $g = u, g_1 = u_1$  and  $g_2 = u_2$ , and simulates all public/private keys as previous game except for the long-term public keys of  $\hat{A}$ :  $A_5, A_6$  and  $A_7$ ;
- set  $X_1^{(s^*)} = e(v, h), X_2^{(s^*)} = e(w, h)$  and  $X_3^{(s^*)} = e(c, h)$ .
- change the computation of  $\sigma_{\hat{A}}^{(s^*)}$  for test oracle  $\pi_{\hat{A}}^{(s^*)}$  to

$$\begin{aligned} & (e(vA_5, g^{c_1^{(s^*)}} g^{c_4^{(s^*)} \beta_{\hat{A}}^{(s^*)}}) \cdot e(wA_6, g^{c_3^{(s^*)}} g^{c_5^{(s^*)} \beta_{\hat{A}}^{(s^*)}}) \cdot e(\Gamma A_7, g^{c_3^{(s^*)}} g^{c_6^{(s^*)} \beta_{\hat{A}}^{(s^*)}}))^r \\ & \cdot (C_1^{(s^*)} C_3^{(s^*) \beta_{\hat{A}}^{(s^*)}})^{a_7} \cdot (C_2^{(s^*)} C_4^{(s^*) \beta_{\hat{A}}^{(s^*)}})^{a_8} \cdot (W_1^{(s)} C_5^{(s^*)})^{a_1+a_4 \beta_{\hat{A}}^{(s^*)}} \\ & \cdot (W_2^{(s^*)} C_6^{(s^*)})^{a_2+a_5 \beta_{\hat{A}}^{(s^*)}} \cdot (W_3^{(s^*)} C_7^{(s^*)})^{a_3+a_6 \beta_{\hat{A}}^{(s^*)}} \cdot (W^{(s^*)})^{x^{(s^*)}}. \end{aligned}$$

Note that those change are purely conceptual. If  $\Gamma$  is true value in each case, then the simulation is equivalent to Game 2, otherwise the simulation is equivalent to Game 3. Now when the algorithm  $\mathcal{A}$  which is able to distinguish Game 3 from Game 2 outputs 1 (meaning this game proceeds as this game),  $\mathcal{B}$  outputs 1 as answer to either the DLIN challenge instance or BDDH challenge instance (meaning  $\Gamma$  is chosen at random), otherwise  $\mathcal{B}$  outputs 0. Therefore, we obtain that

$$|\text{Adv}_2 - \text{Adv}_3| \leq 3 \cdot (d^2 \ell^2 \cdot \epsilon_{\text{BDDH}} + d\ell \cdot (\epsilon_{\text{DLIN}} + 6/2^\lambda)).$$

**Game 4.** We modify Game 3 to Game 4 by changing  $\pi\text{PRF } F$  to a truly random function  $\text{RF}(\cdot)$  for test oracle. If the guessed case is  $C2$  then we make use of the fact that the seed  $\sigma^{(s^*)}$  of  $F$  is chosen uniformly at random. If the adversary  $\mathcal{A}$  distinguishes Game 4 from Game 3 with non-negligible probability, we can use it to construct algorithm  $\mathcal{B}$  that breaks the security of  $\pi\text{PRF } F$ , since such seed is chosen to be independent to other seeds.

As for case  $C5$  and  $C6$ , we first show that key secret  $\sigma_{\hat{A}}^{(s^*)}$  and each the key secret  $\sigma_{\hat{B}}^{(t)}$  of oracle  $\pi_{\hat{B}}^t$  are pairwise independent in this game. We take the simulation for case  $C5$  as example (the simulation for case  $C6$  is quite similar), the tuple  $(B_1, B_2, B_3, B_4, \sigma_{\hat{A}}^{(s^*)}, \sigma_{\hat{B}}^{(t)})$  is denoted by the following equations:

$$\begin{aligned} \log_{e(g,h)} B_1 &\equiv \mu_1 b_1 + b_3 \\ \log_{e(g,h)} B_2 &\equiv \mu_2 b_2 + b_3 \\ \log_{e(g,h)} B_3 &\equiv \mu_1 b_4 + b_6 \\ \log_{e(g,h)} B_4 &\equiv \mu_2 b_5 + b_6 \\ \log_{e(g,h)} \sigma_{\hat{A}}^{(s^*)} &= \mu_1 (x_1^{(s^*)} + a_7) (b_1 + b_4 \beta_{\hat{A}}^{(s^*)}) + \mu_2 (x_2^{(s^*)} + a_8) (b_2 + b_5 \beta_{\hat{A}}^{(s^*)}) + \\ & (x_1^{(s^*)} + x_2^{(s^*)} + a_0) (b_3 + b_6 \beta_{\hat{A}}^{(s^*)}) + \delta^{(s^*)} \\ \log_{e(g,h)} \sigma_{\hat{B}}^{(t)} &= \mu_1 (n_1^{(t)} + d_7^{(t)}) (b_1 + b_4 \beta_{\hat{B}}^{(t)}) + \mu_2 (n_2^{(t)} + d_8^{(t)}) (b_2 + b_5 \beta_{\hat{B}}^{(t)}) + \\ & (n_1^{(t)} + n_2^{(t)} + d_0^{(t)}) (b_3 + b_6 \beta_{\hat{B}}^{(t)}) + \delta_{\hat{B}}^{(t)} \end{aligned}$$

where  $g_1 = g^{\mu_1}, g_2 = g^{\mu_2}, a_0 = a_7 + a_8 + \Delta_a$  (for  $\Delta_a \neq 0$ ),  $d_0^{(t)} = d_7^{(t)} + d_8^{(t)} + \Delta_d^{(t)}$ ,

$$g^{\delta^{(s^*)}} = (X_1^{(s^*)} A_5)^{b_1+b_4 \beta_{\hat{A}}^{(s^*)}} \cdot (X_2^{(s^*)} A_6)^{b_2+b_5 \beta_{\hat{A}}^{(s^*)}} \cdot (X_3^{(s^*)} A_7)^{b_3+b_6 \beta_{\hat{A}}^{(s^*)}} \cdot Y^{(s^*) x^{(s^*)}}$$

, and

$$g^{\delta_{\hat{B}}^{(t)}} = (N_1^{(t)} D_5^{(t)})^{b_1 + b_4 \beta_{\hat{B}}^{(t)}} \cdot (N_2^{(t)} D_6^{(t)})^{b_2 + b_5 \beta_{\hat{B}}^{(t)}} \cdot (N_3^{(t)} D_7^{(t)})^{b_3 + b_6 \beta_{\hat{B}}^{(t)}} \cdot N^{(t) y^{(t)}}.$$

Let  $z_1^{(s^*)} = x_1^{(s^*)} + a_7$ ,  $z_2^{(s^*)} = x_2^{(s^*)} + a_8$ ,  $z_3^{(s^*)} = x_1^{(s^*)} + x_2^{(s^*)} + a_0$ ,  $z_1^{(t)} = n_1^{(t)} + d_7^{(t)}$ ,  $z_2^{(t)} = n_2^{(t)} + d_8^{(t)}$ , and  $z_3^{(t)} = n_1^{(t)} + n_2^{(t)} + d_0^{(t)}$ . We now can obtain a  $6 \times 6$  matrix denoted by  $M$  from the above equations as following:

$$\begin{pmatrix} \log_{e(g,h)} B_1 \\ \log_{e(g,h)} B_2 \\ \log_{e(g,h)} B_3 \\ \log_{e(g,h)} B_4 \\ \log_{e(g,h)} \sigma_{\hat{A}}^{(s^*)} - \delta^{(s^*)} \\ \log_{e(g,h)} \sigma_{\hat{B}}^{(t)} - \delta_{\hat{B}}^{(t)} \end{pmatrix} = \begin{pmatrix} \mu_1 & 0 & 1 & 0 & 0 & 0 \\ 0 & \mu_2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \mu_1 & 0 & 1 \\ 0 & 0 & 0 & 0 & \mu_2 & 1 \\ \mu_1 z_1^{(s^*)} & \mu_2 z_2^{(s^*)} & z_3^{(s^*)} & \mu_1 \beta_{\hat{A}}^{(s^*)} z_1^{(s^*)} & \mu_2 \beta_{\hat{A}}^{(s^*)} z_2^{(s^*)} & \beta_{\hat{A}}^{(s^*)} z_3^{(s^*)} \\ \mu_1 z_1^{(t)} & \mu_2 z_2^{(t)} & z_3^{(t)} & \mu_1 \beta_{\hat{B}}^{(t)} z_1^{(t)} & \mu_2 \beta_{\hat{B}}^{(t)} z_2^{(t)} & \beta_{\hat{B}}^{(t)} z_3^{(t)} \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{pmatrix}$$

One the one hand, if  $(g, g_1, g_2, g_3, g^{z_1^{(t)}}, g^{z_2^{(t)}}, g^{z_3^{(t)}})$  is a linear tuple, then corresponding  $\sigma_{\hat{B}}^{(t)}$  is independent to  $\sigma_{\hat{A}}^{(s^*)}$ , since

$$\log_{e(g,h)} \sigma_{\hat{B}}^{(t)} - \delta_{\hat{B}}^{(t)} = \mu_1 z_1^{(t)} (b_1 + b_4 \beta_{\hat{B}}^{(t)}) + \mu_2 z_2^{(t)} (b_2 + b_5 \beta_{\hat{B}}^{(t)}) + z_3^{(t)} (b_3 + b_6 \beta_{\hat{B}}^{(t)})$$

is linearly dependent on  $\log_{e(g,h)} B_1, \log_{e(g,h)} B_2, \log_{e(g,h)} B_3$  and  $\log_{e(g,h)} B_4$ , while  $\sigma_{\hat{A}}^{(s^*)}$  is linearly independent to  $\log_{e(g,h)} B_1, \log_{e(g,h)} B_2, \log_{e(g,h)} B_3$  and  $\log_{e(g,h)} B_4$  with overwhelming probability (at least  $1 - 6/2^\lambda$ ). On the other hand, consider that

$(g, g_1, g_2, g_3, g^{z_1^{(t)}}, g^{z_2^{(t)}}, g^{z_3^{(t)}})$  is not a linear tuple. We observe that

$$\text{Det}M := \mu_1^2 \mu_2^2 (\beta_{\hat{A}}^{(s^*)} - \beta_{\hat{B}}^{(t)}) (z_3^{(s^*)} - z_1^{(s^*)} - z_2^{(s^*)}) (z_3^{(t)} - z_1^{(t)} - z_2^{(t)}) \neq 0,$$

since  $\beta_{\hat{A}}^{(s^*)} - \beta_{\hat{B}}^{(t)} \neq 0$  (by the rejection rule in Game 2),  $(z_3^{(s^*)} - z_1^{(s^*)} - z_2^{(s^*)}) = \Delta_a \neq 0$  and  $z_3^{(t)} - z_1^{(t)} - z_2^{(t)} = \Delta_z^{(t)} = \Delta_n^{(t)} \beta_{\hat{B}}^{(t)} + \Delta_d^{(t)} \neq 0$ . Hence, the  $\sigma_{\hat{A}}^{(s^*)}$  is independent to  $\sigma_{\hat{B}}^{(t)}$ . Now if there exist adversary  $\mathcal{A}$  that is able to distinguish this game from previous game with non-negligible probability (i.e.  $b = b'$ ). Then we can use it to construct an efficient algorithm  $\mathcal{B}$  to break the security of  $\pi\text{PRF } F$  with index  $\{I_{\mathbb{G}_T}, f_{\mathbb{G}_T}\}$  where  $I_{\mathbb{G}_T} := \{(U, V, \alpha) | (U, V, \alpha) \in \mathbb{G}_T^2 \times \mathbb{Z}_p\}$  and  $f_{\mathbb{G}_T} := (U, V, \alpha) \rightarrow U^{r_1 + \alpha r_2} V$  with  $(r_1, r_2) \xleftarrow{\$} \mathbb{Z}_p^2$ . Algorithm  $\mathcal{B}$  simulates the game for  $\mathcal{A}$  as the challenger in previous game. In particularly,  $\mathcal{B}$  uniformly chooses  $(\mu_1, \mu_2) \xleftarrow{\$} \mathbb{Z}_p^2$  at random, and sets

$$\eta_1 := \mu_1 b_1 + b_3, \eta_2 := \mu_2 b_2 + b_3,$$

$$\eta_3 := \mu_1 b_4 + b_6, \eta_4 := \mu_2 b_5 + b_6,$$

$$U_{\hat{A}}^{(s^*)} := A_7 / (A_5^{\mu_1^{-1}} A_6^{\mu_2^{-1}}), U_{\hat{B}}^{(t)} := N_3^{(t)} D_7^{(t)} / ((N_1^{(t)} A_5)^{\mu_1^{-1}} (N_2^{(t)} A_6)^{\mu_2^{-1}}),$$

$$\begin{aligned} V_{\hat{A}}^{(s^*)} &= (B_1 B_3^{\beta_{\hat{A}}^{(s^*)}})^{x_1^{(s^*)} + a_7} \cdot (B_2 B_4^{\beta_{\hat{A}}^{(s^*)}})^{x_2^{(s^*)} + a_8} \cdot (W_1^{(s^*)} \beta_{\hat{A}}^{(s^*)} B_5)^{a_1 + a_4 \beta_{\hat{A}}^{(s^*)}} \\ &\quad \cdot (W_2^{(s^*)} B_6)^{a_2 + a_5 \beta_{\hat{A}}^{(s^*)}} \cdot (W_3^{(s^*)} B_7)^{a_3 + a_6 \beta_{\hat{A}}^{(s^*)}} \cdot e(g^{x_1^{(s^*)}} g^{a_7}, h)^{\eta_1 + \beta_{\hat{A}}^{(s^*)} \eta_3} \\ &\quad \cdot e(g^{x_2^{(s^*)}} g^{a_8}, h)^{\eta_2 + \beta_{\hat{A}}^{(s^*)} \eta_4} \cdot W^{(s^*) x^{(s^*)}}, \end{aligned}$$

$$V^{(t)} := (D_1^{(t)} D_3^{(t)\beta_{\hat{B}}^{(t)}})^{y_1+b_7} \cdot (D_2^{(t)} D_4^{(t)\beta_{\hat{B}}^{(t)}})^{y_2+b_8} \cdot (N_1^{(t)} e(g^{a_7}, h))^{\eta_1+\beta_{\hat{B}}^{(t)}\eta_3} \cdot (N_2^{(t)} e(g^{a_7}, h))^{\eta_2+\beta_{\hat{B}}^{(t)}\eta_4} \cdot N^{(t)y^{(t)}},$$

and  $(r_1, r_2) := (b_3, b_6)$ . Thereafter,  $\mathcal{B}$  simulates this game for adversary  $\mathcal{A}$  as previous game except for the computations of  $k^{(s^*)}$  and  $k^{(t)} (t \in [d])$ , where  $\mathcal{B}$  gives indices  $(U^{(s^*)}, V^{(s^*)}, \beta_{\hat{A}}^{(s^*)})$  and  $(U^{(t)}, V^{(t)}, \beta_{\hat{B}}^{(t)}) (t \in [d])$  to the oracle  $(F, I_{\mathcal{G}_T})$  in the experiment of the  $\pi$ PRF as Definition 3 and sets the values returned from the oracle as session keys  $k^{(s^*)}$  and  $k^{(t)} (t \in [d])$  respectively. When  $\mathcal{A}$  output 1 (meaning the simulation is as the same as the Game 2), then  $\mathcal{B}$  outputs 1 (meaning the oracle is  $(F, I_{\mathcal{G}_T})$ ); otherwise  $\mathcal{B}$  outputs 0. Since if the oracle is  $(F, I_{\mathcal{G}_T})$ , the simulated game is equivalent to Game 3, otherwise the simulated game is equivalent to Game 4. We therefore obtain that

$$|\text{Adv}_3 - \text{Adv}_4| \leq \epsilon_{\pi\text{PRF}}.$$

Note that, in this game the bit  $b$  is never used in test query, so that we have that  $\text{Adv}_4 = 0$ . Collect the advantages from Game 0 to Game 4, we have that

$$\epsilon \leq \frac{d^2 \ell^2}{2\lambda} + \epsilon_{\text{CRHF}} + 3 \cdot (d^2 \ell^2 \cdot \epsilon_{\text{BDDH}} + d\ell \cdot (\epsilon_{\text{DLIN}} + 6/2^\lambda)) + \epsilon_{\pi\text{PRF}}.$$

## 6 Conclusions

We have presented an efficient eCK-secure key exchange protocols without random oracles (and without NAXOS trick), that the security against chosen public key attacks without KOSK assumption). An open question here is how to construct an eCK secure protocol without  $\pi$ PRF, we leave out this for future work.

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