

# A note on ‘An efficient certificateless aggregate signature with constant pairing computations’

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**Abstract:** Recently, Xiong et al. proposed an efficient certificateless signature (CLS) scheme and used it to construct a certificateless aggregate signature (CLAS) scheme with constant pairing computations. They also demonstrated that both of the two schemes are provably secure in the random oracle model under the computational Diffie-Hellman assumption. Unfortunately, by giving concrete attacks, we point out that Xiong et al.’s schemes are not secure in their security model.

**Key words:** *Certificateless cryptography; Aggregate signature; Bilinear pairing*

## 1. Introduction

An aggregate signature scheme is a signature scheme which allows to aggregate  $n$  signatures on  $n$  distinct messages from  $n$  distinct users into a single signature [1]. The validity of an aggregate signature will convince a verifier that the  $n$  users did indeed sign the  $n$  original messages. Aggregation is useful to reduce bandwidth and storage, and is especially attractive for mobile devices like sensors, cell phones, and PDAs where communication is more power expensive than computation and contributes significantly to reducing battery life.

To satisfy the applications in certificateless environment, certificateless aggregate signature (CLAS) scheme have attracted much attention. Several CLAS schemes [2-5] have proposed by different researchers. However, most of these schemes [2-4] have computational complexity for pairing computations that grows linearly with the number of signers. Besides, both of the schemes [3, 5] of Zhang et al. require certain synchronization, i.e., all signers must share the same synchronized clocks to generate aggregate signature. It is easy to say that it is difficult to achieve synchronization in many communication scenarios. Recently, Xiong et al. [6] proposed an efficient certificateless signature (CLS) scheme and

construct a simple CLAS scheme using the CLS scheme. Compared with previous CLAS schemes, Xiong et al.'s scheme is very efficient, and the verification procedure needs only a very small constant number of pairing computations, independent of the number of aggregated signatures. Besides, their scheme does not require certain synchronization for aggregating randomness. They also demonstrated that both of the two schemes are provably secure in the random oracle model under the computational Diffie-Hellman assumption. Unfortunately, we find that a Type II adversary could forge a legal signature of any message against Xiong et al.'s schemes. The analysis shows Xiong et al.'s schemes are not secure for practical applications.

The organization of the paper is sketched as follows. Section 2 gives a brief review of Xiong et al.'s schemes. The security flaws of Xiong et al.'s schemes are shown in Section 3. Finally, we give some conclusions in Section 4.

## 2. Review of Xiong et al.'s schemes

### 2.1. Xiong et al.'s CLS scheme

In this subsection, we will briefly review Xiong et al.'s CLS scheme. Their CLS scheme consists of five algorithms: *MasterKeyGen*, *PartialKeyGen*, *UserKeyGen*, *Sign* and *Verify*. The detail of these algorithms is described as follows.

*MasterKeyGen*: Given a security parameter  $k$ , KGC runs the algorithm as follows.

1) Generate a cyclic additive group  $G_1$  and a cyclic multiplicative group  $G_2$  with prime order  $q$ .

2) Generate two generators  $P, Q$  of  $G_1$  and an admissible pairing  $e: G_1 \times G_1 \rightarrow G_2$ .

3) Generate a random number  $s \in Z_q^*$  and compute  $P_{pub} = sP$ .

4) Choose cryptographic hash functions  $H_1: \{0,1\}^* \rightarrow G_1$  and  $H_2: \{0,1\}^* \rightarrow Z_q^*$ .

5) KGC publishes the system parameters are  $\{q, G_1, G_2, e, P, Q, P_{pub}, H_1, H_2\}$  and key the master key  $s$  secretly.

*PartialKeyGen*: Given a user's identity  $ID_i$ , KGC computes the user's partial private key  $psk_{ID_i} = sQ_{ID_i}$  and transmits it to the user secretly, where  $Q_{ID_i} = H_1(ID_i)$ .

*UserKeyGen*: The user with identity  $ID_i$  selects a random number  $x_{ID_i} \in Z_q^*$  as his secret key  $usk_{ID_i}$ , and computes his public key as  $upk_{ID_i} = usk_{ID_i} \cdot P$ .

*Sign*: Given a message  $m_i$ , the partial private key  $psk_{ID_i}$ , the secret key  $usk_{ID_i}$ , the user with identity is  $ID_i$  and the corresponding public key is  $upk_{ID_i}$ , performs the following steps to generate a signature.

- 1) Generate a random number  $r_i \in Z_q^*$  and compute  $U_i = r_i P$ .
- 2) Compute  $h_i = H_2(m_i, ID_i, upk_{ID_i}, U_i)$ ,  $V_i = psk_{ID_i} + h_i \cdot r_i \cdot P_{pub} + h_i \cdot x_{ID_i} \cdot Q$ .
- 3) Output  $(U_i, V_i)$  as the signature on  $m_i$ .

*Verify*: Given a signature  $(U_i, V_i)$  of message  $m_i$  on identity  $ID_i$  and corresponding public key  $upk_{ID_i}$ :

- 1) Compute  $Q_{ID_i} = H_1(ID_i)$  and  $h_i = H_2(m_i, ID_i, upk_{ID_i}, U_i)$ .
- 2) Verify  $e(V_i, P) = e(h_i \cdot U_i + Q_{ID_i}, P_{pub})e(h_i \cdot upk_{ID_i}, Q)$  holds or not. If it holds, accept the signature.

## 2.2. Xiong et al.'s CLAS scheme

In this subsection, we will briefly review Xiong et al.'s CLAS scheme. Their CLAS scheme consists of six algorithms: *MasterKeyGen*, *PartialKeyGen*, *UserKeyGen*, *Sign*, *Aggregate* and *AggregateVerify*. The first four algorithms are the same as those in their CLS scheme. The detail of other two algorithms is described as follows.

*Aggregate*: For an aggregating set of  $n$  users  $\{\mathcal{U}_1, \dots, \mathcal{U}_n\}$  with identities  $\{ID_1, \dots, ID_n\}$  and the corresponding public keys  $\{upk_1, \dots, upk_n\}$ , and message-signature pairs  $\{(m_1, \sigma_1 = (U_1, V_1)), \dots, (m_n, \sigma_n = (U_n, V_n))\}$  from  $\{\mathcal{U}_1, \dots, \mathcal{U}_n\}$  respectively, the aggregate signature generator computes  $V = \sum_{i=1}^n V_i$  and outputs  $\sigma = (U_1, \dots, U_n, V)$  as an aggregate signature.

*AggregateVerify* : To verify an aggregate signature  $\sigma = (U_1, \dots, U_n, V)$  signed by  $n$  users  $\{\mathcal{U}_1, \dots, \mathcal{U}_n\}$  with identities  $\{ID_1, \dots, ID_n\}$  and the corresponding public keys  $\{upk_1, \dots, upk_n\}$  on messages  $\{m_1, \dots, m_n\}$ , the verifier performs the following steps:

1) Compute  $Q_{ID_i} = H_1(ID_i)$  and  $h_i = H_2(m_i, ID_i, upk_{ID_i}, U_i)$  for  $i = 1, \dots, n$ .

2) Verify  $e(V, P) = e(\sum_{i=1}^n (h_i \cdot U_i + Q_{ID_i}), P_{pub})e(\sum_{i=1}^n h_i \cdot upk_{ID_i}, Q)$  holds or not.

If it holds, accept the signature.

### 3. Cryptanalysis of Xiong et al.'s scheme

Xiong et al. [6] claimed that both of their schemes are provably secure against two types of adversary in the random oracle model. However, in this section, we shall disprove their claims by giving two concrete attacks.

#### 3.1. Attack against Xiong et al.'s CLS scheme

Xiong et al. [6] claimed their CLS scheme is semantically secure against Type II adversary. Unfortunately, it is not true, since there exists a polynomial time Type II adversary  $\mathcal{A}2$  who can always win **Game I** as below:

1)  $\mathcal{A}2$  submits a user  $\mathcal{U}_i$ 's identity  $ID_i$  to the *RevealPartialKey* oracle and gets  $\mathcal{U}_i$ 's partial private key  $psk_{ID_i} = sQ_{ID_i}$ , where  $Q_{ID_i} = H_1(ID_i)$ .

2)  $\mathcal{A}2$  submits  $ID_i$  and a message  $m_i$  to the *Sign* oracle and gets a legal signature  $(U_i, V_i)$  of message  $m_i$ , where  $U_i = r_i P$ ,  $h_i = H_2(m_i, ID_i, upk_{ID_i}, U_i)$ ,  $V_i = psk_{ID_i} + h_i \cdot r_i \cdot P_{pub} + h_i \cdot x_{ID_i} \cdot Q$  and  $r_i$  is a random number generated by *Sign* oracle.

3)  $\mathcal{A}2$  computes  $T_i = h_i^{-1}(V_i - psk_{ID_i})$ , where  $h_i^{-1}$  satisfy  $h_i^{-1} \cdot h_i \equiv 1 \pmod{q}$ .

4) For any other message  $m'_i$ ,  $\mathcal{A}2$  computes  $U'_i = U_i$ ,  $h'_i = H_2(m'_i, ID_i, upk_{ID_i}, U'_i)$ ,  $V'_i = psk_{ID_i} + h'_i \cdot T_i$ .

5)  $\mathcal{A}2$  outputs  $(U'_i, V'_i)$  as the signature on  $m'_i$ .

Since  $U_i = r_i P$  and  $V_i = psk_{ID_i} + h_i \cdot r_i \cdot P_{pub} + h_i \cdot x_{ID_i} \cdot Q$ , we could have

$$\begin{aligned}
T_i &= h_i^{-1}(V_i - psk_{ID_i}) \\
&= h_i^{-1}(psk_{ID_i} + h_i \cdot r_i \cdot P_{pub} + h_i \cdot x_{ID_i} \cdot Q - psk_{ID_i}) \\
&= r_i \cdot P_{pub} + x_{ID_i} \cdot Q
\end{aligned} \tag{1}$$

$$\begin{aligned}
V'_i &= psk_{ID_i} + h'_i \cdot T_i \\
&= psk_{ID_i} + h'_i \cdot (r_i \cdot P_{pub} + x_{ID_i} \cdot Q) \\
&= psk_{ID_i} + h'_i \cdot r_i \cdot P_{pub} + h'_i \cdot x_{ID_i} \cdot Q
\end{aligned} \tag{2}$$

and

$$\begin{aligned}
e(V'_i, P) &= e(psk_{ID_i} + h'_i \cdot r_i \cdot P_{pub} + h'_i \cdot x_{ID_i} \cdot Q, P) \\
&= e(psk_{ID_i} + h'_i \cdot r_i \cdot P_{pub}, P) e(h'_i \cdot x_{ID_i} \cdot Q, P) \\
&= e(sQ_{ID_i} + h'_i \cdot r_i \cdot sP, P) e(Q, h'_i \cdot x_{ID_i} \cdot P) \\
&= e(h'_i \cdot U_i + Q_{ID_i}, sP) e(h'_i \cdot upk_{ID_i}, Q) \\
&= e(h'_i \cdot U_i + Q_{ID_i}, sP_{pub}) e(h'_i \cdot upk_{ID_i}, Q)
\end{aligned} \tag{3}$$

Then, we know that  $(U'_i, V'_i)$  is a legal signature on  $m'_i$ . Besides,  $ID_i$  has not been submitted to *RevealSecertKey* queries or *ReplaceKey* queries to get the secret key  $usk_{ID_i^*}$  and the oracle *Sign* has never been queried with  $(ID_i, m'_i)$ . So the Tpye II adversary  $\mathcal{A}2$  wins **Game I**.

Therefore, Xiong et al.'s CLS scheme is not secure against attacks of the Type II adversary.

### 3.2. Attack against Xiong et al.'s CLAS scheme

Xiong et al. [6] claimed their CLAS scheme is semantically secure against Tpye II adversary. Unfortunately, it is not true, since there exists a polynomial time Type II adversary  $\mathcal{A}2$  who can always win **Game II** as follows:

Let  $\{\mathcal{U}_1, \dots, \mathcal{U}_n\}$  be an aggregating set of  $n$  users with identities  $\{ID_1, \dots, ID_n\}$  and the corresponding public keys  $\{upk_1, \dots, upk_n\}$ .

1) For  $i=1, 2, \dots, n$ ,  $\mathcal{A}2$  does the following five sub-steps to generate a legal signature  $(U'_i, V'_i)$  on a message  $m'_i$ .

- $\mathcal{A}2$  submits a user  $\mathcal{U}_i$ 's identity  $ID_i$  to the *RevealPartialKey* oracle and gets  $\mathcal{U}_i$ 's partial private key  $psk_{ID_i} = sQ_{ID_i}$ , where  $Q_{ID_i} = H_1(ID_i)$ .

- $\mathcal{A}2$  submits  $ID_i$  and a message  $m_i$  to the *Sign* oracle and gets a legal signature  $(U_i, V_i)$  of message  $m_i$ , where  $U_i = r_i P$ ,  $h_i = H_2(m_i, ID_i, upk_{ID_i}, U_i)$ ,  $V_i = psk_{ID_i} + h_i \cdot r_i \cdot P_{pub} + h_i \cdot x_{ID_i} \cdot Q$  and  $r_i$  is a random number generated by *Sign* oracle.
  - $\mathcal{A}2$  computes  $T_i = h_i^{-1}(V_i - psk_{ID_i})$ , where  $h_i^{-1}$  satisfy  $h_i^{-1} \cdot h_i \equiv 1 \pmod{q}$ .
  - For any other message  $m'_i$ ,  $\mathcal{A}2$  computes  $U'_i = U_i$ ,  $h'_i = H_2(m'_i, ID_i, upk_{ID_i}, U'_i)$ ,  $V'_i = psk_{ID_i} + h'_i \cdot T_i$ .
  - $\mathcal{A}2$  outputs  $(U'_i, V'_i)$  as the signature on  $m'_i$ .
- 2)  $\mathcal{A}2$  computes  $V' = \sum_{i=1}^n V'_i$
  - 3)  $\mathcal{A}2$  outputs  $\sigma' = (U'_1, \dots, U'_2, V')$  as an aggregate signature.

From the analysis in the above subsection, we know that  $(U'_i, V'_i)$  satisfies the equation  $e(V'_i, P) = e(h'_i \cdot U_i + Q_{ID_i}, sP_{pub})e(h'_i \cdot upk_{ID_i}, Q)$  and  $V'_i = psk_{ID_i} + h'_i \cdot r_i \cdot P_{pub} + h'_i \cdot x_{ID_i} \cdot Q$ . Then we could have that

$$\begin{aligned}
e(V', P) &= e\left(\sum_{i=1}^n V'_i, P\right) = e\left(\sum_{i=1}^n (psk_{ID_i} + h'_i \cdot r_i \cdot P_{pub} + h'_i \cdot x_{ID_i} \cdot Q), P\right) \\
&= e\left(\sum_{i=1}^n (psk_{ID_i} + h'_i \cdot r_i \cdot P_{pub}), P\right) e\left(\sum_{i=1}^n h'_i \cdot x_{ID_i} \cdot Q, P\right) \\
&= e\left(\sum_{i=1}^n (h'_i \cdot U_i + Q_{ID_i}), P_{pub}\right) e\left(\sum_{i=1}^n h'_i \cdot upk_{ID_i}, Q\right)
\end{aligned} \tag{3}$$

Thus, we know that  $\sigma' = (U'_1, \dots, U'_2, V')$  is a legal aggregate signature on messages  $\{m'_1, \dots, m'_n\}$ . Besides, for any  $i \in \{1, \dots, n\}$ ,  $ID_i$  has not been submitted to *RevealSecertKey* queries or *ReplaceKey* queries to get the secret key  $usk_{ID_i}$  and the oracle *Sign* has never been queried with  $(ID_i, m'_i)$ . So the Type II adversary  $\mathcal{A}2$  wins **Game II**.

Therefore, Xiong et al.'s CLAS scheme is not secure against attacks of the Type II adversary.

## 4. Conclusion

Recently, Xiong et al. [6] proposed a CLS scheme and used it to construct an efficient CLAS scheme. They claimed that both of their schemes are provably

secure in the random oracle model. However, after review of their scheme and analysis of its security, we demonstrate that both of the schemes cannot withstand the attack of Type II adversary. The analysis shows that their schemes are insecure for practical applications.

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