Constant Ciphertext Length in CP-ABE

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Abstract. Ciphertext policy attribute based encryption (CP-ABE) is a technique in which user with secret key containing attributes, only able to decrypt the message if the attributes in the policy match with the attributes in secret key. The existing methods that use reasonably computable decryption policies produce the ciphertext of size at least linearly varying with the number of attributes with additional pairing operations during encryption and decryption. In this paper, we propose a scheme in which ciphertext remains constant in length, irrespective of the number of attributes. Our scheme works for a threshold case: the number of attributes in a policy must be a subset of attributes in a secret key. The security of propose scheme is based on Decisional Bilinear Diffie-Hellman (DBDH) problem.

Keywords: Attribute, Attribute based encryption, ciphertext policy, constant ciphertext length.

1 Introduction

Encryption is the one of primitive that provides security and confidentiality to the digital communications. In traditional symmetric key cryptography (SKC), the sender and receiver both share the same secret key. However, use of the SKC is besieged with the problems related to the key distribution and management. On the other hand, the Public Key Cryptography proposed to circumvent key management issues is not efficient in a multicast setup as also for bulk encryption/decryption [1]. However, the PKC suffers from the complexity in key assignment and certificate management issues.

Identity Based Encryption (IBE) was proposed to obviate the need for a user to a priori possess a certificate obtained using PKI. IBE, proposed first in [2] relies on using the global identities of a user as his public key, with the corresponding (i.e. associated with his identity) private key being assigned by a globally trusted Key Generation Centre (KGC) after due authentication of a user. Any user could encrypt a message using the global identity of the destined user, whereas a user, whose identity in his secret key matches with the same in the ciphertext, would be alone able to decrypt the same.

In the traditional IBE systems, the identity of a user is specified using either the name, the email ID, or the network address – a string of characters. This makes it cumbersome to establish the necessary correlation between a user's identity (in his private key) and the same associated in the ciphertext that he intends to decrypt. This

is so, because even slight mismatch would render the match as a failure.

Hence, in a variant of the traditional IBE, the identity is specified in the form of descriptive attributes. In the first of such scheme proposed as Fuzzy Identity Based Encryption (FIBE) in [3], a user with identity *W* could decrypt the ciphertext meant for a user with identity *W'*, if and only if |W - W'| > d, where *d* is some threshold value defined initially.

In [4], the authors propose more expressive ABE schemes in the form of two different systems viz. Key Policy Attribute Based Encryption (KP-ABE) and Ciphertext Policy Attribute Based Encryption (CP-ABE). In KP-ABE, a ciphertext is associated with a defined set of attributes and user's secret key is associated with a defined policy containing those attributes. Hence, the secret key could be used successfully only if the attributes in access structure policy defined in the key matches with the attributes in the ciphertext. In [5] authors propose a fully functional Ciphertext Policy Attribute Based Encryption (CP-ABE) in which a user's secret key is associated with a defined set of attributes and the ciphertext is associated with a defined policy. In [6], the authors propose a protocol for conversion from KP-ABE to CP-ABE. One of the limitations of CP-ABE schemes is that the length of ciphertext is dependent on the number of attributes. That is, with *s* being the number of attributes involved in the policy, the ciphertext length is $O(s^3)$.

All of these approaches use a *single authority* while ensuring either variable or constant ciphertext length with/without collusion resistance. In a single authority system, the entire trust is on *the* single authority, so if the authority is compromised then the entire system is compromised as well as there is overhead on CA for key management.

To deal with single point of failure the traditional approach followed in distributed systems is to distribute the responsibility amongst multiple entities. In [20], the authors indeed propose the idea of *multi-authority* system in which there are arbitrary numbers of attribute authorities (AA) with one central authority (CA). Obviously, such schemes require mutual trust between the AAs and the CA. In [23] [24] [25] [26] [27] authors propose different approaches to deal with the limitations of the multi authority system.

1.1 Constant Ciphertext length

In CP-ABE size of ciphertext and secret key will increases linearly with number of attributes in policy, so this will increase the communication overhead. The practical data given in Table 1 shows that for larger number of attributes the computation and communication overhead will create problem for system. The size of plaintext is 350KB.

# attributes in policy	Size of ciphertext (KB)	# Computation operations		
		Pairing	Exponential	multiplication
4	365	17	8	85
5	371	21	10	115
6	376	27	13	160

Table 1. Analysis of CP-ABE scheme

In addition, the number of pairing operation will increase during encryption and decryption, which increase computation overhead on sender and receiver [11]. One of the efficient constructions of the CP-ABE in terms of ciphertext length can be found in the [7] [8]. In that the size of ciphertext depending linearly upon the number of attributes. For example in (t, n) threshold scheme, where there are t or more attributes required to decrypt by user, then the size of ciphertext in [7] is $n + O(1)$ and in [8] $2(n - t) + O(1)$. Both scheme use secret sharing scheme by Shamir [9] and uses the monotonic access structure. All the approaches mentioned so far achieve the security in the generic model. In [10] authors achieve the full security but size of ciphertext was $2n + O(1)$. In [11] authors proposed the constant length ciphertext using the (t, t) threshold system. So this scheme suffer with the problem that number of attributes in user's secret key is same as the number of attributes in policy, this scheme achieves constant length ciphertext as well as constant length secret key length. In [12] authors proposed the constant length ciphertext in threshold ABE based on the dynamic threshold encryption scheme from [13].

One of the essential feature of ABE system is they must be collusion resistance so the users cannot combine to get the decrypted ciphertext which is not entitled for them. In addition, this feature can be very handy in many applications. The notion of ABE without this property under different names: [14] [15]. This notion is somewhat similar to primitive of distributed dynamic IBE [16][17][13][8], in this one sender selects ad-hoc set of identities and define access structure on this one, Users associated with certain identities in the access structure can combine to decrypt the ciphertext.

Our contribution: Our focus here in this paper is on investigating whether is it possible to ensure *constant length* ciphertext in ABE scheme with *collusion resistance* using *single authority*? We attempt to propose the collusion resistant privacy-preserving single authority scheme which contain the constant size ciphertext also it require fixed number of pairing operation during the decryption irrespective of attributes. However, our approach necessitates that the attributes in the ciphertext must be a *subset* of user's attributes in his secret key. For example, if we had one user *Harry* with attributes "Name=Harry", "University = Stanford", "Branch = EE". In this scenario, if some arbitrary sender sends a message to all the *EE* branch students of *Stanford* University, *Harry* would be able to decrypt the message because the number of attributes in his policy is the *subset* of the user's attributes. We propose a protocol for the purpose. The security of our protocol is based on DBDH assumptions. We had modified the approach of [11] to increase the efficiency, so our protocol's security is also based on DBDH assumption. In [12] the number of pairing operations required for encryption and decryption is more as compared to our proposed scheme. The detailed comparison of our scheme with previous schemes is discussed in section 4.

Organization of the paper: The rest of the paper is organized as follows. In the second section, we give the preliminaries which we use throughout the paper and the DBDH problem. In third section, we had given our proposed approach. Fourth section gives the security analysis of our approach. In fifth section the comparison of our approach with existing approaches is given. Last section concludes the paper and references are

at the end.

2 Preliminaries

This section provides the required definitions, the computational hardness assumptions, proposed construction and selective game for it.

2.1 Notations

Most cryptographic protocol requires randomness, for example generating random secret key. We use $x \in_R A$ to represent the operation of selecting element x randomly and uniformly from element set A. At some places we use " ϕ " to denote the NULL output. This paper deals with the computational security setting where security was defined based on the string length. For £ ∈ N where N is the set of natural numbers, $1^{£}$ denotes the strings of length £. If x is a string then $|x|$ denotes its length, e.g. $|1^{£}|=£$.

2.2 Attribute based encryption

2.2.1 Bilinear Group

The security of the CP-ABE system is based on the algebraic group called bilinear groups, which are group with bilinear map.

Definition 2.1 (Bilinear map). Assume G1,G2 and G3 are three multiplicative cyclic group of some prime order p. A bilinear map e : G1 × G2 → G3 is a deterministic function which takes as input one element from G1, one element from G2, and output an element in group G3, which satisfies the following criteria

a) Bilinearity : For all x ∈ G1, y ∈ G2, a,b ∈ Z_p , e (x^a,y^b)=e $(x,y)^{ab}$.
b) Non degeneracy: e $(g1, g2) \neq 1$ where $g1$ and $g2$ are generator of G1 and G2 respectively.
c) e must be computed efficiently.

Definition 2.2 (Discrete Logarithm Problem). Given two group elements g and h, find an integer a ∈ Z_p such that $h= g^a$ whenever such integer exist.

Definition 2.3 (DBDH assumption). The Decision Bilinear Diffie-Hellman (DBDH) problem in G is a problem, for input of a tuple $(g,g^a,g^b,g^c,Z) \in G^4 \times GT$ to decide $Z = e(g,g)^{abc}$ or not. An algorithm A has advantage ϵ in solving DBDH problem in G if $Adv_{DBDH}(A):=|Pr[A(g,g^a,g^b,g^c,e(g,g)^{abc})=0]-Pr[A(g,g^a,g^b,g^c,e(g,g)^z)=0]| \geq \epsilon(\kappa)$, where $e(g,g)^z \in G_T \setminus \{e(g,g)^{abc}\}$. We say that the DBDH assumption holds in G if no PPT algorithm has an advantage of at least ϵ in solving the DBDH problem in G.[11]

Definition 2.4 (Access Structure). Let (A1,A2,…,An) be a set of attributes. A collection $A \subseteq 2^{\{A1,A2,\ldots An\}}$ is monotone if ∀B,C : if $B \in A$ and $B \subseteq A$ then $C \in A$. An (monotone) access structure is a (monotone) collection A of non-empty subsets of (A1,A2,…,An), i.e. $A \subseteq 2^{\{A1,A2,\ldots An\}} \setminus \{ \phi \}$. The sets in A are called authorized and the sets which are not in A called unauthorized sets.

2.2.2 Proposed construction

CP-ABE consists of four polynomial algorithms as follows.

1. **Setup** ($\mathbf{1}^{k}$): It will take implicit security parameter *k* and output public parameter MPK and master key MSK.
2. **KeyGen** (MSK, S): The key generation algorithm run by Central Authority (CA), takes as input the master key of CA and the set of attributes S for user and then generates the secret key SK.
3. **Encrypt** (MPK, M, A): The encryption algorithm takes as input the message M, public parameter MPK and access structure A over the universe of attributes. Generate the output CT such that only those users who had valid set of attributes which satisfy the access policy can only able to decrypt. Assume that the CT implicitly contains access structure A.
4. **Decrypt** (MPK, CT, SK) : The decrypt algorithm run by user takes input the public parameter MPK, the ciphertext CT contains access structure A and the secret key SK containing attribute set S. If S satisfies the access tree then algorithm decrypt the CT and give M otherwise gives "ϕ".

2.2.3 Selective Game setup

Initialization: The adversary A will sends the challenge access structure W* to the challenger.
Setup: The challenger runs **Setup** and generates MPK and MSK. It gives MPK to A.
Phase 1: A sends an attribute list L to the challenger for a **KeyGen** query with attribute list L, where $L \not\models W^*$. The challenger answers with a secret key for these attribute list L. Note that these queries can be repeated adaptively.
Challenge: A sends two equal-length messages M_0 and M_1 to the challenger. The challenger selects $\mu \in_R \{0, 1\}$, and runs C* = **Encrypt** (MPK,M_μ, W*).The challenger gives the ciphertext C* to A.
Phase 2: Same as Phase 1.
Guess: A outputs a guess $\mu' \in \{0, 1\}$.
The advantage of A is defined as $Adv(A) := |Pr(\mu' = \mu) - 1/2|$.

3 Proposed Scheme

In this section we have given the proposed constant ciphertext length CP-ABE scheme where attributes in the policy must be a subset of attributes in user's secret key. Here Z_p= group of large prime order p. Group G and G1 of cyclic multiplicative group of prime order p. Assume $U = \{att_1, att_2, \dots, att_n\}$ be the set of all possible attributes in universe. Assume $S_i = \{v_{i,1}, v_{i,2}, \dots, v_{i,n_i}\}$ be the set of all possible values for att_i where $n_i = |S_i|$. Assume $L = [L_1, L_2, \dots, L_n]$ be a set of attributes for user and $W = [W_1, W_2, \dots, W_k]$ is an access structure. Here $e: G \times G \rightarrow G1$ is the admissible bilinear map function (as per **Definition 2.1**). We assume that *t* and *t'* is the two different universal hash function in random oracle which maps $\{0,1\}^* \times \{0,1\}^* \rightarrow Z_p$ such that $t_{i,j} \neq t'_{i,j}$. *t* is only known to CA.

Setup(1^k) : Base on the implicit security parameter *k*, the CA selects a large prime number p, a bilinear group (G,G1) with order p, a generator $g \in$ G, h $\in$ G, y $\in_R Z_p$ and $t_{i,j} \in Z_p$ (i $\in$ [1,n],j $\in$ [1,n_i]). CA calculates Y = e (g,h)^y and $T_{i,j} = g^{t_{i,j}}$ (i $\in$ [1,n],j $\in$ [1,n_i]).

$$\begin{cases} \text{MPK}=(e, g, h, Y, T_{i,j} (i \in [1,n], j \in [1,n_i])\ ). \\ \text{MSK}=(y, t_{i,j}\ (i \in [1,n], j \in [1,n_i])) \end{cases}$$

KeyGen (MSK,L) : Based on MSK and attribute list L of user *u*, CA generates r $\in_R Z_p$ and calculate the SK of user u as follows.

$$\text{SK}_L = \{ h^{y+r}, \forall\, v_{i,j} \in L\ D_{i,j} = (T_{i,j})^r, g^r, L\}$$

Encrypt (MPK,M,W) : It run by sender. Based on MPK, message M and access structure *A* containing policy W. It selects s $\in_R Z_p$ and calculates ciphertext CT as follows.

$$\begin{cases} C_1 = M\ Y^s \\ C_2 = g^s \\ C_3 = h^s\ (\prod_{v_{i,j} \in W} T_{i,j})^s \end{cases}$$

$$\text{CT} = < C_1, C_2, C_3, W >.$$

Decrypt (MPK, CT, SK_L): Assume AS $\subseteq$ L and AS = W. Therefore, after identifying the AS, user just multiplies all the related values, which are given in the secret key i.e. $\prod_{v_{i,j} \in AS} Dj$.

$$= \frac{C_1\ e(g^r, C_3)}{e(C_2, h^{y+r} \prod_{vi,j \in \text{AS}} (T_{i,j}{}^r))}$$

$$= \frac{M\ e(g,h)^{y\,s}\ e(g,h)^{r\,s}\ e(g,g)^{r\,s\,p}}{}$$

$$= \frac{M\, e(g,h)^{ys}\, e(g,h)^{rs}\, e(g,g)^{r\,s\,p}}{e(g,h)^{ys}\, e(g,h)^{rs}\, e(g,g)^{r\,s\,q}}$$

$$= M$$

Here $p=\sum_{v_{i,j}\in W} t_{i,j}$ and $q=\sum_{v_{i,j}\in AS} t_{i,j}$

4 Security Analysis

4.1 Construction of secret keys $t_{i,j}$

Here we assume that $\sum_{v_{i,j}\in AS} t_{i,j} \neq \sum_{v_{i,j}\in AS'} t_{i,j}$. If there exists AS ⊆ L and AS ⊆ L' such that $\sum_{v_{i,j}\in AS} t_{i,j} = \sum_{v_{i,j}\in AS'} t_{i,j}$ than L' can decrypt W, where L'⊭ W and L ⊨ W. This assumption holds with given probability where $N= \prod_{i=1}^{n} n_i$. p is the group order of G.

$\frac{p(p-1)\ldots(p-(N-1))}{p^N} > \frac{(p-(N-1))^N}{p^N} = (1 - \frac{N-1}{p})^N > (1- \frac{N(N-1)}{p}) > (1- \frac{N^2}{p})$.

Theorem 1: The proposed scheme satisfies the indistinguishability of messages under the DBDH assumption.

Assume that the adversary $\mathbb{A}$ wins the selective game with the advantage ϵ. So we can construct algorithm $\mathbb{X}$ that will break the DBDH assumption with advantage $\frac{\epsilon}{2}(1- \frac{N^2}{p})$ where $N= \prod_{i=1}^{n} n_i$ which is number of access structure. The DBDH challenger generates a,b,c,z $\in_R Z_p$, v $\in_R$ {0,1} and g where g is the generator for group G so

$Z = e(g, g)^{abc}$ if v = 0

$= e(g, g)^{z}$ otherwise

The DBDH challenger gives $(g, g^a, g^b, g^c, z) \in G \times G1$ to $\mathbb{X}$. Now $\mathbb{A}$ gives the challenge access structure W* to $\mathbb{X}$. Let W*=[W1*,W2*,…,Wk*]. $\mathbb{X}$ selects u $\in_R Z_p$ and sets $h=g^u$ and $Y=e(g^a,(g^b)^u)=e(g,h)^{ab}$. For $t'_{i,j}$ {i ∈ [1,n], j ∈ $[1,n_i]$} $\in_R Z_p$, $\mathbb{X}$ computes private keys $t_{i,j}$ {i ∈ [1,n], j ∈ $[1,n_i]$} and public keys $T_{i,j}$ $\{i \in [1,n], j \in [1,n_i]\}$ as follows.

$t_{i,j} = t'_{i,j}$ if($v_{i,j} = W_i^*$)

$= b\, t'_{i,j}$ otherwise

$T_{i,j} = g^{t'_{i,j}}$ if($v_{i,j} = W_i^*$)

$= (g^b)^{t'_{i,j}}$ otherwise.

$\mathbb{X}$ gives MPK=(e, g,h,Y, $T_{i,j}$ {i ∈ [1,n], j ∈ $[1,n_i]$}) to $\mathbb{A}$. For **KeyGen** query L there exists $v_{i,j} = L_i$ and $v_{i,j} \neq W^*$ because L ≠ W*. So we can write $\sum_{v_{i,j}\in L} t_{i,j} = X_1 + bX_2$ where $X_1, X_2 \in Z_p$. Here X_1 and X_2 can be represented as sum of $t'_{i,j}$ value. It means

X can calculate X_1 and X_2, it selects $\beta \in_R Z_p$ and set r = $\frac{\beta - ua}{X2}$ and compute SK_L as follows

$SK_L=\{g^{\frac{\beta}{X2}} (g^a)^{\frac{-u}{X2}}, (g^{ab})^u\ g^{\frac{\beta u}{X2}} (g^a)^{\frac{-u^2}{X2}}, \forall_{v_{i,j} \in L} (g^{t_{i,j}})^{\frac{\beta}{X2}}\ ((g^a)^{t_{i,j}})^{\frac{-u}{X2}}\}$

So SK_L is a valid secret key as follows

$(g^{ab})^u\ g^{\frac{\beta u}{X2}} (g^a)^{\frac{-u^2}{X2}} = (g^u)^{ab}\ (g^u)^{\frac{\beta - ua}{X2}} = h^y\ h^r = h^{y+r}$.

$g^{\frac{\beta}{X2}} (g^a)^{\frac{-u}{X2}} = g^{\frac{\beta - ua}{X2}} = g^r$ and $(g^{t_{i,j}})^{\frac{\beta}{X2}}\ ((g^a)^{t_{i,j}})^{\frac{-u}{X2}} = (g^{t_{i,j}})^r = (T_{i,j})^r$.

Attacker $\mathbb{A}$ will identify set AS $\subseteq$ L and calculate $\prod_{v_{i,j} \in AS} (T_{i,j})^r = g^{r \sum_{v_{i,j} \in AS} t_{i,j}}$

If $X_2 = 0 \bmod p$ holds than there exists AS $\subseteq$ L such that $\sum_{v_{i,j} \in AS} t_{i,j} = \sum_{v_{i,j} \in W^*} t_{i,j}$. Therefore the probability is at most N^2/p as given in previous section.

Now for the **Encrypt**, challenger X chooses $\mu \in_R \{0,1\}$ and computes $C_1^* = M_\mu Z^u$,$C_2^* = g^c$, $C_3^* = h^s (g^c)^{\sum_{v_{i,j} \in W^*} t_{i,j}}$ and sends CT*= < C_1^*,C_2^*,C_3^*, W*> to $\mathbb{A}$. $\mathbb{A}$ outputs guess $\mu' \in \{0,1\}$. X outputs 1 if $\mu' = \mu$ or outputs 0, if $' \neq \mu$. There will be two cases

(i) If $Z=e(g, g)^{abc}$ then $\mathbb{A}$'s advantage is ϵ, so $Pr[x \to 1 | Z= e(g, g)^{abc}] = Pr[\mu' = \mu | Z=e(g, g)^{abc}] = 1/2 + \epsilon$.

(ii) If $Z=e(g,g)^z$ then $\mathbb{A}$ has no advantage to distinguish bit μ, hence $Pr[x \to 0 | Z= e(g, g)^z] = Pr[\mu' \neq \mu | Z= e(g, g)^z] = 1/2$.

From (i) and (ii) it follows that X's advantage in this DBDH game is $\frac{\epsilon}{2}(1 - \frac{N^2}{p})$. ■

Currently we had used symmetric bilinear map in this proof. our scheme can also be proven with asymmetric bilinear map like e: $G_1 \times G_2 \to GT$ over MNT curve [18], where G_1 and G_2 are two different groups, in this case we can also prove the indistinguishability under DBDH assumptions over G_2[19].

5 Analysis of approach

For the sack of clarity we omit the detailed discussion on the previous approach we had given the comparison based on size of public key (MPK), Master key (MSK), secret key of user (SK) and ciphertext (CT) in table 2. Here n is total number of attributes, N'= total number of attributes in the system i.e. $N' = \sum_{i=1}^{n} n_i$ where n_i is the number of possible values for attribute *i*, G1,G2 and GT are bilinear groups, the notation | G| shows the bit-length of the element belongs to group G, the notations kG and kC_e for some k>0, shows the k times calculation over the group G and pairing operations respectively, r_1 is the set of attributes associated with ciphertext and r_2 is the set of associated with secret key length. Here r_1 can be fixed but r_2 will be different for each user. The figures in the table show the maximum value for the given approach. Table 3 shows the expected computational time based on the input parameters for the different approach. Table 4 shows the properties for the approaches. Table 5 shows the type of access structure that used in the policy

construction. The results given in all tables clearly indicate that our scheme is better than any of the previous approaches.

Scheme	MPK	MSK	SK	CT
[3]	$n\|G_1\| + \|G_T\|$	$(n+1)\|Z_p\|$	$r_2\|G_1\|$	$r_1\|G_1\| + \|G_T\|$
[4]	$n\|G_1\| + \|G_T\|$	$(n+1)\| Z_p\|$	$r_2\|G_1\|$	$r_1\|G_1\| + \|G_T\|$
[21]	$(3n+1)\|G_1\| + \| G_T\|$	$(3n+1)\| Z_p\|$	$(2n+1)\|G_1$	$\|(n+1)\|G_1\| + \| G_T\|$
[5]	$3\|G_1\| + \| G_T\|$	$\|Z_p\| + \|G\|$	$(2n+1)\|G_1\|$	$(2r_2+1)\|G_1\| + \| G_T\|$
[21]	$(2N'+1)\|G_1\| + \|G_T\|$	$(2N'+1)\| Z_p\|$	$(3n+1)\|G_1\|$	$(2N'+1)\ \|G_1\| +\|G_T\|$
[22]	$2\|G_1\| + \|G_T\|$	$\|G_1\|$	$(3+n)\|G_1\|$	$(1 + r_1n)\|G_1\| + \|G_T$
[11]	$(2N'+3)\|G_1\| + \|G_T\|$	$(N'+1)\| Z_p\|$	$2\|G_1\|$	$2\|G_1\| + \|G_T\|$
[12]	$(2n)\ \|G_1\|$	$3\|Z_p\|$	$(2n)\ \|G_1\|$	$3\ \|G_1\|$
Our scheme	$(4+n)\ \|G_1\|$	$\|Z_p\|$	$(n+2)\ \|G_1\|$	$4\ \|G_1\|$

Table 2: Size of parameters for ABE schemes

Scheme	Enc.	Dec.
[3]	$r_1G_1 + 2G_T$	$r_1C_e + (r_1+1)G_T$
[4]	$r_1G_1 + 2G_T$	$r_1C_e + (r_1+1)G_T$
[21]	$(n+1)G_1 + 2G_T$	$(n+1)C_e + (n+1)G_T$
[5]	$(2r_1+1)G_1 + 2G_T$	$2r_1C_e + (2r_1+2)G_T$
[21]	$(2N'+1)G_1 + 2G_T$	$(3n+1)C_e + (3n+1)G_T$
[22]	$(1 + 3r_1n)G_1 + 2G_T$	$(1 + n + r_1)C_e + (3r_1 - 1)G_1 + 3G_T$
[11]	$(n+1)G_1 + 2G_T$	$2C_e + 2G_T$
[12]	$(n+t+1)\ G_1$	$3C_e + (t_2)G_T$ + O(n) multiplication for Aggregate function
Our scheme	$(n+4)\ G_1$	$3C_e + 2\ G_1$

Table 3: Computational time for each approach

Scheme	Policy	Recipient Anonymity	Assumption
[3]	Key	No	DMBDH
[4]	Key	No	DBDH
[21]	Ciphertext	No	DBDH
[5]	Ciphertext	No	Generic Group Model
[21]	Ciphertext	Yes	DBDH, D-Linear
[22]	Ciphertext	No	DBDH
[11]	Ciphertext	No	DBDH
[12]	Ciphertext	No	aMSE-DDH
Our scheme	Ciphertext	No	DBDH

Table 4: Properties of different ABE scheme.

Scheme	Nature of Policy
[3]	Threshold Structure
[4]	Tree-based Structure
[21]	AND-gates on positive and negative attributes with wildcards
[5]	Tree-Based Structure
[21]	Linear Structure
[22]	AND-gates on multi-valued attributes with wildcards
[11]	AND-gates on multi-valued attributes
[12]	AND-gates on multi-valued attributes
Our scheme	AND-gates on multi-valued attributes

Table 5: Expressiveness of policy

6 Conclusion and Future Work

In this paper, we propose the constant ciphertext length approach where the number of attributes in ciphertext policy must be a subset of attributes in the receiver's secret key. Our approach is based on the AND-gates with multivalued attributes. Our scheme does not provide recipient's anonymity. In future, we make this scheme for threshold ABE and add feature like the recipient's anonymity to increase the security. One can apply this notion to the KP-ABE scheme to get the better bounds on the size of ciphertext or size of secret key length. All this work is to be considered as future work.

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