

# **A unidirectional conditional proxy re-encryption scheme based on non-monotonic access structure**

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**Abstract:** Recently, Fang et al. [6] introduced an interactive(bidirectional) conditional proxy re-encryption(C-PRE) scheme such that a proxy can only convert ciphertexts that satisfy access policy set by a delegator. Their scheme supports monotonic access policy expressed by “OR” and “AND” gates. In addition, their scheme is called interactive since generation of re-encryption keys requires interaction between the delegator and delegatee. In this paper, we study the problem of constructing a unidirectional(non-interactive) C-PRE scheme supporting non-monotonic access policy expressed by “NOT”, “OR” and “AND” gates. A security model for unidirectional C-PRE schemes is also proposed in this paper. To yield a unidirectional C-PRE scheme supporting non-monotonic access policy, we extend the unidirectional PRE scheme presented by Libert et al. [8] by using the ideas from the non-monotonic attributed based encryption (ABE) scheme presented by Ostrovsky et al. [9]. Furthermore, the security of our C-PRE scheme is proved under the modified 3-weak Decision Bilinear Diffie-Hellman Inversion assumption in the standard model.

**Keywords:** Unidirectional conditional proxy re-encryption, The standard model, Non-monotonic access structure, Chosen ciphertext security, Attributed based encryption

## **1. Introduction**

Encryption is one of the most fundamental cryptographic primitives. The concept of proxy re-encryption (PRE) was introduced by Blaze et al. in 1998 [4]. A proxy in PRE systems can convert a ciphertext encrypted under Alice’s public key (delegator) into a ciphertext of the same message under Bob’s public key (delegatee). Proxy re-encryption has many applications such as email forwarding, distributed file system [2]. A bidirectional PRE scheme allows a proxy to convert ciphertexts encrypted under Alice into ciphertexts under

Bob via a re-encryption key and the same key can also be used to translate from Bob to Alice. On the other hand, if the re-encryption key only allows one-way conversion (e.g., from Alice to Bob), then the corresponding PRE scheme is called unidirectional.

The PRE scheme in [4] is bidirectional and CPA secure under DDH assumption. In 2005, Ateniese et al. [2] presented several CPA secure unidirectional PRE schemes based on bilinear pairing. Then Canetti and Hohenberger [5] presented an appropriate definition of chosen ciphertext security(CCA) for bidirectional PRE schemes and the first CCA secure bidirectional PRE scheme. The work in [5] left an open problem to come up with a CCA secure unidirectional PRE scheme. Libert and Vergnaud [8] presented a definition of chosen ciphertext security (CCA) for unidirectional PRE schemes and the first unidirectional PRE scheme with CCA security in the standard model.

Normal PRE schemes allow a semi-trusted proxy to translate ciphertexts from Alice to Bob unconditionally. It is desirable that a proxy can only convert ciphertexts under certain constraints set by the delegator. Shao et al. [12] designed a PRE scheme with keyword search property, which allows a proxy equipped with trapdoor information to test whether a ciphertext from Alice contains one specified keyword. However, it is pointed out [13] that the trapdoor still allows the proxy to convert ciphertexts from Alice without any restriction. On the other hand, Weng et al. [14, 15] introduced the notion of conditional proxy re-encryption (C-PRE) such that only ciphertexts whose keywords satisfy certain conditions set by Alice can be converted by a proxy. They also left it as an open problem to construct a C-PRE scheme supporting access policy consisting of “OR” and “AND” gates over keywords.

Wang et al. [13] presented a unidirectional PRE scheme supporting conjunctive keywords search and selective delegation such that a proxy can only re-encrypt ciphertexts that contain a set of keywords specified by the delegator. In other words, their construction supports access policy expressed by “AND” gates. By regarding keywords as attributes, Fang et al. [6] presented an interactive(bidirectional) single-hop C-PRE scheme based on access tree used in the attribute based encryption scheme [7], which supports access policy consisting of “OR” and “AND” gates. Their scheme is called interactive since generation of re-encryption keys requires interactions between the delegator and delegatee who take their secret keys as private input. Interactive generation of re-encryption keys is an essential feature

of bidirectional C-PRE scheme defined in [5]. CCA security of their C-PRE scheme was proved under the random oracle model. They also left it as an open problem to construct a *non-interactive(unidirectional)* C-PRE scheme with security in *the standard model*.

Although Wang et al. [13] defined their CCA security model for unidirectional PRE schemes supporting conjunctive keywords search, their security model is coupled tightly with the notion of conjunctive keywords search. Hence the model in [13] is not suitable for C-PRE schemes supporting generic access structure. In addition, the work in [6] considered security model for interactive(bidirectional) C-PRE schemes and proved security of their construction under the random oracle model.

Sahai and Waters [11] introduced the concept of attribute based encryption (ABE), in which a ciphertext is associated with a set of attributes, and a user's private key will reflect an access policy over attributes that controls which ciphertexts a user is able to decrypt. The original construction of Sahai and Waters was limited to express threshold access structure. Goyal et al. [7] presented ABE schemes based on access tree in which the private key supports any monotonic access structure. To increase the expressibility of ABE schemes, Ostrovsky et al. [9] designed an ABE construction that supports non-monotonic access structure represented by "NOT", "OR" and "AND" gates over attributes.

Motivated by the above discussion, we aim to design a *unidirectional(non-interactive)* C-PRE scheme supporting *non-monotonic access structure* to enhance the expressibility of C-PRE schemes. The rest of paper is organized as follows. At first, we provide security definitions for unidirectional C-PRE schemes in which a ciphertext is associated with a set of keywords, and a re-encryption key will reflect an access policy that controls which ciphertexts a proxy is able to re-encrypt. Subsequently, we extend the unidirectional PRE scheme [8] to yield a unidirectional C-PRE scheme supporting non-monotonic access structures. Finally our construction is proved to be CCA secure under the standard model.

A challenge in our security proof lies in the fact that a corrupted user in our model is allowed to obtain re-encryption keys from the target user so long as the access structure associated with these re-encryption keys are not satisfied by the challenge set of attributes associated with the challenge ciphertext. On the other hand, in order to support negation by using the techniques in [9], we have to design two types of re-encryption keys, which also

affects the structure of a user's secret key in our construction.

## 2. Preliminaries

### 2.1 Bilinear pairing

Given a security parameter  $\lambda$ , an efficient algorithm  $PG(1^\lambda)$  outputs  $(e, G, G_T, g, p)$ , where  $G$  is a cyclic group of a prime order  $p$  generated by  $g$ , and  $2^{\lambda-1} < p < 2^\lambda$ .  $G_T$  is a cyclic group of the same order, and let  $e: G \times G \rightarrow G_T$  be a efficiently computable bilinear function with the following properties:

1. Bilinear:  $e(g^a, g^b) = e(g, g)^{ab}$ , for all  $a, b \in \mathbb{Z}_p$ .
2. Non-degenerate:  $e(g, g) \neq 1_{G_T}$ .

### 2.2 Modified 3-wDBDHI assumption

Given  $(e, G, G_T, g, p)$  output by  $PG(1^\lambda)$ , we define two experiments in which an adversary  $A$  outputs 0 or 1.

Experiment 0:  $A$  is given  $(g, g^{\frac{1}{a}}, g^a, g^{a^2}, g^b, e(g, g)^{\frac{b}{a^2}})$ ,  $a, b \leftarrow_R \mathbb{Z}_p^*$ .

Experiment 1:  $A$  is given  $(g, g^{\frac{1}{a}}, g^a, g^{a^2}, g^b, T)$ ,  $a, b \leftarrow_R \mathbb{Z}_p^*$ ,  $T \leftarrow_R G_T$ .

The modified **3-weak Decision Bilinear Diffie-Hellman Inversion** assumption [8] claims for any polynomial time algorithm  $A$ , the probability  $|\Pr[W_0] - \Pr[W_1]|$  is negligible, where  $W_i$  is the event that  $A$  outputs 1 in experiment  $i$ .

### 2.3 One-time signature

A digital signature scheme  $Sig = (\text{Gen}, \text{S}, \text{V})$  consists of the following algorithms:

1. **Gen**( $\lambda$ ): Outputs a secret/public key pair  $(sk, pk)$ .
2. **S**( $sk, m$ ): Given a secret key  $sk$  and a message  $m$ , then outputs a signature  $\sigma$ .
3. **V**( $pk, m, \sigma$ ): Takes as input a public key  $pk$ , a message  $m$  and a signature  $\sigma$ , then outputs either 1 or 0 to denote "accept" or "reject".

We review the definition of strong existential unforgeability for a signature scheme denoted

by  $Sig = (\text{Gen}, S, V)$  in experiment  $Exp_{1^\lambda, SCMA}^{Sig}(A)$ .

$Exp_{1^\lambda, SCMA}^{Sig}(A)$

The challenger  $C$  runs  $(pk, sk) \leftarrow \text{Gen}(1^\lambda)$  and sets  $S_\sigma \leftarrow \emptyset$ .

$(m^*, \sigma^*) \leftarrow A^{\text{O-Sig}}(pk)$ .

The adversary  $A$  wins if  $(m^*, \sigma^*) \notin S_\sigma$  and  $V(pk, m^*, \sigma^*) = 1$ .

Advantage of  $A$  in experiment  $Exp_{1^\lambda, SCMA}^{Sig}(A)$  is defined to be the probability that  $A$  wins in the experiment.

The oracle  $\text{O-Sig}$  is defined as follows:

$\text{O-Sig}(m)$

Returns  $\sigma = S(sk, m)$  and updates  $S_\sigma = S_\sigma \cup \{(m, \sigma)\}$ .

A strongly unforgeable one-time signature scheme  $Sig$  requires that for any PPT adversary  $A$  who can access the oracle  $\text{O-Sig}$  only once, its advantage  $Adv^{OTS}$  in experiment  $Exp_{1^\lambda, SCMA}^{Sig}(A)$  is negligible.

### 3. Security definitions and model

#### 3.1 Syntax of unidirectional C-PRE schemes

A *unidirectional* single-hop C-PRE scheme consists of the following algorithms. A ciphertext is associated with a set of keywords, and a re-encryption key will reflect an access policy over keywords that controls which ciphertexts a proxy is able to re-encrypt.

$\text{Setup}(\lambda)$ : Given the security parameter  $\lambda$ , this algorithm produces a set  $par$  of global public parameters.

$\text{Keygen}(par)$ : Given  $par$ , this algorithm generates a secret/public key pair  $(sk, pk)$ .

$\text{ReKeygen}(par, sk_i, pk_j, \tilde{A})$ : Given  $par$ , the secret key  $sk_i$  of user  $i$ , the public

key  $pk_j$  of user  $j \neq i$  and an access structure  $\tilde{A}$ , this algorithm generates a re-encryption key  $R_{i \rightarrow j, \tilde{A}}$ . We use an algorithm rather than an interactive protocol to implicitly assume that the process of generating re-encryption keys is non-interactive.

$\text{Enc}_1(par, pk_i, m)$ : Given  $par$ , a public key  $pk_i$  and a message  $m$ , this algorithm outputs a first level ciphertext  $CT_1$  that cannot be re-encrypted for another party.

$\text{Enc}_2(par, pk_i, m, \gamma)$ : Given  $par$ , a public key  $pk_i$ , a message  $m$  and a set  $\gamma$  of keywords(attributes), this algorithm outputs a second level ciphertext  $CT_2$  that can be re-encrypted into a first level ciphertext.

$\text{ReEnc}(par, CT_2, \gamma, pk_i, R_{i \rightarrow j, \tilde{A}})$ : Given  $par$ , a re-encryption key  $R_{i \rightarrow j, \tilde{A}}$  and a second level ciphertext  $CT_2$  encrypted under  $pk_i$  and a set  $\gamma$  of keywords, this algorithm outputs a first level ciphertext  $CT_1$  encrypted under  $pk_j$  when  $\gamma$  satisfies access structure  $\tilde{A}$ ; otherwise a message “invalid” is returned.

$\text{Dec}_1(par, sk_i, CT_1)$ : Given  $par$ , a secret key  $sk_i$  and a first level ciphertext  $CT_1$ , this algorithm outputs a message  $m$  or a message “invalid”.

$\text{Dec}_2(par, sk_i, CT_2)$ : Given  $par$ , a secret key  $sk_i$  and a second level ciphertext  $CT_2$ , this algorithm outputs a message  $m$  or a message “invalid”.

In the following, we will take  $par$  as implicit input for simplicity. For any message  $m$ , any couple of secret/public key pair  $(sk_i, pk_i), (sk_j, pk_j)$ , the following conditions of correctness should be satisfied:

$$(1) \text{Dec}_1(sk_i, \text{Enc}_1(pk_i, m)) = m ; \text{Dec}_2(sk_i, \text{Enc}_2(pk_i, m, \gamma)) = m ;$$

(2) If  $\gamma$  satisfies the access structure  $\tilde{A}$ , the following should hold:

$$CT_1 = \text{ReEnc}(\text{Enc}_2(pk_i, m, \gamma), \gamma, pk_i, \text{ReKeygen}(sk_i, pk_j, \tilde{A})),$$

$$\text{Dec}_1(sk_j, CT_1) = m .$$

### 3.2 Security of second level ciphertexts

**Init:** As in [8], the adversary  $Ad$  determines the target user  $i^*$ , the corrupted users and declares a set  $\gamma^*$  of keywords that he wishes to be challenged upon at this stage.

**Setup:** The challenger  $C$  runs  $\text{Setup}(\lambda)$  to produce the global public parameters  $par$  and generates key pairs as follows:

$$\text{KeyGen}(\cdot) \rightarrow (pk^*, sk^*), \text{KeyGen}(\cdot) \rightarrow (pk_x, sk_x), \text{KeyGen}(\cdot) \rightarrow (pk_h, sk_h).$$

$(pk^*, sk^*)$  is the key pair for the honest target user  $i^*$ . Key pairs subscribed by  $h$  or  $h'$  represents honest parties and corrupted key pairs are subscribed by  $x$  or  $x'$ .

**Phase 1:**  $Ad$  takes  $pk^*, \{pk_h\}, \{pk_x, sk_x\}$  as input and issue queries to oracles  $O_{rekey}$ ,  $O_{renc}$  and  $O_{dec-1}$ .

**Challenge:**  $Ad$  outputs two equal-length messages  $(m_0, m_1)$ . The challenger  $C$  flips a random bit  $b$  and returns  $CT_2^* = \text{Enc}_2(pk^*, m_b, \gamma^*)$ .

**Phase 2:**  $Ad$  still issues queries to oracles  $O_{rekey}$ ,  $O_{renc}$  and  $O_{dec-1}$ .

**Guess:**  $Ad$  outputs a bit  $b'$ .

The advantage of the adversary in this game is  $\varepsilon = |\Pr[b' = b] - 0.5|$ . A C-PRE scheme is CCA secure at level 2 if  $\varepsilon$  is negligible.

#### The re-encryption key oracle $O_{rekey}$

Given a tuple  $(pk_i, pk_j, \tilde{A})$ , this oracle proceeds as follows:

(1) If both  $pk_i$  and  $pk_j$  are honest, returns  $R_{i \rightarrow j, \tilde{A}} \leftarrow \text{ReKeygen}(sk_i, pk_j, \tilde{A})$ ;

(2) If the honest  $pk_i = pk^*$ ,  $pk_j$  is corrupted and  $\gamma^*$  does not satisfy  $\tilde{A}$ , returns

$$R_{i^* \rightarrow j, \tilde{A}} \leftarrow \text{ReKeygen}(sk^*, pk_j, \tilde{A});$$

### The re-encryption oracle $O_{renc}$

Given a tuple  $((CT_2, \gamma, pk_i, pk_j, \tilde{A})$ , where  $CT_2$  is a second level ciphertext encrypted under  $(pk_i, \gamma)$ , and  $pk_i, pk_j$  are public keys produced by  $\text{Keygen}$ , this oracle proceeds as follows:

(1) If  $pk_i = pk^*$ ,  $pk_j$  is corrupted and  $\text{Dec}_2(sk^*, CT_2) \in \{m_0, m_1\}$ , returns a message “invalid” since re-encryption may leak information about the challenge bit  $b$  in this case.

(2) If  $\gamma$  satisfies the access structure  $\tilde{A}$ , computes  $R_{i \rightarrow j, \tilde{A}} \leftarrow \text{ReKeygen}(sk_i, pk_j, \tilde{A})$  and returns the first level ciphertext  $CT_1 \leftarrow \text{ReEnc}((CT_2, pk_i, \gamma), R_{i \rightarrow j, \tilde{A}})$ . Otherwise, outputs a message “invalid”.

### First level decryption oracle $O_{dec-1}$

Given a tuple  $(pk_i, CT_1)$ , where  $CT_1$  is a first level ciphertext encrypted under the public key  $pk_i$ , this oracle proceeds as follows:

(1) If  $(pk_i, CT_1)$  is a **derivative** of the challenge pair  $(pk^*, CT_2^*)$ , returns a message “invalid”.

(2) Otherwise, returns  $m \leftarrow \text{Dec}_1(sk_i, CT_1)$ .

A **Derivative**  $(pk_i, CT_1)$  of the challenge pair  $(pk^*, CT_2^*)$  in this game is defined as follows:

If  $CT_1$  is a first level ciphertext and  $pk_i = pk^*$ , or  $pk_i$  belongs to a honest user,  $(pk_i, CT_1)$  is a **derivative** of the challenge pair if  $\text{Dec}_1(sk_i, CT_1) \in \{m_0, m_1\}$ .

### 3.3 Security of first level ciphertexts

**Init:** The adversary  $Ad$  determines the target user  $i^*$  and the corrupted users at this stage.



**Setup:** The challenger  $C$  runs  $\text{Setup}(\lambda)$  to produce the global public parameters  $par$  and generates key pairs in the same way as described previously:

$$\text{KeyGen}(\cdot) \rightarrow (pk^*, sk^*), \text{KeyGen}(\cdot) \rightarrow (pk_x, sk_x), \text{KeyGen}(\cdot) \rightarrow (pk_h, sk_h).$$

**Phase 1:** The adversary  $Ad$  who takes as input  $pk^*, \{pk_h\}, \{pk_x, sk_x\}$  can issue queries to oracles  $O_{rekey}, O_{dec-1}$ .

**Challenge:**  $Ad$  outputs two equal-length message  $(m_0, m_1)$ . The challenger  $C$  flips a random bit  $b$  and returns  $CT_1^* = \text{Enc}_1(pk^*, m_b)$ .

**Phase 2:**  $Ad$  still issues queries to the oracle  $O_{dec-1}$ .

**Guess:** The adversary outputs a bit  $b'$ .

The advantage of the adversary in this game is  $\varepsilon = |\Pr[b' = b] - 0.5|$ . A C-PRE scheme is CCA secure at level 1 if  $\varepsilon$  is negligible.

**The re-encryption key oracle  $O_{rekey}$**

Given a tuple  $(pk_i, pk_j, \tilde{A})$ , this oracle returns  $R_{i \rightarrow j, \tilde{A}} \leftarrow \text{ReKeygen}(sk_i, pk_j, \tilde{A})$ . This means that the adversary is allowed access to all re-encryption keys without any restriction.

**First level decryption oracle  $O_{dec-1}$**

Given a tuple  $(pk_i, CT_1)$ , where  $CT_1$  is a first level ciphertext encrypted under the public key  $pk_i$ , this oracle proceeds as follows:

If  $(pk_i, CT_1)$  is a **derivative** of the challenge pair  $(pk^*, CT_1^*)$ , returns a message “invalid”. Otherwise, returns  $m \leftarrow \text{Dec}_1(sk_i, CT_1)$ .

A **Derivative**  $(pk_i, CT_1)$  of the challenge pair  $(pk^*, CT_1^*)$  in this game is defined as follows:

If  $CT_1$  is a first level ciphertext and  $pk_i = pk^*$ ,  $(pk_i, CT_1)$  is a **derivative** of the

challenge pair if  $\text{Dec}_1(sk_i, CT_1) \in \{m_0, m_1\}$ .

Ateniese et al. [2] defined a security notion called *master secret security* for unidirectional PRE schemes. This notion requires that no coalition of dishonest delegates be able to pool their re-encryption keys in order to expose the secret key of their common delegator. It is discussed in [6, 8] that CCA security at level 1 implies master secret security for single-hop PRE schemes.

#### 4. Our C-PRE scheme

$\text{Setup}(\lambda)$  : Given  $(e, G, G_T, g, p)$  output by  $PG(1^\lambda)$ , picks generators  $(g_1, u, v) \leftarrow G, g_2 = g^w, w \leftarrow Z_p^*$  and a strongly unforgeable one-time signature scheme  $\text{Sig} = (\text{Gen}, S, V)$ . Let parameter  $d$  specifies the exact number of keywords that every second level ciphertext has. We associate each keyword with a unique element in  $Z_p^*$ .

Then chooses two random polynomials  $h(x)$  and  $q(x)$  of degree  $d$  subject to the constraint  $q(0) = w^{-1} \bmod p$ . We also define two functions  $T(x) = g_1^{x^d} \cdot g_2^{h(x)}$  and  $V(x) = g_2^{q(x)}$  that are publicly computable by interpolation. The set *par* of public parameters is  $(g, u, v, g_1, g_2, g_2^{q(0)} = g, \dots, g_2^{q(d)}, g_2^{h(0)}, \dots, g_2^{h(d)}, \text{Sig})$ .

$\text{Keygen}$  : Picks  $(x_{i1}, x_{i2}) \leftarrow Z_p^*$  and sets a secret/public key pair for user  $i$  as  $sk_i = (x_{i1}, x_{i2}), pk_i = (X_{i1} = g^{x_{i1}}, X_{i2} = g^{x_{i2}})$ .

$\text{ReKeygen}(sk_i, pk_j, \tilde{A})$ : Given the secret key  $sk_i$  of user  $i$ , the public key  $pk_j$  of user  $j$  and a non-monotonic access structure  $\tilde{A}$ , user  $i$  generates a re-encryption key  $R_{i \rightarrow j, \tilde{A}}$  as follows:

When dealing with a non-monotonic access structure  $\tilde{A}$  over a set of (unprimed)keywords  $\tilde{P}$ , we proceed similarly as in [9]. For each unprimed keyword  $p \in \tilde{P}$ , we define another

primed keyword  $p'$ . Let  $P' = \{p' \mid p \in \tilde{P}\}$ . Then define a monotonic access structure  $A$  over  $P = P' \cup \tilde{P}$  in such a way that  $S \in \tilde{A}$  if and only if  $N(S) \in A$ , where  $N(\cdot)$  is an operator defined as  $N(S) = S \cup \{p' \in P' \mid p \in P \setminus S\}$ . That is,  $N(S)$  consists of all the keywords in  $S$  plus the primed part of all the keywords that are not in  $S$ .

Let  $A$  be associated with a linear secret sharing mechanism  $\Pi$ . Then user  $i$  applies  $\Pi$  over the set  $P$  to obtain shares  $\{\lambda_k\}$  of the secret  $x_{i2}^{-1}$ . For each keyword  $\tilde{p}_k \in P$  (the underlying unprimed keyword is  $p_k$ ), a random  $r_k \leftarrow Z_p$  is chosen:

If  $\tilde{p}_k = p_k$  is unprimed,  $D_k = (D_k^{(1)} = X_{j1}^{\lambda_k} \cdot T(p_k)^{r_k}, D_k^{(2)} = X_{i2}^{r_k})$ .

If  $\tilde{p}_k = p_k'$  is primed,  $D_k = (D_k^{(3)} = X_{j1}^{\lambda_k} g^{r_k}, D_k^{(4)} = V(p_k)^{r_k}, D_k^{(5)} = X_{i2}^{r_k})$ .

The re-encryption key  $R_{i \rightarrow j, \tilde{A}} = \{D_k\}_{\tilde{p}_k \in P}$ .

$\text{Enc}_1(pk_i, m)$ : Given a public key  $pk_i$  and a message  $m$ , this algorithm proceeds as follows:

(1) Chooses  $r \leftarrow Z_p$  and generates a fresh one-time signature key pair  $(ssk, svk) \leftarrow \text{Gen}(\lambda)$ ;

(2)  $C_1 = svk, C_2' = e(g, X_{i1})^r, C_3 = e(g, g)^r \cdot m, C_4 = (u^{svk} v)^r$ ;

(3) Generates a one-time signature  $\sigma = S(ssk, m \parallel C_3 \parallel C_4)$ ;

The first level ciphertext  $CT_1 = (C_1, C_2', C_3, C_4, \sigma)$ .

$\text{Enc}_2(pk_i, m, \gamma)$ : Given the public key  $pk_i$ , a message  $m$  and a set  $\gamma$  of  $d$  keywords, outputs a second level ciphertext  $CT_2$  that can be re-encrypted into a first level ciphertext as follows:

(1) Chooses  $r \leftarrow Z_p$  and generates a fresh one-time signature key pair  $(ssk, svk)$ ;

(2)  $C_1 = svk, C_2 = X_{i2}^r, C_3 = e(g, g)^r \cdot m, C_4 = (u^{svk} v)^r$

$$C_5^p = \{T(p)^r\}_{p \in \gamma}, C_6^p = \{V(p)^r\}_{p \in \gamma};$$

(3) Generate a one-time signature  $\sigma = S(ssk, m \parallel C_3 \parallel C_4)$ .

The second level ciphertext  $CT_2 = (C_1, C_2, C_3, C_4, C_5^p, C_6^p, \sigma)$ .

$\text{ReEnc}((CT_2, \gamma, pk_i), R_{i \rightarrow j, \tilde{A}})$ : Given a re-encryption key  $R_{i \rightarrow j, \tilde{A}}$  and a second level ciphertext  $CT_2$  encrypted under  $(pk_i, \gamma)$ , if  $\gamma$  satisfies access structure  $\tilde{A}$ , this algorithm outputs a first level ciphertext  $CT_1$  encrypted under  $pk_j$  as follows:

(1) Parses  $CT_2$  as  $C_1 = svk, C_2 = X_{i2}^r, C_3 = e(g, g)^r \cdot m, C_4 = (u^{svk} v)^r$

$$C_5^p = \{T(p)^r\}_{p \in \gamma}, C_6^p = \{V(p)^r\}_{p \in \gamma}, \sigma$$

(2) Recall that  $\tilde{A}$  induces a monotonic access structure  $A$ . Denote  $\gamma' = N(\gamma)$ . As  $\gamma$  satisfies access structure  $\tilde{A}$ ,  $\gamma'$  is authorized in  $A$  by previous definition of the operator  $N(\cdot)$ . Let  $I = \{k : \tilde{p}_k \in \gamma'\}$ . A set of coefficients  $\{\omega_k\}_{k \in I}$  can be efficiently computed such that  $\sum_{k \in I} \omega_k \lambda_k = x_{i2}^{-1}$  [3].

For every unprimed attribute  $\tilde{p}_k = p_k \in \gamma'$ , (so  $p_k \in \gamma$  by definition of the operator  $N(\cdot)$ ), we proceed as follows:

(2.1) Extracts  $D_k = (D_k^{(1)} = X_{j1}^{\lambda_k} \cdot T(p_k)^{r_k}, D_k^{(2)} = X_{i2}^{r_k})$  from the re-encryption key;

(2.2) Computes  $Z_k = e(D_k^{(1)}, C_2) / e(D_k^{(2)}, C_5^{p_k})$

$$= e(g^{x_{j1} \lambda_k} \cdot T(p_k)^{r_k}, g^{x_{i2} r}) / e(g^{x_{i2} r_k}, T(p_k)^r) = e(g, g)^{x_{j1} x_{i2} \lambda_k r}$$

For every primed attributed  $\tilde{p}_k = p'_k \in \gamma'$  (so  $p_k \notin \gamma$  by definition), let  $\gamma_k = \gamma \cup \{p_k\}$ . Note that  $|\gamma_k| = d + 1$  and recall that the degree of the polynomial  $q(\cdot)$  is  $d$ . Using the keywords in  $\gamma_k$  as an interpolation set, we compute lagrangian coefficients  $\{\sigma_p\}_{p \in \gamma_k}$

such that  $\sum_{p \in \gamma_k} \sigma_p q(p) = q(0) = (\log_g g_2)^{-1}$ . Then we proceed as follows:

(2.3) Extracts  $D_k = (D_k^{(3)} = X_{j1}^{\lambda_k} g^{r_k}, D_k^{(4)} = V(p_k)^{r_k}, D_k^{(5)} = X_{i2}^{r_k})$  from the

re-encryption key and computes:

$$\begin{aligned}
Z_k &= \frac{e(D_k^{(3)}, C_2)}{e(D_k^{(5)}, \prod_{p \in \gamma} (C_6^p)^{\sigma_p}) e(D_k^{(4)}, C_2)^{\sigma_{p_k}}} = \frac{e(g^{\lambda_k x_{j1}} g^{r_k}, g^{x_{i2} r})}{e(g^{x_{i2} r_k}, \prod_{p \in \gamma} (V(p)^r)^{\sigma_p}) e(V(p_k)^{r_k}, g^{x_{i2} r})^{\sigma_{p_k}}} \\
&= \frac{e(g^{x_{j1} \lambda_k}, g^{x_{i2} r}) e(g^{r_k}, g^{x_{i2} r})}{e(g^{x_{i2} r_k}, \prod_{p \in \gamma} (g_2^{q(p) \cdot r})^{\sigma_p}) e(g_2^{q(p_k) r_k}, g^{x_{i2} r})^{\sigma_{p_k}}} \\
&= \frac{e(g^{x_{i2} x_{j1} \lambda_k}, g^r) e(g^{r_k}, g^{x_{i2} r})}{e(g, g_2)^{x_{i2} r_k r \sum_{p \in \gamma_k} \sigma_p q(p)}} = e(g, g)^{x_{j1} x_{i2} \lambda_k r}
\end{aligned}$$

Finally we have  $\prod_{k \in I} Z_k^{\omega_k} = e(g, g)^{x_{j1} x_{i2} r (\sum_{k \in I} \omega_k \lambda_k)} = e(g, g)^{\frac{rx_{j1} x_{i2}}{x_{i2}}} = e(g, g)^{rx_{j1}}$ .

The first level ciphertext  $CT_1 = (C_1, C_2' = e(g, X_{j1})^r, C_3, C_4, \sigma)$ .

$\text{Dec}_1(sk_i, CT_1)$ : Given a secret key  $sk_i$  and a first level ciphertext  $CT_1$ , this algorithm proceeds as follows:

(1) Parses  $CT_1$  as  $(C_1, C_2', C_3, C_4, \sigma)$ ;

(2) Computes  $C_2'^{\frac{1}{x_{i1}}} = e(g, g^{x_{i1}})^{\frac{r}{x_{i1}}} = e(g, g)^r$  and  $m = C_3 / e(g, g)^r$ ;

(3) Tests  $V(C_1, m \parallel C_3 \parallel C_4) = 1$  (V1)

If relation V1 does not hold, outputs a message “invalid”; otherwise outputs  $m$ .

$\text{Dec}_2(sk_i, (CT_2, \gamma))$ : Given a secret key  $sk_i$  and a second level ciphertext  $CT_2$ , this algorithm proceeds as follows:

(1) Parses  $CT_2$  as  $(C_1, C_2, C_3, C_4, C_5^p, C_6^p, \sigma)$ ;

(2) Tests  $e(C_2, (u^{C_1} v)) = e(X_{i2}, C_4)$  (V2)

If relation V2 does not hold, outputs a message “invalid”.

(3) Otherwise, computes  $m = \frac{C_3}{e(g, C_2)^{\frac{1}{x_{i2}}}} = \frac{e(g, g)^r \cdot m}{e(g, g^{x_{i2} r})^{\frac{1}{x_{i2}}}}$ ;

(4) If relation V1 does not hold, outputs a message “invalid”; otherwise outputs  $m$ .

**Remark:** Although our construction requires that every second level ciphertext has exactly

$d$  keywords, this restriction can be mitigated by using the method proposed in [9].

**Theorem 1:** Assume that the one-time signature scheme is strongly unforgeable. Our scheme is CCA secure at level 2 under the modified 3-wDBDHI assumption.

**Proof:** Let  $(g, A_{-1} = g^{\frac{1}{a}}, A_1 = g^a, A_2 = g^{a^2}, B = g^b, T)$  be a modified 3-wDBDHI instance. We build an algorithm  $B_A$  deciding whether  $T = e(g, g)^{\frac{b}{a^2}}$  from a successful CCA adversary  $Ad$  at level 2 with advantage  $\varepsilon$ .

**Init:** The adversary  $Ad$  determines the target user  $i^*$ , the corrupted users and declares a set  $\gamma^*$  of  $d$  keywords to be challenged upon.

**Setup:**  $B_A$  picks a one-time signature scheme  $Sig = (Gen, S, V)$  such that the maximal probability  $\delta$  that any public key can be selected should be less than  $2^{-\lambda}$  as in [8].  $B_A$  generates a fresh one-time signature key pair  $(sk^*, svk^*)$  and sets  $u = A_1^{\alpha_1}$ ,  $v = A_1^{-\alpha_1 \cdot svk^*} \cdot A_2^{\alpha_2}$ ,  $g_1 = (A_1)^\mu$ ,  $g_2 = A_2$ ,  $\alpha_1, \alpha_2, \mu \leftarrow Z_p^*$ .

Having chosen a random degree  $d$  polynomial  $f(x)$ , two random degree  $d$  polynomials  $u(x)$  and  $h(x)$  are defined as follows:

Let  $\gamma^* = \{p_1^*, \dots, p_d^*\}$ .  $B_A$  sets  $u(x) = -x^d$  for all  $x \in \gamma^*$  and  $u(x) \neq -x^d$  for some (arbitrary)  $x \notin \gamma^*$ . This ensures that  $u(x) = -x^d$  if and only if  $x \in \gamma^*$ . Let  $h(x) = (a^{-1} \cdot \mu \cdot u(x) + f(x))$ . Hence  $T(x) = g_1^{x^d} \cdot g_2^{h(x)} = g_1^{x^d + u(x)} \cdot g_2^{f(x)}$  can be publicly computed for arbitrary  $x$ .

Then  $B_A$  picks  $\{\theta_1, \dots, \theta_d\} \leftarrow Z_p^*$  and implicitly defines a random degree  $d$  polynomial  $q(x)$  such that  $q(0) = (a^2)^{-1}$ ,  $q(p_i^*) = \theta_i$ ,  $1 \leq i \leq d$ . We have  $g_2^{q(0)} = g$  and  $V(x) = g_2^{q(x)}$  can be computed for arbitrary  $x$  by interpolation. These parameters are distributed identically to that in the real scheme.

**Key generation:** Public key of an honest user  $i \in HU \setminus \{i^*\}$  is defined as  $X_{i1} = g^{ax_{i1}} = A_1^{x_{i1}}$ ,  $X_{i2} = g^{x_{i2}}$  for randomly chosen  $x_{i1}, x_{i2} \leftarrow Z_p^*$ . The target user's public key is set as  $X_{i^*1} = g^{a \cdot x_{i^*1}} = A_1^{x_{i^*1}}$ ,  $X_{i^*2} = g^{a^2 \cdot x_{i^*2}} = A_2^{x_{i^*2}}$  for randomly chosen  $x_{i^*1}, x_{i^*2} \leftarrow Z_p^*$ . Key pair of a corrupted user  $j$  is set as  $X_{j1} = g^{x_{j1}}$ ,  $X_{j2} = g^{x_{j2}}$  for randomly chosen  $x_{j1}, x_{j2} \leftarrow Z_p^*$ .

Given a tuple  $(pk_i, pk_j, \tilde{A})$  chosen by  $Ad$ , the re-encryption key oracle  $O_{rekey}$  is simulated by  $B_A$  as follows:

Let  $\Pi$  be the linear secret sharing mechanism associated with the monotonic access structure  $A$  induced by  $\tilde{A}$  over a set  $P$ . Let  $M$  be the share-generating matrix for  $\Pi$  with  $l$  rows and  $n$  columns. Each row  $M_k$  of  $M$  is labeled by a keyword named  $\tilde{p}_k \in P$  and  $p_k$  is the unprimed keyword underlying  $\tilde{p}_k$ . We list the following propositions:

**Proposition 1** [3]: Assume  $Q$  is not an authorized set in the access structure  $A$ .  $(\underbrace{1, 0, \dots, 0}_n)$  is linearly independent of the rows  $M_Q$ , where  $M_Q$  is the sub-matrix of  $M$  containing those rows labeled by keywords in  $Q$ .

**Proposition 2** [1, 10]: A vector  $\pi$  is linearly independent of a matrix  $N$  if and only if there exist a vector  $\theta$  which can be efficiently computed such that  $N \cdot \theta = \vec{0}$  while  $\pi \cdot \theta = 1$ .

Then we consider the following cases:

(1)  $i = i^*$ ,  $j \in CU$  and  $\gamma^*$  does not satisfy  $\tilde{A}$ :

When  $\tilde{A}$  is not satisfied by  $\gamma^*$ ,  $\gamma^{*/} = N(\gamma^*)$  is not an authorized set in  $A$ . According to **Proposition 1** and 2, there exists a column vector  $\vec{\theta} = (\theta_1, \dots, \theta_n)^T$  such that  $M_{\gamma^{*/}} \cdot \vec{\theta} = \vec{0}$  and  $(\underbrace{1, 0, \dots, 0}_n) \cdot \vec{\theta} = \theta_1 = 1$ .

Given a row vector  $R = (r_1, \dots, r_n) \leftarrow Z_p$ , let  $S = R + (s - r_1) \cdot \vec{\theta}$ , where

$s = (a^2 \cdot x_{i^*2})^{-1}$ . Note that  $S$  is uniformly distributed subject to the constraint that the first component is  $s$ . Let  $M \cdot S^T$  be the vector of  $l$  shares for the secret  $s$ . Let  $M_k$  be the row labeled by  $\widetilde{p}_k \in \gamma^{*/}$ , we have that  $M_k \cdot \vec{\theta} = 0$  by **Proposition 2**. Hence the share  $\lambda_k = M_k \cdot S^T = M_k \cdot R^T$ , which has no dependency on  $s$ .

(1.1) For a unprimed keyword  $\widetilde{p}_k = p_k \in \gamma^* \subseteq \gamma^{*/}$ ,  $B_A$  picks a random  $r_k \leftarrow Z_p$  and outputs the following:

$$D_k = (D_k^{(1)} = X_{j1}^{\lambda_k} \cdot T(p_k)^{r_k}, D_k^{(2)} = X_{i^*2}^{r_k})$$

The above is computable since the share  $\lambda_k$  is independent of  $(a^2 \cdot x_{i^*2})^{-1}$  by the above-mentioned discussion.

(1.2) For a unprimed keyword  $\widetilde{p}_k = p_k \notin \gamma^*$ ,  $\lambda_k$  is linearly dependent on  $(a^2 \cdot x_{i^*2})^{-1}$ .

$B_A$  picks a random  $r'_k \leftarrow Z_p$ , implicitly defines  $r_k = r'_k - \frac{(a\mu)^{-1} \cdot x_{j1} \cdot \lambda_k}{(p_k)^d + u(p_k)}$  and outputs

$D_k = (D_k^{(1)}, D_k^{(2)})$  as follows:

$$\begin{aligned} D_k^{(1)} &= X_{j1}^{\lambda_k} \cdot T(p_k)^{r_k} = (g^{x_{j1}})^{\lambda_k} \cdot (g_1^{(p_k)^d + u(p_k)} \cdot g_2^{f(p_k)})^{r_k} \\ &= (g^{x_{j1}})^{\lambda_k} \cdot (g_1^{(p_k)^d + u(p_k)} \cdot g_2^{f(p_k)})^{r'_k - \frac{(a\mu)^{-1} \cdot x_{j1} \cdot \lambda_k}{(p_k)^d + u(p_k)}} \\ &= (g_1^{(p_k)^d + u(p_k)} \cdot g_2^{f(p_k)})^{r'_k} g^{\frac{f(p_k) \cdot x_{j1} \cdot a \cdot \mu^{-1} \cdot \lambda_k}{(p_k)^d + u(p_k)}} \\ &= (g_1^{(p_k)^d + u(p_k)} \cdot g_2^{f(p_k)})^{r'_k} (g^{a \cdot \lambda_k})^{\frac{f(p_k) \cdot x_{j1} \cdot \mu^{-1}}{(p_k)^d + u(p_k)}} \\ D_k^{(2)} &= X_{i^*2}^{r_k} = (A_2^{x_{i^*2}})^{r'_k - \frac{(a\mu)^{-1} \cdot x_{j1} \cdot \lambda_k}{(p_k)^d + u(p_k)}} = (A_2^{x_{i^*2}})^{r'_k} (g^{a \cdot \lambda_k})^{\frac{\mu^{-1} \cdot x_{j1} \cdot x_{i^*2}}{(p_k)^d + u(p_k)}} \end{aligned}$$

As  $\lambda_k$  is linearly dependent on  $(a^2 \cdot x_{i^*2})^{-1}$ ,  $g^{a \cdot \lambda_k}$  can be computed via  $A_{-1}$  and  $A_1$ .

(1.3) For a primed keyword  $\widetilde{p}_k = p'_k \notin \gamma^{*/}$  (the underlying unprimed keyword  $p_k \in \gamma^*$ ),



$\lambda_k$  is linearly dependent on  $(a^2 \cdot x_{i^*2})^{-1}$ .  $B_A$  picks a random  $r'_k \leftarrow Z_p$ , and implicitly defines  $r_k = r'_k - \lambda_k \cdot x_{j1}$ . Then outputs  $D_k = (D_k^{(3)}, D_k^{(4)}, D_k^{(5)})$  as follows:

$$D_k^{(3)} = X_{j1}^{\lambda_k} g^{r'_k} = g^{r'_k}$$

$$D_k^{(4)} = V(p_k)^{r'_k} = (g_2)^{\theta_{pk} \cdot (r'_k - \lambda_k \cdot x_{j1})} = (A_2)^{r'_k \cdot \theta_{pk}} \cdot (A_2^{\lambda_k})^{-x_{j1} \cdot \theta_{pk}}$$

$$X_{i^*2}^{r_k} = (g^{a^2 \cdot x_{i^*2}})^{(r'_k - \lambda_k \cdot x_{j1})} = (A_2^{r'_k \cdot x_{i^*2}}) \cdot (g^{a^2 \cdot \lambda_k})^{-x_{j1} \cdot x_{i^*2}}$$

$g^{a^2 \cdot \lambda_k}$  can be computed via  $A_2$  since  $\lambda_k$  is linearly dependent on  $(a^2 \cdot x_{i^*2})^{-1}$ .

(1.4) For a primed keyword  $\widetilde{p}_k = p'_k \in \gamma^{*/}$  (the underlying unprimed keyword  $p_k \notin \gamma^*$ ),

$\lambda_k$  is independent of  $(a^2 \cdot x_{i^*2})^{-1}$ .  $B_A$  picks a random  $r_k \leftarrow Z_p$ , Then outputs the following:

$$D_k = (D_k^{(3)} = X_{j1}^{\lambda_k} g^{r_k}, D_k^{(4)} = V(p_k)^{r_k}, D_k^{(5)} = X_{i^*2}^{r_k})$$

(2)  $i = i^*$  and  $j \in HU \setminus \{i^*\}$ :

(2.1) For a primed keyword  $\widetilde{p}_k = p'_k$ ,  $B_A$  picks a random  $r_k \leftarrow Z_p$  and outputs

$D_k = (D_k^{(3)}, D_k^{(4)}, D_k^{(5)})$  as follows:

$$D_k^{(3)} = X_{j1}^{\lambda_k} g^{r_k} = g^{a \cdot x_{j1} \lambda_k} g^{r_k} = (g^{a \cdot \lambda_k})^{x_{j1}} g^{r_k}$$

$$D_k^{(4)} = V(p_k)^{r_k}$$

$$D_k^{(5)} = X_{i^*2}^{r_k} = A_2^{r_k \cdot x_{i^*2}}$$

As  $\lambda_k$  may be linearly dependent on  $(a^2 \cdot x_{i^*2})^{-1}$ ,  $g^{a \cdot \lambda_k}$  can be computed via  $A_{-1}$  and

$A_1$ .

(2.2) For a unprimed keyword  $\widetilde{p}_k = p_k$ ,  $B_A$  picks a random  $r_k \leftarrow Z_p$  and outputs

$D_k = (D_k^{(1)}, D_k^{(2)})$  as follows:

$$D_k^{(1)} = X_{j1}^{\lambda_k} \cdot T(p_k)^{r_k} = (g^{a \cdot \lambda_k})^{x_{j1}} \cdot T(p_k)^{r_k}$$

$$D_k^{(2)} = X_{i^*2}^{r_k} = A_2^{r_k \cdot x_{i^*2}}$$

Similarly,  $g^{a \cdot \lambda_k}$  can be computed via  $A_{-1}$  and  $A_1$ .

(3) If  $i \in HU \setminus \{i^*\}$ : This can be handled easily since we know the shared secret  $x_{i_2}$  in this case.

Finally, returns the re-encryption key  $R_{i \rightarrow j, \tilde{A}} = \{D_k\}$ .

Let the challenge second level ciphertext  $CT_2^* = (svk^*, C_2^*, C_3^*, C_4^*, C_5^{p^*}, C_6^{p^*}, \sigma^*)$  and  $m_b$  be the encrypted challenge message. Then we define an event  $F_{OTS}$  as follows and bound its probability to occur in a way similar to [8].

(1) The adversary issues a re-encryption query in which  $CT_2 = (svk^*, C_2, C_3, C_4, C_5^p, C_6^p, \sigma)$  and  $m$  is the implicit message embedded in  $CT_2$  such that  $(m \parallel C_3 \parallel C_4, \sigma) \neq (m_b \parallel C_3^* \parallel C_4^*, \sigma^*)$  and  $V(sv k^*, m \parallel C_3 \parallel C_4, \sigma) = 1$ .

(2) The adversary issues a first level decryption query in which  $CT_1 = (svk^*, C_2', C_3, C_4, \sigma)$  and  $m$  is the implicit message embedded in  $CT_1$  such that  $(m \parallel C_3 \parallel C_4, \sigma) \neq (m_b \parallel C_3^* \parallel C_4^*, \sigma^*)$  and  $V(sv k^*, m \parallel C_3 \parallel C_4, \sigma) = 1$ .

As the adversary has no information on  $svk^*$  before the challenge phase, the probability of occurrence of  $F_{OTS}$  in phase 1 can be bounded by  $q_o \cdot \delta \leq \frac{q_o}{2^\lambda}$ , where  $q_o$  is the total number of oracle queries made by the adversary and  $\delta$  is the maximal probability that any one-time public key can be selected.

During the guess phase, the event  $F_{OTS}$  could be used to construct an algorithm breaking strong unforgeability of the one-time signature scheme. Therefore  $\Pr[F_{OTS}] \leq \frac{q_o}{2^\lambda} + Adv^{OTS}$ .

The re-encryption oracle  $O_{renc}$  is simulated as follows:

Given a tuple  $((CT_2, \gamma), pk_i, pk_j, \tilde{A})$  chosen by the adversary,  $B_A$  proceeds as follows:

(1) Parses  $CT_2$  as  $(C_1, C_2, C_3, C_4, C_5^p, C_6^p, \sigma)$ ;

(2) If  $\tilde{A}$  is not satisfied by  $\gamma$  or relation V2 does not hold, outputs a message “invalid”;

(3) If  $i \in HU \setminus \{i^*\}$  or  $j \notin CU$ ,  $B_A$  makes a query to the oracle  $O_{rekey}$  and re-encrypts

by using the returned re-encryption key.

(4) If  $i = i^*$ ,  $j \in CU$ , we consider the following sub-cases:

(4.1)  $C_1 = svk^*$ : If  $(m, C_3, C_4, \sigma) = (m_b, C_3^*, C_4^*, \sigma^*)$  in this situation, we have  $m = m_b$ .

But we should return “invalid” by our convention for the re-encryption oracle presented in section

3.2. In addition,  $C_1 = svk^* \wedge (m, C_3, C_4, \sigma) \neq (m_b, C_3^*, C_4^*, \sigma^*)$  implies an occurrence of  $F_{OTS}$ .

Hence  $B_A$  halts when  $C_1 = svk^*$  at step (4.1).

(4.2)  $C_1 \neq svk^*$ : Assuming  $C_4 = (u^{svk}v)^r$ , relation V2 guarantees that  $C_2 = (X_{i^*2})^r$ . So

$C_2^{1/x_{i^*2}} = (A_2^{r \cdot x_{i^*2}})^{1/x_{i^*2}} = A_2^r$ ,  $C_4 = (u^{svk}v)^r = (A_1^{\alpha_1(svk-svk^*)} \cdot A_2^{\alpha_2})^r$ ,  $B_A$  computes

$(\frac{C_4}{C_2^{\alpha_2/x_{i^*2}}})^{\frac{1}{\alpha_1(svk-svk^*)}} = A_1^r$ ,  $e(A_1^r, A_{-1}^{x_{j1}}) = e(g, X_{j1})^r = C_2'$ . Let  $CT_1 = (C_1, C_2', C_3, C_4, \sigma)$ . If

$Dec_1(sk_j, CT_1) \in \{m_0, m_1\}$ , returns a message “invalid”. Otherwise, returns  $CT_1$ .

The first-level decryption oracle  $O_{dec-1}$  is simulated as follows:

Given a tuple  $(pk_i, CT_1)$ , where  $CT_1$  is a first level ciphertext encrypted under a public

key  $pk_i$ ,  $B_A$  proceeds as follows:

(1) Parses  $CT_1$  as  $(C_1, C_2', C_3, C_4, \sigma)$ ;

(2) If  $i \in CU$ , decrypts the ciphertext by the known secret key;

(3)  $i \in HU$ ,  $C_1 = svk^*$ : Assuming  $(m, C_3, C_4, \sigma) = (m_b, C_3^*, C_4^*, \sigma^*)$ , we should output a

message “invalid” to indicate that  $CT_1$  is a Derivative of the challenge  $(pk^*, CT_2^*)$ . Otherwise,

$(m, C_3, C_4, \sigma) \neq (m_b, C_3^*, C_4^*, \sigma^*)$  means that we either face with an occurrence of the event

$F_{OTS}$  or  $B_A$  should output a message “invalid” to indicate that relation V1 does not hold. Hence

$B_A$  halts at step (3).

(4)  $i \in HU$  and  $C_1 \neq \text{svk}^*$ : As  $C_4 = (u^{\text{svk}} v)^r = (A_1^{\alpha_1(\text{svk}-\text{svk}^*)} \cdot A_2^{\alpha_2})^r$ ,  $B_A$  computes:

$$\begin{aligned} e(A_{-1}, C_4) &= e(A_{-1}, A_1^{\alpha_1(\text{svk}-\text{svk}^*)r}) e(A_{-1}, A_2^{\alpha_2 r}) \\ (C_2')^{\frac{\alpha_2}{x_i}} &= e(g, X_{i1})^{\frac{r \cdot \alpha_2}{x_{i1}}} = e(g, g^{a \cdot x_{i1}})^{\frac{r \cdot \alpha_2}{x_{i1}}} = e(g, g^a)^{\alpha_2 r} \\ \left( \frac{e(A_{-1}, C_4)}{(C_2')^{\frac{\alpha_2}{x_i}}} \right)^{\frac{1}{\alpha_1(\text{svk}-\text{svk}^*)}} &= e(g, g)^r, \quad m = C_3 / e(g, g)^r \end{aligned}$$

(5) If relation V1 does not hold or  $m \in \{m_0, m_1\}$ , outputs a message “invalid”; Otherwise outputs  $m$ .

**Challenge:** The adversary outputs two equal-length message  $(m_0, m_1)$ .  $B_A$  flips a random bit  $b$  and outputs the challenge second level ciphertext  $CT_2^*$  as follows:

$$\begin{aligned} C_1^* &= \text{svk}^*, C_2^* = B^{x_{i^*2}^*}, C_3^* = T \cdot m_b, C_4^* = B^{\alpha_2}, \quad C_5^{p^*} = \{B^{f(p_i^*)}\}_{p_i^* \in \gamma^*}, \quad C_6^{p^*} = \{B^{\theta_i^*}\}_{p_i^* \in \gamma^*}, \\ \sigma^* &= S(\text{ssk}^*, m_b \parallel C_3^* \parallel C_4^*) \end{aligned}$$

When  $Ad$  outputs  $b' = b$ ,  $B_A$  outputs 1 to indicate that  $T = e(g, g)^{\frac{b}{a^2}}$ . Otherwise  $B_A$  outputs 0 to indicate that  $T$  is random.

If  $T = e(g, g)^{\frac{b}{a^2}}$ , we have  $r^* = \frac{b}{a^2}$  and the following equations:

$$\begin{aligned} C_2^* &= X_{i^*2}^{r^*} = (g^{a^2 \cdot x_{i^*2}^*})^{\frac{b}{a^2}} = B^{x_{i^*2}^*} \\ C_4^* &= (u^{\text{svk}^*} v)^{r^*} = (A_2^{\alpha_2})^{r^*} = B^{\alpha_2} \\ C_5^{p^*} &= \{T(p_i^*)^{r^*}\}_{p_i^* \in \gamma^*} = \{A_2^{f(p_i^*) \cdot r^*}\}_{p_i^* \in \gamma^*} = \{B^{f(p_i^*)}\}_{p_i^* \in \gamma^*} \\ C_6^{p^*} &= \{V(p_i^*)^{r^*}\}_{p_i^* \in \gamma^*} = \{A_2^{q(p_i^*) \cdot r^*}\}_{p_i^* \in \gamma^*} = \{B^{\theta_i^*}\}_{p_i^* \in \gamma^*} \end{aligned}$$

So  $CT_2^*$  is a valid encryption of  $m_b$  if  $T = e(g, g)^{\frac{b}{a^2}}$ . In contrast, if  $T$  is random,  $CT_2^*$  perfectly hides  $m_b$  and  $Ad$  guesses  $b$  with probability 0.5. Hence the overall advantage of  $B_A$  is  $\varepsilon - \Pr[F_{OTS}]$ .

**Theorem 2:** Assume that the one-time signature scheme is strongly unforgeable, our scheme is CCA secure at level 1 under the modified 3-wDBDHI assumption.

**Proof:** Let  $(g, A_{-1} = g^{\frac{1}{a}}, A_1 = g^a, A_2 = g^{a^2}, B = g^b, T)$  be a modified 3-wDBDHI instance.

We build an algorithm  $B_A$  deciding whether  $T = e(g, g)^{\frac{b}{a^2}}$  from a successful CCA adversary  $Ad$  at level 1 with advantage  $\varepsilon$ .

**Init:** The adversary  $Ad$  determines the target user  $i^*$  and the corrupted users at this stage.

**Setup:**  $B_A$  picks a one-time signature scheme  $Sig = (Gen, S, V)$  and generates a fresh one-time signature key pair  $(ssk^*, svk^*)$ .

$B_A$  sets  $u = g^{\alpha_1}$ ,  $v = g^{-\alpha_1 \cdot svk^*} \cdot X_{i^*_1}^{\alpha_2}$ ,  $\alpha_1, \alpha_2 \leftarrow Z_p^*$ ,  $g_1 \leftarrow G$ ,  $g_2 = g^w$ ,  $w \leftarrow Z_p^*$ . Then chooses two random polynomials  $h(x)$  and  $q(x)$  of degree  $d$  subject to the constraint  $q(0) = w^{-1}$ . Subsequently  $B_A$  defines two publicly computable functions  $T(x) = g_1^{x^d} \cdot g_2^{h(x)}$  and  $V(x) = g_2^{q(x)}$ .

**Key generation:** Key pair of an honest users  $i \in HU \setminus \{i^*\}$  is defined as  $X_{i_1} = g^{x_{i_1}}$ ,  $X_{i_2} = g^{x_{i_2}}$  for randomly chosen  $x_{i_1}, x_{i_2} \leftarrow Z_p^*$ . The target user's public key is set as  $X_{i^*_1} = g^{a^2 \cdot x_{i^*_1}} = A_2^{x_{i^*_1}}$ ,  $X_{i^*_2} = g^{a \cdot x_{i^*_2}}$  for randomly chosen  $x_{i^*_1}, x_{i^*_2} \leftarrow Z_p^*$ . Key pair of a corrupted user  $j$  is set as  $X_{j_1} = g^{x_{j_1}}$ ,  $X_{j_2} = g^{x_{j_2}}$  for randomly chosen  $x_{j_1}, x_{j_2} \leftarrow Z_p^*$ .

Given a tuple  $(pk_i, pk_j, \tilde{A})$  chosen by the adversary, the re-encryption key oracle  $O_{rekey}$  is simulated by  $B_A$  as follows:

Let  $\Pi$  be the linear secret sharing mechanism associated with the monotonic access structure  $A$  induced by  $\tilde{A}$  over a set  $P$ . As the secret keys of corrupt users and honest

users  $i \neq i^*$  are known to  $B_A$ , we only consider how to handle the following case:

(1)  $i = i^*, j \neq i^*$ :

Let  $M$  be the share-generating matrix for  $\prod$  with  $l$  rows and  $n$  columns. Each row  $M_k$  of  $M$  is labeled by a keyword named  $\widetilde{p}_k \in P$  and  $p_k$  is the unprimed keyword underlying  $\widetilde{p}_k$ . Given a vector  $R = ((a \cdot x_{i^*_2})^{-1}, r_2, \dots, r_n)$ , where  $(r_2, \dots, r_n)$  are randomly chosen from  $Z_p$ ,  $M \cdot R^T$  is the vector of  $l$  shares for the secret  $(a \cdot x_{i^*_2})^{-1}$ . For each keyword  $\widetilde{p}_k \in P$  (the underlying unprimed keyword is  $p_k$ ), a random  $r_k \leftarrow Z_p$  is chosen by  $B_A$ .

If  $\widetilde{p}_k$  is unprimed,  $D_k = (D_k^{(1)} = X_{j_1}^{\lambda_k} \cdot T(p_k)^{r_k}, D_k^{(2)} = X_{i_2}^{r_k})$ .

If  $\widetilde{p}_k$  is primed,  $D_k = (D_k^{(3)} = X_{j_1}^{\lambda_k} g^{r_k}, D_k^{(4)} = V(p_k)^{r_k}, D_k^{(5)} = X_{i_2}^{r_k})$

A key point in the above expressions is to compute  $X_{j_1}^{\lambda_k} = (g^{\lambda_k})^{x_{j_1}}$ . As  $\lambda_k = M_k \cdot R^T$  may be linearly dependent on the secret  $(a \cdot x_{i^*_2})^{-1}$ ,  $g^{\lambda_k}$  can be computed via  $A_{-1}$ .

The first level decryption oracle  $O_{dec-1}$  is simulated as follows:

As the secret keys of corrupt users and honest users  $i \neq i^*$  are known to  $B_A$ , given a first level ciphertext  $CT_1$  and a public key  $pk_i$ , we only consider how to handle the case  $i = i^*$ :

(1) Parses  $CT_1$  as  $(C_1, C_2', C_3, C_4, \sigma)$ ;

(2)  $C_1 = svk^*$ : Assume  $(m \parallel C_3 \parallel C_4, \sigma) = (m_b \parallel C_3^* \parallel C_4^*, \sigma^*)$ , which implies the randomness  $r = r^*$ . So  $C_2' = C_2^*$  and we have  $CT_1 = CT_1^*$ . But the challenge ciphertext  $CT_1^*$  is not allowed to be decrypted by our security definition. On the other hand,  $(m \parallel C_3 \parallel C_4, \sigma) \neq (m_b \parallel C_3^* \parallel C_4^*, \sigma^*)$  means that we either face with an occurrence of the event  $F_{OTS}$  or  $B_A$  should output a message “invalid” to indicate that relation V1 does not hold.

Hence  $B_A$  halts at step (2).

(3)  $C_1 \neq svk^*$ : As  $C_4 = (u^{svk} v)^r = (g^{\alpha_1(svk-svk^*)} \cdot X_{i^*1}^{\alpha_2})^r$ ,  $B_A$  computes:

$$e(g, C_4) = e(g, g^{\alpha_1(svk-svk^*)r}) e(g, X_{i^*1}^{\alpha_2 \cdot r})$$

$$(C_2')^{\alpha_2} = e(g, X_{i^*1}^{\alpha_2 \cdot r})$$

$$\frac{e(g, C_4)}{(C_2')^{\alpha_2}} = e(g, g)^r, \quad m = C_3 / e(g, g)^r$$

(4) If relation V1 does not hold or  $m \in \{m_0, m_1\}$ , outputs a message “invalid”; Otherwise outputs  $m$ .

**Challenge:** The adversary outputs two equal-length message  $(m_0, m_1)$ .  $B_A$  flips a random bit  $b$  and outputs the challenge first level ciphertext  $CT_1^*$  as follows:

$$C_1^* = svk^*, C_2'^* = e(g, B^{x_{i^*1}^*}), C_3^* = T \cdot m_b, C_4^* = B^{\alpha_2 \cdot x_{i^*1}^*}, \sigma^* = S(ssk^*, m_b \| C_3^* \| C_4^*)$$

If  $T = e(g, g)^{\frac{b}{a^2}}$ , we have  $r^* = \frac{b}{a^2}$  and the following equations:

$$C_2'^* = e(g, X_{i^*1}^*)^{r^*} = e(g, g^{a^2 \cdot x_{i^*1}^*})^{\frac{b}{a^2}} = e(g, B^{x_{i^*1}^*})$$

$$C_4^* = (u^{svk^*} v)^{r^*} = X_{i^*1}^{\alpha_2 \cdot r^*} = B^{\alpha_2 \cdot x_{i^*1}^*}$$

So  $CT_1^*$  is a valid encryption of  $m_b$  if  $T = e(g, g)^{\frac{b}{a^2}}$ . In contrast, if  $T$  is random,  $CT_1^*$  perfectly hides  $m_b$  and  $Ad$  guesses  $b$  with probability 0.5. Hence the overall advantage of  $B_A$  is  $\varepsilon - \Pr[F_{OTS}]$ .

## 5. Conclusion

Fang et al. [6] presented an interactive bidirectional single-hop C-PRE scheme, which supports access policy consisting of “OR” and “AND” gates. They also left it as an open problem to construct a *non-interactive* C-PRE scheme with security in *the standard model*. In this paper, we present a security model for unidirectional(non-interactive) C-PRE schemes. To yield a unidirectional C-PRE scheme supporting non-monotonic access policy expressed by “NOT”, “OR” and “AND” gates, we extend the unidirectional PRE scheme [8] by using the ideas from the non-monotonic attributed based encryption(ABE) [9]. Hence our C-PRE

scheme enables more flexible access policy set by delegator in comparison with previous works. Non-interactive feature of our unidirectional scheme also simplifies generation of re-encryption keys. Finally we prove our scheme to be CCA secure under the modified 3-weak Decision Bilinear Diffie-Hellman Inversion assumption in the standard model.

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