

# Towards Provably Secure Software Attestation

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**Abstract.** Software attestation has become a popular and challenging research topic at many established security conferences. It aims for verifying the software integrity of (typically) resource-constrained embedded devices. However, for practical reasons, software attestation cannot rely on stored cryptographic secrets or dedicated trusted hardware. Instead, it exploits side-channel information, such as the time that the underlying device needs for a specific computation. Unfortunately, traditional cryptographic solutions and arguments are not applicable in this setting, making new approaches for the design and analysis necessary. This is certainly one of the main reasons why the security properties and assumptions of software attestation have been only vaguely discussed and have never been formally treated, as it is common sense in modern cryptography. Thus, despite its popularity and its expected impact for practice, a sound approach for designing secure software attestation schemes is still an important open problem.

We introduce the first formal security framework for software attestation and formalize various system and design parameters. Moreover, we present a generic software attestation scheme that captures most existing schemes in the literature. Finally, we analyze its security within our framework, yielding sufficient conditions for provably secure software attestation schemes. We regard these results as a first step towards putting software attestation on a solid ground and as a starting point for further research.

**Keywords:** software attestation, keyless cryptography, security model

## 1 Introduction

Embedded systems are increasingly permeating our information society. Potential threats and damages that could be caused by exploiting possible vulnerabilities in these systems have raised the demand for enabling technologies that can validate and verify the integrity of their software state. In this context, *software attestation* has become a popular research topic at many established security conferences with a large body of literature [12,21,23,8,20,22,19,6,18,17,2,7,11,14,16,13,26].

Software attestation is a trust establishment mechanism that allows one system, the *verifier*, to check whether the program memory content of another system, the *prover*, is genuine or has been modified, e.g., by malicious code. As the verifier usually cannot access the memory of the prover directly (without requesting hardware modifications), the common approach in the literature deploys challenge-response-protocols, in which the verifier challenges the prover to compute a checksum over a (random) selection of its memory content. Unfortunately classical cryptographic primitives and protocols cannot be used directly since the adversary may get full control of the prover and its cryptographic secrets. Moreover, software attestation mainly targets resource-constrained embedded systems (such as the Atmel tinyAVR [1] microcontrollers) that cannot afford secure hardware, which precludes the use of solutions based on security hardware [25,15,24,3].

Therefore software attestation follows a radically different approach than most conventional security mechanisms: It exploits the intrinsic physical constraints of the underlying hardware and side-channel information, typically the *computation time* required by the prover to complete the attestation protocol. More detailed, it is typically designed to temporarily utilize all the computing and memory resources of

the prover, aiming to ensure that the prover cannot deviate from the correct execution of the challenge-response-protocol. Summing up, in contrast to common attestation schemes, a malicious prover can *in principle* cheat during the protocol but not without exceeding a certain, concrete *time bound*.

Without question this requires completely different forms of reasoning and likewise demands for other security assumptions on the underlying core functionalities and system properties, representing a challenging task. This may be the main reason that, despite its popularity and expected practical impact, software attestation has not received any formal treatment yet, as it is common sense in modern cryptography. To start with, there exist no common adversary model and precise formalization of the security goals so far. Likewise the underlying security properties and assumptions have been only vaguely discussed. In fact current proposals combine components with unclear and possibly insufficient security properties and multiple attacks against existing software attestation schemes have been documented [21,23,2].

**Contribution.** In this paper, we make a first step towards putting software attestation on a solid ground. Our contributions are as follows:

*Formal framework:* We describe the first formal security framework for software attestation. This includes an adversary model which interestingly fundamentally deviates from the classical cryptographic adversary model. Instead of being a polynomially bounded algorithm, an adversary could be unbounded in principle. However, during the attack it has to specify (or program) a malicious prover with tight resource constraints. The goal is that this malicious prover is able to cheat in the attestation protocol with reasonable success probability but *without* any interaction with the adversary. In other words, an adversary has all the time for *preparing* the attack but only a tight time bound for *executing* the attack.

*Generic software attestation scheme:* We give a description of a *generic* software attestation scheme that covers most existing software attestation protocols in the literature. A benefit is that any results derived on this generic scheme help to investigate and understand the security of concrete schemes.

*Security analysis and new insights:* We identify and formalize several systems parameters and provide an upper bound of the success probability of a malicious prover as a function of these parameters. This requires to use other types of arguments since the typical reduction to an hard problem is not possible anymore. Instead we have to argue *directly* that any attack strategy that is possible within the given time bound fails with a certain probability. The derived bound implies *sufficient* conditions on the parameters. Although some of these aspects have been implicitly assumed and informally discussed in the literature on software attestation, we present a formal treatment of them for the first time. Moreover, our approach provides new insights on how these parameters impact the security of the underlying software attestation scheme. Finally, we stress that our investigation introduces new cryptographic primitives that are similar to established primitives like pseudo-random generators or hash functions but differ in some subtleties: Some cryptographic assumptions can be relaxed while others need to be strengthened.

We see our work as a first step towards provably secure software attestation schemes. Apart from the fact that this topic is of utmost importance for practice, we identify several open research problems, which we believe to be of high interest to the cryptographic research community and hope that our results inspire new research in the area of keyless cryptography.

**Outline.** We give an overview of the related work in [Section 2](#) and introduce our system model in [Section 3](#). We present the formal framework for software attestation in [Section 4](#), describe the generic software attestation scheme and its requirements in [Section 5](#) and formally analyze its security in [Section 6](#). Finally, we discuss our results and conclude in [Section 7](#).

## 2 Related Work

Several works considered the design and extension of software attestation. The existing literature focuses on the design of checksum constructions for different platform architectures and countering platform-specific

attacks [21,20,6,19,14]. Several works consider the strengthening of self-checksumming code against unintended modifications by either limiting the memory available to the prover during attestation [7,26] or by using self-modifying and/or obfuscated attestation algorithms [22,8]. Several works investigated the suitability and extension of software attestation for a variety of computing platforms, including sensors, peripherals and voting machines [18,14,6,20,13]. Furthermore, software attestation has also been proposed as a key establishment mechanism [18]. Since software attestation does not rely on any secrets and thus cannot authenticate the prover to the verifier, multiple works also consider how to combine software attestation with hardware trust anchors such as TPMs and SIM-cards [17,13,11] or intrinsic hardware characteristics such as code execution side-effects [12,21,23] and Physically Unclonable Functions [16]. Interestingly, most proposed implementations employ hash functions and PRNGs that are not cryptographically secure. Further, works that used cryptographically secure algorithms did not consider whether these algorithms maintain their security properties in the “keyless” software attestation scenario, where the underlying secrets such as the PRNG states are known to the adversary. In this respect, our formal analysis provides a first step towards a deeper and more comprehensive understanding of software attestation.

An approach [10,9] related to software attestation uses *Quines*, which is code that outputs its own source-code while it is executed. Further, by imposing a time bound on the execution time of the Quine, the prover can be prevented from emulating the Quine.

Similar to software attestation, *proofs of work* schemes challenge the prover with computationally expensive or memory-bound tasks [4,5]. However, while the goal of these schemes is to mitigate denial-of-service attacks and Spam by imposing artificial load on the service requester, the goal of software attestation schemes is using all of the prover’s resources to prevent it from executing malicious code within a certain time frame. Hence, proofs of work are in general not suitable for software attestation since they are usually less efficient and not designed to achieve the optimality requirements of software attestation algorithms.

### 3 Preliminaries

**Notation.** Let  $A$  and  $B$  be arbitrary algorithms. Then  $y \leftarrow A(x)$  means that on input  $x$ ,  $A$  assigns its output to  $y$ . The expression  $A^B$  means that  $A$  has black-box access to  $B$ . We denote with  $A^{\hat{B}}$  an algorithm  $A$  that does *not* access an algorithm  $B$ . Let  $\mathbb{D}$  be a probability distribution over the set  $X$ , then the term  $x \stackrel{\mathbb{D}}{\leftarrow} X$  means the event of assigning an element of  $X$  to variable  $x$ , which has been chosen according to  $\mathbb{D}$ . Further, we define  $\mathbb{D}(x) := \Pr [x|x \stackrel{\mathbb{D}}{\leftarrow} X]$  for each  $x \in X$  and denote with  $\mathbb{U}$  the uniform distribution.

**System Model.** Software attestation is a protocol between a verifier  $\mathcal{V}$  and a (potentially malicious) prover  $\mathcal{P}$  where the latter belongs to a class of devices with clearly specified characteristics. Typically a prover  $\mathcal{P}$  is a low-end embedded system that consists of memory and a computing engine (CE). The memory is composed of *primary memory* (PM) such as CPU registers and cache, and *secondary memory* (SM) such as RAM and Flash memory. We assume that the memory is divided into *memory words* and denote by  $\Sigma := \{0, 1\}^{l_s}$  the set of all possible memory words (e.g.,  $l_s = 8$  if memory words are bytes). Let  $\mathfrak{s} := 2^{l_a}$  and  $\mathfrak{p} := 2^{l_b}$  be the number of memory words that can be stored in secondary memory (SM) and primary memory (PM), respectively. The *state*  $\text{State}(\mathcal{P}) = S$  of a prover  $\mathcal{P}$  are the memory words stored in SM. Observe that  $S$  includes the program code of  $\mathcal{P}$  and hence,  $S$  specifies the algorithms executed by  $\mathcal{P}$ .

The computing engine (CE) comprises an arithmetics and logic unit that can perform computations on the data in PM and alter the program flow. For performance reasons, PM is typically fast but also expensive. Hence, the size of PM is usually much smaller than the size of SM. To make use of SM, CE includes the *Read* instruction to transfer data from SM to PM and the *Write* instruction to write data from PM to SM. More precisely,  $\text{Read}(S, a, b)$  takes as input a memory address  $a$  of SM and a memory address  $b$  of PM and copies the data word  $x$  stored at address  $a$  in SM to the data word at address  $b$  in PM. For convenience, we write  $\text{Read}(S, a)$  instead of  $\text{Read}(S, a, b)$  whenever the address  $b$  of PM is not relevant. Note that  $\text{Read}(S, a, b)$  overwrites the content  $y$  of PM at address  $b$ . Hence, in case  $y$  should not be lost, it must first be copied to SM using *Write* or copied to another location in PM before *Read* is performed.

It is important to stress that, whenever CE should perform some computation on some value  $x$  stored in SM, it is mandatory that  $x$  is copied to PM before CE can perform the computation. Further, since SM is typically much slower than PM, `Read` and `Write` incur a certain time overhead and delay computations on  $x$ . We denote the time required by the CE to perform some instruction `Ins` with  $\text{Time}(\text{Ins})$ . The program code that determines the behaviour of  $\tilde{\mathcal{P}}$  is encoded as part of the state. Note that we only consider provers as described above while the verifier  $\mathcal{V}$  might be an arbitrary computing platform.

## 4 Secure Software Attestation

Secure software attestation enables the verifier  $\mathcal{V}$  to check whether the prover  $\mathcal{P}$  is in some state  $S$ , i.e., whether  $\text{State}(\mathcal{P}) = S$ . Ideally,  $\mathcal{V}$  could disable the computing engine (CE) of  $\mathcal{P}$  and directly read and verify the software state  $S$  in the secondary memory (SM) of  $\mathcal{P}$ . However, exposing CE and the SM of  $\mathcal{P}$  to  $\mathcal{V}$  in such a way requires hardware extensions on  $\mathcal{P}$ , which contradicts the goal of software attestation to work with no hardware modifications. Instead the common approach in the literature is that  $\mathcal{V}$  and  $\mathcal{P}$  engage in a challenge-response protocol where  $\mathcal{P}$  must answer to a challenge of  $\mathcal{V}$  with a response that depends on  $S$ . However, in contrast to common cryptographic scenarios, software attestation does not rely on any secrets and hence, the adversary has access to the same information as an honest prover. Therefore software attestation follows a fundamentally different approach and leverages side-channel information, typically the time a prover takes for computing the response.

In the following, we provide the first formal specification of the adversary model and the security of software attestation based on a security experiment  $\text{Exp}_{\text{Attest}}^{\mathcal{A}}$  that involves two probabilistic algorithms: an adversary  $\mathcal{A}$  and a challenger  $\mathcal{C}_{\text{Attest}}$ . The task of  $\mathcal{C}_{\text{Attest}}$  is to provide all necessary information to  $\mathcal{A}$  and to play the role of the honest verifier  $\mathcal{V}$ . At the beginning,  $\mathcal{C}_{\text{Attest}}$  receives as input a state  $S$  and a security parameter  $l$  and generates a challenge  $c$ , a response  $r$ , and a time bound  $\delta$ . The adversary  $\mathcal{A}$  gets the same inputs and outputs a malicious prover  $\tilde{\mathcal{P}}$  by specifying its state  $\tilde{S}$ , that is  $\text{State}(\tilde{\mathcal{P}}) = \tilde{S}$ . Afterwards,  $\tilde{\mathcal{P}}$  receives the challenge  $c$  and returns a “guess”  $\tilde{r}$  for the correct response  $r$ . The result of the experiment is `ACCEPT` if  $\tilde{r} = r$  and  $\tilde{\mathcal{P}}$  responded within time  $\delta$  and otherwise `REJECT`. Formally:

$$\begin{aligned} \text{Exp}_{\text{Attest}}^{\mathcal{A}}(S, l) : (c, r, \delta) &\leftarrow \mathcal{C}_{\text{Attest}}(S, l) \\ \tilde{\mathcal{P}} &\leftarrow \mathcal{A}(S, l) \\ \tilde{r} &\leftarrow \tilde{\mathcal{P}}(c) \\ &\text{if } (\tilde{r} = r) \wedge (\text{Time}(\tilde{\mathcal{P}}) < \delta) \text{ return ACCEPT else return REJECT} \end{aligned}$$

$\mathcal{A}$  wins iff the result of  $\text{Exp}_{\text{Attest}}^{\mathcal{A}}$  is `ACCEPT` and loses otherwise. Consequently, a software attestation scheme is secure if all adversaries  $\mathcal{A}$  have a low success probability. Formally:

**Definition 1.** *A software attestation scheme `Attest` is  $\varepsilon$ -secure with respect to a state  $S$  if for all adversaries  $\mathcal{A}$  it holds that  $\Pr[\text{Exp}_{\text{Attest}}^{\mathcal{A}}(S, l) = \text{ACCEPT}] \leq \varepsilon$ .*

## 5 Generic Software Attestation

In this section, we formalize a generic software attestation protocol (with respect to a state  $S$ ) that captures most existing schemes in the literature. In particular, we formally define several aspects and assumptions that were only informally discussed or implicitly defined.

### 5.1 Protocol Specification

The main components of our generic attestation scheme (Figure 1) are two deterministic algorithms:

- Memory address generator       $\text{Gen} : \{0, 1\}^{l_g} \rightarrow \{0, 1\}^{l_g} \times \{0, 1\}^{l_a}, \quad g \mapsto (g', a')$
- Compression function           $\text{Chk} : \{0, 1\}^{l_r} \times \Sigma \rightarrow \{0, 1\}^{l_r}, \quad (r, s) \mapsto r'$

Prover $\mathcal{P}$	Verifier $\mathcal{V}$
$S$	$S$
	$(g_0, r_0) \xleftarrow{\mathbb{U}} \{0, 1\}^{l_g + l_r}$
$r'_0 \leftarrow r_0$	$\xleftarrow{g_0, r_0}$ Store current time $t$
<b>for</b> $i = 1, \dots, N$ <b>do</b>	<b>for</b> $i = 1, \dots, N$ <b>do</b>
$(g_i, a_i) \leftarrow \text{Gen}(g_{i-1})$	$(g_i, a_i) \leftarrow \text{Gen}(g_{i-1})$
$s_i \leftarrow \text{Read}(S, a_i)$	$s_i \leftarrow \text{Read}(S, a_i)$
$r'_i \leftarrow \text{CHK}(r'_{i-1}, s_i)$	$r_i \leftarrow \text{CHK}(r_{i-1}, s_i)$
<b>endfor</b>	<b>endfor</b>
	$\xrightarrow{r'_N}$ Store current time $t'$
	<b>Accept iff</b> $r'_N = r_N \wedge t' - t \leq \delta$

Fig. 1: The Generic Attestation Scheme **Attest**

Here  $l_g$ ,  $l_a$ , and  $l_r$  are the bit length of the state of  $\text{Gen}$ , the memory addresses  $a_i$ , and the attestation response  $r'_N$ , respectively,  $\Sigma$  is the set of possible state entries, and  $N \in \mathbb{N}$  is the number of rounds of **Attest**. Furthermore, we provide an iterative definition of  $\text{Chk}^N$ : For some  $r_0 \in \{0, 1\}^{l_r}$  and  $\mathbf{s} := (s_1, \dots, s_N)$ , we define  $r \leftarrow \text{Chk}^N(c, \mathbf{s})$  as  $r_i := \text{Chk}(r_{i-1}, s_i)$  for  $i = 1, \dots, N$ .

The protocol works as follows: The verifier  $\mathcal{V}$  sends an attestation challenge  $(g_0, r_0)$  to the prover  $\mathcal{P}$ , who iteratively generates a sequence of memory addresses  $(a_1, \dots, a_N)$  based on  $g_0$  using  $\text{Gen}$ . For each  $i \in \{1, \dots, N\}$ ,  $\mathcal{P}$  reads the state entry  $s_i = \text{Read}(S, a_i)$  at address  $a_i$  and iteratively computes  $r'_i = \text{Chk}(r'_{i-1}, s_i)$  using  $r'_0 = r_0$ . Finally,  $\mathcal{P}$  sends  $r'_N$  to  $\mathcal{V}$ , which executes exactly the same computations as  $\mathcal{P}$  using the state  $S$  and compares the final result with the response  $r'_N$  from  $\mathcal{P}$ . Eventually,  $\mathcal{V}$  accepts iff  $r'_N = r_N$  and  $\mathcal{P}$  responded in time  $\delta := N(\delta_{\text{Gen}} + \delta_{\text{Read}} + \delta_{\text{Chk}})$ , where  $\delta_{\text{Gen}}$ ,  $\delta_{\text{Read}}$ , and  $\delta_{\text{Chk}}$  are the time bounds for running  $\text{Gen}$ ,  $\text{Read}$ , and  $\text{Chk}$ , respectively, on a genuine and honest prover.<sup>4</sup>

*Remark 1 (Soundness).* Observe that an honest prover  $\mathcal{P}$  always makes an honest verifier  $\mathcal{V}$  accept since both perform exactly the same computations on the same inputs and the honest prover by assumption requires time  $\delta$ .

## 5.2 Design Criteria and Properties

Next, we discuss the design criteria of the underlying algorithms and formally define the properties required later in the security analysis. Note that, although some of these properties have been informally discussed or implicitly made in prior work, they have never been formally specified and analyzed before.

**Implementation of the Core Functionalities.** The generic protocol deploys three core functionalities:  $\text{Read}$ ,  $\text{Gen}$ , and  $\text{Chk}$ , which of the execution time is of paramount importance for the security of software attestation. Hence, we make the following assumptions that are strongly dependent on the concrete implementation and computing engine of the prover and hard cover in a generic formal framework:

1. There is no implementation of  $\text{Read}$ ,  $\text{Gen}$ , and  $\text{Chk}$  that is more efficient (with respect to time and/or memory) than the implementation used by the honest prover with state  $S$ .
2. It is not possible to execute  $\text{Read}$ ,  $\text{Gen}$ , and  $\text{Chk}$  only partially, e.g., by omitting some of the underlying instructions.

We formally cover these assumptions by modelling  $\text{Read}$ ,  $\text{Gen}$ , and  $\text{Chk}$  as oracles. That is, whenever  $\mathcal{P}$  wants, e.g., to execute the command  $\text{Read}(\text{State}(\mathcal{P}), a)$ ,  $\mathcal{P}$  sends  $a$  to the  $\text{Read}$ -oracle and receives the

<sup>4</sup> In practice the delay for submitting and receiving messages must be considered. However, as this delay should be small compared to the runtime of the protocol, we ignore this aspect.

corresponding result  $s$ . While sending and receiving messages between  $\mathcal{P}$  and the oracles are modelled to take no time, the *determination* of the response does. More precisely if  $\tilde{\mathcal{P}}$  invokes one of these oracles, it takes a certain amount of time before  $\tilde{\mathcal{P}}$  gets the result. Within this time period  $\tilde{\mathcal{P}}$  is inactive and cannot perform any computations. We denote the response time of the **Read**, **Gen**, and **Chk**-oracle by  $\delta_{\text{Read}}$ ,  $\delta_{\text{Gen}}$ , and  $\delta_{\text{Chk}}$ , respectively. Moreover the inputs of the oracles need to be stored in the primary memory of  $\mathcal{P}$ .

*Remark 2 (Order of Computations).* A consequence of this modelling approach is that a malicious prover  $\tilde{\mathcal{P}}$  can compute the outputs of **Gen** and **Chk** only in the right order. For instance, before  $\tilde{\mathcal{P}}$  can determine  $s_i$  it must first determine  $s_{i-1}$ . Given that concrete instantiations are iteratively executed, the limited size of the primary memory (PM) (see below), and the fact that accessing the secondary memory requires significantly more time than accessing PM, we consider this assumption to be reasonable for most practical instantiations.

**System-level Properties.** The size and utilization of the primary memory (PM) plays a fundamental role for assessing the optimality of a software attestation protocol **Attest** with respect to the resources used by the prover  $\mathcal{P}$ . Therefore, we assume that the size of PM is limited to the minimum size required by an honest prover. For example if an honest prover  $\mathcal{P} = \mathcal{P}^{\mathcal{O}}$  is expected to access only an oracle  $\mathcal{O}$  that accepts inputs in  $\Sigma^\ell$ , we assume that  $\tilde{\mathcal{P}}$  can at most store  $\ell$  elements of  $\Sigma$  in its PM at the same time.<sup>5</sup>

Another crucial assumption for any software attestation scheme not explicitly made in most previous works is that the state  $S$  should not be compressible into PM. For instance, consider the extreme case where all entries of  $S$  contain the same value  $s$ . In this case a malicious prover  $\tilde{\mathcal{P}}$  could easily determine the correct attestation response by simply storing  $s$  in PM while having a different state  $\text{State}(\tilde{\mathcal{P}}) \neq S$ . Hence, we require that  $\tilde{\mathcal{P}}$  should not be able to determine a randomly selected entry of  $S$  without accessing the secondary memory with better probability than guessing:

**Definition 2 (State Incompressibility).** For a state  $S$  and for any  $x \in \Sigma$  we denote with  $\mathbb{D}_S$  the probability distribution  $\mathbb{D}_S(x) := \Pr \left[ x = s \mid a \stackrel{\cup}{\leftarrow} \{0, 1\}^{l_a} \wedge s := \text{Read}(S, a) \right]$ .  $S$  is called incompressible if for any algorithm **Alg** that can be executed by  $\mathcal{P}$  and that does not invoke **Read**, i.e.,  $\text{Alg} = \text{Alg}^{\widehat{\text{Read}}}$ , it holds that

$$\Pr \left[ \tilde{s} = s \mid a \stackrel{\cup}{\leftarrow} \{0, 1\}^{l_a} \wedge s = \text{Read}(S, a) \wedge \tilde{s} \leftarrow \text{Alg}(a) \wedge \text{Time}_{\mathcal{P}}(\text{Alg}) \leq \delta_{\text{Read}} \right] \leq \gamma = \max_{x \in \Sigma} \mathbb{D}_S(x).$$

**Cryptographic Properties.** Although it is quite obvious that the security of the software attestation scheme heavily depends on the cryptographic properties of **Gen** and **Chk**, these requirements have not been systematically analyzed and formally specified before. While it would be straightforward to model these functions as pseudo-random number generators (PRNGs) and hash functions (or even random oracles), respectively, there are some subtle differences to the common cryptographic scenario which must be carefully considered. As we elaborate below, this yields a property of **Gen** which is *stronger* than the common security definition of cryptographic PRNGs while for **Chk** a significantly *weaker* condition than the classical security properties of hash functions is sufficient.

*Pseudo-Randomness of Outputs of Gen.* Ideally all possible address combinations should be possible for  $(a_1, \dots, a_N)$ . While this is impossible from an information theoretic point of view, the best one may ask for is that the memory addresses  $a_i$  generated by **Gen** should be computationally indistinguishable from uniformly random values within a certain time bound  $t$ :

**Definition 3 (Pseudo-randomness of Gen).**  $\text{Gen} : \{0, 1\}^{l_g} \rightarrow \{0, 1\}^{l_g + l_a}$  is called  $(t, \varrho)$ -pseudo-random if for any algorithm **Alg** that can be executed by  $\mathcal{P}$  in  $\text{Time}(\text{Alg}) \leq t$  it holds that

$$\left| \Pr \left[ 1 \leftarrow \text{Alg}(a_1, \dots, a_N) \mid g_0 \stackrel{\cup}{\leftarrow} \{0, 1\}^{l_g}, (g_{i+1}, a_{i+1}) \leftarrow \text{Gen}(g_i) : i = 0, \dots, N-1 \right] \right. \\ \left. - \Pr \left[ 1 \leftarrow \text{Alg}(a_1, \dots, a_N) \mid a_i \stackrel{\cup}{\leftarrow} \{0, 1\}^{l_a} : i = 1, \dots, N \right] \right| \leq \varrho.$$

<sup>5</sup> This aspect will become more clear in the security proof.

Observe that this definition requires that  $\text{Alg}$  does not know the seed  $g_0$  of  $\text{Gen}$ , which is not given in the software attestation scheme. In principle nothing prevents  $\tilde{\mathcal{P}}$  to compute the addresses  $(a_1, \dots, a_N)$  on its own, making them easily distinguishably from random values. The best we can do is to require that  $\tilde{\mathcal{P}}$  cannot derive any meaningful information about  $a_{i+1}$  from  $g_i$  without investing a certain minimum amount of time. Specifically, we assume that an algorithm with input  $g$  that does not execute  $\text{Gen}$  cannot distinguish its output  $(g', a') = \text{Gen}(g)$  from uniformly random values. Formally:

**Definition 4 (Unpredictability of Gen).**  $\text{Gen} : \{0, 1\}^{l_g} \rightarrow \{0, 1\}^{l_g} \times \{0, 1\}^{l_a}$  is  $\nu_{\text{Gen}}$ -unpredictable if for any algorithm  $\text{Alg}$  that can be executed by  $\mathcal{P}$  and that does not execute  $\text{Gen}$ , i.e.,  $\text{Alg} = \text{Alg}^{\widehat{\text{Gen}}}$ , it holds that

$$\left| \Pr \left[ 1 \leftarrow \text{Alg}(g, g', a') \mid g \stackrel{\mathbb{U}}{\leftarrow} \{0, 1\}^{l_g} \wedge (g', a') \leftarrow \text{Gen}(g) \right] - \Pr \left[ 1 \leftarrow \text{Alg}(g, g', a') \mid g \stackrel{\mathbb{U}}{\leftarrow} \{0, 1\}^{l_g} \wedge (g', a') \stackrel{\mathbb{U}}{\leftarrow} \{0, 1\}^{l_g} \times \{0, 1\}^{l_a} \right] \right| \leq \nu_{\text{Gen}}.$$

*Weakened Pre-image Resistance of Chk.* The purpose of the compression function  $\text{Chk}^N$  is to map the state  $S$  of the prover  $\mathcal{P}$  to a smaller attestation response  $r_N$ , which reduces the amount of data to be sent from  $\mathcal{P}$  to the verifier  $\mathcal{V}$ . A necessary security requirement on  $\text{Chk}$  is that it should be hard for a malicious prover  $\tilde{\mathcal{P}}$  to replace the correct input  $\mathbf{s} = (s_1, \dots, s_N)$  to  $\text{Chk}$  with some other value  $\tilde{\mathbf{x}} \neq \mathbf{s}$  that yields the same attestation response  $r_N$  as  $\mathbf{s}$ . This is similar to the common notion of *second pre-image resistance* of cryptographic hash functions. However, due to the time bound of  $\text{Attest}$  it is sufficient that  $\text{Chk}^N$  fulfills only a much weaker form of second pre-image resistance, since we need to consider only “blind” adversaries who (in contrast to the classical definition of second pre-image resistance) do not know the correct response  $r_N$  to the verifier’s challenge  $(g_0, r_0)$ . The reason is that, as soon as  $\mathcal{P}$  knows the correct response  $r_N$ , he could send it to  $\mathcal{V}$  and would not bother to determine a second pre-image. Hence, we introduce the definition of blind second pre-image resistance which concerns algorithms that are given only part of the input  $\mathbf{s}$  of  $\text{Chk}^N$  and that have to determine the correct output of  $\text{Chk}^N(r_0, \mathbf{s})$ :

**Definition 5 (Blind Second Pre-image Resistance).**  $\text{Chk} : \{0, 1\}^{l_r} \times \Sigma \rightarrow \{0, 1\}^{l_r}$  is  $\omega$ -blind second pre-image resistant with respect to the distribution  $\mathbb{D}_\Sigma$  (cf. [Definition 2](#)) if for any  $N \in \mathbb{N}$ , any subset of indices  $J \subsetneq \{1, \dots, N\}$ , and for any algorithm  $\text{Alg}$  that can be executed by  $\mathcal{P}$ , it holds that

$$\Pr \left[ \tilde{r} = r \mid r_0 \stackrel{\mathbb{U}}{\leftarrow} \{0, 1\}^{l_r} \wedge \bigwedge_{i=1, \dots, N} \left( s_i \stackrel{\mathbb{D}_\Sigma}{\leftarrow} \Sigma \right) \wedge \tilde{r} \leftarrow \text{Alg}(r_0, (s_j)_{j \in J}, J) \wedge r \leftarrow \text{Chk}^N(r_0, s_1, \dots, s_N) \right] \leq \omega.$$

In addition we also require (similar to [Definition 4](#)) that  $\tilde{\mathcal{P}}$  cannot determine any useful information about  $r_N = \text{Chk}^N(r_0, s_1, \dots, s_N)$  without executing  $\text{Chk}^N$ :

**Definition 6 (Unpredictability of Chk<sup>N</sup>).**  $\text{Chk} : \{0, 1\}^{l_r} \times \Sigma \rightarrow \{0, 1\}^{l_r}$  is  $\nu_{\text{Chk}}$ -unpredictable with respect to the distribution  $\mathbb{D}_\Sigma$  if for any algorithm  $\text{Alg}$  that can be executed by  $\mathcal{P}$  and that does not execute  $\text{Chk}^N$ , i.e.,  $\text{Alg} = \text{Alg}^{\widehat{\text{Chk}^N}}$ , it holds that

$$\left| \Pr \left[ 1 \leftarrow \text{Alg}(r_0, s_1, \dots, s_N, r) \mid r_0 \stackrel{\mathbb{U}}{\leftarrow} \{0, 1\}^{l_r} \wedge s_i \stackrel{\mathbb{D}_\Sigma}{\leftarrow} \Sigma : i \in \{1, \dots, N\} \wedge r = \text{Chk}^N(r_0, s_1, \dots, s_N) \right] - \Pr \left[ 1 \leftarrow \text{Alg}(r_0, s_1, \dots, s_N, r) \mid r_0 \stackrel{\mathbb{U}}{\leftarrow} \{0, 1\}^{l_r} \wedge s_i \stackrel{\mathbb{D}_\Sigma}{\leftarrow} \Sigma : i \in \{1, \dots, N\} \wedge r \stackrel{\mathbb{U}}{\leftarrow} \{0, 1\}^{l_r} \right] \right| \leq \nu_{\text{Chk}}.$$

## 6 Security of the Scheme

In this section we derive an upper bound for the success probability of a malicious prover  $\tilde{\mathcal{P}}$  to make the verifier  $\mathcal{V}$  accept. This bound depends on the parameters defined in [Section 5.2](#) which provides a sufficient condition to prove the scheme secure. The bound is as follows:

**Theorem 1 (Generic Upper Bound).** *Let  $S$  be an incompressible state (Definition 2). Consider the attestation protocol in Figure 1 with functionalities Read, Gen, and Chk such that*

1. *Gen is  $(N(\delta_{\text{Gen}} + \delta_{\text{Read}}), \varrho)$ -pseudo-random (Definition 3) and  $\nu_{\text{Gen}}$ -unpredictable (Definition 4),*
2. *Chk is  $\omega$ -blind second pre-image resistant (Definition 5) and  $\nu_{\text{Chk}}$ -unpredictable (Definition 6).*

*Consider an arbitrary prover  $\tilde{\mathcal{P}}$  as in Section 3 with state  $\text{State}(\tilde{\mathcal{P}}) = \tilde{S}$  that can store  $\mathfrak{p}$  memory words in its primary memory and  $\mathfrak{s}$  memory words in its secondary memory (cf. Section 3). Let*

$$\lambda := 1 - d_H(S, \tilde{S}) = \left| \left\{ a \in \{0, 1\}^{l_a} \mid \text{Read}(\tilde{S}, a) = \text{Read}(S, a) \right\} \right| \cdot 2^{-l_a},$$

*where  $d_H(S, \tilde{S})$  denotes the Hamming distance between  $S$  and  $\tilde{S}$ , i.e., the number of state entries that are different in  $S$  and  $\tilde{S}$ . Then the probability of  $\tilde{\mathcal{P}}$  to win the security experiment  $\mathbf{Exp}_{\text{Attest}}^A$  (Definition 1), i.e.,  $\Pr[\mathbf{Exp}_{\text{Attest}}^A(S, l) = \text{ACCEPT}]$  with  $l := (l_g, l_r)$ , is upper bounded by*

$$\left. \begin{aligned} & \frac{\mathfrak{p} + \mathfrak{s}}{l_s / l_r} \cdot 2^{-(l_g + l_r)} + \max\{\omega, \nu_{\text{Chk}}\} + \\ & \max_{0 \leq M \leq N} (\pi(M, \text{ops}(\delta_{\text{Read}} + \delta_{\text{Gen}})) + \varrho) \cdot \gamma^{N-M} + \nu_{\text{Gen}} \cdot (N - M) \end{aligned} \right\} \quad (1)$$

where

$$\pi(n, x) := \sum_{j=\max\{0, n-2^{l_a}\}}^{n-1} (\max\{\lambda^{x+1}, \gamma\})^{\frac{n}{x+1}-j} \cdot \binom{n}{j} \cdot \left( \prod_{i=0}^{n-j} \frac{2^{l_a} - i}{2^{l_a}} \right) \cdot \left( \frac{n-j}{2^{l_a}} \right)^j \quad (2)$$

and  $\text{ops}(\delta_{\text{Read}} + \delta_{\text{Gen}})$  denotes the number of instructions  $\tilde{\mathcal{P}}$  can execute in time  $\delta_{\text{Read}} + \delta_{\text{Gen}}$ .

*Remark 3.* This result implies that a software attestation scheme is  $\varepsilon$ -secure if the expression in Equation 1 is  $\leq \varepsilon$ , yielding a sufficient condition for security. Note that the bound given in Equation 1 emphasizes the impact of the distribution of the state entries in  $S$  (expressed by  $\gamma$ ) and the similarity between the state  $S$  and the state  $\tilde{S}$  of the prover (expressed by  $\lambda$ ) on the security of the scheme. Both aspects have been either neglected or have been considered only informally in previous work (cf. Section 7).

*Proof of Theorem 1.* Let  $\text{Win}$  denote the event that a malicious prover  $\tilde{\mathcal{P}}$  wins the security experiment  $\mathbf{Exp}_{\text{Attest}}^A$ , i.e.,  $\text{Win}$  means that  $\mathbf{Exp}_{\text{Attest}}^A(S, l) = \text{ACCEPT}$ . We are interested in an upper bound for  $\Pr[\text{Win}]$ . To this end we consider several sub-cases. Let  $\text{Precomp}$  denote the event that the verifier  $\mathcal{V}$  sends a challenge  $(g_0, r_0)$  to  $\tilde{\mathcal{P}}$  for which  $\tilde{\mathcal{P}}$  has precomputed and stored the correct response  $r_N$  in its memory (primary and/or secondary).<sup>6</sup> Then we have

$$\begin{aligned} \Pr[\text{Win}] &= \Pr[\text{Win} | \text{Precomp}] \cdot \Pr[\text{Precomp}] + \Pr[\text{Win} | \neg \text{Precomp}] \cdot \Pr[\neg \text{Precomp}] \\ &\leq \Pr[\text{Precomp}] + \Pr[\text{Win} | \neg \text{Precomp}]. \end{aligned}$$

$\frac{\mathfrak{p} + \mathfrak{s}}{l_s / l_r}$  denotes the maximum number of responses  $\tilde{\mathcal{P}}$  can store in its memory. As the challenge  $(g_0, r_0) \in \{0, 1\}^{l_g + l_r}$  is uniformly sampled, the probability  $\Pr[\text{Precomp}]$  is equal to  $\frac{\mathfrak{p} + \mathfrak{s}}{l_s / l_r} \cdot 2^{-(l_g + l_r)}$ .

We now address the term  $\Pr[\text{Win} | \neg \text{Precomp}]$ , which we abbreviate to  $\overline{\Pr}[\text{Win}]$ . Let  $\text{Correct}$  denote the event that  $\tilde{\mathcal{P}}$  determined all state entries  $(s_1, \dots, s_N)$ , i.e.,  $s_i = \text{Read}(S, a_i)$  and  $(g_i, a_i) = \text{Gen}(g_{i-1})$  for  $i \in \{1, \dots, N\}$ , and that  $\tilde{\mathcal{P}}$  has executed  $\text{Chk}^N$ . Then we have

$$\overline{\Pr}[\text{Win}] \leq \overline{\Pr}[\text{Correct}] + \overline{\Pr}[\text{Win} | \neg \text{Correct}].$$

It follows from the fact that  $\text{Chk}^N$  is  $\omega$ -blind second pre-image resistant (Definition 5) and  $\nu_{\text{Chk}}$ -unpredictable (Definition 6) that  $\overline{\Pr}[\text{Win} | \neg \text{Correct}] \leq \max\{\omega, \nu_{\text{Chk}}\}$ .

For the final term  $\overline{\Pr}[\text{Correct}]$ , we use the following claim, which we prove afterwards.

<sup>6</sup> More precisely,  $\mathcal{A}$  has precomputed this value during the preparation phase and stored the response as part of  $\tilde{S}$ .



**Claim 1.** *The probability  $\overline{\Pr}[\text{Correct}]$  that  $\tilde{\mathcal{P}}$  determines all state entries  $(s_1, \dots, s_N)$  correctly and computes  $r_N = \text{Chk}^N(r_0, s_1, \dots, s_N)$  in the security experiment  $\mathbf{Exp}_{\text{Attest}}^A$  under the assumption that the response to the requested challenge has not been precomputed is upper bounded by*

$$\max_{0 \leq M \leq N} (\pi(M, \text{ops}(\delta_{\text{Read}} + \delta_{\text{Gen}})) + \varrho) \cdot \gamma^{N-M} + \nu_{\text{Gen}} \cdot (N - M)$$

where  $\pi(N, x)$  and  $\text{ops}(\delta_{\text{Read}} + \delta_{\text{Gen}})$  are defined as explained in [Theorem 1](#).

Taking these bounds together concludes the proof.  $\square$

## Proof of Claim 1

In this section, we prove [Claim 1](#) used in the proof of [Theorem 1](#). That is we have to show the claimed upper bound of the probability  $\overline{\Pr}[\text{Correct}]$  that a malicious prover  $\tilde{\mathcal{P}}$  with state  $\tilde{S} := \text{State}(\tilde{\mathcal{P}}) \neq S$  correctly determines all state entries  $(s_1, \dots, s_N)$  in the security experiment  $\mathbf{Exp}_{\text{Attest}}^A$  ([Definition 1](#)) under the assumption that the response for the requested challenge has not been precomputed.

Observe that  $\tilde{\mathcal{P}}$  may decide to deviate from the protocol specification. For example,  $\tilde{\mathcal{P}}$  may skip some instructions with respect to one round  $i$  (probably accepting a lower success probability for determining  $s_i$ ) to save time that could be spent on the determination of another state entry  $s_j$  with  $i \neq j$  (probably aiming for a higher probability to get  $s_j$  right). Hence the challenge is to show that for *any* of these approaches the success probability does not exceed a certain (non-trivial) bound but without being able to reduce it to a single assumption.

We base our proof on a sequence of games played by  $\tilde{\mathcal{P}}$  and an oracle  $\mathcal{O}$  that has access to  $S$ . All these games are divided into two phases: *A setup phase* and a *challenge phase*. In the setup phase  $\mathcal{O}$  generates all addresses  $(a_1, \dots, a_N)$  and determines the corresponding state entries  $s_i = \text{Read}(S, a_i)$ . Afterwards, in the challenge phase,  $\tilde{\mathcal{P}}$  and  $\mathcal{O}$  exchange several messages. In particular  $\tilde{\mathcal{P}}$  must submit its guesses  $\tilde{x}_i$  for the state entries  $s_i$  to  $\mathcal{O}$ .  $\tilde{\mathcal{P}}$  wins the game only if all guesses are correct, i.e.,  $\tilde{x}_i = s_i$  for  $i = 1, \dots, N$ .

The differences between the games lie in the possibilities of  $\tilde{\mathcal{P}}$  to deviate from the protocol specification. While these possibilities are quite limited in the first game (Game 0),  $\tilde{\mathcal{P}}$  gets more and more control with each subsequent game and thus can to perform more powerful attacks. For each transformation between two consecutive games, we show how the success probability of  $\tilde{\mathcal{P}}$  changes. In most cases it turns out that the previous game represents a subset of the possible attack strategies of the current game. Note that  $\mathcal{O}$  only formally represents the honest execution of certain parts of the protocol and should not be confused with a real party. Consequently, we assume that transferring messages between  $\tilde{\mathcal{P}}$  and  $\mathcal{O}$  takes no time.

Observe that the intention of  $\mathcal{O}$  is to have an elegant method for ignoring all computations of  $\tilde{\mathcal{P}}$  which are honestly executed by assumption. Hence to exclude artificial attacks where  $\tilde{\mathcal{P}}$  uses time and/or memory gained by outsourcing the computation to  $\mathcal{O}$ , we restrict the time bound and the size of the primary memory of  $\tilde{\mathcal{P}}$  to what is necessary for executing the computations that are expected from an honest prover.

## Game 0: Randomly Sampling Addresses in Regular Time Intervals

*Game Description.* The purpose of this game is to investigate provers  $\tilde{\mathcal{P}}$  which (1) do not exploit any aspects related to the execution of  $\text{Gen}$ , and (2) that are forced to use exactly time  $\delta_{\text{Read}}$  for the determination of each state entry  $s_i$ . This is captured by modelling the game as follows: Within the setup phase,  $\mathcal{O}$  samples pairwise independent and uniform addresses  $(a_1, \dots, a_N)$  and sets  $s_i := \text{Read}(S, a_i)$  for all  $i \in \{1, \dots, N\}$ . In the challenge phase,  $\mathcal{O}$  iteratively queries  $\tilde{\mathcal{P}}$  with  $a_i$  and  $\tilde{\mathcal{P}}$  returns some response  $\tilde{x}_i$ .

Hereby,  $\tilde{\mathcal{P}}$  can access a  $\text{Read}$  oracle, which on input  $a$  returns  $s = \text{Read}(\tilde{S}, a)$  after time  $\delta_{\text{Read}}$ . As this is the only operation expected from an honest prover, the size of the primary memory only allows to store an address  $a$  and a state entry  $s$ . Moreover the total time bound is limited to  $N \cdot \delta_{\text{Read}}$ , meaning that  $\tilde{\mathcal{P}}$  automatically fails if it needs more time in total than this bound.

Observe that  $\mathcal{O}$  ensures that  $\tilde{\mathcal{P}}$  cannot change the order of the memory addresses, i.e.,  $\mathcal{O}$  only sends  $a_i$  to  $\tilde{\mathcal{P}}$  after  $a_{i-1}$  has been sent.<sup>7</sup> We denote with *round*  $i$  the time-frame between the point in time where  $\tilde{\mathcal{P}}$  receives  $a_i$  and the point in time where  $\tilde{\mathcal{P}}$  receives  $a_{i+1}$  for  $i \in \{1, \dots, N-1\}$ . With *round*  $N$  we denote the time-frame between the point in time where  $\tilde{\mathcal{P}}$  receives  $a_N$  and the point in time where  $\tilde{\mathcal{P}}$  sends the last protocol message  $\tilde{x}_N$  to  $\mathcal{O}$ .  $\tilde{\mathcal{P}}$  wins the game if (1)  $\tilde{x}_i = s_i$  for all  $i \in \{1, \dots, N\}$  and (2) each round took at most time  $\delta_{\text{Read}}$ . Otherwise  $\tilde{\mathcal{P}}$  loses the game.

*Success Probability.* We are interested in an upper bound for the probability  $\Pr[\text{Win}_0]$  that  $\tilde{\mathcal{P}}$  wins Game 0. As  $\tilde{\mathcal{P}}$  loses for sure when he uses more time than  $\delta_{\text{Read}}$  to respond to  $a_i$  in at least one round  $i$ , it is sufficient to restrict to provers that take at most time  $\delta_{\text{Read}}$  in each round. To this end, we derive an upper bound which allows to treat the individual rounds separately. We start with the final round  $N$  and distinguish between two cases.

In Case 1 the response  $\tilde{x}_N$  is the direct result of a query to the Read oracle, i.e.,  $\tilde{x}_N = \text{Read}(\tilde{S}, a)$  for some address  $a$ . If  $a = a_N$  the probability of  $\tilde{x}_N := \text{Read}(\tilde{S}, a_N) = s_N := \text{Read}(S, a_N)$  is  $\lambda$  (cf. [Theorem 1](#)) since  $a_N$  is sampled uniformly and independently from the previous addresses. Now consider that  $a \neq a_N$ . Since  $\tilde{x}_N = \text{Read}(\tilde{S}, a)$  and due to the fact that  $\tilde{\mathcal{P}}$  must respond with  $\tilde{x}_N$  in time  $\delta_{\text{Read}}$  after receiving  $a_N$ ,  $\tilde{\mathcal{P}}$  has no time left to perform any other instructions than Read during round  $N$ . In particular  $a$  could not be chosen in dependence of  $a_N$ , hence being independent of  $a_N$ . Then  $\tilde{x}_N = s_N$  happens with probability of at most  $\gamma$  (cf. [Definition 2](#)). It follows that in Case 1 the probability  $\Pr[\text{Win}_0]$  is upper bounded by  $\max\{\lambda, \gamma\} \cdot \Pr[\tilde{x}_1 = \text{Read}(S, a_1) \wedge \dots \wedge \tilde{x}_{N-1} = \text{Read}(S, a_{N-1})]$ .

Next we consider Case 2, where  $\tilde{x}_N$  is not the result of a query to the Read oracle. It follows from the incompressibility of  $S$  ([Definition 2](#)) and the fact that  $a_N$  has been sampled uniformly and independent of the previous addresses  $a_i$  with  $i < N$ , that the probability of  $\tilde{x}_N = \text{Read}(S, a_N)$  is upper bounded by  $\gamma$ . Hence,  $\gamma \cdot \Pr[\tilde{x}_1 = \text{Read}(S, a_1) \wedge \dots \wedge \tilde{x}_{N-1} = \text{Read}(S, a_{N-1})]$  is an upper bound of  $\Pr[\text{Win}_0]$  in Case 2. It follows from Cases 1 and 2 that  $\Pr[\text{Win}_0] \leq \max\{\lambda, \gamma\} \cdot \Pr[\tilde{x}_1 = \text{Read}(S, a_1) \wedge \dots \wedge \tilde{x}_{N-1} = \text{Read}(S, a_{N-1})]$  and by induction  $\Pr[\text{Win}_0] \leq \pi_0 = \pi_0(N) := (\max\{\lambda, \gamma\})^N$ .

## Game 1: Prover Controls the Address Generation Time

*Game Description.* In this game we increase the power of the malicious prover  $\tilde{\mathcal{P}}$  and allow him to freely choose how much time he devotes for determining each value  $s_i$ , as long as the total time for determining  $(s_1, \dots, s_N)$  does not exceed  $N \cdot \delta_{\text{Read}}$ . This reflects the fact that in the attestation protocol a malicious prover  $\tilde{\mathcal{P}}$  may generate the memory addresses  $(a_1, \dots, a_N)$  on its own whenever it wants to.

Formally, this is captured by introducing a *req* protocol message which  $\tilde{\mathcal{P}}$  needs to send to  $\mathcal{O}$  for receiving the next address  $a_i$  during the challenge phase. More precisely,  $\mathcal{O}$  sends  $a_i$  to  $\tilde{\mathcal{P}}$  only when  $\tilde{\mathcal{P}}$  sent the  $i$ -th request *req* to  $\mathcal{O}$ .

As each round may take a different time period, the winning conditions are relaxed by replacing the time restriction on the individual rounds by an overall time bound for the entire challenge phase. This means that  $\tilde{\mathcal{P}}$  wins Game 1 if (1)  $\tilde{x}_i = s_i$  for all  $i \in \{1, \dots, N\}$  and (2) the duration of the challenge phase does not exceed the time  $N \cdot \delta_{\text{Read}}$ . The size of the primary memory remains as in Game 0.

*Success Probability.* We now upper bound the probability  $\Pr[\text{Win}_1]$  that  $\tilde{\mathcal{P}}$  wins Game 1. To this end, we divide the number of rounds into four distinct sets. Let  $N_{\text{coll}}$  denote the number of rounds where the address sampled by  $\mathcal{O}$  is equal to an address of some previous round by coincidence, i.e.,  $N_{\text{coll}} := |\{i \in \{2, \dots, N\} \mid \exists j \in \{1, \dots, i-1\} : a_i = a_j\}|$ . With respect to the remaining  $N - N_{\text{coll}}$  rounds, let  $N_{\text{equal}}$  (resp.  $N_{\text{more}}$ , resp.  $N_{\text{less}}$ ) be the number of rounds where  $\tilde{\mathcal{P}}$  responds in time equal (resp. more, resp. less) than  $\delta_{\text{Read}}$ . Thus we have  $N = N_{\text{coll}} + N_{\text{equal}} + N_{\text{less}} + N_{\text{more}}$ .

Let  $\text{Coll}(N_{\text{coll}})$  denote the event that exactly  $N_{\text{coll}}$  of the  $N$  addresses are equal to some previous addresses. This implies that in  $N - N_{\text{coll}}$  rounds pairwise different addresses are sampled. Moreover, since

<sup>7</sup> This is a consequence of [Remark 2](#).

there are only  $2^{l_a}$  different addresses,  $N - N_{\text{coll}}$  is upper bound by  $2^{l_a}$ . It follows that  $N - N_{\text{coll}} \leq 2^{l_a} \Leftrightarrow N_{\text{coll}} \geq N - 2^{l_a}$ . Thus it must hold that  $N_{\text{coll}} \geq \max\{0, N - 2^{l_a}\}$  and we have

$$\Pr[\text{Win}_1] = \sum_{N_{\text{coll}} = \max\{0, N - 2^{l_a}\}}^{N-1} \Pr[\text{Win}_1 | \text{Coll}(N_{\text{coll}})] \cdot \Pr[\text{Coll}(N_{\text{coll}})].$$

We now derive upper bounds for  $\Pr[\text{Win}_1 | \text{Coll}(N_{\text{coll}})]$  and  $\Pr[\text{Coll}(N_{\text{coll}})]$ .

In general,  $\Pr[\text{Coll}(N_{\text{coll}})]$  can be expressed by (number combinations of rounds with equal addresses)  $\times$  (probability that addresses in  $N - N_{\text{coll}}$  rounds are pairwise different)  $\times$  (probability that addresses in the remaining rounds are equal to some previous address). The first term is at most  $\binom{N}{N_{\text{coll}}}$  while an upper bound for the last term is  $\left(\frac{N - N_{\text{coll}}}{2^{l_a}}\right)^{N_{\text{coll}}}$ . This gives (for  $\max\{0, N - 2^{l_a}\} \leq N_{\text{coll}} \leq N - 1$ )

$$\Pr[\text{Coll}(N_{\text{coll}})] \leq \binom{N}{N_{\text{coll}}} \cdot \left(\prod_{i=0}^{N - N_{\text{coll}}} \frac{2^{l_a} - i}{2^{l_a}}\right) \cdot \left(\frac{N - N_{\text{coll}}}{2^{l_a}}\right)^{N_{\text{coll}}}. \quad (3)$$

We now fix a value for  $N_{\text{coll}}$  and aim for an upper bound for  $\Pr[\text{Win}_1 | \text{Coll}(N_{\text{coll}})]$ . We do so by giving separate upper bounds on the success probability for the four different types of rounds. Let  $\text{ops} = \text{ops}(\delta_{\text{Read}})$  be the number of operations that can be executed by the computing engine of  $\tilde{\mathcal{P}}$  in time  $\delta_{\text{Read}}$ . Since we are interested in an *upper bound* of  $\tilde{\mathcal{P}}$ 's success probability, we make several assumptions in favor of  $\tilde{\mathcal{P}}$ .

For rounds where  $\tilde{\mathcal{P}}$  invested more time than  $\delta_{\text{Read}}$ , we use the trivial upper bound of 1 even if the time period exceeded  $\delta_{\text{Read}}$  only by the time required to execute one single operation.

For rounds where the requested address coincides with an address previously asked, we likewise use the bound of 1. Moreover we assume that these rounds take *no* time at all and the  $\text{ops}$  instructions saved can be used in  $\text{ops}$  other rounds.

In rounds that take less time than  $\delta_{\text{Read}}$ , it follows from the incompressibility of  $S$  (Definition 2) and the fact that all addresses are pairwise distinct that  $\tilde{x}_i = s_i$  with probability  $\leq \gamma$ . Again, we assume that these rounds take *no* time at all and that the  $\text{ops}$  instructions saved can be used in  $\text{ops}$  other rounds.

In a round that takes exactly time  $\delta_{\text{Read}}$   $\tilde{\mathcal{P}}$  succeeds at most with probability  $\max\{\lambda, \gamma\}$  (cf. Game 0).

While these assumptions strongly exaggerate the possibilities of  $\tilde{\mathcal{P}}$ , they allow to identify optimum strategies. More precisely for each round where  $\tilde{\mathcal{P}}$  uses less time than  $\delta_{\text{Read}}$  or where a previously asked address is requested again, the best approach is to spend the  $\text{ops}$  saved instructions in  $\text{ops}$  other rounds such that for each of these rounds the probability of correctly determining  $s_i$  is equal to 1. It follows that  $N_{\text{more}} = \text{ops} \cdot (N_{\text{coll}} + N_{\text{less}})$  and hence  $N = N_{\text{coll}} + N_{\text{equal}} + N_{\text{less}} + N_{\text{more}} = N_{\text{equal}} + (\text{ops} + 1) \cdot (N_{\text{coll}} + N_{\text{less}})$ . Hence, we have

$$\begin{aligned} \Pr[\text{Win}_1 | \text{Coll}(N_{\text{coll}})] &\leq \pi_0(N_{\text{equal}}) \cdot \gamma^{N_{\text{less}}} \cdot 1^{N_{\text{coll}} + N_{\text{more}}} \\ &= \max_{N_{\text{less}}} \left\{ \lambda^{N - (\text{ops} + 1) \cdot (N_{\text{coll}} + N_{\text{less}})} \cdot \gamma^{N_{\text{less}}}, \gamma^{N - (\text{ops} + 1) \cdot N_{\text{coll}} - \text{ops} \cdot N_{\text{less}}} \right\} \\ &\stackrel{\text{cf. Apx. A}}{=} \left( \max \left\{ \lambda^{\text{ops}(\delta_{\text{Read}}) + 1}, \gamma \right\} \right)^{\frac{N}{\text{ops}(\delta_{\text{Read}}) + 1} - N_{\text{coll}}}. \end{aligned} \quad (4)$$

We get the following upper bound  $\Pr[\text{Win}_1] \leq \pi(N, \text{ops}(\delta_{\text{Read}}))$  where  $\pi(n, x)$  is defined as in Equation 2. Observe that for any fixed value for  $N_{\text{coll}}$ , the probability of having  $N_{\text{coll}}$  collisions (Equation 3) increases with  $N$  (as long as  $N_{\text{coll}} \geq \max\{0, N - 2^{l_a}\}$ ) while the probability to determine the values  $(s_1, \dots, s_N)$  (Equation 4) decreases for  $N$ .

## Game 2: Skipping Address Generation

*Game Description.* So far we covered only provers  $\tilde{\mathcal{P}}$  that honestly generate all addresses  $(a_1, \dots, a_N)$ . Now we change the game such that  $\tilde{\mathcal{P}}$  may decide in each round  $i$  to skip the generation of address  $a_i$ . This allows  $\tilde{\mathcal{P}}$  to “buy” more time for determining the values  $s_i$  but at the “cost” of not knowing  $a_i$ . Formally this is captured by defining a second message *skip* besides *req*. Specifically, in each round  $i$  of the challenge phase,  $\tilde{\mathcal{P}}$  either sends *req* or *skip*. In case of *req*,  $\mathcal{O}$  behaves as in Game 1 and sends the next  $a_i$  to  $\tilde{\mathcal{P}}$ . However, when  $\tilde{\mathcal{P}}$  sends *skip* then  $\mathcal{O}$  does *not* send  $a_i$  to  $\tilde{\mathcal{P}}$  and extends the time bound by  $\delta_{\text{Gen}}$ . That is, at the beginning of the challenge phase, the winning conditions are that (1) all responses  $(\tilde{x}_1, \dots, \tilde{x}_N)$  of  $\tilde{\mathcal{P}}$  are correct, i.e.,  $\tilde{x}_i = s_i \forall i \in \{1, \dots, N\}$  and (2) the challenge phase does not take more time than  $N \cdot \delta_{\text{Read}}$ . However each time  $\tilde{\mathcal{P}}$  sends a *skip* message to  $\mathcal{O}$ , the time bound is extended by  $\delta_{\text{Gen}}$ .

*Success Probability.* We now determine the probability  $\Pr[\text{Win}_2]$  that  $\tilde{\mathcal{P}}$  wins Game 2. To this end we follow the same line of arguments as in Game 1. The only difference is that rounds where collisions in the addresses took place or where either *Read* or *Gen* have been skipped take no time at all and free  $\text{ops}(\delta_{\text{Read}} + \delta_{\text{Gen}})$  operations for other rounds. That is we get a bound with the same structure as in Game 1 but where  $\text{ops}(\delta_{\text{Read}})$  is replaced by  $\text{ops}(\delta_{\text{Read}} + \delta_{\text{Gen}})$ , i.e.,  $\Pr[\text{Win}_2] \leq \pi(N, \text{ops}(\delta_{\text{Read}} + \delta_{\text{Gen}}))$ .

## Game 3: Replacing the Random Sampling with Gen

*Game Description.* Now we consider a variant of Game 2 with the only difference being that the addresses  $(a_1, \dots, a_N)$  are generated by *Gen* instead of being randomly sampled by  $\mathcal{O}$ . That is, during the setup phase  $\mathcal{O}$  randomly samples  $g_0$  and generates  $(a_1, \dots, a_N)$  using *Gen*.

*Success Probability.* Let  $\Pr[\text{Win}_3]$  be the probability that  $\tilde{\mathcal{P}}$  wins Game 3. Using a standard argument, it follows from the pseudo-randomness of the outputs of *Gen* (Definition 3) that  $|\Pr[\text{Win}_3] - \Pr[\text{Win}_2]| \leq \varrho$  and hence  $\Pr[\text{Win}_3] \leq \Pr[\text{Win}_2] + \varrho \leq \pi(N, \text{ops}(\delta_{\text{Read}} + \delta_{\text{Gen}})) + \varrho$ .

## Game 4: Giving Access to Gen

*Game Description.* In the final game  $\mathcal{O}$  no longer generates  $(a_1, \dots, a_N)$  for  $\tilde{\mathcal{P}}$ . Instead  $\tilde{\mathcal{P}}$  now queries the *Gen* oracle, which on input  $g_i$  returns  $(g_i, a_i) = \text{Gen}(g_{i-1})$  after time  $\delta_{\text{Gen}}$ . To this end,  $\mathcal{O}$  samples  $g_0$  in the setup phase and gives this value to  $\tilde{\mathcal{P}}$ .

Observe that the size of the primary memory of  $\tilde{\mathcal{P}}$  is increased to additionally store a value  $g$ . Further, the time bound of the challenge phase is increased to  $N \cdot (\delta_{\text{Gen}} + \delta_{\text{Read}})$ .

*Success Probability.* The only difference between Game 4 and Game 3 is that  $\tilde{\mathcal{P}}$  now knows  $g_0$  and can query the *Gen* oracle. Recall that  $g_0$  is used by *Gen* for computing  $(a_1, \dots, a_N)$ . Hence  $\tilde{\mathcal{P}}$  may decide to skip the generation of one or more addresses and save the time and memory for other computations. However, since *Gen* is assumed to be  $\nu_{\text{Gen}}$ -unpredictable (Definition 4),  $\tilde{\mathcal{P}}$  cannot derive any information on  $a_{i+1}$  or  $g_{i+1}$  from  $g_i$  without querying *Gen*. Thus if  $\tilde{\mathcal{P}}$  never queries *Gen* with some value  $g_i$  it cannot distinguish the subsequent values  $(g_{i+1}, \dots, g_N)$  with a probability better than  $(N - i) \cdot \nu_{\text{Gen}}$ . Therefore we can restrict to provers that compute  $(a_1, g_1), \dots, (a_M, g_M)$  and skip  $(a_{M+1}, g_{M+1}), \dots, (a_N, g_N)$ .

Let  $\Pr[\text{Win}_4]$  be the probability to win Game 4 and  $\Pr[\text{Win}_4(M)]$  be the probability to win Game 4 for a fixed  $M$ . That is we have  $\Pr[\text{Win}_4] \leq \max_M \{\Pr[\text{Win}_4(M)]\}$ . Now consider a variation of Game 4 where  $\mathcal{O}$  replaces the values  $(a_{M+1}, g_{M+1}), \dots, (a_N, g_N)$  by independent and uniformly sampled values and we denote with  $\Pr[\text{Win}'_4(M)]$  the probability that  $\tilde{\mathcal{P}}$  wins this game. As *Gen* is assumed to be  $\nu_{\text{Gen}}$ -unpredictable (cf. Definition 4), it holds that  $\Pr[\text{Win}_4(M)] \leq \Pr[\text{Win}'_4(M)] + \nu_{\text{Gen}} \cdot (N - M)$ .

With respect to  $\Pr[\text{Win}_4(M)]$ , observe that for the first  $M$  rounds the situation is as in Game 3. Hence the success probability for the first  $M$  rounds is upper bounded by  $\pi(M, \text{ops}(\delta_{\text{Read}} + \delta_{\text{Gen}})) + \varrho$ . For the remaining  $N - M$  rounds,  $\mathcal{O}$  uses uniformly sampled the values  $(a_{M+1}, \dots, a_N)$  that are unknown to  $\tilde{\mathcal{P}}$ .

Hence the probability of  $\tilde{\mathcal{P}}$  to derive  $(s_{M+1}, \dots, s_N)$  correctly is upper bounded by  $\gamma^{N-M}$ . This yields  $\Pr[\text{Win}'_4(M)] \leq (\pi(M, \text{ops}(\delta_{\text{Read}} + \delta_{\text{Gen}})) + \varrho) \cdot \gamma^{N-M}$  and hence

$$\overline{\Pr}[\text{Correct}] \leq \max_{0 \leq M \leq N} \{(\pi(M, \text{ops}(\delta_{\text{Read}} + \delta_{\text{Gen}})) + \varrho) \cdot \gamma^{N-M} + \nu_{\text{Gen}} \cdot (N - M)\}.$$

## 7 Discussion and Conclusion

We presented the first formal security framework for software attestation and formalized various of the underlying system and design parameters. Moreover we presented a generic software attestation scheme that encompasses most existing schemes in the literature. For this generic scheme we derived an upper bound on the success probability of a malicious prover that depends on the formalized parameters.

One lesson learned is the impact of these parameters on the security of the generic scheme. The effect of some of them, e.g., the distribution of the state entries and the level of similarity of the states of an honest and a malicious prover, have been implicitly discussed in prior work but have never been explicitly considered and formalized. Our results also show that traditional cryptographic assumptions are partially too strong (second pre-image resistance) and partially too weak (pseudo-randomness).

Further, we identified new (sufficient) conditions on the core functionalities of software attestation. Moreover most previous works require the software attestation algorithm to iterate over all memory words of the secondary memory without giving any formal justification. Our bound allows to identify lower values for  $N$  (if the other parameters are known), allowing for more efficient solutions.

Thus our work represents the first step towards efficient and provably secure software attestation schemes. Still, several open questions remain for future work. One being to relax the presented conditions or to derive necessary conditions. A further task is to determine concrete instantiations. While **Gen** and **Chk** could be easily realized on devices with block ciphers implemented in hardware (similar to the AES instructions in modern CPUs [27]), this becomes more challenging on other platforms.

We are currently working on the following aspects: (1) a practical instantiation of the generic software attestation scheme and its evaluation, and (2) the evaluation of existing software attestation schemes in our framework. These results will be published soon in the full version of this paper.

## References

1. Atmel: tinyAVR homepage. <http://www.atmel.com/tinyavr/> (2013)
2. Castelluccia, C., Francillon, A., Perito, D., Soriente, C.: On the difficulty of software-based attestation of embedded devices. In: Al-Shaer, E., Jha, S., Keromytis, A.D. (eds.) Computer and Communications Security (CCS). pp. 400–409. ACM (Nov 2009)
3. Defrawy, K.E., Francillon, A., Perito, D., Tsudik, G.: SMART: Secure and minimal architecture for (establishing a dynamic) root of trust. In: Network and Distributed System Security Symposium (NDSS). Internet Society (2012)
4. Dwork, C., Naor, M.: Pricing via processing or combatting junk mail. In: Brickell, E. (ed.) Advances in Cryptology – CRYPTO. LNCS, vol. 740, pp. 139–147. Springer, London, UK (1993), <http://dl.acm.org/citation.cfm?id=646757.705669>
5. Dwork, C., Naor, M., Wee, H.: Pebbling and proofs of work. In: Shoup, V. (ed.) Advances in Cryptology – CRYPTO. LNCS, vol. 3621, pp. 37–54. Springer (2005)
6. Franklin, J., Luk, M., Seshadri, A., Perrig, A.: PRISM: Human-verifiable code execution. Tech. rep., Carnegie Mellon University (Feb 2007)
7. Gardner, R.W., Garera, S., Rubin, A.D.: Detecting code alteration by creating a temporary memory bottleneck. Trans. Info. For. Sec. 4(4), 638–650 (2009)
8. Giffin, J.T., Christodorescu, M., Kruger, L.: Strengthening software self-checksumming via self-modifying code. In: Annual Computer Security Applications Conference (ACSAC). pp. 23–32. IEEE, Washington, DC, USA (2005), <http://portal.acm.org/citation.cfm?id=1106808>
9. Graizer, V., Naccache, D.: Alien vs. quine. Security Privacy, IEEE 5(2), 26–31 (march-april 2007)
10. Gratzner, V., Naccache, D.: Alien vs. quine, the vanishing circuit and other tales from the industry’s crypt. In: Advances in Cryptology – EUROCRYPT. pp. 48–58. LNCS, Springer (2007), invited Talk
11. Jakobsson, M., Johansson, K.A.: Retroactive Detection of Malware With Applications to Mobile Platforms. In: Workshop on Hot Topics in Security (HotSec). USENIX, Washington, DC (August 2010), [markus-jakobsson.com/papers/jakobsson-hotsec10.pdf](http://markus-jakobsson.com/papers/jakobsson-hotsec10.pdf)

12. Kennell, R., Jamieson, L.H.: Establishing the genuinity of remote computer systems. In: USENIX Security Symposium. pp. 295–308. USENIX (Aug 2003)
13. Kovah, X., Kallenberg, C., Weathers, C., Herzog, A., Albin, M., Butterworth, J.: New results for Timing-Based attestation. In: Security & Privacy (S&P). IEEE (May 2012)
14. Li, Y., McCune, J.M., Perrig, A.: VIPER: verifying the integrity of PERipherals’ firmware. In: Chen, Y., Danezis, G., Shmatikov, V. (eds.) Computer and Communications Security (CCS). pp. 3–16. ACM (Oct 2011)
15. Parno, B., McCune, J.M., Perrig, A.: Bootstrapping Trust in Commodity Computers. In: Security and Privacy (S&P). pp. 414–429. IEEE, Oakland, CA (May 2010)
16. Sadeghi, A.R., Schulz, S., Wachsmann, C.: Lightweight remote attestation using physical functions. In: Wireless Network Security (WiSec). ACM (Jun 2011), [bit.ly/f40Rr3](http://bit.ly/f40Rr3)
17. Schellekens, D., Wyseur, B., Preneel, B.: Remote attestation on legacy operating systems with Trusted Platform Modules. *Sci. Comput. Program.* 74(1-2), 13–22 (2008)
18. Seshadri, A., Luk, M., Perrig, A.: SAKE: Software attestation for key establishment in sensor networks. *Distributed Computing in Sensor Systems* pp. 372–385 (2008)
19. Seshadri, A., Luk, M., Perrig, A., van Doorn, L., Khosla, P.: SCUBA: Secure code update by attestation in sensor networks. In: Workshop on Wireless security (WiSe). pp. 85–94. ACM, New York, NY, USA (2006)
20. Seshadri, A., Luk, M., Shi, E., Perrig, A., van Doorn, L., Khosla, P.: Pioneer: Verifying integrity and guaranteeing execution of code on legacy platforms. In: Symposium on Operating Systems Principles (SOSP). pp. 1–16. ACM, Brighton, UK (Oct 2005)
21. Seshadri, A., Perrig, A., van Doorn, L., Khosla, P.K.: SWATT: SoftWare-based ATTestation for embedded devices. In: Security and Privacy (S&P). IEEE, Oakland, CA (May 2004)
22. Shaneck, M., Mahadevan, K., Kher, V., Kim, Y.: Remote software-based attestation for wireless sensors. In: Molva, R., Tsudik, G., Westhoff, D. (eds.) Security and Privacy in Ad-hoc and Sensor Networks, LNCS, vol. 3813, chap. 3, pp. 27–41. Springer, Berlin/Heidelberg (2005)
23. Shankar, U., Chew, M., Tygar, J.D.: Side effects are not sufficient to authenticate software. In: USENIX Security Symposium. p. 7. USENIX, Berkeley, CA, USA (Aug 2004)
24. Strackx, R., Piessens, F., Preneel, B.: Efficient isolation of trusted subsystems in embedded systems. In: Jajodia, S., Zhou, J. (eds.) Security and Privacy in Communication Networks (SecureComm), LNCS, vol. 50, chap. 20, pp. 344–361. Springer, Berlin, Heidelberg (Sep 2010)
25. Trusted Computing Group (TCG): TPM Main Specification, Version 1.2 (Mar 2011)
26. Vasudevan, A., Mccune, J., Newsome, J., Perrig, A., Doorn, L.V.: CARMA: A hardware tamper-resistant isolated execution environment on commodity x86 platforms. In: ACM Symposium on Information, Computer and Communications Security (AsiaCCS). ACM (May 2012)
27. Wikipedia: AES instruction set. [http://en.wikipedia.org/wiki/AES\\_instruction\\_set](http://en.wikipedia.org/wiki/AES_instruction_set) (2013)

## A Simplification of the Upper Bound of $\Pr[\text{Win}_1 | \text{Coll}(N_{\text{coll}})]$

In this section, we show how to simplify

$$\Pr[\text{Win}_1 | \text{Coll}(N_{\text{coll}})] \leq \max_{N_{\text{less}}} \left\{ \lambda^{N - (\text{ops} + 1) \cdot (N_{\text{coll}} + N_{\text{less}})} \cdot \gamma^{N_{\text{less}}}, \gamma^{N - (\text{ops} + 1) \cdot N_{\text{coll}} - \text{ops} \cdot N_{\text{less}}} \right\}.$$

Observe that  $0 \leq N_{\text{less}}$  and  $0 \leq N_{\text{equal}} = N - (\text{ops} + 1) \cdot (N_{\text{coll}} + N_{\text{less}}) \Leftrightarrow N_{\text{less}} \leq \frac{N}{\text{ops} + 1} - N_{\text{coll}}$ , i.e.,

$$0 \leq N_{\text{less}} \leq \frac{N}{\text{ops} + 1} - N_{\text{coll}}.$$

To simplify the first term  $\lambda^{N - (\text{ops} + 1) \cdot (N_{\text{coll}} + N_{\text{less}})} \cdot \gamma^{N_{\text{less}}}$ , we define  $e := \log_{\lambda}(\gamma)$  and rephrase the expression as  $\lambda^{N - (\text{ops} + 1) \cdot N_{\text{coll}} - (\text{ops} + 1 - e) \cdot N_{\text{less}}}$ . When  $\text{ops} + 1 - e < 0$ , the maximum value is achieved for  $N_{\text{less}} = 0$ , hence in this case the upper bound is  $\lambda^{N - (\text{ops} + 1) \cdot N_{\text{coll}}}$ . In the other case we get an upper bound for  $N_{\text{less}} = \frac{N}{\text{ops} + 1} - N_{\text{coll}}$ , yielding

$$\lambda^{N - (\text{ops} + 1) \cdot N_{\text{coll}} - (\text{ops} + 1 - e) \cdot \left(\frac{N}{\text{ops} + 1} - N_{\text{coll}}\right)} = \lambda^{N - (\text{ops} + 1) \cdot N_{\text{coll}} - N + e \cdot \frac{N}{\text{ops} + 1} + (\text{ops} + 1) \cdot N_{\text{coll}} - e \cdot N_{\text{coll}}} = \gamma^{\frac{N}{\text{ops} + 1} - N_{\text{coll}}}.$$

With respect to the second term, i.e.,  $\gamma^{N - (\text{ops} + 1) \cdot N_{\text{coll}} - \text{ops} \cdot N_{\text{less}}}$ , the maximum value is achieved if  $N_{\text{less}}$  is as big as possible, i.e.,  $N_{\text{less}} = \frac{N}{\text{ops} + 1} - N_{\text{coll}}$ . This gives an upper bound of

$$\gamma^{N - (\text{ops} + 1) \cdot N_{\text{coll}} - \text{ops} \cdot N_{\text{less}}} = \gamma^{N - (\text{ops} + 1) \cdot N_{\text{coll}} - \text{ops} \cdot \left(\frac{N}{\text{ops} + 1} - N_{\text{coll}}\right)} = \gamma^{\frac{N}{\text{ops} + 1} - N_{\text{coll}}}.$$

Altogether, it follows that

$$\Pr[\text{Win}_1 | \text{Coll}(N_{\text{coll}})] \leq \left( \max \left\{ \lambda^{\text{ops}(\delta_{\text{Read}})+1}, \gamma \right\} \right)^{\frac{N}{\text{ops}(\delta_{\text{Read}})+1} - N_{\text{coll}}}.$$