

Towards Provably Secure Software Attestation

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Abstract. Software attestation has become a popular and challenging research topic at many established security conferences. It aims at verifying the software integrity of (typically) resource-constrained embedded devices. However, for practical reasons, software attestation cannot rely on stored cryptographic secrets or dedicated trusted hardware. Instead, it exploits side-channel information, such as the time that the underlying device needs for a specific computation. Unfortunately, traditional cryptographic solutions and arguments are not applicable in this setting, making new approaches for the design and analysis necessary. This is certainly one of the main reasons why the security properties and assumptions of software attestation have been only vaguely discussed and have never been formally treated, as it is common sense in modern cryptography. Thus, despite its popularity and its expected impact on practice, a sound approach for designing secure software attestation schemes is still an important open problem.

We introduce the first formal security framework for software attestation and formalize various system and design parameters. Moreover, we present a generic software attestation scheme that captures most existing schemes in the literature. Finally, we analyze its security within our framework, yielding sufficient conditions for provably secure software attestation schemes. We regard these results as a first step towards putting software attestation on a solid ground and as a starting point for further research.

Keywords: software attestation, keyless cryptography, security model

1 Introduction

Embedded systems are increasingly permeating our information society. Potential threats and damages that could be caused by exploiting possible vulnerabilities in these systems have raised the demand for enabling technologies that can validate and verify the integrity of a system's software state. In this context, *software attestation* has become a popular research topic at many established security conferences with a large body of literature [12,21,23,8,20,22,19,6,18,17,2,7,11,14,16,13,26].

Software attestation is a trust establishment mechanism that allows a system, the *verifier*, to check whether the program memory content of another system, the *prover*, is genuine or has been modified, e.g., by malicious code. Since the verifier usually cannot access the memory of the prover directly (without requesting hardware modifications), the common approach in the literature deploys challenge-response protocols, in which the verifier challenges the prover to compute a checksum over a (random) selection of its memory content. Unfortunately, classical cryptographic primitives and protocols cannot be used directly since the adversary may get full control of the prover and its cryptographic secrets. That is because software attestation mainly targets resource-constrained embedded systems (such as the Atmel tinyAVR [1] microcontrollers) that cannot afford secure hardware, which precludes the use of solutions based on security hardware [25,15,24,3,13].

Therefore software attestation follows a radically different approach than most conventional security mechanisms: It exploits the intrinsic physical constraints of the underlying hardware and side-channel information, typically the *computation time* required by the prover to complete the attestation protocol. More detailed, software attestation schemes are typically designed to temporarily utilize all the computing and memory resources of the prover, aiming at ensuring that the prover cannot deviate from the correct execution of the challenge-response protocol. Summing up, in contrast to common attestation schemes, a malicious prover can *in principle* cheat during the protocol but not without exceeding a certain concrete *time-bound*.

Without question, this requires completely different forms of reasoning and likewise demands for other security assumptions on the underlying core functionalities and system properties, representing a challenging task. This may be the main reason that, despite its popularity and expected practical impact, software attestation has not received any formal treatment yet as it is common sense in modern cryptography. To start with, there exist no common adversary model and no precise formalization of the security goals so far. Likewise the underlying security properties and assumptions have been only vaguely discussed. In fact, current proposals combine components with unclear and possibly insufficient security properties and multiple attacks against existing software attestation schemes have been documented [21,23,2].

Contribution. In this paper, we make a first step towards putting software attestation on a solid ground. Our contributions are as follows:

First formal software attestation framework: We describe the first formal security framework for software attestation. This includes an adversary model that fundamentally deviates from the classical cryptographic adversary model. Instead of being a polynomially bounded algorithm, the adversary could be unbounded in principle. However, during the attack it has to specify (or program) a malicious prover device with tight resource constraints. The goal is that this malicious prover can cheat in the attestation protocol with reasonable success probability but *without* any interaction with the adversary. In other words, the adversary has unbound resources for *preparing* the attack but only a tight time-bound for *executing* the attack.

Generic software attestation scheme: We present a *generic* software attestation scheme that covers most existing software attestation protocols in the literature. A benefit is that any results derived on this generic scheme help to investigate and understand the security of concrete schemes.

Security analysis and new insights: We identify and formalize several system parameters of software attestation and provide an upper bound of the success probability of a malicious prover as a function of these parameters. This requires to use new types of arguments since the typical reduction to a hard problem is not possible anymore. Instead we have to argue *directly* that any attack strategy that is possible within the given time-bound fails with a certain probability. The derived upper bound of the success probability implies *sufficient* conditions on the system parameters. Although some of these aspects have been implicitly assumed and informally discussed in the literature, we present their first formal treatment. Moreover, our approach provides new insights on how these parameters impact the security of the underlying software attestation scheme, which has never been analyzed before. Finally, we stress that our investigation introduces new cryptographic primitives that are similar to established primitives, such as pseudo-random generators or hash functions, but differ in subtleties: Some cryptographic assumptions can be relaxed while others need to be strengthened.

We see our work as a first step towards provably secure software attestation schemes. Apart from the fact that this topic is of high relevance for practice, we identify several open research problems,

which we believe to be of high interest to the cryptographic research community and hope that our results inspire new research in the area of keyless cryptography.

Outline. We give an overview of the related work in [Section 2](#) and introduce our system model in [Section 3](#). We present the formal framework for software attestation in [Section 4](#), describe the generic software attestation scheme and its requirements in [Section 5](#) and formally analyze its security in [Section 6](#). Finally, we discuss our results and conclude in [Section 7](#).

2 Related Work

Several works consider the design and extension of software attestation. The existing literature focuses on the design of checksum constructions for different platform architectures and countering platform-specific attacks [21,20,6,19,14]. Several works consider the strengthening of self-checksumming code against unintended modifications by either limiting the memory available to the prover during attestation [7,26] or by using self-modifying and/or obfuscated attestation algorithms [22,8]. Several works investigate the suitability and extension of software attestation to a variety of computing platforms, including sensors, peripherals and voting machines [18,14,6,20,13]. Furthermore, software attestation has also been proposed as a key establishment mechanism [18]. Since software attestation does not rely on any secrets and thus cannot authenticate the prover to the verifier, multiple works also consider how to combine software attestation with hardware trust anchors such as TPMs and SIM-cards [17,13,11] or intrinsic hardware characteristics such as code execution side-effects [12,21,23] and Physically Unclonable Functions [16]. Interestingly, most proposed implementations employ hash functions and PRNGs that are not cryptographically secure. Further, works that use cryptographically secure algorithms do not consider whether these algorithms maintain their security properties in the “keyless” software attestation scenario where the underlying secrets, such as the PRNG states, are known to the adversary. In this respect, our formal analysis provides a first step towards a deeper and more comprehensive understanding of software attestation.

An approach [10,9] related to software attestation uses *Quines*, which is code that outputs its own source-code while it is executed. The basic idea is that the device outputs the whole content of the memory such that the verifier can compare it to the expected content. In contrast, software attestation aims to use short outputs only for practical reasons. In that sense, both approaches can be seen as special instantiations of proof-of-knowledge schemes where the proof of knowing the whole expected memory content in the Quines-based approach is given by responding with the whole memory content while in software attestation the response only depends on the memory content. A further difference is that for practical reasons, software attestation typically minimizes the interaction between the prover and the verifier, e.g., to minimize the uncertainties in the running time measurement caused by network jitter. However, the schemes described in [10,9] require significant interaction between the verifier and the device, i.e., rebooting it several times after a few operations have been executed and specifying new inputs. In fact, the authors explicitly mention the development of solutions without the need of rebooting as an open problem. Finally, as opposed to this work, the Quine-based attestation schemes described in [10,9] consider only a very specific system architecture and do not provide formal proofs.

Similar to software attestation, *proofs of work* schemes challenge the prover with computationally expensive or memory-bound tasks [4,5]. However, while the goal of these schemes is to mitigate denial-of-service attacks and Spam by imposing artificial load on the service requester, the goal

of software attestation schemes is using all of the prover’s resources to prevent it from executing malicious code within a certain time frame. Hence, proofs of work are in general not suitable for software attestation since they are usually less efficient and not designed to achieve the optimality requirements of software attestation algorithms.

3 Preliminaries

Notation. Let A and B be arbitrary algorithms. Then $y \leftarrow A(x)$ means that on input x , A assigns its output to y . The expression A^B means that A has black-box access to B . We denote with \widehat{A}^B an algorithm A that does *not* access an algorithm B . Let \mathbb{D} be a probability distribution over the set X , then the term $x \stackrel{\mathbb{D}}{\leftarrow} X$ means the event of assigning an element of X to variable x that has been chosen according to \mathbb{D} . Further, we define $\mathbb{D}(x) := \Pr [x \stackrel{\mathbb{D}}{\leftarrow} X]$ for each $x \in X$ and denote with \mathbb{U} the uniform distribution.

System Model. Software attestation is a protocol between a verifier \mathcal{V} and a (potentially malicious) prover \mathcal{P} where the latter belongs to a class of devices with clearly specified characteristics. That is, whenever we speak about a prover \mathcal{P} , we refer to a device that belong to this class. Typically a prover \mathcal{P} is a low-end embedded system that consists of memory and a computing engine (CE). The memory is composed of *primary memory* (PM), such as CPU registers and cache and *secondary memory* (SM), such as RAM and Flash memory. We assume that the memory is divided into *memory words* and denote by $\Sigma := \{0, 1\}^{l_s}$ the set of all possible memory words (e.g., $l_s = 8$ if memory words are bytes). Let $\mathfrak{s} := 2^{l_a}$ and $\mathfrak{p} := 2^{l_b}$ be the number of memory words that can be stored in the secondary memory (SM) and the primary memory (PM), respectively. An important notion is the state of a prover:

Definition 1 (State). *Consider a prover \mathcal{P} , i.e., a device that belongs to the specified class of devices. The state $\text{State}(\mathcal{P}) = S$ of the prover \mathcal{P} are the memory words stored in SM.*

Note that S includes the program code of \mathcal{P} and hence specifies the algorithms executed by \mathcal{P} .

The computing engine (CE) comprises an arithmetics and logic unit that can perform computations on the data in PM and alter the program flow. For performance reasons, PM is typically fast but also expensive. Hence, the size of PM is usually much smaller than the size of SM. To make use of SM, CE includes the *Read* instruction to transfer data from SM to PM and the *Write* instruction to write data from PM to SM. More precisely, $\text{Read}(S, a, b)$ takes as input a memory address a of SM and a memory address b of PM and copies the data word x stored at address a in SM to the data word at address b in PM. For convenience, we write $\text{Read}(S, a)$ instead of $\text{Read}(S, a, b)$ whenever the address b of PM is not relevant. Note that $\text{Read}(S, a, b)$ overwrites the content y of PM at address b . Hence, in case y should not be lost, it must be first copied to SM using *Write* or copied to another location in PM before *Read* is performed. It is important to stress that, whenever CE should perform some computation on some value x stored in SM, it is mandatory that x is copied to PM before CE can perform the computation. Further, since SM is typically much slower than PM, *Read* and *Write* incur a certain time overhead and delay computations on x . We denote the time required by CE to perform some instruction Ins with $\text{Time}(\text{Ins})$. The program code that determines the behaviour of the prover $\widehat{\mathcal{P}}$ is encoded in the state of \mathcal{P} . Note that we only consider provers as described above while the verifier \mathcal{V} might be an arbitrary computing platform that can interact with \mathcal{P} .

Remark 1 (Platform Architecture). We explicitly exclude provers that are high-end computing platforms with multiple CPUs and/or Direct Memory Access (DMA) since these are typically equipped with secure hardware (such as TPMs) and hence could support common cryptographic solutions based on secrets. Further, their memory architectures are usually more complex than in our system model. In particular, such platforms usually feature additional hardware to predict and fetch memory blocks in advance, making the time-bounded approach much more difficult and its realization highly dependent on the concrete system.

4 Secure Software Attestation

Secure software attestation enables the verifier \mathcal{V} to gain assurance that the state of a prover \mathcal{P} is equal to a particular state S . If this is the case, we say that \mathcal{P} is in state S , i.e., formally $\text{State}(\mathcal{P}) = S$. Consequently a prover \mathcal{P} is called *honest* (with respect to some state S) if $\text{State}(\mathcal{P}) = S$, otherwise it is considered to be *malicious*.

Remark 2 (Distance between Honest and Malicious Prover State). Observe that a prover $\tilde{\mathcal{P}}$ is already considered to be malicious even if its state differs by only one or a few state entries (memory words) from S . This is a necessary consequence from the goal of having a definition of honest and malicious provers that is as generic as possible. Nonetheless, in practice a software attestation scheme could be sufficient that verifies whether the state of a prover coincides with S *almost completely*. We address this issue by considering the Hamming distance λ between the state S of an honest prover and the state \tilde{S} of a malicious prover $\tilde{\mathcal{P}}$. As far as we know, we are the first to formally take into account the impact of λ on the security of software attestation schemes (cf. λ in Theorem 1).

Ideal Approach. Ideally, \mathcal{V} could disable the computing engine (CE) of \mathcal{P} and directly read and verify the software state S stored in the secondary memory (SM) of \mathcal{P} . In fact, this can be seen as the goal of the Quines-based approaches [10,9]. However, exposing CE and the SM of \mathcal{P} to \mathcal{V} in such a way requires hardware extensions⁴ on \mathcal{P} , which contradicts the goal of software attestation to work with no hardware modifications. Instead the common approach in the literature is that \mathcal{V} and \mathcal{P} engage in a challenge-response protocol where \mathcal{P} must answer to a challenge of \mathcal{V} with a response that depends on S .

Practical Approach. In general, software attestation aims to figure out whether the original state S of a device has been replaced by the adversary with a malicious state $\tilde{S} \neq S$. Observe that although \tilde{S} is different from S , we cannot exclude that \tilde{S} may depend on S . This implies an important difference to common cryptographic scenarios: Software attestation cannot rely on any secrets since the adversary has access to the same information as the honest prover \mathcal{P} . Therefore software attestation follows a fundamentally different approach and leverages side-channel information, typically the time δ the prover takes to compute the response. A fundamental requirement of this approach is that it is hard to find any other implementation that can be executed by \mathcal{P} in significantly less time than δ . Otherwise, a malicious prover could use a faster implementation and exploit the time difference to perform additional computations, e.g., to lie about its state.

Furthermore, the communication time jitter between \mathcal{V} and \mathcal{P} is typically much higher than the time needed by the computing engine of \mathcal{P} to perform a few instructions. Hence, to ensure that \mathcal{V}

⁴ Existing testing interfaces such as JTAG cannot be used since they are typically disabled on consumer devices to prevent damage to and unintended reverse-engineering of the device.

can measure also slight changes to the prover’s code (that could be exploited by a malicious prover to lie about its state), \mathcal{V} needs to amplify the effect of such changes. The most promising approach to realize this in practice is designing the attestation protocol as an iterative algorithm with a large number of rounds.

Further, since showing the optimality of complex implementations is a hard problem and since \mathcal{P} must compute the response in a reasonable amount of time, it is paramount that the individual rounds are simple and efficient. As a result, cryptographically secure hash functions and complex Pseudo-Random Number Generators (PRNGs) are not a viable option. Hence, several previous works deployed lightweight ad-hoc designs of compression functions and PRNGs, however, without analyzing the underlying requirements on these components and their interaction. In contrast, we identify concrete requirements and provide a detailed analysis of our instantiation.

Adversary Model and Security Definition. In the following, we provide the first formal specification of the adversary model and the security of software attestation based on a security experiment $\mathbf{Exp}_{\text{Attest}}^{\mathcal{A}}$ that involves two probabilistic algorithms: an adversary \mathcal{A} and a challenger $\mathcal{C}_{\text{Attest}}$. The task of $\mathcal{C}_{\text{Attest}}$ is to provide all necessary information to \mathcal{A} and to play the role of the honest verifier \mathcal{V} . At the beginning, $\mathcal{C}_{\text{Attest}}$ receives as input a state S and a security parameter l and generates a challenge c , a response r and a time-bound δ . The adversary \mathcal{A} gets the same inputs and outputs a (possibly) malicious prover $\tilde{\mathcal{P}}$ by specifying its state \tilde{S} , i.e., $\text{State}(\tilde{\mathcal{P}}) = \tilde{S}$. Afterwards, $\tilde{\mathcal{P}}$ receives the challenge c and returns a “guess” \tilde{r} for the correct response r . The result of the experiment is ACCEPT if $\tilde{\mathcal{P}}$ responded within time δ and $\tilde{r} = r$. Otherwise $\tilde{\mathcal{P}}$ returns REJECT. Formally:

$$\begin{aligned} \mathbf{Exp}_{\text{Attest}}^{\mathcal{A}}(S, l) &: (c, r, \delta) \leftarrow \mathcal{C}_{\text{Attest}}(S, l) \\ &\quad \tilde{\mathcal{P}} \leftarrow \mathcal{A}(S, l) \\ &\quad \tilde{r} \leftarrow \tilde{\mathcal{P}}(c) \\ &\quad \text{if } (\text{Time}(\tilde{\mathcal{P}}) < \delta) \wedge (\tilde{r} = r) \text{ return ACCEPT else return REJECT} \end{aligned}$$

\mathcal{A} wins iff the result of $\mathbf{Exp}_{\text{Attest}}^{\mathcal{A}}$ is ACCEPT and loses otherwise.

Based on this experiment we define *correctness* and *soundness*. Correctness is defined analogously to the common meaning of correctness of challenge-response protocols: In case $\text{State}(\tilde{\mathcal{P}}) = S$, that is $\tilde{\mathcal{P}}$ is in the state expected by \mathcal{V} , the prover $\tilde{\mathcal{P}}$ should always succeed, i.e., the result of the experiment should always be ACCEPT. Soundness means that in case $\text{State}(\tilde{\mathcal{P}}) \neq S$, the probability that the result of the experiment is ACCEPT should be below a certain threshold. Formally:

Definition 2 (Correctness and Soundness). Consider a software attestation scheme *Attest* and a state S . For a given adversary \mathcal{A} we denote by **EqualState** the event that the output of \mathcal{A} during the experiment is a prover $\tilde{\mathcal{P}}$ with $\text{State}(\tilde{\mathcal{P}}) = S$.

The software attestation scheme *Attest* is correct if for all adversaries \mathcal{A} it holds that

$$\Pr [\mathbf{Exp}_{\text{Attest}}^{\mathcal{A}}(S, l) = \text{ACCEPT} | \text{EqualState}] = 1.$$

Furthermore, *Attest* is ε -secure if for all adversaries \mathcal{A} it holds that

$$\Pr [\mathbf{Exp}_{\text{Attest}}^{\mathcal{A}}(S, l) = \text{ACCEPT} | \neg \text{EqualState}] \leq \varepsilon.$$

Remark 3 (Computational Power of \mathcal{A}). Observe that an attack against a software attestation scheme comprises two phases: A *preparation phase* where the adversary \mathcal{A} prepares a state \tilde{S} for

Prover \mathcal{P}	Verifier \mathcal{V}
S	S
$r'_0 \leftarrow r_0$ for $i = 1, \dots, N$ do $(g_i, a_i) \leftarrow \text{Gen}(g_{i-1})$ $s_i \leftarrow \text{Read}(S, a_i)$ $r'_i \leftarrow \text{CHK}(r_{i-1}, s_i)$ endfor	$(g_0, r_0) \xleftarrow{\text{U}} \{0, 1\}^{l_g + l_r}$ Store current time t for $i = 1, \dots, N$ do $(g_i, a_i) \leftarrow \text{Gen}(g_{i-1})$ $s_i \leftarrow \text{Read}(S, a_i)$ $r_i \leftarrow \text{CHK}(r_{i-1}, s_i)$ endfor Store current time t' Accept iff $r'_N = r_N \wedge t' - t \leq \delta$
$\xleftarrow{g_0, r_0}$	$\xrightarrow{r'_N}$

Fig. 1: The Generic Attestation Scheme **Attest**

the prover device and a *attack phase* where the prover device is in state \tilde{S} and runs the attestation protocol with the verifier \mathcal{V} . The security of software attestation significantly differs from common cryptographic models, where the time effort of the adversary is typically bounded (often polynomially bounded in some security parameter). This is not the case for software attestation. More detailed, in the preparation phase, \mathcal{A} can be any *unrestricted* probabilistic algorithm. However, \mathcal{A} has no influence anymore once **Attest** is executed between $\tilde{\mathcal{P}}$ and \mathcal{V} . As $\tilde{\mathcal{P}}$ is executed on a device with the same characteristics as an honest prover, $\tilde{\mathcal{P}}$ has to comply to the same restrictions as \mathcal{P} . In other words, the adversary has unbounded resources for *preparing* the attack but only a tight time-bound for *executing* the attack.

Remark 4 (Difference to Remote Attestation). The goal of *remote attestation* is to verify the integrity of remote provers, e.g., over a network. In particular, in practice a verifier \mathcal{V} usually cannot exclude that a malicious prover may have more computational power than the honest prover. Therefore, remote attestation schemes usually rely on secrets shared between the verifier and the honest prover.

This is fundamentally different to software attestation which cannot rely on cryptographic secrets to authenticate the prover device to \mathcal{V} . Hence, as already elaborated, existing works on software attestation typically assume that \mathcal{V} can authenticate the prover hardware using an out-of-band channel, such as visual authentication.

5 Generic Software Attestation

In this section, we formalize a generic software attestation protocol that captures most existing schemes in the literature. In particular, we formally define several aspects and assumptions that were only informally discussed or implicitly defined.

5.1 Protocol Specification

The main components of our generic attestation scheme (Figure 1) are two deterministic algorithms:

- Memory address generator $\text{Gen} : \{0, 1\}^{l_g} \rightarrow \{0, 1\}^{l_g} \times \{0, 1\}^{l_a}, \quad g \mapsto (g', a')$
- Compression function $\text{Chk} : \{0, 1\}^{l_r} \times \Sigma \rightarrow \{0, 1\}^{l_r}, \quad (r, s) \mapsto r'$

Here l_g , l_a and l_r are the bit length of the state of Gen , the memory addresses a_i and the attestation response r'_N , respectively, Σ is the set of possible state entries and $N \in \mathbb{N}$ is the number of rounds of the attestation algorithm. Furthermore, we provide an iterative definition of Chk^N : For some $r_0 \in \{0, 1\}^{l_r}$ and $\mathbf{s} := (s_1, \dots, s_N)$, we define $r \leftarrow \text{Chk}^N(c, \mathbf{s})$ as $r_i := \text{Chk}(r_{i-1}, s_i)$ for $i = 1, \dots, N$.

The protocol works as follows: The verifier \mathcal{V} sends an attestation challenge (g_0, r_0) to the prover \mathcal{P} , who iteratively generates a sequence of memory addresses (a_1, \dots, a_N) based on g_0 using Gen . For each $i \in \{1, \dots, N\}$, \mathcal{P} reads the state entry $s_i = \text{Read}(S, a_i)$ at address a_i and iteratively computes $r'_i = \text{Chk}(r'_{i-1}, s_i)$ using $r'_0 = r_0$. Finally, \mathcal{P} sends r'_N to \mathcal{V} , which executes exactly the same computations as \mathcal{P} using the state S and compares the final result with the response r'_N from \mathcal{P} . Eventually, \mathcal{V} accepts iff $r'_N = r_N$ and \mathcal{P} responded in time $\delta := N(\delta_{\text{Gen}} + \delta_{\text{Read}} + \delta_{\text{Chk}})$, where δ_{Gen} , δ_{Read} and δ_{Chk} are the time-bounds for running Gen , Read and Chk , respectively, on a genuine and honest prover.⁵

Remark 5 (Correctness). Observe that an honest prover \mathcal{P} always makes an honest verifier \mathcal{V} accept since both perform exactly the same computations on the same inputs and the honest prover by assumption requires at most time δ .

Remark 6 (Generality of the Protocol). Note that the basic concept of our generic scheme and several instantiations for specific platforms can be found in the literature on software attestation (cf. [Section 2](#)). However, we aim at abstracting from the particularities of individual platforms and instead design and analyze a construction that is as generic as possible. Further, some existing software attestation schemes also use the memory addresses a_i and/or the index i as input to the checksum function Chk . However, since there is a dependence between the index i , the memory address a_i and the memory block $s_i = \text{Read}(S, a_i)$ and since the use of simple components is a primary goal of software attestation, we restrict to the case where only the memory blocks are used as input.

5.2 Design Criteria and Properties

Next, we discuss the design criteria of the underlying algorithms and formally define their properties required later in the security analysis. Note that, although some of these properties have been informally discussed or implicitly made in prior work, they have never been formally specified and analyzed before.

Implementation of the Core Functionalities. The generic protocol deploys three core functionalities: Read , Gen and Chk , which of the execution time is of paramount importance for the security of software attestation. Hence, we make the following assumptions that are strongly dependent on the concrete implementation and computing engine of the prover and hard to cover in a generic formal framework:

1. There is no implementation of Read , Gen and Chk (or their combination) that is more efficient (with respect to time and/or memory) than the implementation used by the honest prover in state S .
2. It is not possible to execute Read , Gen and Chk only partially, e.g., by omitting some of the underlying instructions.

⁵ In practice the delay for submitting and receiving messages must be considered. However, as this delay should be small compared to the runtime of the protocol, we ignore this aspect.

We formally cover these assumptions by modelling **Read**, **Gen** and **Chk** as oracles. That is, whenever \mathcal{P} wants, e.g., to execute $\text{Read}(\text{State}(\mathcal{P}), a)$, \mathcal{P} sends a to the **Read**-oracle and receives the corresponding result s . While sending and receiving messages between \mathcal{P} and the oracles are modelled to take no time, the *determination* of the response does. More precisely when \mathcal{P} invokes one of these oracles, it takes a certain amount of time before \mathcal{P} gets the result. Within this time period \mathcal{P} is inactive and cannot perform any computations. We denote the response time of the **Read**, **Gen** and **Chk**-oracle by δ_{Read} , δ_{Gen} and δ_{Chk} , respectively. Moreover the inputs of the oracles need to be stored in the primary memory of \mathcal{P} .

Remark 7 (Order of Computations). A consequence of this modelling approach is that a malicious prover $\tilde{\mathcal{P}}$ can compute the outputs of **Gen** and **Chk** only in the right order. For instance, before $\tilde{\mathcal{P}}$ can determine s_i it must first determine s_{i-1} . Given that concrete instantiations of the generic scheme are iteratively executed, the limited size of the primary memory (PM) (see below) and the fact that accessing the secondary memory requires significantly more time than accessing PM, we consider this assumption to be reasonable for most practical instantiations.

System-Level Properties. The size and utilization of the primary memory (PM) plays a fundamental role for assessing the optimality of a software attestation protocol with regard to the resources used by a prover $\tilde{\mathcal{P}}$. Therefore, we assume that the size of PM is limited to the minimum size required by an honest prover. For example if an honest prover $\mathcal{P} = \mathcal{P}^{\mathcal{O}}$ is expected to access only an oracle \mathcal{O} that accepts inputs in Σ^ℓ , we assume that $\tilde{\mathcal{P}}$ can at most store ℓ elements of Σ in its PM at the same time.⁶

Another crucial assumption for any software attestation scheme not explicitly made in most previous works is that the state S should not be compressible into PM. For instance, consider the extreme case where all entries of S contain the same value s . In this case a malicious prover $\tilde{\mathcal{P}}$ could easily determine the correct attestation response by simply storing s in PM while having a different state $\text{State}(\tilde{\mathcal{P}}) \neq S$. Hence, we require that $\tilde{\mathcal{P}}$ should not be able to determine a randomly selected entry of S without accessing the secondary memory with better probability than guessing:

Definition 3 (State Incompressibility). For a state S and for any state entry $x \in \Sigma$ we denote with \mathbb{D}_S the probability distribution $\mathbb{D}_S(x) := \Pr \left[x = s | a \stackrel{\cup}{\leftarrow} \{0, 1\}^{\ell_a} \wedge s := \text{Read}(S, a) \right]$. S is called incompressible if for any algorithm Alg that can be executed by the prover \mathcal{P} and that does not invoke **Read**, i.e., $\text{Alg} = \text{Alg}^{\widehat{\text{Read}}}$, it holds that

$$\Pr \left[\tilde{s} = s | a \stackrel{\cup}{\leftarrow} \{0, 1\}^{\ell_a} \wedge s = \text{Read}(S, a) \wedge \tilde{s} \leftarrow \text{Alg}(a) \wedge \text{Time}_{\mathcal{P}}(\text{Alg}) \leq \delta_{\text{Read}} \right] \leq \gamma = \max_{x \in \Sigma} \mathbb{D}_S(x).$$

Cryptographic Properties. Although it is quite obvious that the security of the software attestation scheme heavily depends on the cryptographic properties of **Gen** and **Chk**, these requirements have not been systematically analyzed and formally specified before. While it would be straightforward to model these functions as pseudo-random number generators (PRNGs) and hash functions (or even random oracles), respectively, there are some subtle differences to the common cryptographic scenario which must be carefully considered. As we elaborate below, this yields a property of **Gen** which is *stronger* than the common security definition of cryptographic PRNGs while for **Chk** a significantly *weaker* condition than the classical security properties of hash functions is sufficient.

⁶ This aspect will become more clear in the security proof.

Pseudo-Randomness of the Outputs of Gen. To prevent a malicious prover $\tilde{\mathcal{P}}$ from using pre-computed forged attestation responses, the memory addresses a_i generated by **Gen** should be “sufficiently random”. Ideally, all possible address combinations should be possible for (a_1, \dots, a_N) . While this is impossible from an information-theoretic point of view, the best one may ask for is that the memory addresses a_i generated by **Gen** should be computationally indistinguishable from uniformly random values within a certain time-bound t :

Definition 4 (Time-Bounded Pseudo-randomness of Gen). $\text{Gen} : \{0, 1\}^{l_g} \rightarrow \{0, 1\}^{l_g + l_a}$ is called (t, ϱ) -pseudo-random if for any algorithm **Alg** that can be executed by \mathcal{P} in $\text{Time}(\text{Alg}) \leq t$ it holds that

$$\left| \Pr \left[b = 1 \mid g_0 \stackrel{\mathbb{U}}{\leftarrow} \{0, 1\}^{l_g} \wedge (g_{i+1}, a_{i+1}) \leftarrow \text{Gen}(g_i) : i = 0, \dots, N-1 \wedge b \leftarrow \text{Alg}(a_1, \dots, a_N) \right] \right. \\ \left. - \Pr \left[b = 1 \mid a_i \stackrel{\mathbb{U}}{\leftarrow} \{0, 1\}^{l_a} : i = 1, \dots, N \wedge b \leftarrow \text{Alg}(a_1, \dots, a_N) \right] \right| \leq \varrho.$$

Observe that this definition requires that **Alg** does not know the seed g_0 of **Gen**, which is *not* given in the generic software attestation scheme. In principle nothing prevents a malicious prover $\tilde{\mathcal{P}}$ from using g_0 to compute the addresses (a_1, \dots, a_N) on its own, making them easily distinguishable from random values. The best we can do is to require that $\tilde{\mathcal{P}}$ cannot derive any meaningful information about a_{i+1} from g_i without investing a certain minimum amount of time. Specifically, we assume that an algorithm with input g that does not execute **Gen** cannot distinguish $(g', a') = \text{Gen}(g)$ from uniformly random values. Formally:

Definition 5 (Time-Bounded Unpredictability of Gen). $\text{Gen} : \{0, 1\}^{l_g} \rightarrow \{0, 1\}^{l_g} \times \{0, 1\}^{l_a}$ is ν_{Gen} -unpredictable if for any algorithm $\text{Alg}^{\widehat{\text{Gen}}}$ that can be executed by \mathcal{P} and that does not execute **Gen**, it holds that

$$\left| \Pr \left[b = 1 \mid g \stackrel{\mathbb{U}}{\leftarrow} \{0, 1\}^{l_g} \wedge (g', a') \leftarrow \text{Gen}(g) \wedge b \leftarrow \text{Alg}(g, g', a') \right] \right. \\ \left. - \Pr \left[b = 1 \mid g \stackrel{\mathbb{U}}{\leftarrow} \{0, 1\}^{l_g} \wedge (g', a') \stackrel{\mathbb{U}}{\leftarrow} \{0, 1\}^{l_g} \times \{0, 1\}^{l_a} \wedge b \leftarrow \text{Alg}(g, g', a') \right] \right| \leq \nu_{\text{Gen}},$$

where $\text{Alg}^{\widehat{\text{Gen}}}$ has been abbreviated to **Alg**.

Weakened Pre-image Resistance of Chk. The purpose of the compression function Chk^N is to map the state S of the prover \mathcal{P} to a smaller attestation response r_N , which reduces the amount of data to be sent from \mathcal{P} to the verifier \mathcal{V} . Observe that the output of Chk^N depends also on the challenge sent by the verifier to avoid simple replay attacks and the pre-computation of attestation responses. A necessary security requirement on **Chk** is that it should be hard for a malicious prover $\tilde{\mathcal{P}}$ to replace the correct input $\mathbf{s} = (s_1, \dots, s_N)$ to **Chk** with some other value $\tilde{\mathbf{x}} \neq \mathbf{s}$ that yields the same attestation response r_N as \mathbf{s} . This is similar to the common notion of *second pre-image resistance* of cryptographic hash functions. However, due to the time-bound of the software attestation scheme it is sufficient that Chk^N fulfills only a much weaker form of second pre-image resistance since we need to consider only “blind” adversaries who (in contrast to the classical definition of second pre-image resistance) do not know the correct response r_N to the verifier’s challenge (g_0, r_0) . The reason is that, as soon as \mathcal{P} knows the correct response r_N , he could send it to \mathcal{V} and would not bother to determine a second pre-image. Hence, we introduce the definition of blind second pre-image resistance which concerns algorithms that are given only part of the input \mathbf{s} of Chk^N and that have to determine the correct output of $\text{Chk}^N(r_0, \mathbf{s})$:

Definition 6 (Blind Second Pre-image Resistance). $\text{Chk} : \{0, 1\}^{l_r} \times \Sigma \rightarrow \{0, 1\}^{l_r}$ is ω -blind second pre-image resistant with respect to the distribution \mathbb{D}_S (cf. [Definition 3](#)) if for any $N \in \mathbb{N}$, any subset of indices $J \subsetneq \{1, \dots, N\}$ and for any algorithm Alg that can be executed by \mathcal{P} , it holds that

$$\Pr \left[\tilde{r} = r \mid r_0 \stackrel{\cup}{\leftarrow} \{0, 1\}^{l_r} \wedge \bigwedge_{i=1, \dots, N} \left(s_i \stackrel{\mathbb{D}_S}{\leftarrow} \Sigma \right) \wedge \tilde{r} \leftarrow \text{Alg}(r_0, (s_j)_{j \in J}) \wedge r \leftarrow \text{Chk}^N(r_0, s_1, \dots, s_N) \right] \leq \omega.$$

In addition we also require (similar to [Definition 5](#)) that $\tilde{\mathcal{P}}$ cannot determine any useful information about $r_N = \text{Chk}^N(r_0, s_1, \dots, s_N)$ without executing Chk^N :

Definition 7 (Unpredictability of Chk^N). $\text{Chk} : \{0, 1\}^{l_r} \times \Sigma \rightarrow \{0, 1\}^{l_r}$ is ν_{Chk} -unpredictable with respect to the distribution \mathbb{D}_S if for any algorithm $\text{Alg}^{\widehat{\text{Chk}^N}}$ that can be executed by \mathcal{P} and that does not execute Chk^N , it holds that

$$\left| \Pr \left[b = 1 \mid r_0 \stackrel{\cup}{\leftarrow} \{0, 1\}^{l_r} \wedge s_i \stackrel{\mathbb{D}_S}{\leftarrow} \Sigma : i \in \{1, \dots, N\} \wedge r = \text{Chk}^N(r_0, s_1, \dots, s_N) \wedge b \leftarrow \text{Alg}(r_0, s_1, \dots, s_N, r) \right] - \Pr \left[b = 1 \mid r_0 \stackrel{\cup}{\leftarrow} \{0, 1\}^{l_r} \wedge s_i \stackrel{\mathbb{D}_S}{\leftarrow} \Sigma : i \in \{1, \dots, N\} \wedge r \stackrel{\cup}{\leftarrow} \{0, 1\}^{l_r} \wedge b \leftarrow \text{Alg}(r_0, s_1, \dots, s_N, r) \right] \right| \leq \nu_{\text{Chk}},$$

where $\text{Alg}^{\widehat{\text{Chk}^N}}$ has been abbreviated to Alg .

6 Security of the Scheme

In this section we derive an upper bound for the success probability of a malicious prover $\tilde{\mathcal{P}}$ to make the verifier \mathcal{V} accept. This bound depends on the parameters defined in [Section 5.2](#) which provide a sufficient condition to prove the generic attestation scheme secure. The bound is as follows:

Theorem 1 (Generic Upper Bound). Let S be an incompressible state ([Definition 3](#)). Consider the generic attestation scheme in [Figure 1](#) with the components Read , Gen and Chk such that

1. Gen is $(N(\delta_{\text{Gen}} + \delta_{\text{Read}}), \varrho)$ -pseudo-random ([Definition 4](#)) and ν_{Gen} -unpredictable ([Definition 5](#)),
2. Chk is ω -blind second pre-image resistant ([Definition 6](#)) and ν_{Chk} -unpredictable ([Definition 7](#)).

Consider an arbitrary prover $\tilde{\mathcal{P}}$ as in [Section 3](#) with state $\text{State}(\tilde{\mathcal{P}}) = \tilde{S}$ that can store \mathfrak{p} memory words in its primary memory and \mathfrak{s} memory words in its secondary memory (cf. [Section 3](#)). Let

$$\lambda := 1 - d_H(S, \tilde{S}) = \left| \left\{ a \in \{0, 1\}^{l_a} \mid \text{Read}(\tilde{S}, a) = \text{Read}(S, a) \right\} \right| \cdot 2^{-l_a},$$

where $d_H(S, \tilde{S})$ denotes the fractional Hamming distance between S and \tilde{S} , i.e., the fraction of state entries that are different in S and \tilde{S} . Then the probability of $\tilde{\mathcal{P}}$ to win the security experiment $\text{Exp}_{\text{Attest}}^A$ ([Definition 2](#)), i.e., $\Pr [\text{Exp}_{\text{Attest}}^A(S, l) = \text{ACCEPT}]$ with $l := (l_g, l_r)$, is upper bounded by

$$\left. \begin{aligned} & \frac{\mathfrak{p} + \mathfrak{s}}{l_s / l_r} \cdot 2^{-(l_g + l_r)} + \max \{ \omega, \nu_{\text{Chk}} \} + \\ & \max_{0 \leq M \leq N} \left(\pi(M, \text{ops}(\delta_{\text{Read}} + \delta_{\text{Gen}})) + \varrho \right) \cdot \gamma^{N-M} + \nu_{\text{Gen}} \cdot (N - M) \end{aligned} \right\} \quad (1)$$

where

$$\pi(n, x) := \sum_{j=\max\{0, n-2^{l_a}\}}^{n-1} \left(\max \{ \lambda^{x+1}, \gamma \} \right)^{\frac{n}{x+1}-j} \cdot \binom{n}{j} \cdot \left(\prod_{i=0}^{n-j} \frac{2^{l_a} - i}{2^{l_a}} \right) \cdot \left(\frac{n-j}{2^{l_a}} \right)^j \quad (2)$$

and $\text{ops}(\delta_{\text{Read}} + \delta_{\text{Gen}})$ denotes the number of instructions $\tilde{\mathcal{P}}$ can execute in time $\delta_{\text{Read}} + \delta_{\text{Gen}}$.

Remark 8. This result implies that a software attestation scheme is ε -secure if the expression in [Equation 1](#) is $\leq \varepsilon$, yielding a sufficient condition for security. Note that the bound given in [Equation 1](#) emphasizes the impact of the distribution of the state entries in S (expressed by γ) and the similarity between the state S and the state \tilde{S} of the prover (expressed by λ) on the security of the scheme. Both aspects have been either neglected or have been considered only informally in previous work (cf. [Section 7](#)).

Proof of [Theorem 1](#). Let **Win** denote the event that a malicious prover $\tilde{\mathcal{P}}$ wins the security experiment $\mathbf{Exp}_{\text{Attest}}^A$, i.e., **Win** means that $\mathbf{Exp}_{\text{Attest}}^A(S, l) = \text{ACCEPT}$. We are interested in an upper bound for $\Pr[\text{Win}]$. To this end we consider several sub-cases. Let **Precomp** denote the event that the verifier \mathcal{V} sends a challenge (g_0, r_0) to $\tilde{\mathcal{P}}$ for which $\tilde{\mathcal{P}}$ has precomputed and stored the correct response r_N in its memory (primary and/or secondary).⁷ Then we have

$$\begin{aligned} \Pr[\text{Win}] &= \Pr[\text{Win}|\text{Precomp}] \cdot \Pr[\text{Precomp}] + \Pr[\text{Win}|\neg\text{Precomp}] \cdot \Pr[\neg\text{Precomp}] \\ &\leq \Pr[\text{Precomp}] + \Pr[\text{Win}|\neg\text{Precomp}]. \end{aligned}$$

The maximum number of responses $\tilde{\mathcal{P}}$ can store in its memory is $\frac{p+s}{l_s/l_r}$. Since the challenge $(g_0, r_0) \in \{0, 1\}^{l_g+l_r}$ is uniformly sampled, it follows that $\Pr[\text{Precomp}] = \frac{p+s}{l_s/l_r} \cdot 2^{-(l_g+l_r)}$.

We now estimate the term $\Pr[\text{Win}|\neg\text{Precomp}]$, which we abbreviate to $\overline{\Pr}[\text{Win}]$. Let **Correct** denote the event that $\tilde{\mathcal{P}}$ determined all state entries (s_1, \dots, s_N) , i.e., $s_i = \text{Read}(S, a_i)$ and $(g_i, a_i) = \text{Gen}(g_{i-1})$ for $i \in \{1, \dots, N\}$ and that $\tilde{\mathcal{P}}$ has executed Chk^N . Then we have

$$\overline{\Pr}[\text{Win}] \leq \overline{\Pr}[\text{Correct}] + \overline{\Pr}[\text{Win}|\neg\text{Correct}].$$

It follows from the fact that Chk^N is ω -blind second pre-image resistant ([Definition 6](#)) and ν_{Chk} -unpredictable ([Definition 7](#)) that $\overline{\Pr}[\text{Win}|\neg\text{Correct}] \leq \max\{\omega, \nu_{\text{Chk}}\}$.

For the final term $\overline{\Pr}[\text{Correct}]$, we use the following claim, which we prove afterwards.

Claim 1. *The probability $\overline{\Pr}[\text{Correct}]$ that $\tilde{\mathcal{P}}$ determines all state entries (s_1, \dots, s_N) and computes $r_N = \text{Chk}^N(r_0, s_1, \dots, s_N)$ in the security experiment $\mathbf{Exp}_{\text{Attest}}^A$ under the assumption that the response to the requested challenge has not been precomputed is upper bounded by*

$$\max_{0 \leq M \leq N} (\pi(M, \text{ops}(\delta_{\text{Read}} + \delta_{\text{Gen}})) + \varrho) \cdot \gamma^{N-M} + \nu_{\text{Gen}} \cdot (N - M)$$

where $\pi(N, x)$ and $\text{ops}(\delta_{\text{Read}} + \delta_{\text{Gen}})$ are defined as explained in [Theorem 1](#).

Taking these bounds together concludes the proof. \square

Proof of [Claim 1](#)

We now prove [Claim 1](#) used in the proof of [Theorem 1](#). That is we show the claimed upper bound of $\overline{\Pr}[\text{Correct}]$, which is the probability that a malicious prover $\tilde{\mathcal{P}}$ with state $\tilde{S} := \text{State}(\tilde{\mathcal{P}}) \neq S$ correctly determines all state entries (s_1, \dots, s_N) in the security experiment $\mathbf{Exp}_{\text{Attest}}^A$ ([Definition 2](#)) under the assumption that the response for the requested challenge has not been precomputed.

Observe that $\tilde{\mathcal{P}}$ may decide to deviate from the protocol specification. For example, $\tilde{\mathcal{P}}$ may skip some instructions with respect to one round i (probably accepting a lower success probability for determining s_i) to save some time that could be spent on the determination of another state entry

⁷ More precisely, \mathcal{A} has precomputed this value during the preparation phase and stored the response as part of \tilde{S} .

s_j with $i \neq j$ (probably aiming for a higher probability to determine s_j). Hence the challenge is to show that for *any* of these approaches the success probability does not exceed a certain (non-trivial) bound, which cannot be done by a reduction to a single assumption.

We base our proof on a sequence of games played by $\tilde{\mathcal{P}}$ and an oracle \mathcal{O} that has access to S . All these games are divided into two phases: A *setup phase* and a *challenge phase*. In the setup phase \mathcal{O} generates all addresses (a_1, \dots, a_N) and determines the corresponding state entries $s_i = \text{Read}(S, a_i)$. Afterwards, in the challenge phase, $\tilde{\mathcal{P}}$ and \mathcal{O} exchange several messages. In particular $\tilde{\mathcal{P}}$ must submit its guesses \tilde{x}_i for the state entries s_i to \mathcal{O} . $\tilde{\mathcal{P}}$ wins the game only if all guesses are correct, i.e., $\tilde{x}_i = s_i$ for all $i = 1, \dots, N$.

The differences between the games lie in the possibilities of $\tilde{\mathcal{P}}$ to deviate from the protocol specification. While these possibilities are quite limited in the first game (Game 0), $\tilde{\mathcal{P}}$ gets more and more control with each subsequent game and thus can to perform more powerful attacks. For each transformation between two consecutive games, we show how the success probability of $\tilde{\mathcal{P}}$ changes. In most cases it turns out that the previous game represents a subset of the possible attack strategies of the current game. Note that \mathcal{O} formally represents the honest execution of certain parts of the protocol and should not be confused with a real party. Consequently, we assume that transferring messages between $\tilde{\mathcal{P}}$ and \mathcal{O} takes no time.

Observe that the intention of \mathcal{O} is to have an elegant method for ignoring all computations of $\tilde{\mathcal{P}}$ which are honestly executed by assumption. Hence to exclude artificial attacks where $\tilde{\mathcal{P}}$ uses the time and/or memory gained by outsourcing the computation to \mathcal{O} , we restrict the time-bound and the size of the primary memory of $\tilde{\mathcal{P}}$ to what is necessary for honestly executing those computations that are *note* outsourced to \mathcal{O} .

Game 0: Randomly Sampling Addresses in Regular Time Intervals

Game Description. The purpose of this game is to investigate provers $\tilde{\mathcal{P}}$ which (1) do not exploit any aspects related to the execution of **Gen** and (2) that are forced to use exactly time δ_{Read} for the determination of each state entry s_i . This is captured by modelling the game as follows: Within the setup phase, \mathcal{O} samples pairwise independent and uniform addresses (a_1, \dots, a_N) and sets $s_i := \text{Read}(S, a_i)$ for all $i \in \{1, \dots, N\}$. In the challenge phase, \mathcal{O} iteratively queries $\tilde{\mathcal{P}}$ with a_i and $\tilde{\mathcal{P}}$ returns some response \tilde{x}_i .

Hereby, $\tilde{\mathcal{P}}$ can access the **Read** oracle, which on input a returns $s = \text{Read}(\tilde{S}, a)$ after time δ_{Read} . Since this is the only operation expected from an honest prover, the size of the primary memory only allows to store an address a and a state entry s . Moreover the total time-bound is limited to $N \cdot \delta_{\text{Read}}$, meaning that $\tilde{\mathcal{P}}$ automatically fails if it needs more time in total than this bound.

Observe that \mathcal{O} ensures that $\tilde{\mathcal{P}}$ cannot change the order of the memory addresses, i.e., \mathcal{O} only sends a_i to $\tilde{\mathcal{P}}$ after a_{i-1} has been sent.⁸ We denote with *round* i the time-frame between the point in time where $\tilde{\mathcal{P}}$ receives a_i and the point in time where $\tilde{\mathcal{P}}$ receives a_{i+1} for $i \in \{1, \dots, N-1\}$. With *round* N we denote the time-frame between the point in time where $\tilde{\mathcal{P}}$ receives a_N and the point in time where $\tilde{\mathcal{P}}$ sends the last protocol message \tilde{x}_N to \mathcal{O} . $\tilde{\mathcal{P}}$ wins the game if (1) $\tilde{x}_i = s_i$ for all $i \in \{1, \dots, N\}$ and (2) each round took at most time δ_{Read} . Otherwise $\tilde{\mathcal{P}}$ loses the game.

Success Probability. We are interested in an upper bound for the probability $\Pr[\text{Win}_0]$ that $\tilde{\mathcal{P}}$ wins Game 0. Since $\tilde{\mathcal{P}}$ loses for sure when he uses more time than δ_{Read} to respond to a_i in at least one round i , it is sufficient to restrict to provers that take at most time δ_{Read} in each round. To this end,

⁸ This is a consequence of [Remark 7](#).

we derive an upper bound which allows to treat the individual rounds separately. We start with the final round N and distinguish between two cases.

In Case 1 the response \tilde{x}_N is the direct result of a query to the `Read` oracle, i.e., $\tilde{x}_N = \text{Read}(\tilde{S}, a)$ for some address a . If $a = a_N$ the probability of $\tilde{x}_N := \text{Read}(\tilde{S}, a_N) = s_N := \text{Read}(S, a_N)$ is λ (cf. [Theorem 1](#)) since a_N is sampled uniformly and independently from the previous addresses. Now consider that $a \neq a_N$. Since $\tilde{x}_N = \text{Read}(\tilde{S}, a)$ and due to the fact that $\tilde{\mathcal{P}}$ must respond with \tilde{x}_N in time δ_{Read} after receiving a_N , $\tilde{\mathcal{P}}$ has no time left to perform any other instructions than `Read` during round N . In particular a could not be chosen in dependence of a_N , hence being independent of a_N . Then $\tilde{x}_N = s_N$ happens with probability of at most γ (cf. [Definition 3](#)). It follows that in Case 1 the probability $\Pr[\text{Win}_0]$ is upper bounded by $\max\{\lambda, \gamma\} \cdot \Pr[\tilde{x}_1 = \text{Read}(S, a_1) \wedge \dots \wedge \tilde{x}_{N-1} = \text{Read}(S, a_{N-1})]$.

Next we consider Case 2, where \tilde{x}_N is not the result of a query to the `Read` oracle. It follows from the incompressibility of S ([Definition 3](#)) and the fact that a_N has been sampled uniformly and independent of all previous addresses a_i with $i < N$, that the probability of $\tilde{x}_N = \text{Read}(S, a_N)$ is upper bounded by γ . Hence, $\gamma \cdot \Pr[\tilde{x}_1 = \text{Read}(S, a_1) \wedge \dots \wedge \tilde{x}_{N-1} = \text{Read}(S, a_{N-1})]$ is an upper bound of $\Pr[\text{Win}_0]$ in Case 2. It follows from Cases 1 and 2 that $\Pr[\text{Win}_0] \leq \max\{\lambda, \gamma\} \cdot \Pr[\tilde{x}_1 = \text{Read}(S, a_1) \wedge \dots \wedge \tilde{x}_{N-1} = \text{Read}(S, a_{N-1})]$ and by induction $\Pr[\text{Win}_0] \leq \pi_0 = \pi_0(N) := (\max\{\lambda, \gamma\})^N$.

Game 1: Prover Controls the Address Generation Time

Game Description. In this game we increase the power of the malicious prover $\tilde{\mathcal{P}}$ and allow him to freely choose how much time he devotes to determine each value s_i , as long as the total time for determining (s_1, \dots, s_N) does not exceed $N \cdot \delta_{\text{Read}}$. This reflects the fact that in the attestation protocol a malicious prover $\tilde{\mathcal{P}}$ may generate the memory addresses (a_1, \dots, a_N) on its own whenever it wants to.

Formally, this is captured by introducing a *req* protocol message which $\tilde{\mathcal{P}}$ needs to send to \mathcal{O} for receiving the next address a_i during the challenge phase. More precisely, \mathcal{O} sends a_i to $\tilde{\mathcal{P}}$ only when $\tilde{\mathcal{P}}$ sent the i -th request *req* to \mathcal{O} .

Since each round may take a different time period, the winning conditions are relaxed by replacing the time restriction on the individual rounds by an overall time-bound for the entire challenge phase. This means that $\tilde{\mathcal{P}}$ wins Game 1 if (1) $\tilde{x}_i = s_i$ for all $i \in \{1, \dots, N\}$ and (2) the duration of the challenge phase does not exceed the time $N \cdot \delta_{\text{Read}}$. The size of the primary memory remains as in Game 0.

Success Probability. We now upper bound the probability $\Pr[\text{Win}_1]$ that $\tilde{\mathcal{P}}$ wins Game 1. To this end, we divide the number of rounds into four distinct sets. Let N_{coll} denote the number of rounds where the address sampled by \mathcal{O} is equal to an address of some previous round by coincidence, i.e., $N_{\text{coll}} := |\{i \in \{2, \dots, N\} \mid \exists j \in \{1, \dots, i-1\} : a_i = a_j\}|$. With respect to the remaining $N - N_{\text{coll}}$ rounds, let N_{equal} (resp. N_{more} , resp. N_{less}) be the number of rounds where $\tilde{\mathcal{P}}$ responds in time equal (resp. more, resp. less) than δ_{Read} . Thus we have $N = N_{\text{coll}} + N_{\text{equal}} + N_{\text{less}} + N_{\text{more}}$.

Let $\text{Coll}(N_{\text{coll}})$ denote the event that exactly N_{coll} of the N addresses are equal to some previous addresses. This implies that in $N - N_{\text{coll}}$ rounds pairwise different addresses are sampled. Moreover, since there are only 2^{l_a} different addresses, $N - N_{\text{coll}}$ is upper bound by 2^{l_a} . It follows that $N - N_{\text{coll}} \leq 2^{l_a} \Leftrightarrow N_{\text{coll}} \geq N - 2^{l_a}$. Thus it must hold that $N_{\text{coll}} \geq \max\{0, N - 2^{l_a}\}$ and we have

$$\Pr[\text{Win}_1] = \sum_{N_{\text{coll}} = \max\{0, N - 2^{l_a}\}}^{N-1} \Pr[\text{Win}_1 | \text{Coll}(N_{\text{coll}})] \cdot \Pr[\text{Coll}(N_{\text{coll}})].$$

We now derive upper bounds for $\Pr[\text{Win}_1|\text{Coll}(N_{\text{coll}})]$ and $\Pr[\text{Coll}(N_{\text{coll}})]$.

In general, $\Pr[\text{Coll}(N_{\text{coll}})]$ can be expressed by (number combinations of rounds with equal addresses) \times (probability that addresses in $N - N_{\text{coll}}$ rounds are pairwise different) \times (probability that addresses in the remaining rounds are equal to some previous address). The first term is at most $\binom{N}{N_{\text{coll}}}$ while an upper bound for the last term is $\left(\frac{N - N_{\text{coll}}}{2^{l_a}}\right)^{N_{\text{coll}}}$. This gives (for $\max\{0, N - 2^{l_a}\} \leq N_{\text{coll}} \leq N - 1$)

$$\Pr[\text{Coll}(N_{\text{coll}})] \leq \binom{N}{N_{\text{coll}}} \cdot \left(\prod_{i=0}^{N - N_{\text{coll}}} \frac{2^{l_a} - i}{2^{l_a}}\right) \cdot \left(\frac{N - N_{\text{coll}}}{2^{l_a}}\right)^{N_{\text{coll}}}. \quad (3)$$

We now fix a value for N_{coll} and aim for an upper bound for $\Pr[\text{Win}_1|\text{Coll}(N_{\text{coll}})]$. We do so by giving separate upper bounds on the success probability for the four different types of rounds. Let $\text{ops} = \text{ops}(\delta_{\text{Read}})$ be the number of instructions that can be executed by the computing engine of $\tilde{\mathcal{P}}$ in time δ_{Read} . Since we are interested in an *upper bound* of $\tilde{\mathcal{P}}$'s success probability, we make several assumptions in favor of $\tilde{\mathcal{P}}$: (1) For rounds where $\tilde{\mathcal{P}}$ invested more time than δ_{Read} , we use the trivial upper bound of 1 even if the time period exceeded δ_{Read} only by the time required to execute one single instruction. (2) For rounds where the requested address coincides with an address previously asked, we likewise use the bound of 1. Moreover we assume that these rounds take *no* time at all and the ops instructions saved can be used in ops other rounds. (3) In rounds that take less time than δ_{Read} , it follows from the incompressibility of S (Definition 3) and the fact that all addresses are pairwise distinct that $\tilde{x}_i = s_i$ with probability $\leq \gamma$. Again, we assume that these rounds take *no* time at all and that the ops instructions saved can be used in ops other rounds. (4) In a round that takes exactly time δ_{Read} $\tilde{\mathcal{P}}$ succeeds at most with probability $\max\{\lambda, \underline{\gamma}\}$ (cf. Game 0).

While these assumptions strongly exaggerate the possibilities of $\tilde{\mathcal{P}}$, they allow to identify optimum strategies. More precisely for each round where $\tilde{\mathcal{P}}$ uses less time than δ_{Read} or where a previously asked address is requested again, the best approach is to spend the ops saved instructions in ops other rounds such that for each of these rounds the probability of correctly determining s_i is equal to 1. It follows that $N_{\text{more}} = \text{ops} \cdot (N_{\text{coll}} + N_{\text{less}})$ and hence $N = N_{\text{coll}} + N_{\text{equal}} + N_{\text{less}} + N_{\text{more}} = N_{\text{equal}} + (\text{ops} + 1) \cdot (N_{\text{coll}} + N_{\text{less}})$. Hence, we have

$$\begin{aligned} \Pr[\text{Win}_1|\text{Coll}(N_{\text{coll}})] &\leq \pi_0(N_{\text{equal}}) \cdot \gamma^{N_{\text{less}}} \cdot 1^{N_{\text{coll}} + N_{\text{more}}} \\ &= \max_{N_{\text{less}}} \left\{ \lambda^{N - (\text{ops} + 1) \cdot (N_{\text{coll}} + N_{\text{less}})} \cdot \gamma^{N_{\text{less}}}, \gamma^{N - (\text{ops} + 1) \cdot N_{\text{coll}} - \text{ops} \cdot N_{\text{less}}} \right\} \\ &\stackrel{\text{cf. Apx. A}}{=} \left(\max \left\{ \lambda^{\text{ops}(\delta_{\text{Read}}) + 1}, \gamma \right\} \right)^{\frac{N}{\text{ops}(\delta_{\text{Read}}) + 1} - N_{\text{coll}}}. \end{aligned} \quad (4)$$

We get the following upper bound $\Pr[\text{Win}_1] \leq \pi(N, \text{ops}(\delta_{\text{Read}}))$ where $\pi(n, x)$ is defined as in Equation 2. Observe that for any fixed value for N_{coll} , the probability of having N_{coll} collisions (Equation 3) increases with N (as long as $N_{\text{coll}} \geq \max\{0, N - 2^{l_a}\}$) while the probability to determine the values (s_1, \dots, s_N) (Equation 4) decreases for N .

Game 2: Skipping Address Generation

Game Description. So far we covered only provers $\tilde{\mathcal{P}}$ that honestly generate all addresses (a_1, \dots, a_N) . Now we change the game such that $\tilde{\mathcal{P}}$ may decide in each round i to skip the generation of address a_i . This allows $\tilde{\mathcal{P}}$ to “buy” more time for determining the values s_i but at the “cost” of not knowing a_i . Formally this is captured by defining a second message *skip* besides *req*. Specifically, in each

round i of the challenge phase, $\tilde{\mathcal{P}}$ either sends *req* or *skip*. In case of *req*, \mathcal{O} behaves as in Game 1 and sends the next a_i to $\tilde{\mathcal{P}}$. However, when $\tilde{\mathcal{P}}$ sends *skip* then \mathcal{O} does *not* send a_i to $\tilde{\mathcal{P}}$ and extends the time-bound by δ_{Gen} . That is, at the beginning of the challenge phase, the winning conditions are that (1) all responses $(\tilde{x}_1, \dots, \tilde{x}_N)$ of $\tilde{\mathcal{P}}$ are correct, i.e., $\tilde{x}_i = s_i \forall i \in \{1, \dots, N\}$ and (2) the challenge phase does not take more time than $N \cdot \delta_{\text{Read}}$. However each time $\tilde{\mathcal{P}}$ sends a *skip* message to \mathcal{O} , the time-bound is extended by δ_{Gen} .

Success Probability. We now determine the probability $\Pr[\text{Win}_2]$ that $\tilde{\mathcal{P}}$ wins Game 2. To this end we follow the same line of arguments as in Game 1. The only difference is that rounds where collisions in the addresses took place or where either **Read** or **Gen** have been skipped take no time at all and free $\text{ops}(\delta_{\text{Read}} + \delta_{\text{Gen}})$ instructions for other rounds. That is we get a bound with the same structure as in Game 1 but where $\text{ops}(\delta_{\text{Read}})$ is replaced by $\text{ops}(\delta_{\text{Read}} + \delta_{\text{Gen}})$, i.e., $\Pr[\text{Win}_2] \leq \pi(N, \text{ops}(\delta_{\text{Read}} + \delta_{\text{Gen}}))$.

Game 3: Replacing the Random Sampling with Gen

Game Description. Now we consider a variant of Game 2 with the only difference being that the addresses (a_1, \dots, a_N) are generated by **Gen** instead of being randomly sampled by \mathcal{O} . That is, during the setup phase \mathcal{O} randomly samples g_0 and generates (a_1, \dots, a_N) using **Gen**.

Success Probability. Let $\Pr[\text{Win}_3]$ be the probability that $\tilde{\mathcal{P}}$ wins Game 3. Using a standard argument, it follows from the pseudo-randomness of the outputs of **Gen** (Definition 4) that $|\Pr[\text{Win}_3] - \Pr[\text{Win}_2]| \leq \varrho$ and hence $\Pr[\text{Win}_3] \leq \Pr[\text{Win}_2] + \varrho \leq \pi(N, \text{ops}(\delta_{\text{Read}} + \delta_{\text{Gen}})) + \varrho$.

Game 4: Giving Access to Gen

Game Description. In the final game \mathcal{O} no longer generates (a_1, \dots, a_N) for $\tilde{\mathcal{P}}$. Instead $\tilde{\mathcal{P}}$ now queries the **Gen** oracle, which on input g_i returns $(g_i, a_i) = \text{Gen}(g_{i-1})$ after time δ_{Gen} . To this end, \mathcal{O} samples g_0 in the setup phase and gives this value to $\tilde{\mathcal{P}}$.

Observe that the size of the primary memory of $\tilde{\mathcal{P}}$ is increased to additionally store a value g . Further, the time-bound of the challenge phase is increased to $N \cdot (\delta_{\text{Gen}} + \delta_{\text{Read}})$.

Success Probability. The only difference between Game 4 and Game 3 is that $\tilde{\mathcal{P}}$ now knows g_0 and can query the **Gen** oracle. Recall that g_0 is used by **Gen** for computing (a_1, \dots, a_N) . Hence $\tilde{\mathcal{P}}$ may decide to skip the generation of one or more addresses and save the time and memory for other computations. However, since **Gen** is assumed to be ν_{Gen} -unpredictable (Definition 5), $\tilde{\mathcal{P}}$ cannot derive any information on a_{i+1} or g_{i+1} from g_i without querying **Gen**. Thus if $\tilde{\mathcal{P}}$ never queries **Gen** with some value g_i it cannot distinguish the subsequent values (g_{i+1}, \dots, g_N) with a probability better than $(N - i) \cdot \nu_{\text{Gen}}$. Therefore we can restrict to provers that compute $(a_1, g_1), \dots, (a_M, g_M)$ and skip $(a_{M+1}, g_{M+1}), \dots, (a_N, g_N)$.

Let $\Pr[\text{Win}_4]$ be the probability to win Game 4 and $\Pr[\text{Win}_4(M)]$ be the probability to win Game 4 for a fixed M . That is we have $\Pr[\text{Win}_4] \leq \max_M \{\Pr[\text{Win}_4(M)]\}$. Now consider a variation of Game 4 where \mathcal{O} replaces the values $(a_{M+1}, g_{M+1}), \dots, (a_N, g_N)$ by independent and uniformly sampled values and we denote with $\Pr[\text{Win}'_4(M)]$ the probability that $\tilde{\mathcal{P}}$ wins this game. Since **Gen** is assumed to be ν_{Gen} -unpredictable (cf. Definition 5), it holds that $\Pr[\text{Win}_4(M)] \leq \Pr[\text{Win}'_4(M)] + \nu_{\text{Gen}} \cdot (N - M)$.

With respect to $\Pr[\text{Win}_4(M)]$, observe that for the first M rounds the situation is as in Game 3. Hence the success probability for the first M rounds is upper bounded by $\pi(M, \text{ops}(\delta_{\text{Read}} + \delta_{\text{Gen}})) + \varrho$.

For the remaining $N - M$ rounds, \mathcal{O} uses uniformly sampled values (a_{M+1}, \dots, a_N) that are unknown to $\tilde{\mathcal{P}}$. Hence the probability of $\tilde{\mathcal{P}}$ to derive (s_{M+1}, \dots, s_N) correctly is upper bounded by γ^{N-M} . This yields $\Pr[\text{Win}'_4(M)] \leq (\pi(M, \text{ops}(\delta_{\text{Read}} + \delta_{\text{Gen}})) + \varrho) \cdot \gamma^{N-M}$ and hence

$$\overline{\Pr}[\text{Correct}] \leq \max_{0 \leq M \leq N} \{(\pi(M, \text{ops}(\delta_{\text{Read}} + \delta_{\text{Gen}})) + \varrho) \cdot \gamma^{N-M} + \nu_{\text{Gen}} \cdot (N - M)\}.$$

7 Discussion and Conclusion

We presented the first formal security framework for software attestation and formalized various of the underlying system and design parameters. Moreover we presented a generic software attestation protocol that encompasses most existing schemes in the literature. For this generic scheme we derived an upper bound on the success probability of a malicious prover that depends on the formalized parameters.

One lesson learned is the impact of these parameters on the security of the generic scheme. For example, it was observed before that free memory should be filled with pseudo-random data [21], and a later code-compression attack [2] indicated that code redundancy also impacts security. However, the attack was dismissed as impractical [13] or ignored in later works [14,26]. In contrast, we consider the general probability distribution of the state (code and data) in Definition 3 and directly connect it to the adversary advantage. As a result, one can directly evaluate S to determine if additional measures are required for secure attestation. Our results also show that traditional cryptographic assumptions are partially too strong (second pre-image resistance) and partially too weak (pseudo-randomness).

Further, we identified new (sufficient) conditions on the core functionalities of software attestation. Moreover most previous works require the software attestation algorithm to iterate over all memory words of the secondary memory without giving any formal justification. Our bound allows to identify lower values for N (if the other parameters are known), allowing for more efficient solutions. Thus our work represents the first step towards efficient and provably secure software attestation schemes. Still, several open questions remain for future work. One being to relax the presented conditions or to derive necessary conditions. A further task is to determine concrete instantiations. While Gen and Chk could be easily realized on devices with block ciphers implemented in hardware (similar to the AES instructions in modern CPUs [27]), this becomes more challenging on other platforms.

We are currently working on the following aspects: (1) a practical instantiation of the generic software attestation scheme and its evaluation and (2) the evaluation of existing software attestation schemes in our framework.

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A Simplification of the Upper Bound of $\Pr[\text{Win}_1 | \text{Coll}(N_{\text{coll}})]$

In this section, we show how to simplify

$$\Pr[\text{Win}_1 | \text{Coll}(N_{\text{coll}})] \leq \max_{N_{\text{less}}} \left\{ \lambda^{N - (\text{ops} + 1) \cdot (N_{\text{coll}} + N_{\text{less}})} \cdot \gamma^{N_{\text{less}}}, \gamma^{N - (\text{ops} + 1) \cdot N_{\text{coll}} - \text{ops} \cdot N_{\text{less}}} \right\}.$$

Observe that $0 \leq N_{\text{less}}$ and $0 \leq N_{\text{equal}} = N - (\text{ops} + 1) \cdot (N_{\text{coll}} + N_{\text{less}}) \Leftrightarrow N_{\text{less}} \leq \frac{N}{\text{ops} + 1} - N_{\text{coll}}$, i.e.,

$$0 \leq N_{\text{less}} \leq \frac{N}{\text{ops} + 1} - N_{\text{coll}}.$$

To simplify the first term $\lambda^{N - (\text{ops} + 1) \cdot (N_{\text{coll}} + N_{\text{less}})} \cdot \gamma^{N_{\text{less}}}$, we define $e := \log_{\lambda}(\gamma)$ and rephrase the expression as $\lambda^{N - (\text{ops} + 1) \cdot N_{\text{coll}} - (\text{ops} + 1 - e) \cdot N_{\text{less}}}$. When $\text{ops} + 1 - e < 0$, the maximum value is achieved for $N_{\text{less}} = 0$, hence in this case the upper bound is $\lambda^{N - (\text{ops} + 1) \cdot N_{\text{coll}}}$. In the other case we get an upper bound for $N_{\text{less}} = \frac{N}{\text{ops} + 1} - N_{\text{coll}}$, yielding

$$\lambda^{N - (\text{ops} + 1) \cdot N_{\text{coll}} - (\text{ops} + 1 - e) \cdot \left(\frac{N}{\text{ops} + 1} - N_{\text{coll}}\right)} = \lambda^{N - (\text{ops} + 1) \cdot N_{\text{coll}} - N + e \cdot \frac{N}{\text{ops} + 1} + (\text{ops} + 1) \cdot N_{\text{coll}} - e \cdot N_{\text{coll}}} = \gamma^{\frac{N}{\text{ops} + 1} - N_{\text{coll}}}.$$

With respect to the second term, i.e., $\gamma^{N - (\text{ops} + 1) \cdot N_{\text{coll}} - \text{ops} \cdot N_{\text{less}}}$, the maximum value is achieved if N_{less} is as big as possible, i.e., $N_{\text{less}} = \frac{N}{\text{ops} + 1} - N_{\text{coll}}$. This gives an upper bound of

$$\gamma^{N - (\text{ops} + 1) \cdot N_{\text{coll}} - \text{ops} \cdot N_{\text{less}}} = \gamma^{N - (\text{ops} + 1) \cdot N_{\text{coll}} - \text{ops} \cdot \left(\frac{N}{\text{ops} + 1} - N_{\text{coll}}\right)} = \gamma^{\frac{N}{\text{ops} + 1} - N_{\text{coll}}}.$$

Altogether, it follows that

$$\Pr[\text{Win}_1 | \text{Coll}(N_{\text{coll}})] \leq \left(\max \left\{ \lambda^{\text{ops}(\delta_{\text{Read}}) + 1}, \gamma \right\} \right)^{\frac{N}{\text{ops}(\delta_{\text{Read}}) + 1} - N_{\text{coll}}}.$$