Secure and Constant Cost Public Cloud Storage Auditing with Deduplication

Jiawei Yuan University of Arkansas at Little Rock, USA Email: jxyuan@ualr.edu Shucheng Yu University of Arkansas at Little Rock, USA Email: sxyu1@ualr.edu

Abstract-Data integrity and storage efficiency are two important requirements for cloud storage. Proof of Retrievability (POR) and Proof of Data Possession (PDP) techniques assure data integrity for cloud storage. Proof of Ownership (POW) improves storage efficiency by securely removing unnecessarily duplicated data on the storage server. However, trivial combination of the two techniques, in order to achieve both data integrity and storage efficiency, results in non-trivial duplication of metadata (i.e., authentication tags), which contradicts the objectives of POW. Recent attempts to this problem introduce tremendous computational and communication costs and have been proven not secure. It calls for a new solution to support efficient and secure data integrity auditing with storage deduplication for cloud storage. In this paper we solve this open problem with a novel scheme based on techniques including polynomialbased authentication tags and homomorphic linear authenticators. Our design allows deduplication of both files and their corresponding authentication tags. Data integrity auditing and storage deduplication are achieved simultaneously. Our proposed scheme is also characterized by constant realtime communication and computational cost on the user side. Public auditing and batch auditing are both supported. Hence, our proposed scheme outperforms existing POR and PDP schemes while providing the additional functionality of deduplication. We prove the security of our proposed scheme based on the Computational Diffie-Hellman problem and the Strong Diffie-Hellman assumption. Numerical analysis and experimental results on Amazon AWS show that our scheme is efficient and scalable.

I. INTRODUCTION

Cloud storage has been increasingly prevalent because of its advantages [1]. Currently, commercial cloud storage services including Microsoft Skydrive, Amazon S3 and Google Cloud Storage have attracted millions of users. Cloud storage stands for not only the massive computing infrastructure but also the economics of scale. Under such a trend, it becomes urgent to assure the quality of data storage services which involves two frequent concerns from both cloud users and cloud service providers: data integrity and storage efficiency. On one hand, due to the many data loss and corruption events reported for those best-known cloud service providers [2], [3], [4], data owners, who are also cloud users, have the need to periodically audit the integrity of their outsourced data. On the other hand, for cloud service providers it is necessary to improve the efficiency of cloud storage to take advantage of the economics of scale. According to a recent survey by EMC [5], 75% of today's digital data are duplicated copies. To reduce the unnecessarily redundant copies, the cloud storage servers would deduplicate by keeping only one or few copies for each file and making a link to the file for every user who asks to store the file. Cloud users (i.e., data owners) shall always be able to verify the integrity of the file at any time. For storage efficiency, it is desirable to deduplicate both the file and the metadata (e.g., authentication tags) needed for data integrity check. Taking malicious or misbehavior users or cloud servers into consideration, the cloud server needs to verify that the user actually owns the file before creating a link to this file for him/her; the user also needs to confirm that the cloud actually has the file in its memory and audit the integrity of the file throughout its lifetime.

Related Work. Considering only integrity auditing for data outsourced to cloud servers, a number of POR schemes [6], [7], [8], [9] and PDP schemes [10], [11], [12], [13] have been proposed. Among these schemes, Ref.[6] has the best performance which achieves public auditing at a constant communication cost. Similar to other POR or PDP schemes, users in Ref.[6] need to perform O(k) multiplication and addition operations over the underlying field, where k is the number of checking data blocks. Batch auditing for multiple requests scenarios is not supported in Ref.[6].

For secure storage deduplication, Halevi et al. [14] introduced the first POW scheme based on the Merkle hash tree. Pietro et al. [15] enhanced Ref.[14] and proposed a secure POW scheme which reduces the heavy computational cost and high I/O cost in Ref.[14] to a constant number of pseudorandom function operations. Nevertheless, these POW schemes did not consider data integrity auditing.

To achieve both data integrity auditing and storage deduplication, one trivial solution is to directly combine an existing POR/PDP scheme with a POW scheme. This trivial solution, however, will result in a O(W) storage overhead for each file, where W is the number of owners of this file. This is because the data owners, lacking mutual trust, need to separately store their own authentication tags in cloud for file integrity auditing. Since these tags are created for auditing the same file, storing O(W) such copies represents a type of duplication which contradicts the objective of POW for saving storage cost. For efficient proof of storage with deduplication (POSD), Zheng et al. [16] proposed a scheme aiming at providing both public data integrity auditing and secure storage deduplication. In Ref.[16] the communication cost and computational cost on the user side are linear to the number of elements in each

data block as well as the number of checking blocks during the integrity auditing process. With an increasing number of mobile users, who access cloud through mobile apps (e.g., iAWS, iCloud, etc.) and have constrained computational resources and bandwidth (e.g., mobile phones with limited data plan), such a communication and computational complexity could represent a barrier to accessing the cloud storage service. Preferably, computational cost and communication cost on the user side shall be both constant. Moreover, Ref.[16] has been proven not secure [17]. Specifically, by setting the elements in secret keys to some special values, a data owner who outsources data to the cloud server is able to use the server as a malware distribution platform. It still calls for a new solution to support efficient and secure data integrity auditing with storage deduplication for cloud storage.

Our Contribution. In this paper, we solve this open problem and propose the first Public and Constant cost storage integrity Auditing scheme with secure Deduplication (PCAD) based on techniques including polynomial-based authentication tags and homomorphic linear authenticators. The proposed PCAD scheme is characterized by following nice properties: 1) PCAD is able to securely "deduplicate" the authentication tags by aggregating the tags of the same file from different owners, and hence make the storage overhead independent to the number of owners of the file; 2) the communication cost in our PCAD scheme is made constant thanks to our novel design of polynomial-based authentication tags and secure data aggregation; 3) the computational cost on cloud users is also constant because most computational tasks can be securely offloaded to the cloud server; 4) PCAD supports public auditing, i.e., the data integrity auditing operation can be securely performed by any third party other than the owner(s); 5) PCAD allows batch auditing, i.e., multiple auditing requests can be securely aggregated, which substantially reduces the auditing cost for simultaneous requests; 6) in PCAD data integrity auditing and secure deduplication operations can be separately performed by any owner (e.g., a new user) without the help of any other owners (e.g., existing owners). With these salient properties, PCAD outperforms existing POR and PDP schemes while providing an additional functionality of data deduplication. The main idea of our scheme can be summarized as follows: The data owner outsources the erasure-coded file to the cloud server together with the corresponding authentication tags. To audit the integrity of the outsourced file, a user (who may not be the owner) challenges the cloud with a challenging message. On receiving the message, the cloud generates the proof information based on the public key and sends it to the user. With the proof information, the user verifies the data integrity using our verification algorithm. When a user wants to upload a data file to the cloud server which already exists in the cloud, the cloud server performs a checking algorithm to see whether or not this user actually owns the whole file. If the user passes the checking, he/she can directly use the file existed on the server without uploading it again. The security of our proposed scheme is proven based on the Computational Diffie-Hellman problem and the Strong Diffie-Hellman

assumption. Thorough analysis and experimental results on Amazon EC2 Cloud show that our scheme is efficient and scalable. Our main contributions can be summarized as below.

- We proposed the first public and constant cost storage integrity auditing scheme with secure deduplication, which can also efficiently handle multiple auditing requests scenarios with batch operations.
- We formally proved the security of PCAD based on the CDH problem and the SDH assumption. The advantages of PCAD are validated by both numerical analysis and real experiments on Amazon AWS Cloud.
- Our design of polynomial based authentication tag can be used as an independent solution for other related applications, such as verifiable SQL search, encrypted key word search, etc.

The rest of this paper is organized as follows: In Section II, we introduce the models and assumptions of our scheme. Section III provides the construction and security proof of our scheme; Performance evaluations of our scheme are provided in Section IV; We conclude our paper in Section V.

II. MODEL AND ASSUMPTION

A. System Model

In this work, we consider a system consisting of three major entities: Data Owner, Cloud Server and User. The data owner has a number of data files and stores them on the cloud server together with the authentication tags. A user to whom the owner shares the data files can access and check the integrity of data files using the public key. A user can also be a Third Party Authority (TPA), who has capabilities/expertise and can periodically audit the integrity of data files being stored on the behalf of data owners. When a user wants to upload data files which are already stored in the cloud, the cloud server just create a link to this file, instead of storing another copy, for this user if the user has been proven a true owner of the file with our scheme. During the integrity auditing and deduplication processes, the user and the cloud server only use the public key and do not need any help from the data owner. While cloud servers are always equipped with abundant computing resources, data owners and users may have constrained computational power or bandwidth (e.g., mobile phones with limited data plan).

B. Security Model

In our PCAD scheme, we consider the selfish cloud server which may potentially misbehavior in order to save resources. This assumption is consistent with the previous POSD scheme [16]. Specifically, the cloud server may delete rarely accessed data on purpose to save storage cost; the cloud server may not follow the algorithms in our PCAD scheme to save the computational cost or communication cost. Therefore, in our model we need to assure that our scheme is correct and sound. In terms of correctness, we require our scheme accepts all valid secret keys and public keys, all valid authentication tags, all valid proof information generated based on valid public keys and all valid data blocks. In terms of soundness, any selfish

cloud server must actually store the right data files if it can produce the proof information that can be accepted by the verification algorithm in our scheme; if any user can avoid uploading a data file existed on the cloud server and pass the checking of the cloud server, he/she must own the whole data file. W.o.l.g., we define the security game for soundness of our PCAD scheme as below.

Definition II.1. Let $\nabla = (KeyGen, Setup, Prove, Auditing, Deduplication) be a PCAD scheme. Consider the security game among a trust authority <math>TA$, a challenger CHA and a probabilistic polynomial-time adversary Adv.

- The TA runs $KeyGen(1^{\lambda}) \to (PK, SK)$ and sends the PK to the Adv and CHA.
- The Adv chooses a data file F and sends it to the TA. The TA runs $Setup(F, SK, PK) \rightarrow (\sigma, F')$ and responds σ, F' to the Adv, where F' is the erasure coded data file of F and σ is the authentication tags for data blocks in F'.
- To audit the integrity of file F, the CHA generates a challenging message CM and sends it to the Adv.
- The Adv generates a file $F'', F'' \neq F'$ since it may modify or lost some data blocks of F'. Instead of running the Prove algorithm, the Adv runs an arbitrary algorithm $Art(CM, PK, F'', \sigma) \rightarrow Prf$ and sends Prf to the CHA.
- The CHA runs $Audit(Prf, PK) \rightarrow AuditRst$.
- With regard to deduplication process, the Adv claims that he wants to upload F' to the CHA, where F' is already stored on the CHA. CHA runs $Deduplication(PK, \sigma) \rightarrow DedupRst$.
- The Adv wins the game if and only if it can let Audit algorithm output AuditRst as accept with $F'', F'' \neq F'$ or let Deduplication output DedupRst as accept without the whole F'.

We consider ∇ as sound if any Adv can win the above game with at most a negligible probability.

C. Assumptions

Definition II.2. Computational Diffie-Hellman (CDH) Problem [18]

Let x, y be two random numbers. Given (g, g^x, g^y) , it is computationally intractable to compute the value of g^{xy} , where G is a cyclic group of order q and g is a generator of G.

Definition II.3. *t-Strong Diffie-Hellman (t-SDH) Assumption* [19]

Let $\alpha \stackrel{R}{\leftarrow} Z_q^*$. Given input as a (t+1) – tuple $(g, g^{\alpha}, \cdots, g^{\alpha^t}) \in G^{t+1}$, where g is the generator of a cyclic group G of order q. For any probabilistic polynomial time adversary (Adv), the probability $Pr[Adv(g, g^{\alpha}, \cdots, g^{\alpha^t}) = (c, g^{\frac{1}{\alpha+c}})]$ is negligible for any value of $a \in Z_q^*/-\alpha$.

III. CONSTRUCTION OF PCAD

A. Preliminaries and Notation

Bilinear Map: Let G and G_1 be two multiplicative cyclic groups of the same prime order q. A bilinear map is a map that for all $g,h \in G$ and $x,y \stackrel{R}{\leftarrow} Z_q^*$, $e(g^x,h^y)=e(g,h)^{xy}$. For a bilinear map, there exists a computable algorithm that can compute e efficiently and $e(g,g) \neq 1$.

Notation: Let $H(\cdot)$ be the one-way hash function, G be a multiplicative cyclic group of prime order q and g,u be two random generators of G. F' is the erasure coded file to be outsourced and is split into n blocks, each of which has s elements: $\{m_{ij}\}, 1 \le i \le n, 0 \le j \le s-1, f_{\vec{a}(x)}$ is denoted as a polynomial with coefficient vector $\vec{a} = (a_0, a_1, \cdots, a_{s-1})$.

B. Our Construction

In this section, we describe the construction of our PCAD scheme as below.

• $KeyGen(1^{\lambda}) \rightarrow (PK, SK)$: Given a scurity parameter λ , the data owner randomly chooses a prime q (λ bits security) and generates a signing key-pair $((spk, ssk) \stackrel{R}{\leftarrow} Sign())$ [19]. The owner chooses two random numbers $\alpha, \epsilon \stackrel{R}{\leftarrow} Z_q^*$ and computes $\kappa \leftarrow g^{\epsilon}, \{g^{\alpha^j}\}_{j=0}^{s-1}$. The public key and private key are:

$$PK = \{q, \kappa, spk, u, \{g^{\alpha^{j}}\}_{j=0}^{s-1}\}, SK = \{\epsilon, ssk, \alpha\}$$

• $Setup(PK, SK, F) \rightarrow (F', \sigma, \tau)$:

To outsource a file F, the data owner first obtains F' by applying erasure code (e.g., Reed-Solomon code [20]), where F' consists of n data blocks and each block has s elements: $\{m_{ij}\}, 1 \leq i \leq n, 0 \leq j \leq s-1$. The owner then randomly chooses a file name $name \in Z_q^*$ and generates the file tag τ under ssk as $\tau \leftarrow name||n||Sign_{ssk}(name||n)$. For each data block $m_i, 1 \leq i \leq n$, the owner produces an authentication tag as:

$$\sigma_i = (u^{H(name||i)} \cdot \prod_{j=0}^{s-1} g^{m_{ij}\alpha^j})^{\epsilon}$$

$$= (u^{H(name||i)} \cdot g^{f_{\beta_i^s}(\alpha)})^{\epsilon}$$
(1)

where $\vec{\beta_i} = \{\beta_{i,0}, \beta_{i,1}, \cdots, \beta_{i,s-1}\}$ and $\beta_{i,j} = m_{i,j}$. The data owner stores F', file tag τ and corresponding authentication tags σ_i on the cloud server.

- Auditing Phase1(PK, τ) $\rightarrow CM$:
 - To verify the integrity of F', A user first gets the file tag τ from the cloud server and verifies the signature on τ with ssk. If the signature is not valid, the user rejects and halts; otherwise, the user recovers file name name and n. Then the user randomly chooses a k- elements subset K of [1,n] and a random number $\rho \stackrel{R}{\leftarrow} Z_q^*$. Finally, the user produces the challenging message $CM = \{K, \rho\}$ and sends it to the cloud server.
- $Prove(PK, F', CM) \rightarrow Prf$: Based on the challenging message $CM = \{K, \rho\}$, the cloud server produces a k- elements set $Q = \{(i, v_i)\}$,

where $i \in K$ and $v_i = \rho^i \mod q$. Then, the cloud server computes

$$\sigma = \prod_{(i,v_i) \in Q} \sigma_i^{v_i} \tag{2}$$

$$\psi = e(\prod_{j=0}^{s-1} (g^{\alpha^j})^{A_j}, \kappa) = e(g^{f_{\vec{A}}(\alpha)}, \kappa)$$
 (3)

where $\vec{A} = (\sum_{(i,v_i) \in Q} v_i m_{i,0}, \cdots, \sum_{(i,v_i) \in Q} v_i m_{i,s-1})$. The cloud server sends the proof information Prf = $\{\sigma,\psi\}$ to the user.

• Auditing $Phase2(PK, Prf) \rightarrow AuditRst$: On receiving the Prf, the user first computes

$$\vartheta = \sum_{i} H(name||i)v_i, (i, v_i) \in Q'$$

$$\eta = u^{\vartheta}$$
(5)

$$\eta = u^{\vartheta} \tag{5}$$

Based on η , the user verifies the integrity of F' together with $Prf = \{\sigma, \psi\}$ as:

$$e(\eta, \kappa) \cdot \psi \stackrel{?}{=} e(\sigma, g)$$
 (6)

If Eq.6 holds, then the user outputs AuditRst as accept; otherwise, outputs AuditRst as reject.

• $Deduplication(PK, \sigma) \rightarrow DedupRst$: In the *Deduplication* algorithm, a user claims that he has a file F' and wants to store it on the cloud server, where F' is already existed on the server. To check whether or not the user actually owns the whole F', the cloud server randomly chooses a d- elements subset D of [1, n] (we discuss the size of D in Section III-E) and sends D to

the user. On receiving the set D, the user responds the corresponding data block $m_i, i \in D$ to the cloud server. The cloud server computes

$$\sigma' = \prod_{i \in D} \sigma_i \quad \eta' = \prod_{i \in D} u^{H(name||i)}$$
$$\psi' = e(\prod_{j=0}^{s-1} (g^{\alpha^j})^{B_j}, \kappa) = e(g^{f_{\vec{B}}(\alpha)}, \kappa)$$

where $\vec{B} = (\sum_{i \in D} m_{i,0}, \dots, \sum_{i \in D} m_{i,s-1})$. Then the cloud server checks the integrity of uploaded data blocks $m_i, i \in D$ as:

$$e(\eta', \kappa) \cdot \psi' \stackrel{?}{=} e(\sigma', g)$$
 (7)

If Eq.7 holds, the cloud server trusts that the user has the whole F'.

• Correctness: We analyze the correctness of our construction based on Eq.6 and Eq.7 as: Eq.6:

$$e(\sigma, g)$$

$$= e(u^{\epsilon(\sum_{(i,v_i)\in Q} H(name||i)v_i)} \cdot g^{\epsilon f_{\vec{A}}(\alpha)}, g)$$

$$= e(u, g)^{\epsilon(\sum_{(i,v_i)\in Q} H(name||i)v_i)} \cdot e(g, g)^{\epsilon f_{\vec{A}}(\alpha)}$$

$$= e(\eta, \kappa) \cdot \psi$$
(8)

Eq.7

$$e(\sigma', g) \qquad (9)$$

$$= e(u^{\epsilon(\sum_{i \in D} H(name||i))} \cdot g^{\epsilon f_{\vec{B}}(\alpha)}, g)$$

$$= e(u, g)^{\epsilon(\sum_{i \in D} H(name||i))} \cdot e(g, g)^{\epsilon f_{\vec{B}}(\alpha)}$$

$$= e(\eta', \kappa) \cdot \psi'$$

From the above Eq.8 and Eq.9, it is easy to see that our scheme is correct.

C. Auditing After Deduplication

In this section, we describe the auditing of file owned by multiple owners after the deduplication process and show how to aggregate authentication tags for the same file.

After the deduplication process for F' on the cloud server, the user who passes ownership checking also becomes the owner of F'. We define such owner as $owner_w$, where $1 \leq w \leq W$, $owner_0$ as the original owner who uploads F'and W is the total number of $owner_w$. Since these $owner_w$ have no relationship with each other, they need to assure the integrity of F' separately. Specifically, after the deduplication, an $owner_w, w \neq 0$ runs the KeyGen algorithm and generate the public key and the private key as

$$PK_w = \{q, \kappa_w, spk_w, u, \{g^{\alpha_w^j}\}_{j=0}^{s-1}\}$$

$$SK_w = \{\epsilon_w, ssk_w, \alpha_w\}$$

where $\kappa_w = g^{\epsilon_w}$, ϵ_w , $\alpha_w \overset{R}{\leftarrow} Z_q^*$. Then, by running the Setup algorithm, the $owner_w$ generates the file tag τ_w as $\tau_w \leftarrow name||n||Sign_{ssk_w}(name||n)$ and authentication tags for each block m_i in F' as

$$\sigma_{wi} = \left(u^{H(name||i)} \cdot \prod_{j=0}^{s-1} g^{m_{ij}\alpha_w^j}\right)^{\epsilon_w}$$

$$= \left(u^{H(name||i)} \cdot g^{f_{\beta_i}(\alpha_w)}\right)^{\epsilon_w}$$
(10)

where $\vec{\beta_i} = \{\beta_{i,0}, \beta_{i,1}, \dots, \beta_{i,s-1}\}$ and $\beta_{i,j} = m_{i,j}$. The file tag au_w and corresponding authentication tags σ_{wi} are outsourced to the cloud server.

Instead of storing all the tags from different $owner_w$ for the same F' separately, the cloud server aggregates tags for each data block as:

$$\sigma_i = \prod_{w=0}^{W} \sigma_{wi} = u^{\sum_{w=0}^{W} H(name||i)\epsilon_w} \cdot g^{\sum_{w=0}^{W} (f_{\vec{\beta_i}}(\alpha_w) \cdot \epsilon_w)}$$

When a user helps an owner, say $owner_t$, $t \in [0, W]$, to audit the integrity of F', it runs the Auditing Phase1 algorithm to generate the challenging message $CM = \{K, \rho\}$ and sends it to the cloud server. The server runs the Prove algorithm to generate $\sigma = \prod_{(i,v_i) \in Q} \sigma_i^{v_i}$. After that, the server computes $\psi = \prod_{w=0}^W \psi_w$, where $\psi_w = e(g^{f_{\overline{A}}(\alpha_w)}, \kappa_w), 0 \leq w \leq W$. The proof information $Prf = \{\sigma, \psi, \kappa'\}$ is sent to the user, where $\kappa' = \prod \kappa_w, w \in W, w \neq t$. To verify the integrity of F', the user runs the Auditing Phase 2 algorithm and checks

$$e(\eta, \kappa' \cdot \kappa_t) \cdot \psi \stackrel{?}{=} e(\sigma, g)$$
 (11)

If Eq.11 holds, then the user outputs AuditRst as accept; otherwise, outputs AuditRst as reject.

Correctness: We analyze the correctness of our construction based on Eq.11 as:

$$e(\sigma, g)$$

$$= e(u^{\sum_{(i,v_i)\in Q} (t_i \cdot \sum_{w=0}^{W} \epsilon_w)} \cdot g^{\sum_{w=0}^{W} (f_{\vec{\beta_i}}(\alpha_w) \cdot \epsilon_w)}, g)$$

$$= e(u, g)^{\sum_{(i,v_i)\in Q} (t_i \cdot \sum_{w=0}^{W} \epsilon_w)} \cdot e(g, g)^{\sum_{w=0}^{W} (f_{\vec{\beta_i}}(\alpha_w) \cdot \epsilon_w)}$$

$$= e(\eta, \kappa' \cdot \kappa_t) \cdot \psi$$
(12)

where $t_i = H(name||i)v_i$. From the above Eq.12, it is easy to see that our scheme is correct.

D. Batch Auditing

As the TPA has expertise and capabilities that many data owners not have, it can help owners to audit the integrity of their stored files on the cloud periodically. However, when multiple owners delegate their integrity auditing requests at the same time, it is inefficient for the TPA to process these requests one by one. Specifically, given L integrity auditing requests for L different encoded files $F_l' = \{m_{li,j}\}, 1 \leq l \leq L, 1 \leq i \leq n_l, 0 \leq j \leq s_l - 1$ from L different owners, it is desirable for the TPA to handle these requests in batch to reduce both communication cost and computational cost, where n_l is the number of data blocks in encoded file F_l' and s_l is the number of elements in each data block. For this purpose, we design the batch auditing algorithm based on our single request construction as below.

• L data owners run KeyGen algorithm separately. The public keys and private keys are

$$PK_{l} = \{q, \kappa_{l}, spk_{l}, u, \{g^{\alpha_{l}^{j}}\}_{j=0}^{s_{l}-1}\}$$

$$SK_{l} = \{\epsilon_{l}, \alpha_{l}, ssk_{l}\}, 1 \leq l \leq L$$

where $\kappa_l = g^{\epsilon_l}$, ϵ_l , $\alpha_l \stackrel{R}{\leftarrow} Z_q^*$.

- To audit the integrity of these L files, the TPA runs $Auditing\ Phase1$ algorithm and sends the challenging message $CM = \{\rho, K\}$ to the cloud server.
- On receiving the CM, the cloud server first runs Prove algorithm for L files and generates $Prf_l = \{\psi_l, \sigma_l\}, 1 \leq l \leq L$, where

where
$$\psi_l = e(g^{f_{\vec{A}_l}(\alpha_l)}, \kappa_l) \quad \sigma_l = \prod_{(i, v_i) \in Q} \sigma_{li}^{v_i}$$

$$\vec{A}_l = (\sum_{(i, v_i) \in Q} v_i m_{li, 0}, \cdots, \sum_{(i, v_i) \in Q} v_i m_{li, s-1})$$

Then the cloud server computes $\sigma = \prod_{l=1}^L \sigma_l$ and $\psi = \prod_{l=1}^L \psi_l$. The final proof information $Prf = \{\psi, \sigma\}$ is sent to the TPA.

• Based on the received Prf, the TPA first runs $Auditing\ Phase2$ to get $\eta_{li}=u^{H(name_l||i)v_i}, (i,v_i)\in Q$ for each F'_l . Then the TPA checks the integrity of these L files together as

$$\psi \cdot \prod_{l=1}^{L} e(\eta_{li}, \kappa_l) \stackrel{?}{=} e(\sigma, g)$$
 (13)

If Eq.13 holds, the TPA outputs AuditRst as accept; otherwise, outputs AuditRst as reject.

• Correctness:

We analyze the correctness of our construction based on Eq.13 as:

$$e(\sigma, g) \qquad (14)$$

$$= e(u^{\sum_{l=1}^{L} \epsilon_{l}(\sum_{(i,v_{i}) \in Q} t_{li})} \cdot g^{\sum_{l=1}^{L} \epsilon_{l} f_{\vec{A}_{l}}(\alpha_{l})}, g)$$

$$= e(u, g)^{\sum_{l=1}^{L} \epsilon_{l}(\sum_{(i,v_{i}) \in Q} t_{li})} \cdot e(g, g)^{\sum_{l=1}^{L} \epsilon_{l} f_{\vec{A}_{l}}(\alpha_{l})}$$

$$= \prod_{l=1}^{L} (e(u, g)^{\epsilon_{l}(\sum_{(i,v_{i}) \in Q} t_{li})} \cdot e(g, g)^{\epsilon_{l} f_{\vec{A}_{l}}(\alpha_{l})})$$

$$= \prod_{l=1}^{L} (e(u, g)^{\epsilon_{l}(\sum_{(i,v_{i}) \in Q''} t_{li}) + \epsilon_{l}(\sum_{(i,v_{i}) \in Q'} t_{li})}$$

$$\cdot e(g, g)^{\epsilon_{l} f_{\vec{A}_{l}}(\alpha_{l})})$$

$$= \psi \cdot \prod_{l=1}^{L} e(\eta_{li}, \kappa_{l})$$

where $t_{li} = H(name_l||i)v_i$. From the above Eq.14, it is to verify the correctness of our construction.

E. Discussion

In this section, we discuss the error detection probability of our PCAD scheme, the selection of set K' in our Auditing Phase2 algorithm and the selection of set Din our *Deduplication* algorithm. As we mentioned in our Setup algorithm, we adopt Reed-Solomon code to encode the outsourcing file. For an ℓ Reed-Solomon encoded file $(0 < \ell < 1)$, the original file can be recovered from any ℓ fraction of encoded data blocks. Thus, if an ℓ Reed-Solomon encoded file cannot be recovered, the probability of getting an uncorrupted encoded data block will be less than ℓ . In this case, when a user randomly chooses k independently encoded data blocks to challenge, the probability that all these blocks are uncorrupted is less than ℓ^k . When we set $\ell = 0.98$ as previous POR schemes [6], [21] do, the user can achieve at least 99.999% error detection probability when he challenges 600 data blocks for an encoded file.

With regard to the selection of set D in our Deduplication algorithm, the cloud server can determine it based on the number of blocks in the encoded file. For an encoded file with n data blocks, the cloud can randomly choose d blocks to check. In this case, if the d data blocks uploaded by the user pass the verification in Deduplication algorithm, the cloud server will have $1-1/\binom{n}{d}$ confidence that the user owns whole file (e.g., for n=1000, the cloud can randomly chooses d=3, the cloud will have 99.9999% confidence).

F. Security Proof

In this section, we prove the security of our proposed scheme.

Theorem III.1. If $g^{f_{\vec{A}}(\alpha)}$ can be forged by an existed probabilistic polynomial time adversary Adv, we can construct

an algorithm B to efficiently compute the solution to t-SDH problem based on Adv.

Proof: Suppose there exists a probabilistic polynomial time adversity Adv that can generate $f_{\vec{A_1}}(\alpha)$ such that $g^{f_{\vec{A_1}}(\alpha)} = g^{f_{\vec{A}}(\alpha)}$, where $f_{\vec{A}}(x)$ and $f_{\vec{A_1}}(x)$ are known to the Adv. The Adv can construct another polynomial $f_{\vec{A_2}}(x) = f_{\vec{A}}(x) - f_{\vec{A_1}}(x)$. Therefore, $g^{f_{\vec{A_2}}(\alpha)} = g^{f_{\vec{A}}(\alpha)}/g^{f_{\vec{A_1}}(\alpha)} = g^{f_{\vec{A}}(\alpha)-f_{\vec{A_1}}(\alpha)} \in Z_q[x]$. As $f_{\vec{A_1}}(\alpha) = f_{\vec{A}}(\alpha)$ and $f_{\vec{A_2}}(\alpha) = 0$, α becomes a root of polynomial $f_{\vec{A_2}}(x)$. By factoring $f_{\vec{A_2}}(x)[22]$, B can easily find $SK = \alpha$. Based on $SK = \alpha$, B can easily find a number c and get $(c, g^{\frac{1}{\alpha+c}})$ as solution to the instance of the t-SDH problem given by the system parameters.

Theorem III.2. If there exists a probabilistic polynomial time adversary Adv that can pass the verification in our proposed scheme with fake proof information $Prf' = (\psi', \sigma')$, where $Prf' \neq Prf$ and $Prf = (\psi, \sigma)$ is the honest proof information, we can construct an algorithm B that uses Adv to solve the CDH problem.

Proof: Suppose a probabilistic polynomial time adversity Adv can generate a $Prf'=(\psi',\sigma'),(\psi',\sigma')\neq(\psi,\sigma)$ and pass the verification in our proposed scheme, we can get the following two equations:

$$e(\eta, \kappa) \cdot \psi = e(\sigma, g)$$
 (15)

$$e(\eta, \kappa) \cdot \psi' = e(\sigma', g)$$
 (16)

Dividing Eq.15 with Eq.16, we obtain:

$$\frac{\psi}{\psi'} = \frac{e(\sigma, g)}{e(\sigma', g)} \tag{17}$$

Now we do a case analysis on whether $\sigma = \sigma'$.

Case 1: $\sigma \neq \sigma'$. Since $\frac{\psi}{\psi'}$ and $e(\sigma, g)$ are known values to the Adv. We rewrite Eq.17 as

$$e(\sigma', g) = e(\sigma, g) \cdot \frac{\psi'}{\psi} = e(g^{\epsilon f_{\vec{A}}(\alpha)}, g) \cdot \theta$$
 (18)

where $\theta=e(u^{\epsilon(\sum_{(i,v_i)\in Q}H(name||i)v_i)},g)/\frac{\psi'}{\psi}$ as a known value to the Adv. If the Adv can find σ' with non-negligible probability and make Eq.18 hold, we can construct an algorithm B that easily extract $g^{\epsilon f_A(\alpha)}$. With the given information, B can get $g^{\epsilon},g^{f_A(\alpha)}$, where ϵ and $f_A(\alpha)$ are unknown, and thus solve the CDH problem. Therefore, no Adv can find a valid response $Prf',Prf'\neq Prf$ and $\sigma\neq\sigma'$ with non-negligible probability.

Case 2: $\sigma = \sigma'$. In this case, since $\sigma = \sigma'$, from Eq.17, we can infer that $\frac{\psi'}{\psi} = 1$, i.e., $\psi' = \psi$. Therefore, no Adv can find a valid $Prf' = (\psi', \sigma'), Prf' \neq Prf$ with non-negligible probability.

Theorem III.2 is proved.

Now, we prove the security of the data integrity auditing after deduplication.

Theorem III.3. If there exists a probabilistic polynomial time adversary Adv that can collude with W-1 owners and pass owner_t's integrity auditing after deduplication with fake proof information $Prf' = (\psi', \sigma')$, where $Prf' \neq Prf$ and $Prf = (\psi, \sigma)$ is the honest proof information, we can construct an algorithm B that uses Adv to solve the CDH problem.

Proof: As the proof of *Theorem* III.3 is similar to the proof of *Theorem* III.2 above, we just give the idea and key differences here due to the space limitation.

Suppose a probabilistic polynomial time adversity Adv can generate a Prf' and pass the integrity auditing, we can divide the two equations generated based on Prf and Prf' as in the proof of Theorem III.2. Then, we do a case analysis on whether $\sigma = \sigma'$. In case $\sigma \neq \sigma'$, the difference between Theorem III.2 and Theorem III.3 is the known knowledge of the Adv. Since the Adv can collude with the rest W-1 owners of F', we can use θ to denote the known knowledge of the Adv and construct $e(\sigma',g)=e(g^{\epsilon_t f_{\vec{A}}(\alpha_t)},g)\cdot\theta$. Similar to Theorem III.2, we can prove $\sigma'=\sigma$ based on the CDH problem. $\psi'=\psi$ can also be proved similar to the proof of Theorem III.2.

IV. PERFORMANCE EVALUATION

A. Numerical Analysis

In this section, we numerically analyze our PCAD scheme and compare it with Ref.[16], [6], [15]. For simplicity, in the rest of this paper, we use MUL and EXP^1 to denote the complexity of one multiplication operation and one exponentiation operation on Group G respectively.

1) Communication: In our PCAD scheme, the communication cost of the auditing process is caused by the challenging message $CM = \{K, \rho\}$ and the proof information $Prf = \{\psi, \sigma\}$. The CM consists of a set K with k block ids and a random number ρ . As we discussed in Section III-E, the user can randomly challenge k = 600 data blocks to assure at least 99.999% error detection probability. If an error detection probability a fixed parameter, the size of K can be considered as constant and the complexity of challenging message CMis O(1). The proof information is composed of two group elements: ψ and σ . Therefore, the total communication complexity of auditing process in our PCAD scheme is also O(1). In the *Deduplication* process of our scheme, the user only needs to send d encoded data blocks to the cloud server to prove that it actually owns the whole file. As we discussed in Section III-E, the cloud server can set d=3 to achieve at least 99.9999% confidence when there are 1000 or more encoded data blocks for the file. When the number of encoded data blocks for a file is larger than 1000, the cloud server can achieve higher confidence when set d=3. When the number of encoded data blocks for a file is small, say 100, the cloud server just needs to set d=4 to achieve at least 99.9999%confidence. In practice, the number of encoded data blocks for

¹When the operation is on the elliptic curve, EXP means scalar multiplication operation and MUL means one point addition operation.

	Ref.[6] (POR)	Ref.[15] (POW)	Ref.[16] (POSD)	Our PCAD
Public Auditing	Yes	No	Yes	Yes
Deduplication	No	Yes	Yes	Yes
Secure	Yes	Yes	No	Yes
Batch Auditing	No	No	No	Yes
Prepossessing	O(sn)MUL $+O(sn)$ EXP	O(n)PRF	O(sn)MUL $+O(n)$ EXP	O(sn)MUL $+O(sn)$ EXP
Auditing Comp.Cost	O(s+k)MUL $+O(s+2k)$ EXP	N/A	O(ks)MUL $+O(k)$ EXP	O(s+k)MUL $+O(s+k)$ EXP
(Cloud)				
Auditing Comp.Cost	O(k)MUL $+O(k)$ EXP	N/A	O(ks)MUL $+O(k)$ EXP	O(1)MUL+ $O(1)$ EXP
(User)	+O(1)Pairing			+O(1)Pairing
Auditing Comm.Cost	O(1)	N/A	O(s+k)	O(1)
Deduplication Comp.Cost	N/A	O(n)PRF	O(sk)MUL $+O(k)$ EXP	O(s)MUL+ $O(s)$ EXP
(Cloud)				+O(1)Pairing
Deduplication Comp.Cost	N/A	O(1)PRF	O(sk)MUL	0
(User)				
Deduplication Comm.Cost	N/A	O(1)	O(s)	O(1)

Table.1 Complexity Summary: in this table, n is number of encoded blocks for the file, s is the number of elements in each block and k is number of blocks selected for challenging; PRF is one pseudorandom function operation, EXP and MUL are one multiplication operation and one exponentiation operation on Group G respectively

an erasure coded file is very likely to be large, e.g., larger than 100. Therefore, d can be bounded and the total communication complexity of the Deduplication process in our scheme is O(1).

Now, we compare our PCAD scheme with existing schemes [16], [6], [15] and show the result in Table.1. In Ref.[16], the Auditing process requires the cloud server to send kauthentication tags of the challenging blocks and s aggregated data blocks to the user, where s is the number of elements in an encoded block. Thus, its communication complexity during the Auditing process is O(s + k). To perform the Deduplication process, the user needs to sends 2s aggregated data blocks to the cloud server and thus introduces the communication complexity as O(s). Differently, the aggregation of communication information in our design enables our scheme to achieve O(1) communication complexity for both Auditing and Deduplication processes. The POR schemes proposed by Yuan et al. [6] achieves constant communication complexity for the Auditing process same as our PCAD scheme. However, their scheme does not support the Deduplication process and batch auditing, and introduces much higher computational cost on the user side (Discuss later in Section IV-A2). Considering only the deduplication process, Ref.[15] requires O(1) PRF operation on the user side that is more expensive than our PCAD scheme, where PRF is one pseudorandom function operation. In addition, this scheme cannot support the data integrity auditing.

2) Computation: As shown in Section III-B, our PCAD scheme consists of 6 algorithms: KeyGen, Setup, $Auditing\ Phase1$, Prove, $Auditing\ Phase2$ and Deduplication. Among these algorithms, KeyGen and Setup are prepossessing procedures, which are performed by the data owner off-line. In the KeyGen algorithm, the data owner performs (s+1) EXP operations to generate the public key and private key. To produce authentication tags for a encoded file with n blocks, each of which has s elements, the data owner needs (s+2)n EXP and sn MUL operations. Note that the cost in the prepossessing of our scheme is one-time cost for the data owner and will not influence the real-time performance of our scheme.

After these prepossessing procedures, the data owner can go off-line. During the data integrity auditing process, the user performs $Auditing\ Phase1$ to generate the challenging message CM by choosing a constant number of random numbers with negligible cost. On receiving the CM, the cloud server needs (s+k) EXP, (s+k) MUL and 1 Pairing operations to produce the proof information. To verify the integrity of the auditing file, the user performs 1 EXP, 1 MUL and 3 Pairing operations. Therefore, the computational cost for the user to audit the data integrity of a single file is O(1)MUL+O(1)EXP+O(1)Pairing. To perform the Deduplication algorithm in our scheme, no computation cost is required for the user. On the cloud server, it needs O(s)MUL+O(s)EXP+O(1)Pairing.

We now compare our PCAD scheme with existing schemes [16], [6], [15] and summarize the result in Table.1. In Ref.[16], the data integrity auditing process costs a user O(ks)MUL+O(k)EXP operations, and the deduplication processes introduces O(sk)MUL computational complexity to the user, where k is the number of challenging blocks and s is the number of element in a data block. Differently, by outsourcing most computational tasks of both auditing and deduplication processes to the cloud server, our PCAD scheme achieves constant computational cost on users and thus significantly outperforms Ref.[16]. Compared with Ref.[6] that only supports data integrity auditing, our PCAD scheme reduces the computational complexity on the user from O(k)MUL+O(k)EXP+O(1)Pairing to O(1)MUL+O(1)EXP+O(1)Pairing as shown in Table.1. Considering only the deduplication process, Ref.[15] requires O(1)PRF operation on the user side that is comparable to our PCAD scheme, where PRF is one pseudorandom function operation. However, this scheme cannot support the data integrity auditing process.

3) Auditing After Deduplication: In this section, we discuss the storage overhead saved by aggregation of authentication tags in our proposed scheme. Suppose W owners $owner_w, 1 \leq w \leq W$ pass the deduplication checking of the file F' existed on the cloud server. As these owners have no relationship with each other, each $owner_w$ needs to

store n authentication tags on the cloud server separately for future public integrity auditing of F', where n is the number of encoded data blocks in F'. If the cloud server directly store these authentication tags, a O(Wn) storage overhead complexity is introduced to it. Differently, by aggregating the tags for the same data block, the cloud server in our scheme can reduce the storage overhead complexity to O(n). With regard to the computational complexity and communication complexity on an auditing user, it remains the same as the constant level of auditing before deduplication, i.e., O(1) MUL + O(1) EXP + O(1) Pairing computational complexity and O(1) communication complexity.

4) Batch Auditing: In this section, we discuss the communication cost and computational cost saved by our batch auditing design for multiple requests scenarios. Suppose a TPC is hired by L data owners to help them audit the integrity of their outsourced files on the cloud server periodically. If the TPA processes these L auditing requests one by one, it needs L EXP, L MUL and 3L Pairing operations for computation, and 2L group elements and L random numbers for communication. With our batch auditing design, the cloud can server can aggregate L σ_i into one group elements and use 1 random number instead of L ones. Thus, compared with processing requests sequentially, our batch auditing design can help the TPA and the cloud server to save at about 50%communication cost. From the perspective of computational cost, by aggregating σ_i , $1 \le i \le L$, our batch auditing design enables the TPA to reduce number of Pairing operations from 3L to L, which is much more expensive compared with MUL and EXP operations. Therefore, more than 50% computational tasks are saved for the TPA with our batch auditing design.

B. Experimental Result

To show that our proposed PCAD scheme is efficient and scalable, we conducted experiments on Amazon EC2 Cloud Platform using C language with GNU MP library [23] and the Pairing-Based Cryptography library [24]. The machine we used for the TPA is a laptop running Mint Linux 13 with 2.50GHz Intel i5-2520M CPU and 8GB memory. For the cloud server, we utilize nodes that run Red Hat Enterprise Linux 6.3 with 8 Cores CPU and 16GB memory. We set the security parameter $\lambda=160$, which achieves 1024 bits security on Group. All experimental results represent the mean of 10 trials.

To verify our PCAD scheme's constant communication cost and computational cost on the user side, we vary the number of data blocks stored on the cloud server from 1000 to 10000. As shown in Fig.1 (a), the computational cost of users for performing an integrity auditing task almost keeps around 420 ms when the number of data blocks in the auditing files increases. With regard to the communication cost, it also remains stable as about 622 Bytes when the number of data blocks in the auditing files increases as shown in Fig.1 (b). Note that, although we do not perform experiment on more large files, it is easy to obtain that both computational cost and communication cost of our scheme are constant from the analysis in Section IV-A2 and Section IV-A1.

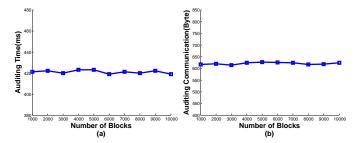


Fig. 1. (a) Auditing Time on Users (b) Auditing Communication Cost on Users

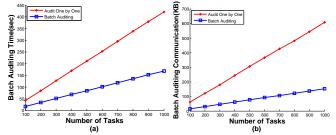


Fig. 2. (a) Auditing Time on TPA (b) Auditing Communication Cost on TPA

To show the benefits of our batch auditing design for multiple auditing tasks scenarios, we change the number of tasks a TPA needs to perform from 100 to 1000. As we demonstrated above, the number of data blocks in each file does not influence the performance of our scheme, we set the number of data blocks to 5000 in each auditing task. Compared with performing these auditing tasks one by one, Fig.2 (a) shows that the TPA can save about 55\% auditing time with batch auditing. From the perspective of communication cost, Fig.2 (b) shows our batch auditing saves about 65% bandwidth for the TPA. Considering the average cost per task, which is computed by dividing total auditing time and total auditing bandwidth cost by the number of tasks respectively, Table.2 shows that our batch reduce the computation cost per task on the TPA from 423 ms to 181 ms and the bandwidth cost per task from 626 Bytes to 209 Bytes.

V. CONCLUSION

To securely fulfill the two important requirements of cloud storage: data integrity and storage efficiency, a number of schemes have been proposed based on the concepts of POR, PDP, POW and POSD. However, most existing schemes only focus on one aspect, because trivial combination of existing POR/PDP schemes with POW schemes can contradict the objects of POW. The only one that simultaneously emphasized both aspects based on the concept of POSD suffers from tremendous computation and computational costs and has been proven not secure. In this work, we filled the gap between POR and POW and proposed a constant cost scheme that achieves secure public data integrity auditing and storage deduplication at the same time. Our proposed scheme enables the deduplication of both files and their corresponding authentication tags. In addition, we extend our design to support batch integrity auditing, and thus substantially save computational

#Tasks = 500	Average Comp.Cost	Average Comm.Cost
Auditing One by One	423 ms	626 Bytes
Batch Auditing	181 ms	209 Bytes

Table.2. Average Computational Cost and Communication Cost for Batch Auditing and Single Auditing

cost and communication cost for multiple requests scenarios. The security of our PCAD scheme is proved based on the CDH problem and the SDH assumption. We validate the efficiency and scalability of our scheme through numerical analysis and experimental results on Amazon EC2 Cloud. Our proposed polynomial based authentication tag can also be used as an independent solution for other related applications, such as verifiable SQL search, encrypted key word search, etc.

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