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Instantiating Random Oracles via UCEs

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Abstract

This paper provides a (standard-model) notion of security for (keyed) hash functions, called UCE, that we show enables instantiation of random oracles (ROs) in a fairly broad and systematic way. Goals and schemes we consider include deterministic PKE; message-locked encryption; hardcore functions; point-function obfuscation; OAEP; encryption secure for key-dependent messages; encryption secure under related-key attack; proofs of storage; and adaptively-secure garbled circuits with short tokens. We can take existing, natural and efficient ROM schemes and show that the instantiated scheme resulting from replacing the RO with a UCE function is secure in the standard model. In several cases this results in the first standard-model schemes for these goals. The definition of UCE-security itself is quite simple, asking that outputs of the function look random given some "leakage," even if the adversary knows the key, as long as the leakage does not permit the adversary to compute the inputs.

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Contents

1 Introduction

The core contribution of this paper is a new notion of security for (keyed) hash functions called UCE (Universal Computational Extractor). UCE-security is the first well-defined, standard-model security attribute of a hash function shown to permit the latter to securely instantiate ROs across a fairly broad spectrum of schemes and goals.

Under the random-oracle paradigm of Bellare and Rogaway (BR93) [21], a "real-world" or instantiated scheme is obtained by implementing the RO of the overlying ROM scheme via a cryptographic hash function. The central (and justified) critique of the paradigm [46] is that the instantiated scheme has only heuristic security. This paper offers *proven* security for the (standard model) instantiated schemes. The proof is based on the (standard-model) assumption that the instantiating function is UCE-secure.

UCE of course does not *always* work.[1] But we show that it works across a fairly large, diverse and interesting spectrum of schemes and goals including deterministic PKE, message-locked encryption, hard-core functions, point-function obfuscation, encryption of key-dependent messages, encryption secure under related-key attack, OAEP, correlated-input secure hashing, adaptively-secure garbled circuits, and proofs of safe storage. In all these cases we can use UCE to obtain standard-model solutions, in most cases instantiating known, natural and efficient schemes, and in several cases getting the first standard-model schemes for the goals in question.

UCE is quite simple and natural, yet powerful. The basic intuition is that the output of a UCE-secure function looks random even given the key and some "leakage," as long as the inputs are not computable from the leakage. Let us now step back to provide some background and then return to our contributions.

1.1 Background

The random-oracle paradigm of BR93 [21] has two steps: (1) Design your scheme, and prove it secure, in the ROM, where the scheme algorithms and adversary have access to a RO denoted RO (2) Instantiate the RO to get the standard model scheme that is actually implemented and used. We will consider instantiation via a family of functions H, which means that the instantiated scheme is obtained by replacing RO calls of the ROM-scheme algorithms by evaluations of the deterministic function $\mathsf{H.Ev}(hk, \cdot)$ specified by a key $hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$, where λ is the security parameter. The key hk is put in the public key of the instantiated scheme if the latter is public key, else enters in some scheme-dependent way. The suggestion of BR93 was that if H "behaved like a RO," the instantiated scheme would be secure in the standard model. They suggested to obtain such instantiations, heuristically, via cryptographic hash functions. The fundamental subsequent concern has been the lack of a proof of security for the instantiated scheme. Canetti, Goldreich and Halevi (CGH98) [46] show that this lack in some cases cannot be overcome because there exist schemes secure in the ROM but which no family of functions can securely instantiate. Advocates for the defense counter by pointing out that the counter-example schemes are artificial, and in-use instantiations of "natural" ROM schemes are unbroken. This has led to examples that are in one way or another less artificial [86, 66, 12, 47, 54, 79].

It is not the purpose of this paper to take sides in this debate. We want instead to make a scientific contribution towards better grounding the security of instantiated ROM schemes.

1.2 The core problem and previous work

The lack of a proof of security for the instantiated scheme is, we submit, a consequence of an even more fundamental lack, namely that of a *definition*, of what it means for a family of functions to "behave like a RO," that could function as an assumption on which to base the proof. The PRF definition [64], which has worked so well in the symmetric setting, is inadequate here because PRF-security relies on the adversary not knowing the key. And collision-resistance (CR) is far from sufficient in any non-trivial usage of a RO.

[1] "Work" means allow the instantiated scheme to be proven secure, and "always works" means works for all schemes secure in the ROM. Indeed, we note that neither UCE nor any other achievable, standard-model security attribute of a family of functions can always work. This is implied by known impossibility results [46, 85].

Figure 1: **Relations between notions of security for families of functions.** On the left we depict relations between different notions of UCE security. On the right we depict relations between UCE1, UCE2 and standard notions. In both, letting S denote the set of all families H that are S-secure, an arrow $\mathsf{A} \to \mathsf{B}$ represents $\mathsf{A} \subseteq \mathsf{B}$, meaning any H that is A-secure is also B-secure. A barred arrow $\mathsf{B} \not\to \mathsf{A}$ represents $\mathsf{B} \not\subseteq \mathsf{A}$, meaning there is an H that is B-secure but not A-secure. (Assuming of course that some B-secure H exists.)

Canetti [44] was the first to articulate this position and seek a standard-model primitive sufficient to capture some usages of a RO. Notions such as Perfectly One-Way Probabilistic Hash Functions (POWHFs) [44, 49, 45] and non-malleable hash functions [27] have however proven of limited applicability [29]. Another direction has been to try to instantiate the RO in particular schemes like OAEP [22], again with limited success [30, 29] or under strong assumptions on RSA [78].

Our position is philosophically different from that of [44, 49]. These works aimed for security notions that they could achieve under standard assumptions. Expectedly, applicability was limited. We aim to maximize applicability and are willing to see our notion (UCE) as an assumption rather than something to achieve under other assumptions.

1.3 UCE

Our definition considers an adversary S, called the source, who is given an oracle HASH, the latter being $\mathsf{H.Ev}(hk, \cdot)$ for key $hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$ if the challenge bit b is 1, and a RO otherwise. If security now asks that S not figure out b, then, if we deny it hk, we would be back to PRFs, and if we give it hk, security would be unachievable. So we don't ask S to figure out b. Instead, it must pass to an accomplice adversary D, called the distinguisher, some information L called the leakage. The distinguisher *is given the key* hk and must figure out b.

Clearly, security is not achievable for arbitrary leakage. (The source could include in L a point x and the result $y = \text{HASH}(x)$ of its oracle on x, and D, having hk, can test whether or not $y = \mathsf{H.Ev}(hk, x)$.) We put an extra condition on the source that we call unpredictability. It requires that it be computationally infeasible for a predictor adversary P, given the leakage produced by the source in the *random* ($b = 0$) game, to find any of the inputs queried by the source to its oracle. Note that unpredictability is a property of the source, not of the family of functions H, the latter not figuring in the definition at all.

Security, finally, requires that for any PT *unpredictable* source S, and any PT distinguisher D, the advantage of S, D in figuring out b is negligible. See Section 4 for a formal definition of this notion that we call UCE1. A variant called UCE2, introduced in Section 6, preserves the source-distinguisher framework of UCE1 but replaces the unpredictability condition with a weaker condition we call reset-security. ("Weaker" because any unpredictable source is reset-secure. This makes UCE stronger: any UCE2-secure family is UCE1-secure.) Both UCE1 and UCE2 involve a single hashing key. We define natural multi-key extensions mUCE1 and mUCE2 as well.

The relations between the different forms of UCE we introduce are depicted on the left side of Fig. 1. As indicated there, UCE2 implies UCE1 but not vice versa, and analogously mUCE2 implies mUCE1 but not vice versa. Of course mUCE1 implies UCE1 and mUCE2 implies UCE2. We do not know whether UCE1 implies mUCE1, and analogously for UCE2 and mUCE2. The right side of the same Figure shows how the basic notions of UCE relate to other, standard security notions for families of functions, namely PRF-security and collision-resistance (CR). As the figure indicates, neither PRF-security nor CR-security imply even the weak form of UCE, namely UCE1. On the other hand, even the strong form of UCE, namely

Goal	Result	UCE
D-PKE	Instantiation of the ROM EwH scheme of [11] to obtain the first standard model deterministic PKE scheme providing full IND [14] and PRIV [11] security. Section 5.3.	UCE1
MLE	Instantiation of the ROM convergent encryption scheme of [56, 17], showing this in-use message-locked encryption scheme meets the IND$-CDA goal of [17]. Section 5.4.	UCE1
HC	Any UCE1-secure family is hardcore for any one-way function and allows for extraction of any number of hardcore bits. Section 5.1.	UCE1
BR93 PKE	Instantiation of a natural ROM PKE scheme from BR93 [21] showing it is IND-CPA-secure. Section 5.2.	UCE1
PFOB	Instantiation of a ROM point-function obfuscation scheme of [48] to obtain a secure standard-model scheme. Section 5.5.	mUCE1
KDM	Instantiation of the ROM BRS scheme [25] to get an efficient and natural standard-model symmetric scheme for encryption of key-dependent messages. Section 5.6.	mUCE1
RKA	An efficient standard-model symmetric encryption scheme providing best-possible security against related-key attacks. Section 5.7.	mUCE1
CIH	Construction from UCE1 of correlation-intractable hash functions meeting the strongest notion of [70]. Section 5.10.	UCE1
STORE	Instantiation of a natural ROM proof of storage scheme from [90]. Section 5.9.	UCE1
OAEP	IND-CPA-KI security of OAEP [22] assuming partial one-wayness (with UCE1) or one-wayness (with UCE2) of the underlying trapdoor function. Sections 5.8 and 6.3.	UCE1/2
GB	Standard-model adaptively secure garbling with short tokens. Section 6.4.	UCE2

Figure 2: **Applications of UCE:** We summarize results for different goals, the last column indicating the form of UCE used.

UCE2, does not imply CR. We can show that UCE1 does not imply PRF, but whether UCE2 implies PRF is open.

1.4 Applications

Fig. 2 summarizes the applications we now discuss.

1. Deterministic PKE. The EwH deterministic PKE (D-PKE) ROM scheme of BBO07 [11] encrypts message m under public key ek by applying the RO to $ek\|m$ to get coins r and then encrypting m with an IND-CPA PKE scheme under ek and coins r. They showed that this achieved their PRIV notion of security in the ROM. Our instantiation adds $hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$ to the public key and then replaces the RO with $\mathsf{H.Ev}(hk, \cdot)$. We show that if H is UCE1-secure then this instantiated D-PKE scheme is PRIV-secure in the standard model. This is not only the first standard-model PRIV-secure scheme (previous standard-model D-PKE schemes achieve only restricted notions of blocksource-PRIV-security [28, 14, 41, 60]) but also the most practical. Our proof makes crucial use of the equivalence between PRIV and an indistinguishability-style notion IND of D-PKE security [14].
2. Message-locked encryption. In convergent encryption (CE) [56, 17], message m is encrypted using a deterministic symmetric encryption scheme with the key derived, via a RO, from the message itself. CE is the most natural and prominent embodiment of message-locked encryption (MLE) and is in current use by commercial cloud-storage providers to provide secure deduplicated storage. The scheme is shown in [17] to meet, in the ROM, a formal notion of MLE-security called PRV$-CDA. We instantiate with a UCE1-family, putting the key in public parameters, and show that the resulting MLE scheme

is PRV$-CDA in the standard model.

3. Hardcore functions. A RO is an ideal hardcore function, with $\mathsf{RO}(x)$ returning any number of bits that remain pseudorandom given $f(x)$ where f is one-way. UCE1 families can securely instantiate the RO here, meaning are secure hardcore functions for any one-way function, able to extract as many bits as desired.
4. BR93 PKE. A simple and natural ROM IND-CPA PKE scheme from [21] encrypts m by picking random x and returning $(f(x), \mathsf{RO}(x) \oplus m)$ where f is a trapdoor function in the public key. We show that instantiating the RO with a UCE1-secure family preserves the IND-CPA security.
5. Point-function obfuscation. A *point function* has non-$\perp$ output on just one point. Canetti, Kalai, Varia, and Wichs [48] give a ROM point-function obfuscation scheme. We mUCE1-instantiate their construction to obtain a standard-model point-function obfuscation scheme.
6. KDM-secure SE. Black, Rogaway and Shrimpton (BRS) [25] showed that the following simple and efficient symmetric encryption (SE) scheme is KDM-secure in the ROM: to encrypt message m under key K, pick a random r and return $(r, \mathsf{RO}(r \| K) \oplus m)$. We instantiate by letting the random value r in the BRS scheme take on the role of a fresh hash key, so that, to encrypt m, we pick $hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$ and return $(hk, \mathsf{H.Ev}(hk, K) \oplus m)$. We prove that if H is mUCE1-secure then this instantiated scheme is KDM secure in the standard model. (We achieve non-adaptive KDM security, but this includes popular cases such as key-cycles.) This scheme is more practical than other standard-model KDM-secure encryption schemes such as [39, 5, 9, 84, 4].
7. RKA-secure SE. Symmetric encryption schemes secure against related-key attack (RKA) must preserve security even when encryption is performed under keys derived from the original key by application of a key-deriving function. Previous schemes [6, 20] provided security for algebraic key-deriving functions such as linear or polynomial functions over a keyspace that is a particular group depending on the scheme. We provide a scheme that has "best possible" security, in that key-deriving functions are arbitrary subject only to a condition necessary for security, namely to have unpredictable outputs. Furthermore, in our scheme, keys are binary strings rather than group elements, so we cover the most common practical attacks, such as XORing a constant to the key. We assume only a mUCE1-secure family of functions.
8. Correlation-intractable secure hashing. Goyal, O'Neill and Rao (GOR) introduced the notion of correlated-input hash (CIH) function families [70] and proposed several notions of security for them. GOR provided constructions achieving limited CIH security from the q-DHI assumption of [33] and from RKA-secure blockciphers, but achieving full CIH security in the standard model has remained open. We solve this problem, showing that UCE1-secure function families are selective (pseudorandomness) CIH secure in the terminology of GOR.
9. Secure storage. Ristenpart, Shacham and Shrimpton [90] give a ROM protocol allowing a client to check that a server is storing its file in its entirety, its interest being that constructions indifferentiable from a RO [85, 51] may fail to securely replace the RO. In contrast, we show that UCE1 instantiation succeeds. (Our instantiation lets the challenge in the protocol be a key naming a member of a UCE1-secure family of functions.)
10. OAEP. OAEP [22] has been a benchmark for RO instantiation [30, 29, 78]. We instantiate OAEP by adding $hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$ to the public key and then implementing both the ROs via $\mathsf{H.Ev}(hk, \cdot)$. Under UCE1, we get IND-CPA-KI security under the partial-domain one-wayness, and hence by [59] under standard one-wayness, of RSA; under UCE2 we get it directly under standard one-wayness. IND-CPA-KI is IND-CPA when challenge messages are not allowed to depend on the public key.[2] Kiltz, O'Neill and Smith (KOS) [78] show that RSA-OAEP is IND-CPA-secure if its two ROs are replaced with t-wise independent hash functions and RSA is Φ-hiding [43]. In comparison our results for RSA are under the standard one-wayness assumption.

[2] More precisely, they are not allowed to depend on hk but are allowed to depend on the RSA part of the public key. This limitation arises because in UCE the strings being hashed by the source cannot depend on the hashing key. We note that this UCE feature does not *always* prevent us from achieving full IND-CPA. Indeed, we do achieve it for the BR93 PKE scheme, because there the inputs to the RO do not depend on the messages.

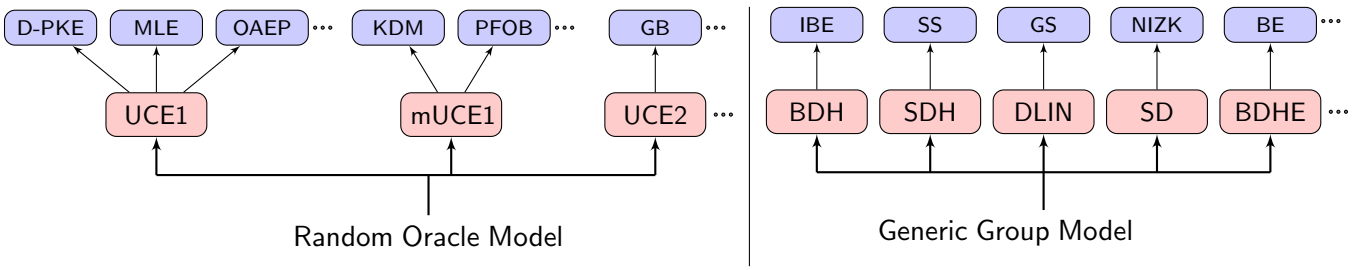

Figure 3: **The layered-cryptography paradigm for the ROM (left) and for pairing-based cryptography (right).** Assumptions are validated in the idealized model and then used to attain end goals entirely in the standard model. SS refers to the short signatures of [32]; BE refers to the broadcast encryption scheme of [37]; NIZK refers to the NIZK arguments of [71]. See text for other abbreviations.

11. Adaptively-secure garbling. Verifiable outsourcing [61], as well as one-time programs [67], call for garbling schemes that are adaptively secure [15]. Standard-model adaptively-secure garbling has however so far been at the cost of large tokens, meaning ones as large as the circuit being garbled [15, 69]. This is not only inefficient but makes the resulting verifiable outsourcing "trivial" in that the client does as much work as the server. We provide a UCE2-based garbling scheme that is adaptively secure and has short tokens. This is the first standard-model garbling scheme with these properties and it results in the first non-trivial instantiation of the outsourcing scheme of [61]. Our garbling scheme is obtained by instantiating a ROM garbled circuit construction of [88].

1.5 Constructing UCE-secure families

We provide a ROM construction of a family of functions shown to achieve both mUCE1 and mUCE2. (And thereby UCE1 and UCE2.)

This at first may seem like a step backwards; wasn't the purpose of UCE to avoid the ROM? As explained in more depth in Section 2, it is a step forward because the security we require from families of functions in implementations has moved from something heuristic and vague, namely to "behave like a RO," to something well defined, namely to be UCE-secure.

In practice we would aim to instantiate UCE-secure families via blockciphers or cryptographic hash functions. We explain that direct instantiation with a blockcipher (e.g. AES) is not secure due to the invertibility of the blockcipher. Cryptographic hash functions, being unkeyed, do not directly provide instantiations either. We suggest instead to use HMAC [13, 10].

2 Perspective and discussion

We explain why UCE is step forward even if we can (currently) only achieve it in the ROM, and how UCE relates to other assumptions.

Layered cryptography. Currently, RO-based design *directly* proves schemes (for end goals) secure in the ROM. We are instead advocating and using what we call a *layered* approach. In this approach, *base primitives* with standard-model security definitions are validated in the ROM. End goals are then reached from the base primitives purely in the standard model, the ROM being entirely dispensed with in the second step. This is illustrated in Fig. 3. We are showing that UCE can function as such a base primitive, and a powerful one at that, since many goals may be reached from it.

In implementations, we would continue to instantiate families assumed UCE-secure via appropriately-keyed cryptographic hash functions, but we claim this layered approach is still an important advance on direct ROM-based design. This is because the property we desire from the object (family of functions) actually being used in the implementation has moved from something heuristic and vague ("behave like a random oracle") to something precise and meaningful (be UCE-secure). Cryptanalytic validation of UCE

security, even if difficult, is at least meaningful, while cryptanalytic evaluation of "behaving like a RO" is not even meaningful because the phrase in quotes is not well defined.

We make an analogy with pairing-based cryptography. Here we have seen the proposal of a large number of standard-model assumptions, including BDH [36], DLIN [35], SDH [35], BDHE [34] and SD (Subgroup Decision) [38] to name just a small fraction. These assumptions are (ubiquitously) validated in the generic-group model, end goals then reached from the assumptions in the standard model. But the generic-group model is subject to issues, critiques and counter-examples analogous to those for the ROM, if not worse [57, 53]. We believe that the (deserved) success and acceptance of pairing-based cryptography, and that it has not come under as much fire as ROM-based cryptography, are due in part to what, in our terminology, is its layered approach (again illustrated in Fig. 3). Namely, schemes for end goals, rather than being directly validated in the generic model (the un-layered or direct approach), are based on standard-model assumptions that are themselves validated in the generic-group model and amenable to cryptanalysis.

It is perhaps curious that the layered approach has not been explicitly articulated and widely used for ROM-based cryptography, while it has been widely used (even if not explicitly articulated) in pairing-based cryptography. The benefits are identical in the two cases. We view our work as making layered cryptography an explicit approach for ROM-based design.

Unification. The ability to UCE-instantiate the RO across different schemes and goals shows that these ROM schemes have something in common, meaning they are in some way relying on the same attributes of the RO for security. This was not obvious to us prior to conceiving UCE. UCE thus leads to a better understanding of what properties of the RO schemes rely on, and enables us to unify different usages under a common umbrella.

Assumption degree and achieving UCE. In the UCE definition, the adversary consists of stages (source and distinguisher) that (due to the unpredictability condition) cannot completely share state. We refer to this as a second-degree assumption, as opposed to a first-degree assumption, where the adversary is a single algorithm. Put another way, a first-degree assumption can be specified via an interaction (game) between an adversary and a challenger. (In some places [74, 87] this is called a "standard" assumption, but we think this is less clear than "first degree.") UCE cannot. This distinction is crucial to its power and to why various negative results are circumvented. Thus, Wichs [92] shows that first-degree assumptions do not suffice for PRIV-secure D-PKE, but our proof that UCE does suffice is not a contradiction because UCE is not first-degree.

A corollary is that UCE itself cannot be achieved based on first-degree assumptions. This does not necessarily mean that UCE is an implausible assumption. (A second-degree assumption does not have to be implied by a first-degree one to be true.)

Without ROs. There is a large body of work on cryptography without random oracles. (A Google Scholar search shows 286 papers with the phrase "without random oracles" in the title, and 3,640 with this phrase somewhere in the paper, as of June 6, 2013.) More often than not, the without-RO schemes of such works are completely different from, and less efficient than, RO ones. While UCE also serves, of course, to get without-RO schemes, it does more, permitting these to be obtained by actual instantiation of the RO in a ROM scheme, so that the efficiency and practicality of the starting ROM scheme is preserved.

Discussion, limitations and related work. That the source adversary in UCE does not get the key is important in avoiding impossibility results like those in [46, 85]. (For example, UCE does not imply correlation intractability as defined, and shown to be unachievable in the standard model, by [46].)

UCE is not a panacea in the sense that it can replace ROs everywhere. UCE helps in cases where the RO is applied to inputs hidden (at least in part) from the adversary. As far as we know, UCE will not help for tasks like instantiating the RO in FDH signatures [23]. This is consistent with impossibility results [54].

Curiously, UCE-based proofs for instantiated schemes are sometimes simpler than the proofs for the starting ROM schemes. This is the case for D-PKE. The intuition for the ROM security of the EwH scheme of [11] is simple enough, but a rigorous ROM proof is in our view less straightforward than our proof of Theorem 5.3 for the UCE1-based instantiation of EwH.

The term "computational extractor" has been used for primitives that extract pseudorandomness from distributions that have computational min-entropy [52, 81, 58]. A UCE-secure family instead extracts pseudorandomness from unpredictable distributions. These may or may not have computational min-entropy in the formal sense the latter is defined [72] but we view unpredictability as we defined it as another computational relaxation of min-entropy so preserved the "extractor" name. "Universal" refers to the ability to do this from *any* starting (unpredictable) distribution.

Programmable hash functions [75] are an information-theoretic tool that in some way mimic the "programmability" of ROs and were used by [75] to build signature schemes with short signatures in the standard model. They do not serve to instantiate ROs in the kinds of applications we consider. Several works [62, 31] define new security properties of hash functions tailored for their own particular applications.

Future directions and open questions. Achieving UCE under other assumptions is an interesting and important direction for future work. We suggest to begin by targeting restricted versions of UCE, starting with independent sources (ones whose oracle queries consist of uniform, independent strings) and moving on to block sources (each oracle query retains high min-entropy even given previous ones). In these cases, we may hope to achieve security under first-degree assumptions. An indication (but not a proof) that this may be possible is that D-PKE that is PRIV-secure for these kinds of sources has been achieved under first-degree assumptions [14, 28, 41, 60]. Full UCE security would, of course, require second-degree assumptions.

Another interesting direction is to find further applications of UCE, in particular to instantiate ROs or build schemes without random oracles. There are many potential targets. As an example, we ask whether it is possible to UCE-instantiate the ROM schemes of [40] for function-private IBE, at least for non-adaptive security, to obtain standard-model schemes.

UCE is a framework permitting definitional variants beyond the four we have formalized. One could define variants with extractability, which may be useful for further applications. A tempting variant is to allow some communication back from the distinguisher to the source. This opens the door to many interesting applications, but is a dangerous path to tread, for any version we, at least, have formalized, we have also broken, even for forms of communication that seemed highly restricted. (By "broken" we mean that we have found attacks showing that *no* family can meet the definition.) Beyond this the larger agenda is to further layered cryptography for the ROM by finding other standard-model definitions for hash families that permit these families to instantiate ROs in applications. A target of particular interest is instantiation of the RO in FDH signatures [23, 50, 54, 77].

Determining the relationship between UCE1 and mUCE1 is an interesting open question. That is, given a family of functions H that is UCE1-secure, is it mUCE1-secure? We conjecture that the answer is "yes" for a version of mUCE1 in which the number of keys is a constant independent of the adversary, but in general the answer is "no." To demonstrate the latter would require a counter-example, meaning a family H that is UCE1-secure but not mUCE1-secure. The corresponding questions can of course be posed for UCE2 as well.

We have shown in Proposition 4.2 that UCE1-security does not imply PRF security. We leave open whether UCE2-security implies PRF security. We conjecture that the answer is "no." Demonstrating this requires exhibiting H that is UCE2-secure but fails to be PRF-secure.

We have shown in Section 4.5 how to build a VOL (variable output length) UCE1 family from a FOL (fixed output length) UCE1 family. An interesting direction is the analog for input lengths, namely the problem usually called domain extension: build a UCE1-secure family taking arbitrary-length inputs from one taking fixed-length inputs. Domain extension has been a popular topic for many primitives in the past.

We have suggested in Section 7.2 that HMAC [13, 82] is a candidate for a practical instantiation of a UCE-secure family. An interesting problem is to either refute this via an attack or validate it in an idealized model, namely prove that HMAC meets our ROM-based UCE definitions of Section 7.1 assuming the compression function underlying the hash function is ideal. Since we have shown in Section 7.1 that a RO is effectively UCE-secure, one might hope to obtain the desired result for HMAC based on the indifferentiability of the latter from a RO [55], but, as per [90], indifferentiability [85, 51] may not suffice since UCE is a second-degree primitive whose security definition is underlain by a multi-stage game. Thus

some other approach or a direct analysis may be needed.

3 Preliminaries

NOTATION. By $\lambda \in \mathbb{N}$ we denote the security parameter. If $n \in \mathbb{N}$ then 1^n denotes its unary representation. We denote the size of a finite set X by $|X|$, the number of coordinates of a vector $\mathbf{x}$ by $|\mathbf{x}|$, and the length of a string $x \in \{0,1\}^*$ by $|x|$. We let ε denote the empty string. If x is a string then $x[i]$ is its i-th bit and $x[1,\ell] = x[1]\ldots x[\ell]$. By $x\|y$ we denote the concatenation of strings x, y. If x is a string and $0 \le \ell \le |x|$, then $y\|_\ell z \leftarrow x$ denotes letting y and z be strings such that $|y| = \ell$ and $y\|z = x$. If X is a finite set, we let $x \leftarrow\!\!\$\, X$ denote picking an element of X uniformly at random and assigning it to x. Algorithms may be randomized unless otherwise indicated. Running time is worst case. "PT" stands for "polynomial-time," whether for randomized algorithms or deterministic ones. If A is an algorithm, we let $y \leftarrow A(x_1,\ldots;r)$ denote running A with random coins r on inputs $x_1,\ldots$ and assigning the output to y. We let $y \leftarrow\!\!\$\, A(x_1,\ldots)$ be the resulting of picking r at random and letting $y \leftarrow A(x_1,\ldots;r)$. We let $[A(x_1,\ldots)]$ denote the set of all possible outputs of A when invoked with inputs $x_1,\ldots$.

For $a, b \in \mathbb{N}$ and $a \le b$, let $[a,b]$ denote the set $\{a, a+1,\ldots,b\}$. We say that $f : \mathbb{N} \to \mathbb{R}$ is negligible if for every polynomial p, there exists $n_p \in \mathbb{N}$ such that $f(n) < 1/p(n)$ for all $n > n_p$. An adversary is an algorithm or a tuple of algorithms.

GAMES. We use the code based game playing framework of [24] augmented with explicit MAIN procedures as in [90]. (See Fig. 4 for an example.) By $\mathrm{G}^A(\lambda) \Rightarrow y$ we denote the event that the execution of game G with adversary A and security parameter λ results in output y, the game output being what is returned by MAIN. We abbreviate $\mathrm{G}^A(\lambda) \Rightarrow \mathsf{true}$ by $\mathrm{G}^A(\lambda)$, the occurrence of this event meaning that A wins the game. The running time of an adversary A in a game G is a function that associates to $\lambda \in \mathbb{N}$ the worst-case number of steps executed in $\mathrm{G}^A(\lambda)$.

4 UCE1

We define UCE1 security of a family of functions. We provide some basic results, including a simplified but equivalent form of unpredictability. We discuss the relation of UCE1 to some other standard notions of security for function families. We also define mUCE1.

4.1 Syntax

A family of functions H specifies the following. On input the unary representation 1^λ of the security parameter $\lambda \in \mathbb{N}$, key generation algorithm $\mathsf{H.Kg}$ returns a key $hk \in \{0,1\}^{\mathsf{H.kl}(\lambda)}$, where $\mathsf{H.kl}\colon \mathbb{N} \to \mathbb{N}$ is the keylength function associated to H. The deterministic, PT evaluation algorithm $\mathsf{H.Ev}$ takes 1^λ, a key $hk \in [\mathsf{H.Kg}(1^\lambda)]$, an input $x \in \{0,1\}^*$ with $|x| \in \mathsf{H.IL}(\lambda)$, and a unary encoding 1^ℓ of an output length $\ell \in \mathsf{H.OL}(\lambda)$ to return an output $\mathsf{H.Ev}(1^\lambda, hk, x, 1^\ell) \in \{0,1\}^\ell$. (The syntax in the Introduction had simplified by dropping the first and last inputs.) Here $\mathsf{H.IL}$ is the input-length function associated to H, so that $\mathsf{H.IL}(\lambda) \subseteq \mathbb{N}$ is the (non-empty) set of allowed input lengths, and similarly $\mathsf{H.OL}$ is the output-length function associated to H, so that $\mathsf{H.OL}(\lambda) \subseteq \mathbb{N}$ is the (non-empty) set of allowed output lengths. The latter allows us to cover fixed output length (FOL) functions, captured by $\mathsf{H.OL}(\lambda)$ being a set of size one, or variable output length (VOL) functions, where $\mathsf{H.OL}(\lambda)$ could be larger and even be $\mathbb{N}$. We say that H has input-length $\ell\colon \mathbb{N} \to \mathbb{N}$ if $\mathsf{H.IL}(\lambda) = \{\ell(\lambda)\}$ for all $\lambda \in \mathbb{N}$, and if such an ℓ exists we denote it by $\mathsf{H.il}$. We say H has output-length $\ell\colon \mathbb{N} \to \mathbb{N}$ if $\mathsf{H.OL}(\lambda) = \{\ell(\lambda)\}$ for all $\lambda \in \mathbb{N}$, and if such an ℓ exists we denote it by $\mathsf{H.ol}$.

4.2 UCE1 security

We define what it means for a family of functions H to be UCE1-secure. Let S be an adversary called the *source* and D an adversary called the *distinguisher*. We associate to them and H the game $\mathrm{UCE}_{\mathsf{H}}^{S,D}(\lambda)$ of

$\underline{\text{MAIN UCE}^{S,D}_{\mathsf{H}}(\lambda)}$	$\underline{\text{MAIN Pred}^{P}_{S}(\lambda)}$	$\underline{\text{MAIN SPred}^{P'}_{S}(\lambda)}$
$b \leftarrow\!\!\$\ \{0,1\}$; $hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$	$\mathsf{done} \leftarrow \mathsf{false}$; $Q \leftarrow \emptyset$	$Q \leftarrow \emptyset$
$L \leftarrow\!\!\$\ S^{\text{HASH}}(1^\lambda)$	$L \leftarrow\!\!\$\ S^{\text{HASH}}(1^\lambda)$; $\mathsf{done} \leftarrow \mathsf{true}$	$L \leftarrow\!\!\$\ S^{\text{HASH}}(1^\lambda)$
$b' \leftarrow\!\!\$\ D(1^\lambda, hk, L)$	$Q' \leftarrow\!\!\$\ P^{\text{HASH}}(1^\lambda, L)$	$x \leftarrow\!\!\$\ P'(1^\lambda, L)$
Return $(b' = b)$	Return $(Q \cap Q' \neq \emptyset)$	Return $(x \in Q)$
$\underline{\text{HASH}(x, 1^\ell)}$	$\underline{\text{HASH}(x, 1^\ell)}$	$\underline{\text{HASH}(x, 1^\ell)}$
If $T[x,\ell] = \perp$ then	If $\mathsf{done} = \mathsf{false}$ then $Q \leftarrow Q \cup \{x\}$	$Q \leftarrow Q \cup \{x\}$
If $b = 1$ then $T[x,\ell] \leftarrow \mathsf{H.Ev}(1^\lambda, hk, x, 1^\ell)$	If $T[x,\ell] = \perp$ then	If $T[x,\ell] = \perp$ then
Else $T[x,\ell] \leftarrow\!\!\$\ \{0,1\}^\ell$	$T[x,\ell] \leftarrow\!\!\$\ \{0,1\}^\ell$	$T[x,\ell] \leftarrow\!\!\$\ \{0,1\}^\ell$
Return $T[x,\ell]$	Return $T[x,\ell]$	Return $T[x,\ell]$

Figure 4: **Games** UCE**,** Pred **used to define UCE1 security of family of functions** H**, and game** SPred **defining the simplified but equivalent form of unpredictability.** Here S is the source, D is the distinguisher, P is the predictor and P' is the simple predictor.

Fig. 4. The source has access to an oracle HASH and we require that any query $x, 1^\ell$ made to this oracle satisfy $|x| \in \mathsf{H.IL}(\lambda)$ and $\ell \in \mathsf{H.OL}(\lambda)$. When the challenge bit b is 1 (the "real" case) the oracle responds via $\mathsf{H.Ev}$ under a key hk that is chosen by the game and *not* given to the source. When $b = 0$ (the "random" case) it responds as a RO. The source communicates to its accomplice distinguisher a string $L \in \{0,1\}^*$ we call the *leakage*. The distinguisher *does* get the key hk as input and must now return its guess $b' \in \{0,1\}$ for b. The game returns true iff $b' = b$, and the UCE advantage of (S, D) is defined for $\lambda \in \mathbb{N}$ via

$$\mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\lambda) = 2\Pr[\text{UCE}^{S,D}_{\mathsf{H}}(\lambda)] - 1\ . \tag{1}$$

One's first thought may now be to say that H is UCE1-secure if $\mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\cdot)$ is negligible for all PT S and all PT D. But an obvious attack shows that no H can meet this definition. Indeed, S can pick some x and ℓ, let $h \leftarrow \text{HASH}(x, 1^\ell)$ and return leakage $L = (x, h, 1^\ell)$ to D. The latter, knowing hk, can return 1 if $h = \mathsf{H.Ev}(1^\lambda, hk, x, 1^\ell)$ and 0 otherwise. We obtain a meaningful and useful definition of UCE1-security for H by restricting attention to sources that are what we call "unpredictable." The formalization considers game $\text{Pred}^P_S(\lambda)$ of Fig. 4 associated to source S and an adversary P called a *predictor*. Given the leakage, the latter outputs a set Q'. It wins if this set contains any HASH-query of the source. For $\lambda \in \mathbb{N}$ we let

$$\mathsf{Adv}^{\mathsf{pred}}_{S,P}(\lambda) = \Pr[\text{Pred}^P_S(\lambda)]\ .$$

We say that source S is *unpredictable* if $\mathsf{Adv}^{\mathsf{pred}}_{S,P}(\cdot)$ is negligible for all PT predictors P. We stress that in the prediction game, the HASH oracle of the source is a RO like in the random game, and the predictor gets the same oracle. The family H is not involved in this definition; unpredictability is a property of the source. Finally, we say that H is UCE1-secure if $\mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\cdot)$ is negligible for all unpredictable, PT sources S and all PT distinguishers D. It is convenient to let $\mathsf{UCE1}$ denote the set of all function families H that are UCE1-secure.

4.3 Simple unpredictability

Applications of UCE1 will involve proving the unpredictability of sources we construct. This task is simplified by using a simpler formulation of unpredictability, called simple unpredictability, that is equivalent to the original. The formalization considers game $\text{SPred}^{P'}_S(\lambda)$ of Fig. 4 associated to source S and an adversary P' called a *simple predictor*. There are two simplifications: the simple predictor does not have access to the RO HASH, and its output is a single string x rather than a set of strings. It wins if x is a HASH-query of the source. For $\lambda \in \mathbb{N}$ we let

$$\mathsf{Adv}^{\mathsf{spred}}_{P',S}(\lambda) = \Pr[\text{SPred}^{P'}_S(\lambda)]\ .$$

$\underline{\text{MAIN PRF}^A_{\mathsf{H}}(\lambda)}$	$\underline{\text{MAIN CR}^A_{\mathsf{H}}(\lambda)}$
$hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$	$hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$
$b \leftarrow\!\!\$\ \{0,1\}$; $b' \leftarrow\!\!\$\ A^{\text{HASH}}(1^\lambda)$	$(x_0, x_1) \leftarrow\!\!\$\ A(1^\lambda, hk)$
Return $(b' = b)$	If $(x_0 = x_1)$ then return false
$\underline{\text{HASH}(x, 1^\ell)}$	Return $(\mathsf{H.Ev}(1^\lambda, hk, x_0, 1^{\mathsf{H.ol}(\lambda)}) = \mathsf{H.Ev}(1^\lambda, hk, x_1, 1^{\mathsf{H.ol}(\lambda)}))$
If $T[x,\ell] = \perp$ then	
If $b = 1$ then	
$T[x,\ell] \leftarrow \mathsf{H.Ev}(1^\lambda, hk, x, 1^\ell)$	
Else $T[x,\ell] \leftarrow\!\!\$\ \{0,1\}^\ell$	
Return $T[x,\ell]$	

Figure 5: **Games defining PRF and CR security of family of functions H.**

We say that source S is *simple unpredictable* if $\mathsf{Adv}^{\mathsf{spred}}_{P',S}(\cdot)$ is negligible for all PT simple predictors P'. The following says that simple unpredictability is equivalent to unpredictability.

Lemma 4.1 Let S be a source. Then S is unpredictable if and only if it is simple unpredictable.

Proof of Lemma 4.1 : Suppose P' is a simple predictor. Let $P^{\text{HASH}}(1^\lambda, L)$ run $x \leftarrow\!\!\$\ P'(1^\lambda, L)$ and return $\{x\}$. Then $\mathsf{Adv}^{\mathsf{spred}}_{P',S}(\cdot) \le \mathsf{Adv}^{\mathsf{pred}}_{S,P}(\cdot)$. This shows that if S is unpredictable then it is also simple unpredictable. Turning to the converse, let P be a PT predictor. We may assume wlog that S and P never repeat HASH queries. We may also assume wlog that the output Q' of P contains every x for which there exists ℓ such that HASH-query $(x, 1^\ell)$ was made by P. (This is wlog because we can modify P to include all such x in Q'.) Let q be a polynomial that bounds the number of elements in the output Q' of P. Game $\text{G}_1^{S,P}(\lambda)$ below includes the boxed code while game $\text{G}_2^{S,P}(\lambda)$ does not:

$\underline{P'(1^\lambda, L)}$	$\underline{\text{MAIN } \boxed{\text{G}_1^{S,P}(\lambda)}, \text{G}_2^{S,P}(\lambda)}$
$Q' \leftarrow\!\!\$\ P^{\text{HASHSIM}}(1^\lambda, L)$; $x \leftarrow\!\!\$\ Q'$; Return x	$Q \leftarrow \emptyset$; $L \leftarrow\!\!\$\ S^{\text{HASH}_1}(1^\lambda)$; $Q' \leftarrow\!\!\$\ P^{\text{HASH}_2}(1^\lambda, L)$
$\underline{\text{HASHSIM}(x, 1^\ell)}$	Return $(Q \cap Q' \neq \emptyset)$
$y \leftarrow\!\!\$\ \{0,1\}^\ell$; Return y	$\underline{\text{HASH}_1(x, 1^\ell)}$
	$Q \leftarrow Q \cup \{x\}$; $T[x,\ell] \leftarrow\!\!\$\ \{0,1\}^\ell$;
	Return $T[x,\ell]$
	$\underline{\text{HASH}_2(x, 1^\ell)}$
	If $x \in Q$ then $\mathsf{bad} \leftarrow \mathsf{true}$; $\boxed{\text{Return } T[x,\ell]}$
	$y \leftarrow\!\!\$\ \{0,1\}^\ell$; Return y

Game $\text{G}_1^{S,P}(\lambda)$ is identical to $\text{Pred}_S^P(\lambda)$, except that it separates the HASH procedures used by S and P, while maintaining consistency. Setting bad has no effect on the outcome of the game. Games $\text{G}_1^{S,P}(\lambda)$ and $\text{G}_2^{S,P}(\lambda)$ are identical-until-bad. From the fundamental lemma of game-playing [24],

$$\mathsf{Adv}^{\mathsf{pred}}_{S,P}(\cdot) = \Pr[\text{G}_1^{S,P}(\cdot)] \le \Pr[\text{G}_2^{S,P}(\cdot)] + \Pr[\text{G}_2^{S,P}(\cdot) \text{ sets } \mathsf{bad}].$$

From the assumption that Q' contains every x such that P queried some $(x, 1^\ell)$ to HASH, if game G_2 sets bad then P will surely win. Hence $\Pr[\text{G}_2^{S,P}(\cdot) \text{ sets } \mathsf{bad}] \le \Pr[\text{G}_2^{S,P}(\cdot)]$. Now, consider the simple predictor P' as above. Then

$$\mathsf{Adv}^{\mathsf{spred}}_{P',S}(\cdot) = \frac{1}{q}\Pr[\text{G}_2^{S,P}(\cdot)] \ge \frac{1}{2q}\mathsf{Adv}^{\mathsf{pred}}_{S,P}(\cdot)\ .$$

This concludes the proof. ∎

4.4 Relations

We look at how UCE1 relates to standard notions of security for families of hash functions, namely PRF and CR.

Definitions. We begin by recalling the definitions. Let H be a hash family with output length $\mathsf{H.ol}$. We say that H is PRF-secure if $\mathsf{Adv}^{\mathsf{prf}}_{\mathsf{H},A}(\cdot)$ is negligible for all PT A, where $\mathsf{Adv}^{\mathsf{prf}}_{\mathsf{H},A}(\lambda) = 2\Pr[\mathrm{PRF}^A_{\mathsf{H}}(\lambda)] - 1$ and game $\mathrm{PRF}^A_{\mathsf{H}}(\lambda)$ is shown in Fig. 5. Here we require that a query (x, ℓ) to Hash satisfy $|x| \in \mathsf{H.IL}(\lambda)$ and $\ell = \mathsf{H.ol}(\lambda)$. We say that H is collision-resistant (CR) if $\mathsf{Adv}^{\mathsf{cr}}_{\mathsf{H},A}(\cdot)$ is negligible for all PT A, where $\mathsf{Adv}^{\mathsf{cr}}_{\mathsf{H},A}(\lambda) = \Pr[\mathrm{CR}^A_{\mathsf{H}}(\lambda)]$ and game $\mathrm{CR}^A_{\mathsf{H}}(\lambda)$ is shown in Fig. 5. Here we require that $|x_0|, |x_1| \in \mathsf{H.IL}(\lambda)$. Let PRF be the set of all families H that are PRFs and CR the set of all H that are collision-resistant.

Results. The following says that UCE1-security neither implies, nor is implied by, PRF-security, and similarly for collision resistance. In any non-containment $\mathsf{B} \not\subseteq \mathsf{A}$, we assume $\mathsf{B} \neq \emptyset$. This establishes some of the claims on the right side of Fig. 1. (The figure does not show the $\mathsf{UCE1} \not\subseteq \mathsf{CR}$ relation that is part (3) in Proposition 4.2, for this is in fact implied by the stronger $\mathsf{UCE2} \not\subseteq \mathsf{CR}$ of Proposition 6.2 coupled with the $\mathsf{UCE2} \subseteq \mathsf{UCE1}$ of Proposition 6.1, but we prove $\mathsf{UCE1} \not\subseteq \mathsf{CR}$ anyway below since the proof is simple and instructive.)

Proposition 4.2 (1) $\mathsf{UCE1} \not\subseteq \mathsf{PRF}$ (2) $\mathsf{PRF} \not\subseteq \mathsf{UCE1}$ (3) $\mathsf{UCE1} \not\subseteq \mathsf{CR}$ (4) $\mathsf{CR} \not\subseteq \mathsf{UCE1}$.

Proof of Proposition 4.2: For simplicity, we only consider hash families of fixed input and output length. Intuitively, the reason (1) is true is that a UCE1-secure family could map a particular input, say $0^{\mathsf{H.il}(\lambda)}$, to $0^{\mathsf{H.ol}(\lambda)}$, under all keys. This clearly violates PRF-security but would not contradict UCE1-security because the "bad" input is predictable. The counter-example for (3) is a family H where $\mathsf{H}(1^\lambda, hk, \cdot, 1^{\mathsf{H.ol}(\lambda)})$ maps $0^{\mathsf{H.il}(\lambda)}$ and $1^{\mathsf{H.il}(\lambda)}$ to the same output. Collision-resistance obviously fails, but since the "bad" inputs are predictable, UCE1-security can be retained. Formally, for parts (1) and (3), let H be a UCE1-secure hash family. Define $\overline{\mathsf{H}}$ as follows. Let $\overline{\mathsf{H}}.\mathsf{il} = \mathsf{H.il}$; let $\overline{\mathsf{H}}.\mathsf{ol} = \mathsf{H.ol}$; let $\overline{\mathsf{H}}.\mathsf{Kg} = \mathsf{H.Kg}$; and let $\overline{\mathsf{H}}.\mathsf{Ev}$ be as shown on the left below:

$\underline{\overline{\mathsf{H}}.\mathsf{Ev}(1^\lambda, hk, x, 1^{\mathsf{H.ol}(\lambda)})}$
$y \leftarrow \mathsf{H.Ev}(1^\lambda, hk, x, 1^{\mathsf{H.ol}(\lambda)})$
If $x \in \{0^{\mathsf{H.il}(\lambda)}, 1^{\mathsf{H.il}(\lambda)}\}$ then
  $y \leftarrow 0^{\mathsf{H.ol}(\lambda)}$
Return y

$\underline{S^{\mathrm{Hash}}(1^\lambda)}$
$L \leftarrow\!\!\$\ \overline{S}^{\mathrm{HashSim}}(1^\lambda)$; Return L
$\underline{\mathrm{HashSim}(x, 1^{\mathsf{H.ol}(\lambda)})}$
If $x \in \{0^{\mathsf{H.il}(\lambda)}, 1^{\mathsf{H.il}(\lambda)}\}$ then
  Return $0^{\mathsf{H.ol}(\lambda)}$
Else return $\mathrm{Hash}(x, 1^{\mathsf{H.ol}(\lambda)})$

$\underline{\mathrm{Main}\ \mathrm{G}_1^{\overline{S},D}(\lambda)\,,\ \boxed{\mathrm{G}_2^{\overline{S},D}(\lambda)}}$
$L \leftarrow\!\!\$\ \overline{S}^{\mathrm{HashSim}}(1^\lambda)$; $b' \leftarrow\!\!\$\ D(1^\lambda, L)$
Return $(b' = 0)$
$\underline{\mathrm{HashSim}(x, 1^{\mathsf{H.ol}(\lambda)})}$
If $x \in \{0^{\mathsf{H.il}(\lambda)}, 1^{\mathsf{H.il}(\lambda)}\}$ then
  $\mathsf{bad} \leftarrow \mathsf{true}$; $\boxed{T[x] \leftarrow 0^{\mathsf{H.ol}(\lambda)}}$
If $T[x] \neq \perp$ then $T[x] \leftarrow\!\!\$\ \{0,1\}^{\mathsf{H.ol}(\lambda)}$
Return $T[x]$

We claim that $\overline{\mathsf{H}} \notin \mathsf{PRF}$ and $\overline{\mathsf{H}} \notin \mathsf{CR}$ but $\overline{\mathsf{H}} \in \mathsf{UCE1}$, which establishes (1) and (3). The reason $\overline{\mathsf{H}} \notin \mathsf{PRF}$ is that an adversary can obtain an advantage of $1/2$ by querying $0^{\mathsf{H.il}(\lambda)}$ to Fn and returning 1 if and only if the first bit of the result is 0. The reason $\overline{\mathsf{H}} \notin \mathsf{CR}$ is that an adversary can output $(0^{\mathsf{H.il}(\lambda)}, 1^{\mathsf{H.il}(\lambda)})$ to win with advantage 1. To see that $\overline{\mathsf{H}} \in \mathsf{UCE1}$, let $\overline{S}$ be an unpredictable PT source, and D be a PT distinguisher. Consider the source S constructed above. Since $\overline{S}$ is unpredictable, so is S. Let $P^{\mathrm{Hash}}(1^\lambda, L)$ be a predictor that always outputs $\{0^{\mathsf{H.il}(\lambda)}, 1^{\mathsf{H.il}(\lambda)}\}$ regardless of the leakage L. Consider games $\mathrm{G}_1^{\overline{S},D}(\lambda)$ and $\mathrm{G}_2^{\overline{S},D}(\lambda)$ above. Let b and c be the challenge bits of game $\mathrm{UCE}^{\overline{S},D}_{\overline{\mathsf{H}}}(\lambda)$ and game $\mathrm{UCE}^{S,D}_{\mathsf{H}}(\lambda)$ respectively. The two

games are identical-until-bad so by the Fundamental Lemma of Game Playing [24] we have:

$$\begin{aligned}
\mathsf{Adv}^{\mathsf{uce}}_{\overline{\mathsf{H}},\overline{S},D}(\cdot) &= \Pr[\mathrm{UCE}^{\overline{S},D}_{\overline{\mathsf{H}}}(\cdot) \mid b=1] + \Pr[\mathrm{UCE}^{\overline{S},D}_{\overline{\mathsf{H}}}(\cdot) \mid b=0] - 1 \\
&= \Pr[\mathrm{UCE}^{\overline{S},D}_{\overline{\mathsf{H}}}(\cdot) \mid b=1] + \Pr[\mathrm{G}^{\overline{S},D}_{1}(\cdot)] - 1 \\
&\leq \Pr[\mathrm{UCE}^{\overline{S},D}_{\overline{\mathsf{H}}}(\cdot) \mid b=1] + \Pr[\mathrm{G}^{\overline{S},D}_{2}(\cdot)] - 1 + \Pr[\mathrm{G}^{\overline{S},D}_{1}(\cdot) \text{ sets } \mathsf{bad}] \\
&= \Pr[\mathrm{UCE}^{S,D}_{\mathsf{H}}(\cdot) \mid c=1] + \Pr[\mathrm{UCE}^{S,D}_{\mathsf{H}}(\cdot) \mid c=0] - 1 + \mathsf{Adv}^{\mathsf{pred}}_{\overline{S},P}(\cdot) \\
&= \mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\cdot) + \mathsf{Adv}^{\mathsf{pred}}_{\overline{S},P}(\cdot) \ .
\end{aligned}$$

The claim then follows from the assumption that $\mathsf{H} \in \mathsf{UCE1}$ and that $\overline{S}$ is unpredictable.

For part (2), the counter-example is a PRP, which is also a PRF but will be efficiently invertible given the key. Formally, the assumption $\mathsf{PRF} \neq \emptyset$ implies that there is an $\mathsf{H} \in \mathsf{PRF}$ that is a PRP. (This follows from [83].) This means $\mathsf{H.il} = \mathsf{H.ol}$ and there is a PT deterministic algorithm $\mathsf{H.Inv}$ which is the inverse of $\mathsf{H.Ev}$, meaning $\mathsf{H.Inv}(1^\lambda, hk, \mathsf{H.Ev}(1^\lambda, hk, x, 1^{\mathsf{H.ol}(\lambda)}), 1^{\mathsf{H.ol}(\lambda)}) = x$ for all $\lambda \in \mathbb{N}$, all $hk \in [\mathsf{H.Kg}(1^\lambda)]$ and all $x \in \{0,1\}^{\mathsf{H.il}(\lambda)}$. To show that $\mathsf{H} \notin \mathsf{UCE1}$, consider source S and distinguisher D below, where $x[1]$ denotes the first bit of x:

$\underline{S^{\mathrm{HASH}}(1^\lambda)}$	$\underline{D(1^\lambda, hk, L)}$
$x \leftarrow\!\!\$\ \{0,1\}^{\mathsf{H.il}(\lambda)}$; $y \leftarrow \mathrm{HASH}(x, 1^{\mathsf{H.ol}(\lambda)})$	$(d, y) \leftarrow L$; $x \leftarrow \mathsf{H.Inv}(1^\lambda, hk, y, 1^{\mathsf{H.ol}(\lambda)})$
$L \leftarrow (x[1], y)$; Return L	If $x[1] = d$ then return 1 else return 0

The source S is unpredictable, but $\mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\cdot) = 1/2$.

For part (4), the counter-example is a collision-resistant family all of whose function-outputs have some common structure, for example beginning with a 1-bit, which is enough to violate UCE1-security. Formally, let $\mathsf{H} \in \mathsf{CR}$ and define $\overline{\mathsf{H}}$ as follows: let $\overline{\mathsf{H}}.\mathsf{il} = \mathsf{H.il}$; let $\overline{\mathsf{H}}.\mathsf{ol} = \mathsf{H.ol} + 1$; let $\overline{\mathsf{H}}.\mathsf{Kg} = \mathsf{H.Kg}$; and let $\overline{\mathsf{H}}.\mathsf{Ev}(1^\lambda, hk, x, 1^{\overline{\mathsf{H}}.\mathsf{ol}(\lambda)}) = 1 \,\|\, \mathsf{H.Ev}(1^\lambda, hk, x, 1^{\overline{\mathsf{H}}.\mathsf{ol}(\lambda)})$ for all $\lambda \in \mathbb{N}$, all $hk \in [\mathsf{H.Kg}(\lambda)]$ and all $x \in \{0,1\}^{\mathsf{H.il}(\lambda)}$. Then $\overline{\mathsf{H}} \in \mathsf{CR}$. On the other hand, we claim that $\overline{\mathsf{H}} \notin \mathsf{UCE1}$. Let source S pick $x \leftarrow\!\!\$\ \{0,1\}^{\mathsf{H.il}(\lambda)}$ and return the first bit of $\mathrm{HASH}(x, 1^{\overline{\mathsf{H}}.\mathsf{ol}(\lambda)})$ as the leakage. Then S is unpredictable. Let D be a distinguisher that returns the leakage L. Then $\mathsf{Adv}^{\mathsf{uce}}_{\overline{\mathsf{H}},S,D}(\lambda) = 1/2$. ▮

We remark that a trivial counter-example for (3) is a family $\mathsf{H} \in \mathsf{UCE1}$ with $\lambda \in \mathsf{H.IL}(\lambda)$ and $\mathsf{H.ol}(\lambda) = 1$ for all $\lambda \in \mathbb{N}$, such a family trivially not being in CR. The above counter-example $\overline{\mathsf{H}}$ is more meaningful because $2^{-\overline{\mathsf{H}}.\mathsf{ol}}$ could be negligible.

4.5 From FOL to VOL

We show that we can build a UCE1-secure family with variable output length (VOL) from a UCE1-secure family with fixed output length (FOL). The construction is simple, namely to run the FOL evaluation algorithm in counter mode. Details follow.

Let H be the given FOL function family, having output length $\mathsf{H.ol}$. For simplicity assume $\mathsf{H.IL}(\lambda) = \mathbb{N}$ for all $\lambda \in \mathbb{N}$, meaning inputs of any length are allowed. We build a VOL function family $\overline{\mathsf{H}} = \mathsf{Extend}[\mathsf{H}]$, also with $\overline{\mathsf{H}}.\mathsf{IL}(\lambda) = \mathbb{N}$, but now with $\overline{\mathsf{H}}.\mathsf{OL}(\lambda)$ also equal to $\mathbb{N}$ for all $\lambda \in \mathbb{N}$, meaning output lengths can be arbitrary. We let $\overline{\mathsf{H}}.\mathsf{Kg} = \mathsf{H.Kg}$, meaning keys for the new family are those of the old family. The new evaluation algorithm $\overline{\mathsf{H}}.\mathsf{Ev}$ is described in Fig. 6.

Theorem 4.3 If $\mathsf{H} \in \mathsf{UCE1}$, then $\mathsf{Extend}[\mathsf{H}] \in \mathsf{UCE1}$.

Proof: Let $\overline{\mathsf{H}} = \mathsf{Extend}[\mathsf{H}]$. Let $\overline{S}$ be an unpredictable source and D a distinguisher. We construct an unpredictable source S such that

$$\mathsf{Adv}^{\mathsf{uce}}_{\overline{\mathsf{H}},\overline{S},D}(\cdot) = \mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\cdot) \ . \tag{2}$$

$\underline{\overline{\mathsf{H}}.\mathsf{Ev}(1^\lambda, hk, x, 1^\ell)}$
$l \leftarrow \lceil \ell / \mathsf{H.ol}(\lambda) \rceil$
For $i = 1, \ldots, l$ do
  $y_i \leftarrow 1^\ell \,\|\, 0 \,\|\, 1^i \,\|\, 0 \,\|\, x$; $h_i \leftarrow \mathsf{H.Ev}(1^\lambda, hk, y_i, 1^{\mathsf{H.ol}(\lambda)})$
$h \leftarrow h_1 \,\|\, \cdots \,\|\, h_l$; Return $h[1, \ell]$

Figure 6: **Construction of a VOL family $\overline{\mathsf{H}}$ from a FOL family H.**

The theorem follows from the assumption that $\mathsf{H} \in \mathsf{UCE1}$. Without loss of generality, we assume that $\overline{S}$ never repeats an oracle query. We build S from $\overline{S}$ as shown below:

$\underline{\text{Source } S^{\text{HASH}}(1^\lambda)}$	$\underline{\text{HASHSIM}(x, 1^\ell)}$
$L \leftarrow\!\!\$\ \overline{S}^{\text{HASHSIM}}(1^\lambda)$	$l \leftarrow \lceil \ell / \mathsf{H.ol}(\lambda) \rceil$
Return L	For $i = 1, \ldots, l$ do
	$y_i \leftarrow 1^\ell \,\|\, 0 \,\|\, 1^i \,\|\, 0 \,\|\, x$; $h_i \leftarrow \text{HASH}(y_i, 1^{\mathsf{H.ol}(\lambda)})$
	$h \leftarrow h_1 \,\|\, \cdots \,\|\, h_l$; Return $h[1, \ell]$

The assumption that $\overline{S}$ never repeats an oracle query implies that the oracle queries made by S are all distinct. (This follows from the way these queries are encoded.) Letting $\overline{b}, b$ denote the challenge bits in games $\text{UCE}^{\overline{S},D}_{\overline{\mathsf{H}}}(\cdot)$ and $\text{UCE}^{S,D}_{\mathsf{H}}(\cdot)$ respectively, we thus have

$$\Pr[\,\text{UCE}^{S,D}_{\mathsf{H}}(\cdot) \,|\, b = 0\,] = \Pr[\,\text{UCE}^{\overline{S},D}_{\overline{\mathsf{H}}}(\cdot) \,|\, \overline{b} = 0\,]$$

$$\Pr[\,\text{UCE}^{S,D}_{\mathsf{H}}(\cdot) \,|\, b = 1\,] = \Pr[\,\text{UCE}^{\overline{S},D}_{\overline{\mathsf{H}}}(\cdot) \,|\, \overline{b} = 1\,]\ .$$

This yields Equation (2). It remains to show that S is unpredictable. By Lemma 4.1 it suffices to show that S is simple-unpredictable. Given a simple predictor P' for S, we build a simple predictor $\overline{P}'$ for $\overline{S}$ such that $\mathsf{Adv}^{\mathsf{spred}}_{P',S}(\cdot) \le \mathsf{Adv}^{\mathsf{spred}}_{\overline{P}',\overline{S}}(\cdot)$. The conclusion follows because we assumed that $\overline{S}$ is unpredictable and Lemma 4.1 says it is thus also simple unpredictable. Predictor $\overline{P}'(1^\lambda, L)$ lets $w \leftarrow\!\!\$\ P'(1^\lambda, L)$. It then parses w as $1^\ell \| 0 \| 1^i \| 0 \| x$ and returns x. The claimed bound is easily verified. ∎

4.6 The UCE framework

UCE1 is part of a broader framework we call UCE. The framework, which we now introduce, allows us to succinctly define many variants.

Let us say that a function family H is $(\mathcal{S}, \mathcal{D})$-UCE-secure, where $\mathcal{S}$ is a class of sources and $\mathcal{D}$ is a class of distinguishers, if the function $\mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\cdot)$ is negligible for all $(S, D) \in \mathcal{S} \times \mathcal{D}$. Let $\mathsf{UCE}[\mathcal{S}, \mathcal{D}]$ be the set of all H that are $(\mathcal{S}, \mathcal{D})$-UCE-secure.

We say that a source S is $\mathcal{P}$-*unpredictable*, where $\mathcal{P}$ is a class of predictors, if $\mathsf{Adv}^{\mathsf{pred}}_{S,P}(\cdot)$ is negligible for all $P \in \mathcal{P}$. We let $\mathsf{Pred}[\mathcal{P}]$ be the class of all $\mathcal{P}$-unpredictable sources. We let $\mathcal{S}^{\mathsf{poly}}, \mathcal{D}^{\mathsf{poly}}$ and $\mathcal{P}^{\mathsf{poly}}$ be, respectively the class of all PT sources, distinguishers and predictors.

Note that a PT source S is unpredictable (as defined above) exactly if it is in the class $\mathcal{S}^{\mathsf{poly}} \cap \mathsf{Pred}[\mathcal{P}^{\mathsf{poly}}]$, and thus $\mathsf{UCE1} = \mathsf{UCE}[\mathcal{S}^{\mathsf{poly}} \cap \mathsf{Pred}[\mathcal{P}^{\mathsf{poly}}], \mathcal{D}^{\mathsf{poly}}]$. This shows how we recover our basic UCE1-security notion for H given above as a special case of this broader framework.

One way in which the framework is useful is to concisely and precisely define relaxations of basic UCE1-security that arise in applications. As an example, above we have allowed the source to query its oracle adaptively. But, for some applications, UCE1 relative to non-adaptive sources suffices. Non-adaptive UCE1 security for H can be formalized simply as $(\mathcal{S}^{\mathsf{na}} \cap \mathcal{S}^{\mathsf{poly}} \cap \mathsf{Pred}[\mathcal{P}^{\mathsf{poly}}], \mathcal{D}^{\mathsf{poly}})$-UCE-security where $\mathcal{S}^{\mathsf{na}}$ is the class of all non-adaptive sources. Similarly, we may define UCE1-security for block sources. Another way in which it is useful is to derive relations between notions of UCE security, for which the following obvious claim is useful:

Proposition 4.4 Suppose $\mathcal{S}_1 \subseteq \mathcal{S}_2$. Then $\mathsf{UCE}[\mathcal{S}_2, \mathcal{D}] \subseteq \mathsf{UCE}[\mathcal{S}_1, \mathcal{D}]$.

$\underline{\text{MAIN mUCE}^{S,D}_{\mathsf{H}}(\lambda)}$	$\underline{\text{MAIN mPred}^{P}_{S}(\lambda)}$	$\underline{\text{MAIN mSPred}^{P'}_{S}(\lambda)}$
$(1^n, t) \leftarrow\!\!\$\ S(1^\lambda, \varepsilon)$	$(1^n, t) \leftarrow\!\!\$\ S(1^\lambda, \varepsilon)$	$(1^n, t) \leftarrow\!\!\$\ S(1^\lambda, \varepsilon)$
For $i = 1$ to n do $\mathbf{hk}[i] \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$	$\mathsf{done} \leftarrow \mathsf{false}$; $Q \leftarrow \emptyset$	$Q \leftarrow \emptyset$
$b \leftarrow\!\!\$\ \{0,1\}$; $L \leftarrow\!\!\$\ S^{\text{HASH}}(1^n, t)$	$L \leftarrow\!\!\$\ S^{\text{HASH}}(1^n, t)$; $\mathsf{done} \leftarrow \mathsf{true}$	$L \leftarrow\!\!\$\ S^{\text{HASH}}(1^n, t)$
$b' \leftarrow\!\!\$\ D(1^\lambda, \mathbf{hk}, L)$	$Q' \leftarrow\!\!\$\ P^{\text{HASH}}(1^\lambda, 1^n, L)$	$x \leftarrow\!\!\$\ P'(1^\lambda, 1^n, L)$
Return $(b' = b)$	Return $(Q \cap Q' \neq \emptyset)$	Return $(x \in Q)$
$\underline{\text{HASH}(x, 1^\ell, i)}$	$\underline{\text{HASH}(x, 1^\ell, i)}$	$\underline{\text{HASH}(x, 1^\ell, i)}$
If $T[x, \ell, i] = \perp$ then	If $\mathsf{done} = \mathsf{false}$ then $Q \leftarrow Q \cup \{x\}$	$Q \leftarrow Q \cup \{x\}$
If $b = 1$ then $T[x, \ell, i] \leftarrow \mathsf{H.Ev}(1^\lambda, \mathbf{hk}[i], x, 1^\ell)$	If $T[x, \ell, i] = \perp$ then	If $T[x, \ell, i] = \perp$ then
Else $T[x, \ell, i] \leftarrow\!\!\$\ \{0,1\}^\ell$	$T[x, \ell, i] \leftarrow\!\!\$\ \{0,1\}^\ell$	$T[x, \ell, i] \leftarrow\!\!\$\ \{0,1\}^\ell$
Return $T[x, \ell, i]$	Return $T[x, \ell, i]$	Return $T[x, \ell, i]$

Figure 7: **Games** mUCE, mPred**, and** mSPred **used to define mUCE1 security of family of functions** H**, and game** mSPred **defining the simplified but equivalent form of unpredictability.** Here S is the multi-key source, D is the distinguisher, P is the predictor and P' is the simple predictor.

4.7 mUCE1 security

In UCE1, there is a single target key hk. Some of our applications will depend on an extension involving multiple keys. Here we define this mUCE1 extension of UCE1.

Let H be a family of functions. Consider game $\text{mUCE}^{S,D}_{\mathsf{H}}(\lambda)$ of Fig. 7 involving a multi-key source S and distinguisher D. Adversary S now begins by returning a unary-encoded integer $n \geq 1$ indicating the number of instances, together with state information t. The game creates n, independent keys. The oracle HASH given to S now allows it to query any instance $i \in [1, n]$ of its choice. As before S returns leakage L based on which the distinguisher D, now given the entire vector $\mathbf{hk}$ of keys, returns its guess bit b'. The mUCE1 advantage of (S, D) is defined for $\lambda \in \mathbb{N}$ by

$$\mathsf{Adv}^{\mathsf{m\text{-}uce}}_{\mathsf{H},S,D}(\lambda) = 2\Pr[\text{mUCE}^{S,D}_{\mathsf{H}}(\lambda)] - 1\ . \tag{3}$$

For $\lambda \in \mathbb{N}$ we let $\mathsf{Adv}^{\mathsf{m\text{-}pred}}_{S,P}(\lambda) = \Pr[\text{mPred}^{P}_{S}(\lambda)]$ where game $\text{mPred}^{P}_{S}(\lambda)$ is in Fig. 7. We say that S is unpredictable if $\mathsf{Adv}^{\mathsf{m\text{-}pred}}_{S,P}(\cdot)$ is negligible for all PT predictors P. We say that H is mUCE-secure if $\mathsf{Adv}^{\mathsf{m\text{-}uce}}_{\mathsf{H},S,D}(\cdot)$ is negligible for all unpredictable PT multi-key sources S and all PT distinguishers D. We let $\mathsf{mUCE1}$ denote the set of all function families H that are mUCE-secure.

We say that S is simple-unpredictable if $\mathsf{Adv}^{\mathsf{m\text{-}spred}}_{P',S}(\cdot)$ is negligible for all PT simple predictors P', where $\mathsf{Adv}^{\mathsf{m\text{-}spred}}_{P',S}(\lambda) = \Pr[\text{mSPred}^{P'}_{S}(\lambda)]$ and game $\text{mSPred}^{P'}_{S}(\lambda)$ is in Fig. 7. The following analogue of Lemma 4.1 shows equivalence of simple unpredictability and unpredictability for multi-key sources.

Lemma 4.5 Let S be a multi-key source. Then S is unpredictable if and only if it is simple unpredictable.

The proof of Lemma 4.5 is similar to the proof of Lemma 4.1 and is omitted.

5 Applications of UCE1

We show how UCE1- or mUCE1-secure families can securely instantiate ROs to yield (new) standard-model solutions for a variety of goals. Specifically, we detail the claims of Section 1.4.

5.1 Hardcore functions for any OWF

A hardcore function for a one-way function f extracts from x bits that are indistinguishable from random even given $f(x)$ [26, 94, 68]. The concept has been central in the development of the theory of public-key encryption, and hardcore functions have been sought and found for many specific one-way functions [26,

$\underline{\text{MAIN } \mathrm{HC}^{A}_{\mathsf{F},\mathsf{H}}(\lambda)}$
$b \leftarrow\!\!\$\ \{0,1\}\ ;\ fk \leftarrow\!\!\$\ \mathsf{F.Kg}(1^\lambda)\ ;\ hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$
$x \leftarrow\!\!\$\ \{0,1\}^{\mathsf{F.il}(\lambda)}\ ;\ y \leftarrow \mathsf{F.Ev}(1^\lambda, fk, x, 1^{\mathsf{F.ol}(\lambda)})$
If $b = 1$ then $r \leftarrow \mathsf{H.Ev}(1^\lambda, hk, x, 1^{\mathsf{H.ol}(\lambda)})$ else $r \leftarrow\!\!\$\ \{0,1\}^{\mathsf{H.ol}(\lambda)}$
$b' \leftarrow\!\!\$\ A(1^\lambda, fk, hk, y, r)\ ;$ Return $(b = b')$

Figure 8: **Game defining security of H as a hardcore function for F.**

94, 68, 73, 3]. Goldreich and Levin [65] present a hardcore function able to extract a single bit from any one-way function. But ROs are "ideal" hardcore functions, able to extract as many pseudorandom bits as desired from any one-way function. We show how the RO here can be UCE1-instantiated. Thus, we show that UCE1-secure families are hardcore functions for any one-way function f, allowing us to extract from x any number of bits that remain indistinguishable from random to an adversary given $f(x)$.

DEFINITIONS. Let F be a family of functions with input length F.il and output length F.ol. We say that F is one-way if $\mathsf{Adv}^{\mathsf{ow}}_{\mathsf{F},I}(\cdot)$ is negligible for all PT I, where $\mathsf{Adv}^{\mathsf{ow}}_{\mathsf{F},I}(\lambda) = \Pr[I(1^\lambda, fk, y) = x]$ in the experiment $fk \leftarrow\!\!\$\ \mathsf{F.Kg}(1^\lambda)\ ;\ x \leftarrow\!\!\$\ \{0,1\}^{\mathsf{F.il}(\lambda)}\ ;\ y \leftarrow \mathsf{F.Ev}(1^\lambda, fk, x)$. Let OW be the set of all F that are one-way. Let H be a family of functions with the same input length as F and output length H.ol. We say that H is hardcore for F if $\mathsf{Adv}^{\mathsf{hc}}_{\mathsf{F},\mathsf{H},A}(\cdot)$ is negligible for all PT A, where $\mathsf{Adv}^{\mathsf{hc}}_{\mathsf{F},\mathsf{H},A}(\lambda) = 2\Pr[\mathrm{HC}^{A}_{\mathsf{F},\mathsf{H}}(\lambda)] - 1$ and game $\mathrm{HC}^{A}_{\mathsf{F},\mathsf{H}}(\lambda)$ is in Fig. 8. Let HC[F] be the set of all H that are hardcore for F.

RESULTS. The following says that if F is one-way and H is UCE1-secure then H is hardcore for F.

Theorem 5.1 If $\mathsf{H} \in \mathsf{UCE1}$ and $\mathsf{F} \in \mathsf{OW}$ then $\mathsf{H} \in \mathsf{HC}[\mathsf{F}]$.

Proof of Theorem 5.1: Given a PT adversary A for game $\mathrm{HC}^{A}_{\mathsf{F},\mathsf{H}}(\cdot)$, we build an unpredictable source S and a distinguisher D such that

$$\mathsf{Adv}^{\mathsf{hc}}_{\mathsf{F},\mathsf{H},A}(\cdot) = \mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\cdot)\ . \tag{4}$$

The assumption $\mathsf{H} \in \mathsf{UCE1}$ implies the right-hand side of Equation (4) is negligible, which yields the theorem. The constructions of S and D are shown below:

$\underline{S^{\text{HASH}}(1^\lambda)}$	$\underline{D(1^\lambda, hk, L)}$	$\underline{I(1^\lambda, fk, y)}$
$fk \leftarrow\!\!\$\ \mathsf{F.Kg}(1^\lambda)\ ;\ x \leftarrow\!\!\$\ \{0,1\}^{\mathsf{F.il}(\lambda)}$	$(fk, y, r) \leftarrow L$	$r \leftarrow\!\!\$\ \{0,1\}^{\mathsf{H.ol}(\lambda)}$
$y \leftarrow \mathsf{F.Ev}(1^\lambda, fk, x)\ ;\ r \leftarrow \text{HASH}(x, 1^{\mathsf{H.ol}(\lambda)})$	$b' \leftarrow\!\!\$\ A(1^\lambda, fk, hk, y, r)$	$x' \leftarrow\!\!\$\ P'(1^\lambda, (fk, y, r))$
$L \leftarrow (fk, y, r)\ ;$ Return L	Return b'	Return x'

Equation (4) is easily verified. It remains to show that S is unpredictable. By Lemma 4.1 it suffices to show that S is simple-unpredictable. Given a simple predictor adversary P' we define I as shown above. Then we have

$$\mathsf{Adv}^{\mathsf{spred}}_{P',S}(\cdot) = \mathsf{Adv}^{\mathsf{ow}}_{\mathsf{F},I}(\cdot)\ .$$

But the assumption $\mathsf{F} \in \mathsf{OW}$ implies the right-hand-side is negligible, which shows that S is simple unpredictable. ∎

5.2 Instantiating the BR93 PKE scheme

BR93 [21] gave a simple PKE scheme which encrypts m by picking x at random and returning $(f(x), \mathsf{RO}(x) \oplus m)$ where the public key f is an injective trapdoor function whose inverse is the secret key. They showed that this is IND-CPA when RO is a RO. We show that instantiating RO with a UCE1 family maintains IND-CPA security. We note that we show full (adaptive) IND-CPA, not IND-CPA-KI. This is because the points to which the RO is applied in this scheme do not depend on the messages.

$\underline{\text{MAIN IND-CPA}^A_{\mathsf{PKE}}(\lambda)}$	$\underline{\mathsf{PKE.Kg}(1^\lambda)}$	$\underline{\mathsf{PKE.Enc}(1^\lambda, (ek, hk), m)}$
$b \leftarrow\!\!\$\ \{0,1\}$	$(ek, dk) \leftarrow\!\!\$\ \mathsf{TF.EKg}(1^\lambda)$	$x \leftarrow\!\!\$\ \{0,1\}^{\mathsf{TF.il}(\lambda)}$
$(ek, dk) \leftarrow\!\!\$\ \mathsf{PKE.Kg}(1^\lambda)$	$hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$	$w \leftarrow \mathsf{H.Ev}(1^\lambda, hk, x, 1^{\mathsf{H.ol}(\lambda)})$
$b' \leftarrow\!\!\$\ A^{\text{LR}}(1^\lambda, ek)$	Return $((ek, hk), (dk, hk))$	Return $(\mathsf{TF.Ev}(1^\lambda, ek, x), w \oplus m)$
Return $(b = b')$		$\underline{\mathsf{PKE.Dec}(1^\lambda, (dk, hk), (y, z))}$
$\underline{\text{LR}(m_0, m_1)}$		$x \leftarrow \mathsf{TF.Inv}(1^\lambda, dk, y)$
$c \leftarrow\!\!\$\ \mathsf{PKE.Enc}(1^\lambda, ek, m_b)$		Return $\mathsf{H.Ev}(1^\lambda, hk, x, 1^{\lvert z\rvert}) \oplus z$
Return c		

Figure 9: **Left:** The IND-CPA game. **Right:** PKE scheme $\mathsf{PKE} = \mathsf{BR93}[\mathsf{H}, \mathsf{TF}]$.

DEFINITIONS. A family of functions TF with input length $\mathsf{TF.il}$ and output length $\mathsf{TF.ol}$ is said to be trapdoor if there are (additional) PT algorithms $\mathsf{TF.EKg}, \mathsf{TF.Inv}$, the second deterministic, such that the following hold. Extended key-generation algorithm $\mathsf{TF.EKg}(1^\lambda)$ returns a pair (ek, dk) of keys, the second called the trapdoor. The usual $\mathsf{TF.Kg}(1^\lambda)$ algorithm lets $(ek, dk) \leftarrow\!\!\$\ \mathsf{TF.EKg}(1^\lambda)$ and returns ek. Finally $\mathsf{TF.Inv}(1^\lambda, dk, \mathsf{TF.Ev}(1^\lambda, ek, x)) = x$ for all $\lambda \in \mathbb{N}$, all $(ek, dk) \in [\mathsf{TF.EKg}(1^\lambda)]$ and all $x \in \{0,1\}^{\mathsf{TF.il}(\lambda)}$.

A PKE scheme PKE as usual specifies a triple of PT algorithms, the last deterministic. Via $(ek, dk) \leftarrow\!\!\$\ \mathsf{PKE.Kg}(1^\lambda)$ we generate keys. Via $c \leftarrow\!\!\$\ \mathsf{PKE.Enc}(1^\lambda, ek, m)$ we can encrypt a message $m \in \{0,1\}^{\mathsf{PKE.il}(\lambda)}$ where $\mathsf{PKE.il}: \mathbb{N} \to \mathbb{N}$ is the message-length function of the scheme. Via $m \leftarrow \mathsf{PKE.Dec}(1^\lambda, dk, c)$ we decrypt. We say that PKE is IND-CPA-secure if $\mathsf{Adv}^{\mathsf{ind\text{-}cpa}}_{\mathsf{PKE},A}(\cdot)$ is negligible for all PT A, where $\mathsf{Adv}^{\mathsf{ind\text{-}cpa}}_{\mathsf{PKE},A}(\lambda) = 2\Pr[\text{IND-CPA}^A_{\mathsf{PKE}}(\lambda)] - 1$ and game $\text{IND-CPA}^A_{\mathsf{PKE}}(\lambda)$ is shown in Fig. 9. Messages m_0, m_1 queried to LR are required to be of the same length and A may be assumed to make only one oracle query. Let $\mathsf{IND\text{-}CPA}$ be the set of all PKE that are IND-CPA secure.

RESULTS. Let TF be a trapdoor family of functions. Let H be a family of functions with the same input length as F and output length $\mathsf{H.ol}$. Our instantiated BR93 scheme is represented by a transform $\mathsf{BR93}$ that associates to H and TF the PKE scheme $\mathsf{PKE} = \mathsf{BR93}[\mathsf{H}, \mathsf{TF}]$ defined in Fig. 9. The message length of the scheme is $\mathsf{PKE.il} = \mathsf{H.ol}$.

Theorem 5.2 If $\mathsf{H} \in \mathsf{UCE1}$ and $\mathsf{TF} \in \mathsf{OW}$ then $\mathsf{BR93}[\mathsf{H}, \mathsf{TF}] \in \mathsf{IND\text{-}CPA}$.

Proof of Theorem 5.2: Theorem 5.2 is a simple corollary of Theorem 5.1, meaning we do not have to use UCE1 directly. Given an adversary A for game $\text{INDCPA}^A_{\mathsf{PKE}}(\lambda)$, where $\mathsf{PKE} = \mathsf{BR93}[\mathsf{H}, \mathsf{TF}]$, we build the following adversary B for game $\text{HC}^B_{\mathsf{TF},\mathsf{H}}(\lambda)$:

$\underline{B(1^\lambda, ek, hk, y, r)}$	$\underline{\text{LRSIM}(m_0, m_1)}$
$d \leftarrow\!\!\$\ \{0,1\}\ ;\ d' \leftarrow\!\!\$\ A^{\text{LRSIM}}(1^\lambda, (ek, hk))$	Return $(y, r \oplus m_d)$
If $(d = d')$ then $b' \leftarrow 1$ else $b' \leftarrow 0$	
Return b'	

Adversary A makes a single oracle query, consisting of a pair $m_0, m_1 \in \{0,1\}^{\mathsf{H.ol}(\lambda)}$ of messages, in response to which B returns the ciphertext shown. Letting b denote the challenge bit in game $\text{HC}^B_{\mathsf{TF},\mathsf{H}}(\lambda)$ we have

$$\Pr[\, d = d' \,|\, b = 1\,] = \frac{1}{2} + \frac{1}{2}\mathsf{Adv}^{\mathsf{ind\text{-}cpa}}_{\mathsf{PKE},A}(\cdot) \qquad \text{and} \qquad \Pr[\, d = d' \,|\, b = 0\,] = \frac{1}{2}\ .$$

Subtracting, we have $\mathsf{Adv}^{\mathsf{hc}}_{\mathsf{TF},\mathsf{H},A}(\lambda) = 0.5 \cdot \mathsf{Adv}^{\mathsf{indcpa}}_{\mathsf{PKE},A}(\lambda)$. ∎

5.3 Deterministic encryption

EwH is a simple and natural D-PKE scheme from [11] that deterministically encrypts m by encrypting m with a randomized IND-CPA scheme with the coins derived by applying a RO to m. In the ROM the scheme is PRIV-secure [11] and equivalently IND-secure [14]. We show that instantiating the RO with a UCE1 hash

$\underline{\text{MAIN } \text{IND}^A_{\mathsf{PKE}}(\lambda)}$	$\underline{\mathsf{DE.Kg}(1^\lambda)}$	$\underline{\mathsf{DE.Enc}(1^\lambda, (ek, hk), m)}$
$b \leftarrow\!\!\$\ \{0,1\}$	$(ek, dk) \leftarrow\!\!\$\ \mathsf{RE.Kg}(1^\lambda)$	$r \leftarrow \mathsf{H.Ev}(1^\lambda, hk, ek \,\|\, m, 1^{\mathsf{RE.rl}(\lambda)})$
$(ek, dk) \leftarrow\!\!\$\ \mathsf{DE.Kg}(1^\lambda)$	$hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$	$c \leftarrow \mathsf{RE.Enc}(1^\lambda, ek, m; r)$
$(\mathbf{m}_0, \mathbf{m}_1) \leftarrow\!\!\$\ A_1(1^\lambda)$	Return $((ek, hk), dk)$	Return c
For $i = 1$ to $\lvert\mathbf{m}_b\rvert$ do		
$\quad \mathbf{c}[i] \leftarrow\!\!\$\ \mathsf{DE.Enc}(1^\lambda, ek, \mathbf{m}_b[i])$		$\underline{\mathsf{DE.Dec}(1^\lambda, dk, c)}$
$b' \leftarrow\!\!\$\ A_2(1^\lambda, ek, \mathbf{c})$		$m \leftarrow \mathsf{RE.Dec}(1^\lambda, dk, c)$
Return $(b = b')$		Return m

Figure 10: **Left:** The IND game. **Right:** D-PKE scheme $\mathsf{DE} = \mathsf{EwH}[\mathsf{H}, \mathsf{RE}]$.

family results in a scheme meeting the same notion of security in the standard model. Previous standard model schemes have met notions providing security only when one assumes messages are drawn from a blocksource, meaning each message has high min-entropy even given previous ones [28, 41]. Instantiated EwH however meets the original and full notions of [11, 14] which only make the necessary assumption that each individual message has high min-entropy, but allow messages to be arbitrarily correlated. This is the first standard-model scheme meeting the PRIV and IND notions.

DEFINITIONS. Let PKE be a PKE scheme as defined in Section 5.2. We say PKE is a D-PKE scheme if the encryption algorithm $\mathsf{PKE.Enc}$ is deterministic. The game defining the IND notion of security for D-PKE scheme DE, following [14], is in Fig. 10. An IND adversary $A = (A_1, A_2)$ is a pair of PT algorithms, where A_1 on input 1^λ returns a pair $(\mathbf{m}_0, \mathbf{m}_1)$ of vectors of messages. It is required that there are functions v, ℓ, depending on the adversary, such that $|\mathbf{m}_0| = |\mathbf{m}_1| = v(\lambda)$ and $|\mathbf{m}_b[i]| = \ell(\lambda)$ for all $b \in \{0,1\}$ and $i \in [1, v(\lambda)]$. It is also required that the strings (messages) $\mathbf{m}_0[1], \ldots, \mathbf{m}_0[|\mathbf{m}_0|]$ are distinct and the strings (messages) $\mathbf{m}_1[1], \ldots, \mathbf{m}_1[|\mathbf{m}_1|]$ are distinct. The guessing probability $\text{Guess}_A(\cdot)$ of A is the function that on input $\lambda \in \mathbb{N}$ returns the maximum, over all b, i, m, of $\Pr[\mathbf{m}_b[i] = m]$, the probability over $(\mathbf{m}_0, \mathbf{m}_1) \leftarrow\!\!\$\ A_1(1^\lambda)$. We say that A has *high min-entropy* if $\text{Guess}_A(\cdot)$ is negligible. We let $\mathsf{Adv}^{\mathsf{ind}}_{\mathsf{DE},A}(\lambda) = 2\Pr[\text{IND}^A_{\mathsf{DE}}(\lambda)] - 1$ and say that DE is IND-secure if $\mathsf{Adv}^{\mathsf{ind}}_{\mathsf{DE},A}(\cdot)$ is negligible for all PT A of high min-entropy. Let IND be the set of all IND-secure D-PKE schemes.

RESULTS. Let RE be a PKE scheme. Let $\mathsf{RE.rl}$: $\mathbb{N} \to \mathbb{N}$ denote its randomness-length function, meaning $\mathsf{RE.Enc}(1^\lambda, \cdot, \cdot)$ draws its coins at random from $\{0,1\}^{\mathsf{RE.rl}(\lambda)}$. Let H be a family of functions with $\mathsf{H.IL} = \mathbb{N}$ and $\mathsf{RE.rl}(\lambda) \in \mathsf{H.OL}(\lambda)$ for all $\lambda \in \mathbb{N}$. Our standard-model instantiation of the ROM encrypt-with-hash transform of BBO07 [11] associates to RE and H the (standard-model) D-PKE scheme $\mathsf{DE} = \mathsf{EwH}[\mathsf{H}, \mathsf{RE}]$ described in Fig. 10. The message length of DE is that of RE. The following theorem says that the transform yields an IND-secure D-PKE scheme if H is UCE1-secure and RE is IND-CPA-secure.

Theorem 5.3 If $\mathsf{H} \in \mathsf{UCE1}$ and $\mathsf{RE} \in \mathsf{IND\text{-}CPA}$ then $\mathsf{EwH}[\mathsf{H}, \mathsf{RE}] \in \mathsf{IND}$.

Proof of Theorem 5.3: Let $\mathsf{DE} = \mathsf{EwH}[\mathsf{H}, \mathsf{RE}]$. Given a high min-entropy adversary $A = (A_1, A_2)$ for game $\text{IND}^A_{\mathsf{DE}}(\cdot)$, we build an unpredictable source S, a distinguisher D, and an adversary B_1 for game $\text{INDCPA}^{B_1}_{\mathsf{RE}}(\cdot)$ such that

$$\mathsf{Adv}^{\mathsf{ind}}_{\mathsf{DE},A}(\cdot) \leq 2\mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\cdot) + \mathsf{Adv}^{\mathsf{ind\text{-}cpa}}_{\mathsf{RE},B_1}(\cdot)\ . \tag{5}$$

The theorem follows from the assumptions $\mathsf{H} \in \mathsf{UCE1}$ and $\mathsf{RE} \in \mathsf{IND\text{-}CPA}$. The constructions of S, D and B_1 are shown below:

$\underline{S^{\text{HASH}}(1^\lambda)}$	$\underline{D(1^\lambda, hk, L)}$	$\underline{B_1^{\text{LR}}(1^\lambda, ek)}$
$(ek, dk) \leftarrow\!\!\$\ \mathsf{RE.Kg}(1^\lambda)\ ;\ d \leftarrow\!\!\$\ \{0,1\}$	$(ek, d, \mathbf{c}) \leftarrow L$	$(\mathbf{m}_0, \mathbf{m}_1) \leftarrow\!\!\$\ A_1(1^\lambda)\ ;\ n \leftarrow \lvert\mathbf{m}_0\rvert$
$(\mathbf{m}_0, \mathbf{m}_1) \leftarrow\!\!\$\ A_1(1^\lambda)\ ;\ n \leftarrow \lvert\mathbf{m}_0\rvert$	$d' \leftarrow\!\!\$\ A_2(1^\lambda, (ek, hk), \mathbf{c})$	For $i = 1, \ldots, n$ do
For $i = 1, \ldots, n$ do	If $(d = d')$ then $b' \leftarrow 1$	$\quad \mathbf{c}[i] \leftarrow \text{LR}(\mathbf{m}_0[i], \mathbf{m}_1[i])$
$\quad \mathbf{r}[i] \leftarrow\!\!\$\ \text{HASH}(ek \Vert \mathbf{m}_d[i], 1^{\mathsf{RE.rl}(\lambda)})$	Else $b' \leftarrow 0$	$hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$
$\quad \mathbf{c}[i] \leftarrow \mathsf{RE.Enc}(1^\lambda, ek, \mathbf{m}_d[i]; \mathbf{r}[i])$	Return b'	$b' \leftarrow\!\!\$\ A_2(1^\lambda, (ek, hk), \mathbf{c})$
$L \leftarrow (ek, d, \mathbf{c})$		Return b'
Return L		

Letting b denote the challenge bit in game $\text{UCE}_{\mathsf{H}}^{S,D}(\cdot)$ we have

$$\Pr[\, d = d' \,|\, b = 1\,] = \frac{1}{2} + \frac{1}{2}\mathsf{Adv}^{\mathsf{ind}}_{\mathsf{DE},A}(\cdot) \qquad \text{and} \qquad \Pr[\, d = d' \,|\, b = 0\,] = \frac{1}{2} + \frac{1}{2}\mathsf{Adv}^{\mathsf{ind\text{-}cpa}}_{\mathsf{RE},B_1}(\cdot)\ .$$

The second equation above exploits the assumption that $\mathbf{m}_d[1], \ldots, \mathbf{m}_d[n]$ are all distinct. Subtracting and re-arranging terms, we have Equation (5).

It remains to show that S is unpredictable. By Lemma 4.1 it suffices to show that S is simple-unpredictable. Since oracle queries of S include messages created by A_1, simple unpredictability may seem at first to follow from the high min-entropy assumption on A. However we will additionally exploit (once again) the assumed IND-CPA security of the randomized RE scheme. This is because the leakage contains the ciphertexts. Thus, letting P' be a simple predictor adversary, and letting v, ℓ be the functions assumed associated to A as per the definitions, we construct B_2 such that

$$\mathsf{Adv}^{\mathsf{spred}}_{S,P'}(\cdot) \le \mathsf{Adv}^{\mathsf{ind\text{-}cpa}}_{\mathsf{RE},B_2}(\cdot) + v(\cdot) \cdot \text{Guess}_A(\cdot)\ . \tag{6}$$

The assumption that A has high min-entropy and that $\mathsf{RE} \in \mathsf{IND\text{-}CPA}$ mean the left-hand-side of Equation (6) is negligible, and thus S is simple unpredictable. We construct B_2 as follows:

$\underline{B_2^{\text{LR}}(1^\lambda, ek)}$
$(\mathbf{m}_0, \mathbf{m}_1) \leftarrow\!\!\$\ A_1(1^\lambda)\ ;\ n \leftarrow \lvert\mathbf{m}_0\rvert\ ;\ d \leftarrow\!\!\$\ \{0,1\}$
For $i = 1, \ldots, n$ do $\mathbf{m}_2[i] \leftarrow\!\!\$\ \{0,1\}^{\ell(\lambda)}\ ;\ \mathbf{c}[i] \leftarrow \text{LR}(\mathbf{m}_2[i], \mathbf{m}_d[i])$
$L \leftarrow (ek, d, \mathbf{c})\ ;\ x \leftarrow P'(1^\lambda, L)$
If $x \in \{\, ek \,\Vert\, \mathbf{m}_d[i]\ :\ 1 \le i \le v(\lambda)\,\}$ then $b' \leftarrow 1$ else $b' \leftarrow 0$
Return b'

Letting b denote the challenge bit in game $\text{IND-CPA}_{\mathsf{RE}}^{B_2}(\cdot)$ we have

$$\Pr[\, b' = 1 \,|\, b = 1\,] = \mathsf{Adv}^{\mathsf{spred}}_{S,P'}(\cdot) \qquad \text{and} \qquad \Pr[\, b' = 1 \,|\, b = 0\,] \le v(\cdot) \cdot \text{Guess}_A(\cdot)\ .$$

Subtracting we obtain Equation (6). ∎

An interesting open question is whether our $\mathsf{EwH}[\mathsf{H}, \mathsf{RE}]$ scheme can also be shown to meet the notions of security for D-PKE with respect to auxiliary inputs from [41], or, more generally, whether UCE allows one to achieve these goals in the standard model. (Here we refer to full auxiliary-input security rather than such security for block sources. The latter is already achieved without ROs in [41].)

D-PKE secure for adaptively-chosen plaintext distributions was considered in [89], who gave ROM solutions. It would be interesting to see if the RO here can be instantiated to obtain standard model schemes. The difficulty in doing this directly with UCE is that in the latter, the points being hashed may not depend on the key.

5.4 Message-locked encryption

Message-locked encryption (MLE) [17] is a form of symmetric encryption in which the key is derived from the message. It allows secure data deduplication. The convergent encryption (CE) MLE scheme of [56, 17] is in use by numerous providers of cloud storage. Its security is justified in the ROM by [17]. Here we

$\underline{\text{MAIN IND\$-CDA}^{A}_{\mathsf{MLE}}(\lambda)}$	$\underline{\mathsf{CE.Pg}(1^\lambda)}$	$\underline{\mathsf{CE.Dec}(1^\lambda, hk, k, c)}$
$p \leftarrow\!\!\$\ \mathsf{MLE.Pg}(1^\lambda)$; $b \leftarrow\!\!\$\ \{0,1\}$	$hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$; Return hk	$m \leftarrow \mathsf{SE.Dec}(1^\lambda, k, c)$
$\mathbf{m} \leftarrow\!\!\$\ A_1(1^\lambda)$	$\underline{\mathsf{CE.Kg}(1^\lambda, hk, m)}$	Return m
For $i = 1$ to $\lvert\mathbf{m}\rvert$ do	$k \leftarrow \mathsf{H.Ev}(1^\lambda, hk, m, 1^{\mathsf{SE.kl}(\lambda)})$	$\underline{\mathsf{CE.Tag}(1^\lambda, hk, c)}$
$\quad \mathbf{k}[i] \leftarrow \mathsf{MLE.Kg}(1^\lambda, p, \mathbf{m}[i])$	Return k	Return c
$\quad \mathbf{c}_1[i] \leftarrow \mathsf{MLE.Enc}(1^\lambda, p, \mathbf{k}[i], \mathbf{m}[i])$	$\underline{\mathsf{CE.Enc}(1^\lambda, hk, k, m)}$	
$\quad \mathbf{c}_0[i] \leftarrow\!\!\$\ \{0,1\}^{\lvert\mathbf{c}_1[i]\rvert}$	$c \leftarrow \mathsf{SE.Enc}(1^\lambda, k, m)$	
$b' \leftarrow\!\!\$\ A_2(1^\lambda, p, \mathbf{c}_b)$	Return c	
Return $(b' = b)$		

Figure 11: **Left:** The IND\$-CDA game. **Right:** MLE scheme $\mathsf{CE}[\mathsf{H}, \mathsf{SE}]$.

instantiate the RO with a UCE1 family to get standard-model security. This results in the most efficient and practical known MLE scheme with a proof in the standard model.

DEFINITIONS. An MLE scheme MLE [17] specifies the following PT algorithms. Via $p \leftarrow\!\!\$\ \mathsf{MLE.Pg}(1^\lambda)$ one generates parameters. Via $k \leftarrow \mathsf{MLE.Kg}(1^\lambda, p, m)$, one deterministically derives a key k from a message $m \in \{0,1\}^*$. Via $c \leftarrow \mathsf{MLE.Enc}(1^\lambda, p, k, m)$ one encrypts m under k to get ciphertext c. Via $m' \leftarrow \mathsf{MLE.Dec}(1^\lambda, p, k, c)$ one deterministically decrypts c under k to get $m' \in \{0,1\}^* \cup \{\perp\}$. Via $t \leftarrow \mathsf{MLE.Tag}(1^\lambda, p, c)$ one deterministically generates a tag t for ciphertext c. Correctness requires the following for all $\lambda \in \mathbb{N}$, all $m \in \{0,1\}^*$, all $p \in [\mathsf{MLE.Pg}(1^\lambda)]$ and all $k_1, k_2 \in [\mathsf{MLE.Kg}(1^\lambda, p, m)]$: (1) $\mathsf{MLE.Tag}(1^\lambda, p, c_1) = \mathsf{MLE.Tag}(1^\lambda, p, c_2)$ for all $c_1 \in [\mathsf{MLE.Enc}(1^\lambda, p, k_1, m)]$ and all $c_2 \in [\mathsf{MLE.Enc}(1^\lambda, p, k_2, m)]$, and (2) $\mathsf{MLE.Dec}(1^\lambda, p, k_2, c) = m$ for all $c \in [\mathsf{MLE.Enc}(1^\lambda, p, k_1, m)]$. The $\text{IND\$-CDA}^{A}_{\mathsf{MLE}}(\lambda)$ game defined in Fig. 11 is a simplification of the one of [17], without side-information. A IND\$-CDA adversary $A = (A_1, A_2)$ is a pair of PT algorithms, where A_1 on input 1^λ returns $\mathbf{m}$, a $v(\lambda)$-vector of distinct $\ell(\lambda)$-bit strings, where v, ℓ depend on A. The guessing probability Guess_A of A is the function that on input $\lambda \in \mathbb{N}$ returns the maximum, over all i, m, of $\Pr[\mathbf{m}[i] = m]$, the probability over $\mathbf{m} \leftarrow\!\!\$\ A_1(1^\lambda)$. We say that A has *high min-entropy* if $\text{Guess}_A(\cdot)$ is negligible. We let $\mathsf{Adv}^{\mathsf{ind\$\text{-}cda}}_{\mathsf{MLE},A}(\lambda) = 2\Pr[\text{IND\$-CDA}^{A}_{\mathsf{MLE}}(\lambda)] - 1$ and say that MLE is IND\$-CDA-secure if $\mathsf{Adv}^{\mathsf{ind\$\text{-}cda}}_{\mathsf{MLE},A}(\cdot)$ is negligible for all PT A that have high min-entropy. We let $\mathsf{IND\$\text{-}CDA}$ be the set of all IND\$-CDA-secure MLE schemes.

A symmetric encryption (SE) scheme SE will be a tool in the construction. Such a scheme specifies the following PT algorithms. Via $k \leftarrow\!\!\$\ \mathsf{SE.Kg}(1^\lambda)$ one generates a key that is a random $\mathsf{SE.kl}(\lambda)$ bit string. Via $c \leftarrow\!\!\$\ \mathsf{SE.Enc}(1^\lambda, k, m)$, one encrypts message $m \in \{0,1\}^*$ under k to get a ciphertext $c \in \{0,1\}^{\mathsf{SE.cl}(\lambda, |m|)}$, where $\mathsf{SE.cl}\colon\ \mathbb{N} \times \mathbb{N} \to \mathbb{N}$ is the ciphertext-length function of SE. Via $m' \leftarrow \mathsf{SE.Dec}(1^\lambda, k, c)$ one deterministically decrypts c under k to get back $m \in \{0,1\}^* \cup \{\perp\}$. Correctness requires that $\mathsf{SE.Dec}(1^\lambda, k, \mathsf{SE.Enc}(1^\lambda, k, m)) = m$ with probability 1 for all $m \in \{0,1\}^*$, all $k \in [\mathsf{SE.Kg}(1^\lambda)]$ and all $\lambda \in \mathbb{N}$, the probability being over the coins of $\mathsf{SE.Enc}$. We say that SE is a D-SE (deterministic SE) scheme if $\mathsf{SE.Enc}$ is deterministic. We will assume a D-SE scheme meeting the following definition of security. Game $\text{ROR}^{A}_{\mathsf{SE}}(\cdot)$ starts by picking $b \leftarrow\!\!\$\ \{0,1\}$. Adversary A is then given access to an oracle ENC that, on input m, picks a fresh key $k \leftarrow\!\!\$\ \mathsf{SE.Kg}(1^\lambda)$ and computes $c \leftarrow \mathcal{E}(1^\lambda, k, m)$. If $b = 1$ then it returns c, otherwise it returns $c' \leftarrow\!\!\$\ \{0,1\}^{|c|}$. When the adversary exits with output b', the game returns $(b' = b)$. We say that SE is ROR-secure if $\mathsf{Adv}^{\mathsf{ror}}_{\mathsf{SE},A}(\cdot)$ is negligible for all PT A, where $\mathsf{Adv}^{\mathsf{ror}}_{\mathsf{SE},A}(\lambda) = 2\Pr[\text{ROR}^{A}_{\mathsf{SE}}(\lambda)] - 1$. We let ROR denote the set of all ROR-secure SE schemes.

RESULTS. Let SE be a D-SE scheme and let H be a family of functions with $\mathsf{H.IL} = \mathbb{N}$ and $\mathsf{SE.kl}(\lambda) \in \mathsf{H.Ol}(\lambda)$ for all $\lambda \in \mathbb{N}$. We describe a standard model instantiation $\mathsf{CE}[\mathsf{H}, \mathsf{SE}]$ of the convergent encryption scheme of [56, 17] in Fig. 11. Correctness is easy to verify. MLE schemes also have an additional security requirement called tag consistency [17], and $\mathsf{CE}[\mathsf{H}, \mathsf{SE}]$ as described here has perfect tag consistency. The following theorem shows that $\mathsf{CE}[\mathsf{H}, \mathsf{SE}]$ is IND\$-CDA-secure.

Theorem 5.4 If $\mathsf{H} \in \mathsf{UCE1}$ and $\mathsf{SE} \in \mathsf{ROR}$, then $\mathsf{CE}[\mathsf{H}, \mathsf{SE}] \in \mathsf{IND\$\text{-}CDA}$.

Proof of Theorem 5.4: Let $\mathsf{MLE} = \mathsf{CE}[\mathsf{H}, \mathsf{SE}]$. Let A be a PT high min-entropy IND\$-CDA adversary.

We'll construct an unpredictable UCE-source S, a distinguisher D, and an adversary B_1 such that

$$\mathsf{Adv}^{\mathsf{ind\$\text{-}cda}}_{\mathsf{MLE},A}(\cdot) \le \mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\cdot) + \mathsf{Adv}^{\mathsf{ror}}_{\mathsf{SE},B_1}(\cdot). \tag{7}$$

Let v, ℓ be functions associated to A as per the definitions. The constructions of S, D, B_1 are shown below:

$\underline{S^{\text{HASH}}(1^\lambda)}$	$\underline{D(1^\lambda, L)}$	$\underline{B_1^{\text{ENC}}(1^\lambda)}$	$\underline{B_2^{\text{ENC}}(1^\lambda)}$
$\mathbf{m} \leftarrow\!\!\$\ A_1(1^\lambda)$	$a' \leftarrow A_2(1^\lambda, hk, L)$	$hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$	$\mathbf{m} \leftarrow\!\!\$\ A_1(1^\lambda)$
For $i = 1$ to $\lvert\mathbf{m}\rvert$ do	Return a'	$\mathbf{m} \leftarrow\!\!\$\ A_1(1^\lambda)$	$Q \leftarrow \{\mathbf{m}[1], \ldots, \mathbf{m}[\lvert\mathbf{m}\rvert]\}$
$\quad \mathbf{k}[i] \leftarrow \text{HASH}(\mathbf{m}[i], 1^{\mathsf{SE.kl}(\lambda)})$		For $i = 1$ to $\lvert\mathbf{m}\rvert$ do	For $i = 1$ to $\lvert\mathbf{m}\rvert$ do
$\quad \mathbf{c}[i] \leftarrow \mathsf{SE.Enc}(1^\lambda, \mathbf{k}[i], \mathbf{m}[i])$		$\quad \mathbf{c}[i] \leftarrow \text{ENC}(\mathbf{m}[i])$	$\quad \mathbf{c}[i] \leftarrow \text{ENC}(\mathbf{m}[i])$
$L \leftarrow \mathbf{c}$		$d' \leftarrow\!\!\$\ A_2(1^\lambda, hk, \mathbf{c})$	$m \leftarrow\!\!\$\ P'(1^\lambda, \mathbf{c})$
Return L		Return d'	If $m \in Q$ then $b' \leftarrow 1$
			Else $b' \leftarrow 0$
			Return b'

Let a, c, d be the challenge bits of games $\text{UCE}^{S,D}_{\mathsf{H}}(\cdot)$, $\text{IND\$-CDA}^{A}_{\mathsf{MLE}}(\cdot)$ and $\text{ROR}^{A}_{\mathsf{SE}}(\cdot)$ respectively. Then

$$\begin{aligned}
\Pr[\,\text{IND\$-CDA}^{A}_{\mathsf{MLE}}(\cdot) \,|\, c = 1\,] &= \Pr[\,\text{UCE}^{S,D}_{\mathsf{H}}(\cdot) \,|\, a = 1\,] \\
\Pr[\,\text{IND\$-CDA}^{A}_{\mathsf{MLE}}(\cdot) \,|\, c = 0\,] &= \Pr[\,\text{ROR}^{B_1}_{\mathsf{SE}}(\cdot) \,|\, d = 0\,] \\
\Pr[\,\text{ROR}^{B_1}_{\mathsf{SE}}(\cdot) \,|\, d = 1\,] &= 1 - \Pr[\,\text{UCE}^{S,D}_{\mathsf{H}}(\cdot) \,|\, a = 0\,]\,.
\end{aligned} \tag{8}$$

So

$$\begin{aligned}
\mathsf{Adv}^{\mathsf{ind\$\text{-}cda}}_{\mathsf{MLE},A}(\cdot) &= \Pr[\,\text{IND\$-CDA}^{A}_{\mathsf{MLE}}(\cdot) \,|\, c = 1\,] + \Pr[\,\text{IND\$-CDA}^{A}_{\mathsf{MLE}}(\cdot) \,|\, c = 0\,] - 1 \\
&= \Pr[\,\text{UCE}^{S,D}_{\mathsf{H}}(\cdot) \,|\, a = 1\,] + \Pr[\,\text{ROR}^{B_1}_{\mathsf{SE}}(\cdot) \,|\, d = 0\,] - 1 \\
&= \Pr[\,\text{UCE}^{S,D}_{\mathsf{H}}(\cdot) \,|\, a = 1\,] + \Big(\Pr[\,\text{UCE}^{S,D}_{\mathsf{H}}(\cdot) \,|\, a = 0\,] - 1 + \Pr[\,\text{ROR}^{B_1}_{\mathsf{SE}}(\cdot) \,|\, d = 1\,]\Big) \\
&\qquad + \Pr[\,\text{ROR}^{B_1}_{\mathsf{SE}}(\cdot) \,|\, d = 0\,] - 1 \\
&= \mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\cdot) + \mathsf{Adv}^{\mathsf{ror}}_{\mathsf{SE},B_1}(\cdot)
\end{aligned}$$

which yields Equation (7). The third equality above is justified by the fact that, by Equation (8), the term in parentheses is zero.

It remains to show that S is unpredictable. By Lemma 4.1 it suffices to show that S is simple-unpredictable. Let P' be a simple predictor. Consider the $\text{ROR}^{A}_{\mathsf{SE}}(\lambda)$ adversary B_2 described above. Letting b denote the challenge bit in game $\text{ROR}^{A}_{\mathsf{SE}}(\lambda)$, we have

$$\Pr[\,b' = 1 \,|\, b = 1\,] = \mathsf{Adv}^{\mathsf{spred}}_{S,P'}(\cdot) \qquad \text{and} \qquad \Pr[\,b' = 1 \,|\, b = 0\,] \le v \cdot \text{Guess}_A(\cdot)\,.$$

Here v is the function associated to A as per the definitions. Subtracting, we have $\mathsf{Adv}^{\mathsf{spred}}_{S,P'}(\cdot) \le \mathsf{Adv}^{\mathsf{ror}}_{\mathsf{SE},B_2}(\cdot) + v \cdot \text{Guess}_A(\cdot)$. The simple unpredictability of S then follows from the assumption that $\mathsf{SE} \in \mathsf{ROR}$ and that A has high min-entropy. ∎

We are not able to meet the MLE security definitions of [1] which allow the messages to depend on the public parameters, since the latter is the key for our UCE family.

5.5 Point-function obfuscation

A *point function* has non-$\perp$ output on just one point. Canetti, Kalai, Varia, and Wichs (CKVW10) [48] showed how to obfuscate point functions in the ROM. We mUCE1-instantiate their construction to obtain a standard-model point-function obfuscation scheme.

$\underline{\text{MAIN PFOB}^{A,T}_{\mathsf{OS}}(\lambda)}$	$\underline{\mathsf{OS.Obf}(1^\lambda, (\alpha,\beta))}$
$b \leftarrow\!\!\$\ \{0,1\}$; $(\boldsymbol{\alpha},\boldsymbol{\beta}) \leftarrow\!\!\$\ A_1(1^\lambda)$	$hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$
For $i = 1$ to $\lvert\boldsymbol{\alpha}\rvert$ do $\mathbf{F}[i] \leftarrow\!\!\$\ \mathsf{OS.Obf}(1^\lambda, (\boldsymbol{\alpha}[i], \boldsymbol{\beta}[i]))$	$\overline{\alpha} \leftarrow \mathsf{H.Ev}(1^\lambda, hk, 0 \,\|\, \alpha, 1^\lambda)$
If $b = 1$ then $w \leftarrow\!\!\$\ A_2(1^\lambda, \mathbf{F})$ else $w \leftarrow\!\!\$\ T^{\text{PROG}}(1^\lambda, \lvert\boldsymbol{\alpha}\rvert)$	$\overline{\beta} \leftarrow \mathsf{H.Ev}(1^\lambda, hk, 1 \,\|\, \alpha, 1^{\lvert\beta\rvert}) \oplus \beta$
$b' \leftarrow\!\!\$\ A_3(1^\lambda, w)$	Return $(hk, \overline{\alpha}, \overline{\beta})$
Return $(b = b')$	
$\underline{\text{PROG}(i, x)}$	$\underline{\mathsf{OS.Ev}(1^\lambda, (hk, \overline{\alpha}, \overline{\beta}), x)}$
Return $\Delta_{\boldsymbol{\alpha}[i],\boldsymbol{\beta}[i]}(x)$	$\alpha^* \leftarrow \mathsf{H.Ev}(1^\lambda, hk, 0 \,\|\, x, 1^\lambda)$
	If $(\alpha^* \neq \overline{\alpha})$ then return $\perp$
	Else return $\overline{\beta} \oplus \mathsf{H.Ev}(1^\lambda, hk, 1 \,\|\, x, 1^{\lvert\overline{\beta}\rvert})$

Figure 12: **Left:** The PFOB game defining security of point-function obfuscator OS. **Right:** Point-function obfuscation scheme $\mathsf{OS} = \mathsf{HTC}[\mathsf{H}]$.

DEFINITIONS. For $(\alpha, \beta) \in \{0,1\}^* \times \{0,1\}^*$ we let $\Delta_{\alpha,\beta}$: $\{0,1\}^* \to \{\beta, \perp\}$ denote the function that on input $x \in \{0,1\}^*$ returns β if $x = \alpha$ and $\perp$ otherwise. A *point-function obfuscator* OS is defined as follows. Via $F \leftarrow\!\!\$\ \mathsf{OS.Obf}(1^\lambda, (\alpha, \beta))$, PT obfuscation algorithm $\mathsf{OS.Obf}$ creates a description F of an obfuscated version of $\Delta_{\alpha,\beta}$. Via $y \leftarrow\!\!\$\ \mathsf{OS.Ev}(1^\lambda, F, x)$, deterministic PT algorithm $\mathsf{OS.Ev}$ evaluates the obfuscated program F at $x \in \{0,1\}^*$ to get output y. Correctness requires that $\mathsf{OS.Ev}(1^\lambda, \mathsf{OS.Obf}(1^\lambda, (\alpha,\beta)), \alpha) = \beta$ for all $\alpha, \beta \in \{0,1\}^*$ and all $\lambda \in \mathbb{N}$. Security is defined via game $\text{PFOB}^{A_1,A_2}_{\mathsf{OS}}(\lambda)$ of Fig. 12. It involves an adversary $A = (A_1, A_2, A_3)$ and a simulator T. Adversary A_1 outputs a pair $(\boldsymbol{\alpha}, \boldsymbol{\beta})$ of vectors of the same length, entries of both being strings, thereby describing a sequence of point functions. It is required that there is a function ℓ, called the function output-length of A, such that all entries of $\boldsymbol{\beta}$ have length $\ell(\lambda)$, and it is required that all entries of $\boldsymbol{\alpha}$ are distinct. We let $\mathsf{Adv}^{\mathsf{obf}}_{\mathsf{OS},A,T}(\lambda) = 2\Pr[\text{PFOB}^{A,T}_{\mathsf{OS}}(\lambda)] - 1$. The guessing probability Guess_A of A is the function that on input $\lambda \in \mathbb{N}$ returns the maximum, over all i, α, of $\Pr[\boldsymbol{\alpha}[i] = \alpha]$, the probability over $(\boldsymbol{\alpha}, \boldsymbol{\beta}) \leftarrow\!\!\$\ A_1(1^\lambda)$. We say that A has *high min-entropy* if $\text{Guess}_A(\cdot)$ is negligible. We say that OS is a secure point-function obfuscator if for all PT high min-entropy A there is a PT simulator T such that $\mathsf{Adv}^{\mathsf{obf}}_{\mathsf{OS},A,T}(\cdot)$ is negligible. Let PFOB denote the set of all secure point-function obfuscators. The high min-entropy condition makes the problem "interesting" in that without it the adversary knows α and thus there is nothing to gain by obfuscation. This definition is from [48], adapted to our notation.

RESULTS. Let H be a family of functions with $\mathsf{H.IL} = \mathsf{H.OL} = \mathbb{N}$. Our Hash-then-Compare point-obfuscation scheme $\mathsf{OS} = \mathsf{HTC}[\mathsf{H}]$ is described in Fig. 12. The following says that multi-key UCE1 security of H suffices for OS to be secure:

Theorem 5.5 If $\mathsf{H} \in \mathsf{mUCE1}$ then $\mathsf{HTC}[\mathsf{H}] \in \mathsf{PFOB}$.

Proof of Theorem 5.5: Let $A = (A_1, A_2, A_3)$ be a PT adversary and let ℓ be its function output-length. We'll construct a multi-key unpredictable source S, a distinguisher D, and a simulator T such that

$$\mathsf{Adv}^{\mathsf{obf}}_{\mathsf{OS},A,T}(\cdot) = \mathsf{Adv}^{\mathsf{m\text{-}uce}}_{\mathsf{H},S,D}(\cdot) \ . \tag{9}$$

The theorem then follows from the assumption that $\mathsf{H} \in \mathsf{mUCE1}$. The constructions of S, D, and T are shown below.

$\underline{T^{\text{PROG}}(1^\lambda, a)}$	$\underline{S^{\text{HASH}}(1^\lambda)}$	$\underline{D(1^\lambda, \mathbf{hk}, L)}$
For $i = 1$ to a do	$(\boldsymbol{\alpha}, \boldsymbol{\beta}) \leftarrow\!\!\$\ A_1(1^\lambda)$	$(\overline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\beta}}) \leftarrow L$
$\quad \mathbf{hk}[i] \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$	For $i = 1$ to $\lvert\boldsymbol{\alpha}\rvert$ do	For $i = 1$ to $\lvert\overline{\boldsymbol{\alpha}}\rvert$ do
$\quad \overline{\boldsymbol{\alpha}}[i] \leftarrow\!\!\$\ \{0,1\}^\lambda$	$\quad \overline{\boldsymbol{\alpha}}[i] \leftarrow\!\!\$\ \text{HASH}(0 \Vert \boldsymbol{\alpha}[i], 1^\lambda, i)$	$\quad \mathbf{F}[i] \leftarrow (\mathbf{hk}[i], \overline{\boldsymbol{\alpha}}[i], \overline{\boldsymbol{\beta}}[i])$
$\quad \overline{\boldsymbol{\beta}}[i] \leftarrow\!\!\$\ \{0,1\}^{\ell(\lambda)}$	$\quad \overline{\boldsymbol{\beta}}[i] \leftarrow\!\!\$\ \boldsymbol{\beta}[i] \oplus \text{HASH}(1 \Vert \boldsymbol{\alpha}[i], 1^{\lvert\boldsymbol{\beta}[i]\rvert}, i)$	$w \leftarrow\!\!\$\ A_2(1^\lambda, \mathbf{F})$
$\quad \mathbf{F}[i] \leftarrow (\mathbf{hk}[i], \overline{\boldsymbol{\alpha}}[i], \overline{\boldsymbol{\beta}}[i])$	$L \leftarrow (\overline{\boldsymbol{\alpha}}, \overline{\boldsymbol{\beta}})$	$b' \leftarrow\!\!\$\ A_3(1^\lambda, w)$
$w \leftarrow\!\!\$\ A_2(1^\lambda, \mathbf{F})$	Return L	Return b'
Return w		

MAIN $\mathrm{KDM}^A_{\mathsf{SE}}(\lambda)$	MAIN $\mathrm{RKA}^A_{\mathsf{SE}}(\lambda)$	$\mathsf{SE.Enc}(1^\lambda, k, m)$
$(1^n, t) \leftarrow\!\!\$\ A_1(1^\lambda, \varepsilon)$	$(\mathbf{m}_0, \mathbf{m}_1, t) \leftarrow\!\!\$\ A_1(1^\lambda, \varepsilon)$	$hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$
For $i = 1$ to n do $\mathbf{k}[i] \leftarrow\!\!\$\ \{0,1\}^{\mathsf{SE.kl}(\lambda)}$	$k \leftarrow\!\!\$\ \{0,1\}^{\mathsf{SE.kl}(\lambda)}$	$h \leftarrow \mathsf{H.Ev}(1^\lambda, hk, k, 1^{\lvert m\rvert})$
$(\mathbf{s}, \mathbf{m}_0, \mathbf{m}_1) \leftarrow\!\!\$\ A_1(1^\lambda, (t, \mathbf{k}))$; $b \leftarrow\!\!\$\ \{0,1\}$	$\mathbf{k} \leftarrow\!\!\$\ A_1(1^\lambda, (t, k))$; $b \leftarrow\!\!\$\ \{0,1\}$	$c \leftarrow (hk, h \oplus m)$; Return c
For $i = 1$ to $\lvert\mathbf{m}_b\rvert$ do	For $i = 1$ to $\lvert\mathbf{m}_b\rvert$ do	$\underline{\mathsf{SE.Dec}(1^\lambda, k, (hk, z))}$
$\quad \mathbf{c}[i] \leftarrow\!\!\$\ \mathsf{SE.Enc}(1^\lambda, \mathbf{k}[\mathbf{s}[i]], \mathbf{m}_b[i])$	$\quad \mathbf{c}[i] \leftarrow\!\!\$\ \mathsf{SE.Enc}(1^\lambda, \mathbf{k}[i], \mathbf{m}_b[i])$	$h \leftarrow \mathsf{H.Ev}(1^\lambda, hk, k, 1^{\lvert z\rvert})$
$b' \leftarrow\!\!\$\ A_2(1^\lambda, t, \mathbf{c})$; Return $(b = b')$	$b' \leftarrow\!\!\$\ A_2(1^\lambda, t, \mathbf{c})$; Return $(b = b')$	$m \leftarrow\!\!\$\ h \oplus z$; Return m

Figure 13: **Left:** The KDM game. **Middle:** The RKA game. **Right:** The SE scheme $\mathsf{SE} = \mathsf{HtX}[\mathsf{H}]$.

Note that T does not call its oracle, the latter being in fact unnecessary to prove security. Let b and c be the challenge bits of games $\mathrm{mUCE}^{S,D}_{\mathsf{H}}(\lambda)$ and $\mathrm{PFOB}^{A,T}_{\mathsf{OS}}(\lambda)$ respectively. Then

$$\begin{aligned}\Pr[\mathrm{mUCE}^{S,D}_{\mathsf{H}}(\cdot) \mid b = 1] &= \Pr[\mathrm{PFOB}^{A,T}_{\mathsf{OS}}(\cdot) \mid c = 1] \\ \Pr[\mathrm{mUCE}^{S,D}_{\mathsf{H}}(\cdot) \mid b = 0] &= \Pr[\mathrm{PFOB}^{A,T}_{\mathsf{OS}}(\cdot) \mid c = 0]\end{aligned}$$

Summing yields Equation (9). What's left is to show that S is unpredictable. By Lemma 4.5 it suffices to show that S is simple unpredictable. Let P' be a PT simple predictor. Let v be a polynomial such that $|\boldsymbol{\alpha}| \le v(\lambda)$ in game $\mathrm{PFOB}^{A,T}_{\mathsf{OS}}(\lambda)$, for all $\lambda \in \mathbb{N}$. Then $\mathsf{Adv}^{\mathsf{spred}}_{S,P'}(\cdot) \le v \cdot \mathrm{Guess}_A(\cdot)$, so the high min-entropy assumption on A implies that S is simple unpredictable. ∎

5.6 Security for key-dependent messages

Black, Rogaway, and Shrimpton (BRS) [25] formalized security in the presence of key-dependent messages (KDM) and described a simple and efficient KDM-secure symmetric encryption scheme in the ROM. We now instantiate the RO in the BRS scheme with a mUCE1 family and obtain an efficient KDM-secure symmetric encryption scheme in the standard model. There are several other standard-model KDM-secure encryption schemes [39, 5, 9, 84, 4] but they are significantly more complex and less efficient than our instantiated BRS scheme.

DEFINITIONS. Let SE be a symmetric encryption (SE) scheme as defined in Section 5.4. In game $\mathrm{KDM}^A_{\mathsf{SE}}(\lambda)$ of Fig. 13, an adversary $A = (A_1, A_2)$ is a pair of algorithms. Algorithm A_1, when invoked with $(1^\lambda, \varepsilon)$, returns $(1^n, t)$ where n is the number of keys it is requesting be created, and t is state information. Then when invoked with $(1^\lambda, (t, \mathbf{k}))$ where $\mathbf{k} \in (\{0,1\}^{\mathsf{SE.kl}(\lambda)})^n$ is a vector of keys, it outputs a triple of vectors $\mathbf{s}, \mathbf{m}_0, \mathbf{m}_1$ satisfying the following: (1) $|\mathbf{s}| = |\mathbf{m}_0| = |\mathbf{m}_1|$, and (2) $\mathbf{s}[i] \in [1, n]$ and $|\mathbf{m}_0[i]| = |\mathbf{m}_1[i]|$ for all $i \in [1, |\mathbf{s}|]$. We say that SE is KDM-secure if $\mathsf{Adv}^{\mathsf{kdm}}_{\mathsf{SE},A}(\cdot)$ is negligible for every PT KDM adversary A, where $\mathsf{Adv}^{\mathsf{kdm}}_{\mathsf{SE},A}(\lambda) = 2\Pr[\mathrm{KDM}^A_{\mathsf{SE}}(\lambda)] - 1$. We let KDM denote the set of all KDM-secure schemes. Our definitions capture non-adaptive security, but this includes the cases that have been most prominent in past work, namely key cycles and cliques [39, 5, 2, 42].

RESULTS. BRS [25] showed that encrypting a message m under key k by picking a random r and returning $(r, \mathsf{RO}(r \,\|\, k) \oplus m)$ is KDM secure when RO is a random oracle. The natural first attempt to instantiate via a family H would be to add $hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$ to the encryption key and then replace RO with $\mathsf{H.Ev}(1^\lambda, hk, \cdot, 1^{|m|})$, but this fails because in the KDM setting the messages are chosen by A_1 as a function of the encryption key(s), and UCE1-security will not apply if the messages depend on hk. Instead, we leave the key unchanged relative to the BRS scheme and view the random value r of the BRS scheme as a key for H, so that a fresh key hk is chosen for each encryption. Given H with $\mathsf{H.OL}(\lambda) = \mathbb{N}$ and $\lambda \in \mathsf{H.IL}(\lambda)$ for all $\lambda \in \mathbb{N}$, our instantiated transform produces the SE scheme $\mathsf{SE} = \mathsf{HtX}[\mathsf{H}]$ whose encryption and decryption algorithms are described in Fig. 13. (Here "HtX" stands for "Hash-then-XOR.") Its key length is defined by $\mathsf{SE.kl}(\lambda) = \lambda$ for all $\lambda \in \mathbb{N}$. The following theorem says that $\mathsf{HtX}[\mathsf{H}]$ is KDM secure if H is mUCE1-secure.

Theorem 5.6 If $\mathsf{H} \in \mathsf{mUCE1}$, then $\mathsf{HtX}[\mathsf{H}] \in \mathsf{KDM}$.

Proof of Theorem 5.6: Let $\mathsf{SE} = \mathsf{HtX}[\mathsf{H}]$. Let $A = (A_1, A_2)$ be a PT KDM adversary. We will construct a multi-key unpredictable source S and a distinguisher D such that

$$\mathsf{Adv}^{\mathsf{kdm}}_{\mathsf{SE},A}(\cdot) \le 2 \cdot \mathsf{Adv}^{\mathsf{m\text{-}uce}}_{\mathsf{H},S,D}(\cdot) \ . \tag{10}$$

The theorem then follows from the assumption that $\mathsf{H} \in \mathsf{mUCE1}$. Let q and $\overline{n}$ be polynomials such that, in game $\mathrm{KDM}^A_{\mathsf{SE}}(\lambda)$, we have $|\mathbf{m}_0| \le q(\lambda)$ and $n \le \overline{n}(\lambda)$ for all $\lambda \in \mathbb{N}$. The constructions of S and D are shown below:

$\underline{S^{\mathrm{HASH}}(1^\lambda, t)}$
If $t = \varepsilon$ then
  $(1^n, t') \leftarrow\!\!\$\ A_1(1^\lambda, \varepsilon)$; Return $(1^{q(\lambda)}, (1^n, t'))$
Else
  $(1^n, t') \leftarrow t$; $d \leftarrow\!\!\$\ \{0,1\}$
  For $i = 1$ to n do $\mathbf{k}[i] \leftarrow\!\!\$\ \{0,1\}^\lambda$
  $(\mathbf{s}, \mathbf{m}_0, \mathbf{m}_1) \leftarrow\!\!\$\ A_1(1^\lambda, (t', \mathbf{k}))$
  For $i = 1$ to $|\mathbf{m}_d|$ do $\mathbf{c}'[i] \leftarrow \mathrm{HASH}(\mathbf{k}[\mathbf{s}[i]], 1^{|\mathbf{m}_d[i]|}, i) \oplus \mathbf{m}_d[i]$
  $L \leftarrow (\mathbf{c}, t', d)$; Return L

$\underline{D(1^\lambda, \mathbf{hk}, L)}$
$(\mathbf{c}', t', d) \leftarrow L$
For $i = 1$ to $|\mathbf{c}'|$ do
  $\mathbf{c}[i] \leftarrow (\mathbf{hk}[i], \mathbf{c}'[i])$
$d' \leftarrow\!\!\$\ A_2(1^\lambda, t', \mathbf{c})$
If $(d = d')$ then $b' \leftarrow 1$ else $b' \leftarrow 0$
Return b'

Let b denote the challenge bit in game $\mathrm{mUCE}^{S,D}_{\mathsf{H}}(\cdot)$. Then

$$\Pr[\,\mathrm{mUCE}^{S,D}_{\mathsf{H}}(\cdot) \mid b = 1\,] = \Pr[\mathrm{KDM}^A_{\mathsf{SE}}(\cdot)]$$
$$\Pr[\,\mathrm{mUCE}^{S,D}_{\mathsf{H}}(\cdot) \mid b = 0\,] = \frac{1}{2} \ .$$

Summing yields Equation (10). It remains to show that S is unpredictable. By Lemma 4.5, it suffices to show that S is simple unpredictable. Consider an arbitrary PT simple predictor P'. The leakage $(\mathbf{c}, t', d)$ that P' receives is independent of $\mathbf{k}$ and the components of the latter are uniformly and independently distributed λ-bit strings. Hence $\mathsf{Adv}^{\mathsf{m\text{-}spred}}_{P',S}(\lambda) \le \overline{n}(\lambda)/2^\lambda$ for every $\lambda \in \mathbb{N}$. ▌

5.7 Security against related-key attack

Symmetric encryption schemes secure against related-key attack (RKA) must preserve security even when encryption is performed under keys $k' = \phi(k)$ derived from the original key by application of a key-deriving function ϕ [19, 6]. Previous schemes [6, 20] provided security for algebraic key-deriving functions ϕ such as linear or polynomial functions over a keyspace that is a particular group depending on the scheme. We provide a scheme that has "best possible" security, in that key-deriving functions are arbitrary subject only to a condition necessary for security, namely to have unpredictable outputs. (If the output can be predicted, an adversary can guess the key k' and decrypt.) Furthermore, in our scheme, keys are binary strings rather than group elements, so we cover the most common practical transforms, such as XORing a constant to the key. The scheme itself is in fact the same $\mathsf{HtX}[\mathsf{H}]$ scheme that we showed KDM secure in Section 5.6 and is thus quite simple and natural. We continue to assume only a mUCE1-secure family of functions.

DEFINITIONS. Let SE be a symmetric encryption (SE) scheme as defined in Section 5.4. In game $\mathrm{RKA}^A_{\mathsf{SE}}(\lambda)$ of Fig. 13, an adversary $A = (A_1, A_2)$ is a pair of PT algorithms. Algorithm A_1, when invoked with $(1^\lambda, \varepsilon)$, returns a vectors $\mathbf{m}_0, \mathbf{m}_1$ of messages, along with state information t. It is required that $|\mathbf{m}_0| = |\mathbf{m}_1|$ and that $|\mathbf{m}_0[i]| = |\mathbf{m}_1[i]|$ for all $i \in [1, |\mathbf{m}_0|]$. Then, when invoked with $(1^\lambda, (t, k))$, it produces a vector $\mathbf{k} \in (\{0,1\}^{\mathsf{SE.kl}(\lambda)})^{|\mathbf{m}_0|}$, the entries of this vector being the derived, or related keys. RKA-security is not achievable if A_1 can produce arbitrary keys, as shown by impossibility results in [18], so, following the latter, one usually parametrizes security via a class Φ of transforms that the adversary is allowed to apply to the base key to obtain the related keys, and restricts this class appropriately to obtain results. We will not take this Φ-parametrized approach because we can achieve security for key-deriving functions that are arbitrary subject only to the necessary condition of being unpredictable. Define the guessing probability Guess_A of

A as the function that on input $\lambda \in \mathbb{N}$ returns the maximum, over all k', i and $(\mathbf{m}_0, \mathbf{m}_1, t) \in [A_1(1^\lambda, \varepsilon)]$, of $\Pr[\mathbf{k}[i] = k']$, the probability being over $k \leftarrow\!\!\$\ \{0,1\}^{\mathsf{SE.kl}(\lambda)}$; $\mathbf{k} \leftarrow\!\!\$\ A_1(1^\lambda, t, k)$. We say that A has high min-entropy if $\mathrm{Guess}_A(\cdot)$ is negligible. We say that SE is RKA-secure if $\mathsf{Adv}^{\mathsf{rka}}_{\mathsf{SE},A}(\cdot)$ is negligible for all PT A that have high min-entropy, where $\mathsf{Adv}^{\mathsf{rka}}_{\mathsf{SE},A}(\lambda) = 2\Pr[\mathrm{RKA}^A_{\mathsf{SE}}(\lambda)] - 1$. We let RKA denote the set of all RKA-secure symmetric encryption schemes.

Results. Let H be a family of functions with $\mathsf{H.OL}(\lambda) = \mathbb{N}$ and $\lambda \in \mathsf{H.IL}(\lambda)$ for all $\lambda \in \mathbb{N}$. The following theorem states that the SE scheme $\mathsf{SE} = \mathsf{HtX}[\mathsf{H}]$, defined in Section 5.6 and depicted in Fig. 13, is RKA-secure, assuming only that H is mUCE1-secure.

Theorem 5.7 If $\mathsf{H} \in \mathsf{mUCE1}$, then $\mathsf{HtX}[\mathsf{H}] \in \mathsf{RKA}$.

Proof: Let $\mathsf{SE} = \mathsf{HtX}[\mathsf{H}]$. Let $A = (A_1, A_2)$ be a PT RKA adversary of high min-entropy. We will construct a multi-key unpredictable source S and a distinguisher D such that

$$\mathsf{Adv}^{\mathsf{rka}}_{\mathsf{HtX}[\mathsf{H}],A}(\cdot) \le 2 \cdot \mathsf{Adv}^{\mathsf{m\text{-}uce}}_{\mathsf{H},S,D}(\cdot) \ . \tag{11}$$

The theorem then follows from the assumption that $\mathsf{H} \in \mathsf{mUCE1}$. Let m be a polynomial such that, in game $\mathrm{RKA}^A_{\mathsf{SE}}(\lambda)$, we have $|\mathbf{m}_0| \le m(\lambda)$ for all $\lambda \in \mathbb{N}$. The constructions of S and D are shown below:

$\underline{S^{\mathrm{Hash}}(1^\lambda, t)}$	$\underline{D(1^\lambda, \mathbf{hk}, L)}$
If $t = \varepsilon$ then	$(\mathbf{c}', t', d) \leftarrow L$
$\quad(\mathbf{m}_0, \mathbf{m}_1, t') \leftarrow\!\!\$\ A_1(1^\lambda)$; $n \leftarrow \lvert\mathbf{m}_0\rvert$; Return $(1^n, (\mathbf{m}_0, \mathbf{m}_1, t'))$	For $i = 1$ to $\lvert\mathbf{c}'\rvert$ do
Else	$\quad\mathbf{c}[i] \leftarrow (\mathbf{hk}[i], \mathbf{c}'[i])$
$\quad(\mathbf{m}_0, \mathbf{m}_1, t') \leftarrow t$; $k \leftarrow\!\!\$\ \{0,1\}^\lambda$; $\mathbf{k} \leftarrow\!\!\$\ A_1(1^\lambda, t', k)$; $d \leftarrow\!\!\$\ \{0,1\}$	$d' \leftarrow\!\!\$\ A_2(1^\lambda, t', \mathbf{c})$
$\quad$For $i = 1$ to $\lvert\mathbf{m}_d\rvert$ do $\mathbf{c}'[i] \leftarrow \mathrm{Hash}(\mathbf{k}[i], 1^{\lvert\mathbf{m}_d[i]\rvert}, i) \oplus \mathbf{m}_d[i]$	If $(d = d')$ then $b' \leftarrow 1$ else $b' \leftarrow 0$
$\quad L \leftarrow (\mathbf{c}', t', d)$; Return L	Return b'

Let b denote the challenge bit in game $\mathrm{mUCE}^{S,D}_{\mathsf{H}}(\cdot)$. Then

$$\Pr[\,\mathrm{mUCE}^{S,D}_{\mathsf{H}}(\cdot) \mid b = 1\,] = \Pr[\mathrm{RKA}^A_{\mathsf{SE}}(\cdot)]$$
$$\Pr[\,\mathrm{mUCE}^{S,D}_{\mathsf{H}}(\cdot) \mid b = 0\,] = \frac{1}{2} \ .$$

Summing yields Equation (11). It remains to show that S is unpredictable. By Lemma 4.5, it suffices to show that S is simple unpredictable. Consider an arbitrary PT simple predictor P'. The leakage $(\mathbf{c}, t', d)$ that P' receives is independent of $\mathbf{k}$. Hence $\mathsf{Adv}^{\mathsf{m\text{-}spred}}_{P',S}(\cdot) \le m \cdot \mathrm{Guess}_A(\cdot)$. ∎

5.8 OAEP

OAEP [22] is a ROM transform of a trapdoor permutation to a PKE scheme. If the trapdoor permutation is one-way then the associated OAEP PKE scheme is IND-CPA in the ROM [22]. We would like to instantiate the RO in OAEP in a way that retains this result in the standard model. Here we show that a UCE1 instantiation gets us IND-CPA-KI (IND-CPA for messages that do not depend on the public key) assuming the trapdoor permutation is partially one-way. (Later we will see that a UCE2 instantiation will reach the same conclusion assuming only one-wayness, but we note that for RSA, the most popular choice of trapdoor permutation, one-wayness implies partial one-wayness anyway [59].) Compared to KOS [78], we have relaxed the assumption on the trapdoor permutation from lossiness to plain one-wayness. In the particular case of RSA we have relaxed the assumption from Φ-hiding to standard one-wayness. We note that RSA-OAEP is a widely used and implemented standard.

Definitions. Let TF be a family of functions with input length $\mathsf{TF.il}$ and output length $\mathsf{TF.ol}$. We say that TF is *p-partially one-way*, where p: $\mathbb{N} \to \mathbb{N}$, if $\mathsf{Adv}^{\mathsf{pow}}_{\mathsf{TF},p,I}(\cdot)$ is negligible for all PT I, where

MAIN IND-CPA-KI$^A_{\mathsf{PKE}}(\lambda)$	$\mathsf{OAEP.Kg}(1^\lambda)$	$\mathsf{OAEP.Dec}(1^\lambda, (dk, hk), c)$
$b \leftarrow\!\!\$\ \{0,1\}$	$(ek, dk) \leftarrow\!\!\$\ \mathsf{TF.EKg}(1^\lambda)$	$c' \leftarrow \mathsf{TF.Inv}(1^\lambda, dk, c, 1^{\mathsf{TF.ol}(\lambda)})$
$(ek, dk) \leftarrow\!\!\$\ \mathsf{PKE.Kg}(1^\lambda)$	$hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$	$x \Vert_{\ell_3(\lambda)} y \leftarrow c'$
$t \leftarrow\!\!\$\ A^{\mathrm{LR}}(1^\lambda, \varepsilon)$	Return $((ek, hk), (dk, hk))$	$t_2 \leftarrow \mathsf{H.Ev}(1^\lambda, hk, 1 \Vert x, 1^{\ell_3(\lambda)})$
$b' \leftarrow\!\!\$\ A(1^\lambda, t, ek)$	$\mathsf{OAEP.Enc}(1^\lambda, (ek, hk), m)$	$r \leftarrow y \oplus t_2$
Return $(b = b')$	$r \leftarrow\!\!\$\ \{0,1\}^{\ell_3(\lambda)}$	$t_1 \leftarrow \mathsf{H.Ev}(1^\lambda, hk, 0 \Vert r, 1^{\ell_1(\lambda)+\ell_2(\lambda)})$
$\mathrm{LR}(m_0, m_1)$	$t_1 \leftarrow \mathsf{H.Ev}(1^\lambda, hk, 0 \Vert r, 1^{\ell_1(\lambda)+\ell_2(\lambda)})$	$m \Vert_{\ell_2(\lambda)} z \leftarrow x \oplus t_1$
$c \leftarrow\!\!\$\ \mathsf{PKE.Enc}(1^\lambda, ek, m_b)$	$x \leftarrow (m \,\Vert\, 0^{\ell_2(\lambda)}) \oplus t_1$	If $(z = 0^{\ell_2(\lambda)})$ then return m else return $\perp$
Return c	$t_2 \leftarrow \mathsf{H.Ev}(1^\lambda, hk, 1 \Vert x, 1^{\ell_3(\lambda)})$	
	$y \leftarrow t_2 \oplus r$	
	$c \leftarrow \mathsf{TF.Ev}(1^\lambda, ek, x \,\Vert\, y, 1^{\mathsf{TF.ol}(\lambda)})$	
	Return c	

Figure 14: **Left:** Game defining IND-CPA-KI security of public-key encryption scheme PKE. **Right:** $\mathsf{OAEP}[\mathsf{H}, \mathsf{TF}, \ell_1, \ell_2, \ell_3]$ scheme.

$\mathsf{Adv}^{\mathsf{pow}}_{\mathsf{TF},p,I}(\lambda) = \Pr[I(1^\lambda, ek, y) = x[1, p(\lambda)]]$ in the experiment $ek \leftarrow\!\!\$\ \mathsf{TF.Kg}(1^\lambda)\ ;\ x \leftarrow\!\!\$\ \{0,1\}^{\mathsf{TF.il}(\lambda)}\ ;\ y \leftarrow \mathsf{TF.Ev}(1^\lambda, ek, x, 1^{\mathsf{TF.ol}(\lambda)})$. We let OW_p be the set of all TF that are p-partially one-way.

We say that a PKE scheme PKE is IND-CPA-KI secure if $\mathsf{Adv}^{\mathsf{indcpa\text{-}ki}}_{\mathsf{PKE},A}(\cdot)$ is negligible for all PT adversaries A, where $\mathsf{Adv}^{\mathsf{indcpa\text{-}ki}}_{\mathsf{PKE},A}(\lambda) = 2\Pr[\text{IND-CPA-KI}^A_{\mathsf{PKE}}(\lambda)] - 1$ and game IND-CPA-KI$^A_{\mathsf{PKE}}(\lambda)$ is in Fig. 14. Messages m_0, m_1 queried to LR are required to be of the same length. We let $\mathsf{IND\text{-}CPA\text{-}KI}$ denote the set of IND-CPA-KI-secure PKE schemes.

RESULTS. Let TF be a trapdoor family of functions (as defined in Section 5.2) with $\mathsf{TF.il} = \mathsf{TF.ol}$. Let ℓ_1, ℓ_2, ℓ_3: $\mathbb{N} \to \mathbb{N}$ satisfy $\ell_1 + \ell_2 + \ell_3 = \mathsf{TF.il}$. Let H be a family of functions such that $1 + \ell_1(\lambda) + \ell_2(\lambda), 1 + \ell_3(\lambda) \in \mathsf{H.IL}(\lambda)$ and $\ell_3(\lambda), \ell_1(\lambda) + \ell_2(\lambda) \in \mathsf{H.OL}(\lambda)$ for all $\lambda \in \mathbb{N}$. Our instantiated OAEP transform associates to these the PKE scheme $\mathsf{PKE} = \mathsf{OAEP}[\mathsf{H}, \mathsf{TF}, \ell_1, \ell_2, \ell_3]$ whose algorithms are described in Fig. 14. The scheme has message-length function $\mathsf{PKE.il} = \ell_1$. We note that we use the fact that our family H allows variable output lengths. Pre-pending a 0 bit to r and a 1 bit to x before hashing is for domain separation. The following says that if TF is $(\ell_1+\ell_2)$-partially one-way and H is UCE1-secure then $\mathsf{OAEP}[\mathsf{H}, \mathsf{TF}, \ell_1, \ell_2, \ell_3]$ is IND-CPA-KI secure as long as ℓ_2, ℓ_3 are super-logarithmic.

Theorem 5.8 Let $\mathsf{TF}, \mathsf{H}, \ell_1, \ell_2, \ell_3$ be as above, and let $\mathsf{PKE} = \mathsf{OAEP}[\mathsf{H}, \mathsf{TF}, \ell_1, \ell_2, \ell_3]$. Assume $2^{-\ell_1-\ell_2}, 2^{-\ell_3}$ are negligible. If $\mathsf{H} \in \mathsf{UCE1}$ and $\mathsf{TF} \in \mathsf{OW}_{\ell_1+\ell_2}$, then $\mathsf{PKE} \in \mathsf{IND\text{-}CPA\text{-}KI}$.

Proof of Theorem 5.8: Let A be a PT adversary for game IND-CPA-KI$^A_{\mathsf{PKE}}(\lambda)$. Let q be a polynomial such that the number of LR-queries of A in this game is at most $q(\lambda)$. Let

$$\epsilon(\lambda) = \frac{q(\lambda)^2}{2^{1+\ell_3(\lambda)}} + \frac{q(\lambda)^2}{2^{1+\ell_1(\lambda)+\ell_2(\lambda)}}$$

for all $\lambda \in \mathbb{N}$. We'll construct an unpredictable source S and a distinguisher D such that

$$\mathsf{Adv}^{\mathsf{indcpa\text{-}ki}}_{\mathsf{PKE},A}(\cdot) \le 2\mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\cdot) + 2\epsilon\ . \tag{12}$$

The theorem follows from the assumption that $\mathsf{H} \in \mathsf{UCE1}$ and that $2^{-\ell_1-\ell_2}, 2^{-\ell_3}$ are negligible. The constructions of S and D are shown below:

$\underline{S^{\text{HASH}}(1^\lambda)}$
$ek \leftarrow\!\$\ \mathsf{TF.Kg}(1^\lambda)\ ;\ d \leftarrow\!\$\ \{0,1\}$
$t \leftarrow\!\$\ A^{\text{LRSIM}}(1^\lambda, \varepsilon)\ ;\ L \leftarrow (ek, t, d)$
Return L

$\underline{\text{LRSIM}(m_0, m_1)}$
$r \leftarrow\!\$\ \{0,1\}^{\ell_3(\lambda)}\ ;\ t_1 \leftarrow \text{HASH}(0 \| r, 1^{\ell_1(\lambda)+\ell_2(\lambda)})$
$x \leftarrow (m_d \,\|\, 0^{\ell_2(\lambda)}) \oplus t_1\ ;\ t_2 \leftarrow \text{HASH}(1 \| x, 1^{\ell_3(\lambda)})$
$y \leftarrow t_2 \oplus r\ ;\ c \leftarrow \mathsf{TF.Ev}(1^\lambda, ek, x \,\|\, y, 1^{\mathsf{TF.ol}(\lambda)})$
Return c

$\underline{D(1^\lambda, hk, L)}$
$(ek, t, d) \leftarrow L$
$d' \leftarrow\!\$\ A(1^\lambda, t, (ek, hk))$
If $(d = d')$ then $b' \leftarrow 1$ else $b' \leftarrow 0$
Return b'

Let b denote the challenge bit in game $\text{UCE}^{S,D}_{\mathsf{H}}(\lambda)$. We claim that

$$\Pr[\,\text{UCE}^{S,D}_{\mathsf{H}}(\cdot) \,|\, b = 1\,] = \frac{1}{2} + \frac{1}{2}\mathsf{Adv}^{\mathsf{indcpa\text{-}ki}}_{\mathsf{PKE},A}(\cdot) \tag{13}$$

$$1 - \Pr[\,\text{UCE}^{S,D}_{\mathsf{H}}(\cdot) \,|\, b = 0\,] \leq \frac{1}{2} + \epsilon\ . \tag{14}$$

Subtracting and re-arranging terms, we have Equation (12). We turn to justifying the two equations above. The first follows from the adversary definitions. For the second, consider the following games, where G_1 includes the boxed code and G_2 does not:

$\underline{\text{MAIN } \boxed{\text{G}^A_1(\lambda)},\ \text{G}^A_2(\lambda)}$
$ek \leftarrow\!\$\ \mathsf{TF.Kg}(1^\lambda)$
$hk \leftarrow\!\$\ \mathsf{Kg}(1^\lambda)$
$d \leftarrow\!\$\ \{0,1\}$
$Q_0, Q_1 \leftarrow \emptyset$
$t \leftarrow\!\$\ A^{\text{LR}}(1^\lambda, \varepsilon)$
$d' \leftarrow\!\$\ A(1^\lambda, t, (ek, hk))$
Return $(d = d')$

$\underline{\text{LR}(m_0, m_1)}$
$r \leftarrow\!\$\ \{0,1\}^{\ell_3(\lambda)}\ ;\ x \leftarrow\!\$\ \{0,1\}^{\ell_1(\lambda)+\ell_2(\lambda)}\ ;\ z \leftarrow m_d \,\|\, 0^{\ell_2(\lambda)}\ ;\ t_1 \leftarrow z \oplus x$
If $(r \in Q_0)$ then $\mathsf{bad} \leftarrow \mathsf{true}\ ;\ \boxed{t_1 \leftarrow T_0[r]\ ;\ x \leftarrow t_1 \oplus z}$
$T_0[r] \leftarrow t_1\ ;\ Q_0 \leftarrow Q_0 \cup \{r\}$
$y \leftarrow\!\$\ \{0,1\}^{\ell_3(\lambda)}\ ;\ t_2 \leftarrow y \oplus r$
If $(x \in Q_1)$ then $\mathsf{bad} \leftarrow \mathsf{true}\ ;\ \boxed{t_2 \leftarrow T_1[x]\ ;\ y \leftarrow t_2 \oplus r}$
$T_1[x] \leftarrow t_2\ ;\ Q_1 \leftarrow Q_1 \cup \{x\}$
$c \leftarrow \mathsf{TF.Ev}(1^\lambda, ek, x \,\|\, y, 1^{\mathsf{TF.ol}(\lambda)})\ ;$ Return c

The games are identical-until-bad so by the Fundamental Lemma of Game Playing [24] we have

$$\begin{aligned}
1 - \Pr[\,\text{UCE}^{S,D}_{\mathsf{H}}(\cdot) \,|\, b = 0\,] &= \Pr[\text{G}^A_1(\cdot)] \\
&\leq \Pr[\text{G}^A_2(\cdot)] + \Pr[\text{G}^A_2(\cdot) \text{ sets } \mathsf{bad}] \\
&= \frac{1}{2} + \Pr[\text{G}^A_2(\cdot) \text{ sets } \mathsf{bad}] \\
&\leq \frac{1}{2} + \frac{q(q-1)}{2^{1+\ell_3}} + \frac{q(q-1)}{2^{1+\ell_1+\ell_2}}\ ,
\end{aligned}$$

which establishes Equation (14). It remains to show that S is unpredictable. By Lemma 4.1, it suffices to show that S is simple-unpredictable. Let P' be a PT simple predictor and consider the invertor defined below:

$\underline{I(1^\lambda, ek, c')}$
$d \leftarrow\!\$\ \{0,1\}\ ;\ j \leftarrow\!\$\ \{1, \ldots, q(\lambda)\}\ ;\ i \leftarrow 0$
$t \leftarrow\!\$\ A^{\text{LRSIM}}(1^\lambda, \varepsilon)\ ;\ L \leftarrow (ek, t, d)\ ;\ u \leftarrow\!\$\ P'(1^\lambda, L)$
Return $u[2, |u|]$

$\underline{\text{LRSIM}(m_0, m_1)}$
$i \leftarrow i + 1$
If $(i = j)$ then return c'
$x_i \leftarrow\!\$\ \{0,1\}^{\ell_1(\lambda)+\ell_2(\lambda)}\ ;\ y_i \leftarrow\!\$\ \{0,1\}^{\ell_3(\lambda)}$
$c_i \leftarrow \mathsf{TF.Ev}(1^\lambda, ek, x_i \,\|\, y_i, 1^{\mathsf{TF.ol}(\lambda)})$
Return c_i

Then we claim that

$$\mathsf{Adv}^{\mathsf{spred}}_{P',S}(\cdot) \leq q \cdot \mathsf{Adv}^{\mathsf{pow}}_{\mathsf{TF},\ell_1+\ell_2,I}(\cdot) + \epsilon + \frac{q}{2^{\ell_3}}\ . \tag{15}$$

The simple unpredictability of S follows from the assumption that $\mathsf{TF} \in \mathsf{OW}_{\ell_1+\ell_2}$ and $2^{-\ell_1-\ell_2}, 2^{-\ell_3}$ are negligible. To justify Equation (15), consider the games whose MAIN procedures are below, with LR continuing to be the same as for games $\mathrm{G}_1^A(\lambda), \mathrm{G}_2^A(\lambda)$ above:

$\underline{\text{MAIN } \boxed{\mathrm{G}_3^{A,P'}(\lambda)},\ \mathrm{G}_4^{A,P'}(\lambda)}$	$\underline{\text{MAIN } \mathrm{G}_5^{A,P'}(\lambda)}$
$ek \leftarrow\!\!\$\ \mathsf{TF.Kg}(1^\lambda)\ ;\ hk \leftarrow\!\!\$\ \mathsf{Kg}(1^\lambda)$	$ek \leftarrow\!\!\$\ \mathsf{TF.Kg}(1^\lambda)\ ;\ hk \leftarrow\!\!\$\ \mathsf{Kg}(1^\lambda)$
$d \leftarrow\!\!\$\ \{0,1\}\ ;\ Q_0, Q_1 \leftarrow \emptyset\ ;\ t \leftarrow\!\!\$\ A^{\mathrm{LR}}(1^\lambda, \varepsilon)$	$d \leftarrow\!\!\$\ \{0,1\}\ ;\ Q_0, Q_1 \leftarrow \emptyset\ ;\ t \leftarrow\!\!\$\ A^{\mathrm{LR}}(1^\lambda, \varepsilon)$
$L \leftarrow (ek, t, d)\ ;\ u \leftarrow\!\!\$\ P'(1^\lambda, L)$	$L \leftarrow (ek, t, d)\ ;\ u \leftarrow\!\!\$\ P'(1^\lambda, L)$
Return $(u[2, \lvert u\rvert] \in Q_0 \cup Q_1)$	Return $(u[2, \lvert u\rvert] \in Q_1)$

Then

$$\begin{aligned}
\mathsf{Adv}^{\mathsf{spred}}_{P',S,P'}(\cdot) &\le \Pr[\mathrm{G}_3^{A,P'}(\cdot)] \\
&\le \Pr[\mathrm{G}_4^{A,P'}(\cdot)] + \Pr[\mathrm{G}_4^{A,P'}(\cdot) \text{ sets } \mathsf{bad}] \\
&\le \Pr[\mathrm{G}_4^{A,P'}(\cdot)] + \epsilon \\
&\le \Pr[\mathrm{G}_5^{A,P'}(\cdot)] + \epsilon + \frac{q}{2^{\ell_3}} \\
&\le q \cdot \mathsf{Adv}^{\mathsf{pow}}_{\mathsf{TF},\ell_1+\ell_2,I}(\cdot) + \epsilon + \frac{q}{2^{\ell_3}}\ ,
\end{aligned}$$

establishing Equation (15). ∎

We remark that the assumption that TF is partially one-way, rather than merely one-way, is crucial to the proof above. This is because there is a one-way TF that makes the source S above predictable. For example, let $\mathsf{TF.EKg} = \mathsf{F.EKg}$ and $\mathsf{TF.Ev}(1^\lambda, ek, x \,\|\, y, 1^{\mathsf{TF.ol}(\lambda)}) = x \,\|\, \mathsf{F.Ev}(1^\lambda, ek, y, 1^{\mathsf{F.ol}(\lambda)})$, where F is a trapdoor permutation family with $\mathsf{F.il} = \ell_3$. This counter-example TF is one-way but not $(\ell_1+\ell_2)$-partially one-way, and makes S predictable. Theorem 6.3 shows that making the stronger UCE2 assumption on H will allow us to relax the assumption on TF to plain one-wayness.

5.9 Proofs of storage

Client Alice has uploaded her file x to a server in the cloud. She is worried that the server is malicious and, to save space, is not actually storing x. A storage-auditing protocol [8, 76, 91] allows Alice to efficiently verify that the server stores her file. A particularly natural and canonical protocol for this task, embodied in the SafeStore system [80], is for Alice to send the server a random challenge hk and expect in response $H(x\|hk)$ where H is a hash function. Ristenpart, Shacham and Shrimpton (RSS) [90] show that this protocol is secure if H is a RO. Interestingly, however, they show that H being indifferentiabile from a RO [85] is *not* enough for the protocol to be secure. We show that UCE1 *is* enough. Our instantiation interprets hk as a key for a UCE1 family H, meaning we let $H(x\|hk) = \mathsf{H.Ev}(1^\lambda, hk, x, 1^\lambda)$. This results in a natural, simple standard model, secure storage-auditing protocol.

DEFINITIONS. The security definition of RSS [90] assumes the message (file) x is a uniformly random string as a function of which the adversary has computed some information s that it stores. In asymptotic terms, the requirement is then that the adversary cannot defeat the audit (meaning, provide the correct hash value) as long as $2^{|s|-|x|}$ is negligible. Real files are, however, not uniformly random strings. We accordingly provide a stronger and more general definition that implies that of RSS.

Let H be a family of functions with $\mathsf{H.IL}(\lambda) = \mathbb{N}$ and $\lambda \in \mathsf{H.OL}(\lambda)$ for all $\lambda \in \mathbb{N}$. In game $\mathrm{Store}^A_{\mathsf{H}}(\lambda)$ of Fig. 15, the adversary $A = (A_1, A_2)$ is a pair of algorithms. Algorithm A_1 produces the message x together with the information s about x that it will store. (So s can be a function of x.) Then a challenge hk is picked at random, and, to win, A_2, given s and hk, must be able to provide $y = \mathsf{H.Ev}(1^\lambda, hk, x, 1^\lambda)$. We let $\mathsf{Adv}^{\mathsf{store}}_{\mathsf{H},A}(\lambda) = \Pr[\mathrm{Store}^A_{\mathsf{H}}(\lambda)]$ for all $\lambda \in \mathbb{N}$. This advantage cannot be small for all A, for A_1 can let $s = x$

$\underline{\text{MAIN Store}^{A}_{\mathsf{H}}(\lambda)}$
$(x,s) \leftarrow\!\!\$\ A_1(1^\lambda)\ ;\ hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)\ ;\ y' \leftarrow\!\!\$\ A_2(1^\lambda, hk, s)$
$y \leftarrow \mathsf{H.Ev}(1^\lambda, hk, x, 1^\lambda)\ ;$ Return $(y = y')$

Figure 15: **Game defining storage-auditing security.**

and then A_2 can compute the correct hash, but this corresponds to a server who does, indeed, store the file. So let us define the guessing probability $\text{Guess}_A(\cdot)$ of A via

$$\text{Guess}_A(\lambda) = \sum_{s'} \Pr[s = s'] \cdot \max_{x'} \Pr[\, x = x' \,|\, s = s' \,]$$

for all $\lambda \in \mathbb{N}$, where the probability is over $(x,s) \leftarrow\!\!\$\ A_1(1^\lambda)$. We say that A has *high min-entropy* if $\text{Guess}_A(\cdot)$ is negligible. Having high min-entropy corresponds to A cheating, meaning not storing information sufficient to recover the file. We say that a hash family H is *storage-auditing secure* if $\mathsf{Adv}^{\mathsf{store}}_{\mathsf{H},A}(\cdot)$ is negligible for all PT adversaries A of high min-entropy. This captures the requirement that adversaries that fail to store enough information to recover the file will also fail to pass the audit protocol. Let STORE denote the set of all storage-auditing secure hash families.

RESULTS. The following says that UCE1 security implies storage-auditing security.

Theorem 5.9 Let H be a hash family. If $\mathsf{H} \in \mathsf{UCE1}$ then $\mathsf{H} \in \mathsf{STORE}$.

Proof: Let A be a PT adversary of high min-entropy. We'll construct an unpredictable source S and a distinguisher D such that

$$\mathsf{Adv}^{\mathsf{store}}_{\mathsf{H},A}(\lambda) \le \mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\lambda) + 2^{-\lambda} \tag{16}$$

for every $\lambda \in \mathbb{N}$. The theorem follows from the assumption that $\mathsf{H} \in \mathsf{UCE1}$. The constructions of S and D are shown below:

$\underline{S^{\text{HASH}}(1^\lambda)}$	$\underline{D(1^\lambda, hk, L)}$
$(x,s) \leftarrow\!\!\$\ A_1(1^\lambda)\ ;\ y \leftarrow \text{HASH}(x, 1^\lambda)$	$(y,s) \leftarrow L\ ;\ y' \leftarrow A_2(1^\lambda, hk, s)$
$L \leftarrow (y,s)\ ;$ Return L	If $(y = y')$ then $b' \leftarrow 1$ else $b' \leftarrow 0\ ;$ Return b'

Let b denote the challenge bit in game $\text{UCE}^{S,D}_{\mathsf{H}}(\cdot)$. Then for all $\lambda \in \mathbb{N}$ we have

$$\Pr[\, \text{UCE}^{S,D}_{\mathsf{H}}(\lambda) \,|\, b = 1 \,] = \mathsf{Adv}^{\mathsf{store}}_{\mathsf{H},A}(\lambda)$$
$$\Pr[\, \text{UCE}^{S,D}_{\mathsf{H}}(\lambda) \,|\, b = 0 \,] = 1 - 2^{-\lambda}\ .$$

Summing yields Equation (16). It remains to show that S is unpredictable. By Lemma 4.5, it suffices to show that S is simple unpredictable. Consider an arbitrary PT simple predictor P'. Then $\mathsf{Adv}^{\mathsf{spred}}_{S,P'}(\cdot) \le \text{Guess}_A(\cdot)$. From the assumption that A is of high min-entropy, the claim follows. ▌

5.10 Correlated-input hash functions

Goyal, O'Neill, and Rao (GOR) [70] introduced the notion of correlated-input hash (CIH) function families and proposed several notions of security for them. They provided constructions achieving limited CIH security from the q-DHI assumption of [33] and from RKA secure blockciphers. But achieving full CIH security in the standard model has remained open. Here, we show that UCE1-secure function families achieve this goal, being selective (pseudorandomness) CIH secure in the terminology of GOR.

DEFINITIONS. Let H be a function family with input length $\mathsf{H.il}$ and output length $\mathsf{H.ol}$. Game $\text{CIH}^{A}_{\mathsf{H}}(\lambda)$ of Fig. 16 captures the selective pseudorandom correlated-input hash security notion of [70] adapted to

$\underline{\text{MAIN } \mathrm{CIH}^A_{\mathsf{H}}(\lambda)}$
$hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)\ ;\ b \leftarrow\!\!\$\ \{0,1\}\ ;\ \mathbf{m} \leftarrow\!\!\$\ A_1(1^\lambda)$
For $i = 1$ to $|\mathbf{m}|$ do $\mathbf{h}_1[i] \leftarrow \mathsf{H.Ev}(1^\lambda, hk, \mathbf{m}[i], 1^{\mathsf{H.ol}(\lambda)})\ ;\ \mathbf{h}_0[i] \leftarrow\!\!\$\ \{0,1\}^{\mathsf{H.ol}(\lambda)}$
$b' \leftarrow\!\!\$\ A_2(1^\lambda, hk, \mathbf{h}_b)\ ;$ Return $(b' = b)$

Figure 16: **The** CIH **game.**

our setting and notation. An adversary $A = (A_1, A_2)$ is a pair of algorithms. Algorithm A_1 on input 1^λ returns $\mathbf{m}$, a $v(\lambda)$-vector of distinct $\mathsf{H.il}(\lambda)$-bit strings, where the polynomial v depends on A. The guessing probability Guess_A of A is the function that on input $\lambda \in \mathbb{N}$ returns the maximum, over all i, m, of $\Pr[\mathbf{m}[i] = m]$, the probability over $\mathbf{m} \leftarrow\!\!\$\ A_1(1^\lambda)$. We say that A has *high min-entropy* if $\mathrm{Guess}_A(\cdot)$ is negligible. We let $\mathsf{Adv}^{\mathsf{cih}}_{\mathsf{H},A}(\lambda) = 2\Pr[\mathrm{CIH}^A_{\mathsf{H}}(\lambda)] - 1$. We say that H is CIH-secure if $\mathsf{Adv}^{\mathsf{cih}}_{\mathsf{H},A}$ is negligible for all PT A that have high min-entropy. Note that the high min-entropy assumption on A is necessary to achieve security. We let CIH be the set of all CIH-secure function families.

RESULTS. Let H be a function family with input length $\mathsf{H.il}$ and output length $\mathsf{H.ol}$. The following theorem says that if H is UCE-secure then it is also CIH-secure.

Theorem 5.10 Let H be a FOL function family. If $\mathsf{H} \in \mathsf{UCE1}$ then $\mathsf{H} \in \mathsf{CIH}$.

Proof: Let $A = (A_1, A_2)$ be a PT CIH adversary. we will construct an unpredictable source S and a distinguisher D such that

$$\mathsf{Adv}^{\mathsf{cih}}_{\mathsf{H},A}(\cdot) = \mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\cdot)\ . \tag{17}$$

The theorem then follows from the assumption that $\mathsf{H} \in \mathsf{mUCE1}$. The constructions of S and D are shown below:

$\underline{S^{\mathrm{HASH}}(1^\lambda)}$	$\underline{D(1^\lambda, hk, L)}$
$\mathbf{m} \leftarrow\!\!\$\ A_1(1^\lambda)$	$\mathbf{h} \leftarrow L$
For $i = 1$ to $\lvert\mathbf{m}\rvert$ do $\mathbf{h}[i] \leftarrow \mathrm{HASH}(\mathbf{m}[i], 1^{\mathsf{H.ol}(\lambda)})$	$b' \leftarrow\!\!\$\ A_2(1^\lambda, hk, \mathbf{h})$
$L \leftarrow \mathbf{h}\ ;$ Return L	Return b'

It remains to show that S is unpredictable. By Lemma 4.1, it suffices to show that S is simple unpredictable. Consider an arbitrary PT simple predictor P'. The leakage it receives consists of random strings unrelated to the messages that S queries to its oracle, so $\mathsf{Adv}^{\mathsf{spred}}_{S,P'}(\cdot) \le v \cdot \mathrm{Guess}_A(\cdot)$ where v is the polynomial associated to A as per the definition. ∎

We note that UCE1 security does not imply adaptive CIH security, where the inputs can depend on the key.

6 UCE2 security and applications

We define a form of UCE, called UCE2, that is stronger than UCE1. We apply it to improve the UCE1-based OAEP results and to obtain standard-model adaptively-secure garbling schemes with short tokens.

6.1 UCE2 security notion

UCE2 maintains the basic source-distinguisher framework of UCE1. However it weakens the unpredictability condition to a condition we call reset security. This results in a stronger notion.

We say that a source S is *reset-secure* if $\mathsf{Adv}^{\mathsf{reset}}_{R,S}(\cdot)$ is negligible for any PT reset adversary R, where $\mathsf{Adv}^{\mathsf{reset}}_{R,S}(\lambda) = 2\Pr[\mathrm{Reset}^R_S(\lambda)] - 1$ and game $\mathrm{Reset}^R_S(\lambda)$ is defined in Fig. 17. The idea here is that R cannot distinguish between the RO HASH used by the source S and an independent RO. We say that H is UCE2-secure if $\mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\cdot)$ is negligible for every PT reset-secure S and every PT D, where the UCE advantage

$\underline{\text{MAIN Reset}_S^R(\lambda)}$	$\underline{\text{MAIN mReset}_S^R(\lambda)}$
$\mathsf{Dom} \leftarrow \emptyset$; $L \leftarrow\!\!\$\ S^{\text{HASH}}(1^\lambda)$; $b \leftarrow\!\!\$\ \{0,1\}$	$\mathsf{Dom} \leftarrow \emptyset$; $(1^n, t) \leftarrow\!\!\$\ S(1^\lambda, \varepsilon)$; $L \leftarrow\!\!\$\ S^{\text{HASH}}(1^\lambda, t)$; $b \leftarrow\!\!\$\ \{0,1\}$
If $b = 0$ then // reset the array T	If $b = 0$ then // reset the array T
For all $(x, \ell) \in \mathsf{Dom}$ do $T[x, \ell] \leftarrow\!\!\$\ \{0,1\}^\ell$	For all $(x, \ell, i) \in \mathsf{Dom}$ do $T[x, \ell, i] \leftarrow\!\!\$\ \{0,1\}^\ell$
$b' \leftarrow R^{\text{HASH}}(1^\lambda, L)$; Return $(b' = b)$	$b' \leftarrow\!\!\$\ R^{\text{HASH}}(1^\lambda, 1^n, L)$; Return $(b = b')$
$\underline{\text{HASH}(x, 1^\ell)}$	$\underline{\text{HASH}(x, 1^\ell, i)}$
$\mathsf{Dom} \leftarrow \mathsf{Dom} \cup \{(x, \ell)\}$	$\mathsf{Dom} \leftarrow \mathsf{Dom} \cup \{(x, \ell, i)\}$
If $T[x, \ell] = \perp$ then $T[x, \ell] \leftarrow\!\!\$\ \{0,1\}^\ell$	If $T[x, \ell, i] = \perp$ then $T[x, \ell, i] \leftarrow\!\!\$\ \{0,1\}^\ell$
Return $T[x, \ell]$	Return $T[x, \ell, i]$

Figure 17: **Left:** Game defining reset security of a source. **Right:** Game defining reset security of a multi-key source.

here continues to be that of Equation (1). Let $\mathsf{UCE2}$ denote the set of all function families H that are UCE2-secure.

For some intuition, recall that unpredictability was introduced as a way to rule out unpreventable success. The example we gave is the source S_1 that queries to its HASH oracle a point x to get back y and returns $L = (x, y)$ as the leakage. S_1 is "insecure" in the sense that there is PT distinguisher D such that $\mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}$ is high. But this does not violate UCE1 since S_1 is unpredictable. Now, let source S_2 query x and get back y but return only $L = x$ as the leakage. UCE1 puts this source out of consideration because it is predictable. Yet, this source is "secure" in the sense that a PT distinguisher, given L, has no advantage in determining the challenge bit. This indicates that unpredictability is sometimes too strong a requirement. However S_2 is reset secure. Thus, UCE2 allows us to "use" S_2. This ability to use a larger class of sources strengthens the notion and is useful in some applications.

Recall that in Section 4.6 we defined $\mathsf{UCE}[\mathcal{S}, \mathcal{D}]$ as the set of all H such that $\mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\cdot)$ is negligible for all $(S, D) \in \mathcal{S} \times \mathcal{D}$. We could recover UCE1 as a special case, namely $\mathsf{UCE1} = \mathsf{UCE}[\mathcal{S}^{\mathsf{poly}} \cap \mathsf{Pred}[\mathcal{P}^{\mathsf{poly}}], \mathcal{D}^{\mathsf{poly}}]$. Similarly, we can see that $\mathsf{UCE2} = \mathsf{UCE}[\mathcal{S}^{\mathsf{poly}} \cap \mathsf{Reset}[\mathcal{R}^{\mathsf{poly}}], \mathcal{D}^{\mathsf{poly}}]$, where $\mathsf{Reset}[\mathcal{R}]$ is the set of all sources S such that $\mathsf{Adv}^{\mathsf{reset}}_{R,S}(\cdot)$ is negligible for all $R \in \mathcal{R}$ and $\mathcal{R}^{\mathsf{poly}}$ is the set of all PT reset adversaries. This shows how UCE2 continues to be part of the broader UCE framework, and will also be useful in understanding how UCE2 relates to UCE1.

We can likewise define mUCE2, the multi-key extension of UCE2. A multi-key source S is *reset-secure* if $\mathsf{Adv}^{\mathsf{m\text{-}reset}}_{R,S}(\cdot)$ is negligible for any PT reset adversary R, where $\mathsf{Adv}^{\mathsf{m\text{-}reset}}_{R,S}(\lambda) = 2\Pr[\text{mReset}^R_S(\lambda)] - 1$ and game $\text{mReset}^R_S(\lambda)$ is defined in Fig. 17. A family H is mUCE2-secure if $\mathsf{Adv}^{\mathsf{m\text{-}uce}}_{\mathsf{H},S,D}(\cdot)$ is negligible for every PT reset-secure multi-key source S and every PT D, where the mUCE advantage here continues to be that of Equation (3). Let $\mathsf{mUCE2}$ denote the set of all function families H that are mUCE2-secure.

6.2 Relations

The following says that UCE2-security implies UCE1-security but the converse doesn't hold. By using a similar proof, it also follows that mUCE2-security implies mUCE1-security but the converse doesn't hold. This establishes some of the relations claimed in Fig. 1. Part (2) below assumes $\mathsf{UCE1} \neq \emptyset$.

Proposition 6.1 (1) $\mathsf{UCE2} \subseteq \mathsf{UCE1}$ and (2) $\mathsf{UCE1} \not\subseteq \mathsf{UCE2}$.

Proof of Proposition 6.1: Recall that $\mathsf{UCE1} = \mathsf{UCE}[\mathcal{S}^{\mathsf{poly}} \cap \mathsf{Pred}[\mathcal{P}^{\mathsf{poly}}], \mathcal{D}^{\mathsf{poly}}]$ and $\mathsf{UCE2} = \mathsf{UCE}[\mathcal{S}^{\mathsf{poly}} \cap \mathsf{Reset}[\mathcal{R}^{\mathsf{poly}}], \mathcal{D}^{\mathsf{poly}}]$. For part (1), by Proposition 4.4 it suffices to show that $\mathsf{Pred}[\mathcal{P}^{\mathsf{poly}}] \subseteq \mathsf{Reset}[\mathcal{R}^{\mathsf{poly}}]$. (That is, any unpredictable source is reset secure.) So let S be an unpredictable source. We want to show that S is reset secure. Intuitively, this is because a reset adversary will be unable to query its oracle at any point where the source queried its oracle, except with negligible probability, and hence will usually have the same view both when its oracle is reset and when it is not. Formally, given a reset adversary R we build a predictor P such that $\mathsf{Adv}^{\mathsf{reset}}_{R,S}(\cdot) \leq \mathsf{Adv}^{\mathsf{pred}}_{S,P}(\cdot)$. Predictor P is on the left below:

$\underline{P^{\text{HASH}}(1^\lambda, L)}$
$Q' \leftarrow \emptyset$; $b' \leftarrow\!\!\$\ R^{\text{HASHSIM}}(1^\lambda, L)$
Return Q'

$\underline{\text{HASHSIM}(x, \ell)}$
$Q' \leftarrow Q' \cup \{x\}$; $T[x, \ell] \leftarrow\!\!\$\ \{0,1\}^\ell$
Return $T[x, \ell]$

$\underline{\text{MAIN } \mathrm{G}_0^{S,R}(\lambda) \ /\ \boxed{\mathrm{G}_1^{S,R}(\lambda)}}$
$Q \leftarrow \emptyset$; $L \leftarrow\!\!\$\ S^{\text{HASH}}(1^\lambda)$; $b' \leftarrow R^{\text{HASH}}(1^\lambda, L)$
Return $(b' = 1)$

$\underline{\text{HASH}(x, \ell)}$
If $T[x, \ell] = \perp$ then $T[x, \ell] \leftarrow\!\!\$\ \{0,1\}^\ell$
Else $\mathsf{bad} \leftarrow \mathsf{true}$; $\boxed{T[x, \ell] \leftarrow\!\!\$\ \{0,1\}^\ell}$
Return $T[x, \ell]$

For the analysis, assume neither S nor R repeat an oracle query, and consider games $\mathrm{G}_0^{S,R}(\lambda), \mathrm{G}_1^{S,R}(\lambda)$ above, where $\mathrm{G}_1^{S,R}(\lambda)$ includes the boxed code and $\mathrm{G}_0^{S,R}(\lambda)$ does not. The games are identical-until-bad, so by the Fundamental Lemma of Game Playing [24],

$$\mathsf{Adv}^{\mathsf{reset}}_{R,S}(\cdot) = \Pr[\mathrm{G}_0^{S,R}(\cdot)] - \Pr[\mathrm{G}_1^{S,R}(\cdot)] \le \Pr[\mathrm{G}_1^{S,R}(\cdot) \text{ sets } \mathsf{bad}] \le \mathsf{Adv}^{\mathsf{pred}}_{S,P}(\cdot)$$

as desired.

For part (2), let H be a UCE1-secure hash family. Define a hash function family $\overline{\mathsf{H}}$ as follows: $\overline{\mathsf{H}}.\mathsf{Kg} = \mathsf{H}.\mathsf{Kg}$ and $\overline{\mathsf{H}}.\mathsf{Ev}(1^\lambda, hk, 0^\lambda, 1^\lambda) = 0^\lambda$, and $\overline{\mathsf{H}}.\mathsf{Ev}(1^\lambda, hk, x, 1^\ell) = \mathsf{H}.\mathsf{Ev}(1^\lambda, hk, x, 1^\ell)$ otherwise. The family $\overline{\mathsf{H}}$ is still UCE1-secure, as an unpredictable source cannot query $\text{HASH}(0^\lambda, 1^\lambda)$. However, $\overline{\mathsf{H}}$ is UCE2-insecure. A source S can get $y = \text{HASH}(0^\lambda, 1^\lambda)$, and leak 1 if $y = 0^\lambda$, and leak 0 otherwise. The distinguisher simply echoes the leakage. We claim that the source above is reset-secure. Let R be an arbitrary PT reset adversary. In game $\text{Reset}^R_S(\lambda)$, the leakage is 0 with probability $1 - 2^{-\lambda}$, regardless of the challenge bit. Moreover, when the challenge bit is 0, if R queries $(0^\lambda, 1^\lambda)$ to HASH, the chance that it gets 0^λ is at most $2^{-\lambda}$. Hence $\mathsf{Adv}^{\mathsf{reset}}_{R,S}(\lambda) \le 2^{1-\lambda}$. ∎

The following is a UCE2 analog of Proposition 4.2, showing that UCE2 security neither implies, nor is implied by collision resistance. PRF security doesn't imply UCE2 security, but whether UCE2 security implies PRF security remains open. As usual, any non-containment $\mathsf{B} \not\subseteq \mathsf{A}$ assumes $\mathsf{B} \neq \emptyset$.

Proposition 6.2 (1) $\mathsf{PRF} \not\subseteq \mathsf{UCE2}$, (2) $\mathsf{UCE2} \not\subseteq \mathsf{CR}$, and (3) $\mathsf{CR} \not\subseteq \mathsf{UCE2}$.

Proof: Parts (1) and (3) are direct corollaries of Propositions 6.1 and 4.2. For part (2), a trivial counter-example is a family $\mathsf{H} \in \mathsf{UCE2}$ with $\lambda \in \mathsf{H}.\mathsf{IL}(\lambda)$ and $\mathsf{H}.\mathsf{ol}(\lambda) = 1$ for all $\lambda \in \mathbb{N}$, such a family trivially not being in CR. We'd like however to give a more meaningful counter-example, namely an $\mathsf{H} \in \mathsf{UCE2} \setminus \mathsf{CR}$ with $2^{-\mathsf{H}.\mathsf{ol}}$ negligible. Towards this, let $\mathsf{H} \in \mathsf{UCE2}$ be a family with output length $\mathsf{H}.\mathsf{ol}$ such that $2^{-\mathsf{H}.\mathsf{ol}}$ is negligible and $\mathsf{H}.\mathsf{IL}(\cdot) = \mathbb{N}$. Define a hash family $\overline{\mathsf{H}}$ as follows: (i) $\overline{\mathsf{H}}.\mathsf{Kg} = \mathsf{H}.\mathsf{Kg}$ and (ii) $\overline{\mathsf{H}}.\mathsf{Ev}(1^\lambda, hk, x, 1^\ell) = \mathsf{H}.\mathsf{Ev}(1^\lambda, hk, x, 1^\ell)$ if $x \neq hk$, and $\overline{\mathsf{H}}.\mathsf{Ev}(1^\lambda, hk, hk, 1^\ell) = \overline{\mathsf{H}}.\mathsf{Ev}(1^\lambda, hk, \overline{hk}, 1^\ell)$, where $\overline{hk}$ is the complement of hk. Note $\overline{\mathsf{H}}.\mathsf{ol} = \mathsf{H}.\mathsf{ol}$ and $\overline{\mathsf{H}}.\mathsf{IL} = \mathsf{H}.\mathsf{IL}$. Then $\overline{\mathsf{H}} \notin \mathsf{CR}$, as $\overline{\mathsf{H}}.\mathsf{Ev}(1^\lambda, hk, hk, 1^\ell) = \overline{\mathsf{H}}.\mathsf{Ev}(1^\lambda, hk, \overline{hk}, 1^\ell)$. To prove that $\overline{\mathsf{H}} \in \mathsf{UCE2}$, consider arbitrary PT reset-secure source $\overline{S}$ and PT distinguisher $\overline{D}$. Assume that $\overline{S}$ never repeats an oracle query. Let q be a polynomial such that the number of oracle queries of $\overline{S}$ in game $\mathrm{UCE}^{\overline{S},\overline{D}}_{\overline{\mathsf{H}}}(\lambda)$ is at most $q(\lambda)$ for all $\lambda \in \mathbb{N}$. We'll construct reset-secure source S and distinguisher D such that

$$\mathsf{Adv}^{\mathsf{uce}}_{\overline{\mathsf{H}},\overline{S},\overline{D}}(\cdot) \le \mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\cdot) + \frac{q}{2^{\mathsf{H}.\mathsf{ol}}} \quad . \tag{18}$$

The claim then follows from the assumption that $\mathsf{H} \in \mathsf{UCE2}$ and $2^{-\mathsf{H}.\mathsf{ol}}$ is negligible. The constructions of S and D are shown below:

$\underline{S^{\text{HASH}}(1^\lambda)}$
$\mathsf{done} \leftarrow \mathsf{false}$; $\overline{L} \leftarrow\!\!\$\ \overline{S}^{\text{HASHSIM}}(1^\lambda)$
If done then return 1
Return $0 \,\|\, \overline{L}$

$\underline{\text{HASHSIM}(x, 1^\ell)}$
$y \leftarrow \text{HASH}(x, 1^\ell)$
If $(|x| = \mathsf{H.kl}(\lambda)$ and $y = \overline{\mathsf{H}}.\mathsf{Ev}(1^\lambda, x, x, 1^{\mathsf{H.ol}(\lambda)}))$ then $\mathsf{done} \leftarrow \mathsf{true}$
Return y

$\underline{D^{\text{HASH}}(1^\lambda, hk, L)}$
If $L[1] = 0$ then
  $\overline{L} \leftarrow L[2, |L|]$
  $b' \leftarrow\!\!\$\ \overline{D}(1^\lambda, hk, \overline{L})$
  Return b'
Return 1

Let $\overline{b}$ and b be the challenge bits of game $\text{UCE}^{\overline{S},\overline{D}}_{\overline{\mathsf{H}}}(\lambda)$ and game $\text{UCE}^{S,D}_{\mathsf{H}}(\lambda)$ respectively. Then

$$\begin{aligned}
\Pr[\,\text{UCE}^{S,D}_{\mathsf{H}}(\cdot) \,|\, b = 1\,] &\geq \Pr[\,\text{UCE}^{\overline{S},\overline{D}}_{\overline{\mathsf{H}}}(\cdot) \,|\, \overline{b} = 1\,] \\
\Pr[\,\text{UCE}^{S,D}_{\mathsf{H}}(\cdot) \,|\, b = 0\,] &\geq \Pr[\,\text{UCE}^{\overline{S},\overline{D}}_{\overline{\mathsf{H}}}(\cdot) \,|\, \overline{b} = 0\,] - \frac{q}{2^{\mathsf{H.ol}}} \ .
\end{aligned}$$

Summing yields Equation (18). What's left is to prove that S is reset-secure. Let R be a PT reset-adversary. Consider the following reset-adversary $\overline{R}$:

$\underline{\overline{R}^{\text{HASH}}(1^\lambda, \overline{L})}$
$b' \leftarrow\!\!\$\ R^{\text{HASH}}(1^\lambda, 0 \,\|\, \overline{L})$; Return b'

Let $\overline{c}$ and c be the challenge bits of game $\text{Reset}^{\overline{R}}_{\overline{S}}(\lambda)$ and $\text{Reset}^{R}_{S}(\lambda)$ respectively. Then

$$\begin{aligned}
\Pr[\,\text{Reset}^{R}_{S}(\cdot) \,|\, c = 1\,] &\leq \Pr[\,\text{Reset}^{\overline{R}}_{\overline{S}}(\cdot) \,|\, \overline{c} = 1\,] + \frac{q}{2^{\mathsf{H.ol}}} \\
\Pr[\,\text{Reset}^{R}_{S}(\cdot) \,|\, c = 0\,] &\leq \Pr[\,\text{Reset}^{\overline{R}}_{\overline{S}}(\cdot) \,|\, \overline{c} = 0\,] + \frac{q}{2^{\mathsf{H.ol}}} \ .
\end{aligned}$$

Hence $\mathsf{Adv}^{\mathsf{reset}}_{R,S}(\cdot) \leq \mathsf{Adv}^{\mathsf{reset}}_{\overline{R},\overline{S}}(\cdot) + 2q/2^{\mathsf{H.ol}}$. ∎

6.3 OAEP revisited

Recall that in Section 5.8 we showed that $\mathsf{OAEP}[\mathsf{H}, \mathsf{TF}]$ is IND-CPA-KI secure, assuming TF is partial one-way and H is UCE1-secure. Here we show that if we strengthen the assumption on H to UCE2-security then we can relax the assumption on TF to be just one-wayness.

Theorem 6.3 Let $\mathsf{TF}, \mathsf{H}, \ell_1, \ell_2, \ell_3$ be as in Theorem 5.8, and let $\mathsf{PKE} = \mathsf{OAEP}[\mathsf{H}, \mathsf{TF}, \ell_1, \ell_2, \ell_3]$. Assume $2^{-\ell_1-\ell_2}, 2^{-\ell_3}$ are negligible. If $\mathsf{H} \in \mathsf{UCE2}$ and $\mathsf{TF} \in \mathsf{OW}$ then $\mathsf{PKE} \in \mathsf{IND\text{-}CPA\text{-}KI}$.

Proof of Theorem 6.3: Let A be a PT adversary for game $\text{IND-CPA-KI}^A_{\mathsf{PKE}}(\lambda)$. Let q, ϵ be as in the proof of Theorem 5.8. We construct source S and distinguisher D exactly as in the proof of Theorem 5.8, so that Equation (12) continues to be true. The difference is that, rather than showing S is unpredictable assuming $\mathsf{TF} \in \mathsf{OW}_{\ell_1+\ell_2}$, we will now show it is reset secure assuming only that $\mathsf{TF} \in \mathsf{OW}$. The theorem then follows from the assumption that $\mathsf{H} \in \mathsf{UCE2}$ and that $2^{-\ell_1-\ell_2}, 2^{-\ell_3}$ are negligible.

To show that S is reset-secure, let R be an arbitrary PT reset adversary. Wlog, assume that R never repeats a query to its HASH oracle. Assume further that each of R's queries $(u, 1^\ell)$ to HASH satisfies the following: (i) If $u[1] = 0$ then $|u| = \ell_3(\lambda) + 1$ and $\ell = \ell_1(\lambda) + \ell_2(\lambda)$, and (ii) If $u[1] = 1$ then $|u| = \ell_1(\lambda) + \ell_2(\lambda) + 1$ and $\ell = \ell_3(\lambda)$. Let p be a polynomial such that the number of HASH queries of R in game $\text{Reset}^R_S(\lambda)$ is at most $p(\lambda)$ for all $\lambda \in \mathbb{N}$. We'll construct a PT adversary I such that

$$\mathsf{Adv}^{\mathsf{reset}}_{S,R}(\cdot) \leq q \cdot \mathsf{Adv}^{\mathsf{ow}}_{\mathsf{TF},I}(\cdot) + \frac{p \cdot q}{2^{\ell_3}} + 2\epsilon \ . \tag{19}$$

$\underline{\text{MAIN } \boxed{\mathrm{G}_1^{A,R}(\lambda)},\ \mathrm{G}_2^{A,R}(\lambda)}$
$ek \leftarrow\!\!\$\ \mathsf{TF.Kg}(1^\lambda)\ ;\ b \leftarrow\!\!\$\ \{0,1\}\ ;\ M, Q_0, Q_1 \leftarrow \emptyset$
$t \leftarrow\!\!\$\ A^{\mathrm{LR}}(1^\lambda)\ ;\ L \leftarrow (ek, b, t)\ ;\ d' \leftarrow\!\!\$\ R^{\text{HASH}}(1^\lambda, L)$
Return $(d' = 1)$

$\underline{\mathrm{LR}(m_0, m_1)}$
$r \leftarrow\!\!\$\ \{0,1\}^{\ell_3(\lambda)}\ ;\ x \leftarrow\!\!\$\ \{0,1\}^{\ell_1(\lambda)+\ell_2(\lambda)}\ ;\ y \leftarrow\!\!\$\ \{0,1\}^{\ell_3(\lambda)}$
$z \leftarrow m_b \,\|\, 0^{\ell_2(\lambda)}\ ;\ t_1 \leftarrow x \oplus z$
If $r \in Q_0$ then $\mathsf{bad} \leftarrow \mathsf{true}$; $\boxed{t_1 \leftarrow T_0[r]\ ;\ x \leftarrow t_1 \oplus z}$
$T_0[r] \leftarrow t_1\ ;\ Q_0 \leftarrow Q_0 \cup \{r\}\ ;\ t_2 \leftarrow y \oplus r$
If $x \in Q_1$ then $\mathsf{bad} \leftarrow \mathsf{true}$; $\boxed{t_2 \leftarrow T_1[x]\ ;\ y \leftarrow t_2 \oplus r}$
$Q_1 \leftarrow Q_1 \cup \{x\}\ ;\ T_1[x] \leftarrow t_2$
$c \leftarrow \mathsf{TF.Ev}(1^\lambda, ek, x \,\|\, y, 1^{\mathsf{TF.ol}(\lambda)})$; Return c

$\underline{\text{HASH}(u, 1^\ell)}$
$v \leftarrow u[2, |u|]\ ;\ s \leftarrow u[1]$
If $T_s[v] = \perp$ then $T_s[v] \leftarrow\!\!\$\ \{0,1\}^\ell$
Return $T_s[v]$

$\underline{\text{MAIN } \boxed{\mathrm{G}_3^{A,R}(\lambda)},\ \mathrm{G}_4^{A,R}(\lambda)}$
$ek \leftarrow\!\!\$\ \mathsf{TF.Kg}(1^\lambda)\ ;\ b \leftarrow\!\!\$\ \{0,1\}\ ;\ M, Q_0, Q_1 \leftarrow \emptyset$
$t \leftarrow\!\!\$\ A^{\mathrm{LR}}(1^\lambda)\ ;\ L \leftarrow (ek, b, t)\ ;\ d' \leftarrow\!\!\$\ R^{\text{HASH}}(1^\lambda, L)$
Return $(d' = 1)$

$\underline{\mathrm{LR}(m_0, m_1)}$
$x \leftarrow\!\!\$\ \{0,1\}^{\ell_1(\lambda)+\ell_2(\lambda)}\ ;\ y \leftarrow\!\!\$\ \{0,1\}^{\ell_3(\lambda)}$
$z \leftarrow m_b \,\|\, 0^{\ell_2(\lambda)}\ ;\ T_1[x] \leftarrow\!\!\$\ \{0,1\}^{\ell_3(\lambda)}\ ;\ t_2 \leftarrow T_1[x]$
$t_1 \leftarrow x \oplus z\ ;\ r \leftarrow y \oplus t_2$
$V_0[r] \leftarrow t_1\ ;\ Q_0 \leftarrow Q_0 \cup \{r\}$
$V_1[x] \leftarrow r\ ;\ Q_1 \leftarrow Q_1 \cup \{x\}$
$c \leftarrow \mathsf{TF.Ev}(1^\lambda, ek, x \,\|\, y, 1^{\mathsf{TF.ol}(\lambda)})$; Return c

$\underline{\text{HASH}(u, 1^\ell)}$
$v \leftarrow u[2, |u|]\ ;\ s \leftarrow u[1]$
If $s = 0$ and $v \in Q_0$ then
  If $v \in M$ then $\mathsf{coll} \leftarrow \mathsf{true}$; $T_0[v] \leftarrow V_0[v]$
  Else $\mathsf{bad} \leftarrow \mathsf{true}$; $\boxed{T_0[v] \leftarrow V_0[v]}$
If $s = 1$ and $v \in Q_1$ then $r \leftarrow V_1[v]\ ;\ M \leftarrow M \cup \{r\}$
If $T_s[v] = \perp$ then $T_s[v] \leftarrow\!\!\$\ \{0,1\}^\ell$
Return $T_s[v]$

$\underline{\text{MAIN } \mathrm{G}_5^{A,R}(\lambda)}$
$ek \leftarrow\!\!\$\ \mathsf{TF.Kg}(1^\lambda)\ ;\ b \leftarrow\!\!\$\ \{0,1\}\ ;\ M, Q_0, Q_1 \leftarrow \emptyset$
$t \leftarrow\!\!\$\ A^{\mathrm{LR}}(1^\lambda)\ ;\ L \leftarrow (ek, b, t)\ ;\ d' \leftarrow\!\!\$\ R^{\text{HASH}}(1^\lambda, L)$
Return $(d' = 1)$

$\underline{\mathrm{LR}(m_0, m_1)}$
$x \leftarrow\!\!\$\ \{0,1\}^{\ell_1(\lambda)+\ell_2(\lambda)}\ ;\ y \leftarrow\!\!\$\ \{0,1\}^{\ell_3(\lambda)}$
$z \leftarrow m_b \,\|\, 0^{\ell_2(\lambda)}\ ;\ T_1[x] \leftarrow\!\!\$\ \{0,1\}^{\ell_3(\lambda)}\ ;\ t_2 \leftarrow T_1[x]$
$t_1 \leftarrow x \oplus z\ ;\ r \leftarrow y \oplus t_2$
$V_0[r] \leftarrow t_1\ ;\ Q_0 \leftarrow Q_0 \cup \{r\}\ ;\ V_1[x] \leftarrow r\ ;\ Q_1 \leftarrow Q_1 \cup \{x\}$
$c \leftarrow \mathsf{TF.Ev}(1^\lambda, ek, x \,\|\, y, 1^{\mathsf{TF.ol}(\lambda)})$; Return c

$\underline{\text{HASH}(u, 1^\ell)}$
$v \leftarrow u[2, |u|]\ ;\ s \leftarrow u[1]$
If $s = 0$ and $v \in Q_0$ then
  If $v \in M$ then $\mathsf{coll} \leftarrow \mathsf{true}$
If $s = 1$ and $v \in Q_1$ then $r \leftarrow V_1[v]\ ;\ M \leftarrow M \cup \{r\}$
If $T_s[v] = \perp$ then $T_s[v] \leftarrow\!\!\$\ \{0,1\}^\ell$
Return $T_s[v]$

$\underline{\text{MAIN } \mathrm{G}_6^{A,R}(\lambda),\ \boxed{\mathrm{G}_7^{A,R}(\lambda)}}$
$ek \leftarrow\!\!\$\ \mathsf{TF.Kg}(1^\lambda)\ ;\ b \leftarrow\!\!\$\ \{0,1\}\ ;\ M, Q_0, Q_1 \leftarrow \emptyset$
$t \leftarrow\!\!\$\ A^{\mathrm{LR}}(1^\lambda)\ ;\ L \leftarrow (ek, b, t)\ ;\ d' \leftarrow\!\!\$\ R^{\text{HASH}}(1^\lambda, L)$
Return $(d' = 1)$

$\underline{\mathrm{LR}(m_0, m_1)}$
$r \leftarrow\!\!\$\ \{0,1\}^{\ell_3(\lambda)}\ ;\ x \leftarrow\!\!\$\ \{0,1\}^{\ell_1(\lambda)+\ell_2(\lambda)}$
$y \leftarrow\!\!\$\ \{0,1\}^{\ell_3(\lambda)}\ ;\ z \leftarrow m_b \,\|\, 0^{\ell_2(\lambda)}\ ;\ t_1 \leftarrow x \oplus z$
If $r \in Q_0$ then $\mathsf{bad} \leftarrow \mathsf{true}$; $\boxed{t_1 \leftarrow T_0[r]\ ;\ x \leftarrow t_1 \oplus z}$
$T_0[r] \leftarrow t_1\ ;\ Q_0 \leftarrow Q_0 \cup \{r\}\ ;\ t_2 \leftarrow y \oplus r$
If $x \in Q_1$ then $\mathsf{bad} \leftarrow \mathsf{true}$; $\boxed{t_2 \leftarrow T_1[x]\ ;\ y \leftarrow t_2 \oplus r}$
$Q_1 \leftarrow Q_1 \cup \{x\}\ ;\ T_1[x] \leftarrow t_2$
$c \leftarrow \mathsf{TF.Ev}(1^\lambda, ek, x \,\|\, y, 1^{\mathsf{TF.ol}(\lambda)})$; Return c

$\underline{\text{HASH}(u, 1^\ell)}$
$z \leftarrow\!\!\$\ \{0,1\}^\ell$; Return z

Figure 18: **Games for the proof of Theorem 6.3.** Games $\mathrm{G}_1, \mathrm{G}_3$, and G_7 contain the corresponding boxed statements but the other games do not.

The reset security of S then follows from the assumption that $\mathsf{TF} \in \mathsf{OW}$ and that $2^{-\ell_1-\ell_2}, 2^{-\ell_3}$ are negligible. Proceeding, consider games G_1–G_7 in Fig. 18. Let a be the challenge bit of game $\mathrm{Reset}_S^R(\lambda)$. Then

$$\Pr[\mathrm{G}_1^{A,R}(\lambda)] = \Pr[\,\mathrm{Reset}_S^R(\lambda) \,|\, a = 1\,] \text{ and } \Pr[\mathrm{G}_7^{A,R}(\lambda)] = 1 - \Pr[\,\mathrm{Reset}_S^R(\lambda) \,|\, a = 0\,]$$

We explain the game chain up to the terminal one. In game $\mathrm{G}_2^{A,R}(\lambda)$, we no longer maintain consistency among queries: in each query to LR, the strings x and y are uniformly random, independent of anything

else. The two games $\mathrm{G}_1^{A,R}(\lambda)$ and $\mathrm{G}_2^{A,R}(\lambda)$ are identical-until-bad. Then

$$\Pr[\mathrm{G}_1^{A,R}(\cdot)] - \Pr[\mathrm{G}_2^{A,R}(\cdot)] \le \Pr[\mathrm{G}_2^{A,R}(\cdot) \text{ sets } \mathsf{bad}] \le \sum_{i=0}^{q-1} \frac{i}{2^{\ell_3}} + \frac{i}{2^{\ell_1+\ell_2}} \le \epsilon \ .$$

In game $\mathrm{G}_3^{A,R}(\lambda)$, we delay the writing to $T_0[r]$ until R queries HASH at this point. Then

$$\Pr[\mathrm{G}_2^{A,R}(\cdot)] = \Pr[\mathrm{G}_3^{A,R}(\cdot)] \ .$$

In game $\mathrm{G}_4^{S,R}(\lambda)$, we won't write to $T_0[r]$ if R queries $(0 \,\|\, r, 1^{\ell_1(\lambda)+\ell_2(\lambda)})$ before querying $(1 \,\|\, x, 1^{\ell_3(\lambda)})$. Games $\mathrm{G}_3^{S,R}(\lambda)$ and $\mathrm{G}_4^{S,R}(\lambda)$ are identical-until-bad. Before R queries $(1 \,\|\, x, 1^{\ell_3(\lambda)})$, the string r is independent of whatever R receives, and thus

$$\Pr[\mathrm{G}_3^{S,R}(\cdot)] - \Pr[\mathrm{G}_4^{S,R}(\cdot)] \le \Pr[\mathrm{G}_4^{S,R}(\cdot) \text{ sets } \mathsf{bad}] \le \frac{q \cdot p}{2^{\ell_3}}.$$

In game $\mathrm{G}_5^{A,R}(\lambda)$, we drop the writing to $T_0[r]$ entirely. This may cause inconsistency with game $\mathrm{G}_4^{S,R}(\lambda)$ only if R queries $(1 \,\|\, x, 1^{\ell_3(\lambda)})$ and then $(0 \,\|\, r, 1^{\ell_1(\lambda)+\ell_2(\lambda)})$ to HASH. Games $\mathrm{G}_4^{S,R}(\lambda)$ and $\mathrm{G}_5^{S,R}(\lambda)$ are identical-until-coll. Then

$$\Pr[\mathrm{G}_4^{S,R}(\cdot)] - \Pr[\mathrm{G}_5^{S,R}(\cdot)] \le \Pr[\mathrm{G}_5^{S,R}(\cdot) \text{ sets } \mathsf{coll}] \ .$$

Note that in game $\mathrm{G}_5^{S,R}(\lambda)$, the answers of HASH and LR are independent. We now construct adversary I such that

$$\mathsf{Adv}^{\mathsf{ow}}_{\mathsf{TF},I}(\cdot) \ge \frac{1}{q} \Pr[\mathrm{G}_5^{S,R}(\cdot) \text{ sets } \mathsf{coll}] \ .$$

The construction of I is shown below:

$\underline{I(1^\lambda, ek, c')}$
$b \leftarrow\$ \{0,1\}$; $j \leftarrow\$ \{1, \ldots q(\lambda)\}$; $i \leftarrow 0$
$t \leftarrow\$ A^{\mathrm{LRSim}}(1^\lambda)$; $Q_0, Q_1 \leftarrow \emptyset$; $R^{\mathrm{HashSim}}(1^\lambda, (ek, b, t))$
For $r \in Q_0, x \in Q_1$ do
  $y \leftarrow T_1[x] \oplus r$; $c \leftarrow \mathsf{TF.Ev}(1^\lambda, ek, x \,\|\, y, 1^{\mathsf{TF.ol}(\lambda)})$
  If $c = c'$ then return $x \,\|\, y$

$\underline{\mathrm{HashSim}(u, 1^\ell)}$
$v \leftarrow u[2, |u|]$; $s \leftarrow u[1]$
$Q_s \leftarrow Q_s \cup \{v\}$; $T_s[v] \leftarrow\$ \{0,1\}^\ell$; Return $T_s[v]$

$\underline{\mathrm{LRSim}(m_0, m_1)}$
$i \leftarrow i + 1$; $x \leftarrow\$ \{0,1\}^{\ell_1(\lambda)+\ell_2(\lambda)}$
$y \leftarrow\$ \{0,1\}^{\ell_3(\lambda)}$
$c \leftarrow \mathsf{TF.Ev}(1^\lambda, ek, x \,\|\, y, 1^{\mathsf{TF.ol}(\lambda)})$
If $i = j$ then $c \leftarrow c'$
Return c

In game $\mathrm{G}_6^{A,R}(\lambda)$, procedure HASH explicitly returns random answers, independent of LR. Then

$$\Pr[\mathrm{G}_5^{A,R}(\cdot)] = \Pr[\mathrm{G}_6^{A,R}(\cdot)] \ .$$

In game $\mathrm{G}_7^{A,R}(\lambda)$, procedure LR needs to maintain consistency among its answers, but these are still independent of HASH. Games $\mathrm{G}_6^{A,R}(\lambda)$ and $\mathrm{G}_7^{A,R}(\lambda)$ are identical-until-bad. Then

$$\Pr[\mathrm{G}_6^{A,R}(\cdot)] - \Pr[\mathrm{G}_7^{A,R}(\cdot)] \le \Pr[\mathrm{G}_7^{A,R}(\cdot) \text{ sets } \mathsf{bad}] \le \epsilon \ .$$

Hence

$$\begin{aligned}
\mathsf{Adv}^{\mathsf{reset}}_{S,R}(\cdot) &= \Pr[\,\mathrm{Reset}_S^R(\lambda) \,|\, a = 1\,] + \Pr[\,\mathrm{Reset}_S^R(\lambda) \,|\, a = 0\,] - 1 \\
&= \Pr[\mathrm{G}_1^{A,R}(\cdot)] - \Pr[\mathrm{G}_7^{A,R}(\cdot)] \\
&= \sum_{i=1}^{6} \Pr[\mathrm{G}_i^{A,R}(\cdot)] - \Pr[\mathrm{G}_{i+1}^{A,R}(\cdot)] \\
&\le 2\epsilon + \frac{p \cdot q}{2^{\ell_3}} + q \cdot \mathsf{Adv}^{\mathsf{ow}}_{\mathsf{TF},I}(\cdot)
\end{aligned}$$

yielding Equation (19). ∎

6.4 Adaptively secure garbling with short tokens

BHR1 [16] introduce garbling schemes as an abstraction of the garbled circuit technique that originates with Yao [93]. They provide definitions for privacy, obliviousness and authenticity. These however capture the standard, static-security requirements met by the standard Yao-style constructions. BHR2 [15] point out that applications like one-time programs [67] and secure-outsourcing [61] require adaptive security, and provide corresponding definitions of adaptive security (again for privacy, obliviousness and authenticity) for garbling schemes.

Ideally, we want garbling schemes where the garbled inputs are short. (Garbling schemes take a function to produce a garbled function, and also associate to an input a garbled input. We say that the scheme has short garbled inputs if the size of the garbled inputs depends only on the security parameter and the input and output lengths of the original function. We note that the secure-outsourcing protocol of [61] requires short garbled inputs in order to be non-trivial.) Statically-secure schemes naturally have this feature, but retaining it while providing adaptive security is challenging. Schemes with this feature have been provided in the ROM [15, 7]. But in the standard model, known constructions achieving adaptive security [15, 69] have long garbled inputs, meaning ones of size proportional to the size of the circuit being garbled. Standard-model adaptively-secure garbling with short tokens has remained open.

Here we resolve this problem with schemes based on a UCE2-secure family. We give two garbling schemes, GaP and GaAO, that have short tokens. The former provides adaptive privacy, while the latter provides adaptive obliviousness and authenticity. Both assume only a UCE2-secure hash family.

We note that the secure-outsourcing protocol of [61] uses Fully Homomorphic Encryption [63], requiring the garbling scheme it (also) uses to be secure in the standard model. (A ROM garbling scheme does not work for them.) Thus we obtain the first non-trivial instantiation of the secure-outsourcing protocol of [61].

Definitions. BHR1 [16] give the following formalization of circuits. A *circuit* is defined as a 6-tuple $f = (n, m, q, A, B, G)$ where $n \geq 2$ is the number of *inputs*, $m \geq 1$ is the number of *outputs*, $q \geq 1$ is the number of *gates*, and $r = n + q$ the number of *wires*. We let $\text{Inputs} = [1\,..\,n]$, $\text{Wires} = [1\,..\,n+q]$, $\text{OutputWires} = [n+q-m+1\,..\,n+q]$, and $\text{Gates} = [n+1\,..\,n+q]$. Then A: $\text{Gates} \to \text{Wires}\backslash\text{OutputWires}$ is a function to identify each gate's *first* incoming wire and B: $\text{Gates} \to \text{Wires}\backslash\text{OutputWires}$ is a function to identify each gate's *second* incoming wire. Finally G: $\text{Gates} \times \{0,1\}^2 \to \{0,1\}$ is a function that determines the *functionality* of each gate. We require $A(g) < B(g) < g$ for all $g \in \text{Gates}$.

The conventions above embody all of the following. Gates have two inputs, arbitrary functionality, and arbitrary fan-out. The wires are numbered 1 to $n + q$. Every non-input wire is the outgoing wire of some gate. The ith bit of input is presented along wire i. The ith bit of output is collected off wire $n + q - m + i$. The outgoing wire of each gate serves as the name of that gate. Circuit output wires, i.e., wires in OutputWires, cannot be circuit input wires and cannot be incoming wires to gates. No output wire can be used twice in the output. Requiring $A(g) < B(g) < g$ ensures that the directed graph corresponding to f is acyclic, and that no wire feeds a gate twice. Numbering the gates produces a topological sort when the circuit is viewed as a directed graph.

A *garbling scheme* GS is specified via algorithms $\mathsf{GS.Gb}, \mathsf{GS.En}, \mathsf{GS.De}, \mathsf{GS.Ev}$, and $\mathsf{GS.ev}$ [16]. The first algorithm $\mathsf{GS.Gb}$ is probabilistic; the others are deterministic. Algorithm $\mathsf{GS.Gb}$ takes input a string f describing a function and outputs a triple of strings (F, e, d). Here, f describes a function $\mathsf{GS.ev}(f, \cdot) : \{0,1\}^n \to \{0,1\}^m$. The values $n = f.n$ and $m = f.m$ should be efficiently computable from f. The strings e and d describe functions $\mathsf{GS.En}(e, \cdot) : \{0,1\}^n \to \{0,1\}^*$ and $\mathsf{GS.De}(d, \cdot) : \{0,1\}^* \to \{0,1\}^m \cup \{\perp\}$ respectively. We call λ, f, F, e, d the *security parameter, initial function, garbled function, encoding function* and *decoding function*, respectively. For any $x \in \{0,1\}^n$, we require that $\mathsf{GS.ev}(f, x) = \mathsf{GS.De}(d, Y)$, where $(F, e, d) \leftarrow\!\!\$\ \mathsf{GS.Gb}(1^\lambda, f), X \leftarrow \mathsf{GS.En}(e, x)$ and $Y \leftarrow \mathsf{GS.Ev}(F, X)$. The strings X and Y are called *garbled input* and *garbled output* respectively.

We restrict attention to *projective circuit-garbling schemes* following the terminology of BHR1. Evaluation $\mathsf{GS.ev}$ is the canonical circuit-evaluation function cev below. Encoding $\mathsf{GS.En}(e, \cdot)$ uses the bits of

$\underline{\textsc{Main Prv2}_{\mathsf{GS},\Phi,\mathrm{Sim}}(\lambda)}$ $b \leftarrow\!\!\$\ \{0,1\}\ ;\ b' \leftarrow\!\!\$\ \mathcal{A}^{\textsc{Garble},\textsc{Input}}(1^\lambda)\ ;$ Return $(b=b')$ $\underline{\textsc{Garble}(f)}$ $n \leftarrow f.n\ ;\ Q \leftarrow \emptyset\ ;\ \tau \leftarrow \varepsilon$ If $b=1$ then $\quad (F,(X_1^0,X_1^1,\ldots,X_n^0,X_n^1),d) \leftarrow\!\!\$\ \mathsf{GS.Gb}(1^\lambda,f)$ Else $(F,d) \leftarrow\!\!\$\ \mathrm{Sim}(1^\lambda,\Phi(f),0)$ Return (F,d)	$\underline{\textsc{Input}(i,c)}$ If $i \notin \{1,\ldots,n\} \setminus Q$ then return $\bot$ $x_i \leftarrow c\ ;\ Q \leftarrow Q \cup \{i\}$ If $\lvert Q\rvert = n$ then $\quad x \leftarrow x_1 \cdots x_n\ ;\ y \leftarrow\!\!\$\ \mathsf{GS.ev}(f,x)\ ;\ \tau \leftarrow y$ If $b=1$ then $X_i \leftarrow X_i^{x_i}$ Else $X_i \leftarrow\!\!\$\ \mathrm{Sim}(1^\lambda,\tau,i,\lvert Q\rvert)$ Return X_i
$\underline{\textsc{Main Obv2}_{\mathsf{GS},\Phi,\mathrm{Sim}}(\lambda)}$ $b \leftarrow\!\!\$\ \{0,1\}\ ;\ b' \leftarrow\!\!\$\ \mathcal{A}^{\textsc{Garble},\textsc{Input}}(1^\lambda)\ ;$ Return $(b=b')$ $\underline{\textsc{Garble}(f)}$ $n \leftarrow f.n;\ \ Q \leftarrow \emptyset;\ \ \sigma \leftarrow \varepsilon$ If $b=1$ then $\quad (F,(X_1^0,X_1^1,\ldots,X_n^0,X_n^1),d) \leftarrow \mathsf{GS.Gb}(1^\lambda,f)$ Else $F \leftarrow \mathrm{Sim}(1^\lambda,\Phi(f),0)$ Return F	$\underline{\textsc{Input}(i,c)}$ If $i \notin \{1,\ldots,n\} \setminus Q$ then return $\bot$ $x_i \leftarrow c;\ \ Q \leftarrow Q \cup \{i\}$ If $b=1$ then $X_i \leftarrow X_i^{x_i}$ Else $X_i \leftarrow \mathrm{Sim}(1^\lambda,i,\lvert Q\rvert)$ Return X_i
$\underline{\textsc{Main Aut2}_{\mathsf{GS}}(\lambda)}$ $Y \leftarrow\!\!\$\ \mathcal{A}^{\textsc{Garble},\textsc{Input}}(1^\lambda)$ Return $(\mathsf{GS.De}(d,Y) \neq \bot$ and $Y \neq \mathsf{GS.Ev}(F,X))$ $\underline{\textsc{Garble}(f)}$ $n \leftarrow f.n\ ;\ Q \leftarrow \emptyset\ ;\ \sigma \leftarrow \varepsilon$ $(F,(X_1^0,X_1^1,\ldots,X_n^0,X_n^1),d) \leftarrow \mathsf{GS.Gb}(1^\lambda,f)\ ;$ Return F	$\underline{\textsc{Input}(i,c)}$ If $i \notin \{1,\ldots,n\} \setminus Q$ then return $\bot$ $x_i \leftarrow c;\ \ Q \leftarrow Q \cup \{i\},\ \ X_i \leftarrow X_i^{x_i}$ If $\lvert Q\rvert = n$ then $X \leftarrow (X_1,\ldots,X_n)$ Return X_i

Figure 19: **Security notions of a garbling scheme GS.** The scheme is required to be projective. The adversary $\mathcal{A}$ makes a single query to Garble, followed by multiple queries to Input.

$x = x_1 \cdots x_n$ to select from $e = (X_1^0, X_1^1, \ldots, X_n^0, X_n^1)$ the subvector $X = (X_1^{x_1}, \ldots, X_n^{x_n})$. We say that a garbling scheme has *short* garbled inputs if garbled input lengths depend only on the security parameter λ, the input length n, and output length m of f.

$\underline{\mathsf{cev}(f,x)}$
$(n,m,q,A,B,G) \leftarrow f\ ;\ x_1 \cdots x_n \leftarrow x$
For $g \leftarrow n+1$ to $n+q$ do
$\quad a \leftarrow A(g)\ ;\ b \leftarrow B(g)\ ;\ x_g \leftarrow G_g(x_a,x_b)$
Return $x_{n+q-m+1} \cdots x_{n+q}$

We parametrize privacy by a "knob" that measures what we allow to be revealed. The *side-information function* Φ maps f to some information about it, $\Phi(f)$. We require that $f.n$ and $f.m$ be efficiently computable from $\Phi(f)$. In this work, we consider only the side-information function Φ_{topo} that maps a circuit $f = (n,m,q,A,B,G)$ to its *topological circuit* (n,m,q,A,B).

BHR2 [15] identify *privacy, obliviousness,* and *authenticity* as relevant security notions for garbling schemes, and go on to show that privacy is sufficient for one-time programs [67] while obliviousness and authenticity suffice for secure-outsourcing [61].

Let GS be a garbling scheme and let Φ be a side-information function. The first notion *privacy* is defined via game $\mathrm{Prv2}_{\mathsf{GS},\Phi,\mathrm{Sim}}$ of Fig. 19, where Sim, the *simulator*, is an always-terminating algorithm that maintains state across invocations. We define $\mathsf{Adv}^{\mathrm{prv2},\Phi,\mathrm{Sim}}_{\mathsf{GS},\mathcal{A}}(\lambda) = 2\Pr[\mathrm{Prv2}^{\mathcal{A}}_{\mathsf{GS},\Phi,\mathrm{Sim}}(\lambda)] - 1$. Garbling scheme GS is prv2-secure with respect to Φ if for any PT adversary $\mathcal{A}$ there exists a simulator Sim such that $\mathsf{Adv}^{\mathrm{prv2},\Phi,\mathrm{Sim}}_{\mathsf{GS},\mathcal{A}}(\lambda)$ is negligible. We let $\mathsf{PRV2}[\Phi]$ denote the set of all garbling schemes prv2-secure over Φ.

The next notion, *obliviousness*, is defined through game $\mathrm{Obv2}_{\mathsf{GS},\Phi,\mathrm{Sim}}$ of Fig. 19. Advantage is defined through $\mathsf{Adv}^{\mathrm{obv2},\Phi,\mathrm{Sim}}_{\mathsf{GS},\mathcal{A}}(\lambda) = 2\Pr[\mathrm{Obv2}^{\mathcal{A}}_{\mathsf{GS},\Phi,\mathrm{Sim}}(\lambda)] - 1$. Garbling scheme GS is obv2-secure with respect to Φ

$\underline{\mathsf{Ga}[\mathsf{h}].\mathsf{Gb}(1^\lambda, f)}$
$(n, m, q, A', B', G) \leftarrow f$
For $i \leftarrow 1$ to $n+q$ do
  $t \leftarrow\!\!\$\ \{0,1\}$; $X_i^0 \leftarrow\!\!\$\ \{0,1\}^{\lambda-1}t$; $X_i^1 \leftarrow\!\!\$\ \{0,1\}^{\lambda-1}\bar{t}$
For $g \leftarrow n+1$ to $n+q$, $i \leftarrow 0$ to 1, $j \leftarrow 0$ to 1 do
  $a \leftarrow A'(g)$; $b \leftarrow B'(g)$
  $A \leftarrow X_a^i$; $\mathsf{a} \leftarrow \mathrm{lsb}(A)$; $B \leftarrow X_b^j$; $\mathsf{b} \leftarrow \mathrm{lsb}(B)$
  $T[g, \mathsf{a}, \mathsf{b}] \leftarrow \mathsf{h}(A \,\|\, B \,\|\, g, 1^\lambda) \oplus X_g^{G_g(i,j)}$
$F \leftarrow (n, m, q, A', B', T)$
$e \leftarrow (X_1^0, X_1^1, \ldots, X_n^0, X_n^1)$
$d \leftarrow (X_{n+q-m+1}^0, X_{n+q-m+1}^1, \ldots, X_{n+q}^0, X_{n+q}^1)$
Return (F, e, d)

$\underline{\mathsf{Ga}[\mathsf{h}].\mathsf{Ev}(F, X)}$
$(n, m, q, A', B', T) \leftarrow F$, $(X_1, \ldots, X_n) \leftarrow X$
For $g \leftarrow n+1$ to $n+q$ do
  $a \leftarrow A'(g)$; $b \leftarrow B'(g)$
  $A \leftarrow X_a$; $\mathsf{a} \leftarrow \mathrm{lsb}(A)$; ; $B \leftarrow X_b$; $\mathsf{b} \leftarrow \mathrm{lsb}(B)$
  $X_g \leftarrow \mathsf{h}(X_a \,\|\, X_b \,\|\, g, 1^\lambda) \oplus T[g, \mathsf{a}, \mathsf{b}]$
Return $(X_{n+q-m+1}, \ldots, X_{n+q})$

$\underline{\mathsf{Ga}[\mathsf{h}].\mathsf{De}(d, Y)}$
$(Y_1, \ldots, Y_m) \leftarrow Y$; $(Y_1^0, Y_1^1, \ldots, Y_m^0, Y_m^1) \leftarrow d$
For $i \leftarrow 1$ to m do
  If $Y_i = Y_i^0$ then $y_i \leftarrow 0$
  Elsif $Y_i = Y_i^1$ then $y_i \leftarrow 1$ else return $\perp$
Return $y \leftarrow y_1 \cdots y_m$

$\underline{\mathsf{GaAO}[\mathsf{H}].\mathsf{Gb}(1^\lambda, f)}$
$hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$
$(F, e, d) \leftarrow\!\!\$\ \mathsf{Ga}[\mathsf{H.Ev}(1^\lambda, hk, \cdot, \cdot)].\mathsf{Gb}(1^\lambda, f)$
$(X_1^0, X_1^1, \ldots, X_n^0, X_n^1) \leftarrow e$
For $i \leftarrow 1$ to $n-1$ do $V_i \leftarrow\!\!\$\ \{0,1\}^{|hk|}$
$V_n \leftarrow V_1 \oplus \cdots \oplus V_{n-1} \oplus hk$
For $i \leftarrow 1$ to n do $T_i^0 \leftarrow (X_i^0, V_i)$; $T_i^1 \leftarrow (X_i^1, V_i)$
Return $\big(F, (T_1^0, T_1^1, \ldots, T_n^0, T_n^1), d\big)$

$\underline{\mathsf{GaAO}[\mathsf{H}].\mathsf{Ev}(F, X^*)}$
$\big((X_1, V_1), \ldots, (X_n, V_n)\big) \leftarrow X^*$
$X \leftarrow (X_1, \ldots, X_n)$
$hk \leftarrow V_1 \oplus \cdots V_n$
$Y \leftarrow \mathsf{Ga}[\mathsf{H.Ev}(1^\lambda, hk, \cdot, \cdot)].\mathsf{Ev}(F, X)$
Return Y

$\underline{\mathsf{GaP}[\mathsf{H}].\mathsf{Gb}(1^\lambda, f)}$
$hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$
$(F, e, d) \leftarrow\!\!\$\ \mathsf{Ga}[\mathsf{H.Ev}(1^\lambda, hk, \cdot, \cdot)].\mathsf{Gb}(1^\lambda, f)$
$(X_1^0, X_1^1, \ldots, X_n^0, X_n^1) \leftarrow e$
$(Y_1^0, Y_1^1, \ldots, Y_m^0, Y_m^1) \leftarrow d$
For $i \leftarrow 1$ to m do $u_i \leftarrow \mathrm{lsb}(Y_i^0)$
$U \leftarrow u_1 \cdots u_m$; $K \leftarrow (U, hk)$
For $i \leftarrow 1$ to $n-1$ do $V_i \leftarrow\!\!\$\ \{0,1\}^{|K|}$
$V_n \leftarrow V_1 \oplus \cdots \oplus V_{n-1} \oplus K$
For $i \leftarrow 1$ to n do $T_i^0 \leftarrow (X_i^0, V_i)$; $T_i^1 \leftarrow (X_i^1, V_i)$
Return $\big(F, (T_1^0, T_1^1, \ldots, T_n^0, T_n^1), \varepsilon\big)$

$\underline{\mathsf{GaP}[\mathsf{H}].\mathsf{Ev}(F, X^*)}$
$\big((X_1, V_1), \ldots, (X_n, V_n)\big) \leftarrow X^*$
$X \leftarrow (X_1, \ldots, X_n)$
$(hk, U) \leftarrow V_1 \oplus \cdots V_n$
$Y \leftarrow \mathsf{Ga}[\mathsf{H.Ev}(1^\lambda, hk, \cdot, \cdot)].\mathsf{Ev}(F, X)$
Return (Y, U)

$\underline{\mathsf{GaP}[\mathsf{H}].\mathsf{De}(d^*, Y^*)}$
$((Y_1, \ldots, Y_m), U) \leftarrow Y^*$; $u_1 \ldots u_m \leftarrow U$
For $i \leftarrow 1$ to m do $y_i \leftarrow \mathrm{lsb}(Y_i) \oplus u_i$
Return $y \leftarrow y_1 \cdots y_m$

Figure 20: **Top:** A conventional circuit-garbling scheme Ga. Here $\bar{t}$ denotes the complement bit of t. **Middle:** Scheme GaAO that achieves aut2 and obv2 security. **Bottom:** Scheme GaP that achieves prv2 security. We omit their encoding functions, as they are projective. The evaluation functions of these schemes are the canonical circuit evaluation cev.

if for all PT adversaries $\mathcal{A}$ there exists a simulator Sim such that $\mathsf{Adv}^{\mathrm{obv2},\Phi,\mathrm{Sim}}_{\mathsf{GS},\mathcal{A}}(\lambda)$ is negligible. We let $\mathsf{OBV2}[\Phi]$ denote the set of all garbling schemes obv2-secure over Φ.

The third notion *authenticity* is defined through game $\mathrm{Obv2}_{\mathsf{GS},\Phi,\mathrm{Sim}}$ of Fig. 19. We define $\mathsf{Adv}^{\mathrm{aut2}}_{\mathsf{GS},\mathcal{A}}(\lambda) = \Pr[\mathrm{Aut2}^{\mathcal{A}}_{\mathsf{GS}}(\lambda)]$. Garbling scheme GS is aut2-secure if $\mathsf{Adv}^{\mathrm{aut2}}_{\mathsf{GS},\mathcal{A}}(\lambda)$ is negligible for any PT adversary $\mathcal{A}$. We let $\mathsf{AUT2}$ denote the set of all aut2 secure garbling schemes.

Results. Let H be a hash family such that $\mathsf{H.IL}(\lambda) = \mathbb{N}$ and $\mathsf{H.ol}(\lambda) = \lambda$ for every $\lambda \in \mathbb{N}$. Fig. 20 describes the $\mathsf{GaP}[\mathsf{H}]$ and $\mathsf{GaAO}[\mathsf{H}]$ garbling schemes, based on the RO-model scheme $\mathsf{Ga}[\mathsf{h}]$ of Pinkas, Schneider, Smart, and Williams [88] that depends on a keyless hash h such that $\mathsf{h}(x, 1^\ell) \in \{0,1\}^\ell$ for every $x \in \{0,1\}^*$ and $\ell \in \mathbb{N}$.

In scheme Ga, wires carry λ-bit tokens (strings) the last bit of each is the token's *type*. To garble a circuit, we begin selecting two tokens for each wire, one of each type. One of these will represent 0—the token is said to have *semantics* of 0—while the other will represent 1. The variable X_i^b names the token

of wire i with semantics of b. For every wire i, we select random tokens of opposite type, making the association between a token's type and its semantics random. We then compute q garbled tables, one for each gate g. Table $T[g,\cdot,\cdot]$ has four rows, entry $\mathtt{a},\mathtt{b}$ the row to use when the left incoming token is of type $\mathtt{a}$ and the right incoming token is of type $\mathtt{b}$. The token that gets encrypted for this row is the token for the outgoing-wire with the correct semantics. Given two tokens X_a and X_b we use their types to determine which entry of the garbled table we need to decrypt. The description of the decoding function d is the list of the tokens at output wires.

Informally, scheme GaAO generates a key hk of a UCE-secure hash H and runs $\mathsf{Ga}[\mathsf{H.Ev}(1^\lambda, hk, \cdot, \cdot)].\mathsf{Gb}$ to produce (F, e, d). We then secret-share the key hk to n shares, distributing the shares among the input tokens. Scheme GaP also creates a hash key hk and runs $\mathsf{Ga}[\mathsf{H.Ev}(1^\lambda, hk, \cdot, \cdot)].\mathsf{Gb}$ to generate (F, e, d). However, it instead uses a vacuous decoding function. Let $d = (Y_1^0, Y_1^1, \ldots, Y_m^0, Y_m^1)$, $u_i = \mathrm{lsb}(Y_i^0)$ for every $i \le m$, and $U = u_1 \ldots u_m$, where function lsb maps a string to its last bit. It then secret-shares (U, hk) to n shares and distribute them among the input tokens. Later, to decode the garbled output $Y = (Y_1, \ldots, Y_m)$ from procedure $\mathsf{Ga.Ev}$, it will output $y_1 \cdots y_m$ as the final output, where $y_i = \mathrm{lsb}(Y_i) \oplus u_i$ for every $i \le m$.

The following says that $\mathsf{GaP}[\mathsf{H}]$ is prv2-secure, and $\mathsf{GaAO}[\mathsf{H}]$ is obv2 and aut2-secure. The proof is in Appendix A.

Theorem 6.4 Let H be a hash function family such that $\mathsf{H.IL}(\lambda) = \mathbb{N}$ and $\mathsf{H.ol}(\lambda) = \lambda$ for every $\lambda \in \mathbb{N}$. If $\mathsf{H} \in \mathsf{UCE2}$, then (1) $\mathsf{GaAO}[\mathsf{H}] \in \mathsf{AUT2}$, (2) $\mathsf{GaAO}[\mathsf{H}] \in \mathsf{OBV2}[\Phi_{\mathrm{topo}}]$, and (3) $\mathsf{GaP}[\mathsf{H}] \in \mathsf{PRIV2}[\Phi_{\mathrm{topo}}]$.

7 Constructions of UCE families

In this section, we describe constructions of UCE-secure function families. We provide a ROM construction and prove that it is mUCE2 secure. (And hence it is also UCE2, mUCE1 and UCE1 secure.) We refer the reader to Section 2 for a discussion of the value of validating UCE in the ROM as part of the layered-cryptography approach for ROM-based design. Then, we go on to explore practical instantiations of UCE secure functions.

7.1 Achieving UCE in the ROM

A random oracle RO is a stateful algorithm that maintains a table H, initially empty. When invoked with inputs $(m, 1^\ell)$, it returns $H[m,\ell]$ if $H[m,\ell]$ is already defined. Otherwise it picks $y \leftarrow\!\!\$\ \{0,1\}^\ell$, sets $H[m,\ell]$ to y and then returns y. A game in the ROM would implement RO and present an interface to access RO for its routines as well as adversaries.

Definitions. The first step is to extend the syntax. In a ROM family of functions H, the algorithm $\mathsf{H.Ev}$ has oracle access to RO. The rest is as before. We now define mUCE2 security of H in the ROM. The multi-key source S now has access to RO in addition to Hash, and the distinguisher gets access to RO as well. We continue to define m-uce advantage via Equation (3), with game $\mathrm{mUCE}_{\mathsf{H}}^{S,D}(\lambda)$ now being that of Fig. 21. A reset adversary now gets oracle access to RO in addition to Hash. We say that S is reset-secure if $\mathsf{Adv}^{\mathsf{m\text{-}reset}}_{R,S}(\cdot)$ is negligible for any PT reset adversary R, where $\mathsf{Adv}^{\mathsf{m\text{-}reset}}_{R,S}(\lambda) = 2\Pr[\mathrm{mReset}^S_R(\lambda)] - 1$ and game $\mathrm{mReset}^S_R(\lambda)$ is in Fig. 21. We say that H is mUCE2-secure in the ROM if $\mathsf{Adv}^{\mathsf{m\text{-}uce}}_{\mathsf{H},S,D}(\cdot)$ is negligible for every PT reset-securemulti-key source S and every PT D. Let $\mathsf{mUCE2^{ro}}$ denote the set of all ROM function families H that are mUCE2-secure in the ROM. Analogously, one can define UCE2, UCE1 and mUCE1 security in the ROM, and let $\mathsf{UCE2^{ro}}, \mathsf{UCE1^{ro}}, \mathsf{mUCE1^{ro}}$ be the set of all ROM function families H that are, respectively, UCE2, UCE1, and mUCE1-secure in the ROM. The relations depicted on the left of Fig. 1 still hold in the ROM.

Results. We now describe a mUCE2-secure ROM family of functions. The construction H is as follows. Let $\mathsf{H.Kg}(1^\lambda)$ return $hk \leftarrow\!\!\$\ \{0,1\}^\lambda$ for every $\lambda \in \mathbb{N}$. Let $\mathsf{H.IL} = \mathbb{N}$ and $\mathsf{H.OL} = \mathbb{N}$. Let $\mathsf{H.Ev^{RO}}(1^\lambda, hk, m, 1^\ell)$ return $\mathsf{RO}(hk \,\|\, m, 1^\ell)$ for every $hk \in \{0,1\}^\lambda$, every $m \in \{0,1\}^*$, every $\ell \in \mathbb{N}$ and every $\lambda \in \mathbb{N}$. The following says that H is mUCE2-secure in the ROM.

Theorem 7.1 Let H be the ROM function family defined above. Then $\mathsf{H} \in \mathsf{mUCE2^{ro}}$.

MAIN $\mathrm{mUCE}^{S,D}_{\mathsf{H}}(\lambda)$	MAIN $\mathrm{mReset}^{R}_{S}(\lambda)$
$(1^n, t) \leftarrow\!\!\$\ S^{\mathsf{RO}}(1^\lambda, \varepsilon)$	$\mathsf{Dom} \leftarrow \emptyset\ ;\ (1^n, t) \leftarrow\!\!\$\ S^{\mathsf{RO}}(1^\lambda, \varepsilon)$
For $i = 1$ to n do $\mathbf{hk}[i] \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$	$L \leftarrow\!\!\$\ S^{\mathsf{RO},\mathrm{HASH}}(1^n, t)\ ;\ b \leftarrow\!\!\$\ \{0,1\}$
$b \leftarrow\!\!\$\ \{0,1\}\ ;\ L \leftarrow\!\!\$\ S^{\mathsf{RO},\mathrm{HASH}}(1^n, t)$	If $b = 0$ then // reset the array T
$b' \leftarrow\!\!\$\ D^{\mathsf{RO}}(1^\lambda, \mathbf{hk}, L)$	For $(x, \ell, i) \in \mathsf{Dom}$ do
Return $(b' = b)$	$T[x, \ell, i] \leftarrow\!\!\$\ \{0,1\}^\ell$
HASH$(x, 1^\ell, i)$	$b' \leftarrow\!\!\$\ R^{\mathsf{RO},\mathrm{HASH}}(1^\lambda, L)\ ;$ Return $(b = b')$
If $T[x, \ell, i] = \perp$ then	HASH$(x, 1^\ell, i)$
If $b = 1$ then $T[x, \ell, i] \leftarrow \mathsf{H.Ev}^{\mathsf{RO}}(1^\lambda, \mathbf{hk}[i], x, 1^\ell)$	$\mathsf{Dom} \leftarrow \mathsf{Dom} \cup \{(x, \ell, i)\}$
Else $T[x, \ell, i] \leftarrow\!\!\$\ \{0,1\}^\ell$	If $T[x, \ell, i] = \perp$ then $T[x, \ell, i] \leftarrow\!\!\$\ \{0,1\}^\ell$
Return $T[x, \ell, i]$	Return $T[x, \ell, i]$
$\mathsf{RO}(v, 1^\ell)$	$\mathsf{RO}(v, 1^\ell)$
If $H[v, \ell] = \perp$ then $H[v, \ell] \leftarrow\!\!\$\ \{0,1\}^\ell$	If $H[v, \ell] = \perp$ then $H[v, \ell] \leftarrow\!\!\$\ \{0,1\}^\ell$
Return $H[v, \ell]$	Return $H[v, \ell]$

Figure 21: **Left:** Game mUCE defines multi-key UCE security in the ROM. **Right:** Game mReset defines multi-key reset-security in the ROM.

Proof: Let S be a PT, reset-secure multi-key source and let D be a PT distinguisher. Let $\overline{n}, q$ be polynomials such that $n \le \overline{n}(\lambda)$ and S, D between them make at most $q(\lambda)$ RO-queries in game $\mathrm{mUCE}^{S,D}_{\mathsf{H}}(\lambda)$, for all $\lambda \in \mathbb{N}$. Assume $\overline{n}(\lambda) < 2^\lambda$ for all $\lambda \in \mathbb{N}$. Wlog, assume that S doesn't repeat a query to HASH or RO, and D does not repeat a query to RO. We'll construct a PT reset-adversary R such that for all $\lambda \in \mathbb{N}$ we have

$$\mathsf{Adv}^{\mathsf{m\text{-}uce2}}_{S,D,\mathsf{H}}(\lambda) \le \mathsf{Adv}^{\mathsf{mreset}}_{S,R}(\lambda) + \frac{3\overline{n}(\lambda) \cdot q(\lambda) + \overline{n}(\lambda)^2}{2^\lambda} \ . \tag{20}$$

The theorem follows from the assumption that S is reset secure.

Consider games G_1–G_7 in Fig. 22. We let RO_1 be the interface of S to access to the random oracle, and RO_2 be that of D. Let d be the challenge bit of game $\mathrm{mUCE}^{S,D}_{\mathsf{H}}(\cdot)$. Then

$$\Pr[\mathrm{G}^{S,D}_1(\lambda)] = \Pr[\mathrm{mUCE}^{S,D}_{\mathsf{H}}(\lambda) \mid d = 1] \text{ and } \Pr[\mathrm{G}^{S,D}_7(\lambda)] = 1 - \Pr[\mathrm{mUCE}^{S,D}_{\mathsf{H}}(\lambda) \mid d = 0] \ .$$

We explain the game chain up to the terminal one. In game $\mathrm{G}^{S,D}_2(\lambda)$, we sample the hash keys so that they are distinct. Next, for each query to HASH, if the corresponding entry $H[v, \ell]$ is already defined then game $\mathrm{G}^{S,D}_2(\lambda)$ overwrites $H[v, \ell]$ by a fresh $y \leftarrow\!\!\$\ \{0,1\}^\ell$, and outputs y. Since S doesn't repeat a prior query to HASH and the keys are distinct, the overwriting occurs only if S queries $(\mathbf{k}[i] \,\|\, x, 1^\ell)$ to RO, and then queries $(x, 1^\ell, i)$ to HASH. The two games $\mathrm{G}^{S,D}_1(\lambda)$ and $\mathrm{G}^{S,D}_2(\lambda)$ are identical-until-bad. Then, for all $\lambda \in \mathbb{N}$, we have

$$\Pr[\mathrm{G}^{S,D}_1(\lambda)] - \Pr[\mathrm{G}^{S,D}_2(\lambda)] \le \Pr[\mathrm{G}^{S,D}_2(\lambda) \text{ sets } \mathsf{bad}] \le \frac{\overline{n}(\lambda) \cdot q(\lambda)}{2^\lambda} + \frac{\overline{n}(\lambda)^2}{2^{\lambda+1}} \ .$$

In game $\mathrm{G}^{S,D}_3(\lambda)$, for each string v, if there is $i \le n$ such that $v[1, \lambda] = \mathbf{k}[i]$, instead of reading/writing to $H[v, \ell]$, we'll use $T\big[v[\lambda + 1, |v|], \ell, i\big]$. Since the keys are distinct, for every $\lambda \in \mathbb{N}$,

$$\Pr[\mathrm{G}^{S,D}_2(\lambda)] = \Pr[\mathrm{G}^{S,D}_3(\lambda)] \ .$$

In game $\mathrm{G}^{S,D}_4(\lambda)$, the keys now are sampled independently. The two games $\mathrm{G}^{S,D}_3(\lambda)$ and $\mathrm{G}^{S,D}_4(\lambda)$ are identical-until-bad. Then for every $\lambda \in \mathbb{N}$,

$$\Pr[\mathrm{G}^{S,D}_3(\lambda)] - \Pr[\mathrm{G}^{S,D}_4(\lambda)] \le \Pr[\mathrm{G}^{S,D}_4(\lambda) \text{ sets } \mathsf{bad}] \le \frac{\overline{n}(\lambda)^2}{2^{\lambda+1}} \ .$$

In game $\mathrm{G}^{S,D}_5(\lambda)$, replies from RO_2 are no longer consistent with HASH replies. The two games $\mathrm{G}^{S,D}_4(\lambda)$

$\underline{\text{MAIN } \mathrm{G}_1^{S,D}(\lambda),\ \boxed{\mathrm{G}_2^{S,D}(\lambda)}}$
$(1^n, t) \leftarrow\!\!\$\ S(1^\lambda, \varepsilon)$; $M \leftarrow \emptyset$
For $i = 1$ to n do
  $\mathbf{k}[i] \leftarrow\!\!\$\ \{0,1\}^\lambda$
  If $\mathbf{k}[i] \in M$ then $\mathsf{bad} \leftarrow \mathsf{true}$; $\boxed{\mathbf{k}[i] \leftarrow\!\!\$\ \{0,1\}^\lambda \backslash M}$
  $M \leftarrow M \cup \{\mathbf{k}[i]\}$
$L \leftarrow\!\!\$\ S^{\text{HASH},\mathsf{RO}_1}(1^n, t)$; $b \leftarrow\!\!\$\ D^{\mathsf{RO}_2}(1^\lambda, \mathbf{k}, L)$
Return $(b = 1)$

$\underline{\text{HASH}(x, 1^\ell, i)}$
$v \leftarrow \mathbf{k}[i] \,\|\, x$; $y \leftarrow\!\!\$\ \{0,1\}^\ell$
If $H[v, \ell] = \perp$ then $H[v, \ell] \leftarrow y$
Else $\mathsf{bad} \leftarrow \mathsf{true}$; $\boxed{H[v, \ell] \leftarrow y}$
Return $H[v, \ell]$

$\underline{\mathsf{RO}_1(v, 1^\ell)}$
If $H[v, \ell] = \perp$ then $H[v, \ell] \leftarrow\!\!\$\ \{0,1\}^\ell$
Return $H[v, \ell]$

$\underline{\mathsf{RO}_2(v, 1^\ell)}$
If $H[v, \ell] = \perp$ then $H[v, \ell] \leftarrow\!\!\$\ \{0,1\}^\ell$
Return $H[v, \ell]$

$\underline{\text{MAIN } \boxed{\mathrm{G}_3^{S,D}(\lambda)},\ \mathrm{G}_4^{S,D}(\lambda)}$
$(1^n, t) \leftarrow\!\!\$\ S(1^\lambda, \varepsilon)$; $M \leftarrow \emptyset$
For $i = 1$ to n do
  $\mathbf{k}[i] \leftarrow\!\!\$\ \{0,1\}^\lambda$
  If $\mathbf{k}[i] \in M$ then $\mathsf{bad} \leftarrow \mathsf{true}$; $\boxed{\mathbf{k}[i] \leftarrow\!\!\$\ \{0,1\}^\lambda \backslash M}$
  $M \leftarrow M \cup \{\mathbf{k}[i]\}$
$L \leftarrow\!\!\$\ S^{\text{HASH},\mathsf{RO}_1}(1^n, t)$; $b \leftarrow\!\!\$\ D^{\mathsf{RO}_2}(1^\lambda, \mathbf{k}, L)$; Return $(b = 1)$

$\underline{\text{HASH}(x, 1^\ell, i)}$
If $T[x, \ell, i] = \perp$ then $T[x, \ell, i] \leftarrow\!\!\$\ \{0,1\}^\ell$
Return $T[x, \ell, i]$

$\underline{\mathsf{RO}_1(v, 1^\ell)}$
$x \leftarrow v[\lambda + 1, |v|]$; $K \leftarrow v[1, \lambda]$
For $i = 1$ to n do
  If $K = \mathbf{k}[i]$ then
    $\mathsf{coll} \leftarrow \mathsf{true}$
    If $T[x, \ell, i] = \perp$ then $T[x, \ell, i] \leftarrow\!\!\$\ \{0,1\}^\ell$
    Return $T[x, \ell, i]$
If $H[v, \ell] = \perp$ then $H[v, \ell] \leftarrow\!\!\$\ \{0,1\}^\ell$
Return $H[v, \ell]$

$\underline{\mathsf{RO}_2(v, 1^\ell)}$
$x \leftarrow v[\lambda + 1, |v|]$; $K \leftarrow v[1, \lambda]$
For $i = 1$ to n do
  If $K = \mathbf{k}[i]$ then
    If $T[x, \ell, i] = \perp$ then $T[x, \ell, i] \leftarrow\!\!\$\ \{0,1\}^\ell$
    Return $T[x, \ell, i]$
If $H[v, \ell] = \perp$ then $H[v, \ell] \leftarrow\!\!\$\ \{0,1\}^\ell$
Return $H[v, \ell]$

Figure 22: **Games G_1–G_4 for the proof of Theorem 7.1.** Games $\mathrm{G}_2, \mathrm{G}_3$ include the corresponding boxed statement, while the other games do not.

and $\mathrm{G}_5^{S,D}(\lambda)$ are identical-until-coll. Then for every $\lambda \in \mathbb{N}$,

$$\Pr[\mathrm{G}_4^{S,D}(\lambda)] - \Pr[\mathrm{G}_5^{S,D}(\lambda)] \le \Pr[\mathrm{G}_5^{S,D}(\lambda) \text{ sets } \mathsf{coll}] \le \frac{\overline{n}(\lambda) \cdot q(\lambda)}{2^\lambda},$$

where the last inequality is due to the fact that the keys now are uniformly random, and independent of whatever S receives. In game $\mathrm{G}_6^{S,D}(\lambda)$, in procedure RO_2, we reset the entries of T before giving answers to D. Now consider the reset-adversary R constructed below:

$\underline{R^{\text{HASH},\mathsf{RO}}(1^\lambda, 1^n, L)}$
For $i = 1$ to n do $\mathbf{k}[i] \leftarrow\!\!\$\ \{0,1\}^\lambda$
$b \leftarrow\!\!\$\ D^{\mathsf{ROSim}}(1^\lambda, \mathbf{k}, L)$
Return b

$\underline{\mathsf{ROSim}(v, 1^\ell)}$
$x \leftarrow v[\lambda + 1, |v|]$; $K \leftarrow v[1, \lambda]$
For $i = 1$ to n do
  If $K = \mathbf{k}[i]$ then then return $\text{HASH}(x, 1^\ell, i)$
Else return $\mathsf{RO}(v, 1^\ell)$

Let a be the challenge bit of game $\mathrm{mReset}_S^R(\lambda)$. Then for every $\lambda \in \mathbb{N}$,

$$\Pr[\mathrm{G}_5^{S,D}(\lambda)] = \Pr[\,\mathrm{mReset}_S^P(\lambda) \,|\, a = 1\,] \text{ and } \Pr[\mathrm{G}_6^{S,D}(\lambda)] = 1 - \Pr[\,\mathrm{mReset}_S^P(\lambda) \,|\, a = 0\,] \ .$$

In game $\mathrm{G}_7^{S,D}(\lambda)$, replies from RO_1 and RO_2 are consistent. Games $\mathrm{G}_6^{S,D}(\lambda)$ and $\mathrm{G}_7^{S,D}(\lambda)$ are identical-

$\underline{\text{MAIN } \mathrm{G}_5^{S,D}(\lambda)}$
$(1^n, t) \leftarrow\!\!\$\ S(1^\lambda, \varepsilon)$
For $i = 1$ to n do $\mathbf{k}[i] \leftarrow\!\!\$\ \{0,1\}^\lambda$
$L \leftarrow\!\!\$\ S^{\text{HASH},\mathsf{RO}_1}(1^n, t)$; $b \leftarrow\!\!\$\ D^{\mathsf{RO}_2}(1^\lambda, \mathbf{k}, L)$
Return $(b = 1)$

$\underline{\text{HASH}(x, 1^\ell, i)}$
If $T[x,\ell,i] = \perp$ then $T[x,\ell,i] \leftarrow\!\!\$\ \{0,1\}^\ell$
Return $T[x,\ell,i]$

$\underline{\mathsf{RO}_1(v, 1^\ell)}$
$x \leftarrow v[\lambda+1, |v|]$; $K \leftarrow v[1,\lambda]$
For $i = 1$ to n do
  If $K = \mathbf{k}[i]$ then $\mathsf{coll} \leftarrow \mathsf{true}$
If $H[v,\ell] = \perp$ then $H[v,\ell] \leftarrow\!\!\$\ \{0,1\}^\ell$
Return $H[v,\ell]$

$\underline{\mathsf{RO}_2(v, 1^\ell)}$
$x \leftarrow v[\lambda+1, |v|]$; $K \leftarrow v[1,\lambda]$
For $i = 1$ to n do
  If $K = \mathbf{k}[i]$ then
    If $T[x,\ell,i] = \perp$ then $T[x,\ell,i] \leftarrow\!\!\$\ \{0,1\}^\ell$
    Return $T[x,\ell,i]$
If $H[v,\ell] = \perp$ then $H[v,\ell] \leftarrow\!\!\$\ \{0,1\}^\ell$
Return $H[v,\ell]$

$\underline{\text{MAIN } \mathrm{G}_6^{S,D}(\lambda),\ \boxed{\mathrm{G}_7^{S,D}(\lambda)}}$
$(1^n, t) \leftarrow\!\!\$\ S(1^\lambda, \varepsilon)$
For $i = 1$ to n do $\mathbf{k}[i] \leftarrow\!\!\$\ \{0,1\}^\lambda$
$L \leftarrow\!\!\$\ S^{\text{HASH},\mathsf{RO}_1}(1^n, t)$; $b \leftarrow\!\!\$\ D^{\mathsf{RO}_2}(1^\lambda, \mathbf{k}, L)$
Return $(b = 1)$

$\underline{\text{HASH}(x, 1^\ell, i)}$
If $T[x,\ell,i] = \perp$ then $T[x,\ell,i] \leftarrow\!\!\$\ \{0,1\}^\ell$
Return $T[x,\ell,i]$

$\underline{\mathsf{RO}_1(v, 1^\ell)}$
If $H[v,\ell] = \perp$ then $H[v,\ell] \leftarrow\!\!\$\ \{0,1\}^\ell$
Return $H[v,\ell]$

$\underline{\mathsf{RO}_2(v, 1^\ell)}$
$x \leftarrow v[\lambda+1, |v|]$; $K \leftarrow v[1,\lambda]$
For $i = 1$ to n do
  If $K = \mathbf{k}[i]$ then
    If $H[v,\ell] \neq \perp$ then $\mathsf{bad} \leftarrow \mathsf{true}$; $\boxed{\text{Return } H[v,\ell]}$
    $T[x,\ell,i] \leftarrow\!\!\$\ \{0,1\}^\ell$; Return $T[x,\ell,i]$
If $H[v,\ell] = \perp$ then $H[v,\ell] \leftarrow\!\!\$\ \{0,1\}^\ell$
Return $H[v,\ell]$

Figure 23: **Games $\mathrm{G}_5, \mathrm{G}_6$, and G_7 for the proof of Theorem 7.1.** Game G_7 includes the corresponding boxed statement, while the other games do not.

until-bad. The flag bad is set only if S queries $(\mathbf{k}[i] \,\|\, x, 1^\ell)$ to RO_1, for some $i \le n$. Then

$$\Pr[\mathrm{G}_6^{S,D}(\lambda)] - \Pr[\mathrm{G}_7^{S,D}(\lambda)] \le \Pr[\mathrm{G}_7^{S,D}(\lambda) \text{ sets } \mathsf{bad}] \le \frac{\overline{n}(\lambda) \cdot q(\lambda)}{2^\lambda} \ .$$

Hence, for every $\lambda \in \mathbb{N}$,

$$\begin{aligned}
\mathsf{Adv}^{\mathsf{m\text{-}uce2}}_{S,D,\mathsf{H}}(\lambda) &= \Pr[\,\mathrm{mUCE}^{S,D}_{\mathsf{H}}(\lambda) \,|\, d = 1\,] + \Pr[\,\mathrm{mUCE}^{S,D}_{\mathsf{H}}(\lambda) \,|\, d = 0\,]\,] - 1 \\
&= \Pr[\mathrm{G}_1^{S,D}(\lambda)] - \Pr[\mathrm{G}_7^{S,D}(\lambda)] \\
&\le \sum_{i=1}^{6} \Pr[\mathrm{G}_i^{S,D}(\lambda)] - \Pr[\mathrm{G}_{i+1}^{S,D}(\lambda)] \\
&\le \mathsf{Adv}^{\mathsf{mreset}}_{S,R}(\lambda) + \frac{3\overline{n}(\lambda) \cdot q(\lambda) + \overline{n}(\lambda)^2}{2^\lambda}
\end{aligned}$$

yielding Equation (20). ∎

7.2 Practical constructions

We consider practical, heuristic instantiations for families of functions assumed UCE secure. The UCE framework and security definitions are asymptotic, while these real-world instantiations are based on non-asymptotic blockciphers and hash functions, so we make no formal claims about security. We ignore the security parameter and consider FOL families, so that we view $\mathsf{H.Kg}$ as taking no inputs and we view $\mathsf{H.Ev}$ as taking only a key and an input.

A natural instantiation is via a block cipher, for example AES. Here, H.Kg would pick a random 128-bit string K, and $\mathsf{H.Ev}(K, X)$ would return $\mathsf{AES}(K, X)$. However, as we saw in part (2) of the proof of Proposition 4.2, this construction fails to provide UCE1 security since AES is efficiently invertible given the key and this can be exploited to mount an attack. This is interesting in the light of the fact that it is standard to use AES as a PRF or PRP.

One could consider instantiations based on cryptographic hash functions such as SHA256, but UCE security requires a keyed function, and SHA256 is not keyed. This suggests that we use the HMAC construction of [13, 82]. This is indeed our leading suggestion for a practical way to instantiate families assumed UCE secure.

An interesting open question is whether the assumption that HMAC provides (say) mUCE2-security can be validated in an idealized model where one assumes the compression function is ideal. (If not, the suggestion that it be used to instantiate UCE families in practice should be reconsidered.) Since we have provided in Section 7.1 a ROM-based construct, one might hope to validate HMAC based on its indifferentiability from a RO [55], but as per [90] indifferentiability may not be enough because UCE is a multi-stage game, so a different approach or a direct analysis may be needed.

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A Proof of Theorem 6.4

For part (1), consider an arbitrary adversary $\mathcal{A}$ attacking the aut2 security of $\mathsf{GaAO}[\mathsf{H}]$. Assume that $\mathcal{A}(1^\lambda)$ uses $\rho(\lambda)$ coins. We'll construct a reset-secure source S and a distinguisher D such that

$$\mathsf{Adv}^{\mathrm{aut2}}_{\mathsf{GaAO}[\mathsf{H}],\mathcal{A}}(\lambda) \leq \mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\lambda) + 2^{1-\lambda} \tag{21}$$

for every $\lambda \in \mathbb{N}$. The theorem then follows from the assumption that $\mathsf{H} \in \mathsf{UCE2}$. The constructions of S and D are shown below.

$\underline{S^{\text{HASH}}(1^\lambda)}$
$r \leftarrow\!\!\$\ \{0,1\}^{\rho(\lambda)}$; $\mathsf{cnt} \leftarrow 0$; $L \leftarrow \bot$
$\mathcal{A}^{\text{GARBLE},\text{INPUT}}(1^\lambda; r)$; Return L

$\underline{\text{GARBLE}(f)}$
$(F, (X_1^0, X_1^1, \ldots, X_n^0, X_n^1), d) \leftarrow\!\!\$\ \mathsf{Ga}[\text{HASH}].\mathsf{Gb}(1^\lambda, f)$
Return F

$\underline{\text{INPUT}(i, c)}$
$\mathsf{cnt} \leftarrow \mathsf{cnt} + 1$; $X_i \leftarrow X_i^c$; $V_i \leftarrow\!\!\$\ \{0,1\}^{\mathsf{H.kl}(\lambda)}$
If $\mathsf{cnt} < n$ then return (X_i, V_i)
$V_i \leftarrow \bot$; $X \leftarrow (X_1, \ldots, X_n)$; $V \leftarrow (V_1, \ldots, V_n)$
$(Y_1^0, Y_1^1, \ldots, Y_m^0, Y_m^1) \leftarrow d$
For $j = 1$ to m do $d_j \leftarrow Y_j^{1-y_j}[1, \lambda - 1]$
$L \leftarrow (F, r, X, V, (d_1, \ldots, d_m))$; Return $\bot$

$\underline{D(1^\lambda, hk, L)}$
$(F, r, X, V, (d_1, \ldots, d_m)) \leftarrow L$; $(V_1, \ldots, V_n) \leftarrow V$
$(Y_1, \ldots, Y_m) \leftarrow Y \leftarrow \mathsf{Ga}[\mathsf{H.Ev}(1^\lambda, hk, \cdot, \cdot)].\mathsf{Ev}(F, X)$
$(X_1, \ldots, X_n) \leftarrow X$; $Y' \leftarrow\!\!\$\ \mathcal{A}^{\text{GARBLE},\text{INPUT}}(1^\lambda; r)$
$x \leftarrow x_1 \cdots x_n$; $y_1 \cdots y_m \leftarrow y \leftarrow \mathsf{cev}(f, x)$
For $j = 1$ to m do
  $t_j \leftarrow \mathrm{lsb}(Y_j)$; $Y_j^{y_j} \leftarrow Y_j$; $Y_j^{1-y_j} \leftarrow d_j \,\|\, (1 - t_j)$
$d \leftarrow (Y_1^0, Y_1^1, \ldots, Y_m^0, Y_m^1)$
$y' \leftarrow \mathsf{Ga}[\mathsf{H.Ev}(1^\lambda, hk, \cdot, \cdot)].\mathsf{De}(d, Y')$
If $y' \neq \bot$ and $Y' \neq Y$ then return 1 else return 0

$\underline{\text{GARBLE}(f)}$
Return F

$\underline{\text{INPUT}(i, c)}$
If $V_i = \bot$ then $V_i \leftarrow 0^{\mathsf{H.kl}(\lambda)}$; $V_i \leftarrow V_1 \oplus \cdots \oplus V_n \oplus hk$
$x_i \leftarrow c$; Return (X_i, V_i)

Let a be the challenge bit of game $\mathrm{UCE}_{\mathsf{H}}^{S,D}(\lambda)$. Then

$$\mathsf{Adv}^{\mathrm{aut2},\mathcal{A}}_{\mathsf{GaAO[H]}}(\lambda) = \Pr[\,\mathrm{UCE}_{\mathsf{H}}^{S,D}(\lambda) \,|\, a = 1\,] \ .$$

On the other hand, in game $\mathrm{UCE}_{\mathsf{H}}^{S,D}(\lambda)$ with $a = 0$, only if $\mathcal{A}$ can specify some $j \leq m$ and the correct d_j does D output 1. However, since each d_j is a uniformly random string that is independent of whatever $\mathcal{A}$ receives, $\Pr[\mathrm{UCE}_{\mathsf{H}}^{S,D}(\lambda) \mid a = 0] \geq 1 - 2^{-\lambda}$. Hence

$$\begin{aligned} \mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\lambda) &= \Pr[\mathrm{UCE}_{\mathsf{H}}^{S,D}(\lambda) \mid a = 1] + \Pr[\mathrm{UCE}_{\mathsf{H}}^{S,D}(\lambda) \mid a = 0] - 1 \\ &\geq \mathsf{Adv}^{\mathrm{aut2}}_{\mathsf{GaAO[H]},\mathcal{A}}(\lambda) - 2^{-\lambda}, \end{aligned}$$

yielding Equation (21). Finally, Lemma A.1 below shows that S is reset-secure.

Lemma A.1 For any PT adversary R and any polynomial p that bounds the number of R's HASH queries, $\mathsf{Adv}^{\mathsf{reset}}_{S,R}(\lambda) \leq 2p(\lambda)/2^\lambda$ for every $\lambda \in \mathbb{N}$.

Proof: Consider games G_1–G_4 in Fig. 24. Let c be the challenge bit of game $\mathrm{Reset}_S^P(\lambda)$. Then

$$\Pr[\mathrm{G}_1^{\mathcal{A},R}(\lambda)] = \Pr[\,\mathrm{Reset}_S^P(\lambda) \,|\, c = 1\,] \text{ and } \Pr[\mathrm{G}_7^{\mathcal{A},R}(\lambda)] = 1 - \Pr[\,\mathrm{Reset}_S^P(\lambda) \,|\, c = 0\,]$$

for every $\lambda \in \mathbb{N}$. We explain the game chains up until the terminal game. In game $\mathrm{G}_2^{\mathcal{A},R}(\lambda)$, we postpone sampling the tokens and defining $4q$ points of the array H until they are needed. Hence

$$\Pr[\mathrm{G}_1^{\mathcal{A},R}(\lambda)] = \Pr[\mathrm{G}_2^{\mathcal{A},R}(\lambda)] \ .$$

After last INPUT query, one can run $\mathsf{Ga}[\text{HASH}(\cdot, 1^\lambda)].\mathsf{Ev}(F, X)$ to obtain a token per wire. We say that these tokens are *visible*, and the other tokens are *invisible*. A query $(w, 1^\lambda)$ is *illegitimate* if (i) $w = A \,\|\, B \,\|\, g$, (ii) A and B are tokens of the left- and right-incoming wires of g respectively, and (iii) at least one of A and B is invisible. In game $\mathrm{G}_3^{\mathcal{A},R}(\lambda)$, for illegitimate queries, procedure HASH gives answers independent of GARBLE and INPUT. The two games $\mathrm{G}_2^{\mathcal{A},R}(\lambda)$ and $\mathrm{G}_3^{\mathcal{A},R}(\lambda)$ are identical-until-bad. Triggering bad means specifying a non-output wire and its invisible token. As long as bad is not set, the first $\lambda - 1$ bits of each invisible token are uniformly random and independent of whatever $(\mathcal{A}, R)$ receives. Hence

$$\Pr[\mathrm{G}_2^{\mathcal{A},R}(\lambda)] - \Pr[\mathrm{G}_3^{\mathcal{A},R}(\lambda)] \leq \Pr[\mathrm{G}_3^{\mathcal{A},R}(\lambda) \text{ sets } \mathsf{bad}] \leq \frac{2p(\lambda)}{2^\lambda} \ .$$

In game $\mathrm{G}_4^{\mathcal{A},R}(\lambda)$, the visible tokens at non-input wires and all invisible tokens are unused, and thus the code generating them can be removed. Hence

$$\Pr[\mathrm{G}_3^{\mathcal{A},R}(\lambda)] = \Pr[\mathrm{G}_4^{\mathcal{A},R}(\lambda)] \ .$$

$\underline{\text{MAIN } \mathrm{G}_1^{\mathcal{A},R}(\lambda)}$
$r \leftarrow\!\!\$\ \{0,1\}^{\rho(\lambda)}$; $\mathsf{cnt} \leftarrow 0$; $L \leftarrow \bot$
$\mathcal{A}^{\text{GARBLE},\text{INPUT}}(1^\lambda)$; $b' \leftarrow R^{\text{HASH}}(1^\lambda, L)$
Return $(b' = 1)$

$\underline{\text{INPUT}(i, c)}$
If $i > n$ or $x_i \neq \bot$ then return $\bot$
$V_i \leftarrow\!\!\$\ \{0,1\}^{\mathsf{H.kl}(\lambda)}$; $\mathsf{cnt} \leftarrow \mathsf{cnt} + 1$; $X_i \leftarrow X_i^c$
If $\mathsf{cnt} < n$ then return (X_i, V_i)
For $j \leftarrow 1$ to m do $d_j \leftarrow\!\!\$\ \{0,1\}^{\lambda-1}$
$V_i \leftarrow \bot$; $V \leftarrow (V_1, \ldots, V_n)$
$X \leftarrow (X_1, \ldots, X_n)$
$L \leftarrow (F, r, X, V, (d_1, \ldots, d_m))$
Return $\bot$

$\underline{\text{GARBLE}(1^\lambda, f)}$
$(n, m, q, A', B', G) \leftarrow f$; $\mathsf{cnt} \leftarrow 0$
For $i \leftarrow 1$ to $n + q$ do
  $t \leftarrow\!\!\$\ \{0,1\}$; $X_i^0 \leftarrow\!\!\$\ \{0,1\}^{\lambda-1} t$; $X_i^1 \leftarrow\!\!\$\ \{0,1\}^{\lambda-1} \bar{t}$
For $g \leftarrow n + 1$ to $n + q$, $i \leftarrow 0$ to 1, $j \leftarrow 0$ to 1 do
  $a \leftarrow A'(g)$; $b \leftarrow B'(g)$
  $A \leftarrow X_a^i$; $\mathsf{a} \leftarrow \mathrm{lsb}(A)$; $B \leftarrow X_b^j$; $\mathsf{b} \leftarrow \mathrm{lsb}(B)$
  $T[g, \mathsf{a}, \mathsf{b}] \leftarrow\!\!\$\ \{0,1\}^\lambda$; $H[A \| B \| g, \lambda] \leftarrow T[g, \mathsf{a}, \mathsf{b}] \oplus X_g^{G_g(i,j)}$
$F \leftarrow (n, m, q, A', B', T)$
Return F

$\underline{\text{HASH}(w, 1^\ell)}$
If $H[w, \ell] = \bot$ then $H[w, \ell] \leftarrow\!\!\$\ \{0,1\}^\ell$
Return $H[w, \ell]$

$\underline{\text{MAIN } \boxed{\mathrm{G}_2^{\mathcal{A},R}(\lambda)}, \mathrm{G}_3^{\mathcal{A},R}(\lambda)}$
$r \leftarrow\!\!\$\ \{0,1\}^{\rho(\lambda)}$; $\mathsf{cnt} \leftarrow 0$; $L \leftarrow \bot$
$\mathcal{A}^{\text{GARBLE},\text{INPUT}}(1^\lambda)$; $b' \leftarrow R^{\text{HASH}}(1^\lambda, L)$
Return $(b' = 1)$

$\underline{\text{INPUT}(i, c)}$
If $i > n$ or $x_i \neq \bot$ then return $\bot$
$x_i \leftarrow c$; $\mathsf{cnt} \leftarrow \mathsf{cnt} + 1$; $X_i^{x_i} \leftarrow X_i \leftarrow\!\!\$\ \{0,1\}^\lambda$
If $\mathsf{cnt} < n$ then
  $V_i \leftarrow\!\!\$\ \{0,1\}^{\mathsf{H.kl}(\lambda)}$; Return (X_i, V_i)
For $g \leftarrow n + 1$ to $n + q$ do
  $a \leftarrow A'(g)$; $b \leftarrow B'(g)$; $x_g \leftarrow G_g(x_a, x_b)$
  $Z \leftarrow H[X_a \| X_b \| g, \lambda] \leftarrow\!\!\$\ \{0,1\}^\lambda$
  $X_g^{x_g} \leftarrow X_g \leftarrow Z \oplus T[g, x_a, x_b]$
For $j \leftarrow 1$ to $n + q - m$ do
  $t \leftarrow \mathrm{lsb}(X_j)$; $X_j^{1-x_j} \leftarrow\!\!\$\ \{0,1\}^{\lambda-1} \| \bar{t}$
For $j \leftarrow 1$ to m do
  $g \leftarrow n + q - m + j$; $d_j \leftarrow\!\!\$\ \{0,1\}^{\lambda-1}$
  $t \leftarrow \mathrm{lsb}(X_g)$; $X_g^{1-x_g} \leftarrow d_j \| \bar{t}$
$V_i \leftarrow \bot$; $V \leftarrow (V_1, \ldots, V_n)$
$X \leftarrow (X_1, \ldots, X_n)$
$L \leftarrow (F, r, X, V, (d_1, \ldots, d_m))$; Return $\bot$

$\underline{\text{GARBLE}(1^\lambda, f)}$
$(n, m, q, A', B', G) \leftarrow f$; $\mathsf{cnt} \leftarrow 0$
For $g \leftarrow n + 1$ to $n + q$, $\mathsf{a} \leftarrow 0$ to 1, $\mathsf{b} \leftarrow 0$ to 1 do
  $T[g, \mathsf{a}, \mathsf{b}] \leftarrow\!\!\$\ \{0,1\}^\lambda$
$F \leftarrow (n, m, q, A', B', T)$
Return F

$\underline{\text{HASH}(w, 1^\ell)}$
If $H[w, \ell] \neq \bot$ then return $H[w, \ell]$
$H[w, \ell] \leftarrow\!\!\$\ \{0,1\}^\ell$
If $w = A \| B \| g$ and $\ell = \lambda$ then
  $a \leftarrow A'(g)$; $b \leftarrow B'(g)$; $\mathsf{a} \leftarrow \mathrm{lsb}(A)$; $\mathsf{b} \leftarrow \mathrm{lsb}(B)$
  If $A = X_a^i$ and $B = X_b^j$ then
    $\mathsf{bad} \leftarrow \mathsf{true}$; $\boxed{H[w, \ell] \leftarrow T[g, \mathsf{a}, \mathsf{b}] \oplus X_g^{G_g(i,j)}}$
Return $H[w, \ell]$

$\underline{\text{MAIN } \mathrm{G}_4^{\mathcal{A},R}(\lambda)}$
$r \leftarrow\!\!\$\ \{0,1\}^{\rho(\lambda)}$; $\mathsf{cnt} \leftarrow 0$; $L \leftarrow \bot$
$\mathcal{A}^{\text{GARBLE},\text{INPUT}}(1^\lambda)$; $b' \leftarrow R^{\text{HASH}}(1^\lambda, L)$
Return $(b' = 1)$

$\underline{\text{INPUT}(i, c)}$
If $i > n$ or $x_i \neq \bot$ then return $\bot$
$x_i \leftarrow c$; $\mathsf{cnt} \leftarrow \mathsf{cnt} + 1$; $X_i \leftarrow\!\!\$\ \{0,1\}^\lambda$
If $\mathsf{cnt} < n$ then $V_i \leftarrow\!\!\$\ \{0,1\}^{\mathsf{H.kl}(\lambda)}$; Return (X_i, V_i)
For $j \leftarrow 1$ to m do $d_j \leftarrow\!\!\$\ \{0,1\}^{\lambda-1}$
$V_i \leftarrow \bot$; $V \leftarrow (V_1, \ldots, V_n)$; $X \leftarrow (X_1, \ldots, X_n)$
$L \leftarrow (F, r, X, V, (d_1, \ldots, d_m))$; Return $\bot$

$\underline{\text{GARBLE}(1^\lambda, f)}$
$(n, m, q, A', B', G) \leftarrow f$; $\mathsf{cnt} \leftarrow 0$
For $g \leftarrow n + 1$ to $n + q$, $\mathsf{a} \leftarrow 0$ to 1, $\mathsf{b} \leftarrow 0$ to 1 do
  $T[g, \mathsf{a}, \mathsf{b}] \leftarrow\!\!\$\ \{0,1\}^\lambda$
$F \leftarrow (n, m, q, A', B', T)$
Return F

$\underline{\text{HASH}(w, 1^\ell)}$
If $H[w, \ell] = \bot$ then $H[w, \ell] \leftarrow\!\!\$\ \{0,1\}^\ell$
Return $H[w, \ell]$

Figure 24: **Games for the proof of Lemma A.1.** Game G_2 contains the boxed statement but game G_3 does not.

$\underline{\mathrm{Sim}(1^\lambda, \phi, 0)}$	$\underline{\mathrm{Sim}(1^\lambda, i, \mathsf{cnt})}$
$(n, m, q, A, B) \leftarrow \phi$	$X_i \leftarrow\!\!\$\ \{0,1\}^\lambda$; $V_i \leftarrow 0^{\mathsf{H.kl}(\lambda)}$
For $g \leftarrow n+1$ to $n+q$, $\mathsf{a} \leftarrow 0$ to 1, $\mathsf{b} \leftarrow 0$ to 1 do	If $\mathsf{cnt} < n$ then $V_i \leftarrow\!\!\$\ \{0,1\}^{\mathsf{H.kl}(\lambda)}$
$\quad T[g, \mathsf{a}, \mathsf{b}] \leftarrow\!\!\$\ \{0,1\}^\lambda$	Else $V_i \leftarrow V_1 \oplus \cdots \oplus V_n \oplus hk$
$hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$, $F \leftarrow (n, m, q, A, B, T)$; Return F	Return (X_i, V_i)

Figure 25: **Constructed simulator for part (2) of Theorem 6.4.**

Hence

$$\begin{aligned}
\mathsf{Adv}^{\mathsf{reset}}_{R,S}(\lambda) &= \Pr[\,\mathrm{Reset}^P_S(\lambda) \,|\, c = 1\,] + \Pr[\,\mathrm{Reset}^P_S(\lambda) \,|\, c = 0\,] - 1 \\
&= \Pr[\mathrm{G}_1^{\mathcal{A},R}(\lambda)] - \Pr[\mathrm{G}_4^{\mathcal{A},R}(\lambda)] \\
&= \Pr[\mathrm{G}_2^{\mathcal{A},R}(\lambda)] - \Pr[\mathrm{G}_3^{\mathcal{A},R}(\lambda)] \leq \frac{2p(\lambda)}{2^\lambda}
\end{aligned}$$

for every $\lambda \in \mathbb{N}$. ▌

For part (2), let Sim be the simulator constructed in Fig. 25. We'll construct a reset-secure source S and a distinguisher D such that

$$\mathsf{Adv}^{\mathrm{obv2},\,\Phi_{\mathrm{topo}},\,\mathrm{Sim}}_{\mathsf{GaAO[H]},\mathcal{A}}(\cdot) \leq \mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\cdot)\ . \tag{22}$$

The theorem then follows from the assumption that $\mathsf{H} \in \mathsf{UCE2}$. The constructions of S and D are shown below.

$\underline{S^{\textsc{Hash}}(1^\lambda)}$	$\underline{D(1^\lambda, hk, L)}$
$r \leftarrow\!\!\$\ \{0,1\}^{\rho(\lambda)}$; $\mathsf{cnt} \leftarrow 0$; $L \leftarrow \perp$	$(F, r, X, V) \leftarrow L$; $(V_1, \ldots, V_n) \leftarrow V$
$\mathcal{A}^{\textsc{Garble},\textsc{Input}}(1^\lambda; r)$; Return L	$(X_1, \ldots, X_n) \leftarrow X$
	$b' \leftarrow \mathcal{A}^{\textsc{Garble},\textsc{Input}}(1^\lambda; r)$; Return b'
$\underline{\textsc{Garble}(f)}$	
$(F, (X_1^0, X_1^1, \ldots, X_n^0, X_n^1), d) \leftarrow\!\!\$\ \mathsf{Ga}[\textsc{Hash}].\mathsf{Gb}(1^\lambda, f)$	$\underline{\textsc{Garble}(f)}$
Return F	Return F
$\underline{\textsc{Input}(i, c)}$	$\underline{\textsc{Input}(i, c)}$
$\mathsf{cnt} \leftarrow \mathsf{cnt} + 1$; $X_i \leftarrow X_i^c$; $V_i \leftarrow\!\!\$\ \{0,1\}^{\mathsf{H.kl}(\lambda)}$	If $V_i = \perp$ then $V_i \leftarrow 0^{\mathsf{H.kl}(\lambda)}$; $V_i \leftarrow V_1 \oplus \cdots \oplus V_n \oplus hk$
If $\mathsf{cnt} < n$ then return (X_i, V_i)	Return (X_i, V_i)
$V_i \leftarrow \perp$; $X \leftarrow (X_1, \ldots, X_n)$; $V \leftarrow (V_1, \ldots, V_n)$	
$L \leftarrow (F, r, X, V)$; Return $\perp$	

Let a be the challenge bit of game $\mathrm{UCE}^{S,D}_{\mathsf{H}}(\lambda)$, and b be the challenge bit of game $\mathrm{Obv2}^{\mathcal{A}}_{\mathsf{GaAO[H]},\Phi_{\mathrm{topo}},\mathrm{Sim}}(\lambda)$. Then

$$\begin{aligned}
\Pr[\,\mathrm{Obv2}^{\mathcal{A}}_{\mathsf{GaAO[H]},\Phi_{\mathrm{topo}},\mathrm{Sim}}(\cdot) \,|\, b = 1\,] &= \Pr[\,\mathrm{UCE}^{S,D}_{\mathsf{H}}(\cdot) \,|\, a = 1\,], \text{ and} \\
\Pr[\,\mathrm{Obv2}^{\mathcal{A}}_{\mathsf{GaAO[H]},\Phi_{\mathrm{topo}},\mathrm{Sim}}(\cdot) \,|\, b = 0\,] &= \Pr[\,\mathrm{UCE}^{S,D}_{\mathsf{H}}(\cdot) \,|\, a = 0\,]\ .
\end{aligned}$$

Summing up yields Equation (22). What's left is to show that S is reset-secure. Indeed, whatever this source leaks is also leaked by the source in part (1). Hence from Lemma A.1, for any PT adversary R and any polynomial p that bounds the number of R's queries to Hash, it follows that $\mathsf{Adv}^{\mathsf{reset}}_{S,R}(\lambda) \leq 2p(\lambda)/2^\lambda$ for every $\lambda \in \mathbb{N}$.

For part (3), let Sim be the simulator constructed in Fig. 26. We'll construct a reset-secure source S and a distinguisher D such that

$$\mathsf{Adv}^{\mathrm{prv2},\,\Phi_{\mathrm{topo}},\,\mathrm{Sim}}_{\mathsf{GaP[H]},\mathcal{A}}(\cdot) \leq \mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\cdot)\ . \tag{23}$$

The theorem then follows from the assumption that $\mathsf{H} \in \mathsf{UCE2}$. The constructions of S and D are shown below.

$\underline{\mathrm{Sim}(1^\lambda, \phi, 0)}$
$(n, m, q, A, B) \leftarrow \phi$
For $g \leftarrow n+1$ to $n+q$, $\mathsf{a} \leftarrow 0$ to 1, $\mathsf{b} \leftarrow 0$ to 1 do
  $T[g, \mathsf{a}, \mathsf{b}] \leftarrow\!\!\$\ \{0,1\}^\lambda$
$hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$, $F \leftarrow (n, m, q, A, B, T)$
Return (F, ε)

$\underline{\mathrm{Sim}(1^\lambda, i, \tau, \mathsf{cnt})}$
$X_i \leftarrow\!\!\$\ \{0,1\}^\lambda$; $K \leftarrow (0^m, hk)$; $V_i \leftarrow 0^{|K|}$
If $\mathsf{cnt} < n$ then $V_i \leftarrow\!\!\$\ \{0,1\}^{|K|}$
Else
  $X \leftarrow (X_1, \ldots, X_n)$
  $Y \leftarrow \mathsf{Ga}[\mathsf{H.Ev}(1^\lambda, hk, \cdot, 1^\lambda)].\mathsf{Ev}(F, X)$
  $(Y_1, \ldots, Y_m) \leftarrow Y$; $y_1 \cdots y_m \leftarrow \tau$
  For $j \leftarrow 1$ to m do $u_j \leftarrow y_j \oplus \mathrm{lsb}(Y_j)$
  $U \leftarrow u_1 \cdots u_m$; $K \leftarrow (U, hk)$
  $V_i \leftarrow V_1 \oplus \cdots \oplus V_n \oplus K$
Return (X_i, V_i)

Figure 26: **Constructed simulator for part (3) of Theorem 6.4.**

$\underline{S^{\mathrm{Hash}}(1^\lambda)}$
$r \leftarrow\!\!\$\ \{0,1\}^{\rho(\lambda)}$; $\mathsf{cnt} \leftarrow 0$; $L \leftarrow \perp$; $K \leftarrow (0^m, 0^{\mathsf{H.kl}(\lambda)})$
$\mathcal{A}^{\mathrm{Garble},\mathrm{Input}}(1^\lambda; r)$; Return L

$\underline{\mathrm{Garble}(f)}$
$(F, (X_1^0, X_1^1, \ldots, X_n^0, X_n^1), d) \leftarrow\!\!\$\ \mathsf{Ga}[\mathrm{Hash}].\mathsf{Gb}(1^\lambda, f)$
Return (F, ε)

$\underline{\mathrm{Input}(i, c)}$
$\mathsf{cnt} \leftarrow \mathsf{cnt} + 1$; $X_i = X_i^c$; $V_i \leftarrow\!\!\$\ \{0,1\}^{|K|}$
If $\mathsf{cnt} < n$ then return (X_i, V_i)
$V_i \leftarrow \perp$; $X = (X_1, \ldots, X_n)$; $V = (V_1, \ldots, V_n)$
$L \leftarrow (F, r, X, V)$; Return $\perp$

$\underline{D(1^\lambda, hk, L)}$
$(F, r, X, V) \leftarrow L$; $(V_1, \ldots, V_n) \leftarrow V$
$(X_1, \ldots, X_n) \leftarrow X$
$b' \leftarrow \mathcal{A}^{\mathrm{Garble},\mathrm{Input}}(1^\lambda; r)$; Return b'

$\underline{\mathrm{Garble}(f)}$
Return (F, ε)

$\underline{\mathrm{Input}(i, c)}$
$x_i \leftarrow c$
If $V_i = \perp$ then
  $x \leftarrow x_1 \cdots x_n$; $y_1 \cdots y_m \leftarrow y \leftarrow \mathsf{cev}(f, x)$
  $Y \leftarrow \mathsf{Ga}[\mathsf{H.Ev}(1^\lambda, hk, \cdot, \cdot)].\mathsf{Ev}(F, X)$
  $(Y_1, \ldots, Y_m) \leftarrow Y$
  For $j = 1$ to m do $u_j \leftarrow \mathrm{lsb}(Y_j) \oplus y_j$
  $U \leftarrow u_1 \cdots u_m$
  $V_i \leftarrow V_1 \oplus \cdots V_{i-1} \oplus V_i \oplus \cdots \oplus V_n \oplus (U, hk)$
Return (X_i, V_i)

Let a be the challenge bit of game $\mathrm{UCE}_{\mathsf{H}}^{S,D}(\lambda)$, and b be the challenge bit of game $\mathrm{Prv2}^{\mathcal{A}}_{\mathsf{GaAO[H]},\Phi_{\mathrm{topo}},\mathrm{Sim}}(\lambda)$. Then

$$\begin{aligned}
\Pr[\,\mathrm{Prv2}^{\mathcal{A}}_{\mathsf{GaAO[H]},\Phi_{\mathrm{topo}},\mathrm{Sim}}(\cdot) \,|\, b = 1\,] &= \Pr[\,\mathrm{UCE}_{\mathsf{H}}^{S,D}(\cdot) \,|\, a = 1\,], \text{ and} \\
\Pr[\,\mathrm{Prv2}^{\mathcal{A}}_{\mathsf{GaAO[H]},\Phi_{\mathrm{topo}},\mathrm{Sim}}(\cdot) \,|\, b = 0\,] &= \Pr[\,\mathrm{UCE}_{\mathsf{H}}^{S,D}(\cdot) \,|\, a = 0\,]\ .
\end{aligned}$$

Summing up yields Equation (23). What's left is to show that S is reset-secure. Indeed, whatever this source leaks is also leaked by the source in part (1). Hence from Lemma A.1, for any PT adversary R and any polynomial p that bounds the number of R's queries to Hash, it follows that $\mathsf{Adv}^{\mathsf{reset}}_{S,R}(\lambda) \le 2p(\lambda)/2^\lambda$ for every $\lambda \in \mathbb{N}$.