A note on high-security general-purpose elliptic curves

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Abstract. In this note we describe some general-purpose, high-efficiency elliptic curves tailored for security levels beyond 2^{128}. For completeness, we also include legacy-level curves at standard security levels. The choice of curves was made to facilitate state-of-the-art implementation techniques.

1 Introduction

General-purpose elliptic curves are necessary to attain high-efficiency implementations of the most common cryptographic protocols like asymmetric encryption and plain digital signatures (but setting aside less conventional application like identity-based encryption). The standard NIST curves [14], though fairly efficient overall, arguably no longer represent the state of the art in the area [4, 7].

More efficient general-purpose curves have been recently proposed to address this situation [3, 4, 8], but for the 2^{128} security level at most, which corresponds to the expected security level of the standard NIST curve P-256 (or its binary counterpart, B-283). This is the case of Curve25519 [3] and Curve1174 [4]. However, while there is reason to look for higher security curves [15], no similar curves seem to have been proposed in the literature for higher security levels[5], matching the presumed levels of the standard NIST curves P-384 and P-521, as well as their binary counterparts, B-409 and B-571.

In this note we address this need, adopting the same settings as Curve25519 and Curve1174, respectively. Specifically, we describe Elligator (1 and 2) curves

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[5] However, since the initial version of this note was made public the SafeCurves site [5] has appeared with an extensive treatment of the subject and independent verification of most of the curves listed in this document.

with target security levels roughly matching the presumed levels of all standard NIST curves.

2 Curve choice

The curves Curve25519 and Curve1174 have been engineered to facilitate simple, efficient and secure implementation of general-purpose elliptic curve cryptosystems, with impressive results [8]. On these grounds, it makes sense to look for similar curves at higher security levels. At the same time, one can take the opportunity to provide curves matching the expected security levels of other standard NIST curves, the most prominent case being P-224 (and its binary counterpart, B-233), corresponding approximately to the security level of 3DES [12].

Curve25519 [3] is an Elligator type 2 curve with the following properties (among others):

- It is a Montgomery curve [13] over a large prime field $\mathbb{F}_p$;
- The prime p has the form $p = 2^m - \delta$ where $0 < \delta < m = \lceil \lg(p) \rceil$;
- The prime p satisfies $p \equiv 5 \pmod 8$, hence square root computation in $\mathbb{F}_p$ can be done with the Atkin method [2];
- The value $\xi = 2$ is a quadratic non-residue in $\mathbb{F}_p$, and hence can used to define a non-trivial quadratic twist of an elliptic curve over $\mathbb{F}_p$;
- The curve equation is $E : y^2 = x^3 + Ax^2 + x$ and the twist equation is $E' : v^2 = u^3 + 2Au^2 + 4u$, where $|(A-2)/4| > 0$ is as small as possible (so as to improve arithmetic performance), and $A^2 - 4$ is not a square (so that the curve has a unique point of order 2 and the Montgomery ladder yields a complete addition law).
- The curve order has the form $n = 8r$ where $r > 2^{m-3}$ is prime;
- The order of the non-trivial quadratic twist of the curve has the form $n' = 4r'$ where $r' > 2^{m-3}$ is prime, with $|r'| = |r| + 1$;

Curve1174 [4] is an Elligator type 1 curve with the following properties (among others):

- It is an Edwards curve [6, 10] over a large prime field $\mathbb{F}_p$;
- The prime p has the form $p = 2^m - \delta$ where $0 < \delta < m = \lceil \lg(p) \rceil$;
- The prime p satisfies $p \equiv 3 \pmod 4$, hence square root computation in $\mathbb{F}_p$ can be done with the Cippolla-Lehmer method [11];
- The curve equation is $E : x^2 + y^2 = 1 + dx^2y^2$ and the equation of a non-trivial quadratic twist of E is $E' : u^2 + v^2 = 1 + (1/d)u^2v^2$, where $|d| > 1$ is as small as possible (so as to improve arithmetic performance);
- The curve order has the form $n = 4r$ where $r > 2^{m-3}$ is prime;
- The order of the non-trivial quadratic twist of the curve has the form $n' = 4r'$ where $r' > 2^{m-3}$ is prime, with $|r'| = |r|$;

Besides, for the most part the bit sizes m were chosen to be the largest values strictly smaller than the commonplace sizes $\{224, 256, 384, 512\}$ such that a prime $p = 2^m - \delta$ as required above exists (except for the previously existing Curve1174 which was defined with a slightly smaller bit size, namely, $m = 251$ rather than $m = 254$).

3 The curves

We now list curves for several security levels, up to the level roughly comparable to the presumed security level of the NIST curves P-521 and B-571. The primes have the general form $p = 2^m - \delta$ for δ as small as possible. While it would be desirable that $\delta < 32$ (see [4]), this is not always possible. Yet, insisting that $\delta < \lg p$ increases the likeliness that any attack advantage this setting might cause is negligible (exponentially small).

The naming convention we adopt follows the pattern "M-m" for the Montgomery curve uniquely defined by the conditions in Section 2 over an m-bit prime field, and "E-m" for the similarly unique Edwards curve. This is reminiscent of the NIST naming convention for curves. The literature [4, 7] suggests two other naming conventions, namely, "Curve$m\delta$" for the uniquely defined Montgomery curve over $\mathbb{F}_{2^m-\delta}$ (where δ is, in turn, uniquely defined by m), and "Curve$|d|$" for an Edwards curve with coefficient d (in this case, the underlying field is not apparent in the name).

Table 1 contains curves in the Montgomery model, while Table 2 contains curves in the Edwards model. For completeness, we include Curve25519 and Curve1174 (which, by definition, satisfy the requirements in Section 2, but are labelled here as originally published). The prime group order is r. For the Edwards (Elligator 1) curves, the suggested base point of order r is given by its y coordinate while for Montgomery (Elligator 2) curves, is given by its x-coordinate. In each case, the given coordinate is as small as possible when viewed as a natural number. The 'ρ-sec' column indicates the security level as measured by the cost of a Pollard ρ attack.

Table 1. Montgomery curves

curve	p	A	ρ-sec	x	r
M-221	$2^{221} - 3$	117050	$2^{108.8}$	4	421249166674228746791672110734682167926895081980396304944335052891
Curve25519	$2^{255} - 19$	486662	$2^{125.8}$	9	723700557733226221397318656304299424085711635937990760600195093828545425098 9
M-383	$2^{383} - 187$	2065150	$2^{189.8}$	12	246262538727465495076744000625897586281748370440409041674693457404128898423468088300832718308361526678487001100744 7
M-511	$2^{511} - 187$	530438	$2^{253.8}$	5	837987995621412318723376562387865382967460363787024586107722590232610251879607410804876779383055508762141059258497448934987052508775626162460930737942299

Table 2. Edwards curves

curve	p	d	ρ-sec	y	r
E-222	$2^{222} - 117$	160102	$2^{109.8}$	28	168499666669691498 71666884429387\ 267355697374567600 58294185521417\ 407
Curve1174	$2^{251} - 9$	-1174	$2^{124.3}$	2	904625697166532776 74664832038037\ 428009233903527949 54740234892617\ 73642975601
E-382	$2^{382} - 105$	-67254	$2^{189.8}$	17	246262538727465495 07674400062589\ 758628174837044040 90416745738034\ 557663054564649171 26265932668324\ 4604346084081047321
E-521	$2^{521} - 1$	-376014	$2^{259.3}$	12	171619941503265242 87454751997703\ 483043173588250358 26352348615864\ 796385795849413675 47587665166365\ 784963669365906523 41426043192829\ 487025423179934212 93670108523

An implementation of all these curves is available as part of the RELIC library [1]. An independent implementation of E-521 by Mike Scott is available from `http://indigo.ie/~mscott/ed521.cpp` (or alternatively from `https://github.com/coruus/E521`).

See [5] for more properties of these and other similarly safe curves.

4 Conclusion

We have described general-purpose high-efficiency curves roughly matching the expected security of the standard NIST curves P-384 and P-521, and as a bonus, also curves roughly matching the expected security of the standard NIST curve P-224. All curves provided herein follow the Elligator (1 and 2) strategy, which is arguably the state of the art for the design of cryptographically-oriented elliptic curves.

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A Legacy-level curves

Under certain circumstances, typically on legacy systems, where one needs a lower but still reasonable security level, or must adhere to layout constraints of

existing applications, or simply cannot afford higher-security curves for lack of computational resources (as may be the case on certain very constrained platforms typical of the Internet of Things), curves matching the expected security level of (say) the NIST curves B-163 or P-192 may be useful.

Though the primary purpose of this note is to suggest some curves at high security levels, for completeness we list a few possible alternatives for those legacy-level curves on Tables 3 and 4. It is noteworthy to mention that one of the examples we provide here was inspired by the Telex system [16], where arithmetic is performed over the field $\mathbb{F}_p$ with $p = 2^{168} - 2^8 - 1$. Although this prime does not strictly satisfy the criterion $2^m - p < m$, it fails to do so only slightly, and it has the merit of enabling very efficient modular reduction, so we relax the criterion to $2^m - p < \max(m, 2^8 + 2)$ in that case.

Table 3. Legacy-level Montgomery curves

curve	p	A	ρ-sec	x	r
M-159	$2^{159} - 91$	197782	$2^{77.8}$	3	91343852333181432387730411159116\ 468190437625759
M-191	$2^{191} - 19$	−281742	$2^{93.8}$	11	39231885846166754773973683895833\ 39089424349757045953115 97

Table 4. Legacy-level Edwards curves

curve	p	d	ρ-sec	y	r
E-157	$2^{157} - 133$	−42000	$2^{77.3}$	20	45671926166590716193865246478592\ 509883108923719
E-168	$2^{168} - 257$	−715	$2^{82.8}$	2	93536104789177786765035835538253\ 283032607241942227
E-190	$2^{190} - 33$	−15584	$2^{93.8}$	9	39231885846166754773973683896145\ 32905756843180909199007 73

B Verifying the curves

The following Magma [9] script checks that the curves presented in this note do indeed satisfy the requirements in Section 2, except the condition that the coefficients A and d in the curve equations $E : y^2 = x^3 + Ax^2 + x$ and $E : x^2 + y^2 = 1 + dx^2y^2$ are as small as possible in absolute integer value. Extending the script so as to check this last condition is straightforward, but the processing time can be very long (several weeks for the highest security levels, if run sequentially). Independent verification has been kindly provided by Samuel Neves in SageMath, and by Mike Scott in C/C++ using the Miracl™ library.

```
procedure MontyGenBasept(m, A)
    p := 2^m;
    repeat
        p := PreviousPrime(p : Proof := false);
    until p mod 8 eq 5;
    F := GF(p);
    A := F!A;
    E := EllipticCurve([0, A, 0, 1, 0]);
    n := #E;
    r := n div 8;
    assert n mod 8 eq 0 and IsProbablePrime(r);

    // find generator:
    x := F!0;
    repeat
        repeat
            x +:= F!1;
        until IsSquare(x^3 + A*x^2 + x);
        G := E![x, Sqrt(x^3 + A*x^2 + x)];
    until not IsZero(2*G) and not IsZero(4*G) and not IsZero(8*G)
        and not IsZero(r*G) and not IsZero(2*r*G) and not IsZero(4*r*G)
        and IsZero(n*G);
    "Monty Generator:", G;

    // find base point:
    x := F!0;
    repeat
        repeat
            x +:= F!1;
        until IsSquare(x^3 + A*x^2 + x);
        P := E![x, Sqrt(x^3 + A*x^2 + x)];
    until not IsZero(8*P) and IsZero(r*P);
    "Monty Base Point:", P;
end procedure;

function MontyCurve(m, A)
    p := 2^m;
    repeat
        p := PreviousPrime(p : Proof := false);
    until p mod 8 eq 5;
    delta := 2^m - p;
    if delta gt m then
        return false;
    end if;
    F := GF(p);
```

```
sgnA := (A lt 0) select "- " * Sprint(-A) else "+ " * Sprint(A);
z := F!2;
assert not IsSquare(z);
if IsSquare(F!A - 2) or IsSquare(F!A^2 - 4) then
    return false;
end if;
// NB: now (A - 2)/(A + 2) is not a square either
// check curve y^2 = x^3 + A*x^2 + x:
ok, E := IsEllipticCurve([0, F!A, 0, 1, 0]);
if not ok then
    return false;
end if;
n := #E;
if (n mod 8 ne 0) or not IsProbablePrime(n div 8) then
    return false;
end if;
r := n div 8;
if r lt 2^(m - 3) then
    return false;
end if;
// check twist v^2 = u^3 + A*z*u^2 + z^2*u:
ok, Et := IsEllipticCurve([0, A*z, 0, z^2, 0]);
if not ok then
    return false;
end if;
nt := #Et;
if (nt mod 4 ne 0) or not IsProbablePrime(nt div 4) then
    return false;
end if;
rt := nt div 4;
if rt lt 2^(m - 3) then
    return false;
end if;
t := p + 1 - n;
if nt ne p + 1 + t then
    return false;
end if;
r := n div 8; // "|r| =", Round(Log(2, r));
sec := Log(2, Sqrt(Pi(RealField())*r/4));
"Good Elligator 2 curve: y^2 = x^3 " * sgnA * "*x^2 + x",
    "over GF(2^" * Sprint(m) * " - " * Sprint(delta) *")",
    "at sec level 2^" * Sprint(sec),
    "with r =", r;
assert IsProbablePrime(r);
return true;
```

```
end function;

procedure MontyTests()
    "================";
    m := 159; A := 197782;
    if MontyCurve(m, A) then
        MontyGenBasept(m, A);
    else
        "LOGIC ERROR!";
    end if;
    "================";
    m := 191; A := -281742;
    if MontyCurve(m, A) then
        MontyGenBasept(m, A);
    else
        "LOGIC ERROR!";
    end if;
    "================";
    m := 221; A := 117050;
    if MontyCurve(m, A) then
        MontyGenBasept(m, A);
    else
        "LOGIC ERROR!";
    end if;
    "================";
    m := 255; A := 486662;
    if MontyCurve(m, A) then
        MontyGenBasept(m, A);
    else
        "LOGIC ERROR!";
    end if;
    "================";
    m := 383; A := 2065150;
    if MontyCurve(m, A) then
        MontyGenBasept(m, A);
    else
        "LOGIC ERROR!";
    end if;
    "================";
    m := 511; A := 530438;
    if MontyCurve(m, A) then
        MontyGenBasept(m, A);
    else
        "LOGIC ERROR!";
    end if;
```

```
    "================";
end procedure;

MontyTests();

function EdAbs(F, d, y)
    ok, x := IsSquare((F!1 - y^2)/(F!1 - d*y^2));
    return (ok) select x else F!0;
end function;

function EdAdd(d, P1, P2)
    X1 := P1[1]; Y1 := P1[2]; Z1 := P1[3];
    X2 := P2[1]; Y2 := P2[2]; Z2 := P2[3];
    A := Z1*Z2; B := A^2; C := X1*X2; D := Y1*Y2;
    E := d*C*D; F := B - E; H := B + E;
    X3 := A*F*((X1 + Y1)*(X2 + Y2) - C - D); Y3 := A*H*(D - C); Z3 := F*H;
    return [X3, Y3, Z3];
end function;

function EdDbl(P1)
    X1 := P1[1]; Y1 := P1[2]; Z1 := P1[3];
    B := (X1 + Y1)^2; C := X1^2; D := Y1^2; E := C + D;
    H := Z1^2; J := E - 2*H;
    X2 := (B - E)*J; Y2 := E*(C - D); Z2 := E*J;
    return [X2, Y2, Z2];
end function;

function EdMul(F, d, k, P)
    V := [F!0, F!1, F!1];
    if k lt 0 then
        k := -k;
        P := [-P[1], P[2], P[3]];
    end if;
    while k ne 0 do
        if k mod 2 eq 1 then
            V := EdAdd(d, V, P);
        end if;
        P := EdDbl(P);
        k div:= 2;
    end while;
    iZV := V[3]^-1;
    return [V[1]*iZV, V[2]*iZV, F!1];
end function;

procedure EddieGenBasept(m, d)
```

```
    p := 2^m;
    repeat
        p := PreviousPrime(p : Proof := false);
    until p mod 4 eq 3;
    F := GF(p);
    d := F!d;
    E := EllipticCurve([0, 2*d + 2, 0, (1 - d)^2, 0]);
    n := #E;
    r := n div 4;
    assert n mod 4 eq 0 and IsProbablePrime(r);
    O := [F!0, F!1, F!1];

    // find generator:
    y := F!0;
    repeat
        repeat
            y +:= F!1;
            x := EdAbs(F, d, y);
        until x ne F!0;
        G := [x, y, F!1];
    until EdMul(F, d, 2, G) ne O and EdMul(F, d, 4, G) ne O
        and EdMul(F, d, r, G) ne O and EdMul(F, d, 2*r, G) ne O;
    assert EdMul(F, d, 4*r, G) eq O;
    assert x^2 + y^2 eq 1 + d*x^2*y^2;
    "Eddie Generator: (", x, ",", y, ")";

    // find base point:
    y := F!0;
    repeat
        repeat
            y +:= F!1;
            x := EdAbs(F, d, y);
        until x ne F!0;
        P := [x, y, F!1];
    until EdMul(F, d, 4, P) ne O and EdMul(F, d, r, P) eq O;
    assert x^2 + y^2 eq 1 + d*x^2*y^2;
    "Eddie Base Point: (", x, ",", y, ")";
end procedure;

function EddieCurve(m, d)
    p := 2^m;
    repeat
        p := PreviousPrime(p);
    until p mod 4 eq 3;
    delta := 2^m - p;
```

```
m0 := (m ne 168) select m else Max(m, 2^8 + 2); // see discussion on Telex
if delta gt m0 then
    return false;
end if;
F := GF(p);
curveEq := "x^2 + y^2 = 1 " * ((d lt 0) select "- "*Sprint(-d) else "+ "*Sprint(d)) * "
d := F!d;
if IsSquare(d) /* not complete Edwards */ then
    return false;
end if;
if IsSquare(1 - d) /* trivial quad twist */ then
    return false;
end if;
assert IsSquare(-d);
u := Sqrt(-d);
if not IsSquare(2*(u - 1)/(u + 1)) /* injective map undefined */ then
    return false;
end if;
// check curve x^2 + y^2 = 1 + dx^2y^2,
// or equivalently y^2 = x^3 + (2d + 2) x^2 + (1 - d)^2 x:
ok, E := IsEllipticCurve([0, (2*d + 2), 0, (1 - d)^2, 0]);
if not ok then
    return false;
end if;
n := SEA(E : AbortLevel := 2, MaxSmooth := 4);
if (n mod 4 ne 0) or not IsProbablePrime(n div 4) then
    return false;
end if;
r := n div 4;
if r lt 2^(m - 3) then
    return false;
end if;
// check twist x^2 + y^2 = 1 + (1/d)x^2y^2,
// or equivalently v^2 = u^3 + 2(1 + d)/(1 - d)u^2 + u:
ok, Et := IsEllipticCurve([0, 2*(1 + d)/(1 - d), 0, 1, 0]);
if not ok then
    return false;
end if;
nt := #Et;
if (nt mod 4 ne 0) or not IsProbablePrime(nt div 4) then
    return false;
end if;
rt := nt div 4;
if rt lt 2^(m - 3) then
    return false;
```

```
    end if;
    t := p + 1 - n;
    if nt ne p + 1 + t then
        return false;
    end if;
    r := n div 4; // "|r| =", Round(Log(2, r));
    sec := Log(2, Sqrt(Pi(RealField())*r/4));
    "Good Elligator 1 curve: " * curveEq,
        "over GF(2^" * Sprint(m) * " - "* Sprint(delta) * ")",
        "at sec level 2^" * Sprint(sec),
        "with r =", r;
    assert IsProbablePrime(r);
    return true;
end function;

procedure EddieTests()
    "================";
    m := 157; d := -42000;
    if EddieCurve(m, d) then
        EddieGenBasept(m, d);
    else
        "LOGIC ERROR!";
    end if;
    "================";
    m := 168; d := -715;
    if EddieCurve(m, d) then
        EddieGenBasept(m, d);
    else
        "LOGIC ERROR!";
    end if;
    "================";
    m := 190; d := -15584;
    if EddieCurve(m, d) then
        EddieGenBasept(m, d);
    else
        "LOGIC ERROR!";
    end if;
    "================";
    m := 222; d := 160102;
    if EddieCurve(m, d) then
        EddieGenBasept(m, d);
    else
        "LOGIC ERROR!";
    end if;
    "================";
```

```
    m := 251; d := -1174;
    if EddieCurve(m, d) then
        EddieGenBasept(m, d);
    else
        "LOGIC ERROR!";
    end if;
    "================";
    m := 382; d := -67254;
    if EddieCurve(m, d) then
        EddieGenBasept(m, d);
    else
        "LOGIC ERROR!";
    end if;
    "================";
    m := 521; d := -376014;
    if EddieCurve(m, d) then
        EddieGenBasept(m, d);
    else
        "LOGIC ERROR!";
    end if;
    "================";
end procedure;

EddieTests();
```