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Cryptography from Compression Functions: The UCE Bridge to the ROM

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Abstract

This paper suggests and explores the use of UCE security for the task of turning VIL-ROM schemes into FIL-ROM ones. The benefits we offer over indifferentiability, the current leading method for this task, are the ability to handle multi-stage games and greater efficiency. The paradigm consists of (1) Showing that a VIL UCE function can instantiate the VIL RO in the scheme, and (2) Constructing the VIL UCE function given a FIL random oracle. The main technical contributions of the paper are domain extension transforms that implement the second step. Leveraging known results for the first step we automatically obtain FIL-ROM constructions for several primitives whose security notions are underlain by multi-stage games. Our first domain extender exploits indifferentiability, showing that although the latter does not work directly for multi-stage games it can be used indirectly, through UCE, as a tool for this end. Our second domain extender targets performance. It is parallelizable and shown through implementation to provide significant performance gains over indifferentiable domain extenders.

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1 Introduction

Two forms of the random oracle model (ROM) of BR [9] have emerged, namely the VIL-ROM and FIL-ROM. In the VIL-ROM, the random oracle, denoted RO, is variable input length (VIL), meaning takes inputs of arbitrary length. In the FIL-ROM, the random oracle, denoted ro, is fixed input length (FIL), meaning only takes inputs of one, particular length. The VIL-ROM is preferable for the design and analysis of ROM schemes and reflects the original view of BR [9] that random oracles would be instantiated by cryptographic hash functions that, like SHA-256, take variable length inputs. However hash functions are built in a very structured way from their underlying compression functions. This lead researchers beginning with Coron, Dodis, Malinaud and Puniya [15] to suggest that it should be the compression function, rather than the hash function, that is treated as "ideal," leading to the FIL-ROM. Indeed, SHA-256 is built from its compression function sha-256 in a way that renders SHA-256 subject to the extension attack, which can lead to attacks when SHA-256 is used to instantiate a VIL random oracle. Treating the compression function (rather than the full hash function) as the ideal object is more reflective of the design goals and intuition of practitioners and leads to better security.

The consensus then is that we should design schemes in the FIL-ROM. The question is how best to do this. One option is to directly design and analyze schemes in this model, but this is difficult and ad hoc. A better option is to provide a construction E^{ro} of a VIL function that can substitute a VIL RO, meaning we would design schemes secure in the VIL-ROM as usual and then automatically replace RO with E^{ro} to obtain security in the FIL-ROM. We refer to such an E as a domain extension construction or domain extender.

For this to work in some broad and useful way, we need a *definition* of some property, call it X, that, if satisfied by E^{ro}, allows the latter to securely replace RO in the VIL-ROM and thus provide security in the FIL-ROM, for some useful and hopefully large set of schemes that are proven secure in the VIL-ROM. The leading proposal for X is "indifferentiability from a random oracle" as defined by Maurer, Renner and Holenstein (MRH) [21] and advocated by [15].

This paper suggests, and explores, an alternative X. We suggest that X be the notion of UCE (Universal Computational Extractor) security defined by BHK [6]. Our results will show both theoretical and practical benefits of X=UCE over X=indifferentiability in this role. On the theoretical side, UCE allows us to move from the VIL-ROM to the FIL-ROM for primitives whose security is defined via multi-stage games, a setting where indifferentiability fails [25]. On the practical side, we exhibit UCE domain extenders E that are significantly more efficient than known indifferentiability ones, in particular parallelizable to take advantage of modern multi-core machines, our efficiency claims being not just asymptotic but supported by implementations and experiments. Conceived as a way to remove random oracles, UCE now becomes a bridge to better security in the ROM.

Limitations of indifferentiability. While indifferentiability works well in some settings [21], it has two major limitations. The first is that indifferentiable-from-random functions do not suffice to securely replace a VIL random oracle for primitives whose security definition is underlain by multi-stage games [25]. This gap is more than academic, for we are seeing the emergence of numerous primitives and security notions of practical importance whose definitions are inherently multi-stage. Examples, all but the last listed by RSS [25], include Deterministic PKE (D-PKE) [4], Hedged PKE [5], encryption secure against key-dependent messages [12], primitives secure against related-key attack [8], proofs of storage [25] and Message-Locked Encryption (MLE) [7]. In each case there are natural, efficient and canonical solutions in the VIL-ROM that we would like to implement in the FIL-ROM, but indifferentiability offers no way to do this. The second limitation of indifferentiability is performance. Typical indifferentiable domain extenders iterate the compression function sequentially. This means that instantiations are left unable to take advantage of modern multi-core processors to provide performance gains. This reduces the potential for high volume usage and deployment of cryptography based on compression functions.

Our perspective. We conceptualize the goal that motivated the use of indifferentiability as aiming to design an X-secure domain extender —this being a construction E^{ro} that, given the FIL random oracle ro,

Method	Notions	Performance	Applications
Keyed-Indiff	$\mathsf{UCE}[\mathcal{S}^{\mathrm{srs}}]$ $\mathsf{UCE}[\mathcal{S}^{\mathrm{crs}}]$	About $m/(m-n)$ times the speed of M	All schemes in [6]
AU-then-Hash	$\mathsf{UCE}[\mathcal{S}^{\mathrm{sup}}]$	Parallelizable ~ 0.4 cycles per byte	MLE, key derivation, storage auditing

Figure 1: **Our UCE domain extension constructions and their properties.** The second column gives the UCE notion that is achieved. M is the indifferentiable domain extender used in the first construction. The numbers n and m are the key length of the hash function and the input length of the ideal compression function, respectively. Typically, $n = 128$ and $m = 512$.

computes a VIL, X-secure function— for a "good" choice of X, meaning one that allows E^{ro} to securely replace RO in the VIL-ROM for some significant set of applications. The composition theorem of [21] shows that X=indifferentiability is able to do this for single-stage games, which is certainly important. However, as discussed above, indifferentiability also has important limitations. We ask if there are alternative definitions X that can overcome these limitations and complement indifferentiability in its role.

The core limitation of indifferentiability is the inability to handle multi-stage games. We suggest that a natural route around this is that X-*security itself be multi-stage*. The particular candidate X we suggest is the UCE notion of [6], which is indeed multi-stage. Our suggested UCE-based paradigm to move schemes from the VIL-ROM to the FIL-ROM has two steps: (1) Show that instantiating the VIL random oracle in the scheme with a VIL UCE function preserves security, and (2) Implement the VIL UCE function as E^{ro} to obtain a FIL-ROM scheme. Prior work has already given us the first step for many constructions: UCE-secure hash functions are shown in [6] to be able to securely instantiate VIL random oracles for diverse multi-stage applications including the important practical ones noted above and all examples of multi-stage schemes listed in [25]. The missing element is UCE domain extenders E for the second step. If we had those, we could immediately harvest the existing results to get FIL-ROM constructions for many multi-stage primitives. The concrete quest that emerges, then, is for UCE domain extenders.

Our results. Our core contribution is two domain extenders for UCE that together allow us to reach the above goals of security and speed. These are constructions E that take a FIL random oracle ro and return a VIL, keyed function E^{ro} that meets UCE security notions of BHK in the FIL-ROM.[1] See Fig. 1 for a summary of the two domain extenders and their properties.

Our first construction is generic, turning any indifferentiable domain extender into a UCE domain extender. Given an indifferentiable domain extender M, we show (Theorem 4.2) that the hash family H defined for key hk and input x by $\mathsf{H}_{hk}(x) = \mathsf{M}^{\mathsf{ro}(hk\,\|\,\cdot)}(x)$, is UCE secure. The forms of UCE achieved by H are what BHK call $\mathsf{UCE}[\mathcal{S}^{\mathrm{crs}}]$-security and $\mathsf{UCE}[\mathcal{S}^{\mathrm{srs}}]$-security, namely UCE security for computationally or statistically reset-secure sources. This is enough to get FIL-ROM instantiations for all applications obtained in [6] and discussed above, including the storage-auditing scheme used in [25] as a counterexample for the failure of the indifferentiability framework in multi-stage settings, encryption secure for key-dependent messages, encryption secure against related-key attack, MLE, D-PKE and more.

This construction illustrates what we believe is an interesting relation between UCE and indifferentiability. Indifferentiability cannot *directly* yield the applications we have obtained for multi-stage primitives. However, it can be used, in a blackbox way, to create a domain extender that meets a *particular* multi-stage notion of security, namely UCE. Then, exploiting known UCE results, we can obtain FIL-ROM security for *many* multi-stage primitives. Thus our construction shows how to use UCE to leverage indifferentiability to solve a problem that indifferentiability could not solve directly.

While our first construction delivers, we believe, important advances on the theoretical front, its per-

[1] UCE hash functions are keyed, whence the introduction of a key in this setting. Also, UCE is not a monolithic or single security notion, but rather a framework in which one parameterizes notions of security by classes of "sources." Applications rely on different choices of the starting class. The framework is recalled in Section 3. Here we will avoid the details beyond noting for which classes each of our constructions is secure and what this entails for applications.

formance is that of the underlying indifferentiable construction. Our second construction targets speed. It follows the Carter-Wegman paradigm [14]. We show (Theorem 5.2) that if F is almost-universal, then the hash family H defined for key (hk, K) and input x by $\mathsf{H}_{hk}(x) = \mathsf{ro}(K \,\|\, \mathsf{F}_{fk}(x))$, is UCE secure. Here the form of UCE achieved is what BHK call $\mathsf{UCE}[\mathcal{S}^{\mathrm{sup}}]$-security. We can instantiate this to obtain highly efficiently, fully parallelizable hash functions. The most important application here is a FIL-ROM instantiation of the CE MLE scheme of [18, 7], leading to efficient systems for secure deduplicated storage.

General domain extension. Above we presented the domain extension problem for notion X as being to design E such that E^{ro} is a VIL X-secure function in the FIL-ROM. More generally, the problem is to design E such that if $\overline{\mathsf{H}}$ is a FIL X-secure function then $\mathsf{E}^{\overline{\mathsf{H}}}$ is a VIL X-secure function. Here $\overline{\mathsf{H}}$ can be a FIL-ROM function, and thus the prior formulation is a special case. Our domain extenders discussed above generalize to solve this problem. In the first case, we show (Theorem 4.1) that if $\overline{\mathsf{H}}$ is UCE secure then so is the family H defined for key hk and input x by $\mathsf{H}_{hk}(x) = \mathsf{M}^{\overline{\mathsf{H}}(hk\,\|\,\cdot)}(x)$, where M, as before, is an indifferentiable domain extender, the UCE classes for which this is true being $\mathsf{UCE}[\mathcal{S}^{\mathrm{crs}}]$ and $\mathsf{UCE}[\mathcal{S}^{\mathrm{srs}}]$ as before. Setting $\overline{\mathsf{H}}_{hk}(\cdot) = \mathsf{ro}(hk\|\cdot)$ recovers the result stated above as a special case. The generalization however yields something new, namely a *standard model* domain extender for UCE. This follows by letting $\overline{\mathsf{H}}$ be a standard model FIL UCE function. This is interesting because it shows that indifferentiability, which so far has been a ROM notion and tool, can be leveraged to get results purely in the standard model, and allows us to relax UCE assumptions in the standard model, from being on VIL functions to being on FIL functions. Our second result likewise generalizes to show (Theorem 5.1) that if F is almost-universal and $\overline{\mathsf{H}}$ is UCE secure then the hash family H defined for key (hk, K) and input x by $\mathsf{H}_{hk}(x) = \overline{\mathsf{H}}(K \,\|\, \mathsf{F}_{fk}(x))$ is also UCE secure, the UCE class for which this is true being $\mathsf{UCE}[\mathcal{S}^{\mathrm{sup}}]$ as before.

Instantiation and experimental results. We give a very fast instantiation of F for our second construction discussed above, based on reduced-round AES and polynomial evaluation. Our construction makes use of the fact that four-round AES, with the four subkeys chosen uniformly and independently, is an almost-xor-universal hash function [20]. We stress that our universal hashing construction is unconditional, making no assumption on AES. This leads to a highly efficient, parallelizable UCE-secure hash $\mathsf{FastHash}$. Our experiments show that even in the sequential setting, $\mathsf{FastHash}$ is about 5.3 times faster than SHA-256. When parallelism is employed, $\mathsf{FastHash}$ achieves a much better speedup, about 24 times faster than SHA-256. Finally, we demonstrate the utility of $\mathsf{FastHash}$ by giving an extremely fast MLE scheme based on it.

Related work and discussion. Mittelbach [23] defines restrictions on a multi-stage game so that the indifferentiability composition theorem of MRH still holds for a subclass of indifferentiable domain extenders called iterative domain extenders, and is thereby able to show that the latter suffice for applications like D-PKE and MLE. He also shows that if M is an iterative domain extender then M^{ro} is UCE-secure. In comparison, our first construction is more general: It is able to use any indifferentiable domain extender, and as a result our applications are able to use a broader class of domain extenders; it turns any FIL UCE function into a VIL one; and it works both in the standard model and the ROM. On the other hand, Mittelbach's construction is about $m/(m-n)$ times faster than our first construction, where m is the input length of the compression function, and n is the key length.

Dodis, Ristenpart, and Shrimpton [17] define preimage-awareness (PrA) as a strengthening of collision resistance and show that the plain Merkle-Damgård is a PrA extender. PrA can also be used in multi-stage games: Ristenpart, Shacham, and Shrimpton [25] show how to compose a PrA-secure hash with a FIL RO to achieve D-PKE.

Demay, Gazi, Hirt and Maurer [16] introduce a more fine-grained indifferentiability framework in which simulator resources are parameterized, and explain the failure of indifferentiability for multi-stage games, discovered by [25], in terms of simulator memory. They also present some more general negative results for indifferentiable domain extension.

The end products of our above-outlined two-stage UCE-based paradigm is FIL-ROM schemes for a variety of primitives, based typically on standard assumptions. For example our FIL-ROM MLE scheme

makes no assumptions and our FIL-ROM D-PKE scheme assumes only a standard IND-CPA PKE scheme. In particular, while the UCE notion is used to obtain these results, it does not show up in the end product as an assumption. One may go a step further and ask about instantiating the FIL random oracle in these schemes to obtain standard-model schemes. Our results imply that UCE security of the instantiating functions suffices. This is another benefit over the use of indifferentiability, where there is no well-defined property of a function that allows it to securely instantiate the random oracle in the final FIL-ROM scheme. One has to be careful here that some forms of UCE are not achievable in the standard model if indistinguishability obfuscation for all circuits exists [13], but most of the applications use other forms of UCE, and, even for the few that do not, the results of [13] do not rule out the possibility of *some* (non-UCE) secure instantiation of the FIL random oracle.

We clarify that UCE complements, rather than replaces, indifferentiability. Indifferentiability, unlike UCE, can handle all single stage games [21]. UCE can handle some (but not all) of these, and when it can may be preferable due to the performance gain. On the other hand UCE can handle a swathe of popular multi-stage games, which indifferentiability cannot. Overall the task of moving VIL-ROM schemes to FIL-ROM ones requires and benefits from the availability of multiple tools, currently available ones including indifferentiability [21], restricted indifferentiability [23] and, now, UCE.

2 Preliminaries

Concrete security bounds are important for applications. However, notions in the current domain, involving simulators and multiple conditions and adversaries, are complex. The result is that when theorems are stated purely concretely, it is hard to understand the (much more simple) conceptual import. We will try to achieve the "best of both worlds." We formulate definitions asymptotically. The first cut theorem statements are asymptotic so that one can quickly see the core implication and result. This is followed by a concrete statement with bounds.

NOTATION. By $\lambda \in \mathbb{N}$ we denote the security parameter. If $n \in \mathbb{N}$ then 1^n denotes its unary representation. We denote the size of a finite set X by $|X|$, the number of coordinates of a vector $\mathbf{x}$ by $|\mathbf{x}|$, and the length of a string $x \in \{0,1\}^*$ by $|x|$. We let ε denote the empty string. If x is a string then $x[i]$ is its i-th bit and $x[1,\ell] = x[1]\ldots x[\ell]$. By $x\|y$ we denote the concatenation of strings x, y. If X is a finite set, we let $x \leftarrow\!\!\$\ X$ denote picking an element of X uniformly at random and assigning it to x. Algorithms may be randomized unless otherwise indicated. Running time is worst case. "PT" stands for "polynomial-time," whether for randomized algorithms or deterministic ones. If A is an algorithm, we let $y \leftarrow A(x_1,\ldots;r)$ denote running A with random coins r on inputs $x_1,\ldots$ and assigning the output to y. We let $y \leftarrow\!\!\$\ A(x_1,\ldots)$ be the resulting of picking r at random and letting $y \leftarrow A(x_1,\ldots;r)$. We let $[A(x_1,\ldots)]$ denote the set of all possible outputs of A when invoked with inputs $x_1,\ldots$. We say that $f : \mathbb{N} \to \mathbb{R}$ is negligible if for every positive polynomial p, there exists $n_p \in \mathbb{N}$ such that $f(n) < 1/p(n)$ for all $n > n_p$.

GAMES. We use the code based game playing framework of [10]. (See Fig. 3 for an example.) By $\mathrm{G}^{A_1,A_2,\cdots}(\lambda) \Rightarrow y$ we denote the event that the execution of game G with adversaries $A_1, A_2, \ldots$ and security parameter λ results in output y. We abbreviate $\mathrm{G}^{A_1,A_2,\cdots}(\lambda) \Rightarrow \mathsf{true}$ by $\mathrm{G}^{A_1,A_2,\cdots}(\lambda)$, the occurrence of this event meaning that $A_1, A_2, \ldots$ win the game.

For concrete security assessments, let the *number of queries* of A to an oracle Proc be the function $\mathbf{Q}_A^{\mathsf{Proc}}$ that on input λ returns the maximum number of queries that A makes to Proc when executed with security parameter λ, the maximum over all coins and all possible replies to queries to all oracles of A. Time assessments are simplified by the convention that running time is that of the game rather than merely the adversary, and we let $\mathbf{T}(\mathrm{G}^{A_1,A_2,\cdots})$ denote the function of λ that returns the maximum execution time of game G with adversaries $A_1, A_2, \ldots$ and security parameter λ, the maximum over all coins, and the time being all inclusive, meaning the time taken by game procedures to compute replies is included.

RANDOM ORACLES. A random oracle $\mathsf{RO} : U \to \{0,1\}^n$ is a procedure that maintains a table H, initially empty, and is defined by

$\underline{\mathsf{RO}(x)}$
If $H[x] \neq \perp$ then $H[x] \leftarrow\!\!\$\ \{0,1\}^n$; Return $H[x]$

We say that RO is variable-input length (VIL) if $U = \{0,1\}^*$ and fixed-input length (FIL) if there is $m \in \mathbb{N}$ such that $U = \{0,1\}^m$. Formally, any random oracle referred to in a game should appear explicitly in the game as a procedure defined as above, but for the same of brevity of game descriptions, we omit writing it explicitly, instead only indicating the domain and range of each random oracle. By convention, RO indicates a VIL random oracle, and ro a FIL random oracle.

3 UCE framework

The Universal Computational Extractor (UCE) framework of BHK [6] is intended to define security notions for families of hash functions in the standard model, but BHK also lift this to the ROM to show its achievability there. We use the latter with the random oracle being FIL. We note that the standard-model definition is the special case where parties and algorithms make no queries to the random oracle.

BHK first give a single-key version of the definition and then extend it to a multi-key one. We will work directly with the multi-key version, calling it UCE rather than mUCE as in [6].

Function families. Our syntax for function families follows [6], in particular allowing variable output lengths. A family of functions H specifies the following. On input the unary representation 1^λ of the security parameter $\lambda \in \mathbb{N}$, key generation algorithm $\mathsf{H.Kg}$ returns a key $hk \in \{0,1\}^{\mathsf{H.kl}(\lambda)}$, where $\mathsf{H.kl}\colon \mathbb{N} \to \mathbb{N}$ is the keylength function associated to H. The deterministic, PT evaluation algorithm $\mathsf{H.Ev}$ takes 1^λ, a key $hk \in [\mathsf{H.Kg}(1^\lambda)]$, an input $x \in \{0,1\}^*$ with $|x| \in \mathsf{H.IL}(\lambda)$, and a unary encoding 1^ℓ of an output length $\ell \in \mathsf{H.OL}(\lambda)$ to return $\mathsf{H.Ev}(1^\lambda, hk, x, 1^\ell) \in \{0,1\}^\ell$. Here $\mathsf{H.IL}$ is the input-length function associated to H, so that $\mathsf{H.IL}(\lambda) \subseteq \mathbb{N}$ is the set of allowed input lengths, and similarly $\mathsf{H.OL}$ is the output-length function associated to H, so that $\mathsf{H.OL}(\lambda) \subseteq \mathbb{N}$ is the set of allowed output lengths. The latter allows us to cover functions of variable output length. If H has fixed input length then let $\mathsf{H.il}$ denote the function such that $\mathsf{H.IL}(\lambda) = \{\mathsf{H.il}(\lambda)\}$ for every $\lambda \in \mathbb{N}$. If H has fixed output length, define $\mathsf{H.ol}$ likewise. In the ROM, we allow $\mathsf{H.Ev}$ access to a FIL random oracle denoted ro. We write $\mathsf{H.Ev}^{\mathsf{ro}}$ to indicate explicitly that $\mathsf{H.Ev}$ needs access to a FIL random oracle ro.

Framework. Let H be a family of functions. Let S be an adversary called the *source* and D an adversary called the *distinguisher*. We associate to them and H the game $\mathrm{UCE}^{S,D}_{\mathsf{H}}(\lambda)$ in the left panel of Fig. 2. Initially, the source specifies a unary-encoded integer $n \geq 1$ to indicate the number of hash keys that it wants to use. The game then chooses a secret vector $\mathbf{hk}$ of n uniformly random hash keys and grants the source access to an oracle Hash. We require that any query $(x, 1^\ell, i)$ made to this oracle satisfy $|x| \in \mathsf{H.IL}(\lambda)$, $\ell \in \mathsf{H.OL}(\lambda)$ and $i \in \{1, \ldots, n\}$. When the challenge bit b is 1 (the "real" case) the oracle responds via $\mathsf{H.Ev}$ under $\mathbf{hk}[i]$. When $b = 0$ (the "random" case) it responds via the ith random-oracle procedure. The source then leaks a string L to its accomplice distinguisher. The latter *does* get the keys $\mathbf{hk}$ as input and must now return its guess $b' \in \{0,1\}$ for b. The game returns true iff $b' = b$, and the uce-advantage of (S, D) is defined for $\lambda \in \mathbb{N}$ via

$$\mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\lambda) = 2\Pr[\mathrm{UCE}^{S,D}_{\mathsf{H}}(\lambda)] - 1 \ .$$

If $\mathcal{S}$ is a class (set) of sources, we say that H is $\mathsf{UCE}[\mathcal{S}]$-secure if $\mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\cdot)$ is negligible for all sources $S \in \mathcal{S}$ and all PT distinguishers D. Trivial attacks from [6] show that $\mathsf{UCE}[\mathcal{S}]$-security is not achievable if $\mathcal{S}$ is the class of all PT sources. To obtain meaningful notions of security, BHK [6] impose restrictions on the source. There are many ways to do this; below we'll focus on what they call unpredictable and reset-secure sources. To discuss the concrete security of constructions it will be useful to say that S is a N-key source if we always have $n \leq N(\lambda)$ when $(1^n, t) \leftarrow\!\!\$\ S(1^\lambda, \varepsilon)$.

Unpredictable sources. A source is unpredictable if it is hard to guess the source's Hash queries even given the leakage, in the *random case* of UCE game. Formally, let S be a source and P an adversary called a predictor. Consider game $\mathrm{Pred}^P_S(\lambda)$ in the middle panel of Fig. 2 associated to S, P. Given 1^n and the

$\underline{\text{Game UCE}^{S,D}_{\mathsf{H}}(\lambda)}$	$\underline{\text{Game Pred}^{P}_{S}(\lambda)}$	$\underline{\text{Game Reset}^{R}_{S}(\lambda)}$
$(1^n, t) \leftarrow\!\!\$\ S(1^\lambda, \varepsilon)$	$(1^n, t) \leftarrow\!\!\$\ S(1^\lambda, \varepsilon)$	$U \leftarrow \emptyset\ ;\ (1^n, t) \leftarrow\!\!\$\ S(1^\lambda, \varepsilon)$
For $i = 1, \ldots, n$ do $\mathbf{hk}[i] \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$	$Q \leftarrow \emptyset$	$L \leftarrow\!\!\$\ S^{\mathsf{Hash},\mathsf{ro}}(1^n, t)\ ;\ b \leftarrow\!\!\$\ \{0,1\}$
$b \leftarrow\!\!\$\ \{0,1\}\ ;\ L \leftarrow\!\!\$\ S^{\mathsf{Hash},\mathsf{ro}}(1^\lambda, t)$	$L \leftarrow\!\!\$\ S^{\mathsf{Hash},\mathsf{ro}}(1^n, t)$	If $b = 0$ then // reset the array T
$b' \leftarrow\!\!\$\ D^{\mathsf{ro}}(1^\lambda, \mathbf{hk}, L)\ ;$ Return $(b' = b)$	$Q' \leftarrow\!\!\$\ P^{\mathsf{ro}}(1^\lambda, 1^n, L)$	For $(x, \ell, i) \in U$ do
$\underline{\mathsf{Hash}(x, 1^\ell, i)}$	Return $(Q' \cap Q \neq \emptyset)$	$T[x, \ell, i] \leftarrow\!\!\$\ \{0,1\}^\ell$
If $T[x, \ell, i] = \perp$ then	$\underline{\mathsf{Hash}(x, 1^\ell, i)}$	$b' \leftarrow\!\!\$\ R^{\mathsf{Hash},\mathsf{ro}}(1^\lambda, L)\ ;$ Return $(b = b')$
If $b = 0$ then $T[x, \ell, i] \leftarrow\!\!\$\ \{0,1\}^\ell$	$Q \leftarrow Q \cup \{x\}$	$\underline{\mathsf{Hash}(x, 1^\ell, i)}$
Else $T[x, \ell, i] \leftarrow \mathsf{H.Ev}^{\mathsf{ro}}(1^\lambda, \mathbf{hk}[i], x, 1^\ell)$	If $T[x, \ell, i] = \perp$ then	If $T[x, \ell, i] = \perp$ then $T[x, \ell, i] \leftarrow\!\!\$\ \{0,1\}^\ell$
Return $T[x, \ell, i]$	$T[x, \ell, i] \leftarrow\!\!\$\ \{0,1\}^\ell$	$U \leftarrow U \cup \{(x, \ell, i)\}$
	Return $T[x, \ell, i]$	Return $T[x, \ell, i]$

Figure 2: **Games** UCE **(left),** Pred **(middle), and** Reset **(right) to define UCE security.** Here $\mathsf{ro} : \{0,1\}^{\mathsf{ro.il}(\lambda)} \to \{0,1\}^{\mathsf{ro.ol}(\lambda)}$ is a random oracle.

leakage, the predictor outputs a set Q'. The predictor wins if Q' contains a Hash-query of the source. For $\lambda \in \mathbb{N}$ we let

$$\mathsf{Adv}^{\mathsf{pred}}_{S,P}(\lambda) = \Pr[\text{Pred}^{P}_{S}(\lambda)] \ .$$

We require that the size of Q', as well as the number of queries that P makes to ro, be bounded by a polynomial (allowed to depend on P) in λ. We say that S is computationally (respectively, statistically) unpredictable if $\mathsf{Adv}^{\mathsf{pred}}_{S,P}(\cdot)$ is negligible for all PT (respectively, all, even computationally unbounded) predictors P. We let $\mathcal{S}^{\text{cup}}$ be the class of computationally unpredictable PT sources, and $\mathcal{S}^{\text{sup}}$ the class of statistically unpredictable PT sources. The corresponding security notions for H are $\mathsf{UCE}[\mathcal{S}^{\text{cup}}]$ and $\mathsf{UCE}[\mathcal{S}^{\text{sup}}]$.

Reset-secure sources. We recall the second restriction on sources from [6], called reset security. Let S be a source and R an adversary called a reset adversary. The source again is executed with its Hash being a random oracle. The reset adversary is either given access to the same random oracle or to an *independent* one. The requirement is that it should not be able to tell which. Formally, consider game $\text{Reset}^{R}_{S}(\lambda)$ at the right panel of Fig. 2 associated to S, R. For $\lambda \in \mathbb{N}$ we let

$$\mathsf{Adv}^{\mathsf{reset}}_{S,R}(\lambda) = 2\Pr[\text{Reset}^{R}_{S}(\lambda)] - 1 \ .$$

We require that the number of queries that P makes to Hash and ro be bounded by a polynomial (allowed to depend on R) in λ. We say S is computationally (respectively, statistically) reset-secure if $\mathsf{Adv}^{\mathsf{reset}}_{S,R}(\cdot)$ is negligible for all PT (respectively, all, even computationally unbounded) reset adversaries R. We let $\mathcal{S}^{\text{crs}}$ be the class of all PT computationally reset-secure sources, and $\mathcal{S}^{\text{srs}}$ the class of all PT statistically reset-secure sources. The corresponding security notions for H are $\mathsf{UCE}[\mathcal{S}^{\text{crs}}]$ and $\mathsf{UCE}[\mathcal{S}^{\text{srs}}]$.

Relations and achievability. Reset security is a relaxation of unpredictability. In particular BHK [6] show that $\mathsf{UCE}[\mathcal{S}^{\text{crs}}]$-security of H implies $\mathsf{UCE}[\mathcal{S}^{\text{cup}}]$-security of H and $\mathsf{UCE}[\mathcal{S}^{\text{srs}}]$-security of H implies $\mathsf{UCE}[\mathcal{S}^{\text{sup}}]$-security of H. The converses are not necessarily true. BFM [13] show that if indistinguishability obfuscation for all circuits is possible then $\mathsf{UCE}[\mathcal{S}^{\text{crs}}]$-security is not achievable in the standard model. In the ROM however BHK [6] show that both $\mathsf{UCE}[\mathcal{S}^{\text{crs}}]$-security and $\mathsf{UCE}[\mathcal{S}^{\text{srs}}]$-security are achievable.

4 UCE from indifferentiability

We first review necessary definitions of the indifferentiability framework [21].

Indifferentiability. We consider an algorithm M that, given a FIL random oracle ro, attempts to have input-output behavior approximating that of a VIL random oracle. Indifferentiability provides one definition of what it means for M to succeed at this task. Consider game $\text{Indiff}^{A}_{\mathsf{M},\overline{\mathsf{M}}}(\lambda)$ of Fig. 3 associated

$\underline{\text{Game Indiff}^A_{\mathsf{M},\overline{\mathsf{M}}}(\lambda)}$	$\underline{\mathsf{Func}(x)}$	$\underline{\mathsf{Prim}(x)}$
$b \leftarrow\!\!\$\ \{0,1\}$; st $\leftarrow \varepsilon$	If $b = 1$ then return $\mathsf{M}^{\mathsf{ro}}(1^\lambda, x)$	If $b = 1$ then return $\mathsf{ro}(x)$
$b' \leftarrow\!\!\$\ A^{\mathsf{Prim},\mathsf{Func}}(1^\lambda)$	Else return $\mathsf{RO}(x)$	$(y, \text{st}) \leftarrow\!\!\$\ \overline{\mathsf{M}}^{\mathsf{RO}}(1^\lambda, \text{st}, x)$
Return $(b = b')$		Return y

Figure 3: **Game** Indiff **defining indifferentiability.** Here $\mathsf{RO} : \{0,1\}^* \to \{0,1\}^{\mathsf{M.fol}(\lambda)}$ and $\mathsf{ro} : \{0,1\}^{\mathsf{M.pil}(\lambda)} \to \{0,1\}^{\mathsf{M.pol}(\lambda)}$ are random oracles.

to M, an algorithm $\overline{\mathsf{M}}$ called a simulator, and an adversary A. In the first world ($b = 1$), oracle Prim implements the FIL random oracle ro while oracle Func implements the construction, namely M^{ro}, that aims to approximate a VIL random oracle. In the second world ($b = 0$), oracle Func implements a true VIL random oracle RO while replies to Prim queries are determined by the simulator that itself has access to RO. The simulator is stateful, its state st being maintained by the game. The input x to M has arbitrary length, the oracle provided to M maps $\mathsf{M.pil}(\lambda)$-bit inputs to $\mathsf{M.pol}(\lambda)$-bit outputs, and M returns outputs of length $\mathsf{M.fol}(\lambda)$, where $\mathsf{M.pil}, \mathsf{M.pol}, \mathsf{M.fol} : \mathbb{N} \to \mathbb{N}$ are functions associated to M called the input-length of M's primitive, output-length of M's primitive, and output-length of M's functionality, respectively. For $\lambda \in \mathbb{N}$ we let

$$\mathsf{Adv}^{\mathsf{indiff}}_{\mathsf{M},\overline{\mathsf{M}},A}(\lambda) = 2\Pr[\text{Indiff}^A_{\mathsf{M},\overline{\mathsf{M}}}(\lambda)] - 1 \ .$$

We require that the number of queries that A makes to its oracles be bounded by a polynomial (allowed to depend on A) in λ. Then we say that M is a *pseudorandom oracle* (PRO) if there is a PT simulator $\overline{\mathsf{M}}$ such that $\mathsf{Adv}^{\mathsf{indiff}}_{\mathsf{M},\overline{\mathsf{M}},A}(\cdot)$ is negligible for every (even computationally unbounded) adversary A.

For concrete security assessments we let $Q_{\overline{\mathsf{M}},q}$ be the function that on input λ returns the maximum, over all $x_1, \ldots, x_q \in \{0,1\}^{\mathsf{M.pil}(\lambda)}$, of the total number of oracle queries that $\overline{\mathsf{M}}$ makes when run sequentially on inputs $x_1, \ldots, x_q$, starting from state ε. Also let $T_{\overline{\mathsf{M}},q}$ be the function that on input λ returns the maximum, over all $x_1, \ldots, x_q \in \{0,1\}^{\mathsf{M.pil}(\lambda)}$, of the total running time of $\overline{\mathsf{M}}$ when run sequentially on inputs $x_1, \ldots, x_q$, starting from state ε, the time for an oracle query being taken as linear in the length of the query and reply.

The Keyed-Indiff extender. Let $\overline{\mathsf{H}}$ be a FIL function family that is $\mathsf{UCE}[\mathcal{S}^{\text{xxx}}]$-secure for some xxx. We want to build a VIL family of functions H that is also $\mathsf{UCE}[\mathcal{S}^{\text{xxx}}]$-secure. Our construction uses as a tool any PRO M with $\mathsf{M.pil} = \overline{\mathsf{H}}.\mathsf{il}$ and $\mathsf{M.pol} = \overline{\mathsf{H}}.\mathsf{ol}$. We associate to M and $\overline{\mathsf{H}}$ the family of functions $\mathsf{H} = \mathsf{Keyed\text{-}Indiff}[\mathsf{M}, \overline{\mathsf{H}}]$ defined as follows. We let $\mathsf{H.IL} = \mathbb{N}$, meaning H is VIL. The output length of H is $\mathsf{H.ol} = \mathsf{M.fol}$. We let $\mathsf{H.Kg} = \overline{\mathsf{H}}.\mathsf{Kg}$, meaning keys for H are the same as for $\overline{\mathsf{H}}$. Finally for any $\lambda \in \mathbb{N}$, any $hk \in [\mathsf{H.Kg}(1^\lambda)]$ and any $x \in \{0,1\}^*$ we let

$$\mathsf{H.Ev}^{\mathsf{ro}}(1^\lambda, hk, x, 1^{\mathsf{H.ol}(\lambda)}) = \mathsf{M}^{\overline{\mathsf{H}}.\mathsf{Ev}^{\mathsf{ro}}(1^\lambda, hk, \cdot, 1^{\overline{\mathsf{H}}.\mathsf{ol}(\lambda)})}(1^\lambda, x) \ . \tag{1}$$

This needs some explanation. Begin by ignoring ro, so that we are looking at a standard-model construction. Recall that M takes an oracle mapping $\{0,1\}^{\mathsf{M.pil}(\lambda)}$ to $\{0,1\}^{\mathsf{M.pol}(\lambda)}$. In the indifferentiability setting, this is a random oracle. Our construction however does something different. It implements M's oracle via the given $\mathsf{UCE}[\mathcal{S}^{\text{xxx}}]$-secure family $\overline{\mathsf{H}}$. The key hk is held fixed. Our claim will be that H is itself $\mathsf{UCE}[\mathcal{S}^{\text{xxx}}]$-secure for xxx $\in \{\text{crs}, \text{srs}\}$. Something we consider interesting is that this result is entirely standard model, yet uses ROM theory, in the form of a PRO, for the construction and proof. Finally the ro in the construction simply reflects that the result lifts to the ROM. In case $\overline{\mathsf{H}}$ was a ROM family of functions, H will be as well. This extension, together with known applications of $\mathsf{UCE}[\mathcal{S}^{\text{xxx}}]$-security, allow us to implement in the FIL-ROM many constructions given in the VIL-ROM.

Result. We view $\mathsf{Keyed\text{-}Indiff}[\mathsf{M}, \cdot]$ as a domain extension transform taking a FIL family $\overline{\mathsf{H}}$ and returning a VIL family $\mathsf{H} = \mathsf{Keyed\text{-}Indiff}[\mathsf{M}, \overline{\mathsf{H}}]$. The following says that this transform preserves $\mathsf{UCE}[\mathcal{S}^{\text{xxx}}]$-security for xxx $\in \{\text{crs}, \text{srs}\}$. The proof is in Appendix A.

Theorem 4.1 Let $\overline{\mathsf{H}}$ be a hash function family. Let M be a PRO such that $\mathsf{M.pil} = \overline{\mathsf{H}}.\mathsf{il}$ and $\mathsf{M.pol} = \overline{\mathsf{H}}.\mathsf{ol}$.

Let $\mathsf{H} = \mathsf{Keyed\text{-}Indiff}[\mathsf{M}, \overline{\mathsf{H}}]$. Let xxx $\in \{\mathrm{crs}, \mathrm{srs}\}$.

Asymptotic result: If $\overline{\mathsf{H}}$ is $\mathsf{UCE}[\mathcal{S}^{\mathrm{xxx}}]$-secure then so is H.

Concrete result: Let $\overline{\mathsf{M}}$ be a simulator for M. Let S be an N-key source, D a distinguisher and $\overline{R}$ a reset adversary. Then we construct an N-key source $\overline{S}$, indifferentiability adversaries A, B and a reset adversary R such that

$$\mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\lambda) \leq \mathsf{Adv}^{\mathsf{uce}}_{\overline{\mathsf{H}},\overline{S},D}(\lambda) + N(\lambda) \cdot \mathsf{Adv}^{\mathsf{indiff}}_{\mathsf{M},\overline{\mathsf{M}},A}(\lambda) \tag{2}$$

$$\mathsf{Adv}^{\mathsf{reset}}_{\overline{S},\overline{R}}(\lambda) \leq \mathsf{Adv}^{\mathsf{reset}}_{S,R}(\lambda) + 3N(\lambda) \cdot \mathsf{Adv}^{\mathsf{indiff}}_{\mathsf{M},\overline{\mathsf{M}},B}(\lambda) \tag{3}$$

for all $\lambda \in \mathbb{N}$. Furthermore:

$\mathbf{Q}^{\mathsf{Prim}}_A = 0$; $\mathbf{Q}^{\mathsf{Func}}_A, \mathbf{Q}^{\mathsf{Func}}_B \leq \mathbf{Q}^{\mathsf{Hash}}_S$; $\mathbf{Q}^{\mathsf{Prim}}_B \leq \mathbf{Q}^{\mathsf{Hash}}_{\overline{R}}$
$\mathbf{Q}^{\mathsf{ro}}_R = \mathbf{Q}^{\mathsf{ro}}_{\overline{R}}$; $\mathbf{Q}^{\mathsf{Hash}}_R \leq Q_{\overline{\mathsf{M}},q}$ where $q = \mathbf{Q}^{\mathsf{Hash}}_{\overline{R}}$; $\mathbf{Q}^{\mathsf{ro}}_{\overline{S}} = \mathbf{Q}^{\mathsf{ro}}_S$
$\mathbf{Q}^{\mathsf{Hash}}_{\overline{S}}$ is bounded by the number of oracle queries of M in the execution of $\mathrm{UCE}^{S,D}_{\mathsf{H}}$
$\mathbf{T}(\mathrm{Indiff}^{A}_{\mathsf{M},\overline{\mathsf{M}}}) \leq \mathbf{T}(\mathrm{UCE}^{S,D}_{\mathsf{H}})$; $\mathbf{T}(\mathrm{UCE}^{\overline{S},D}_{\overline{\mathsf{H}}}) \leq \mathbf{T}(\mathrm{UCE}^{S,D}_{\mathsf{H}})$
$\mathbf{T}(\mathrm{Reset}^{R}_{S}) \leq \mathbf{T}(\mathrm{Reset}^{\overline{R}}_{\overline{S}}) + T_{\overline{\mathsf{M}},q}$ where $q = \mathbf{Q}^{\mathsf{Hash}}_{\overline{R}}$
$\mathbf{T}(\mathrm{Indiff}^{B}_{\mathsf{M},\overline{\mathsf{M}}}) \leq \mathbf{T}(\mathrm{Reset}^{R}_{S}) + \mathbf{T}(\mathrm{Reset}^{\overline{R}}_{\overline{S}})$ ▮

We emphasize that Keyed-Indiff works in both the standard and the random oracle models. In particular if FIL family $\overline{\mathsf{H}}$ is $\mathsf{UCE}[\mathcal{S}^{\mathrm{xxx}}]$-secure in the standard model, then so is $\mathsf{Keyed\text{-}Indiff}[\mathsf{M}, \overline{\mathsf{H}}]$, for xxx $\in \{\mathrm{crs}, \mathrm{srs}\}$. This resolves an open problem from [6] to construct UCE domain extenders in the standard model.

Some applications in [6] use only a single hash key. In other words, they only need $\mathsf{UCE}[\mathcal{S}^{\mathrm{crs}} \cap \mathcal{S}^{\mathrm{one}}]$ and $\mathsf{UCE}[\mathcal{S}^{\mathrm{srs}} \cap \mathcal{S}^{\mathrm{one}}]$ security, where $\mathcal{S}^{\mathrm{one}}$ is the class of 1-key sources. $\mathsf{Keyed\text{-}Indiff}[\mathsf{M}, \cdot]$ is also a domain extender for $\mathsf{UCE}[\mathcal{S}^{\mathrm{crs}} \cap \mathcal{S}^{\mathrm{one}}]$ and $\mathsf{UCE}[\mathcal{S}^{\mathrm{srs}} \cap \mathcal{S}^{\mathrm{one}}]$ because the value of N is preserved.

Instantiation. To obtain a concrete result that can be used in applications, we now instantiate $\overline{\mathsf{H}}$ above in a simple way, namely (1) $\overline{\mathsf{H}}.\mathsf{Kg}(1^\lambda)$ returns $hk \leftarrow\!\!\$\ \{0,1\}^\lambda$, and (2) $\overline{\mathsf{H}}.\mathsf{Ev}^{\mathsf{ro}}(1^\lambda, hk, x, 1^{\overline{\mathsf{H}}.\mathsf{ol}(\lambda)})$ returns $\mathsf{ro}(hk \,\|\, x)$. This is shown by BHK [6] to be UCE secure in the FIL-ROM for all forms of UCE they define. For illustration, Fig. 4 describes the PRO ChopMD (truncated Merkle-Damgård [15]) and shows how one would implement $\mathsf{Keyed\text{-}Indiff}[\mathrm{ChopMD}, \overline{\mathsf{H}}]$ from an iterative hash like SHA-256. From Theorem 4.1 we obtain the following.

Theorem 4.2 Let $\overline{\mathsf{H}}$ be constructed as above. Let M be a PRO such that $\mathsf{M}.\mathsf{pil} = \overline{\mathsf{H}}.\mathsf{il}$ and $\mathsf{M}.\mathsf{pol} = \overline{\mathsf{H}}.\mathsf{ol}$. Let $\mathsf{H} = \mathsf{Keyed\text{-}Indiff}[\mathsf{M}, \overline{\mathsf{H}}]$.

Asymptotic result: H is $\mathsf{UCE}[\mathcal{S}^{\mathrm{crs}}]$-secure.

Concrete result: Let $\overline{\mathsf{M}}$ be a simulator for M. Let S be an N-key source and D a distinguisher. We can construct a reset adversary R and an indifferentiability adversary A such that

$$\mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\lambda) \leq \mathsf{Adv}^{\mathsf{reset}}_{S,R}(\lambda) + 4N(\lambda) \cdot \mathsf{Adv}^{\mathsf{indiff}}_{\mathsf{M},\overline{\mathsf{M}},A}(\lambda) + \frac{2N(\lambda) \cdot q(\lambda) + N^2(\lambda)}{2^\lambda}$$

for every $\lambda \in \mathbb{N}$. Furthermore,

$\mathbf{Q}^{\mathsf{Prim}}_A \leq \mathbf{Q}^{\mathsf{Hash}}_S$; $\mathbf{Q}^{\mathsf{Func}}_A, \mathbf{Q}^{\mathsf{ro}}_R \leq \mathbf{Q}^{\mathsf{ro}}_D$; and $\mathbf{Q}^{\mathsf{Hash}}_R \leq Q_{\overline{\mathsf{M}},q}$, where $q = \mathbf{Q}^{\mathsf{ro}}_D$
$\mathbf{T}(\mathrm{Indiff}^{A}_{\mathsf{M},\overline{\mathsf{M}}}), \mathbf{T}(\mathrm{Reset}^{R}_{S}) \leq \mathbf{T}(\mathrm{UCE}^{S,D}_{\mathsf{H}}) + T_{\overline{\mathsf{M}},q}$, where $q = \mathbf{Q}^{\mathsf{ro}}_D$ ▮

Theorem 4.2 is the one that can be used for the applications, namely to obtain FIL-ROM constructions for (possibly multi-stage) primitives that have been constructed using a VIL UCE function, such as those in BHK [6]. We simply instantiate the VIL UCE function with H given by Theorem 4.2. The broader paradigm to move from the VIL-ROM to the FIL-ROM is thus the following. Take a primitive with a VIL-ROM proof, and show that the random oracle can be UCE-instantiated. Then apply Theorem 4.2.

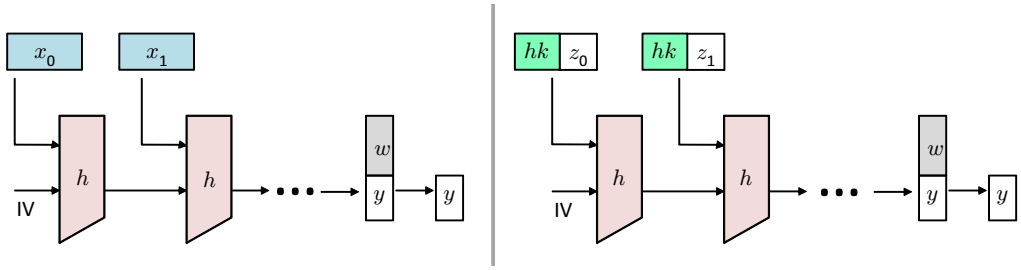

Figure 4: **An instantiation of Keyed-Indiff domain extender:** The PRO ChopMD (truncated Merkle-Damgård [15]) on the left, and $\mathsf{Keyed\text{-}Indiff}[\mathrm{ChopMD}, \overline{\mathsf{H}}]$ on the right, where h is a compression function and $\overline{\mathsf{H}} = h(hk \,\|\, \cdot)$.

$\underline{\mathsf{H.Kg}(1^\lambda)}$	$\underline{\mathsf{H.Ev^{ro}}(1^\lambda, hk, x, 1^\ell)}$
$fk \leftarrow\!\!\$\ \mathsf{F.Kg}(1^\lambda)$; $\overline{hk} \leftarrow\!\!\$\ \overline{\mathsf{H}}.\mathsf{Kg}(\lambda)$	$(\overline{hk}, fk) \leftarrow hk$; $u \leftarrow \mathsf{F.Ev}(1^\lambda, fk, x, 1^{\mathsf{F.ol}(\lambda)})$
$hk \leftarrow (\overline{hk}, fk)$; Return hk	$y \leftarrow \overline{\mathsf{H}}.\mathsf{Ev^{ro}}(1^\lambda, \overline{hk}, u, 1^\ell)$; Return y

Figure 5: **The $\mathsf{H} = \mathsf{AU\text{-}then\text{-}Hash}[\mathsf{F}, \overline{\mathsf{H}}]$ construction, built from a AU hash F and a FIL UCE-secure hash $\overline{\mathsf{H}}$.**

5 UCE from universal hashing

In this section, we show how almost universal hash functions can be used to build a domain extender for UCE.

AU HASH FAMILIES. For any function family F let

$$\mathbf{Coll1}_{\mathsf{F}}(\lambda, m) = \max_{|y|=\mathsf{F.ol}(\lambda), |x|\le m} \Big\{ \Pr_{fk \leftarrow\!\!\$\ \mathsf{F.Kg}(1^\lambda)} [y = \mathsf{F.Ev}(1^\lambda, fk, x, 1^{\mathsf{F.ol}(\lambda)})] \Big\},$$

and define $\mathbf{Coll2}_{\mathsf{F}}(\lambda, m_0, m_1)$ as

$$\max \Big\{ \Pr_{fk \leftarrow\!\!\$\ \mathsf{F.Kg}(1^\lambda)} [\mathsf{F.Ev}(1^\lambda, fk, x_0, 1^{\mathsf{F.ol}(\lambda)}) = \mathsf{F.Ev}(1^\lambda, fk, x_1, 1^{\mathsf{F.ol}(\lambda)})] \Big\};$$

the maximum is taken over distinct strings x_0, x_1 such that each $|x_i| \le m_i$. Let

$$\mathbf{Coll}_{\mathsf{F}}(\lambda, m_0, m_1) = \max\big\{ \mathbf{Coll2}_{\mathsf{F}}(\lambda, m_0, m_1), \mathbf{Coll1}_{\mathsf{F}}(\lambda, \min\{m_0, m_1\}) \big\} \ .$$

A hash family F is *almost universal* (AU) if $f(\lambda) = \mathbf{Coll}_{\mathsf{F}}(\lambda, m_0, m_1)$ is negligible for all polynomials m_0, m_1. This generalizes the Carter-Wegman notion of universal hashing [14].

A similar definition is given in [11], which is very useful when one needs to work with arbitrarily large input and short hash keys. In Section 6, we'll show how to concretely instantiate a very fast AU hash for $\lambda = 128$, from reduced-round AES and a classic polynomial-based universal hash. Define

$$\mathsf{Adv}^{\mathsf{coll}}_{\mathsf{F}}(\lambda, p, \sigma) = \max_{\ell \le p, \ell' \le p, m_1 + \cdots + m_\ell \le \sigma, m'_1 + \cdots + m'_{\ell'} \le \sigma} \Big\{ \sum_{i=1}^{\ell} \sum_{j=1}^{\ell'} \mathbf{Coll}_{\mathsf{F}}(\lambda, m_i, m'_j) \Big\} \ .$$

If F is AU then $\mathsf{Adv}^{\mathsf{coll}}_{\mathsf{F}}(\lambda, p, \sigma)$ is negligible for all polynomials p and σ: since $\mathbf{Coll}(\lambda, \cdot, \cdot)$ is increasing in both arguments, it follows that $\mathsf{Adv}^{\mathsf{coll}}_{\mathsf{F}}(\lambda, p, \sigma) \le p^2 \mathbf{Coll}_{\mathsf{F}}(\lambda, \sigma, \sigma)$.

UCE EXTENDER FROM AN AU HASH. We now describe a UCE extender from AU hash. Intuitively, one first uses the AU hash to condense the input, and then applies the resulting string to the (keyed) compression function. Formally, let $\overline{\mathsf{H}}$ be a hash function family of fixed input length, and F be a universal hash function family with $\mathsf{F.ol} = \overline{\mathsf{H}}.\mathsf{il}$ and $\mathsf{F.IL} = \mathbb{N}$. Consider the hash function family $\mathsf{H} = \mathsf{AU\text{-}then\text{-}Hash}[\mathsf{F}, \overline{\mathsf{H}}]$ as given in Fig. 5, with $\mathsf{H.OL} = \overline{\mathsf{H}}.\mathsf{OL}$ and $\mathsf{H.IL} = \mathbb{N}$. The construction essentially follows the widely used Carter-Wegman paradigm [26] Below, we show that $\mathsf{AU\text{-}then\text{-}Hash}[\mathsf{F}, \cdot]$ is also a domain extender for $\mathsf{UCE}[\mathcal{S}^{\mathrm{sup}}]$ security. The proof is in Appendix B.

Theorem 5.1 Let $\overline{\mathsf{H}}$ be a function family of fixed input length, and F be an AU hash function family with $\mathsf{F.ol} = \overline{\mathsf{H}}.\mathsf{il}$ and $\mathsf{F.IL} = \mathbb{N}$. Let $\mathsf{H} = \mathsf{AU\text{-}then\text{-}Hash}[\mathsf{F}, \overline{\mathsf{H}}]$.

Asymptotic result: If $\overline{\mathsf{H}}$ is $\mathsf{UCE}[\mathcal{S}^{\rm sup}]$-secure then so is H.

Concrete result: Let S be a N-key source, D a distinguisher, and $\overline{P}$ a predictor. We can construct a source $\overline{S}$, a distinguisher $\overline{D}$, and a predictor P such that

$$\mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\lambda) \le \mathsf{Adv}^{\mathsf{uce}}_{\overline{\mathsf{H}},\overline{S},\overline{D}}(\lambda) + \mathsf{Adv}^{\mathsf{coll}}_{\mathsf{F}}(\lambda, p, \sigma) \tag{4}$$

$$\mathsf{Adv}^{\mathsf{pred}}_{\overline{S},\overline{P}}(\lambda) \le \sqrt{2q\mathsf{Adv}^{\mathsf{coll}}_{\mathsf{F}}(\lambda, p, \sigma)} + \sqrt{q\mathsf{Adv}^{\mathsf{pred}}_{S,P}(\lambda)} \tag{5}$$

where $p = \mathbf{Q}^{\mathsf{Hash}}_{S}$, q is the maximum of the size of $\overline{P}$'s output in the execution of $\mathrm{Pred}^{\overline{P}}_{\overline{S}}$, and σ is the maximum of the total length of Hash queries that S makes in $\mathrm{UCE}^{S,D}_{\mathsf{H}}$. Furthermore,

$\mathbf{Q}^{\mathsf{ro}}_{\overline{S}} = \mathbf{Q}^{\mathsf{ro}}_{S}$; $\mathbf{Q}^{\mathsf{Hash}}_{\overline{S}} = \mathbf{Q}^{\mathsf{Hash}}_{S}$; $\mathbf{Q}^{\mathsf{ro}}_{\overline{D}} = \mathbf{Q}^{\mathsf{ro}}_{D}$
$\mathbf{T}(\mathrm{UCE}^{\overline{S},\overline{D}}_{\overline{\mathsf{H}}}) \le \mathbf{T}(\mathrm{UCE}^{S,D}_{\mathsf{H}})$, and P outputs a set of size at most $\mathbf{Q}^{\mathsf{Hash}}_{S}$ ∎

We emphasize that $\mathsf{AU\text{-}then\text{-}Hash}$ works in both the standard and the random-oracle models. In particular If FIL family $\overline{\mathsf{H}}$ is $\mathsf{UCE}[\mathcal{S}^{\rm sup}]$-secure in the standard model then so is $\mathsf{AU\text{-}then\text{-}Hash}[\mathsf{F}, \overline{\mathsf{H}}]$. The intended applications for the $\mathsf{AU\text{-}then\text{-}Hash}[\mathsf{F}, \cdot]$ transform, as listed in Fig. 1, use only a single hash key, that is, they only need $\mathsf{UCE}[\mathcal{S}^{\rm sup} \cap \mathcal{S}^{\rm one}]$ security, where $\mathcal{S}^{\rm one}$ is the class of 1-key sources. $\mathsf{AU\text{-}then\text{-}Hash}[\mathsf{F}, \cdot]$ is also a domain extender for $\mathsf{UCE}[\mathcal{S}^{\rm sup} \cap \mathcal{S}^{\rm one}]$ security because the value of N is preserved.

Instantiation. So far we have assumed the existence of a fixed-input-length UCE-secure hash $\overline{\mathsf{H}}$. For the MLE application in [6], one needs to hash a short session key to produce a big one-time pad. We aim to make this process as fast as AES-CTR. It's possible to build a ROM-based $\overline{\mathsf{H}}$ without any other cryptographic primitive, but this won't give us the desired speed, if one instantiates ro from, say the compression function of SHA-256. Our construction therefore makes use of a PRP E, which will be instantiated by AES.

Recall that a PRP is a function family E such that $\mathsf{E.il} = \mathsf{E.ol}$, and $\mathsf{E.Ev}(1^\lambda, K, \cdot, 1^{\mathsf{E.ol}(\lambda)})$ is in $\mathrm{Perm}(\mathsf{E.ol}(\lambda))$ for every $\lambda \in \mathbb{N}$ and $\mathsf{Adv}^{\mathsf{prp}}_{\mathsf{E},A}(\lambda) = 2\Pr[\mathrm{PRP}^A_{\mathsf{E}}(\lambda)] - 1$ is negligible for every PT adversary A, where $\mathrm{Perm}(\ell)$ is the set of permutation on $\{0,1\}^\ell$ and game $\mathrm{PRP}^A_{\mathsf{E}}$ is defined as follows.

Game $\mathrm{PRP}^A_{\mathsf{E}}(\lambda)$	$\mathrm{RR}(x)$
$b \leftarrow\!\!\$\ \{0,1\}$; $K \leftarrow\!\!\$\ \{0,1\}^{\mathsf{E.kl}(\lambda)}$	If $b = 1$ then $y \leftarrow \mathsf{E.Ev}(1^\lambda, K, x, 1^{\mathsf{E.ol}(\lambda)})$
$\pi \leftarrow\!\!\$\ \mathrm{Perm}(\mathsf{E.ol}(\lambda))$; $b' \leftarrow\!\!\$\ A^{\mathrm{RR}}(1^\lambda)$	Else $y \leftarrow \pi(x)$
Return $(b = b')$	Return y

We now describe our ROM-based UCE construction. Assume that $\mathsf{ro.il}(\lambda) \ge 2\lambda$, and $\mathsf{ro.ol}(\lambda) = \mathsf{E.kl}(\lambda)$, and $\mathsf{E.ol}(\lambda) = \lambda$ for every $\lambda \in \mathbb{N}$. The construction $\mathsf{H}_{\mathsf{rom}}$ is given as follows, where $\mathsf{H}_{\mathsf{rom}}.\mathsf{OL}(\lambda) = \{1, 2, 3, \ldots, 2^{\lceil \lambda/2 \rceil}\}$ and $\mathsf{H}_{\mathsf{rom}}.\mathsf{il}(\lambda) = \mathsf{ro.il}(\lambda) - \lambda$ for every $\lambda \in \mathbb{N}$.

$\mathsf{H}_{\mathsf{rom}}.\mathsf{Kg}(1^\lambda)$	$\mathsf{H}_{\mathsf{rom}}.\mathsf{Ev}^{\mathsf{ro}}(1^\lambda, hk, x, 1^\ell)$
$hk \leftarrow\!\!\$\ \{0,1\}^\lambda$; Return hk	$K \leftarrow \mathsf{ro}(hk \,\|\, x \,\|\, \ell)$; $m \leftarrow \lceil \ell/\lambda \rceil$
	For $i = 1$ to m do $y_i \leftarrow \mathsf{E.Ev}(1^\lambda, K, \ell \,\|\, i, 1^\lambda)$
	$y \leftarrow y_1 \,\|\, \cdots \,\|\, y_m$; $y \leftarrow y[1, \ell]$; Return y

Here when we call $\mathsf{ro}(hk \,\|\, x \,\|\, \ell)$ and $\mathsf{E.Ev}(1^\lambda, K, \ell \,\|\, i, 1^\lambda)$, the string ℓ is encoded as a $\lceil \lambda/2 \rceil$-bit string, and i is encoded as a $\lfloor \lambda/2 \rfloor$-bit string. The hash $\mathsf{H}_{\mathsf{rom}}$ is indeed UCE-secure; the concrete security statement and the proof are in Appendix C. We conclude the following.

Theorem 5.2 Let F be an AU hash function family with $\mathsf{F.ol} = \mathsf{H_{rom}.il}$ and $\mathsf{F.IL} = \mathbb{N}$. Let $\mathsf{H} = \mathsf{AU\text{-}then\text{-}Hash}[\mathsf{F}, \mathsf{H_{rom}}]$.

Asymptotic result: H is $\mathsf{UCE}[\mathcal{S}^{\mathrm{sup}}]$-secure.

Concrete result: Let S be an N-key source and D a distinguisher. We can construct a predictor P and a PRP adversary A such that

$$\mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\lambda) \le 2\sqrt{q(\lambda)\mathsf{Adv}^{\mathsf{coll}}_{\mathsf{F}}(\lambda, p(\lambda), \sigma(\lambda))} + \sqrt{q(\lambda)\mathsf{Adv}^{\mathsf{pred}}_{S,P}(\lambda)} + 2p(\lambda)\cdot\mathsf{Adv}^{\mathsf{prp}}_{\mathsf{E},A}(\lambda) + \frac{2s^2(\lambda) + N^2(\lambda) + q^2(\lambda)}{2^\lambda}$$

for every $\lambda \in \mathbb{N}$, where $p = \mathbf{Q}^{\mathsf{Hash}}_S$; $q = \mathbf{Q}^{\mathsf{ro}}_S + \mathbf{Q}^{\mathsf{ro}}_D$; σ and s are the maximum of the total length of the first components and the total number of λ-bit blocks in the second components, respectively, of Hash queries in the execution of $\mathrm{UCE}^{S,D}_{\mathsf{H}}$. Furthermore

$\mathbf{Q}^{\mathsf{LR}}_A$ is maximum of the number of λ-bit blocks in the second component of a Hash query in $\mathrm{UCE}^{S,D}_{\mathsf{H}}$
$\mathbf{T}(\mathrm{PRP}^A_{\mathsf{E}}) \le \mathbf{T}(\mathrm{UCE}^{S,D}_{\mathsf{H}})$, and P outputs a set of size at most $\mathbf{Q}^{\mathsf{Hash}}_S$. ∎

6 Fast, parallelizable AU hash from reduced-round AES

We now show how to construct a fast parallelizable AU hash, which we call $\mathsf{F_{aes4}}$. In this section, let $n = 128$, $C = 2^{15}$, and let r be a small integer, say $r = 5$. All function families in this section are concrete; the security parameter λ is hidden in the formulas, but implicitly, it is $\lambda = 128$. For any integer m, let $\|m\|_n$ denote $\lfloor m/n \rfloor + 1$. We'll first describe two building blocks: $\mathsf{F_{poly}}$, a polynomial-based AU hash that operates on $\{0,1\}^*$, and $\mathsf{F_{tree}}$, a highly efficient AU hash based on reduced-round AES that operates on $\{x \in (\{0,1\}^n)^+ : |x| \le 2^r n\}$. We then show how to combine them to produce a highly efficient AU hash $\mathsf{F_{aes4}}$ whose domain is $\{0,1\}^*$.

THE $\mathsf{F_{poly}}$ CONSTRUCTION. We now describe a variant of a classic polynomial-based universal hash [14], which we call $\mathsf{F_{poly}}$. Let $\mathsf{F_{poly}.ol} = n$. As described in the pseudocode below, the key fk is picked as a random element of $\mathrm{GF}(2^n)$. To hash, we parse the input string $x \in \{0,1\}^*$ to a unique sequence $(w_0, \ldots, w_m)$, where each $w_i \in \mathrm{GF}(2^n)$ and w_m is not the zero element. This is performed by (i) parse $v_0 \,\|\, \cdots \,\|\, v_m \leftarrow x \,\|\, 10^s 1$, where $s \in \mathbb{N}$ is the smallest number such that $s + |x| \equiv -2 \pmod n$ and each $|w_i| = n$, and (ii) let each w_i be the encoding of v_i in $\mathrm{GF}(2^n)$. Then, the hash is computed as $\sum_{i=0}^{m} w_i \cdot fk^i$.

$\mathsf{F_{poly}.Kg}()$	$\mathsf{F_{poly}.Ev}(fk, x, 1^n)$
$fk \leftarrow\!\!\$\ \mathrm{GF}(2^n)$	$(w_0, \ldots, w_m) \leftarrow x$; $y \leftarrow w_0$
Return fk	For $i = 1$ to m do $y \leftarrow y + w_i \cdot fk^i$
	Return y

The proof of Proposition 6.1 below is in Appendix D.

Proposition 6.1 (a) For any $m \in \mathbb{N}$, we have $\mathbf{Coll1}_{\mathsf{F_{poly}}}(m) \le \|m\|_n / 2^n$, and (b) for any $m_0, m_1 \in \mathbb{N}$, we have $\mathbf{Coll2}_{\mathsf{F_{poly}}}(m_0, m_1) \le \max\{\|m_0\|_n, \|m_1\|_n\}/2^n$.

THE $\mathsf{F_{tree}}$ CONSTRUCTION. Let $E : \{0,1\}^{4n} \times \{0,1\}^n \to \{0,1\}^n$ denote a function based on 4-round AES which works as follows. Parse the key K as the concatenation of n-bit substrings S_0, S_1, S_2, S_3, and let $S_4 = 0^n$. The input is initially xored with S_0, and each S_i is used as the subkey of the i-th AES round, for $i \in \{1,2,3,4\}$. One can build from E a hash of domain $\{n, 2n, 3n, \ldots, 2^r n\}$ as illustrated in Fig. 6. Formally, let Halve denote the following operation. On input $(K, x) \in \{0,1\}^{4n} \times (\{0,1\}^n)^*$, we partition x into n-bit blocks $x_1 \cdots x_m$. For every two consecutive blocks x_{2i-1} and x_{2i}, we compute $y_i \leftarrow E_K(x_{2i-1}) \oplus x_{2i}$. If m is odd then let $y_{\lceil m/2 \rceil} \leftarrow x_m$. Finally output $y_1 \,\|\, \cdots \,\|\, y_{\lceil m/2 \rceil}$. Consider the following tree-hash construction $\mathsf{F_{tree}}$, with $\mathsf{F_{tree}.IL} = \{n, 2n, 3n, \ldots, 2^r n\}$ and $\mathsf{F_{tree}.ol} = n$:

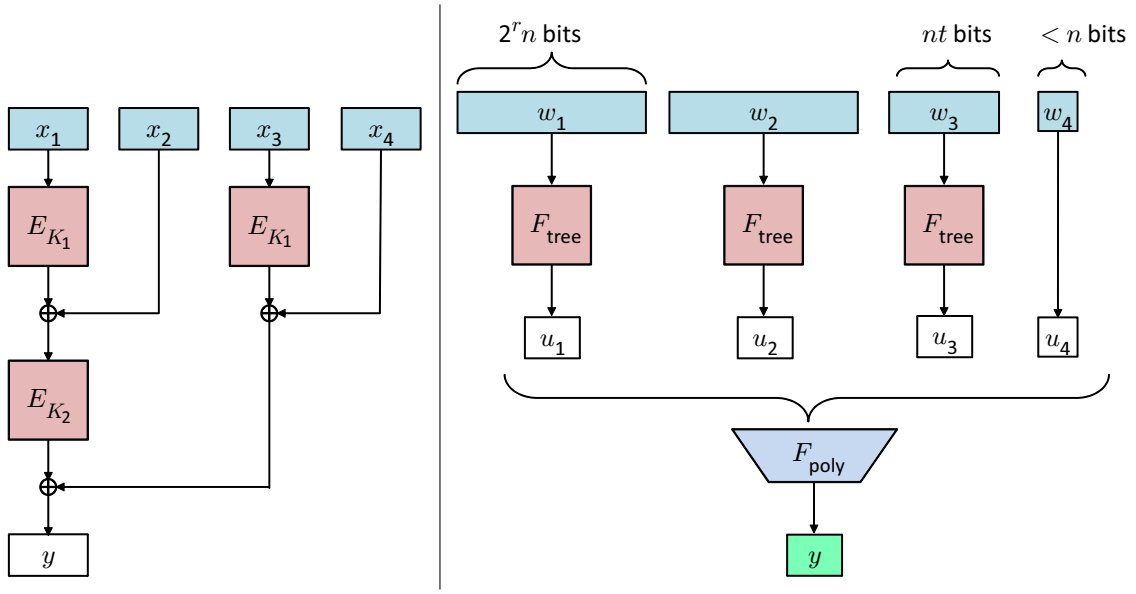

Figure 6: **Left:** Illustration of $\mathsf{F}_{\mathsf{tree}}$ for $r = 2$. **Right:** Illustration of $\mathsf{F}_{\mathsf{fast}}$.

$\underline{\mathsf{F}_{\mathsf{tree}}.\mathsf{Kg}()}$	$\underline{\mathsf{F}_{\mathsf{tree}}.\mathsf{Ev}(fk, x, 1^n)}$
For $i = 1$ to r do $K_i \leftarrow\!\!\$\ \{0,1\}^{4n}$	$z_0 \leftarrow x$; $(K_1, \ldots, K_r) \leftarrow fk$
$hk \leftarrow (K_1, \ldots, K_r)$; Return fk	For $i = 1$ to r do $z_i \leftarrow \mathsf{Halve}(K_i, z_{i-1})$
	Return z_r

Minematsu and Tsunoo [22] show that

$$\mathbf{Coll2}_{\mathsf{F}_{\mathsf{tree}}}(m_0, m_1) \leq \frac{Cr}{2^n} \tag{6}$$

for any $m_0, m_1 \leq 2^r$. We stress that the result in [22] makes no assumption on AES. This is based on the fact that four-round AES, with the subkeys chosen uniformly and independently, is an almost-xor-universal hash [20].

Combining $\mathsf{F}_{\mathsf{tree}}$ and $\mathsf{F}_{\mathsf{poly}}$. One can "cascade" $\mathsf{F}_{\mathsf{tree}}$ and $\mathsf{F}_{\mathsf{poly}}$ to produce a hash $\mathsf{F}_{\mathsf{fast}}$ of domain $\{0,1\}^*$ as shown in Fig. 6. The formal code is shown below.

$\underline{\mathsf{F}_{\mathsf{fast}}.\mathsf{Kg}()}$	$\underline{\mathsf{F}_{\mathsf{fast}}.\mathsf{Ev}(fk, x, 1^n)}$	$\underline{\mathsf{Shrink}(fk_1, x)}$
$fk_1 \leftarrow\!\!\$\ \mathsf{F}_{\mathsf{tree}}.\mathsf{Kg}()$	$(fk_1, fk_2) \leftarrow fk$	$w_1 w_2 \cdots w_k \leftarrow x$; $u_k \leftarrow w_k$
$fk_2 \leftarrow\!\!\$\ \mathsf{F}_{\mathsf{poly}}.\mathsf{Kg}()$	$y \leftarrow \mathsf{Shrink}(fk_1, x)$	For $i = 1$ to $k-1$ do
Return (fk_1, fk_2)	$z \leftarrow \mathsf{F}_{\mathsf{poly}}.\mathsf{Ev}(fk_2, y, 1^n)$	$\quad u_i \leftarrow \mathsf{F}_{\mathsf{tree}}.\mathsf{Ev}(fk_1, w_i, 1^n)$
	Return z	$y \leftarrow u_1 \,\|\, \cdots \,\|\, u_k$; Return y

In the procedure Shrink above, we parse a string x as the concatenation of substrings $w_1, \ldots, w_k$, where the length of each w_i, with $i \leq k-2$, is exactly $2^r n$, and $|w_{k-1}| > 0$ is a multiple of n but does not exceed $2^r n$, and $0 \leq |w_k| < n-1$. Note that on a large input x, the hash F will make at most $(1-2^{-r})\lceil x/n \rceil$ calls on E, and then run $\mathsf{F}_{\mathsf{poly}}$ on a string of length about $|x|/2^r$. The proof of Proposition 6.2 below is in Appendix E.

Proposition 6.2 For any $m_0, m_1 \in \mathbb{N}$, we have $\mathbf{Coll}_{\mathsf{F}_{\mathsf{fast}}}(m_0, m_1) \leq (Cr + \max\{\|m_0\|_n, \|m_1\|_n\})/2^n$.

Using with AU-then-Hash. The hash $\mathsf{F}_{\mathsf{fast}}$ can't be used directly with the AU-then-Hash transform in Section 5, because the term $(q\mathsf{Adv}^{\mathsf{coll}}_{\mathsf{F}_{\mathsf{fast}}}(p,\sigma))^{1/2}$ in Theorem 5.1 is about $(\sqrt{qp\sigma} + Crp\sqrt{q})/2^{n/2}$, which is inferior. The reason for this is that the output length of this hash is only n bits, which is too short. We

therefore need to "double" the output length. Formally, given a hash family $\overline{\mathsf{F}}$, the family $\mathsf{F} = \mathsf{Double}[\overline{\mathsf{F}}]$, with $\mathsf{F.IL} = \overline{\mathsf{F}}.\mathsf{IL}$ and $\mathsf{F.ol} = 2\overline{\mathsf{F}}.\mathsf{ol}$, is constructed as follows.

$\underline{\mathsf{F.Kg}()}$	$\underline{\mathsf{F.Ev}(fk, x, 1^{\mathsf{F.ol}})}$
$fk_1, fk_2 \leftarrow\!\!\$\ \overline{\mathsf{F}}.\mathsf{Kg}()$	$(fk_1, fk_2) \leftarrow fk$
$fk \leftarrow (fk_1, fk_2)$; Return fk	For $i = 1$ to 2 do $y_i \leftarrow \overline{\mathsf{F}}.\mathsf{Ev}(fk_i, x, 1^{\overline{\mathsf{F}}.\mathsf{ol}})$
	Return $y_1 \,\|\, y_2$

Let $\mathsf{F}_{\mathsf{aes4}}$ denote $\mathsf{Double}[\mathsf{F}_{\mathsf{fast}}]$. As shown in the term $(q\mathsf{Adv}^{\mathsf{coll}}_{\mathsf{F}_{\mathsf{fast}}}(p,\sigma))^{1/2}$ in Theorem 5.1 is bounded by $(Crp\sqrt{2q} + 2(\|\sigma\|_n + p)\sqrt{pq})/2^n$, which is good. The proof is in Appendix F.

Proposition 6.3 For any p and σ, we have $\mathsf{Adv}^{\mathsf{coll}}_{\mathsf{F}_{\mathsf{aes4}}}(p,\sigma) \le \frac{2C^2r^2p^2+4p(\|\sigma\|_n+p)^2}{2^{2n}}$.

KEY LENGTH. The key material of $\mathsf{FastHash} = \mathsf{AU\text{-}then\text{-}Hash}[\mathsf{F}_{\mathsf{aes4}}, \mathsf{H}_{\mathsf{rom}}]$ is relatively large: 672B for $r = 5$. It's slightly bigger than that of some widely used schemes such as RSA [24] (256B). This is acceptable because the key is used as a public parameter.

7 An example application

In Section 1, we introduced a two-step paradigm for using UCE to move VIL-ROM schemes to the FIL-ROM: (1) Show that instantiating the VIL random oracle in the scheme with a VIL UCE function preserves security, and (2) Implement the VIL UCE function as E^{ro} to obtain a FIL-ROM scheme, where E is a UCE domain extender. We explained that step (1) has already been done for many primitives [6], and what remained was to build domain extenders E allowing step (2). At this point we have reached the latter goal, providing two such constructions for E, namely Keyed-Indiff of Section 4 and AU-then-Hash of Section 5. We now illustrate how to put things together to obtain a final FIL ROM scheme for an example application, namely Message-locked Encryption (MLE) [7], by exploiting the results of [6] for the first step and our results for the second step. Other applications may be obtained in the same way.

MLE DEFINITIONS. A MLE scheme can be used to provide space-efficient secure outsourced storage. Formally an MLE scheme MLE specifies the following PT algorithms. Algorithm $\mathsf{MLE.Pg}(1^\lambda)$ generates a parameter p. Next, given a parameter p and a message m, algorithm $\mathsf{MLE.Kg}(1^\lambda, p, m)$ deterministically generates a key K. To encrypt a message m under key K, one calls $c \leftarrow\!\!\$\ \mathsf{MLE.Enc}(1^\lambda, p, K, m)$. One then can decrypt a ciphertext c via $m \leftarrow \mathsf{MLE.Dec}(1^\lambda, p, K, c)$. The tag-generation algorithm $\mathsf{MLE.Tag}(1^\lambda, p, c)$ deterministically derives a tag t from a ciphertext c. The correctness requirement demands that for all $\lambda \in \mathbb{N}, m \in \{0,1\}, p \in [\mathsf{MLE.Pg}(1^\lambda)]$, and $K_1, K_2 \in [\mathsf{MLE.Kg}(1^\lambda, p, m)]$ we have (1) For all $c_1 \in [\mathsf{MLE.Enc}(1^\lambda, p, K_1, m)]$ and $c_2 \in [\mathsf{MLE.Enc}(1^\lambda, p, K_2, m)]$, we have $\mathsf{MLE.Tag}(1^\lambda, p, c_1) = \mathsf{MLE.Tag}(1^\lambda, p, c_2)$, and (2) $\mathsf{MLE.Dec}(1^\lambda, p, K_2, c) = m$ for all $c \in [\mathsf{MLE.Enc}(1^\lambda, p, K_1, m)]$. For privacy, the IND$-CDA security is defined via the game in the left panel of Fig. 7. A IND$-CDA adversary A is a pair of PT algorithms (A_1, A_2), where $A_1(1^\lambda)$ returns a string vector $\mathbf{m}$ that satisfies the following: (1) There is a polynomial v and a function $\mathsf{len} : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$, depending solely on A, such that $|\mathbf{m}| = v(\lambda)$ and $|\mathbf{m}[i]| = \mathsf{len}(\lambda, i)$, for every $\lambda \in \mathbb{N}, m \in [A_1(1^\lambda)]$, and $i \le |\mathbf{m}|$, and (2) the strings $\mathbf{m}[1], \ldots, \mathbf{m}[|\mathbf{m}|]$ are distinct. Define the guessing probability Guess_A of A as the function that on input $\lambda \in \mathbb{N}$ returns the maximum, over all i, m, of $\Pr[\mathbf{m}[i] = m]$, the probability over $\mathbf{m} \leftarrow\!\!\$\ A_1(1^\lambda)$. We say that A has *high min-entropy* if $\mathrm{Guess}_A(\cdot)$ is negligible. We let $\mathsf{Adv}^{\mathsf{cda}}_{\mathsf{MLE},A}(\lambda) = 2\Pr[\text{IND\$-CDA}^A_{\mathsf{MLE}}(\lambda)] - 1$ and say that MLE is IND$-CDA-secure if $\mathsf{Adv}^{\mathsf{cda}}_{\mathsf{MLE},A}(\cdot)$ is negligible for all PT A that have high min-entropy.

A FIL-ROM MLE SCHEME. BHK [6] describe a variant of the Convergent Encryption (CE) scheme of [18] as shown in the right panel of Fig. 7. They then give the following result.

Proposition 7.1 [6] Let H be a $\mathsf{UCE}[\mathcal{S}^{\mathrm{sup}}]$-secure hash with $\mathsf{H.IL} = \mathsf{H.OL} = \mathbb{N}$.

Asymptotic result: $\mathsf{CE}[\mathsf{H}]$ is IND$-CDA secure.

$\underline{\mathsf{Game\ IND\$\text{-}CDA}^{A}_{\mathsf{MLE}}(\lambda)}$	$\underline{\mathsf{CE.Pg}(1^\lambda)}$	$\underline{\mathsf{CE.Enc}(1^\lambda, hk, K, m)}$
$p \leftarrow\!\!\$\ \mathsf{MLE.Pg}(1^\lambda)\ ;\ b \leftarrow\!\!\$\ \{0,1\}$	$hk \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$	$c \leftarrow m \oplus \mathsf{H.Ev}(1^\lambda, hk, K, 1^{\lvert m\rvert})$
$\mathbf{m} \leftarrow\!\!\$\ A_1(1^\lambda)$	Return hk	Return c
For $i = 1$ to $\lvert\mathbf{m}\rvert$ do	$\underline{\mathsf{CE.Kg}(1^\lambda, hk, m)}$	$\underline{\mathsf{CE.Dec}(1^\lambda, hk, K, c)}$
$\quad \mathbf{K}[i] \leftarrow \mathsf{MLE.Kg}(1^\lambda, p, \mathbf{m}[i])$	$K \leftarrow \mathsf{H.Ev}(1^\lambda, hk, m, 1^{2\lambda})$	$m \leftarrow c \oplus \mathsf{H.Ev}(1^\lambda, hk, K, 1^{\lvert c\rvert})$
$\quad \mathbf{c}_1[i] \leftarrow \mathsf{MLE.Enc}(1^\lambda, p, \mathbf{K}[i], \mathbf{m}[i])$	Return K	Return m
$\quad \mathbf{c}_0[i] \leftarrow\!\!\$\ \{0,1\}^{\lvert\mathbf{c}_1[i]\rvert}$		$\underline{\mathsf{CE.Tag}(1^\lambda, hk, c)}$
$b' \leftarrow\!\!\$\ A_2(1^\lambda, p, \mathbf{c}_b)$		Return c
Return $(b' = b)$		

Figure 7: **Left:** The IND$-CDA game. **Right:** MLE scheme CE[H].

Concrete result: Let $A = (A_1, A_2)$ be an IND$-CDA adversary. We can construct a 1-source S and a distinguisher D such that for any predictor P,

$$\begin{aligned}\mathsf{Adv}^{\mathsf{cda}}_{\mathsf{CE[H]},A}(\lambda) &\le \mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\lambda) + \frac{2v^2}{2^{2\lambda}} \\ \mathsf{Adv}^{\mathsf{pred}}_{S,P}(\lambda) &\le v\ell \cdot \mathrm{Guess}_A(\lambda) + \frac{2v^2 + v\ell}{2^{2\lambda}}\end{aligned}$$

for every $\lambda \in \mathbb{N}$, where v and ℓ are the maximum of the size of the vectors that A_1 and P output, respectively. Furthermore $\mathbf{Q}^{\mathsf{ro}}_S = \mathbf{Q}^{\mathsf{ro}}_{A_1}$; $\mathbf{Q}^{\mathsf{ro}}_D = \mathbf{Q}^{\mathsf{ro}}_{A_2}$; $\mathbf{T}(\mathrm{UCE}^{S,D}_{\mathsf{H}}) \le \mathbf{T}(\text{IND\$-CDA}^{A}_{\mathsf{CE[H]}})$, and $\mathbf{Q}^{\mathsf{Hash}}_S \le 2v$. ▮

By combining Theorem 5.2 and Proposition 7.1, we have the following result. This is the final end-product of our methodology, namely a FIL-ROM scheme for the application in question.

Proposition 7.2 Construct $\mathsf{H}_{\mathsf{rom}}$ from a PRP E as in Section 5. Let F be an AU hash function family with $\mathsf{F.ol} = \mathsf{H}_{\mathsf{rom}}.\mathsf{il}$ and $\mathsf{F.IL} = \mathbb{N}$. Let $\mathsf{H} = \mathsf{AU\text{-}then\text{-}Hash}[\mathsf{F}, \mathsf{H}_{\mathsf{rom}}]$.

Asymptotic result: CE[H] is IND$-CDA-secure.

Concrete result: Let $A = (A_1, A_2)$ be an IND$-CDA adversary. We then can construct an adversary B attacking E such that

$$\mathsf{Adv}^{\mathsf{cda}}_{\mathsf{CE[H]},A}(\lambda) \le v\sqrt{2q \cdot \mathrm{Guess}_A(\lambda)} + 2v \cdot \mathsf{Adv}^{\mathsf{prp}}_{\mathsf{E},B}(\lambda) + 2\sqrt{q\mathsf{Adv}^{\mathsf{coll}}_{\mathsf{F}}(\lambda, 2v, \lambda \cdot (v+s))} + \frac{2s^2 + q^2 + 4v}{2^\lambda}$$

for all $\lambda \in \mathbb{N}$, where $q = \mathbf{Q}^{\mathsf{ro}}_A$, s is the total λ-bit blocks in the string vector $\mathbf{m}$ that A_1 outputs, and $v = \lvert\mathbf{m}\rvert$. Moreover, $\mathbf{T}(\mathrm{PRP}^{B}_{\mathsf{E}}) \le \mathbf{T}(\text{IND\$-CDA}^{A}_{\mathsf{CE[FastHash]}})$ and $\mathbf{Q}^{\mathsf{LR}}_B$ is the maximum of the number of λ-bit blocks in the components of $\mathbf{m}$. ▮

Finally we can obtain a concrete scheme by instantiating F and E in Proposition 7.2. Namely, let $\lambda = 128$ and $\mathsf{H} = \mathsf{FastHash} = \mathsf{AU\text{-}then\text{-}Hash}[\mathsf{F}_{\mathsf{aes4}}, \mathsf{H}_{\mathsf{rom}}]$. Let $\mathsf{E} = \mathrm{AES}$. Then from Proposition 7.2 we have the following corollary. For any IND$-CDA adversary $A = (A_1, A_2)$, one can construct another adversary B attacking AES such that

$$\mathsf{Adv}^{\mathsf{cda}}_{\mathsf{CE[FastHash}],A} \le v\sqrt{2q \cdot \mathrm{Guess}_A} + 2v \cdot \mathsf{Adv}^{\mathsf{prp}}_{\mathrm{AES},B} + \frac{2s^2 + q^2 + (16v + 4s)\sqrt{2vq}}{2^{128}} + \frac{v\sqrt{q}}{2^{107}}$$

where $q = \mathbf{Q}^{\mathsf{ro}}_A$, s is the total 128-bit blocks in the string vector $\mathbf{m}$ that A_1 outputs, and $v = \lvert\mathbf{m}\rvert$. Moreover, $\mathbf{T}(\mathrm{PRP}^{B}_{\mathrm{AES}}) \le \mathbf{T}(\text{IND\$-CDA}^{A}_{\mathsf{CE[FastHash]}})$ and $\mathbf{Q}^{\mathsf{LR}}_B$ is the maximum of the number of 128-bit blocks in the components of $\mathbf{m}$.

Hash function	Setting	Speed (cycles per byte)		
		1MB	16MB	128MB
SHA-256 [1]		11.5	12.0	12.0
FastHash	sequential	2.1	2.2	2.2
	parallel - 12 threads	0.4	0.4	0.5

Figure 8: **Running time of the hash constructions.** The first column lists the hash names, the second column lists the setting, namely sequential or parallel, along with the number of threads, and the last three columns list the running time on messages of sizes 1MB, 16MB, and 128MB respectively.

8 Implementation

In this section, we'll describe how to instantiate the AU hash $\mathsf{F}_{\mathsf{aes4}}$ in Section 6, and the FIL UCE-secure hash $\mathsf{H}_{\mathsf{rom}}$ in Section 5. We then compare the speed of FastHash, the resulting instantiation of AU-then-Hash$[\mathsf{F}_{\mathsf{aes4}}, \mathsf{H}_{\mathsf{rom}}]$, with a standard hash function, SHA-256. We first describe our choices for components and parameters to instantiate the construction, and then provide an overview of the implementation, before outlining the testing environment and test specifications. We also compare the convergent encryption (CE) MLE scheme[2]from FastHash and SHA-256. Our results indicate a speedup of 5.3x for our hash function over SHA-256 and 6.3x for CE in the sequential setting, and 24x and 20x speedups, respectively, once parallelism is enabled.

Instantiations. To instantiate $\mathsf{F}_{\mathsf{aes4}}$, we use the standard irreducible polynomial $p(x) = x^{127} + x^7 + x^2 + x + 1$ for multiplication over GF(2^{128}). For $\mathsf{H}_{\mathsf{rom}}$, the FIL RO is instantiated by the compression function of SHA-256, and the PRP by AES128.

Implementation. We implemented FastHash in C with inline assembly. We used Intel's library for multiplication over $\mathbb{GF}(2^{128})$ [3], Intel's optimized SHA256 implementation [1], and Intel's AES-NI library [2] for the code involving AES operations. We used the pthreads library for implementing threads for parallelization.

Setup. We performed experiments on an Intel Core i7-970 processor clocking at 3201 MHz with a 12288 KB L1 cache. The machine provides hardware support for SSE4 vector instructions, AES operations (AES-NI), and multiplication in $\mathbb{GF}(2^{128})$. Tests were compiled with gcc version 4.6 optimization level -O3, with support for SSE4 via -msse4 flag, AES-NI instructions through the -maes flag, $\mathbb{GF}(2^{128})$ multiplications via the -mpcmulqdq flag, and parallelization via the -pthread flag. We ran the tests in isolation, after turning off processor frequency scaling. We used the rdtsc instruction to count cycles.

Experiments. We measured the performance of instantiations of the hash functions (i.e. FastHash and SHA-256) as well as CE schemes based on these hash functions on messages of lengths 1MB, 16MB and 128MB. In each case, we measured the median running times of the different hash functions over 100 iterations, repeated this process 100 times and obtained the mean of the medians.

In the case of parallelizable constructions, viz. FastHash and CE[FastHash], we ran tests with multiple levels of parallelism, starting from single-threaded, serial constructions, and increasing the number of threads until we reached a point of thrashing where the performance starts to deteriorate because of other bottlenecks in the system. We report both the single-thread sequential running time, and the optimal parallel running time along with the optimal number of threads. In the latter case, the reported time does not include the time to create and destroy the threads.

In Fig. 8, we report the median running times of the hash function instantiations, in cycles per byte. We compare these times with the best times reported for SHA-256 on similar processors [1]. Our construction

[2] In CE [7], one first hashes the message x to derive a key K, and then runs AES-CTR on key K to encrypt x. To use FastHash on CE, one needs to use the CE variant of [6], in which AES-CTR on message m is replaced by FastHash$(hk, K, 1^{|x|}) \oplus x$. Note that this doesn't give us any speed advantage over the standard version of CE, as the masking via FastHash is essentially AES-CTR. The only thing we gain is the abstraction of AES as part of the hash, so that one can apply UCE$[\mathcal{S}^{\text{sup}}]$.

MLE Scheme	Setting	Speed (cycles per byte)		
		1MB	16MB	128MB
CE implementation in [7]		22.1	22.3	22.6
CE[FastHash]	sequential	3.5	3.6	3.7
	parallel - 12 threads	1.2	1.1	1.1

Figure 9: **Running time of CE instantiations.** The first column lists the instantiations, the second column lists the setting, namely sequential or parallel, along with the number of threads, and the last three columns list the running time (key generation + encryption) on messages of size 1MB, 16MB, and 128MB respectively.

achieves substantially better running times. On messages of 1MB, SHA runs at 11.5 cycles per byte, but our instantiation runs more than 5.3 times faster, at a cost of 2.1 cycles per byte. With parallelism, we achieve much better speeds, below one cycle per byte.

In Fig. 9, we demonstrate the benefits of having faster hash functions by comparing the speeds of CE implemented with FastHash with the implementation of CE by SHA-256 and AES-CTR in [7]. Our experiments show that CE[FastHash], even in the sequential setting, is about 6.3x faster than the speeds reported in [7]. When parallelism enabled, we achieve about 20x speedup.

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A Proofs of Theorem 4.1

For simplicity in analyzing concrete security, we'll assume that for any $N, q_1, \ldots, q_N \in \mathbb{N}$, we have

$$\sum_{i=1}^{N} T_{\overline{\mathsf{M}},q_i} \le T_{\overline{\mathsf{M}},q} \quad \text{and} \quad \sum_{i=1}^{N} Q_{\overline{\mathsf{M}},q_i} \le Q_{\overline{\mathsf{M}},q},$$

where $q = q_1 + \cdots + q_N$. That is, the maximum cost of running N separate instances of $\overline{\mathsf{M}}$ for $q_1, \ldots, q_N$ inputs, all starting from states $\mathrm{st} = \varepsilon$, doesn't exceed the cost of running a single instance of $\overline{\mathsf{M}}$ starting from state $\mathrm{st} = \varepsilon$ for q inputs.

In this proof, we'll use the following conventions. First, ro refers to the random oracle that everybody has access in UCE/Reset games, $\overline{\mathsf{RO}}$ and $\overline{\mathsf{ro}}$ refer to the VIL and FIL random oracles in the definition of indifferentiability, and roSim refers to the simulated random-oracle interface implemented by our constructed adversaries, respectively.

We'll give a proof for $\mathsf{UCE}[\mathcal{S}^{\mathrm{crs}}]$; the proof for $\mathsf{UCE}[\mathcal{S}^{\mathrm{srs}}]$ is the same. Let S be a PT computationally reset-secure N-key source and D be a PT distinguisher. Consider the following source $\overline{S}$.

$\underline{\overline{S}^{\mathsf{ro}}(1^\lambda, \varepsilon)}$
$(1^n, t) \leftarrow\!\!\$\ S^{\mathsf{ro}}(1^\lambda, \varepsilon)$; Return $(1^n, t)$
$\underline{\overline{S}^{\mathsf{Hash},\mathsf{ro}}(1^\lambda, t)}$
$L \leftarrow\!\!\$\ S^{\mathsf{HashSim},\mathsf{ro}}(1^\lambda, t)$; Return L

$\underline{\mathsf{HashSim}(x, 1^\ell, i)}$
$y \leftarrow \mathsf{M}^{\mathsf{Hash}(\cdot, 1^{\mathsf{H.ol}(\lambda)}, i)}(1^\lambda, x)$; Return y

Next, we'll reduce (S, D) to an adversary A attacking the indifferentiability of M. Note that here S may have access up to $N(\lambda)$ oracles Hash, yet A has only two oracles Prim and Func. To resolve this, we'll employ a hybrid argument. Adversary A picks $r \leftarrow\!\!\$\ \{1, \ldots, N(\lambda)\}$, and implements VIL random oracles $\mathsf{RO}_1, \ldots, \mathsf{RO}_{N(\lambda)} : \{0,1\}^* \to \{0,1\}^{\mathsf{M.fol}(\lambda)}$ and FIL random oracle $\mathsf{ro}_1, \ldots, \mathsf{ro}_{N(\lambda)} : \{0,1\}^{\mathsf{M.pil}(\lambda)} \to \{0,1\}^{\mathsf{M.pol}(\lambda)}$. For a query on the ith oracle, if $i = r$ then it queries its Func oracle. Otherwise, it responds by querying RO_i if $i < r$, and running $\mathsf{M}^{\mathsf{ro}_i}$ if $i > r$. The code of A is shown below.

$\underline{A^{\mathsf{Prim},\mathsf{Func}}(1^\lambda)}$
$r \leftarrow\!\!\$\ \{1, \ldots, N(\lambda)\}$
$(1^n, t) \leftarrow\!\!\$\ S^{\mathsf{roSim}}(1^\lambda, \varepsilon)$; $L \leftarrow\!\!\$\ S^{\mathsf{HashSim},\mathsf{roSim}}(1^\lambda, t)$
For $i = 1$ to n do $\mathbf{hk}[i] \leftarrow\!\!\$\ \mathsf{H.Kg}(1^\lambda)$
$b' \leftarrow\!\!\$\ D^{\mathsf{roSim}}(1^\lambda, \mathbf{hk}, L)$; Return b'

$\underline{\mathsf{HashSim}(x, 1^\ell, i)}$
If $i = r$ then $(y \leftarrow \mathsf{Func}(x)$; Return $y)$
If $i < r$ then return $\mathsf{RO}_i(x)$
Else return $\mathsf{M}^{\mathsf{ro}_i}(1^\lambda, x)$

Let $\overline{\mathsf{M}}$ be the corresponding PT universal simulator for M. Let a, b, and $\overline{b}$ be the challenge bits of games $\mathrm{Indiff}^{A,\overline{\mathsf{M}}}_{\mathsf{M}}$, $\mathrm{UCE}^{S,D}_{\mathsf{H}}$, and $\mathrm{UCE}^{\overline{S},D}_{\overline{\mathsf{H}}}$ respectively. Then

$$\Pr[\,\mathrm{UCE}^{\overline{S},D}_{\overline{\mathsf{H}}}(\cdot) \Rightarrow \mathsf{true} \,|\, \overline{b} = 1\,] = \Pr[\,\mathrm{UCE}^{S,D}_{\mathsf{H}}(\cdot) \Rightarrow \mathsf{true} \,|\, b = 1\,] \ .$$

Moreover,

$$\begin{aligned} & \Pr[\,\mathrm{UCE}^{\overline{S},D}_{\overline{\mathsf{H}}}(\cdot) \Rightarrow \mathsf{false} \,|\, \overline{b} = 0\,] - \Pr[\,\mathrm{UCE}^{S,D}_{\mathsf{H}}(\cdot) \Rightarrow \mathsf{false} \,|\, b = 0\,] \\ = \quad & N \cdot \big(\Pr[\,\mathrm{Indiff}^{A,\overline{\mathsf{M}}}_{\mathsf{M}}(\cdot) \Rightarrow \mathsf{true} \,|\, a = 1\,] - \Pr[\,\mathrm{Indiff}^{A,\overline{\mathsf{M}}}_{\mathsf{M}}(\cdot) \Rightarrow \mathsf{false} \,|\, a = 0\,]\big) \ . \end{aligned}$$

Subtracting, we have $\mathsf{Adv}^{\mathsf{uce}}_{\overline{\mathsf{H}},\overline{S},D}(\cdot) = \mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\cdot) - N \cdot \mathsf{Adv}^{\mathsf{indiff}}_{A,\overline{\mathsf{M}},\mathsf{M}}(\cdot)$, thus the UCE security of H follows from the UCE security of $\overline{\mathsf{H}}$ and the indifferentiability of M. What's left is to prove that $\overline{S}$ is computationally reset-secure. Let $\overline{R}$ be a PT reset adversary. Consider the following reset adversary R.

$\underline{R^{\mathsf{Hash},\mathsf{ro}}(1^\lambda, 1^n, L)}$
For $i = 1$ to n do $\mathrm{st}_i \leftarrow \varepsilon$
$b' \leftarrow\!\!\$\ \overline{R}^{\mathsf{HashSim},\mathsf{ro}}(1^\lambda, 1^n, L)$; Return b'

$\underline{\mathsf{HashSim}(x, 1^\ell, i)}$
$(y, \mathrm{st}_i) \leftarrow\!\!\$\ \overline{\mathsf{M}}^{\mathsf{Hash}(\cdot, 1^{\mathsf{H.ol}(\lambda)}, i)}(1^\lambda, \mathrm{st}_i, x)$; Return y

In the code above, R maintains $N(\lambda)$ independent instances of $\overline{\mathsf{M}}$. We now describe how to reduce $(S, \overline{R})$ to an adversary B attacking the indifferentiability of M. Adversary B implements VIL random oracles $\mathsf{RO}_1, \ldots, \mathsf{RO}_{2N(\lambda)} : \{0,1\}^* \to \{0,1\}^{\mathsf{M.fol}(\lambda)}$ and FIL random oracles $\mathsf{ro}_1, \ldots, \mathsf{ro}_{2N(\lambda)} : \{0,1\}^{\mathsf{M.pil}(\lambda)} \to \{0,1\}^{\mathsf{M.pol}(\lambda)}$. It also maintains $N(\lambda)$ independent instances of $\overline{\mathsf{M}}$. The code of B is shown below.

$\underline{B^{\mathsf{Prim},\mathsf{Func}}(1^\lambda)}$
$s \leftarrow\!\!\$\ \{1, 2, 3\}$; $r \leftarrow\!\!\$\ \{1, \ldots, N(\lambda)\}$
$(1^n, t) \leftarrow\!\!\$\ S^{\mathsf{roSim}}(1^\lambda, \varepsilon)$
For $i = 1$ to n do $\mathrm{st}_i \leftarrow \varepsilon$
$L \leftarrow\!\!\$\ S^{\mathsf{Hash},\mathsf{roSim}}(1^n, t)$
$b' \leftarrow\!\!\$\ \overline{R}^{\mathsf{HashSim},\mathsf{roSim}}(1^\lambda, 1^n, L)$
If $s = 1$ then return b'
Else return $1 - b'$

$\underline{\mathsf{Hash}(x, 1^\ell, i)}$
If $s = 2$ then $j \leftarrow i + N(\lambda)$ else $j \leftarrow i$
If $i < r$ then return $\mathsf{RO}_j(x)$ elsif $i > r$ then return $\mathsf{M}^{\mathsf{ro}_j}(1^\lambda, x)$
If $s = 2$ then return $\mathsf{RO}_j(x)$ else return $\mathsf{Func}(x)$

$\underline{\mathsf{HashSim}(x, 1^\ell, i)}$
If $s = 3$ then $j \leftarrow i + N(\lambda)$ else $j \leftarrow i$
If $i < r$ then $(y, \mathrm{st}_i) \leftarrow\!\!\$\ \overline{\mathsf{M}}^{\mathsf{RO}_j}(1^\lambda, \mathrm{st}_i, x)$; Return y
Elsif $i > r$ return $\mathsf{ro}_j(x)$
If $s = 3$ then return $\mathsf{ro}_j(x)$ else return $\mathsf{Prim}(x)$

Let c, d, and $\overline{d}$ be challenge bits of game $\mathrm{Indiff}^{B,\overline{\mathsf{M}}}_{\mathsf{M}}$, Reset^{R}_{S}, and $\mathrm{Reset}^{\overline{R}}_{\overline{S}}$ respectively. Let $s \in \{1,2,3\}$ be the random coin that B samples. Then

$$\begin{aligned} & \Pr[\,\mathrm{Reset}^{R}_{S}(\cdot) \Rightarrow \mathsf{true} \,|\, d = 1\,] - \Pr[\,\mathrm{Reset}^{\overline{R}}_{\overline{S}}(\cdot) \Rightarrow \mathsf{true} \,|\, \overline{d} = 1\,] \\ = \;& N \cdot (\Pr[\,\mathrm{Indiff}^{B,\overline{\mathsf{M}}}_{\mathsf{M}}(\cdot) \Rightarrow \mathsf{false} \,|\, (c = 0) \wedge (s = 1)\,] - \Pr[\,\mathrm{Indiff}^{B,\overline{\mathsf{M}}}_{\mathsf{M}}(\cdot) \Rightarrow \mathsf{true} \,|\, (c = 1) \wedge (s = 1)\,]) \end{aligned}$$

On the other hand,

$$\begin{aligned} & \Pr[\,\mathrm{Reset}^{R}_{S}(\cdot) \Rightarrow \mathsf{false} \,|\, d = 0\,] - \Pr[\,\mathrm{Reset}^{\overline{R}}_{\overline{S}}(\cdot) \Rightarrow \mathsf{false} \,|\, \overline{d} = 0\,] \\ = \;& N \cdot (\Pr[\,\mathrm{Indiff}^{B,\overline{\mathsf{M}}}_{\mathsf{M}}(\cdot) \Rightarrow \mathsf{true} \,|\, (c = 0) \wedge (s = 2)\,] - \Pr[\,\mathrm{Indiff}^{B,\overline{\mathsf{M}}}_{\mathsf{M}}(\cdot) \Rightarrow \mathsf{false} \,|\, (c = 1) \wedge (s = 3)\,]) \end{aligned}$$

Moreover,

$$\Pr[\,\mathrm{Indiff}^{B,\overline{\mathsf{M}}}_{\mathsf{M}}(\cdot) \Rightarrow \mathsf{false} \,|\, (c = 1) \wedge (s = 2)\,] = \Pr[\,\mathrm{Indiff}^{B,\overline{\mathsf{M}}}_{\mathsf{M}}(\cdot) \Rightarrow \mathsf{true} \,|\, (c = 0) \wedge (s = 3)\,] \ .$$

Summing up,

$$\begin{aligned} \mathsf{Adv}^{\mathsf{reset}}_{S,R}(\cdot) &= \mathsf{Adv}^{\mathsf{reset}}_{\overline{S},\overline{R}}(\cdot) - N \sum_{j=1}^{3} \big(2\Pr[\,\mathrm{Indiff}^{B,\overline{\mathsf{M}}}_{\mathsf{M}}(\cdot) \Rightarrow \mathsf{true} \,|\, s = j\,] - 1\big) \\ &= \mathsf{Adv}^{\mathsf{reset}}_{\overline{S},\overline{R}}(\cdot) - 3N\mathsf{Adv}^{\mathsf{indiff}}_{B,\overline{\mathsf{M}},\mathsf{M}}(\cdot) \ . \end{aligned}$$

The computational reset-security of $\overline{S}$ then follows from the computational reset-security of S and the fact that M is a PRO.

B Proof of Theorem 5.1

Let S be a PT statistically unpredictable source and D be a PT distinguisher. Consider the following source $\overline{S}$ and distinguisher $\overline{D}$.

$\underline{\overline{S}^{\mathsf{ro}}(1^\lambda, \varepsilon)}$
$(1^n, t) \leftarrow\!\!\$\ S^{\mathsf{ro}}(1^\lambda, \varepsilon)$; $\overline{t} \leftarrow (1^\lambda, t)$; Return $(1^n, \overline{t})$
$\underline{\overline{S}^{\mathsf{Hash},\mathsf{ro}}(1^n, \overline{t})}$
For $i = 1$ to n do $\mathbf{fk}[i] \leftarrow\!\!\$\ \mathsf{F.Kg}(1^\lambda)$
$(1^\lambda, t) \leftarrow \overline{t}$; $L \leftarrow\!\!\$\ S^{\mathsf{HashSim},\mathsf{ro}}(1^n, t)$; Return $(\mathbf{fk}, L)$
$\underline{\mathsf{HashSim}(x, 1^\ell, i)}$
$u \leftarrow \mathsf{F.Ev}(1^\lambda, \mathbf{fk}[i], x, 1^{\mathsf{F.ol}(\lambda)})$; Return $\mathsf{Hash}(u, 1^\ell, i)$

$\underline{\overline{D}^{\mathsf{ro}}(1^\lambda, \overline{\mathbf{hk}}, \overline{L})}$
$(\mathbf{fk}, L) \leftarrow \overline{L}$; $\mathbf{hk} \leftarrow (\mathbf{fk}, \overline{\mathbf{hk}})$
$b' \leftarrow\!\!\$\ D^{\mathsf{ro}}(1^\lambda, \mathbf{hk}, L)$; Return b'

Let b and $\overline{b}$ be the challenge bits of games $\mathrm{UCE}^{S,D}_{\mathsf{H}}$, and $\mathrm{UCE}^{\overline{S},\overline{D}}_{\overline{\mathsf{H}}}$ respectively. Then

$$\Pr[\,\mathrm{UCE}^{\overline{S},\overline{D}}_{\overline{\mathsf{H}}}(\cdot) \Rightarrow \mathsf{true} \,|\, \overline{b} = 1\,] = \Pr[\,\mathrm{UCE}^{S,D}_{\mathsf{H}}(\cdot) \Rightarrow \mathsf{true} \,|\, b = 1\,] \ .$$

Wlog, assume that S doesn't repeat a prior query to Hash. Consider the following games H_1 and H_2, where game H_1 contains the boxed statement but game H_2 does not.

$\underline{\mathsf{Game}\ \boxed{H^{S,D}_1(\lambda)},\ H^{S,D}_2(\lambda)}$
$(1^n, t) \leftarrow\!\!\$\ S^{\mathsf{ro}}(1^\lambda)$
For $i = 1$ to n do
  $\mathbf{fk}[i] \leftarrow\!\!\$\ \mathsf{F.Kg}(1^\lambda)$; $\overline{\mathbf{hk}}[i] \leftarrow\!\!\$\ \overline{\mathsf{H}}.\mathsf{Kg}(1^\lambda)$; $\mathbf{hk}[i] \leftarrow (\mathbf{fk}[i], \overline{\mathbf{hk}}[i])$
$L \leftarrow\!\!\$\ S^{\mathsf{HashSim},\mathsf{ro}}(1^n, t)$; $b' \leftarrow\!\!\$\ D^{\mathsf{ro}}(1^\lambda, \mathbf{hk}, L)$; Return $(b' = 1)$

$\underline{\mathsf{HashSim}(x, 1^\ell, i)}$
$u \leftarrow \mathsf{F.Ev}(1^\lambda, \mathbf{fk}[i], x, 1^{\mathsf{F.ol}(\lambda)})$
$y \leftarrow\!\!\$\ \{0,1\}^\ell$
If $H[u, \ell, i] \neq \perp$ then
  $\mathsf{bad} \leftarrow \mathsf{true}$; $\boxed{y \leftarrow H[u, \ell, i]}$
$H[u, \ell, i] \leftarrow y$; Return y

Then

$$\Pr[\,\mathrm{UCE}^{\overline{S},\overline{D}}_{\overline{\mathsf{H}}}(\cdot) \Rightarrow \mathsf{false} \,|\, \overline{b} = 0\,] = \Pr[H_1^{S,D}(\cdot)] \quad \text{and} \quad \Pr[\,\mathrm{UCE}^{S,D}_{\mathsf{H}}(\cdot) \Rightarrow \mathsf{false} \,|\, b = 0\,] = \Pr[H_2^{S,D}(\cdot)] \ .$$

Let σ be a polynomial that bounds the total length of the first components in Hash queries in $\mathrm{UCE}^{S,D}_{\mathsf{H}}$, and $p = \mathbf{Q}_S^{\mathsf{Hash}}$. Since the games H_1 and H_2 are identical-until-bad, by the Fundamental lemma of game-playing [10],

$$\Pr[H_1^{S,D}(\cdot)] - \Pr[H_2^{S,D}(\cdot)] \le \Pr[H_2^{S,D}(\cdot) \text{ sets } \mathsf{bad}] \ .$$

Suppose that S makes ℓ queries whose first components have length $m_1, \ldots, m_\ell$. Then the chance that game $H_2^{S,D}$ sets bad is at most

$$\sum_{i=1}^{\ell} \sum_{j=1}^{\ell} \mathbf{Coll2}_{\mathsf{F}}(\lambda, m_i, m_j) \le \mathsf{Adv}^{\mathsf{coll}}_{\mathsf{F}}(\lambda, p, \sigma) \ .$$

Hence

$$\mathsf{Adv}^{\mathsf{uce}}_{\overline{\mathsf{H}},\overline{S},\overline{D}}(\lambda) \ge \mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\lambda) - \mathsf{Adv}^{\mathsf{coll}}_{\mathsf{F}}(\lambda, p, \sigma) \ .$$

We claim that $\overline{S}$ is statistically unpredictable, and thus the UCE-security of H follows from the UCE-security of $\overline{\mathsf{H}}$. To justify this claim, let $\overline{P}$ be an arbitrary predictor. Since $\overline{P}$ is computationally-unbounded and can make as many ro queries as it likes, wlog, assume that $\overline{P}$ is deterministic. Let ρ_1 and ρ_2 be polynomials that bound the number of coins that S uses in the first and the second phases in executing $\mathrm{UCE}^{S,D}_{\mathsf{H}}$, respectively. Let μ be a polynomial that bounds the total output length of Hash queries in executing $\mathrm{UCE}^{S,D}_{\mathsf{H}}$. Let T be the set of (x,i) in from the queries $(x, 1^\ell, i)$ that S makes. Consider the predictor P below. Here $\mathsf{Sample}^{\mathsf{ro}}(1^\lambda, n, L)$ is an algorithm that computes the multiset $\mathcal{T}$ such that the random variable T, given that the leakage L of S and the number of hash keys n, is uniformly distributed over $\mathcal{T}$, and then return a uniformly random element T' of $\mathcal{T}$. Then $|T'| \le p(\lambda)$.

$\underline{P^{\mathsf{ro}}(1^\lambda, 1^n, L)}$
$T' \leftarrow\!\!\$\ \mathsf{Sample}^{\mathsf{ro}}(1^\lambda, n, L)$; $Q' \leftarrow \emptyset$
For $(x,i) \in T$ do $Q' \leftarrow Q' \cup \{x\}$
Return Q'

$\underline{\mathsf{Sample}^{\mathsf{ro}}(1^\lambda, n, L)}$
$\mathcal{T} \leftarrow \emptyset$ // $\mathcal{T}$ is a multiset
For $(r_1, r_2, R) \in \{0,1\}^{\rho_1(\lambda)} \times \{0,1\}^{\rho_2(\lambda)} \times \{0,1\}^{\mu(\lambda)}$ do
  $(1^d, t) \leftarrow S^{\mathsf{ro}}(1^\lambda, \varepsilon; r_1)$; $Q \leftarrow \emptyset$; $j \leftarrow 1$
  $L' \leftarrow\!\!\$\ S^{\mathsf{HashSim},\mathsf{ro}}(1^d, t; r_2)$; $R \leftarrow R[1, j-1]$
  If $(L' = L) \wedge (M[r_1, r_2, R] = \perp) \wedge (d = n)$ then $\mathcal{T} \leftarrow \mathcal{T} \cup \{\mathcal{T}\}$
  $M[r_1, r_2, R] \leftarrow 1$
$T' \leftarrow\!\!\$\ \mathcal{T}$; Return T'

$\underline{\mathsf{HashSim}(x, 1^\ell, i)}$
$T \leftarrow T \cup \{(x,i)\}$; $y \leftarrow R[j, j+\ell-1]$; $j \leftarrow j + \ell$; Return y

Consider games G_1–G_4 in Fig. 10, where games G_1 and G_4 contain the corresponding boxed statements, but games G_2 and G_3 do not. We explain the game chain up to the terminal one. Game $G_1^{S,\overline{P}}$ coincides with game $\mathrm{Pred}^{\overline{P}}_{\overline{S}}$. Game $G_2^{S,\overline{P}}$ is identical to $G_1^{S,\overline{P}}$, except for the following difference. In game G_1, the simulated $\mathsf{HashSim}$ oracle of S will return the same answer for queries $(x_0, 1^\ell, i)$ and $(x_1, 1^\ell, i)$ such that $\mathsf{F.Ev}(1^\lambda, \mathbf{fk}[i], x_0, 1^{\mathsf{F.ol}(\lambda)}) = \mathsf{F.Ev}(1^\lambda, \mathbf{fk}[i], x_1, 1^{\mathsf{F.ol}(\lambda)})$. In game G_2, $\mathsf{HashSim}$ acts as a random oracle. The two games are identical-until-bad, and thus

$$\Pr[G_1^{S,\overline{P}}(\lambda)] - \Pr[G_2^{S,\overline{P}}(\lambda)] \le \Pr[G_2^{S,\overline{P}}(\lambda) \text{ sets } \mathsf{bad}] \le \mathsf{Adv}^{\mathsf{coll}}_{\mathsf{F}}(\lambda, p, \sigma) \ .$$

Game $G_3^{S,\overline{P}}$ is identical to game $G_2^{S,\overline{P}}$, except for the following difference. Each game samples a vector $\mathbf{fk}$ of n keys for F and maintains an (initially empty) set U. For each query $(x, 1^\ell, i)$ of S, we add $u = \mathsf{F.EvF}(1^\lambda, hk[i], x, 1^{\mathsf{F.ol}(\lambda)})$ to U. Let the multiset $\mathcal{U}$ be computed as follows. Initially, $\mathcal{U} = \emptyset$. For every $T' \in \mathcal{T}$, we add the set $U' = \{\ \mathsf{F.Ev}(1^\lambda, \mathbf{fk}[i], z, 1^{\mathsf{F.ol}(\lambda)}) \ :\ (z,i) \in T'\ \}$ as a new element of $\mathcal{U}$. Then the

$\underline{\mathsf{Game}\ \boxed{G_1^{S,\overline{P}}(\lambda)},\ G_2^{S,\overline{P}}(\lambda)}$	$\underline{\mathsf{Game}\ G_3^{S,\overline{P}}(\lambda),\ \boxed{G_4^{S,\overline{P}}(\lambda)}}$
$(1^n,t) \leftarrow\!\!\$\ S^{\mathsf{ro}}(1^\lambda,\varepsilon)\ ;\ T,U \leftarrow \emptyset$	$(1^n,t) \leftarrow\!\!\$\ S^{\mathsf{ro}}(1^\lambda,\varepsilon)\ ;\ Q,T,U,U' \leftarrow \emptyset$
For $i=1$ to n do $\mathbf{fk}[i] \leftarrow\!\!\$\ \mathsf{F.Kg}(1^\lambda)$	For $i=1$ to n do $\mathbf{fk}[i] \leftarrow\!\!\$\ \mathsf{F.Kg}(1^\lambda)$
$L \leftarrow\!\!\$\ S^{\mathsf{HashSim},\mathsf{ro}}(1^n,t)$	$L \leftarrow\!\!\$\ S^{\mathsf{HashSim},\mathsf{ro}}(1^n,t)\ ;\ \overline{L} \leftarrow (\mathbf{fk},L)$
$\overline{L} \leftarrow (\mathbf{fk},L)\ ;\ V \leftarrow \overline{P}^{\mathsf{ro}}(1^\lambda,1^n,\overline{L})\ ;$ Return $(V \cap U \neq \emptyset)$	$T' \leftarrow\!\!\$\ \mathsf{Sample}^{\mathsf{ro}}(1^\lambda,n,L)\ ;\ V \leftarrow \overline{P}^{\mathsf{ro}}(1^\lambda,1^n,\overline{L})$
	For $(x,i) \in T'$ do $Q' \leftarrow Q' \cup \{x\}$
$\underline{\mathsf{Hash}(x,1^\ell,i)}$	If $Q \cap Q' \neq \perp$ then $\mathsf{bad} \leftarrow \mathsf{true}\ ;\ \boxed{T' \leftarrow T' \backslash T}$
If $H[x,\ell,i] = \perp$ then $H[x,\ell,i] \leftarrow\!\!\$\ \{0,1\}^\ell$	For $(x,i) \in T'$ do
Return $H[x,\ell,i]$	$\quad u \leftarrow \mathsf{F.Ev}(1^\lambda,\mathbf{fk}[i],x,1^{\mathsf{F.ol}(\lambda)})\ ;\ U' \leftarrow U' \cup \{u\}$
	Return $(V \cap U \cap U' \neq \emptyset)$
$\underline{\mathsf{HashSim}(x,1^\ell,i)}$	
$u \leftarrow \mathsf{F.Ev}(1^\lambda,\mathbf{fk}[i],x,1^{\mathsf{F.ol}(\lambda)})$	$\underline{\mathsf{Hash}(x,1^\ell,i)}$
$U \leftarrow U \cup \{u\}\ ;\ T \leftarrow T \cup \{(x,i)\}$	If $H[x,\ell,i] = \perp$ then $H[x,\ell,i] \leftarrow\!\!\$\ \{0,1\}^\ell$
For $(z,i) \in T$ do	Return $H[x,\ell,i]$
$\quad$ If $(u = \mathsf{F.Ev}(1^\lambda,\mathbf{fk}[i],z,1^{\mathsf{F.ol}(\lambda)})) \wedge (H[z,\ell,i] \neq \perp)$ then	
$\quad\quad \mathsf{bad} \leftarrow \mathsf{true}\ ;\ \boxed{\text{Return } H[z,\ell,i]}$	$\underline{\mathsf{HashSim}(x,1^\ell,i)}$
Return $\mathsf{Hash}(x,1^\ell,i)$	$T \leftarrow T \cup \{(x,i)\}\ ;\ u \leftarrow \mathsf{F.Ev}(1^\lambda,\mathbf{fk}[i],x,1^{\mathsf{F.ol}(\lambda)})$
	$U \leftarrow U \cup \{u\}\ ;\ Q \leftarrow Q \cup \{x\}\ ;$ Return $\mathsf{Hash}(x,1^\ell,i)$

Figure 10: **Games G_1–G_4 in the proof of Theorem 5.1.**

random variable U, given that the leakage L of S and the key vectors $\mathbf{fk}$ for F, is uniformly distributed over $\mathcal{U}$. In game G_2, we return $(V \cap U \neq \emptyset)$, where V is the output of $\overline{P}$. In game G_3, we choose $U' \leftarrow\!\!\$\ \mathcal{U}$ and return $(V \cap U \cap U' \neq \emptyset)$. Let q be a polynomial that bounds the size of $\overline{P}$'s output in $\mathrm{Pred}_{\overline{S}}^{\overline{P}}$, and let $V = \{v_1,\ldots,v_k\}$, with $k \leq q(\lambda)$. For each $i \leq k$, let $\mathcal{V}_i = \{\, W \in \mathcal{U} \,:\, v_i \in W \,\}$, and let $p_i = |\mathcal{V}_i|/|\mathcal{U}|$. Then

$$\Pr[v_i \in U] = \Pr[U \in \mathcal{V}_i] = p_i,$$

whereas

$$\Pr[v_i \in U \cap U'] = \Pr[(U \in \mathcal{V}_i) \wedge (U' \in \mathcal{V}_i)] = p_i^2\ .$$

In other words,

$$\big(\Pr[G_2^{S,\overline{P}}(\cdot)]\big)^2 = \Big(\sum_{i=1}^k p_i\Big)^2 \leq k \sum_{i=1}^k p_i^2 \leq q \sum_{i=1}^k p_i^2 = q \Pr[G_3^{S,\overline{P}}(\cdot)],$$

where the first inequality is due to Cauchy-Schwartz inequality. Game $G_4^{S,\overline{P}}$ is identical to game $G_3^{S,\overline{P}}$, except for the following difference. Let Q be the set of input x in Hash queries $(x,1^\ell,i)$ of S, and let Q' be the output of P, which is derived from $T' \leftarrow\!\!\$\ \mathsf{Sample}^{\mathsf{ro}}(1^\lambda,n,L)$. In game G_4, if $Q \cap Q' \neq \emptyset$, then we replace the set T' by $T' \backslash T$, before we compute the set U' from T'. Since $Q \cap Q' \neq \emptyset$ whenever $T \cap T' \neq \emptyset$, it follows that in game G_4, when we compute U' from T', we always have $T \cap T' = \emptyset$. The two games are identical-until-bad, and thus

$$\Pr[G_3^{S,\overline{P}}(\cdot)] - \Pr[G_4^{S,\overline{P}}(\cdot)] \leq \Pr[G_4^{S,\overline{P}}(\cdot) \text{ sets } \mathsf{bad}] = \Pr[\mathrm{Pred}_{\mathsf{H}}^{S,P}(\cdot)]\ .$$

As game $G_4^{S,\overline{P}}$ returns $(V \cap U \cap U' \neq \emptyset)$, it follows that $\Pr[G_4^{S,\overline{P}}(\cdot)]$ is bounded by the chance that $U \cap U' \neq \emptyset$. The latter happens if there are $(x_0,i) \in T$ and $(x_1,j) \in T'$ such that

$$\mathsf{F.Ev}(1^\lambda,\mathbf{fk}[i],x_0,1^{\mathsf{F.ol}(\lambda)}) = \mathsf{F.Ev}(1^\lambda,\mathbf{fk}[j],x_1,1^{\mathsf{F.ol}(\lambda)})\ . \tag{7}$$

If $i = j$, since $T \cap T' = \emptyset$, it follows that $x_0 \neq x_1$, and thus Equation (7) happens with probability at most $\mathbf{Coll2}_{\mathsf{F}}(\lambda,|x_0|,|x_1|) \leq \mathbf{Coll}_{\mathsf{F}}(\lambda,|x_0|,|x_1|)$. If $i \neq j$, wlog, suppose that $|x_0| \leq |x_1|$. Since $\mathbf{fk}[i]$ is

independent of $\mathsf{F.Ev}(1^\lambda, \mathbf{fk}[j], x_1, 1^{\mathsf{F.ol}(\lambda)})$, it follows that Equation (7) happens with probability at most $\mathbf{Coll1}_{\mathsf{F}}(\lambda, |x_0|) \le \mathbf{Coll}_{\mathsf{F}}(\lambda, |x_0|, |x_1|)$. Suppose that the first components in the elements of T and T' have length $m_1, \ldots, m_\ell$ and $m'_1, \ldots, m'_{\ell'}$ respectively. Then

$$\Pr[G_4^{S,\overline{P}}(\lambda)] \le \sum_{i=1}^{\ell} \sum_{j=1}^{\ell'} \mathbf{Coll}_{\mathsf{F}}(\lambda, m_i, m'_j) \le \mathsf{Adv}^{\mathsf{coll}}_{\mathsf{F}}(\lambda, p, \sigma) \ .$$

Summing up,

$$\begin{aligned} \mathsf{Adv}^{\mathsf{pred}}_{\overline{S},\overline{P}}(\lambda) &\le \mathsf{Adv}^{\mathsf{coll}}_{\mathsf{F}}(\lambda, p, \sigma) + \sqrt{q\mathsf{Adv}^{\mathsf{pred}}_{S,P}(\lambda) + q\mathsf{Adv}^{\mathsf{coll}}_{\mathsf{F}}(\lambda, p, \sigma)} \\ &\le \sqrt{2q\mathsf{Adv}^{\mathsf{coll}}_{\mathsf{F}}(\lambda, p, \sigma)} + \sqrt{q\mathsf{Adv}^{\mathsf{pred}}_{S,P}(\lambda)} \ . \end{aligned}$$

Hence the unpredictability of $\overline{S}$ follows from the unpredictability of S.

C Constructing FIL UCE-secure hash in the FIL-ROM

BHK's CONSTRUCTION. Bellare, Hoang, and Keelveedhi [6] show how to build a $\mathsf{UCE}[\mathcal{S}^{\mathrm{crs}}]$-secure hash $\overline{\mathsf{H}}$ in the ROM via (i) $\overline{\mathsf{H}}.\mathsf{Kg}(1^\lambda)$ returns $hk \leftarrow\!\!\$\ \{0,1\}^\lambda$, and (ii) $\overline{\mathsf{H}}.\mathsf{Ev}^{\mathsf{ro}}(1^\lambda, hk, x, 1^{\overline{\mathsf{H}}.\mathsf{ol}(\lambda)})$ returns $\mathsf{ro}(hk \,\|\, x)$.

Proposition C.1 [6] The family $\overline{\mathsf{H}}$ above is $\mathsf{UCE}[\mathcal{S}^{\mathrm{crs}}]$-secure. Concretely, for any N-key source S and distinguisher D, we can construct a reset adversary R such that

$$\mathsf{Adv}^{\mathsf{uce}}_{\overline{\mathsf{H}},S,D}(\lambda) \le \mathsf{Adv}^{\mathsf{reset}}_{S,R}(\lambda) + \frac{2N(\lambda) \cdot q(\lambda) + N^2(\lambda)}{2^\lambda}$$

for every $\lambda \in \mathbb{N}$, where q is a polynomial that bounds the total number of ro queries of both S and D. The reset adversary R makes at most $q(\lambda)$ Hash queries and $q(\lambda)$ random-oracle queries, and uses the same running time as D.

FAST VOL $\mathsf{UCE}[\mathcal{S}^{\mathrm{sup}}]$-SECURE HASH. In Section 5, we described how to construct a FIL, VOL hash $\mathsf{H}_{\mathsf{rom}}$ from a PRP E in FIL-ROM. Below, we'll show that $\mathsf{H}_{\mathsf{rom}}$ is $\mathsf{UCE}[\mathcal{S}^{\mathrm{sup}}]$-secure.

Proposition C.2 Let E be a PRP. Let $\mathsf{H} = \mathsf{H}_{\mathsf{rom}}$ as constructed in Section 5.

Asymptotic result: H is $\mathsf{UCE}[\mathcal{S}^{\mathrm{sup}}]$-secure.

Concrete result: For any N-key source S and any distinguisher D, we can construct a predictor P and a PRP adversary A such that

$$\mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\lambda) \le 2p(\lambda) \cdot \mathsf{Adv}^{\mathsf{prp}}_{\mathsf{E},A}(\lambda) + \mathsf{Adv}^{\mathsf{pred}}_{S,P}(\lambda) + \frac{2s^2(\lambda) + N^2(\lambda) + q^2(\lambda)}{2^\lambda},$$

where $p = \mathbf{Q}^{\mathsf{Hash}}_S$; $q = \mathbf{Q}^{\mathsf{ro}}_S + \mathbf{Q}^{\mathsf{ro}}_D$; s is the maximum of the total of λ-blocks in the second components of Hash queries in the execution of $\mathrm{UCE}^{S,D}_{\mathsf{H}_{\mathsf{rom}}}$. Furthermore

$\mathbf{Q}^{\mathsf{LR}}_A$ is maximum of the number of λ-bit blocks in the second component of a Hash query in $\mathrm{UCE}^{S,D}_{\mathsf{H}}$
$\mathbf{T}(\mathrm{PRP}^A_{\mathsf{E}}) = \mathbf{T}(\mathrm{UCE}^{S,D}_{\mathsf{H}})$, and P outputs a set of at most $\mathbf{Q}^{\mathsf{ro}}_S + \mathbf{Q}^{\mathsf{ro}}_D$ elements. ▌

Proof: Let S be a PT statistically unpredictable N-key source and D be a PT distinguisher. Let $p = \mathbf{Q}^{\mathsf{Hash}}_S$; $q = \mathbf{Q}^{\mathsf{ro}}_S + \mathbf{Q}^{\mathsf{ro}}_D$; and s be the maximum of the total of λ-blocks in the second components of Hash queries in the execution of $\mathrm{UCE}^{S,D}_{\mathsf{H}_{\mathsf{rom}}}$.

Wlog, assume that S and D never repeat a prior query to their oracles, and assume that $q(\lambda) \le 2^{\lambda-1}$. Consider games G_1–G_8 in Figures 11 and 12. For clarity, in each game, we write ro_1 and ro_2 to denote

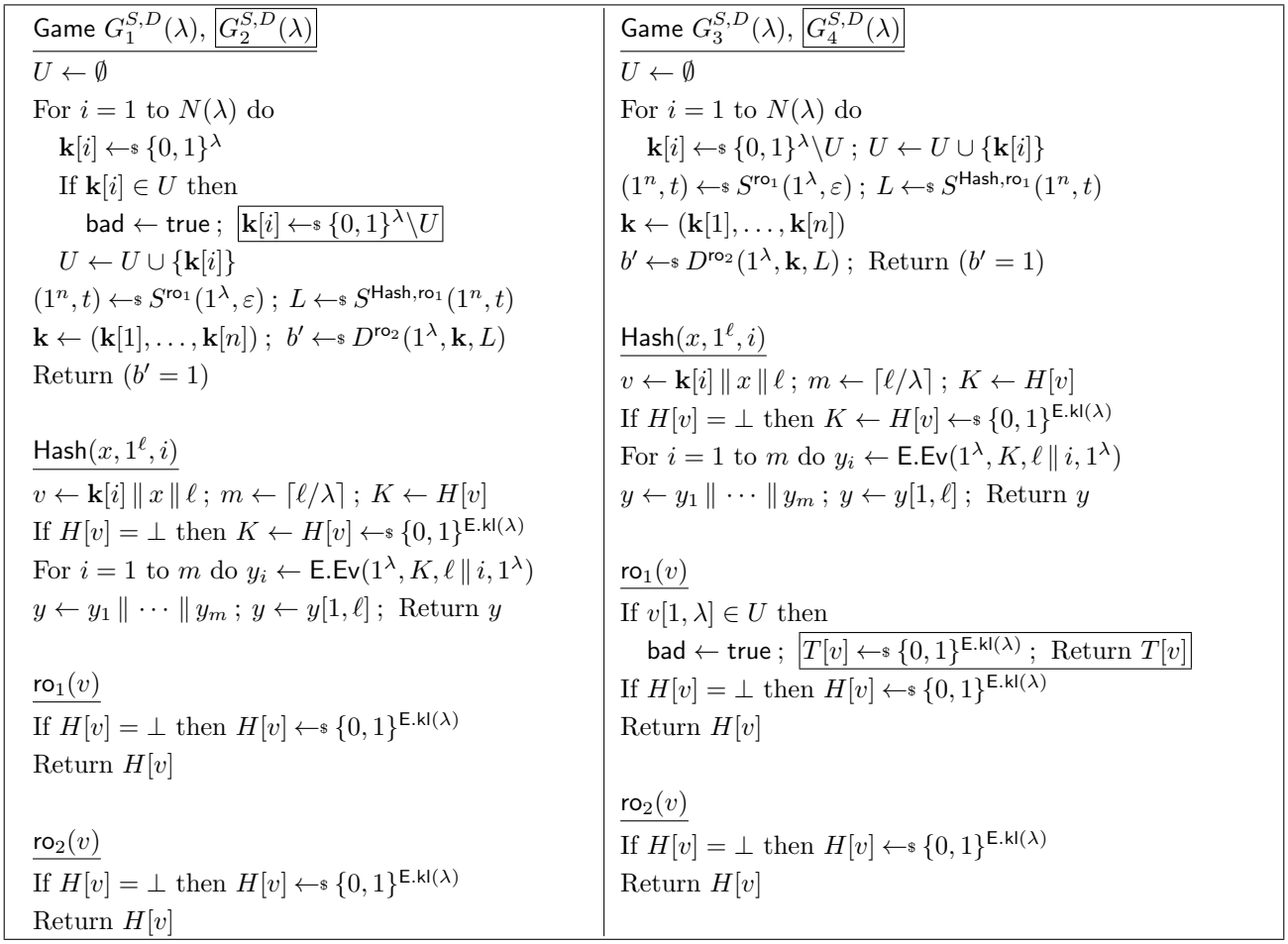

Figure 11: **Games G_1–G_4 in the proof of Proposition C.2.** Games G_2 and G_4 contain the corresponding boxed statements but game G_1 and G_3 do not.

the random oracles of S and D respectively. Game $G_1^{S,D}$ corresponds to game $\mathrm{UCE}_{\mathsf{H}_{\mathsf{rom}}}^{S,D}$ with challenge bit $b = 1$, and $\Pr[G_8^{S,D}(\cdot)] = \Pr[\,\mathrm{UCE}_{\mathsf{H}_{\mathsf{rom}}}^{S,D}(\cdot) \Rightarrow \mathsf{false} \,|\, b = 0\,]$. We explain the game chain up to the terminal one. In game $G_2^{S,D}$, instead of sampling the hash keys independently, we sample so that they are distinct. Games $G_1^{S,D}$ and $G_2^{S,D}$ are identical-until-bad, and thus

$$\Pr[G_1^{S,D}(\lambda)] - \Pr[G_2^{S,D}(\lambda)] \leq \Pr[G_2^{S,D}(\lambda) \text{ sets } \mathsf{bad}] \leq \frac{N^2(\lambda)}{2^{\lambda+1}}$$

for every $\lambda \in \mathbb{N}$. Game $G_3^{S,D}$ is the simplified version of $G_2^{S,D}$. In game $G_4^{S,D}$, if S makes a ro_1 query whose prefix is a hash key then it will get a random answer, independent of Hash and ro_2. Since whatever S receives will be independent of the hash keys and games $G_3^{S,D}$ and $G_4^{S,D}$ are identical-until-bad, it follows that

$$\Pr[G_3^{S,D}(\lambda)] - \Pr[G_4^{S,D}(\lambda)] \leq \Pr[G_4^{S,D}(\lambda) \text{ sets } \mathsf{bad}] \leq \sum_{i=0}^{q(\lambda)-1} \frac{i}{2^\lambda - i} \leq \sum_{i=0}^{q(\lambda)-1} \frac{i}{2^{\lambda-1}} \leq \frac{q^2(\lambda)}{2^\lambda}$$

for every $\lambda \in \mathbb{N}$. Game $G_5^{S,D}$ is identical to game $G_4^{S,D}$, making it explicit that for each Hash query, we sample a fresh key for E that is independent of whatever S has received so far. The reason is that inside Hash the queries to ro always have different λ-bit prefixes. In game $G_6^{S,D}$, we sample the hash keys independently. Games $G_5^{S,D}$ and $G_6^{S,D}$ are identical-until-bad, and thus

$$\Pr[G_5^{S,D}(\lambda)] - \Pr[G_6^{S,D}(\lambda)] \leq \Pr[G_6^{S,D}(\lambda) \text{ sets } \mathsf{bad}] \leq \frac{N^2(\lambda)}{2^{\lambda+1}}$$

$\underline{\mathsf{Game}\ \boxed{G_5^{S,D}(\lambda)},\ G_6^{S,D}(\lambda)}$

$U \leftarrow \emptyset$
For $i = 1$ to $N(\lambda)$ do
  $\mathbf{k}[i] \leftarrow\!\!\$\ \{0,1\}^\lambda$
  If $\mathbf{k}[i] \in U$ then
    $\mathsf{bad} \leftarrow \mathsf{true}$; $\boxed{\mathbf{k}[i] \leftarrow\!\!\$\ \{0,1\}^\lambda \backslash U}$
  $U \leftarrow U \cup \{\mathbf{k}[i]\}$
$(1^n, t) \leftarrow\!\!\$\ S^{\mathsf{ro}_1}(1^\lambda, \varepsilon)$; $L \leftarrow\!\!\$\ S^{\mathsf{Hash},\mathsf{ro}_1}(1^n, t)$
$\mathbf{k} \leftarrow (\mathbf{k}[1], \ldots, \mathbf{k}[n])$
$b' \leftarrow\!\!\$\ D^{\mathsf{ro}_2}(1^\lambda, \mathbf{k}, L)$; Return $(b' = 1)$

$\underline{\mathsf{Hash}(x, 1^\ell, i)}$
$v \leftarrow \mathbf{k}[i] \,\|\, x \,\|\, \ell$; $m \leftarrow \lceil \ell/\lambda \rceil$
$K \leftarrow H[v] \leftarrow\!\!\$\ \{0,1\}^{\mathsf{E.kl}(\lambda)}$
For $i = 1$ to m do $y_i \leftarrow \mathsf{E.Ev}(1^\lambda, K, \ell \,\|\, i, 1^\lambda)$
$y \leftarrow y_1 \,\|\, \cdots \,\|\, y_m$; $y \leftarrow y[1, \ell]$; Return y

$\underline{\mathsf{ro}_1(v)}$
If $v[1, \lambda] \in U$ then
  $T[v] \leftarrow\!\!\$\ \{0,1\}^{\mathsf{E.kl}(\lambda)}$; Return $T[v]$
If $H[v] = \perp$ then $H[v] \leftarrow\!\!\$\ \{0,1\}^{\mathsf{E.kl}(\lambda)}$
Return $H[v]$

$\underline{\mathsf{ro}_2(v)}$
If $H[v] = \perp$ then $H[v] \leftarrow\!\!\$\ \{0,1\}^{\mathsf{E.kl}(\lambda)}$
Return $H[v]$

$\underline{\mathsf{Game}\ G_7^{S,D}(\lambda),\ \boxed{G_8^{S,D}(\lambda)}}$

$Q, U \leftarrow \emptyset$
For $i = 1$ to $N(\lambda)$ do $\mathbf{k}[i] \leftarrow\!\!\$\ \{0,1\}^\lambda$; $U \leftarrow U \cup \{\mathbf{k}[i]\}$
$(1^n, t) \leftarrow\!\!\$\ S^{\mathsf{ro}_1}(1^\lambda, \varepsilon)$; $L \leftarrow\!\!\$\ S^{\mathsf{Hash},\mathsf{ro}_1}(1^n, t)$
$\mathbf{k} \leftarrow (\mathbf{k}[1], \ldots, \mathbf{k}[n])$
$b' \leftarrow\!\!\$\ D^{\mathsf{ro}_2}(1^\lambda, \mathbf{k}, L)$; Return $(b' = 1)$

$\underline{\mathsf{Hash}(x, 1^\ell, i)}$
$v \leftarrow \mathbf{k}[i] \,\|\, x \,\|\, \ell$; $m \leftarrow \lceil \ell/\lambda \rceil$; $K \leftarrow H[v] \leftarrow\!\!\$\ \{0,1\}^{\mathsf{E.kl}(\lambda)}$
For $i = 1$ to m do
  $y_i \leftarrow \mathsf{E.Ev}(1^\lambda, K, \ell \,\|\, i, 1^\lambda)$; $\boxed{y_i \leftarrow\!\!\$\ \{0,1\}^\lambda}$
$y \leftarrow y_1 \,\|\, \cdots \,\|\, y_m$; $y \leftarrow y[1, \ell]$; $Q \leftarrow Q \cup \{x\}$; Return y

$\underline{\mathsf{ro}_1(v)}$
If $v[1, \lambda] \in U$ then $T[v] \leftarrow\!\!\$\ \{0,1\}^{\mathsf{E.kl}(\lambda)}$; Return $T[v]$
If $H[v] = \perp$ then $H[v] \leftarrow\!\!\$\ \{0,1\}^{\mathsf{E.kl}(\lambda)}$
Return $H[v]$

$\underline{\mathsf{ro}_2(v)}$
$x \leftarrow v[\lambda + 1, \mathsf{ro.il}(\lambda) - \lceil \lambda/2 \rceil]$
If $(v[1, \lambda] \in U) \wedge (x \in Q)$ then
  $\mathsf{bad} \leftarrow \mathsf{true}$; $y \leftarrow\!\!\$\ \{0,1\}^{\mathsf{E.kl}(\lambda)}$
  If $T[v] \neq \perp$ then return $T[v]$ else return y
If $H[v] = \perp$ then $H[v] \leftarrow\!\!\$\ \{0,1\}^{\mathsf{E.kl}(\lambda)}$
Return $H[v]$

Figure 12: **Games G_5–G_6 in the proof of Proposition C.2.** Games G_5 and G_8 contain the corresponding boxed statements but game G_6 and G_7 do not.

$\underline{A_0^{\mathrm{RR}}(1^\lambda),\ \boxed{A_1^{\mathrm{RR}}(1^\lambda)}}$

$j \leftarrow\!\!\$\ \{1, \ldots, p(\lambda)\}$; $U, Q \leftarrow \emptyset$
For $i = 1$ to $N(\lambda)$ do
  $\mathbf{k}[i] \leftarrow\!\!\$\ \{0,1\}^\lambda$; $U \leftarrow U \cup \{\mathbf{k}[i]\}$
$\mathsf{cnt} \leftarrow 0$; $(1^n, t) \leftarrow\!\!\$\ S^{\mathsf{roSim}_1}(1^\lambda, \varepsilon)$
$L \leftarrow\!\!\$\ S^{\mathsf{HashSim},\mathsf{roSim}_1}(1^n, t)$; $\mathbf{k} \leftarrow (\mathbf{k}[1], \ldots, \mathbf{k}[n])$
$a' \leftarrow 0$; $b' \leftarrow\!\!\$\ D^{\mathsf{roSim}_2}(1^\lambda, \mathbf{k}, L)$
$\boxed{a' \leftarrow b' ;}$ Return a'

$\underline{\mathsf{roSim}_1(v)}$
If $H[v] = \perp$ then $H[v] \leftarrow\!\!\$\ \{0,1\}^{\mathsf{E.kl}(\lambda)}$
Return $H[v]$

$\underline{\mathsf{HashSim}(x, 1^\ell, i)}$
$\mathsf{cnt} \leftarrow \mathsf{cnt} + 1$; $m \leftarrow \lceil \ell/\lambda \rceil$; $K \leftarrow\!\!\$\ \{0,1\}^{\mathsf{E.kl}(\lambda)}$; $Ran \leftarrow \emptyset$
For $i = 1$ to m do
  If $\mathsf{cnt} = j$ then $y_i \leftarrow \mathrm{RR}(\ell \,\|\, i)$
  Elsif $\mathsf{cnt} > j$ then $y_i \leftarrow\!\!\$\ \{0,1\}^\lambda \backslash Ran$; $Ran \leftarrow Ran \cup \{y_i\}$
  Else $y_i \leftarrow \mathsf{E.Ev}(\lambda, K, \ell \,\|\, i, 1^\lambda)$
$y \leftarrow y_1 \,\|\, \cdots \,\|\, y_m$; $y \leftarrow y[1, \ell]$; Return y

$\underline{\mathsf{roSim}_2(v)}$
$x \leftarrow v[\lambda + 1, \mathsf{ro.il}(\lambda) - \lceil \lambda/2 \rceil]$
If $(v[1, \lambda] \in U) \wedge (x \in Q)$ then $a' \leftarrow 1$
If $H[v] = \perp$ then $H[v] \leftarrow\!\!\$\ \{0,1\}^{\mathsf{E.kl}(\lambda)}$
Return $H[v]$

Figure 13: **Adversaries A_0, A_1 in the proof of Proposition C.2.** Adversary A_1 contains the boxed statement but A_0 does not.

for every $\lambda \in \mathbb{N}$. In game $G_7^{S,D}$, if D make a ro_2 query v whose prefix is a hash key then there are two possibilities. If S previously made the same query to ro_1 then we return the consistent answer to D.

Otherwise, if the string $v[\lambda+1, \mathsf{ro.il}(\lambda) - \lceil \lambda/2 \rceil]$ is a Hash query of S then we return a random answer, independent of Hash and ro_1. Then

$$\Pr[G_6^{S,D}(\lambda)] - \Pr[G_7^{S,D}(\lambda)] \leq \Pr[G_7^{S,D}(\lambda) \text{ sets } \mathsf{bad}] \ .$$

In game $G_8^{S,D}$, instead of invoking E to compute the outputs of Hash, we return independent, uniformly random strings. We now construct an PRP adversary A. It first samples a uniformly random bit. If the bit is 0 then it runs an adversary A_0; otherwise it runs another adversary A_1. The code of A_0 and A_1 is given in Fig. 13. Then

$$2\mathsf{Adv}^{\mathsf{prp}}_{\mathsf{E},A}(\cdot) = \mathsf{Adv}^{\mathsf{prp}}_{\mathsf{E},A_0}(\cdot) + \mathsf{Adv}^{\mathsf{prp}}_{\mathsf{E},A_1}(\cdot) \ .$$

Let us now study the adversaries A_0 and A_1 in details. Recall that in games G_1–G_8, we use E on several keys, whereas in the PRP game, there is a single key. Hence in constructing A_0, we employ the following hybrid argument. The adversary A_0 samples $j \leftarrow \{1, \ldots, p(\lambda)\}$ and emulates game $\mathrm{UCE}^{S,D}_{\mathsf{H}_{\mathsf{rom}}}$ on its simulated random oracle roSim, with the following differences. First, A_0 doesn't sample the challenge bit. Next, on the ith Hash query, if $i < j$ then it samples a fresh key for E, and uses E to generate the output. Otherwise, if $i > j$ then instead of calling E, it will simulate a truly random permutation. Otherwise, if $i = j$ then each invocation of E is replaced by the corresponding query to the oracle RR. If D can make a roSim query v whose prefix is a hash key and $v[\lambda+1, \mathsf{ro.il}(\lambda) - \lceil \lambda/2 \rceil]$ is a Hash query of S, then A_0 returns 1. It returns 0 otherwise. The adversary A_1 is identical to A_0, except that it returns D's guess. Then

$$\begin{aligned}
\Pr[\,\mathrm{PRP}^{A_0}_{\mathsf{E}}(\cdot) \Rightarrow \mathsf{true} \,|\, a = 1\,] &= \frac{1}{p}\Pr[G_7^{S,D}(\cdot) \text{ sets } \mathsf{bad}], \\
\Pr[\,\mathrm{PRP}^{A_1}_{\mathsf{E}}(\cdot) \Rightarrow \mathsf{true} \,|\, d = 1\,] &= \frac{1}{p}\Pr[G_7^{S,D}(\cdot)],
\end{aligned}$$

where a and d are the challenge bits in games $\mathrm{PRP}^{A_0}_{\mathsf{E}}$ and $\mathrm{PRP}^{A_1}_{\mathsf{E}}$ respectively. Now suppose that in game $G_8^{S,D}$, the source makes queries to Hash of output lengths $\ell_1, \ldots, \ell_t$, with $\ell_1 + \cdots + \ell_t \leq \lambda \cdot s(\lambda)$ and $t \leq p(\lambda)$. Due to the PRP/PRF Switching Lemma [19],

$$p(\lambda) \cdot \Pr[\,\mathrm{PRP}^{A_1}_{\mathsf{E}}(\lambda) \Rightarrow \mathsf{false} \,|\, d = 0\,] \leq \Pr[G_8^{S,D}(\lambda)] + \sum_{i=1}^{t} \frac{(\lceil \ell_i/\lambda \rceil - 1)\lceil \ell_i/\lambda \rceil}{2^{\lambda+1}} \leq \Pr[G_8^{S,D}(\lambda)] + \frac{s^2(\lambda)}{2^\lambda}$$

for all $\lambda \in \mathbb{N}$. Hence

$$p(\lambda) \cdot \mathsf{Adv}^{\mathsf{prp}}_{\mathsf{E},A_1}(\lambda) \geq \Pr[G_7^{S,D}(\lambda)] - \Pr[G_8^{S,D}(\lambda)] - \frac{s^2(\lambda)}{2^\lambda}$$

Now consider the following predictor P.

$\underline{P^{\mathsf{ro}}(1^\lambda, 1^n, L)}$	$\underline{\mathsf{roSim}(v)}$
$U, Q' \leftarrow \emptyset$	$x \leftarrow v[\lambda+1, \mathsf{ro.il}(\lambda) - \lceil \lambda/2 \rceil]$
For $i = 1$ to $N(\lambda)$ do	If $v[1,\lambda] \in U$ then $Q' \leftarrow Q' \cup \{x\}$
$\quad \mathbf{k}[i] \leftarrow\!\!\$\ \{0,1\}^\lambda$; $U \leftarrow U \cup \{\mathbf{k}[i]\}$	Return $\mathsf{ro}(v)$
$\mathbf{k} \leftarrow (\mathbf{k}[1], \ldots, \mathbf{k}[n])$; $b' \leftarrow\!\!\$\ D^{\mathsf{roSim}}(1^\lambda, \mathbf{k}, L)$	
Return Q'	

Again, by PRP/PRF Switching Lemma,

$$\begin{aligned}
p(\lambda) \cdot \Pr[\,\mathrm{PRP}^{A_0}_{\mathsf{E}}(\lambda) \Rightarrow \mathsf{false} \,|\, a = 0\,] &\leq \Pr[G_8^{S,D}(\lambda) \text{ sets } \mathsf{bad}] + \frac{s^2(\lambda)}{2^\lambda} \\
&\leq \Pr[\mathrm{Pred}^P_S(\lambda)] + \frac{s^2(\lambda)}{2^\lambda}
\end{aligned}$$

for all $\lambda \in \mathbb{N}$. Hence

$$p(\lambda) \cdot \mathsf{Adv}^{\mathsf{prp}}_{\mathsf{E},A_0}(\lambda) + \mathsf{Adv}^{\mathsf{pred}}_{S,P}(\lambda) \geq \Pr[G_7^{S,D}(\lambda) \text{ sets } \mathsf{bad}] - \frac{s^2(\lambda)}{2^\lambda}$$

for all $\lambda \in \mathbb{N}$. Summing up,

$$\begin{aligned} \mathsf{Adv}^{\mathsf{uce}}_{\mathsf{H},S,D}(\lambda) &= \Pr[G_1^{S,D}(\lambda)] - \Pr[G_8^{S,D}(\lambda)] \\ &\leq 2p(\lambda) \cdot \mathsf{Adv}^{\mathsf{prp}}_{\mathsf{E},A}(\lambda) + \mathsf{Adv}^{\mathsf{pred}}_{S,P}(\lambda) + \frac{2s^2(\lambda) + N^2(\lambda) + q^2(\lambda)}{2^\lambda} \end{aligned}$$

for all $\lambda \in \mathbb{N}$. The predictor outputs a set of at most $q(\lambda)$ elements, $\mathbf{Q}_A^{\mathsf{LR}}$ is maximum of the number of λ-bit blocks in the second component of a Hash query in $\mathrm{UCE}_{\mathsf{H}}^{S,D}$, and $\mathbf{T}(\mathrm{PRP}_{\mathsf{E}}^{A}) = \mathbf{T}(\mathrm{UCE}_{\mathsf{H}}^{S,D})$. ∎

D Proof of Proposition 6.1

For part (a), consider an arbitrary string x such that $|x| \leq m$. Parse $(w_0, w_1, \ldots, w_k) \leftarrow x$, where each w_i is in $\mathrm{GF}(2^n)$. Consider an arbitrary $y \in \mathrm{GF}(2^n)$. Consider polynomial $f(t) \in \mathrm{GF}(2^n)[t]$ defined as

$$f(t) = \sum_{i=0}^{k} w_i \cdot t^i - y \ .$$

The polynomial f has degree $k \leq \|m\|_n$, and thus it has at most $\|m\|_n$ roots. On the other hand, $\mathsf{F}_{\mathsf{poly}}.\mathsf{Ev}(fk, x) = y$ if and only if fk is a root of f. Hence

$$\Pr_{fk \xleftarrow{\$} \mathrm{GF}(2^n)}[f(fk) = 0] \leq \frac{\|m\|_n}{2^n}$$

and thus $\mathbf{Coll1}_{\mathsf{F}_{\mathsf{poly}}}(m) \leq \|m\|_n / 2^n$, as claimed.
For part (b), consider arbitrary distinct string x_0, x_1 such that each $|x_i| \leq m_i$. Parse $(w_0, w_1, \ldots, w_k) \leftarrow x_0$ and $(v_0, \ldots, v_\ell) \leftarrow x_1$, where each w_i and v_j are in $\mathrm{GF}(2^n)$. Consider the polynomial $f(t) \in \mathrm{GF}(2^n)[t]$ defined as follows:

$$f(t) = \sum_{i=0}^{k} w_i \cdot t^i - \sum_{j=0}^{\ell} v_j \cdot t^j \ .$$

We claim that f is not the zero polynomial. If $k = \ell$ then the polynomial $f(t) = \sum_{i=0}^{k} (w_i \oplus v_i) t^i$ is nonzero because $(w_0, \ldots, w_k) \neq (v_0, \ldots, v_\ell)$. If $k \neq \ell$ then wlog, assume that $k > \ell$. Then the leading coefficient of $f(t)$ is w_k that is not the zero element of $\mathrm{GF}(2^n)$, and the claim follows. Since the degree of f is at most $\max\{k, \ell\} \leq \max\{\|m_0\|_n, \|m_1\|_n\}$, it follows that f has at most $\max\{\|m_0\|_n, \|m_1\|_n\}$ roots. On the other hand,

$$\mathsf{F}_{\mathsf{poly}}.\mathsf{Ev}(fk, x_0) = \mathsf{F}_{\mathsf{poly}}.\mathsf{Ev}(fk, x_1)$$

if and only if $f(fk) = 0$. Hence

$$\Pr_{fk \xleftarrow{\$} \mathrm{GF}(2^n)}[f(fk) = 0] \leq \frac{\max\{\|m_0\|_n, \|m_1\|_n\}}{2^n}$$

and thus $\mathbf{Coll2}_{\mathsf{F}_{\mathsf{poly}}}(m_0, m_1) \leq (\max\{\|m_0\|_n, \|m_1\|_n\})/2^n$, as claimed.

E Proof of Proposition 6.2

From Proposition 6.1, $\mathbf{Coll1}_{\mathsf{F}_{\mathsf{poly}}}(m) \leq \|m\|_n / 2^n$. It follows that $\mathbf{Coll1}_{\mathsf{F}_{\mathsf{fast}}}(m) \leq \|m\|_n / 2^n$. It remains to show that $\mathbf{Coll2}_{\mathsf{F}_{\mathsf{fast}}}(m_0, m_1) \leq (Cr + \max\{\|m_0\|_n, \|m_1\|_n\})/2^n$. Consider two arbitrary distinct strings x_0

and x_1 such that each $|x_i| \le m_i$. Fix $fk_1 \in [\mathsf{F}_{\mathsf{tree}}.\mathsf{Kg}()]$. Let $y_i = \mathsf{Shrink}(fk_1, x_i)$ and $\ell_i = |y_i|$, for $i \in \{0,1\}$. If $y_0 \neq y_1$ then from Proposition 6.1,

$$\Pr_{fk_2 \xleftarrow{\$} \mathsf{F}.\mathsf{Kg}()}[\mathsf{F}_{\mathsf{poly}}.\mathsf{Ev}(fk_2, y_0, 1^n) = \mathsf{F}_{\mathsf{poly}}.\mathsf{Ev}(fk_2, y_1, 1^n)] \le \frac{\max\{\|\ell_0\|_n, \|\ell_1\|_n\}}{2^n} \le \frac{\max\{\|m_0\|_n, \|m_1\|_n\}}{2^n} \ .$$

What's left is to prove that

$$\Pr_{fk_1 \xleftarrow{\$} \mathsf{F}_{\mathsf{tree}}.\mathsf{Kg}()}[\mathsf{Shrink}(fk_1, x_0) = \mathsf{Shrink}(fk_1, x_1)] \le \frac{Cr}{2^n} \ .$$

Parse $w_1 \ldots w_k \leftarrow x_0$ and $v_1 \cdots v_d \leftarrow x_1$. Note that $k = \|\ell_0\|_n$ and $d = \|\ell_1\|_n$. This means that if $\mathsf{Shrink}(fk_1, x_0) = \mathsf{Shrink}(fk_1, x_1)$, we'll have $\ell_0 = \ell_1$, and thus $k = d$. It there is some $i \le k-1$ such that $w_i \neq v_i$ then the claim follows from Equation (6). Otherwise, since $x_0 \neq x_1$, we must have $w_k \neq v_k$. When we break $\mathsf{Shrink}(fk_1, x_0)$ into n-bit blocks such that the last block is strictly shorter than n bits, v_k is the last block of $\mathsf{Shrink}(fk_1, x_0)$. The same holds for w_k and $\mathsf{Shrink}(fk_1, x_1)$. Hence $\mathsf{Shrink}(fk_1, x_0) \neq \mathsf{Shrink}(fk_1, x_1)$ for every fk_1.

F Proof of Proposition 6.3

Note that

$$\begin{aligned}
\mathbf{Coll}_{\mathsf{F}_{\mathsf{aes4}}}(m_0, m_1) \le (\mathbf{Coll}_{\mathsf{F}_{\mathsf{fast}}}(m_0, m_1))^2 &\le \frac{\left(Cr + \max\{\|m_0\|_n, \|m_1\|_n\}\right)^2}{2^{2n}} \\
&\le \frac{2C^2r^2 + 2(\max\{\|m_0\|_n, \|m_1\|_n\})^2}{2^{2n}} \ .
\end{aligned}$$

Then

$$\begin{aligned}
\mathsf{Adv}^{\mathsf{coll}}_{\mathsf{F}_{\mathsf{aes4}}}(p, \sigma) &\le \max_{\ell \le p, \ell' \le p, m_1 + \cdots + m_\ell \le \sigma, m'_1 + \cdots + m'_{\ell'} \le \sigma} \left\{ \sum_{i=1}^{\ell} \sum_{j=1}^{\ell'} \frac{2C^2r^2 + 2(\max\{\|m_i\|_n, \|m'_j\|_n\})^2}{2^{2n}} \right\} \\
&\le \frac{2C^2r^2p^2}{2^{2n}} + \max_{\ell \le p, \ell' \le p, m_1 + \cdots + m_\ell \le \sigma, m'_1 + \cdots + m'_{\ell'} \le \sigma} \left\{ \sum_{i=1}^{\ell} \sum_{j=1}^{\ell'} \frac{2(\|m_i\|_n)^2 + 2(\|m'_j\|_n)^2}{2^{2n}} \right\} \\
&\le \frac{2C^2r^2p^2}{2^{2n}} + \frac{2p}{2^{2n}} \cdot \max_{\ell \le p, \ell' \le p, m_1 + \cdots + m_\ell \le \sigma, m'_1 + \cdots + m'_{\ell'} \le \sigma} \left\{ \sum_{i=1}^{\ell} (\|m_i\|_n)^2 + \sum_{j=1}^{\ell'} (\|m'_j\|_n)^2 \right\} \\
&\le \frac{2C^2r^2p^2 + 4p(\|\sigma\|_n + p)^2}{2^{2n}},
\end{aligned}$$

as claimed.