Template Attacks Based On Priori Knowledge

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Abstract. Template attacks are widely accepted as the *strongest* side-channel attacks from the information theoretic point of view, and they can be used as a very *powerful* tool to evaluate the physical security of cryptographic devices. Template attacks consist of two stages, the profiling stage and the extraction stage. In the profiling stage, the attacker is assumed to have a large number of power traces measured from the reference device, using which he can accurately characterize signals and noises in different points. However, in practice, the number of profiling power traces may not be sufficient. In this case, signals and noises are not accurately characterized, and the key-recovery efficiency of template attacks is significantly influenced. We show that, the attacker can still make template attacks powerfully enough in practice as long as the priori knowledge about the reference device be obtained. We note that, the priori knowledge is just a prior distribution of the signal component of the instantaneous power consumption, which the attacker can easily obtain from his previous experience of conducting template attacks, from Internet and many other possible ways. Evaluation results show that, the priori knowledge, even if not accurate, can still help increase the power of template attacks, which poses a serious threat to the physical security of cryptographic devices.

Keywords: Side-Channel Attacks, Power Analysis Attacks, Template Attacks, Priori Knowledge.

1 Introduction

Pervasive devices such as mobile phones, smart cards, RFIDs, and sensor nodes are now closely integrated into our lives. The devices typically operate in hostile environments and hence the data contained might be relatively easy compromised. The physical accessibility has led to a number of very powerful attacks targeting implementations. In these powerful attacks, side-channel attacks is an effective means to derive secrets stored in a security device from measurements of a leaking physical signal such as power consumption [9], execution time [13], electromagnetic radiation [14], and many more [24]. Traditional security notions (such as chosen-ciphertext security for public-key encryption schemes) do not provide any security guarantee against such attacks, and many implementations of provably secure cryptosystems were broken by side-channel attacks.

Side-channel attacks based on power consumption have received such a large amount of attention because they are very powerful and can be conducted relatively easily. As an important kind of power analysis attacks, template attacks were proposed by Chari et al. in 2002 [1], which consist of two stages, i.e. the profiling stage and the extraction stage. In the profiling stage, the attacker has a reference device identical or similar to the target device, and he can use the reference device to characterize the leakage of the target device. In the extraction stage, the attacker can exploit a small number of power traces measured from the target device to recover the correct (sub)key.

In order to make template attacks powerfully enough, the attacker needs to use a large number of power traces to accurately characterize signals and noises in different interesting points. However, in practice, the number of profiling traces may be limited (e.g. the attacker can only obtain less than 5,000 profiling traces from the reference device.). Many scenarios will lead to the attacker only obtain limited profiling traces. For example, a common countermeasure is to limit the operation times of the reference device, or the key used by the reference device will be refreshed after being used several times. In these scenarios, the attacker can only obtain a limited number of power traces in the profiling stage, and signals and noises are not accurately characterized, which significantly influences the key-recovery efficiency of template attacks.

1.1 Motivations

A natural question is whether or not it is possible to further increase the power of template attacks even if the number of profiling traces is limited? We anticipate that using the priori knowledge about the reference device may be a possible way. The priori knowledge is just a kind of prior distribution of the actual value of the signal component in the instantaneous power consumption. There are many ways that the attacker can obtain the priori knowledge in practice. We show three typical examples here.

Example 1: Assume that the attacker has characterized the power leakages of some cryptographic devices whose leakage characterizations are similar to the reference device. Then, he may obtain the priori knowledge about the reference device. For example, noises in different interesting points are usually assumed to follow the normal distribution. If the attacker can estimate the mean value and the variance of the normal distribution using power traces measured from previous cryptographic devices, then the priori knowledge about the reference device can be obtained.

Example 2: From Internet (e.g. [20,21]), the attacker may obtain some power traces or other potential useful information (e.g. Signal-to-Noise Ratio pp. 73 in [9]) of different devices which are similar to the reference device, using which he can infer the priori knowledge of the reference device (similarly to Example 1).

Example 3: For a sophisticated attacker, after obtaining power traces from the reference device in the profiling stage, he can use the power traces to obtain an interval estimation of the actual value of the signal component and roughly infer the prior distribution is a kind of distribution (e.g. normal distribution) over the interval.

To sum up, for a seasoned attacker, it is not only reasonable but also realistic for him to possess the priori knowledge about the reference device from a practical point of view. Therefore, we need to consider the power of template attacks when the attacker can not obtain enough power traces from the reference device in the profiling stage *but* has the priori knowledge about the reference device. Specifically, two questions need to be answered. The first question is how can the attacker exploit the priori knowledge during the profiling stage in a theoretically correct and practically feasible way to make template attacks more powerful (i.e. achieve better classification performance)? The second question is whether or not the priori knowledge (even if it may not be very accurate) will make template attacks more powerful really?

Of course, one may ask such question: Why not the attacker exploits the power traces obtained from the similar devices (from his previous experience of conducting template attacks or from Internet) together with the power traces obtained from the reference device to build the templates to make template attacks more powerful? In fact, if one *directly* exploits power traces from the similar devices and the reference device to build the templates, the classification performance of template attacks will be decreased [23]. The reason is that the acquisition campaigns about the devices are different[1] even if the leakage distributions of the similar devices and the reference device are similar [23].

If we can give positive answers to the above two important questions, then in order to make template attacks more powerful in the above scenarios, the attacker can first *extract* the priori knowledge from the power traces obtained from the different but similar devices and then conduct template attacks with the priori knowledge as well as the limited power traces obtained from the reference device. From this point of view, these two questions are worth researching.

1.2 Contributions

Main contributions of our work are two-folds. Firstly, based on the method of Bayes estimation [15], we give a theoretically correct and practically feasible way of exploiting the priori knowledge when the attacker conducts template attacks with limited power traces obtained from the reference device in the profiling stage. Our new way fits the property of instantaneous power consumption of most cryptographic devices. Hence, it can be exploited widely in practice. Moreover, we present how to easily obtain the priori knowledge for practical attackers.

Secondly, we verify our way of exploiting the priori knowledge using both simulated and practical experiments. Evaluation results show that, both in simulated and practical experiments, template attacks will be more powerful if the attacker can possess accurate priori knowledge. Additionally, the more accurate the priori knowledge is, the more powerful template attacks will be. Therefore, with the priori knowledge we can further increase the power of template attacks.

[1] For example, there exist offsets in the different acquisition campaigns.

1.3 Related Work

Answers to some practical issues of template attacks were provided by [2], such as how to choose interesting points in an efficient way and how to preprocess noisy data. Choudary et al. proposed efficient methods to avoid possible numerical obstacles when implementing template attacks in [4]. In [10], Hanley et al. presented a variant of template attacks which can be applied to block ciphers when the plaintext and ciphertext are unknown. In [7], template attacks were used to attack a masked implementation. Recently, a simple pre-processing technique of template attacks, normalizing the sample values using the means and variances was evaluated [6]. Standaert et al. [22] showed how to best evaluate profiling and extraction of profiled attacks by using the information theoretic metric and the security metric.

Template attacks are also utilized to attack the implementation of public-key cryptographic algorithms. For example, in [25], Hanley et al. used template attacks to distinguish multiplications from squaring operations for RSA signature generation. In [26], Medwed et al. investigate how template attacks can be applied to implementation of the elliptic curve digital signature algorithm, which is particularly suitable for 32-bit platforms.

Principal Component Analysis (PCA)-based template attacks were investigated in [3]. However, this kind of template attacks may not improve the classification performance [6] and has high computational requirements [2]. Therefore, PCA-based template attacks are not used widely in practice and we briefly review them in Section 2.3. Linear Discriminant Analysis (LDA)-based template attacks were introduced in [8] and depend on the condition of equal covariances [4] (Please see Section 2.1 for more details.), which does not hold in most settings. Therefore, LDA-based template attacks are not better than PCA-based template attacks [4] and we ignore them in this paper. Up to now, no previous work considered our important questions.

1.4 Organization of This Paper

The rest of this paper is organized as follows. In Section 2, we review the concept of template attacks and Bayes estimation. In Section 3, we give a reasonable way of exploiting the priori knowledge to make template attacks more powerful. In Section 4, we verify the way of exploiting the priori knowledge for template attacks by both simulated and practical experiments. In Section 5, we conclude the whole paper and introduce future work.

2 Preliminaries

In this section, we will briefly review classical template attacks [1], reduced template attacks (pp. 108 in [9]), PCA-based template attacks, and the method of Bayes estimation.

2.1 Classical Template Attacks

We will introduce the two stages of classical template attacks: the profiling stage and the extraction stage.

2.1.1 The Profiling Stage Assume that there exist K different (sub)keys $key_i, i = 0, 1, \ldots, K-1$ which need to be classified. Also, there exist K different key-dependent operations O_i, $i = 0, 1, \ldots, K-1$. Usually, one will generate K templates, one for each key-dependent operation O_i. One can exploit some methods to choose N interesting points $(P_0, P_1, \ldots, P_{N-1})$. The interesting points are those time samples that contain the most information about the characterized key-dependent operations. Each template is composed of a mean vector and a covariance matrix. The mean vector is used to estimate the signal component of side-channel leakages. It is the average signal vector $\mathbf{M}_i = (M_i[P_0], \ldots, M_i[P_{N-1}])$ for each one of the key-dependent operations. The covariance matrix is used to estimate the probability density of the noise component at different interesting points. It is assumed that noises at different interesting points approximately follow the multivariate normal distribution. A N dimensional noise vector $\mathbf{n}_i(\mathbf{S})$ is extracted from each actual power trace $\mathbf{S} = (S[P_0], \ldots, S[P_{N-1}])$ representing the template's key dependency O_i as $\mathbf{n}_i(\mathbf{S}) = (S[P_0] - M_i[P_0], \ldots, S[P_{N-1}] - M_i[P_{N-1}])$. One computes the $(N \times N)$ covariance matrix $\mathbf{C}_i$ from these noise vectors. The probability density of the noises occurring under key-dependent operation O_i is given by the N dimensional multivariate Gaussian distribution $p_i(\cdot)$, where the probability of observing a noise vector $\mathbf{n}_i(\mathbf{S})$ is:

$$p_i(\mathbf{n}_i(\mathbf{S})) = \frac{1}{\sqrt{(2\pi)^N |\mathbf{C}_i|}} exp\Big(-\frac{1}{2}\mathbf{n}_i(\mathbf{S})\mathbf{C}_i^{-1}\mathbf{n}_i(\mathbf{S})^T \Big) \quad \mathbf{n}_i(\mathbf{S}) \in \mathbb{R}^N. \qquad (1)$$

In equation (1), the symbol $|\mathbf{C}_i|$ denotes the determinant of $\mathbf{C}_i$ and the symbol $\mathbf{C}_i^{-1}$ denotes its inverse. We know that the matrix $\mathbf{C}_i$ is the estimation of the true covariance $\mathbf{\Sigma}_i$. The condition of equal covariances [4] means that the leakages from different key-dependent operations have the same true covariance $\mathbf{\Sigma} = \mathbf{\Sigma}_0 = \mathbf{\Sigma}_1 = \cdots = \mathbf{\Sigma}_{K-1}$. In most settings, the condition of equal covariances does not hold. Therefore, in this paper, we only consider the device in which the condition of equal covariances does not hold.

2.1.2 The Extraction Stage Assume that one obtains t power traces (denoted by $\mathbf{S}_1, \mathbf{S}_2, \ldots, \mathbf{S}_t$) from the target device in the extraction stage. When the power traces are statistically independent, one will apply maximum likelihood approach on the product of conditional probabilities (pp. 156 in [9]), i.e.

$$key_{ck} := argmax_{key_i}\Big\{ \prod_{j=1}^{t} \mathsf{Pr}(\mathbf{S}_j | key_i), i = 0, 1, \ldots, K-1 \Big\},$$

where $\mathsf{Pr}(\mathbf{S}_j|key_i) = p_{f(\mathbf{S}_j,key_i)}(n_{f(\mathbf{S}_j,key_i)}(\mathbf{S}_j))$. The key_{ck} is considered to be the correct (sub)key. The output of the function $f(\mathbf{S}_j, key_i)$ is the index of a key-dependent operation. For example, when the output of the first S-box in the first round of AES-128 is chosen as the target intermediate value, one builds templates for each output of the S-box. In this case, $f(\mathbf{S}_j, key_i) = Sbox(m_j \oplus key_i)$, where m_j is the input plaintext corresponding to the actual power trace $\mathbf{S}_j$.

2.2 Reduced Template Attacks

In order to avoid numerical obstacles with the inversion of the covariance matrix $\mathbf{C}_i$, one can set the covariance matrix equal to the identity matrix. This essentially means that one does not take the covariances between different interesting points into consideration. A template that only consists of a mean vector is called a *reduced template* (pp. 108 in [9]). Correspondingly, template attacks based on reduced templates are called as reduced template attacks. In reduced template attacks, the probability density of the noises occurring under key-dependent operation O_i is given by the distribution $p'_i(\cdot)$, where the probability of observing a noise vector $\mathbf{n}_i(\mathbf{S})$ is:

$$p'_i(\mathbf{n}_i(\mathbf{S})) = \frac{1}{\sqrt{(2\pi)^N}} exp\Big(-\frac{1}{2}\mathbf{n}_i(\mathbf{S})\mathbf{n}_i(\mathbf{S})^T\Big) \quad \mathbf{n}_i(\mathbf{S}) \in \mathbb{R}^N.$$

2.3 PCA-based Template Attacks

PCA-based template attacks [3] exploit a continual point fragment correspond to the target intermediate value in actual power traces (We assume the length of the continual point fragment is N.). One first computes the empirical covariance matrix, which is given by

$$\mathbf{ECM} = \frac{1}{K}\sum_{i=0}^{K-1}(\mathbf{M}_i - \overline{\mathbf{M}})(\mathbf{M}_i - \overline{\mathbf{M}})^T.$$

The quantity $\overline{\mathbf{M}} = \sum_{i=0}^{K-1}\mathbf{M}_i/K$ is the average of the mean vectors. Let us denote the matrixes of eigenvectors and eigenvalues of $\mathbf{ECM}$ by $\mathbf{U}$ and Δ, i.e. $\mathbf{ECM} = \mathbf{U}\Delta\mathbf{U}^T$. The principal directions $\{w_i\}_{i=1}^{L}$ are the columns of $\mathbf{U}$ that correspond to the L largest eigenvalues of Δ. The corresponding matrix of principal directions is denoted $\mathbf{W} \in \mathbb{R}^{N\times L}$. One uses projected mean vectors $\{\mathbf{W}^T\mathbf{M}_i^T\}_{i=0}^{K-1}$ and projected covariance matrices $\{\mathbf{W}^T\mathbf{C}_i\mathbf{W}\}_{i=0}^{K-1}$ to conduct this kind of attacks. The probability of observing a noise vector when one assumes the key-dependent operation is O_i is computed by

$$p''_i(\mathbf{n}_i(\mathbf{S})) = \frac{exp(-\frac{1}{2}\mathbf{n}_i(\mathbf{S})\mathbf{W}(\mathbf{W}^T\mathbf{C}_i\mathbf{W})_i^{-1}(\mathbf{n}_i(\mathbf{S})\mathbf{W})^T)}{\sqrt{(2\pi)^L|\mathbf{W}^T\mathbf{C}_i\mathbf{W}|}} \quad \mathbf{n}_i(\mathbf{S}) \in \mathbb{R}^N.$$

Then, one classifies the correct (sub)key based on the probability computed by equation $p''_i(\mathbf{n}_i(\mathbf{S}))$.

2.4 Bayes Estimation

In the following, we briefly introduce the method of Bayes estimation [15]. We firstly introduce the definition of Bayes estimators. Then, we introduce how to compute a Bayes estimator.

Suppose an unknown parameter θ is known to have a prior distribution Λ (The prior distribution can be discrete or continuous distribution. In this paper, we only assume the prior distribution is continuous.). Quite generally, suppose that the consequences of estimating $g(\theta)$ by a value $\delta(X)$ (based on some measurements X) are measured by $L(\theta, \delta(X))$. As of the *loss function* L, we shall assume that

$$L(\theta, \delta(X)) \geq 0 \text{ for all } \theta \text{ and } \delta(X),$$

and $L[\theta, g(\theta)] = 0$ for all θ, so that the loss is zero when the correct value is estimated. The accuracy, or rather inaccuracy, of an estimator δ is then measured by the *risk function*

$$R(\theta, \delta) = E_{\theta}\{L[\theta, \delta(X)]\},$$

the long-term average loss resulting from the use of $\delta(X)$. This defines the risk function as a function of $\delta(X)$. An estimator $\delta(X)$ minimizing

$$r(\Lambda, \delta) = \int R(\theta, \delta) d\Lambda(\theta)$$

is called a *Bayes estimator* with respect to the prior distribution Λ. Note that, the prior distribution Λ is a probability distribution of the parameter θ, that is,

$$\int d\Lambda(\theta) = 1.$$

Now, we will introduce how to compute a Bayes estimator of an unknown parameter θ. Let $\lambda(\theta)$ denote the prior probability density of the parameter θ. The probability density of the population (or discrete probability function) is denoted by $f(X;\theta)$. If one extracts n samples $(X_1, X_2, \ldots, X_n)$ from the population, then the probability density of this group of samples is

$$f(X_1;\theta)f(X_2;\theta)\cdots f(X_n;\theta).$$

Thereby, we can compute the marginal density

$$p(X_1, X_2, \ldots, X_n) = \int \lambda(\theta) f(X_1;\theta) f(X_2;\theta) \cdots f(X_n;\theta) d\theta.$$

Then, the following posterior probability density is computed:

$$\lambda(\theta|X_1, \ldots, X_n) = \lambda(\theta) f(X_1;\theta) \cdots f(X_n;\theta) / p(X_1, X_2, \ldots, X_n). \quad (2)$$

In general, the Bayes estimator of the parameter θ is set to be the mean value of $\lambda(\theta|X_1, \ldots, X_n)$.

3 Using Priori Knowledge to Improve Template Attacks

In this section, we introduce how to use the priori knowledge about the reference device for template attacks. The usage of the priori knowledge is same for classical template attacks, reduced template attacks, and PCA-based template attacks.

It is well known that the instantaneous power consumption PC_{total} can be modeled as the sum of an operation-dependent component PC_{op}, a data-dependent component PC_{data}, the electronic noise $PC_{el.noise}$, and a constant component PC_{const} (pp. 62-65 in [9]), i.e.

$$PC_{total} = PC_{op} + PC_{data} + PC_{el.noise} + PC_{const}.$$

The value $PC_{op} + PC_{data}$ (or $PC_{op} + PC_{data} + PC_{const}$) can be viewed as the signal component and the value $PC_{el.noise}$ can be viewed as the noise component. Usually, for each point P_j in an actual power trace, when the operation and the data are all fixed, its power consumption PC_{total} follows a normal distribution $\mathcal{N}(\mu_j, \sigma_j^2)$ and the electronic noise $PC_{el.noise}$ follows the normal distribution $\mathcal{N}(0, \sigma_j^2)$ (pp. 62-65 in [9]). For example, Figure 1 shows the histogram of the instantaneous power consumption for a fixed output of the 1^{st} S-box in the 1^{st} round of our unprotected AES-128 software implementation with 400 samples (Please see Section 4.3 for more details.). The shape of the histogram shown in Figure 1 indicates that the instantaneous power consumption follow a normal distribution. For fixed operation on fixed data, due to $Var(PC_{op}) = Var(PC_{data}) = Var(PC_{const}) = 0$, we have $PC_{op} + PC_{data} + PC_{const} = \mu_j$. The priori knowledge is a kind of prior distribution of the actual value of the signal component μ_j. Due to the existence of the electronic noise, we can reasonably assume the prior distribution of the actual value of μ_j obtained by the attacker is a normal distribution.

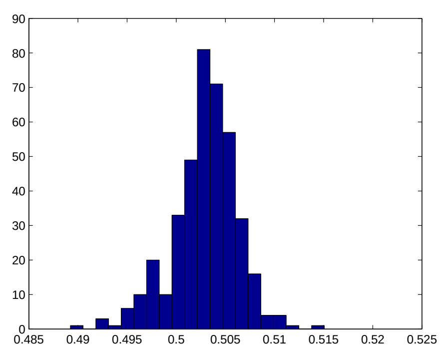


Fig. 1. The histogram of the instantaneous power consumption

There are many ways that the attacker can obtain the prior distribution and we just give out two possible ways of them. Considering Example 1 in Section 1, for the same position about the target intermediate value, the attacker obtains n samples (For convenience, the samples are denoted by $X_1, \ldots, X_n$.) from power traces obtained from his previous experience of conducting template attacks against different devices which are similar to the reference device.

Possible Way 1: After obtaining the n samples, the attacker computes

$$\theta_1 = \frac{1}{n} \cdot \sum_{i=1}^{n} X_i,\ \theta_2^2 = \frac{1}{n-1} \cdot \sum_{i=1}^{n} (X_i - \theta_1)^2.$$

Then, the attacker can easily obtain the prior distribution which is the normal distribution $\mathcal{N}(\theta_1, \theta_2^2)$.

Possible Way 2: The attacker can repeat a process U times. Every time, the attacker chooses m samples (denoted by $X_{i_1}, \ldots, X_{i_m}$) from the n samples uniformly at random and computes $\sum_{k=1}^{m} X_{i_k}/m$. Therefore, there are U different values about $\sum_{k=1}^{m} X_{i_k}/m$. The mean value of the U different values are set to be θ_1 and the variance of the U different values are set to be θ_2^2. In this way, according to the central limit theorem, the prior distribution $\mathcal{N}(\theta_1, \theta_2^2)$ was got. Clearly, when the value m is larger, the estimation of θ_1 and θ_2^2 is more accurate.

Because the leakage distributions of the devices are very similar to that of the reference device, the prior distribution can be used for the interesting points correspond to the same position about the target intermediate value for the reference device. We note that, compared with traditional template attacks, the computational price of obtaining the priori knowledge about the reference device is very small. This implies that the attacker can obtain the prior distribution easily in practice.

The more accurate the signal component (the value of μ_j) is estimated, the more accurate the noise component (the value $PC_{total} - \mu_j$) will be estimated. For an interesting point, if the signal component and the noise component are accurately estimated, accurate templates will be built and template attacks will be more powerful. In the classical way of building templates, for an interesting point, the attacker computes the mean value of the samples to estimate the actual value of the signal component μ_j. Specifically, for the key-dependent operation O_i, the point P_j is an interesting point and the attacker obtains n power traces $(\mathbf{S}_1, \mathbf{S}_2, \ldots, \mathbf{S}_n)$ from the reference device in the profiling stage (Note that, the value of n is small.). Therefore, the attacker obtains n values of the power consumption of the point P_j, one from each power trace. The n values are denoted by $S_1[P_j], S_2[P_j], \ldots, S_n[P_j]$. The actual value of μ_j is estimated by μ_j':

$$\mu_j' = M_i[P_j] = \sum_{k=1}^{n} S_k[P_j]/n.$$

However, in our scenario, the attacker not only has n power traces from reference device, but also possesses the priori knowledge about the reference device which can be used to estimate the actual value of μ_j more accurately. Let's consider the most common case. Assume that the attacker knows that the actual value of μ_j follows the normal distribution $\mathcal{N}(\theta_1, \theta_2^2)$ from priori knowledge[1] but does not know what the actual value of μ_j accurately is. The attacker can use the method of Bayes estimation to estimate the actual value of μ_j with the priori knowledge $\mathcal{N}(\theta_1, \theta_2^2)$ in the profiling stage as follows: The attacker computes the probability density of the actual value of the signal component μ_j from priori knowledge as

$$\lambda(\mu_j) = (\sqrt{2\pi}\theta_2)^{-1} exp\Big[-\frac{1}{2\theta_2^2}(\mu_j - \theta_1)^2\Big].$$

Moreover, the power consumption of the point P_j satisfies the following probability density function:

$$f(x;\mu_j,\sigma_j) = (\sqrt{2\pi}\sigma_j)^{-1} exp\Big[-\frac{1}{2\sigma_j^2}(x - \mu_j)^2\Big].$$

From equation (2), the attacker computes the posterior probability density:

$$\lambda(\mu_j|S_1[P_j],\ldots,S_n[P_j]) = C_1 exp\Big[-\frac{1}{2\theta_2^2}(\mu_j - \theta_1)^2 - \frac{1}{2\sigma_j^2}\sum_{k=1}^{n}(S_k[P_j] - \mu_j)^2\Big],$$

the constant C_1 only has relation with $\theta_1, \theta_2, \sigma_j, S_1[P_j], \ldots, S_n[P_j]$ and has no relation with μ_j. It has that

$$-\frac{1}{2\theta_2^2}(\mu_j - \theta_1)^2 - \frac{1}{2\sigma_j^2}\sum_{k=1}^{n}(S_k[P_j] - \mu_j)^2 = -\frac{1}{2A^2}(\mu_j - B)^2 + C_2,$$

where

$$A^2 = \sigma_j^2\theta_2^2/(\sigma_j^2 + n\theta_2^2),$$
$$B = (nM_i[P_j] + \sigma_j^2\theta_1/\theta_2^2)/(n + \sigma_j^2/\theta_2^2),$$

and C_2 has no relation with μ_j. Furthermore, the attacker can obtain

$$\lambda(\mu_j|S_1[P_j],\ldots,S_n[P_j]) = C_3 exp\Big[-\frac{1}{2A^2}(\mu_j - B)^2\Big],$$

where $C_3 = C_1 e^{C_2}$. Because it has that

$$\int_{-\infty}^{+\infty}\lambda(\mu_j|S_1[P_j],\ldots,S_n[P_j])d\mu_j = 1,$$

hence $C_3 = (\sqrt{2\pi}A)^{-1}$. Up to now, the attacker obtains the Bayes estimator of the actual value of μ_j as

$$\mu_j'' = \frac{n}{n + \sigma_j^2/\theta_2^2}\left(\frac{\sum_{k=1}^{n} S_k[P_j]}{n}\right) + \frac{\sigma_j^2/\theta_2^2}{n + \sigma_j^2/\theta_2^2}\theta_1. \quad (3)$$

[1] Note that, the normal distribution $\mathcal{N}(\theta_1, \theta_2^2)$ itself may not be very accurate. However, from the priori knowledge, the parameters θ_1, θ_2^2 are all known to the attacker.

The equation (3) shows that if the attacker does not have the priori knowledge (i.e. the prior distribution $\mathcal{N}(\theta_1, \theta_2^2)$), he can only use $\sum_{k=1}^{n} S_k[P_j]/n$ to estimate the actual value of μ_j. If the attacker does not have power traces obtained from the reference device, he can only use the priori knowledge (i.e. the value θ_1) to estimate the actual value of μ_j. If the attacker has power traces obtained from the reference device as well as the priori knowledge, by equation (3), he will use the weighted average of $\sum_{k=1}^{n} S_k[P_j]/n$ and θ_1 to estimate the actual value of μ_j under the ratio $n : \sigma_j^2/\theta_2^2$ in the profiling stage. This ratio is reasonable and the relevant reasons are as follows. On one hand, when more power traces are obtained from the reference device by the attacker, the proportion of $\sum_{k=1}^{n} S_k[P_j]/n$ should be larger. On the other hand, when the value θ_2^2 is smaller (This implies that the prior distribution of the actual value of μ_j is more accurate.), the proportion of θ_1 should be larger.

From equation (3), when n is fixed and the accuracy of the priori knowledge remains unchanged (This means that the values of θ_1 and θ_2^2 are all fixed.), we also find that the proportion of the priori knowledge will be larger with the increase of the electronic noise level (i.e., the value of σ_j is larger.). This implies that the Bayes estimation of the actual value of μ_j will more rely on the priori knowledge in high electronic noise level. For example, in Table 1, we show the proportion of the value $\sum_{k=1}^{n} S_k[P_j]/n$ and the priori knowledge under different electronic noise levels by assuming $n = 5$ and $\theta_2^2 = 0.5$. Table 1 shows that the Bayes estimation almost not rely on the value $\sum_{k=1}^{n} S_k[P_j]/n$ when the electronic noise level is very high (e.g. $\sigma_j = 5$).

When the number of profiling stage is limited, the attacker may not know the actual value of σ_j^2 in practice. In this case, the attacker can still compute the Bayes estimator about the actual value of μ_j by reasonably assuming that the actual value of σ_j^2 equals to a constant value. Of course, when the attacker knows the actual value of σ_j^2, more accurate Bayes estimation about the actual value of μ_j can be obtained. In Section 4.2, we will discuss this scenario in detail.

Other details of building templates remain unchanged. Our way only exploits the priori knowledge to estimate the actual value of the signal component more accurately by using the Bayes estimation method. We note that, due to the computational price of both obtaining and exploiting the priori knowledge is low, the priori knowledge can easily be used by practical attackers.

Table 1. The proportion of the value $\sum_{k=1}^{n} S_k[P_j]/n$ and the priori knowledge

Noise Level	n	θ_2^2	$n/(n+\sigma_j^2/\theta_2^2)$	$(\sigma_j^2/\theta_2^2)/(n+\sigma_j^2/\theta_2^2)$
$\sigma_j = 1$	5	0.5	0.71	0.29
$\sigma_j = 2$	5	0.5	0.38	0.62
$\sigma_j = 3$	5	0.5	0.22	0.78
$\sigma_j = 4$	5	0.5	0.14	0.86
$\sigma_j = 5$	5	0.5	0.09	0.91

4 Experimental Evaluations

For the implementation of a cryptographic algorithm with countermeasures, one usually tries his best to use some approaches to delete the countermeasures from power traces at first. If the countermeasures can be deleted, then one tries to recover the correct (sub)key using some attacks against unprotected implementation. For example, if one has power traces with random delays [11], he may first use the approach proposed in [12] to remove the random delays from power traces and then uses some attacks to recover the correct (sub)key. The approaches of deleting countermeasures from power traces are beyond the scope of this paper. Moreover, considering power traces without any countermeasures shows the upper bound of the physical security of the target cryptographic device. Therefore, we take unprotected AES-128 implementation as an example.

Experiments verified that the priori knowledge can be exploited to improve PCA-based template attacks. However, due to the fact that PCA-based template attacks are not used widely in practice, we only show simulated and practical experiments about classical template attacks and reduced template attacks. Moreover, the work [19] showed that reduced template attacks are more powerful compared with classical template attacks when the number of power traces used in the profiling stage is limited. Therefore, we mainly exploit reduced template attacks to exhibit our discoveries in this paper.

In both simulated and practical experiments, we tried to attack the outputs of the S-boxes in the 1^{st} round of AES-128. Before introducing the specific experiment details, we first introduce how to get the prior distribution of the actual value of the signal component for every interesting point for both simulated and practical experiments.

For simplicity, for both simulated and practical experiments, let n_p denote the number of traces used in the profiling stage and let n_e denote the number of traces used in the extraction stage. In this paper, we use the typical metric *Guessing Entropy* [5] as the metric about the classification performance of template attacks (Many other papers also used Guessing Entropy (e.g. [4, 16, 17]) as the metric.).

4.1 How to Get The Priori Knowledge

In order to get the priori knowledge, we simulated the cases that the attacker can obtain the priori knowledge from his previous experience of conducting template attacks against devices which are similar to the reference device.

For both simulated and practical experiments, we get the prior distribution of the actual value of the signal component for every interesting point using the traces which were generated in the same way as those were used in the two stages of template attacks. In this way, we can clearly give out an upper bound of how powerful template attacks will become by exploiting the priori knowledge. In both simulated and practical experiments, for each key-dependent operation O_i and each interesting point P_j , we considered the prior distribution under four different levels of accuracy and assumed the prior distribution is a normal distribution $\mathcal{N}(\theta_1, \theta_2^2)$ (For different interesting points, the corresponding prior distributions are different.).

For each key-dependent operation O_i, we generated 400 traces (simulated traces or actual power traces). The 400 traces were used to estimate the prior distributions for every interesting point as follows. We repeated the following process 300 times. Every time, we chose m traces (denoted by $S_1, \ldots, S_m$) from the 400 traces uniformly at random and computed $\sum_{k=1}^{m} S_k[P_j]/m$. Therefore, there were 300 different values about $\sum_{k=1}^{m} S_k[P_j]/m$. The mean value of the 300 different values was set to be θ_1 and the variance of the 300 different values was set to be θ_2^2. In this way, the prior distribution $\mathcal{N}(\theta_1, \theta_2^2)$ was got. Note that, in practice, the attacker has many ways to get the prior distribution $\mathcal{N}(\theta_1, \theta_2^2)$. Our method which was used in this paper is just one of them. We respectively let $m =$ 16, 32, 64, 128 and obtained four different estimation of the prior distribution. Clearly, when the value m is larger, the estimation of θ_1 and θ_2^2 is more accurate. Therefore, we obtained four different prior distributions under different levels of accuracy, which represent the priori knowledge that the attacker can possess in practical attack scenarios.

We considered many kinds of template attacks and define the following symbols to denote them. In all the experiments, we let the symbol "CTA" denotes the classical template attacks without any priori knowledge. The symbol "CTA-16" denotes classical template attacks based on priori knowledge which is obtained when the value m equals to 16. Similarly, we define the symbols "CTA-32", "CTA-64", and "CTA-128" to denote the cases that the value m equals to 32, 64, and 128 respectively. We let the symbol "RTA" denotes the reduced template attacks without any priori knowledge. The symbol "RTA-16" denotes reduced template attacks based on priori knowledge which is obtained when the value m equals to 16. Similarly, we define the symbols "RTA-32", "RTA-64", and "RTA-128" to denote the cases that the value m equals to 32, 64, and 128.

4.2 Simulated Experiments

Simulated experiments are divided into two groups. In the first group, we assume that the attacker knows the actual value of the electronic noise level of each interesting point. Due to the fact that the number of profiling traces obtained by the attacker is limited in our scenario, in the second group, we assume that the attacker does not know the actual value of the electronic noise level and reasonably assumes it to be a constant value for every interesting point. In all the simulated experiments, we chose 4 interesting points and the typical Hamming-Weight power model (pp. 40-41 in [9]) was adopted to describe the power consumption.

4.2.1 Group 1

In this group of simulated experiments, we assume that the attacker knows the actual value of the electronic noise level of each interesting point. For the sake of simplicity, we use the standard deviation of simulated Gaussian noise which is denoted by σ to represent the electronic noise level. We employed four different electronic noise levels to test the influence of noises on the classification performance of template attacks. The standard deviations of simulated Gaussian noise for the four electronic noise levels were 2, 3, 4, and 5.

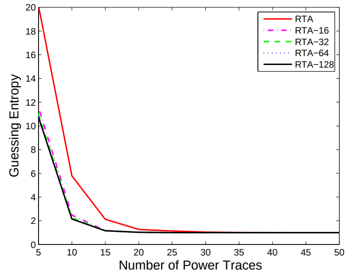
For each electronic noise level, we respectively used 2,000 and 4,000 simulated traces to build the 256 reduced templates in the profiling stage for the five kinds of reduced template attacks (RTA, RTA-16, RTA-32, RTA-64, and RTA-128). This means that the attacker respectively obtained 2,000 and 4,000 traces from the reference device in the profiling stage. The simulated traces used in the profiling stage were generated with a fixed subkey and random plaintext inputs. We generated additional 100,000 simulated traces with another fixed subkey and random plaintext inputs under each electronic noise level. The 100,000 simulated traces were used in the extraction stage. For fixed n_p and σ, we tested the Guessing Entropy of the five kinds of reduced template attacks when the attacker could use n_e simulated traces in the extraction stage as follows. We respectively repeated the five kinds of reduced template attacks 1,000 times. For each time, we chose n_e simulated traces from the 100,000 simulated traces uniformly at random and the five kinds of reduced template attacks were conducted with the same n_e simulated traces. We respectively computed the Guessing Entropy of the five kinds of reduced template attacks with the results of the 1,000 times attacks. The Guessing Entropy of the five kinds of reduced template attacks for different values of n_p and σ is shown in Figure 2.

Table 2. The simulated experiment results for the case $n_p = 2,000, n_e = 20, \sigma = 5$

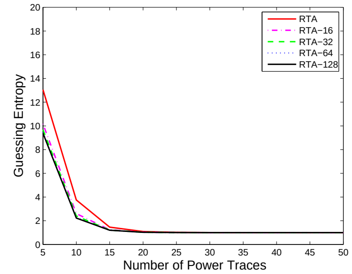
RTA	RTA-16	RTA-32	RTA-64	RTA-128
43.51	15.04	12.46	11.43	11.15

The Guessing Entropy of the five kinds of reduced template attacks for the case $\{n_p = 2,000, n_e = 20, \sigma = 5\}$ is shown in Table 2. From Figure 2 and Table 2, we find that the classification performance of reduced template attacks with accurate priori knowledge will be obvious better than that of reduced template attacks without priori knowledge. For example, in Table 2, the Guessing Entropy of RTA equals to 43.51, while the Guessing Entropy of RTA-128 equals to 11.15. Moreover, if the priori knowledge is more accurate, the classification performance of reduced template attacks with priori knowledge will be better. For example, in Table 2, the Guessing Entropy of RTA-16 equals to 15.04, while the Guessing Entropy of RTA-128 obviously reduces to 11.15.

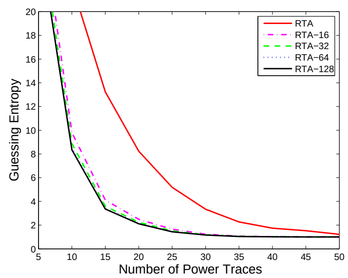
Table 3 shows the Guessing Entropy of RTA and RTA-128 for different levels of noises when n_p is fixed to $2,000$ and n_e is fixed to 20. From Figure 2 and Table 3, we further find that, when the electronic noise level is higher, reduced template attacks with priori knowledge will achieve larger advantage over reduced template attacks without priori knowledge. For example, in Table 3, the Guessing Entropy of RTA and RTA-128 is almost equal when σ equals to 2 (1.27 and



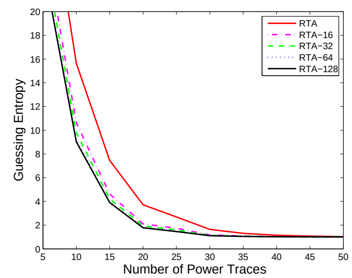
(a) $n_p = 2,000, \sigma = 2$



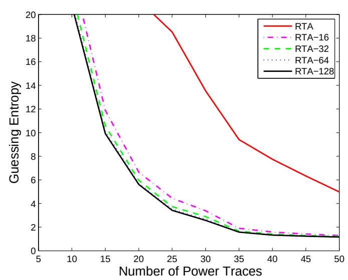
(b) $n_p = 4,000, \sigma = 2$



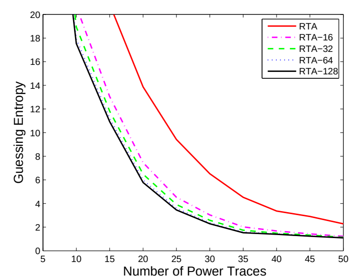
(c) $n_p = 2,000, \sigma = 3$



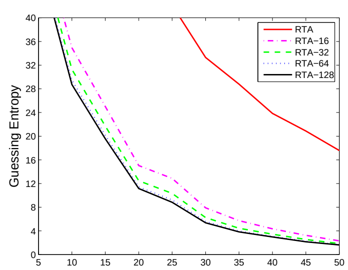
(d) $n_p = 4,000, \sigma = 3$



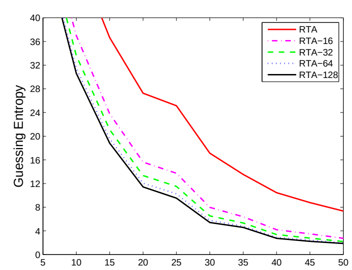
(e) $n_p = 2,000, \sigma = 4$



(f) $n_p = 4,000, \sigma = 4$



(g) $n_p = 2,000, \sigma = 5$



(h) $n_p = 4,000, \sigma = 5$

Fig. 2. The simulated experiment results

Table 3. The simulated experiment results for different levels of noises

$n_p = 2,000, n_e = 20$	$\sigma = 2$	$\sigma = 3$	$\sigma = 4$	$\sigma = 5$
RTA	1.27	8.23	21.22	43.51
RTA-128	1.03	2.11	5.61	11.15

1.03). However, when σ equals to 5, the Guessing Entropy of RTA-128 (11.15) is much lower than that of RTA (43.51).

When more simulated traces can be obtained from the reference device (e.g. $n_p = 4,000$) in the profiling stage, the advantages of reduced template attacks with priori knowledge over template attacks without priori knowledge will be smaller. For classical template attacks, we computed the Guessing Entropy of the five kinds of classical template attacks (CTA, CTA-16, CTA-32, CTA-64, and CTA-128) similarly. The simulated experiment results show that classical template attacks with accurate priori knowledge have advantages over classical template attacks without priori knowledge.

4.2.2 Group 2

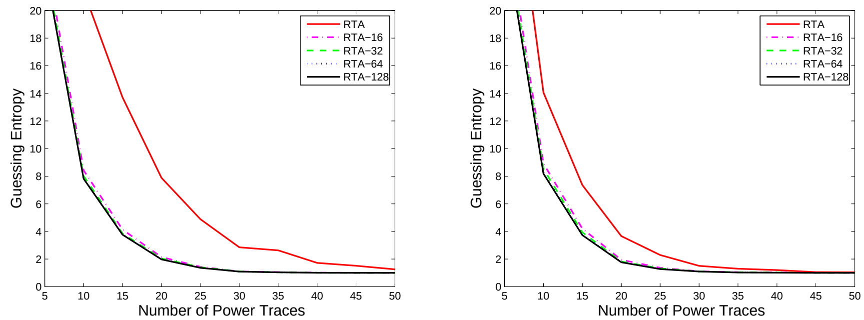
In this group of simulated experiments, we assume that the attacker does not know the actual value of the electronic noise level of each interesting point and simulate the case that the attacker reasonably assumes the electronic noise level of each interesting point to be a constant value.

We also use the standard deviation of simulated Gaussian noise to represent the electronic noise level. The actual value of the electronic noise level is denoted by σ and the electronic noise level which is reasonably assumed by the attacker is denoted by $\hat{\sigma}$. To be specific, the priori knowledge was got from simulated traces whose standard deviation of simulated Gaussian noise equalled to σ. Moreover, the standard deviation of simulated Gaussian noise of the simulated traces which were used both in the profiling stage and the extraction stage equalled to σ. However, the Bayes estimation of the actual value of the signal component μ_j was computed by

$$\mu_j'' = \frac{n}{n + \hat{\sigma}_j^2/\theta_2^2}\left(\frac{\sum_{k=1}^{n} S_k[P_j]}{n}\right) + \frac{\hat{\sigma}_j^2/\theta_2^2}{n + \hat{\sigma}_j^2/\theta_2^2}\theta_1.$$

Two cases in this scenario were considered by us. We assumed $\sigma < \hat{\sigma}$ in the first case and assumed $\sigma > \hat{\sigma}$ in the second case. In both the two cases, we also used 2,000 and 4,000 simulated traces to build the 256 reduced templates in the profiling stage for the five kinds of reduced template attacks. Moreover, we tested the Guessing Entropy of the five kinds of reduced template attacks when the attacker could use n_e simulated traces in the extraction stage similarly to the simulated experiments in Group 1. In the following, we will introduce the specific results of the simulated experiments of the above two cases.

Case 1: $\sigma < \hat{\sigma}$, $\sigma = 3$ and $\hat{\sigma} = 4$

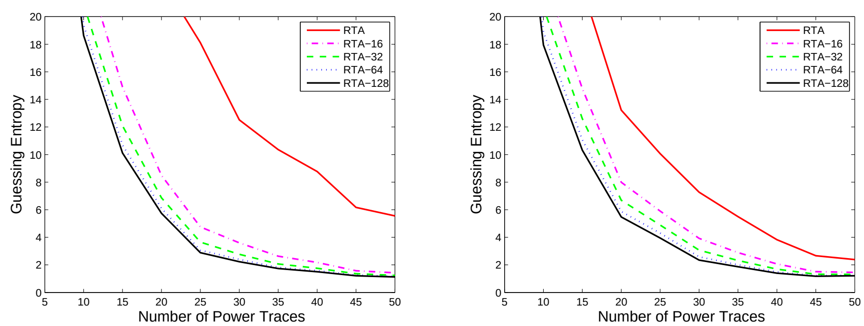


(a) $n_p = 2,000$ (b) $n_p = 4,000$

Fig. 3. The results of simulated experiments when $\sigma = 3$ and $\hat{\sigma} = 4$

Figure 3 shows the Guessing Entropy of the five kinds of reduced template attacks when $\sigma = 3$ and $\hat{\sigma} = 4$.

Case 2: $\sigma > \hat{\sigma}$, $\sigma = 4$ and $\hat{\sigma} = 3$



(a) $n_p = 2,000$ (b) $n_p = 4,000$

Fig. 4. The results of simulated experiments when $\sigma = 4$ and $\hat{\sigma} = 3$

Figure 4 shows the Guessing Entropy of the five kinds of reduced template attacks when $\sigma = 4$ and $\hat{\sigma} = 3$.

From Figure 3 and Figure 4, we find that the classification performance of reduced template attacks with priori knowledge will be obvious better than that of reduced template attacks without priori knowledge even if the attacker does not know the actual value of the electronic noise level for each interesting point. This means that it is not necessary to know the actual value of the electronic noise level for each interesting point for the practical attacker when he wants to exploit the priori knowledge. Therefore, it is more easily for the practical attacker to conduct template attacks with priori knowledge.

4.3 Practical Experiments

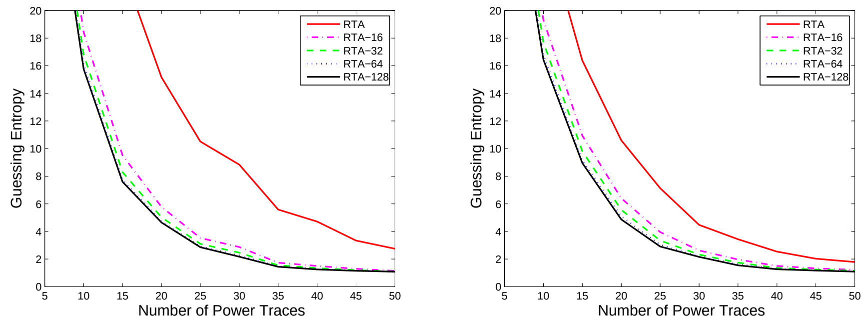
We tried to attack the outputs of all the S-boxes in the 1^{st} round of an unprotected AES-128 software implementation on an typical 8-bit microcontroller STC89C58RD+ whose operating frequency is 11MHz. The actual power traces were acquired with a sampling rate of 50MS/s. The average number of actual power traces during the sampling process was 10 times. For our device, the condition of equal covariances [4] does not hold.

We generated two sets of actual power traces, Set A and Set B. The Set A captured 10,000 power traces which were generated with a fixed main key and random plaintext inputs. The Set B captured 100,000 power traces which were generated with another fixed main key and random plaintext inputs. The power traces in Set A were used in the profiling stage and the power traces in Set B were used in the extraction stage. We used the same device as that was used to get the prior distribution in Section 4.1 to generate the two sets of actual power traces, which provides a good setting for the focuses of our research. For each S-box of the unprotected AES-128 software implementation, we chose 4 interesting points. Both classical template attacks and reduced template attacks were conducted based on the same 4 interesting points. We only show the practical experiment results of the 1^{st} and the 2^{nd} S-box in this paper. For other S-boxes in the 1^{st} round, similar evaluation results were obtained by us. In all practical experiments, when the Bayes estimation of the actual value of the signal component was computed, we reasonably assumed that the actual value of the electronic noise level equals to a constant value for each interesting point and each target intermediate value.

For reduced template attacks, we respectively chose 2,000 and 4,000 different power traces from Set A to build the 256 templates for the five kinds of reduced template attacks (RTA, RTA-16, RTA-32, RTA-64, and RTA-128). The 100,000 power traces of Set B were used in the extraction stage for the five kinds of reduced template attacks. For fixed n_p, we tested the Guessing Entropy of the five kinds of reduced template attacks when one uses n_e power traces in the extraction stage similarly to that of the simulated experiments but used actual power traces. The Guessing Entropy of the five kinds of reduced template attacks for the 1^{st} S-box are shown in Figure 5. The Guessing Entropy of the five kinds of reduced template attacks for the 1^{st} S-box when n_p is fixed to $2,000$ and n_e is fixed to 20 is shown in Table 4.

From Figure 5 and Table 4, we find that the classification performance of reduced template attacks with accurate priori knowledge will be obvious better than that of reduced template attacks without priori knowledge. For example, in Table 4, the Guessing Entropy of RTA equals to 15.16, while the Guessing Entropy of RTA-16 reduces to 5.78.

For classical template attacks, in order to avoid numerical obstacles with the inversion of the covariance matrix, we respectively chose 5,000 and 10,000 different power traces from Set A to build the 256 templates for the five kinds of classical template attacks (CTA, CTA-16, CTA-32, CTA-64, and CTA-128). Moreover, using power traces from Set B, we computed the Guessing Entropy of



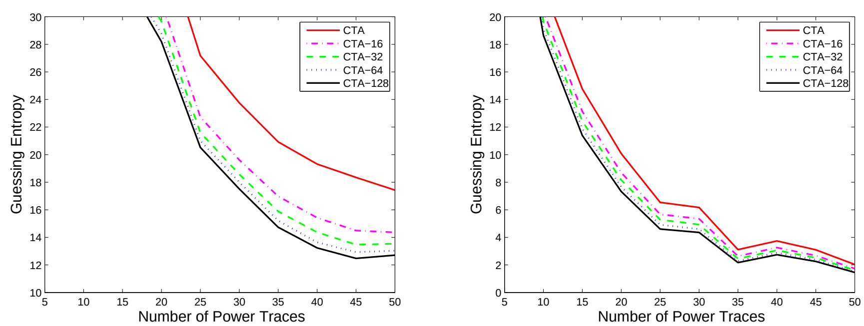
(a) $n_p = 2,000$ (b) $n_p = 4,000$

Fig. 5. The experiment results of reduced template attacks for the 1^{st} S-box

Table 4. The experiment results of reduced template attacks for the 1^{st} S-box

$n_p = 2,000$	RTA	RTA-16	RTA-32	RTA-64	RTA-128
$n_e = 20$	15.16	5.78	5.03	4.73	4.65

the five kinds of classical template attacks when one uses n_e power traces in the extraction stage similarly. The Guessing Entropy of the five kinds of classical template attacks for the 1^{st} S-box are shown in Figure 6.



(a) $n_p = 5,000$ (b) $n_p = 10,000$

Fig. 6. The experiment results of classical template attacks for the 1^{st} S-box

For the 2^{nd} S-box, we also used the actual power traces in Set A and Set B to compute the Guessing Entropy of the five kinds of reduced template attacks and the five kinds of classical template attacks similarly. The practical experiment results for the 2^{nd} S-box which can also verify our discoveries are shown in Figure 7, Figure 8, and Table 5 in Appendix A.

The practical experiment results show that, for both reduced template attacks and classical template attacks, if the priori knowledge is more accurate, the classification performance will be better. For example, in Table 4, the Guessing Entropy of RTA-16 equals to 5.78, while the Guessing Entropy of RTA-128 reduces to 4.65. When more power traces can be obtained from the reference device, the advantages of template attacks with priori knowledge over template attacks without priori knowledge will be smaller.

5 Conclusion and Future Work

In this paper, we show that leaking the priori knowledge about the reference device poses serious threat to the physical security of cryptographic devices. Therefore, we suggest that the designers of a cryptographic device should take the priori knowledge into consideration when he uses template attacks to evaluate the physical security of the cryptographic device. The future work is as follows. First, our discoveries show that the approach to infer (estimate) the priori knowledge as accurately as possible is crucial and is worth being researched from the attacker's point of view. Second, it would be interesting to research how to prevent the attacker to obtain the priori knowledge (Using countermeasures such as the random delays [11] may be a good choice.). We should also concern on how to exploit the priori knowledge to make other profiled side-channel attacks (such as stochastic model based attacks [18] etc.) become more powerful in a reasonable way. It is also necessary to further verify our discoveries in other devices such as ASIC and FPGA.

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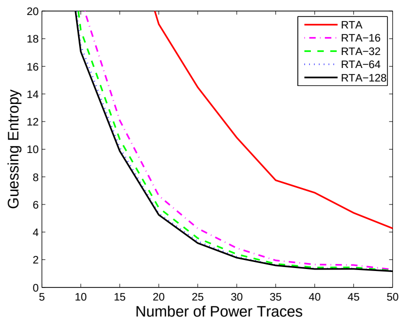
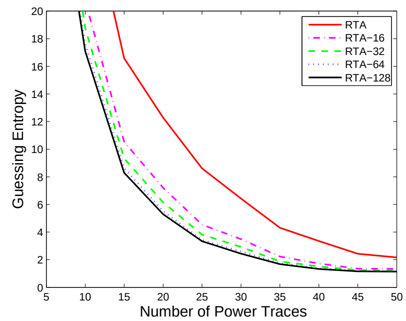
Appendix A: Practical Experiments for The 2^{nd} S-box

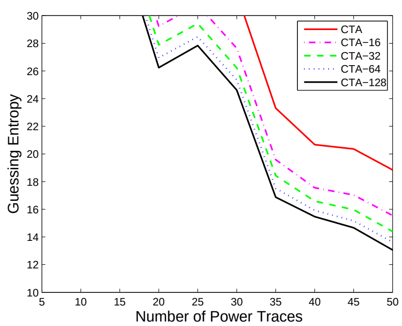


(a) $n_p = 2,000$

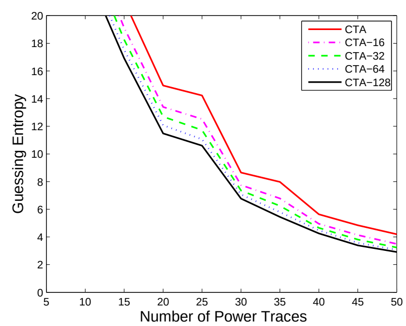


(b) $n_p = 4,000$

Fig. 7. The experiment results of reduced template attacks for the 2^{nd} S-box



(a) $n_p = 5,000$



(b) $n_p = 10,000$

Fig. 8. The experiment results of classical template attacks for the 2^{nd} S-box

Table 5. The experiment results of reduced template attacks for the 2^{nd} S-box

$n_p = 2,000$	RTA	RTA-16	RTA-32	RTA-64	RTA-128
$n_e = 20$	19.05	6.64	5.76	5.34	5.25