

# On the Security of a access polynomial based self-healing key management schemes in wireless sensor networks

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## Abstract

Secure group communication in wireless sensors networks (WSN) is a critical issue. Self-healing mechanism is that nodes in WSN can recover the lost session keys without requiring anything to the group manager (GM). Self-healing can be applied to key management, which can efficiently achieve the secure group communication. In 2015, Sun *et al.* proposed a self-healing group key distribution (SGKD) scheme based on access polynomial and sliding-window in WSN, and they claim that the proposed scheme can achieve any-wise backward secrecy, any-wise forward secrecy and  $\delta$  collusion resistance. However, we find the scheme can not resist the collusion attack of new joined nodes and revoked nodes, and the legitimate cannot achieve session key recovery in some case.

Keywords: wireless sensors networks, self-healing group key distribution, cryptanalysis

## 1 Introduction

Recently, wireless sensor networks (WSN) draws more and more attention because of its wide application such as intelligence, rescue missions and military operations. There are large number of sensor nodes, which are powered by batteries and thus, have limited storage and communication capabilities. Besides, the WSN is confronted with some security challenges such as the unreliability, the attack. In order to solve the security challenges, key management is a critical method, which includes key distribution and key updating.

Because of the unreliability of WSN, some group key distribution and key updating messages may not be able to receive the sensors nodes. The direct way is that the node requires GM to retransmit the missing message. The retransmission of large number of nodes will aggravate the communication overhead, which may be a disaster. Staddon [1] proposed a SGKD, which can efficiently reduce the communication overhead of retransmission. The property of self-healing can guarantee that if node misses the key distribution or key updating messages, he can recover the lost group keys just by combining the previous and the subsequent messages.

Staddon [1] first proposed the concept of self-healing and constructed two scheme using lagrange interpolation. The scheme in [1] guarantees that the group nodes can recover the session keys but

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the revoked nodes can not, and the lost session key can be recovered by combining the previous and subsequent broadcast message, namely, self-healing. However, Blundo *et al.* [2] found an attack in the construction 1 [1]. Later, Liu *et al.* proposed a novel SGKD scheme based on revocation polynomial, which is more efficient in terms of the storage overhead and communication overhead. More [4] proposed a sliding-window SGKD scheme, which can balance the communication overhead and the self-healing capability. Hong *et al.* simplified the SGKD scheme based on revocation polynomial [3], which communication overhead is further to be reduced. Later, some SGKD schemes based on revocation polynomial are proposed [6]-[20]. Zou *et al.* first proposed the SGKD scheme based on access polynomial, which has constant storage overhead and protects the privacy of nodes' identifies. Later, some SGKD schemes based on access polynomial are proposed [22, 23, 24].

Recently, Sun *et al.* proposed a SGKD scheme based on access polynomial, they claimed that their scheme has  $\delta$  collusion resistance. However, we find the scheme I and II [25] can not resist collusion attack of the new joined node and the revoked node. Besides, the legitimate user can not recover the session key in some case.

The reminder of the paper is arranged as follows. Section 2 briefly introduces Sun *et al.*'s scheme. The cryptanalysis of Sun *et al.*'s scheme is presented in Section 3. Section 4 concludes the whole paper.

## 2 Overview of Sun *et al.*'s scheme [25]

In this section, we mainly introduce security model and the proposed scheme I and II in [25]. We take the same notations as Sun *et al.*'s scheme. For readability, some notations are listed in Table 1.

### 2.1 Security model

The scheme in [25] adopts a modified security model. Next we briefly introduce the security model.

**Definition 1** (*Key independence*)  $\{GK_t\}_{t \in Z_+} \subset F_q$  is the set of session keys which are distributed independently and uniformly. The scheme  $D$  satisfies session key independence if  $\mathbb{h}(GK_t | \{GK_i\}_{i \in Z_+ \setminus t}) = \mathbb{h}(GK_t)$  for any  $t \in Z_+$ .

**Definition 2** (*Session key distribution with any-wise revocation capability*) Suppose that  $j \in Z_+$ ,  $l \in \{1, 2, \dots, n\}$ ,  $n$  is the total group members,  $SK_l^i$  is the secret information of  $U_l$  in session  $i$ . The scheme  $D$  is a session key distribution scheme with any-wise revocation capability if

(1) For any node  $U_l$  in  $G_i$ ,  $GK_i$  is determined by  $B_i$  and  $SK_l^i$ . That is  $\mathbb{h}(GK_i | B_i, SK_l^i) = 0$ .

(2)  $GK_i$  cannot be derived from either  $\{B_t\}_{t \in Z_+}$  or  $\{SK_l^i\}_{i \in \{1, 2, \dots, n\}}$ . That is

$$\mathbb{h}(GK_i | \{B_t\}_{t \in Z_+}) = \mathbb{h}(GK_i | \{SK_l^i\}_{i \in \{1, 2, \dots, n\}}) = \mathbb{h}(GK_i).$$

(3) Let  $\tilde{R} = R_1 \cup R_2 \cup \dots \cup R_\infty$  be the set of revoked nodes the lifetime of the group.  $D$  has any-wise revocation capability if any nodes belonging to  $\tilde{R}$  cannot get the secret information  $SK_l^i$  of node  $l$  not belonging to  $\tilde{R}$ ,  $i \in Z_+$ . That is

$$\mathbb{h}(\{SK_l^i\}_{i \in Z_+, l: U_l \in G_i} | \{B_t\}_{t \in Z_+}, \{SK_r^k\}_{k \in Z_+, r: U_r \in \tilde{R}}) = \mathbb{h}(\{SK_l^i\}_{i \in Z_+, l: U_l \in G_i})$$

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$D$	The proposed scheme
$q, F_q$	A large primer number, and a finite field of order $q$
$GM$	Group manager
$G_i$	The set of all legitimate group members during session $i, i = 1, 2, \dots$
$U_l$	Ordinary sensor nodes in $G_i, l = 1, 2, \dots,  G_i $
$J_i$	The set of group members joining the group in session $i, i = 1, 2, \dots$
$R_i$	The set of group members revoked in session $i, i = 1, 2, \dots$
$GK_i$	Group key in session $i, i = 1, 2, \dots$
$UK_l^i$	The pairwise key between $U_l$ and GM in session $i, U_l \in G_i$
$SK_l^i$	The shared secret information between $U_l$ and GM in session $i, U_l \in G_i$
$H(\cdot), H^k(\cdot)$	One-way hash chain, and continuously hash operation $k$ times
$S^F$	The seed of the forward key chain
$FK_i$	The $j$ th element of forward key chain, where $FK_i = H^i(S^F)$
$B_i$	The broadcast message for key updating in session $i, i = 1, 2, \dots$
$\phi_i(x)$	Access polynomial generated by GM in session $i, i = 1, 2, \dots$
$\delta$	The size of the self-healing window
$\hbar(\cdot)$	The entropy function in information theory
$MAC_K(M)$	HMAC operation on $M$ using the key $K$ to ensure message integrity and authentication
$S_l^i$	The total secret information stored at node $l$ in the group in session $i$

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Table 1: Notations

**Definition 3** ( $\delta$ -self-healing capability)  $D$  has  $\delta$ -self-healing capability if

$$\hbar(GK_i | B_{i_1}, B_{i_2}, SK_l^{i_1}, SK_l^{i_2}) = \begin{cases} 0 & \text{if } i_2 - i_1 \leq \delta \\ \hbar(GK_i) & \text{if } i_2 - i_1 > \delta. \end{cases}$$

where  $1 < i_1 \leq i < i_2$  and  $U_l \in G_{i_1} \cup G_i \cup G_{i_2}$ .

**Definition 4** (Any-wise forward secrecy) Let  $\tilde{R}_i = R_1 \cup R_2 \cup \dots \cup R_i$  be the set of revoked nodes before and during session  $i$ .  $D$  guarantees any-wise forward security if every node belonging to  $\tilde{R}_i$  cannot get any information about  $GK_i, GK_{i+1}, \dots, GK_\infty$  for any  $i \in \mathbb{Z}_+$  and  $|\tilde{R}_i| \leq \infty$ . That is

$$\hbar(\{GK_t\}_{t \in \{i, i+1, \dots\}} | \{B_j\}_{j \in \mathbb{Z}_+}, \{SK_r^k\}_{k \in \mathbb{Z}_+, r: U_r \in \tilde{R}_i}) = \hbar(\{GK_t\}_{t \in \{i, i+1, \dots\}})$$

**Definition 5** (Any-wise backward secrecy) Let  $\tilde{J}_i = J_i \cup J_{i+1} \cup \dots \cup J_\infty$  be the set of nodes joining the group during and after session  $i$ .  $D$  guarantees any-wise backward security if all nodes belonging to  $\tilde{J}_i$  cannot get any information about  $GK_1, GK_2, \dots, GK_{i-1}$  for any  $i \in \mathbb{Z}_+$  and  $|\tilde{J}_i| \leq \infty$ . That is

$$\hbar(\{GK_t\}_{t \in \{1, 2, \dots, i-1\}} | \{B_j\}_{j \in \mathbb{Z}_+}, \{SK_r^k\}_{k \in \mathbb{Z}_+, r: U_r \in \tilde{J}_i}) = \hbar(\{GK_t\}_{t \in \{1, 2, \dots, i-1\}})$$

**Definition 6** ( $\lambda$ -collusion resistance) Let  $\tilde{R}_r \cup \tilde{J}_s$  be the set of nodes revoked before and during session  $r$  and nodes joining the group during and after session  $s$ , where  $1 < r < s$ .  $D$  has  $\lambda$ -collusion resistance if the following condition is true. That is

$$\hbar(\{GK_t\}_{t \in \{r, r+1, \dots, s-1\}} | \{B_j\}_{j \in \mathbb{Z}_+}, \{SK_j^k\}_{k \in \mathbb{Z}_+, j: U_j \in \tilde{R}_r \cup \tilde{J}_s}) = \begin{cases} 0 & \text{if } s - r \leq \lambda \\ \hbar(\{GK_t\}_{t \in \{r, r+1, \dots, s-1\}}) & \text{if } s - r > \delta. \end{cases}$$

## 2.2 Sun *et al.* proposed scheme I

In this subsection, we mainly introduce the proposed scheme I in [25]. The scheme I is composed of five parts: **Initialization**, **Broadcast**, **GroupKeyRecoveryandPairwiseKeyUpdating**, **NodeAddition** and **NodeRevocation**.

- **Part1 : Initialization**

The offline sever generates a random seek  $S^F$  for the forward key chain and a secret  $\{SK_l, UK_l^0\}$  only shared between GM and  $U_l$ , and send them to GM and  $U_l$ , where  $l = 1, 2, \dots, |G_0|$  and  $G_0$  is the initial set of the group. GM keeps the secret information  $\{S^F, UK_1^0, SK_1, UK_2^0, SK_2, \dots, UK_{|G_0|}^0, SK_{|G_0|}\}$ , while  $U_l$  keeps  $\{S^F, UK_l^0, SK_l\}$ . As  $i \geq \delta$ ,  $U_l$  will store  $\{H(GK_{i-\delta}), H(GK_{i-\delta+1}), \dots, H(GK_{i-1}), GK_i\}$ , which takes up  $(\delta + 1)\log_2 q$  bits. The size of  $\delta$  meets  $\delta \geq MaxSessions$  is the maximum value of consecutive sessions during which group members lost their group keys under unreliable links.

- **Part2 : Broadcast**

Key updating and recover will be launched by GM if one of the following cases happens: (a) new nodes want to join the group; (b) malicious nodes are revoked from the group; (c) any intrusion is detected; (d) the current session runs out.

Consider the process for session  $i$ , where  $i \in Z_+$ .

- (1) GM generates random  $\theta_i, \zeta_i, RK_i \in F_q$  and a modified access polynomial

$$\phi_i(x) = (\theta_i x - \zeta_i) \prod_{l: U_l \in G_i} (x - UK_l^{i1}) + RK_i$$

- (2) Group key  $GK_i$  is computed as

$$GK_i = FK_i \oplus RK_i$$

- (3) Using the key  $RK_i$ , GM encrypts  $RK_{i-1}, RK_{i-2}, \dots, RK_{i-\delta}$  in Exclusive OR (XOR) cipher using one-way hash operations, and gets  $RK_{i-1} \oplus H(RK_i), RK_{i-2} \oplus H(RK_{i-1}), \dots, RK_{i-\delta} \oplus H(RK_{i-\delta-1})$ . Here,  $\delta$  is the size of sliding window proposed by More *et al.* in [4], and the length of  $H(RK_i + j)$  should be equal to the length of  $RK_i$ , where  $0 \leq j \leq \delta - 1$ .
- (4) GM computes the hash message authentication code (HMAC) with  $RK_i$  as  $MAC_{RK_i}(\phi_i(x) | RK_{i-1} \oplus H(RK_i + 1), \dots, RK_{i-\delta} \oplus H(RK_{i-\delta-1}))$ .
- (5) GM updates the secret information as follows.
  - (a) Pairwise key  $UK_l^i$  between GM and group member  $U_l$ :

$$UK_l^i = \begin{cases} H(i \oplus SK_l) & \text{if } 1 \leq i \leq \delta \\ H(H(GK_{i-\delta}) \oplus SK_l) & \text{if } i > \delta. \end{cases}$$

- (b) The forward key chain:

$$FK_{i+1} = \begin{cases} S^F & \text{if } i = 0 \\ H(FK_i) & \text{if } i \geq 1. \end{cases}$$

(c) The secret information  $S_0^i$  is stored in GM as  $\{FK_{i+1}, \{UK_l^i, SK_l\}_{l:U_l \in G_i}, \{RK_t\}_{1 \leq t \leq i}, \{H(GK_k)\}_{i-\delta \leq k \leq i-1}, GK_i\}$ , where  $(\delta + 2 + i + 2|G_i|) \log_2 q$  bits space are used.

(6) Finally, GM broadcasts the message  $B_i$ ,

$$B_i = \phi_i(x) | RK_{i-1} \oplus H(RK_i), RK_{i-2} \oplus H(RK_i + 1), \dots, RK_{i-\delta} \oplus H(RK_i + \delta - 1) | \\ MAC_{RK_i}(\phi_i(x) | RK_{i-1} \oplus H(RK_i), RK_{i-2} \oplus H(RK_i + 1), \dots, RK_{i-\delta})$$

• **Part3 : GroupKeyRecoveryandPairwiseKeyUpdating**

Suppose that key synchronization for the legitimate node  $U_l \in G_i$  gets broken after session  $i_1$ , and  $0 < i - i_1 \leq \delta + 1$  holds, and the information stored in node  $U_l$  is  $SK_l^{i_1} = \{FK_{i_1+1}, UK_l^{i_1-1}, SK_l, \{H(GK_k)\}_{i_1-\delta \leq k \leq i_1-1}, GK_{i_1}\}$ . Once node  $U_l$  receives  $B_i$ , it recovers lost group session keys as follows.

(1) If  $i - 1$  is equal to  $i_1$ . then execute step(2); otherwise, pairwise key  $UK_l^{i-1}$  between GM and  $U_l$  is computed as,

$$UK_l^{i-1} = \begin{cases} H((i-1) \oplus SK_l) & \text{if } 2 \leq i \leq \delta + 1 \\ H(H(GK_{i-\delta-1}) \oplus SK_l) & \text{if } i > \delta + 1. \end{cases}$$

(2) Compute  $RK_i = \phi(UK_l^{i-1})$ ;

(3) The integrity and broadcast authentication will be checked by using HMAC. First,  $MAC_{RK_i}(\phi_i(x) | RK_{i-1} \oplus H(RK_i), RK_{i-2} \oplus H(RK_i + 1), \dots, RK_{i-\delta} \oplus H(RK_i + \delta - 1))$  is calculated by using  $RK_i$  and  $\phi_i(x) | RK_{i-1} \oplus H(RK_i), RK_{i-2} \oplus H(RK_i + 1), \dots, RK_{i-\delta} \oplus H(RK_i + \delta - 1)$ ; Then, if the computed HMAC and the received HMAC are equal, the message is not tampered by attacks and broadcast authentication get passed; otherwise, discard the message.

(4) Using the key  $RK_i$  and one-way hash operations,  $RK_{i-1}, RK_{i-2}, \dots, RK_{i-\delta}$  can be got as  $RK_j = H(RK + i - j + 1) \oplus (RK_j \oplus H(RK + i - j + 1))$ , where  $i - \delta \leq j \leq i - 1$ .

(5) Node  $U_l$  gets group keys  $GK_i$  and  $GK_{i-1}, GK_{i-2}, \dots, GK_{i_1+1}$  as follows:

$$\begin{aligned} GK_{i_1+1} &= FK_{i_1+1} \oplus RK_{i_1+1}, \\ FK_{i_1+2} &= H(FK_{i_1+1}), \\ &\dots, \\ GK_i &= FK_i \oplus RK_i, \\ FK_{i+1} &= H(FK_i), \end{aligned}$$

(6) Pairwise key  $UK_l^i$  is updated as

$$UK_l^i = \begin{cases} H(i \oplus SK_l) & \text{if } 1 \leq i \leq \delta \\ H(H(GK_{i-\delta}) \oplus SK_l) & \text{if } i > \delta. \end{cases}$$

The secret information  $S_l^i$  is updated as

$$S_l^i = \{FK_{i+1}, UK_l^i, SK_l, \{H(GK_k)\}_{k=i-\delta, i-\delta+1, \dots, i-1}, GK_i\},$$

where  $(\delta + 4) \log_2 q$  bits space are used. It should be noted that when  $i - i_1 > \delta$  holds, node  $U_l$  has lost more than  $\delta$  session keys consecutively and will request group session keys from GM using  $SK_l$ .

- **Part4 : NodeAddition**

If node  $U_w$  wants to join the group  $G_i$  in session  $i$ , it is first pre-loaded with secret  $\{FK_i, UK_w^{i-1}, SK_w, \{H(GK_k)\}_{i-1-\delta \leq k \leq i-1}\}$ . Then, a key updating process is launched by GM which has been loaded with  $\{UK_w^{i-1}, SK_w\}$  by the sink or offline sever. Once receiving  $B_i$ , node  $U_w$  computes  $GK_i$  and updates  $UK_w^{i-1}$  as Part 3 above.

- **Part5 : NodeRevocation**

When GM detects that  $U_r \in G_{i-1}$  is compromised in session  $i - 1$ , thus GM will revoke  $U_r$  in session  $i$  by only removing the item  $(x - UK_r^{i-1})$  from the polynomial  $\prod_{l:U_l \in G_{i-1}} (x - UK_l^{i-1})$ .

### 2.3 Sun *et al.* proposed scheme II

Scheme II removes the key chain  $FK_i$ , but strengthens collusion resistance capability, and becomes more efficient. Similarly, scheme II is also composed of 5 parts. The difference lies in the processes of group keys calculation, recovery and node addition as well.

(A) Group key  $GK_i$  is computed as

$$GK_i = \begin{cases} RK_i & \text{if } i = 1 \\ H(GK_1) \oplus RK_i & \text{if } 1 < i \leq \delta + 1 \\ H(GK_{i-\delta-1}) \oplus RK_i & \text{if } i > \delta + 1 \end{cases}$$

(B) Group key recovery Similar to Part 3 in scheme I, only difference exists in step (5). For session  $i$ , node  $U_l \in G_i$  will calculate current group key  $GK_i$  and lost group  $GK_{i_1+1}, \dots, GK_{i-1}$ , where  $0 < i - i_1 \leq \delta + 1$ . The case  $i_1 > \delta + 1$  is only considered, while other simple cases can be derived easily, and is not given here.

$$\begin{aligned} GK_{i_1} &= H(GK_{i_1-\delta-1}) \oplus RK_{i_1}, \\ GK_{i_1+1} &= H(GK_{i_1-\delta}) \oplus RK_{i_1+1}, \\ &\dots, \\ GK_i &= H(GK_{i-\delta-1}) \oplus RK_i, \end{aligned}$$

After session  $i$ , GM updates the secret information  $S_0^i$  as  $\{\{UK_l^i, SK_l\}_{l:U_l \in G_i}, \{RK_t\}_{1 \leq t \leq i}, \{H(GK_k)\}_{i-\delta \leq k \leq i-1}, GK_i\}$  occupying  $(\delta + 1 + i + 2|G_i|) \log_2 q$  bits space, while node  $U_l \in G_i$  updates the secret information  $S_l^i$  as  $\{UK_l^i, SK_l, \{H(GK_k)\}_{i-\delta \leq k \leq i-1}, GK_i\}$  occupying  $(\delta + 3) \log_2 q$  bits space.

(C) Node addition If node  $U_w$  wants to join the group  $G_i$  in session  $i$ , it is first pre-loaded with secret  $\{UK_w^{i-1}, SK_w, \{H(GK_k)\}_{i-1-\delta \leq k \leq i-1}\}$ . Then, a key updating process is launched by GM which has been loaded with  $\{UK_w^{i-1}, SK_w\}$  by the sink or offline sever. Once receiving  $B_i$ , node  $U_w$  computes  $GK_i$  and updates  $UK_w^{i-1}$  as Part 3 in scheme I [25].

### 3 Cryptanalysis of Sun *et al.*'s Scheme

In this section, we mainly introduce the two flaws of Sun *et al.*'s scheme I and II.

#### 3.1 Flaws of Scheme I in [25]

##### 3.1.1 Lack of collusion resistance

Let  $U_l$  denote the group node who joined the group in session  $i_1$  and revoked in session  $i_2$ . Let  $U_w$  denote the node who joined the group in session  $i$ , where  $i_1 < i_2 < i - \delta < i$ . When  $U_l$  joined the group, he received the secret  $FK_{i_1}$ . Thus, he can computes  $\{FK_{i_1+1}, \dots, FK_{i-\delta}, \dots, FK_i\}$  as

$$\begin{aligned} FK_{i_1+1} &= H(FK_{i_1}), \\ FK_{i_1+2} &= H(FK_{i_1+1}) = H(H(FK_{i_1})) = H^2(FK_{i_1}), \\ &\dots, \\ FK_i &= H(FK_{i-1}) = \dots = H^{i-i_1}(FK_{i_1}), \end{aligned}$$

$U_w$  is a legitimate node in session  $U_w$ . Thus, he can evaluate  $RK_i$  by  $\phi_i(UK_w^{i-1})$ . The broadcast message composes of  $RK_{i-1} \oplus H(RK_i), RK_{i-2} \oplus H(RK_{i-1}), \dots, RK_{i-\delta} \oplus H(RK_{i-\delta+1})$ . Then,  $U_w$  can compute  $\{RK_{i-1}, RK_{i-2}, \dots, RK_{i-\delta}\}$  as

$$\begin{aligned} RK_{i-1} &= H(RK_i) \oplus (RK_{i-1} \oplus H(RK_i)), \\ RK_{i-2} &= H(RK_{i-1}) \oplus (RK_{i-2} \oplus H(RK_{i-1})), \\ &\dots, \\ RK_{i-\delta} &= H(RK_{i-\delta+1}) \oplus (RK_{i-\delta} \oplus H(RK_{i-\delta+1})), \end{aligned}$$

If  $U_l$  colludes with  $U_w$ , they provide the  $\{FK_{i_1+1}, \dots, FK_{i-\delta}, \dots, FK_i\}$  by  $U_l$  and  $\{RK_{i-1}, RK_{i-2}, \dots, RK_{i-\delta}\}$  by  $U_w$  respectively.

Thus,  $GK_{i-\delta}, GK_{i-\delta+1}, \dots, GK_{i-1}$  can be evaluated as

$$\begin{aligned} GK_{i-1} &= FK_{i-1} \oplus RK_{i-1}, \\ GK_{i-2} &= FK_{i-2} \oplus RK_{i-2}, \\ &\dots, \\ GK_{i-\delta} &= FK_{i-\delta} \oplus RK_{i-\delta}, \end{aligned}$$

The collusion of  $U_l$  and  $U_w$  can recover the session keys from session  $i - \delta$  to  $i - 1$ . However, both of them are not the legitimate node in these sessions. Therefore, the scheme I in [25] can not resist the attack of the new joined node and the revoked node collusion.

### 3.1.2 Lack of session key recovery capability

In this subsection, we will show the flaw of Sun *et al.*'s Scheme. The flaw makes the scheme cannot achieve session key recovery for a legitimate user. Obviously, it can not be tolerated for a SGKD scheme. We specifically introduce as follows.

Let  $U_l$  denote a user who joined the group in session  $i$ , where  $i > \delta$ . When  $U_l$  joined the group, he will receive the secret  $\{FK_i, UK_l^{i-1}, SK_l, \{H(GK_k)\}_{i-1-\delta \leq k \leq i-1}\}$ . If  $U_l$  does not receive the broadcast message from session  $i$  to session  $j$ , where  $j - i > \delta$ . In session  $j + 1$ , suppose  $U_l$  is a legitimate.  $U_l$  first computes  $UK_l^j$  according to step (1) in the Sch-I's part 3.

$$UK_l^j = H(H(GK_{j-\delta}) \oplus SK_l)$$

Because  $j - \delta > i$  and  $U_l$  does not receive the broadcast message from session  $i$  to session  $j$ ,  $U_l$  can not compute  $H(GK_{j-\delta})$ . Thus,  $U_l$  cannot compute  $UK_l^j$ . In this case, even if  $U_l$  is a legitimate user, he cannot achieve session key recovery.

## 3.2 Attack to Scheme II in [25]

### 3.2.1 Lack of collusion resistance

The scheme II [25] includes the same problem.

Suppose that  $U_l$  denotes the node who joined the group in session  $j_1$  and revoked in session  $j_2$ , and  $U_w$  denotes the node who joins the group in session  $i$ , where  $i_1 < i - \delta < i_2 < i$ . When  $U_l$  joined the group, he received the secret  $\{H(GK_k)\}_{i-1-\delta \leq k \leq i-1}$ . He is a legitimate node from session  $i_1$  to  $i_2$ . He has access to the session keys  $\{GK_{j'}\}_{i_1 \leq j' \leq i_2}$ . He can compute  $\{H(GK_{j'})\}_{i_1 \leq j' \leq i_2}$ . For  $u_w$ , he is a legitimate node in session  $i$ . Then, he can obtain  $\{RK_{i-1}, RK_{i-2}, \dots, RK_{i-\delta}\}$  as above.

If  $U_l$  colludes with  $U_w$ ,  $U_l$  provide  $\{H(GK_{j'})\}_{i_1 \leq j' \leq i_2}$  and  $U_w$  provides  $\{RK_{i-1}, RK_{i-2}, \dots, RK_{i-\delta}\}$ . Thus,  $\{GK_{i-1}, GK_{i-2}, \dots, GK_{i_2}\}$  can be evaluated as

$$\begin{aligned} GK_{i-1} &= H(GK_{i-\delta-2}) \oplus RK_{i-1}, \\ GK_{i-2} &= H(GK_{i-\delta-3}) \oplus RK_{i-2}, \\ &\dots, \\ GK_{i_2-1} &= H(GK_{i_2-\delta-2}) \oplus RK_{i_2-1}. \end{aligned}$$

The collusion of  $U_l$  and  $U_w$  can recover the session keys from session  $i_2$  to  $i - 1$ . However, both of them are not the legitimate node in these sessions. Therefore, the scheme II in [25] can not resist the attack of the new joined node and the revoked node collusion.

The group key is composed of two parts, namely,  $FK_i, RK_i$ . For a revoked user, he can compute the  $FK_i$  after the session he joined. For a new joined user, he can use  $RK_i$  of the current session to obtain  $\{RK_{i-1}, \dots, RK_{i-\delta}\}$  by decrypting  $\{RK_{i-1} \oplus H(RK_i), RK_{i-2} \oplus H(RK_i + 1), \dots, RK_{i-\delta} \oplus$



$H(RK_i + \delta - 1)$ . Thus, the scheme cannot resist collusion attack. The reason why this attack exists is that the  $FK_i, RK_i$  can easily be deduced from the personal secret and broadcast message.

Note that the flaw of Sch-I in 3.1.2 also hold on for the Sch-II.

## 4 Conclusion

In this paper, we show that two flaws of Sun *et al.*'s scheme. Specially speaking, it can not resist collusion attack of the new joined node and the revoked node. Besides, it can not achieve session key recovery for a legitimate node in some case.

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