# Security Attack on CloudBI: Practical privacy-preserving outsourcing of biometric identification in the cloud 

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## I. Introduction

In ESORICS 2015 [1], Wang et al. proposed a privacy-preserving outsourcing design for biometric identification using public cloud platforms, namely CloudBI. CloudBI introduces two designs: CloudBI-I and CloudBI-II. CloudBI$I$ is more efficient and CloudBI-II has stronger privacy protection. Based on the threat model of CloudBI, CloudBI-II is claimed to be secure even when the cloud service provider can act as a user to submit fingerprint information for identification. However, this security argument is not hold and CloudBI-II can be completely broken when the cloud service provider submit a small number of identification requests. In this technical report, we will review the design of CloudBI-II and introduce the security attack that can efficiently break it.

## II. Brief review of CloudBI-II

In the data encryption phase of CloudBI-II, each FingerCode $b_{i}=\left[b_{i 1}, b_{i 2}, \cdots, b_{i n}\right]$ are extended as $B_{i}^{\prime}$

$$
B_{i}^{\prime}=\left[\begin{array}{cccccc}
b_{i 1} & 0 & \cdots & 0 & 0 & 0 \\
0 & b_{i 2} & \cdots & 0 & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & 0 & b_{i n} & 0 & 0 \\
0 & \cdots & 0 & 0 & -0.5 \sum_{j=1}^{n} b_{i j}^{2} & 0 \\
0 & \cdots & 0 & 0 & 0 & 1
\end{array}\right]
$$

Each $B_{i}^{\prime}$ is encrypted as

$$
C_{i}=M_{1} Q_{i} B_{i}^{\prime} M_{2}
$$

where $M_{1}, M_{2}$ are two random $(n+2) \times(n+2)$ invertible matrices, and $Q_{i}$ is a random $(n+2) \times(n+2)$ lower triangular matrix with diagonal entries set as 1 . All $C_{i}$ will be outsourced to cloud servers.

$$
Q_{i}=\left[\begin{array}{ccccc}
1 & 0 & \cdots & 0 & 0 \\
r_{21} & 1 & \cdots & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots \\
r_{(n+1) 1} & \cdots & r_{(n+1) n} & 1 & 0 \\
r_{(n+2) 1} & \cdots & r_{(n+2) n} & r_{(n+2)(n+1)} & 1
\end{array}\right]
$$

When the user submit a candidate FingerCode $b_{c}=\left[b_{c 1}, b_{c 2}, \cdots, b_{c n}\right]$ for identification, the biometric database owner extends it as $B_{c}^{\prime}$

$$
B_{c}^{\prime}=\left[\begin{array}{cccccc}
b_{c 1} & 0 & \cdots & 0 & 0 & 0 \\
0 & b_{c 2} & \cdots & 0 & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & 0 & b_{c n} & 0 & 0 \\
0 & \cdots & 0 & 0 & 1 & 0 \\
0 & \cdots & 0 & 0 & 0 & r_{c}
\end{array}\right]
$$

where $r_{c}$ is a random number generated for each identification request. The owner then encrypts $B_{c}^{\prime}$ as

$$
C_{F}=M_{2}^{-1} B_{c}^{\prime} Q_{c} M_{1}^{-1}
$$

where $M_{1}^{-1}$ and $M_{2}^{-1}$ are inverse matrices of $M_{1}$ and $M_{2}$ respectively, $Q_{c}$ is a random $(n+2) \times(n+2)$ lower triangular matrix with diagonal entries set as $1 . C_{F}$ is finally submitted to cloud servers for identification.

## III. Security Attack on CloudBI-II

We now show that the cloud server only needs to submit more than 3 identification requests to break the ciphertext $C_{i}$ of any FingerCode $b_{i}$ in the owner's database. For expression simplicity, we use $n^{\prime}$ to denote $n+2$ in the rest part of this section.

After submitting an identification request, the cloud server has access to $C_{i}$ of any FingerCode $b_{i}$ and $C_{F}$ of the submitted FingerCode $b_{c}$. Then, the cloud server can compute

$$
P_{i}=C_{i} C_{F}=M_{1} Q_{i} B_{i}^{\prime} M_{2} M_{2}^{-1} B_{c}^{\prime} Q_{c} M_{1}^{-1}=M_{1} Q_{i} B_{i}^{\prime} B_{c}^{\prime} Q_{c} M_{1}^{-1}
$$

We now use $P_{1}$ of FingerCode $b_{1}$ as an example to show our attack, which can also be applied to any other FingerCode $b_{i}$ in the same manner. In $P_{1}$, there are $n^{\prime 2}$ unknowns in $M_{1}, n^{\prime}-1$ unknowns in $B_{1}^{\prime}, \frac{n^{\prime 2}-n^{\prime}}{2}$ unknowns in $Q_{1}, \frac{n^{\prime 2}-n^{\prime}}{2}$ unknowns in $Q_{c}$. As $b_{c}$ is submitted by the cloud server, there is only one unknown $r_{c}$ in $B_{c}^{\prime}$. $M_{1}^{-1}$ can be expressed with elements in $M_{1}$ since it is the inverse matrix of $M_{1}$. Among these unknowns, $M_{1}$, $Q_{1}, B_{1}^{\prime}$ are fixed for all identification requests, $B_{c}^{\prime}$ and $Q_{c}$ are randomly generated for each identification request. Therefore, after the first identification request, each new identification request only introduces $\frac{n^{\prime 2}-n^{\prime}}{2}+1$ unknowns to the computation of $P_{1}$. However, as $M_{1}, Q_{i}, B_{i}^{\prime}, B_{c}^{\prime}, Q_{c}, M_{1}^{-1}$ are all $n^{\prime} \times n^{\prime}$ matrices, it is easy to see that the cloud server can construct $n^{\prime 2}$ equations for $P_{1}$ from each new identification request. As shown in Table III, when the cloud server submits more than 3 identification requests, it can construct more equations than the number of unknowns in $P_{1}$. Thus, all unknowns in $P_{1}$ decrypted by solving their corresponding equations. Once unknowns in $B_{i}^{\prime}$ are decrypted, the cloud can easily extract the actual FingerCode $b_{1}$. To decrypt any other FingerCode $b_{i}$, the cloud server just needs to perform the same attack as that for $b_{1}$.

To this end, we have demonstrated that CloudBI-II can be completely broken when the cloud server can submit more then 3 identification requests.

| \# of Requests <br> 1 | \# of Unknowns in $P_{1}$ <br> $2 n^{\prime 2}$ | \# of Equations from $P_{1}$ <br> $n^{\prime 2}$ |
| :---: | :---: | :---: |
| 2 | $\frac{5 n^{\prime 2}}{2}-\frac{n^{\prime}}{2}+1$ | $2 n^{\prime 2}$ |
| 3 | $3 n^{\prime 2}-n^{\prime}+2$ | $3 n^{\prime 2}$ |
| 4 | $\frac{7 n^{\prime 2}}{2}-\frac{3 n^{\prime}}{2}+3$ | $4 n^{\prime 2}$ |

TABLE I
Unknowns vs Equations

## IV. Example of Security Attack on CloudBI-II

In this example, we set $\mathrm{n}=2$ and $\mathrm{n}^{\prime}=\mathrm{n}+2=4$. For $b_{1}=[2,2]$, the owner extends it as

$$
B_{1}^{\prime}=\left[\begin{array}{cccc}
2 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 \\
0 & 0 & -4 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]
$$

The owner randomly generates $M_{1}, M_{2}, Q_{1}$ as

$$
M_{1}=\left[\begin{array}{llll}
1 & 2 & 1 & 0 \\
1 & 2 & 0 & 0 \\
0 & 2 & 2 & 1 \\
1 & 1 & 0 & 1
\end{array}\right] M_{2}=\left[\begin{array}{llll}
3 & 4 & 1 & 1 \\
1 & 3 & 3 & 0 \\
1 & 4 & 2 & 2 \\
2 & 2 & 0 & 1
\end{array}\right] Q_{1}=\left[\begin{array}{llll}
1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 \\
3 & 2 & 1 & 0 \\
4 & 1 & 0 & 1
\end{array}\right]
$$

$B_{1}^{\prime}$ is encrypted as $C_{1}=M_{1} Q_{i} B_{1}^{\prime} M_{2}$ and outsourced to cloud servers. Now the cloud server selects $b_{c}=(1,3)$ for identification and submits it 3 times. We denote the extended $B_{c}^{\prime}$ for 3 identification requests as $B_{c 1}^{\prime}, B_{c 2}^{\prime}, B_{c 3}^{\prime}$ respectively.

$$
B_{c 1}^{\prime}=\left[\begin{array}{llll}
1 & 0 & 0 & 0 \\
0 & 3 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 5
\end{array}\right] B_{c 2}^{\prime}=\left[\begin{array}{llll}
1 & 0 & 0 & 0 \\
0 & 3 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 3
\end{array}\right] B_{c 3}^{\prime}=\left[\begin{array}{llll}
1 & 0 & 0 & 0 \\
0 & 3 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 6
\end{array}\right]
$$

The owner encrypts $B_{c 1}^{\prime}, B_{c 2}^{\prime}$ and $B_{c 3}^{\prime}$ as $C_{F 1}=M_{2}^{-1} B_{c 1}^{\prime} Q_{c 1} M_{1}^{-1}, C_{F 2}=M_{2}^{-1} B_{c 2}^{\prime} Q_{c 2} M_{1}^{-1}$ and $C_{F 3}=$ $M_{2}^{-1} B_{c 3}^{\prime} Q_{c 3} M_{1}^{-1}$ respectively, where $Q_{c 1}, Q_{c 2}$ and $Q_{c 3}$ are randomly generated as

$$
Q_{c 1}=\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
2 & 1 & 0 & 0 \\
5 & 3 & 1 & 0 \\
8 & 11 & 2 & 1
\end{array}\right] Q_{c 2}=\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
5 & 12 & 1 & 0 \\
2 & 8 & 3 & 1
\end{array}\right] Q_{c 3}=\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
12 & 1 & 0 & 0 \\
10 & 0 & 1 & 0 \\
3 & 2 & 1 & 1
\end{array}\right]
$$

After $C_{F 1}, C_{F 2}$ and $C_{F 3}$ are sent to the cloud, the cloud compute

$$
\begin{align*}
& P_{11} M_{1}=C_{1} C_{F 1} M_{1}=M_{1} Q_{1} B_{1}^{\prime} B_{c 1}^{\prime} Q_{c 1} M_{1}^{-1} M_{1}=M_{1} Q_{1} B_{1}^{\prime} B_{c 1}^{\prime} Q_{c 1}  \tag{1}\\
& P_{12} M_{1}=C_{1} C_{F 2} M_{1}=M_{1} Q_{1} B_{1}^{\prime} B_{c 2}^{\prime} Q_{c 2} M_{1}^{-1} M_{1}=M_{1} Q_{1} B_{1}^{\prime} B_{c 2}^{\prime} Q_{c 2}  \tag{2}\\
& P_{13} M_{1}=C_{1} C_{F 3} M_{1}=M_{1} Q_{1} B_{1}^{\prime} B_{c 3}^{\prime} Q_{c 3} M_{1}^{-1} M_{1}=M_{1} Q_{1} B_{1}^{\prime} B_{c 3}^{\prime} Q_{c 3} \tag{3}
\end{align*}
$$

Based on Eq. 1-3, the cloud can construct the following equations to solve all unknowns in $M_{1}, Q_{1}, B_{1}^{\prime}, B_{c 1}^{\prime}, B_{c 2}^{\prime}$, $B_{c 2}^{\prime}, Q_{c 1}, Q_{c 2}$ and $Q_{c 3}$.

$$
\begin{aligned}
& P_{11} M_{1}=\left[\begin{array}{cccc}
\frac{124}{3} & -24 & \frac{-68}{3} & \frac{68}{3} \\
32 & -18 & -16 & 16 \\
94 & -37 & -46 & 51 \\
\frac{190}{3} & -19 & \frac{-80}{3} & \frac{95}{3}
\end{array}\right] \times\left[\begin{array}{cccc}
x_{1} & x_{2} & x_{3} & x_{4} \\
x_{5} & x_{6} & x_{7} & x_{8} \\
x_{9} & x_{10} & x_{11} & x_{12} \\
x_{13} & x_{14} & x_{15} & x_{16}
\end{array}\right]=\left[\begin{array}{cccc}
\frac{124}{3} x_{1}-24 x_{5}-\frac{68}{3} x_{9}+\frac{68}{3} x_{13} & \cdots & \ldots & \frac{124}{3} x_{4}-24 x_{8}-\frac{68}{3} x_{12}+\frac{68}{3} x_{16} \\
32 x_{1}-18 x_{5}-16 x_{9}+16 x_{13} & \cdots & \cdots & 32 x_{4}-18 x_{8}-16 x_{12}+16 x_{16} \\
\frac{190}{3} x_{1}-19 x_{5}-\frac{80}{3} x_{9}+\frac{95}{3} x_{13} & \cdots & \cdots & \cdots \\
\frac{190}{3} x_{4}-19 x_{8}-\frac{80}{3} x_{12}+\frac{95}{3} x_{16}
\end{array}\right] \\
& =\left[\begin{array}{cccc}
x_{1} & x_{2} & x_{3} & x_{4} \\
x_{5} & x_{6} & x_{7} & x_{8} \\
x_{9} & x_{10} & x_{11} & x_{12} \\
x_{13} & x_{14} & x_{15} & x_{16}
\end{array}\right] \times\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
x_{17} & 1 & 0 & 0 \\
x_{18} & x_{19} & 1 & 0 \\
x_{20} & x_{21} & x_{22} & 1
\end{array}\right] \times\left[\begin{array}{cccc}
x_{23} & 0 & 0 & 0 \\
0 & x_{24} & 0 & 0 \\
0 & 0 & x_{25} & 0 \\
0 & 0 & 0 & 1
\end{array}\right] \times\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 3 & 0 & 0 \\
0 & 0 & 3 & 0 \\
0 & 0 & 0 & x_{26}
\end{array}\right] \times\left[\begin{array}{ccc}
1 & 0 & 0 \\
x_{27} & 1 & 0 \\
x_{28} & x_{29} & 1 \\
x_{30} & x_{31} & x_{32} \\
1
\end{array}\right] \\
& P_{12} M_{1}=\left[\begin{array}{cccc}
\frac{4}{3} & -12 & \frac{-8}{3} & \frac{8}{3} \\
0 & 6 & 0 & 0 \\
\frac{0}{3} & -21 & \frac{-7}{2} & \frac{16}{3} \\
7 & 9 & 1 & 2
\end{array}\right] \times\left[\begin{array}{cccc}
x_{1} & x_{2} & x_{3} & x_{4} \\
x_{5} & x_{6} & x_{7} & x_{8} \\
x_{9} & x_{10} & x_{11} & x_{12} \\
x_{13} & x_{14} & x_{15} & x_{16}
\end{array}\right]=\left[\begin{array}{cccc}
\frac{4}{3} x_{1}-12 x_{5}-\frac{8}{3} x_{9}+\frac{8}{3} x_{13} & \ldots & \cdots & \frac{4}{3} x_{4}-12 x_{8}-\frac{8}{3} x_{12}+\frac{8}{3} x_{16} \\
6 x_{9} & \ldots & \cdots & 6 x_{12} \\
7 x_{1}+9 x_{5}+x_{9}+2 x_{13} & \cdots & \cdots & \cdots \\
7 x_{4}+9 x_{8}+x_{12}+2 x_{16}
\end{array}\right] \\
& =\left[\begin{array}{cccc}
x_{1} & x_{2} & x_{3} & x_{4} \\
x_{5} & x_{6} & x_{7} & x_{8} \\
x_{9} & x_{10} & x_{11} & x_{12} \\
x_{13} & x_{14} & x_{15} & x_{16}
\end{array}\right] \times\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
x_{17} & 1 & 0 & 0 \\
x_{18} & x_{19} & 1 & 0 \\
x_{20} & x_{21} & x_{22} & 1
\end{array}\right] \times\left[\begin{array}{cccc}
x_{23} & 0 & 0 & 0 \\
0 & x_{24} & 0 & 0 \\
0 & 0 & x_{25} & 0 \\
0 & 0 & 0 & 1
\end{array}\right] \times\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 3 & 0 & 0 \\
0 & 0 & 3 & 0 \\
0 & 0 & 0 & x_{33}
\end{array}\right] \times\left[\begin{array}{ccc}
1 & 0 & 0 \\
x_{34} & 1 & 0 \\
x_{35} & x_{36} & 1 \\
x_{37} & x_{38} & x_{39} \\
1
\end{array}\right] \\
& P_{31} M_{1}=\left[\begin{array}{cccc}
\frac{980}{3} & -232 & \frac{-496}{3} & \frac{496}{3} \\
192 & -138 & -96 & 96 \\
\frac{1792}{3} & -410 & \frac{-872}{3} & \frac{890}{3} \\
218 & -156 & -106 & 112
\end{array}\right] \times\left[\begin{array}{cccc}
x_{1} & x_{2} & x_{3} & x_{4} \\
x_{5} & x_{6} & x_{7} & x_{8} \\
x_{9} & x_{10} & x_{11} & x_{12} \\
x_{13} & x_{14} & x_{15} & x_{16}
\end{array}\right]=\left[\begin{array}{cccc}
\frac{980}{3} x_{1}-232 x_{5}-\frac{496}{3} x_{9}+\frac{496}{3} x_{13} & \ldots & \ldots & \frac{980}{3} x_{4}-232 x_{8}-\frac{496}{3} x_{12}+\frac{496}{3} x_{16} \\
192 x_{1}-138 x_{5}-96 x_{9}+96 x_{13} & \ldots & \ldots & 192 x_{4}-138 x_{8}-96 x_{12}+96 x_{16} \\
218 x_{1}-156 x_{5}-106 x_{9}+112 x_{13} & \ldots & \ldots & \cdots \\
218 x_{4}-156 x_{8}-106 x_{12}+112 x_{16}
\end{array}\right] \\
& =\left[\begin{array}{cccc}
x_{1} & x_{2} & x_{3} & x_{4} \\
x_{5} & x_{6} & x_{7} & x_{8} \\
x_{9} & x_{10} & x_{11} & x_{12} \\
x_{13} & x_{14} & x_{15} & x_{16}
\end{array}\right] \times\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
x_{17} & 1 & 0 & 0 \\
x_{18} & x_{19} & 1 & 0 \\
x_{20} & x_{21} & x_{22} & 1
\end{array}\right] \times\left[\begin{array}{cccc}
x_{23} & 0 & 0 & 0 \\
0 & x_{24} & 0 & 0 \\
0 & 0 & x_{25} & 0 \\
0 & 0 & 0 & 1
\end{array}\right] \times\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 3 & 0 & 0 \\
0 & 0 & 3 & 0 \\
0 & 0 & 0 & x_{40}
\end{array}\right] \times\left[\begin{array}{ccc}
1 & 0 & 0 \\
x_{41} & 1 & 0 \\
x_{42} & x_{43} & 1 \\
x_{44} & x_{45} & x_{46} \\
1
\end{array}\right]
\end{aligned}
$$

Based on above matrix multiplications, it is clear that the cloud server can construct 16 equations for $P_{11} M_{1}, 16$ equations for $P_{12} M_{1}$, and 16 equations for $P_{12} M_{1}$. Meanwhile, there are 46 total unknowns in $P_{11} M_{1}, P_{21} M_{1}$ and $P_{31} M_{1}$. Thus, when the cloud server submit 3 identification requests, it will have sufficient information to solve all unknowns in $M_{1}, Q_{1}, B_{1}^{\prime}, B_{c 1}^{\prime}, B_{c 2}^{\prime}, B_{c 3}^{\prime}, Q_{c 1}, Q_{c 2}$ and $Q_{c 3}$. Once the cloud server gets $B_{1}^{\prime}$, it can easily recover $b_{1}=[2,2]$.

## References

[1] Qian Wang, Shengshan Hu, Kui Ren, Meiqi He, Minxin Du, and Zhibo Wang. Cloudbi: Practical privacy-preserving outsourcing of biometric identification in the cloud. In ESORICS 2015, volume 9327, pages 186-205. Springer International Publishing, 2015.

