# Automating Fast and Secure Translations from Type-I to Type-III Pairing Schemes

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**Abstract**

Pairing-based cryptography has exploded over the last decade, as this algebraic setting offers good functionality and efficiency. However, there is a huge security gap between how schemes are usually analyzed in the academic literature and how they are typically implemented. The issue at play is that there exist multiple types of pairings: Type-I called "symmetric" is typically how schemes are presented *and proven secure* in the literature, because it is simpler and the complexity assumptions can be weaker; however, Type-III called "asymmetric" is typically the most efficient choice for an implementation in terms of bandwidth and computation time.

There are two main complexities when moving from one pairing type to another. First, the change in algebraic setting invalidates the original security proof. Second, there are usually multiple (possibly thousands) of ways to translate from a Type-I to a Type-III scheme, and the "best" translation may depend on the application.

Our contribution is the design, development and evaluation of a new software tool, AutoGroup+, that automatically translates from Type-I to Type-III pairings. The output of AutoGroup+ is: (1) "secure" provided the input is "secure" and (2) optimal based on the user's efficiency constraints (excluding software and run-time errors). Prior automation work for pairings was either not guaranteed to be secure or only partially automated and impractically slow. This work addresses the pairing security gap by realizing a fast and secure translation tool.

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## Contents

## 1 Introduction

Automation is increasingly being explored as a means of assisting in the design or implementation of a cryptographic scheme. The benefits of using computer assistance include speed, accuracy, and cost.

Recently, automation for *pairing* (also called *bilinear*) cryptographic constructions (e.g., [AGHP12, AGH13, AGOT14, BFF$^+$14]) has been under exploration. Since the seminal work of Boneh and Franklin [BF01], interest in pairings is strong: they have become a staple at top cryptography and security conferences, the free Charm library has been downloaded thousands of times worldwide (see public Github records of [AGM$^+$13]), and recently pairing-commercializer Voltage Security was acquired by a major US company (HP) [Kri15].

Pairings are algebraic groups with special properties (see Section 2.1), which are often employed for their functionality and efficiency. There are different types of pairings: Type-I called "symmetric" is typically how schemes are presented *and proven secure* in the literature, because it is simpler and the complexity assumptions can be weaker; however, Type-III called "asymmetric" is typically the most efficient choice for an implementation in terms of bandwidth and computation time.

Unfortunately, translating a Type-I scheme into the Type-III scheme is complicated. First, there may be thousands of different Type-III translations of a Type-I scheme and the "best" translation may depend on the application. For instance, one translation might optimize ciphertext size while another offers the fastest decryption time. Second, each new translation requires a new proof under Type-III assumptions. Exploring and analyzing all possible translations is clearly a great burden on a human cryptographer. Indeed a small subset of manual translations of a scheme or particular set of schemes is regarded as a publishable result in its own right, e.g., [RCS12, CLL$^+$13, CLL$^+$14].

Given this translation hurdle, common practice today is to analyze a Type-I scheme, but then use ad-hoc means to derive a Type-III translation that is unproven and possibly non-optimal. The goal of this work is to address this problem by covering new ground in cryptographic automation.

**Our Contribution: The AutoGroup+ Software Tool.** Our primary contribution is the design, development, and performance evaluation of a new publicly-available[1] software tool, AutoGroup+, that automatically translates pairing schemes from Type-I to Type-III. The output of AutoGroup+ is: (1) "secure" provided the input is "secure" (see Section 3.2) and (2) optimal based on the user's efficiency constraints (see Section 3.1.5).[2] The input is a computer-readable format of the Type-I construction, metadata about its security analysis, and user-specified efficiency constraints. The output is a translated Type-III construction (in text, C++, Python, or LaTeX) with metadata about its security analysis. (See Figure 1.)

The audience for this tool is: (1) anyone wanting to implement a pairing construction, and (2) pairing construction designers. We highlight some features.

New SDL Database. The input to AutoGroup+ requires a computer-readable format of the Type-I construction, the Type-I complexity assumption(s), and the Type-I security proof. It was a challenge to create a means of translating human-written security proofs into a computer-readable format. We focused on a common type of proof exhibiting a certain type of black-box reduction.[3] We created a Scheme Description Language (SDL) for this and did the tedious work of carefully

[1]Software will be made available shortly at `https://github.com/jhuisi/auto-tools`.

[2]These claims regard the cryptographic transformation and exclude any software or run-time errors.

[3]The theoretical translation security results of [AGOT14] on which we will base our security are also limited to this class of proof.

transcribing eight popular constructions, five assumptions and eight reductions. (See Appendix B for an example of a simple case.) We believe the future of cryptographic automation research will involve processing the assumptions and proofs; thus our database will be made public as a testbed for future automation research.

*Speed of Tool.* AutoGroup+ took less than 21 seconds to process any of the test set, which included six simple schemes (16 or less solutions), three medium schemes (256 to 512 solutions), and three complex schemes (1024 to 2048 solutions). (The preference for simple schemes was to compare with prior work.) This measures from SDL input to a C++ (or alternative) output. Speed is very important here for usage, because we anticipate that designers may iteratively use this tool like a compiler and implementors may want to try out many different efficiency optimizations.

In contrast, in CRYPTO 2014, Abe, Groth, Ohkubo and Tango [AGOT14] laid out an elegant theoretical framework for doing pairing translations in four steps. It left open the issue of whether their framework was practical to implement for a few reasons: (1) they automated only one of four steps (code not released), (2) their algorithm for this step was exponential time, and (3) they tested it on only simple and medium schemes, but their medium scheme took over 1.75 hours for *one* step. Our fully automated translation of that scheme took 6.5 seconds, which is much more in line with the "compiler"-like usage we anticipate.

We attribute our drastic efficiency improvement in part to our use of the Z3 SMT Solver. As described in Section 3, we found a way to encode a scheme, its assumption(s) and its reduction as an SMT problem and then use Z3 to help find the set of possible translations quickly.

*New Results.* We evaluated AutoGroup+ on 8 distinct constructions (plus 4 additional variations of one scheme), with various optimization priorities, for 45 bandwidth-optimizing translations. In Figure 8, we report the sizes compared to the symmetric case, which are significantly smaller. In Figure 9, we report on over 130 timing experiments resulting from the translations. Due both to the asymmetric setting and AutoGroup+'s optimizations, in most cases, the running times were reduced to less than 10% of the symmetric case. In Figure 10, we report on the effect that different levels of complexity have on translation time for a single scheme.

In Section 5, we compare the performance of AutoGroup+ to prior automation works, published manual translations, and translations existing as source code in the Advanced Crypto Software Collection [Con] and Charm library [AGM$^+$13]. We discovered a few things. In 12 points of comparison with AutoGroup, AutoGroup+ matches those solutions and provides a security validation and new assumptions, adding only a few additional seconds of running time. In one point of comparison with Abe et al., AutoGroup+ appears to find a solution that is one element shorter in the public key, but this is a complex case as we discuss in Section 5 and strengthens the argument that a fully-automated tool is needed.

In the four points of overlap with ACSC and Charm, we are able to confirm the security and ciphertext-size optimality of one broadcast encryption and one hierarchical identity-based encryption implementation. We are also able to confirm the security and signature-size optimality of one signature implementation. These confirmations are new results. Our tool was able to confirm the ciphertext-size optimality, but not the security of the Charm implementation of Dual System Encryption [Wat09]. That implementation made changes to the keys outside the scope of the translations here or in [AGH13, AGOT14]. However, our tool did find a secure translation with the same ciphertext-size.

Overall, our tests show that the tool can produce high-quality solutions in just seconds, demonstrating that pairing translations can be practically and securely performed by computers.

## 1.1 Prior Work

The desirability of translating Type-I to Type-III pairings is well documented. First, this is an exercise that cryptographers are still actively doing by hand. In PKC 2012, Ramanna, Chatterjee and Sarkar [RCS12] nicely translated the dual system encryption scheme of Waters [Wat09] from the Type-I pairing setting to a number of different Type-III possibilities. Recently, Chen, Lim, Ling, Wang and Wee [CLL$^+$13, CLL$^+$14] presented an elegant semi-general framework for (re-)constructing various IBE, Inner-Product Encryption and Key-Policy Functional Encryption schemes in the Type-III setting, assuming the SXDH assumption holds.[4] These works go into deeper creative detail (changing the scheme or adding assumptions) than our automator, and thus mainly get better results, but then, these works appear to have taken significant human resources. In contrast, our work offers a computerized translation as a starting point.

The Advanced Crypto Software Collection (ACSC) [Con], which includes the Charm pairing library [AGM$^+$13], contains many Type-III implementations of schemes that were published and analyzed in the Type-I format. To the best of our knowledge, there is no formal analysis of these converted schemes and thus also no guarantees that the translations are secure or optimal efficiency-wise for a user's specific application. (We remark that ACSC/Charm makes no claims that they *are* secure or optimal.) The public Github records for Charm show that it has been downloaded thousands of times; thus, it would be prudent to verify these implementations. (See our results on this in Section 5.)

In ACM CCS 2013, Akinyele, Green and Hohenberger [AGH13] presented a publicly-available software tool called AutoGroup, which offered an automated translation from Type-I to Type-III pairing schemes. This work employed sophisticated tools, such as the Z3 Satisfiability Modulo Theories (SMT) solver produced by Microsoft Research (see Section 2), to quickly find a set of *possible* assignments of elements into $\mathbb{G}_1$ or $\mathbb{G}_2$. There was not, however, any guarantee that the resulting translation remained secure. Indeed, Akinyele et al. [AGH13] explicitly framed their results as follows: translation has two parts: (1) the search for an efficient translation, and (2) a security analysis of it. They automated the first part and left the security analysis to a human cryptographer. Since they made their source code public, we used it as a starting point and thus named our work after theirs.

While using AutoGroup is certainly faster than a completely manual approach, the lack of a security guarantee is a real drawback. At that time, there was simply no established theory on how to generalize these translations.

Fortunately, in CRYPTO 2014, Abe, Groth, Ohkubo and Tango [AGOT14] pushed the theory forward in this area. They elegantly formalized the notion that if certain dependencies from the Type-I complexity assumption(s) and the reduction in the security analysis were added to the dependencies imposed by the scheme itself, then there was a generic way to reason about the security of the translated scheme. Their main theorem, which we will later use, can informally be stated as:

**Theorem 1.1** (Informal [AGOT14])**.** *Following the conversion method of [AGOT14], if the Type-I scheme is correct and secure in the generic Type-I group model, then its converted Type-III scheme is correct and secure in the generic Type-III group model.*

[4]Informally, the SXDH assumption asserts that in a Type-III pairing group, there exist no efficient isomorphisms from $\mathbb{G}_1$ to $\mathbb{G}_2$ or from $\mathbb{G}_2$ to $\mathbb{G}_1$.

There are four steps in their translation: (1) build a dependency graph between the group elements for each algorithm in the construction, the complexity assumption(s) and the security reduction (In the graph, elements are nodes and a directed edge goes from $g$ to $h$ to if $h$ is derived from $g$, such as $h = g^x$.), (2) merge all graphs into a single graph, (3) split this graph into two graphs (where elements of the first graph will be assigned to $\mathbb{G}_1$ and elements of the second assigned to $\mathbb{G}_2$), and (4) derive the converted scheme.

For the four schemes tested in [AGOT14], steps (1), (2), and (4) were done by hand. The algorithm for step (3) was *exponential* in two variables[5] and the Java program to handle step (3) reported taking 1.75 hours on a medium scheme. Thus, this is a great theory advance, but it left open the question of whether the entire translation could be efficiently automated as a "real-time" tool.

AutoGroup+ **in a Nutshell**. In short, prior work admitted a public software tool that is fast, but possibly insecure [AGH13], and a cryptographic framework that is slow, but secure [AGOT14]. Our goal was to realize the best of both worlds. Even though the implementations differed, we discovered that both works began by tracing generator to pairing dependencies, where [AGH13] did this bottom up and [AGOT14] used a top down approach. Since both of these representations can be helpful for different optimizations, AutoGroup+ does both. It also traces these dependencies for the complexity assumptions and reductions. The pairings and hash variables in the combined dependency graph are translated into a formula and constraints, and then fed into a SMT solver. The output set is then efficiently searched for an optimal solution using the SMT solver again, then verified as a valid graph split (as formalized in [AGOT14]). Finally, if the split is valid, then a converted scheme and complexity assumption(s) are output.

## 2 Background

### 2.1 Pairings

Let $\mathbb{G}_1$, $\mathbb{G}_2$ and $\mathbb{G}_T$ be groups of prime order $p$. A map $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ is an admissible *pairing* (also called a *bilinear map*) if it satisfies the following three properties:

1. Bilinearity: for all $g \in \mathbb{G}_1$, $h \in \mathbb{G}_2$, and $a, b \in \mathbb{Z}_p$, it holds that $e(g^a, h^b) = e(g^b, h^a) = e(g, h)^{ab}$.
2. Non-degeneracy: if $g$ and $h$ are generators of $\mathbb{G}_1$ and $\mathbb{G}_2$, resp., then $e(g, h)$ is a generator of $\mathbb{G}_T$.
3. Efficiency: there exists an efficient method that given any $g \in \mathbb{G}_1$ and $h \in \mathbb{G}_2$, computes $e(g, h)$.

A pairing generator is an algorithm that on input a security parameter $1^\lambda$, outputs the parameters for a pairing group $(p, g, h, \mathbb{G}_1, \mathbb{G}_2, \mathbb{G}_T, e)$ such that $p$ is a prime in $\Theta(2^\lambda)$, $\mathbb{G}_1$, $\mathbb{G}_2$ and $\mathbb{G}_T$ are groups of order $p$ where $g$ generates $\mathbb{G}_1$, $h$ generates $\mathbb{G}_2$ and $e : \mathbb{G}_1 \times \mathbb{G}_2 \to \mathbb{G}_T$ is an admissible pairing.

The above pairing is called an *asymmetric* or Type-III pairing. This type of pairing is generally preferred in implementations for its efficiency. We also consider *symmetric* or Type-I pairings, where there is an efficient isomorphism $\psi : \mathbb{G}_1 \to \mathbb{G}_2$ (and vice versa) such that a symmetric map

[5]Their splitting algorithm runs exponentially in both the number of pairings and the bottom nodes (without outgoing edges) of the dependency graph. Thus, scalability is a real concern.

is defined as $e : \mathbb{G}_1 \times \psi(\mathbb{G}_1) \to \mathbb{G}_T$. We generally treat $\mathbb{G} = \mathbb{G}_1 = \mathbb{G}_2$ for simplicity and write $e : \mathbb{G} \times \mathbb{G} \to \mathbb{G}_T$. These types of pairings are typically preferred for presenting constructions in the academic literature for two reasons. First, they are simpler from a presentation perspective, requiring fewer subscripts and other notations. More importantly, they are sometimes preferred because the underlying symmetric assumption on which the proof is based may be viewed as simpler or weaker than the corresponding asymmetric assumption.

We include current efficiency numbers for Type-I and Type-III groups in Appendix A, demonstrating the significant advantages of the latter.

### 2.2 The Z3 Satisfiability Modulo Theories (SMT) Solver

Our implementation also relies on the power of the state-of-the-art Z3 SMT solver [DMB08] developed at Microsoft Research. SMT is a generalization of boolean satisfiability (or SAT) solving where the goal is to decide whether solutions exist to a given logical formula. The publicly available Z3 is one such tool that is highly efficient in solving constraint satisfaction problems and used in many different applications.

### 2.3 A Scheme Description Language (SDL) and Toolchain

This work builds on the efforts of prior automation works [AGHP12, AGH13] which include several tools such as a scheme description language (or SDL), an accompanying parser for SDL, a code generator that translates SDL schemes into executable code in either C++ or Python, and a LaTeX generator for SDL descriptions. We obtained all these prior tools from the publicly-available AutoTools GitHub repository[6]. Our code and SDL database will be made public in this repository shortly as well. Since we used the code of AutoGroup as a starting point, we derived our tool name from it.

## 3 The AutoGroup+ System

As described in Section 1, AutoGroup+ is a new software tool built to realize the best of both worlds from a prior tool called AutoGroup [AGH13] (fast, but no security guarantees) and new theoretical insights [AGOT14] (secure, but exponential time and no public tool.)

### 3.1 How It Works

We begin with an illustration of the AutoGroup+ system in Figure 1. This system takes in the description of a symmetric (Type-I) pairing-based scheme $S$, together with metadata about its security and user-desired efficiency constraints, and outputs an asymmetric (Type-III) pairing-based translation $S'$, together with metadata about its security. Informally, if $S$ was secure, then $S'$ will be both secure and optimal for the constraints set by the user over the space of "basic" translations.

[6]Project link: `https://github.com/jhuisi/auto-tools`

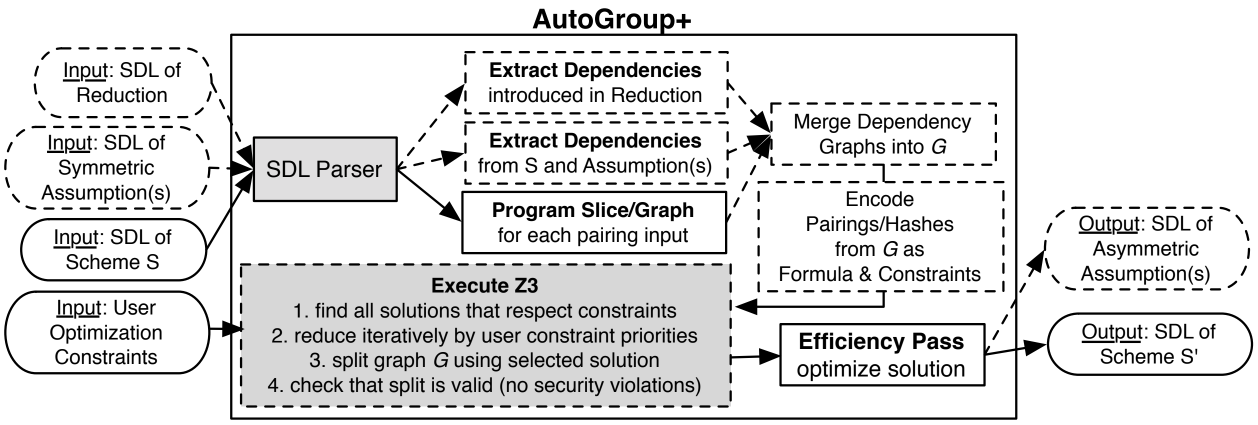

AutoGroup+
Input: SDL of Reduction
Input: SDL of Symmetric Assumption(s)
Input: SDL of Scheme S
Input: User Optimization Constraints
SDL Parser
Extract Dependencies introduced in Reduction
Extract Dependencies from S and Assumption(s)
Program Slice/Graph for each pairing input
Merge Dependency Graphs into G
Encode Pairings/Hashes from G as Formula & Constraints
Execute Z3
1. find all solutions that respect constraints
2. reduce iteratively by user constraint priorities
3. split graph G using selected solution
4. check that split is valid (no security violations)
Efficiency Pass optimize solution
Output: SDL of Asymmetric Assumption(s)
Output: SDL of Scheme S'

Figure 1: A high-level presentation of the AutoGroup+ tool. Components that are new or improved, over AutoGroup, are included with dashed lines. Both AutoGroup+ and AutoGroup use external tools Z3, SDL Parser and Code generator (omitted from the figure).

### 3.1.1 Step 1: Generating Computer-Readable Inputs

AutoGroup+ operates on four inputs: an abstract description of the (1) scheme itself, (2) the complexity assumption(s) on which the scheme is based, (3) the black-box reduction in the scheme's proof of security, and (4) a set of efficiency optimization constraints specified by the user (e.g., optimize for smallest key or ciphertext size.). The abstract descriptions are all specified in a Scheme Description Language (SDL) [AGHP12, AGH13].

The need for SDL representations of the complexity assumptions and security reductions are new challenges for this work. To run our Section 5 tests, we had to translate the text in the published papers to the SDL format by hand. This was a time-consuming and tedious task. However, we maximize the benefit of doing this, by making these SDL files publicly available. This enables anyone to check their correctness and provides a ready-made base of test files for any future automation exercises that require this deeper scheme analysis.

One novel and curious observation we made during these experiments was that *how* group elements were derived in the symmetric group impacted the dependency graphs and therefore the asymmetric results. To say this another way, two schemes computing the exact same elements, but in different ways, could have different dependency graphs and therefore different asymmetric translations. As a toy example, suppose a scheme has $\mathrm{PK} = (g, A = g^a, B = g^b)$ and $\mathrm{SK} = (\mathrm{PK}, a, b)$. Now suppose that as part of a signing algorithm, the holder of SK must compute the value $C = g^{ab}$. Suppose in Scheme 1, the signer computes this as $x = ab \mod p$ and $C = g^x$. Suppose in Scheme 2, the signer computes this as $C = A^b$. Then in the dependency graph for Scheme 1, there is a root node $g$, with nodes $A$ and $C$ hanging off it. Whereas for the graph of Scheme 2, there is a root node $g$ with $A$ off it, and $C$ off of $A$. The importance of these differences comes alive when we attempt to split the graph (see Step 3.1.4). Suppose there is the pairing $e(A, C)$. Then in Scheme 1, the generator $g$ must be split, but $A$ can be assigned to $\mathbb{G}_1$ and $C$ to $\mathbb{G}_2$, resulting in a 4 element public key. However, in Scheme 2, the generator $g$ *and* the element $A$ must be split, with $A_1 \in \mathbb{G}_1$ and $A_2 \in \mathbb{G}_2$, so that one can compute $C = (A_2)^b \in \mathbb{G}_2$. This results in a 5 element public key. The general rule is that the fewer unnecessary dependencies the better.

Interestingly, Abe et al. [AGOT14] sometimes *added* dependencies that did not exist in the original schemes. For instance, for the Waters 2005 IBE [Wat05], Waters clearly states to choose $g_2, u', u_i$ as fresh random generators, but Abe et al. explicitly "assume" that they are generated from a separate generator $g$. For this particular scheme, this does not impact the asymmetric translations, but in theory it could.

Our experiments did not add any dependencies. We note that in this step, a human is not being tasked with any job but simple transcription of the input into a language the computer can understand.

**System Limitations and Allowable Inputs** This system shares some of the same limitations as prior works [AGH13, AGOT14]. First, this is a junk-in-makes-junk-out system. AutoGroup+ assumes that the security reduction is correct, the complexity assumptions are true, and that the SDL was typed in correctly. If any of these turn out to be false, the output cannot be depended on. Fortunately, we can mitigate these risks as follows. The correctness of the security reductions might be verified automatically using a number of tools, such as EasyCrypt [Tea]. The pairing-based assumptions may be sanity-checked in the generic group model using the recently developed tool by Barthe et al. [BFF+14] from CRYPTO 2014. Finally, the SDL transcriptions can be verified in the usual crowd-based manner which we encourage by making them public.

Second, the system does not accept all possible schemes that might appear in the literature. For starters, it supports only prime-order symmetric pairing schemes with a "standard" reduction analysis[7] that is based on non-interactive assumptions. It can support dynamic (also called $q$-based) assumptions, where the size of the assumption may grow depending on the usage of the scheme. But it currently does not support interactive (also called oracle-based) assumptions, such as the LSRW Assumption behind the popular Camenisch-Lysyanskaya pairing-based signatures [CL04], because it isn't clear to us how to build a dependency graph when the adversary can arbitrarily manipulate prior responses into a new oracle input.

Moreover, how the scheme hashes into pairing groups also may disqualify it from being translated. We now give an example of how to alter the Setup algorithm of the Waters 2005 IBE scheme [Wat05], so that AutoGroup+ cannot translate it. (Indeed, it is not clear to us if a translation even exists.) In the original Setup algorithm, the master authority chooses a generator $g \in \mathbb{G}$ at random. Then public parameter $g_1$ is derived from $g$, while parameters $g_2, u_0, \ldots, u_n \in \mathbb{G}$ are chosen independently at random. Instead, suppose we treat the hash function $H : \{0,1\}^* \to \mathbb{G}$ as a random oracle. Let generator $g \in \mathbb{G}$ be computed as $g = H(ID)$, where $ID$ is some string describing the master authority. Then $g_1$ is derived from $g$ as before, but we set $g_2 = g^r, u_0 = g^{r_0}, \ldots, u_n = g^{r_n}$ for random $r, r_0, \ldots, r_n \in \mathbb{Z}_p$ (where $p$ is the order of $\mathbb{G}$). It is easy to see that the public parameters have the same distribution as before (assuming the random oracle model); all we have changed is *how* the master authority samples these parameters. Thus, this variant of the Waters IBE remains secure in the symmetric setting, and yet it is not clear how to translate it to the asymmetric setting. We return to this example in Section 5.

These assumption and hashing limitations also appear in the theoretical work of Abe et al. [AGOT14], and fortunately, these issues seem relatively rare and did not come up for any of the schemes we tested (except our hand-made counterexample). As in [AGH13, AGOT14], we note that if Au-

[7]We refer the reader to Abe et al. [AGOT14] for a formal definition of the allowed reductions. Roughly, we mean an analysis where there is an efficient algorithm called a *reduction* that is successful in solving the hard problem (underlying the complexity assumption) given black-box access to an adversary that successfully attacks the scheme.

toGroup+ cannot produce a translation, it does not imply that a translation does not exist. A characterization of untranslatable schemes is an open theoretical problem.

### 3.1.2 Step 2: Extracting Algebraic Dependencies

Once AutoGroup+ has parsed all its input files, it begins processing them to graph the algebraic dependencies between source group elements in a scheme, assumption and reduction. All source group elements are nodes in the graph and a directed edge exists if there is a direct dependency between two elements. E.g., if $h = g^x$, then $h$ is derived from $g$ and we place an edge from $g$ and $h$.

AutoGroup+ extracts the dependency graphs *automatically from the SDL* for each input file and builds a distinct graph from the SDL representations and metadata. AutoGroup+ defines two new procedures that programmatically extract the dependency graph for the assumption(s) as well as the reduction(s) (see Section 4 for an example). Then, AutoGroup+ reuses logic from AutoGroup to programmatically build the graph of the scheme by tracking the generators in the setup algorithm and by tracing backward from each pairing in the scheme. It merges the program slice (or trace) extracted for each pairing input into one dependency graph for the scheme. The resulting graphs are the same as those produced by Abe et al. [AGOT14] (except where we reduced dependencies by computing elements more directly as discussed in the last step.)

The work of Abe et al. [AGOT14] required a human to build (and later merge) these dependency graphs by hand and the graphs were constructed starting from the common generators downward. The AutoGroup work of Akinyele et al.[AGH13] automatically derived these graphs *for the scheme only* from the SDL description of the scheme. They did not consider the assumptions or reduction dependencies. Indeed, AutoGroup only graphed the dependencies as a traceback from the pairings, whereas AutoGroup+ also adds a top-down analysis from the assumption down to the pairings for the security logic.

### 3.1.3 Step 3: Merge Dependency Graphs

After extracting the dependencies, AutoGroup+ has a set of distinct graphs: one graph that represents dependencies from the setup, key generation, encryption and decryption algorithms, as well as a graph for each complexity assumption and one or more graphs for the reduction. These graphs are then systematically merged together using the metadata provided with the SDL inputs. The metadata includes a reduction map which relates the names of source group elements in the reduction to those in the assumption. We require this map to understand which nodes represent the same group element (across the scheme, assumption and reduction) to simplify merging into a single node. See the example in Section 4. AutoGroup+ programmatically checks the type information in the reduction map across all SDL inputs to ensure correctness during the merge.

### 3.1.4 Step 4: Assign Variables using the SMT Solver

This is the most complex step in the automation. In the symmetric setting, all group elements in the scheme were in $\mathbb{G}$. To move to the asymmetric setting, we must assign elements to either $\mathbb{G}_1$ or $\mathbb{G}_2$ in such a way that the dependencies between elements are not violated (*e.g.*, if $h = g^x$, then both $g, h$ must be in the same group) and such that for all variables $a, b$, if we have a pairing between them $e(a, b)$, then $a$ and $b$ must be in distinct source groups (*e.g.*, $a \in \mathbb{G}_1$ and $b \in \mathbb{G}_2$ or vice versa). Such an assignment may not be feasible (see such an example in Section 3.1.1) or it may require that one or more variables in the symmetric scheme be duplicated in the asymmetric scheme with one assigned to $\mathbb{G}_1$ and another to $\mathbb{G}_2$. E.g., in the symmetric setting if $g \in \mathbb{G}$, $a = g^x$ and $b = g^y$ and these elements are paired as $e(a, b)$, then in the asymmetric setting, $g$ will be split into $g_1 \in \mathbb{G}_1$ and $g_2 \in \mathbb{G}_2$, where $a = g_1^x$ and $b = g_2^y$, so that one can compute $e(a, b)$.

To *efficiently* make these variable assignments, AutoGroup+ follows the approach of AutoGroup in that it uses a powerful Z3 Satisfiability Modulo Theories (SMT) solver produced by Microsoft Research (see Section 2) to compute the set of all possible splits (*i.e.*, all possible variable assignment combinations) and then later identifies the best one. Z3 takes as input a logical formula and determines whether valid variable assignments exist that evaluate that formula to *true*. Similar to AutoGroup, AutoGroup+ expresses the pairing equations as a logical formula of conjunctions and inequality operations over binary variables. E.g., $e(a, b) \cdot e(c, d)$ is translated to the logical formula $A \neq B \wedge C \neq D$ where $A$ is a reference to $a$, $B$ to $b$, etc.

One major difference between AutoGroup+ and AutoGroup is that the former's dependency graphs include dependencies based on the assumptions and reductions. The formula is derived from the pairings that occur in the graph (from the construction, reduction and assumption(s)) with a conjunction joining each pairing piece, plus extra constraints added for variables that cannot be duplicated (regarding hashing). This formula is then fed into the solver. The solver returns a set of 0 or 1 assignments for each variable. We then apply each solution to the merged dependency graph to generate the split (variables assigned to 0 on one side and the rest on the other).

### 3.1.5 Step 5: Search for Optimal Solution

There are often many (possibly thousands) of ways to translate a symmetric scheme into an asymmetric scheme; thus, we can end up with many feasible graph splits. Indeed, the output of the SMT solver in the last step is a *set* of assignments of the variables. In this step, we again use the SMT solver to deduce which assignment from this set is "best". AutoGroup+ allows selection of assignments based on a number of user-specified optimization constraints. For public-key encryption, the user can choose to minimize the public-key, assumption, secret key and/or ciphertext size. Similarly for signature schemes, the user can mimize the public-key parameters, assumption, and/or the signature size.

To select an optimal assignment, AutoGroup+ encodes these user requirements as parameters of some *objective function*. We then call the solver a second time with this objective function set to rank/narrow the given solutions to one. Depending on the optimization goal, the objective function can be specified in one of two ways. If reducing public-key size or the assumption, then we are concerned with minimizing the duplication of source group elements. As such, we first specify an EvalGraph function that the solver uses to compute the splits for each element in the public key or assumption: $\mathsf{EvalGraph}(A_j, B, G) = S$, where $A_j = a_1, \ldots, a_n$ represents pairing input variable assignments for the $j$-th solution (each $a_i$ variable is either $0 = \mathbb{G}_1$ or $1 = \mathbb{G}_2$), $B = b_1, \ldots, b_m$ represents the source group elements to minimize either in the assumption or public-key, and $G$ represents the merged dependency graph.

Our search algorithm first applies the EvalGraph function to determine how the $b_i$ values are assigned for each solution. Once the $b_i$ values are assigned, we then compute $S = s_1, \ldots, s_m$ where each $s_i$ corresponds to one of three values for each $b_i$ assignment. That is, let a $w_1$ value denote a $\mathbb{G}_1$ only assignment, $w_2$ is $\mathbb{G}_2$ only, and $w_3 = w_1 + w_2$ is both a $\mathbb{G}_1$ and $\mathbb{G}_2$ assignment (or simply a split). We then set $w_1$ and $w_2$ to the group size of $\mathbb{G}_1$ and $\mathbb{G}_2$ for Type-III pairing curves (*e.g.*, BN256). Each solution is ranked in terms of splits and the total size of group elements in $B$. Our search returns the $j$-th solution that results in the fewest splits in $B$ with the smallest overall size $S_j$. This overall size breaks ties between multiple solutions with the same number of splits.

$$\min_{j \in |A|} (\mathsf{CountSplits}(S_j), \sum_{i=1}^{m} S_{j,i}) \tag{1}$$

For the other optimization options (*i.e.*, secret-key, ciphertext, etc), we can reuse the objective function specified by AutoGroup as is:

$$\min_{j \in |A|} F(A_j, C, w_1, w_2) = \sum_{i=1}^{n} ((1 - a_i) \cdot w_1 + a_i \cdot w_2) \cdot c_i \tag{2}$$

where the $A_j$ represents the $j$-th solution as before, $C = \{c_1, \ldots, c_n\}$ represent some *cost* associated with each $a_i$ variable reference, and $w_1$ and $w_2$ correspond to *weights* (for different Type-III pairing curves) over groups $\mathbb{G}_1$ and $\mathbb{G}_2$. By encoding these cost values, it is feasible to create different weight functions that adhere to the user specified constraints. Once these functions are specified correctly, we minimize it across the set of assignments and return the solution that yields the lowest value. Thus, the combination of equations 1 and 2 yield all the possible ways a current user can optimize a given symmetric scheme. Further optimizations are future work.

Once the "best" solution is found, we have a CheckValidSplit procedure that verifies that the conditions (1) and (2) of a "valid split" hold as defined in Definition 3.1. If this solution satisfies these conditions, we are done. If not, we simply test the next best solution, because the solver caches all solutions and we record metadata about each solution in terms of efficiency and security.

#### 3.1.6 Step 6: Evaluate and Process the Solution

Once a split is chosen, AutoGroup+ must reconstruct SDL for the asymmetric scheme *and assumption*. It reuses the functionality provided by AutoGroup to construct the SDL as dictated by the split.[8] To output the new asymmetric assumptions, AutoGroup+ follows the logic of Abe et al. [AGOT14] (although they did not implement this step) and implements a new procedure that uses the graph split to reconstruct the asymmetric assumption(s). For each element in the asymmetric assumption, we learn the new assignments of the elements using the graph split and mechanically generate the asymmetric assumption SDL. Finally, we rely on existing tools [AGHP12, AGH13] to translate the new asymmetric SDL representation into executable code for C++ or Python, or simply LaTeX.

### 3.2 Analysis of AutoGroup+

We analyze AutoGroup+'s security and optimizations.

**Security.** At a high-level, the Abe et al. [AGOT14] security argument works as follows. In the Type-I setting, we treat $\mathbb{G}_1 = \mathbb{G}_2$ because there are efficient isomorphisms between these two groups. However, suppose we work in the generic Type-I group model, where elements are a

[8]We further perform an efficiency check on the final scheme as previously done in AutoGroup.

black box and to compute this isomorphism, a party must utilize an oracle $\mathcal{O}$. Next, consider moving to a Type-III group, where every element (for which the discrete logarithm is known with respect to the base generators) is duplicated; that is, for $h = g^x \in \mathbb{G}$, we have $h_1 = g_1^x \in \mathbb{G}_1$ and $h_2 = g_2^x \in \mathbb{G}_2$. Then in the generic Type-III group model, we can simulate having efficiently computable isomorphisms between these groups by exposing an oracle $\mathcal{O}'$ that on input $d_1 \in \mathbb{G}_1$ outputs $d_2 \in \mathbb{G}_2$ (or vice versa). In essence, by exposing the "corresponding" group element (through the oracle in the Type-III setting), we "allow" all necessary isomorphism computations for the scheme itself to operate, however, at the same time, we can argue that any adversary that breaks this scheme (with these elements exposed) can be turned into an attacker against the Type-I scheme, where these isomorphisms are natively computable. The resulting theorem was summarized in Theorem 1.1: namely, the Type-III conversion will be secure in the generic group model, if one follows the conversion criteria in [AGOT14] and the Type-I input was secure in the generic group model.

Thus, we must argue that the AutoGroup+ implementation satisfies the criteria in [AGOT14]. The dependency graphs are created and merged according to the same algorithm. (AutoGroup+ tracks some additional information on the side for optimization purposes.) What is required is that the splitting of the merged dependency graph satisfies Abe et al.'s notion of a "valid split."

**Definition 3.1** (Valid Split [AGOT14])**.** *Let $\Gamma = (V, E)$ be a dependency graph for $\Pi = (\mathsf{S}, \mathsf{R}, \mathsf{A})$, a tuple representing a scheme, reduction and assumption(s) that are in the set covered by the [AGOT14] translation. Let $P = (p_1[0], \ldots, p_n[1]) \subset V$ be pairing nodes. A pair of graphs $\Gamma_0 = (V_0, E_0)$ and $\Gamma_1 = (V_1, E_1)$ is a valid split of $\Gamma$ with respect to $\mathsf{NoDup} \subseteq V$ if the following hold:*

1. *merging $\Gamma_0$ and $\Gamma_1$ recovers $\Gamma$,*
2. *for each $i \in \{0, 1\}$ and every $X \in V_i \backslash P$, the ancestor subgraph of $X$ in $\Gamma$ is included in $\Gamma_i$.*
3. *for each $i \in \{1, \ldots, n_p\}$ pairing nodes $p_i[0]$ and $p_i[1]$ are separately included in $V_0$ and $V_1$,*
4. *No node in $V_0 \cap V_1$ is included in $\mathsf{NoDup}$. $\mathsf{NoDup}$ is a list of nodes that cannot be assigned to both $V_0$ and $V_1$.*

In terms of AutoGroup+ security, conditions (1) and (2) are satisfied in the search procedure (step 5). That is, before we admit a split, we do these simple tests. Condition (3) is satisfied by the SMT solver with the logical formula encoding of pairing nodes (step 4). Condition (4) is also satisfied by the SMT solver (step 4). We encode the output of hashes as constraints over the logical formula; specifically, we ask the solver to find splits that keep hashes in $\mathbb{G}_1$. This is the only place we differ slightly. Abe et al. allow $\mathbb{G}_1$ or $\mathbb{G}_2$ assignment for hashes but not both. Our approach prioritizes solutions that preserve efficiency. We could give the user the option of relaxing this to match Abe et al. though. The translation back to SDL is fairly straightforward from the split.

**Optimizations.** In terms of optimality over the set of solutions admitted by the "valid split" method, AutoGroup+ finds the "best" one by searching over the entire set. It does this efficiently by turning the user-specified optimizations into the appropriate objective function and passing this function into the SMT solver. Our experiments in Section 5 provide evidence that the tool is, indeed, finding the optimal solutions over the space of valid translations.

As discussed in Section 1.1, we do not rule out the existence of even better solutions that employ insights outside of this method (such as altering the construction or adding "stronger" assumptions, such as SXDH.)

## 4 An Automation Example with BB-HIBE

In this section, we illustrate each phase of the AutoGroup+ implementation described in Section 3 by showing the step-by-step translation of the Boneh-Boyen hierarchical identity-based encryption [BB04a] (or BB HIBE) scheme. We begin by recalling the scheme: an efficient HIBE scheme (with $\ell = 2$) [BB04b, §4.1] that is selective identity secure based on the standard Decisional Bilinear-Diffie Hellman (DBDH) assumption.

This scheme consists of four algorithms: Setup, KeyGen, Encrypt and Decrypt. The Setup algorithm takes as input a security parameter and defines public keys (ID) of depth $\ell$ as vectors of elements in $\mathbb{Z}_p^\ell$. We define $\ell = 2$, thus the identity is comprised of $\mathsf{ID} = (ID_1, ID_2) \in \mathbb{Z}_p^2$. The algorithm generates system parameters as follows. First, select a random generator $g \in \mathbb{G}$, a random $\alpha \in \mathbb{Z}_p$, and sets $g_1 = g^\alpha$. Then, pick random $h_1, h_2, g_2 \in \mathbb{G}$. Set the master public parameters $params = (g, g_1, g_2, h_1, h_2)$ and the master secret key $msk = {g_2}^\alpha$.
The KeyGen algorithm takes as input an $\mathsf{ID} = (ID_1, ID_2) \in {\mathbb{Z}_p}^2$, picks random $r_1, r_2 \in Z_p$ and outputs:

$$d_1 = {g_2}^\alpha \cdot ({g_1}^{ID_1} \cdot h_1)^{r_1} \cdot ({g_1}^{ID_2} \cdot h_2)^{r_2}, d_2 = g^{r_1}, d_3 = g^{r_2}$$

and the algorithm outputs $d_{ID} = (d_1, d_2, d_3)$
The Encrypt algorithm takes as input the public parameters $params$, an identity ID and a message $M \in \mathbb{G}_T$. To encrypt the message $M$ under the public key $\mathsf{ID} = (ID_1, ID_2)$, picks a random $s \in \mathbb{Z}_p$ and computes:

$$C = (e(g_1, g_2)^s \cdot M, g^s, ({g_1}^{ID_1} \cdot h_1)^s, ({g_1}^{ID_2} \cdot h_2)^s)$$

and the algorithm outputs $C = (C_1, C_2, C_3, C_4)$.
The Decrypt algorithm takes as input a private key $d_{ID} = (d_1, d_2, d_3)$ and a ciphertext $C$ and computes $M$ as:

$$M = C_1 \cdot \frac{e(C_3, d_2) \cdot e(C_4, d_3)}{e(C_2, d_1)}$$

The scheme is based on the DBDH assumption.

**Assumption 1** (Decisional Bilinear Diffie-Hellman)**.** *Let $g$ generate group $\mathbb{G}$ of prime order $p \in \Theta(2^\lambda)$ with mapping $e : \mathbb{G} \times \mathbb{G} \to \mathbb{G}_T$. For all p.p.t. adversaries $\mathcal{A}$, the following probability is negligible in $\lambda$:*

$$\begin{aligned} |\tfrac{1}{2} - \Pr[a, b, c \leftarrow \mathbb{Z}_p, z \leftarrow \{0, 1\}, A = g^a,& \\ B = g^b, C = g^c, T_0 = e(g, g)^{abc}, T_1 \leftarrow \mathbb{G}_T;& \\ z' \leftarrow \mathcal{A}(g, A, B, C, T_z) : z = z']|.& \end{aligned}$$

**Step 1: Generating SDL Inputs** In order for AutoGroup+ to perform the translation, we first begin by transcribing the scheme, reduction and the DBDH assumption into SDL. We provide the SDL description of the above scheme, reduction and assumption in Appendix B. The reader will notice that the SDL descriptions closely and concisely follow the paper counterpart. This design is on purpose as to reduce the burden of transcribing these constructions for AutoGroup+ users. Indeed, in our experience the most time consuming and tedious part is in specifying the reductions accurately.

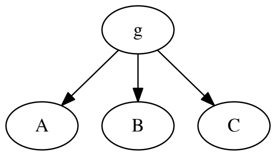


Figure 2: Dependency graph for the DBDH instance generated by AutoGroup+.

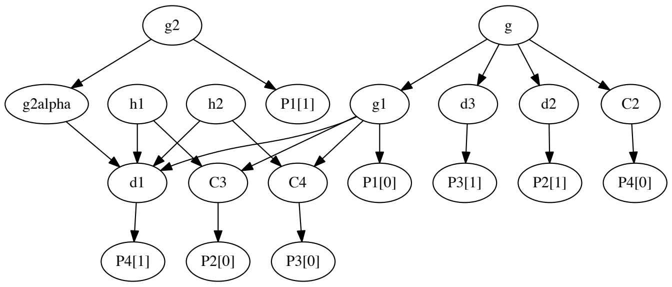


Figure 3: Dependency graph that merges Setup, KeyGen, Encrypt and Decrypt algorithms in BB HIBE and generated by AutoGroup+. For brevity, we only show the combined scheme graph and omit the smaller ones for each routine in the scheme. Note that nodes $P1$ through $P4$ represent unique pairing identifiers.

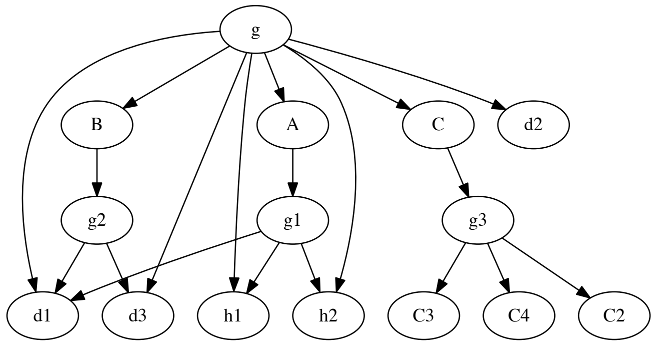


Figure 4: Dependency graph for the reduction to DBDH in BB HIBE. This graph was generated by AutoGroup+.

**Step 2: Extracting the Dependencies** Once the SDLs have been generated along with the metadata and the user's desired optimization goal, the user can proceed with executing AutoGroup+ to begin deriving the dependency graphs for each input file. AutoGroup+ programmatically extracts the dependencies from the SDL descriptions starting with the assumption(s), then the reduction(s) and finally, the scheme. The dependency graph diagrams for BB HIBE [BB04b, §4.1] are included in Figures 2, 3, and 4. Note that these diagrams were generated automatically by our tool; we believe this feature provides more transparency to make it easier for humans to that the software is operating correctly.

**Step 3: Merge the Graphs** In Figure 5, we show the third step in AutoGroup+ which is to merge the multiple dependency graphs (assumption, reduction and scheme graphs) into one single graph.

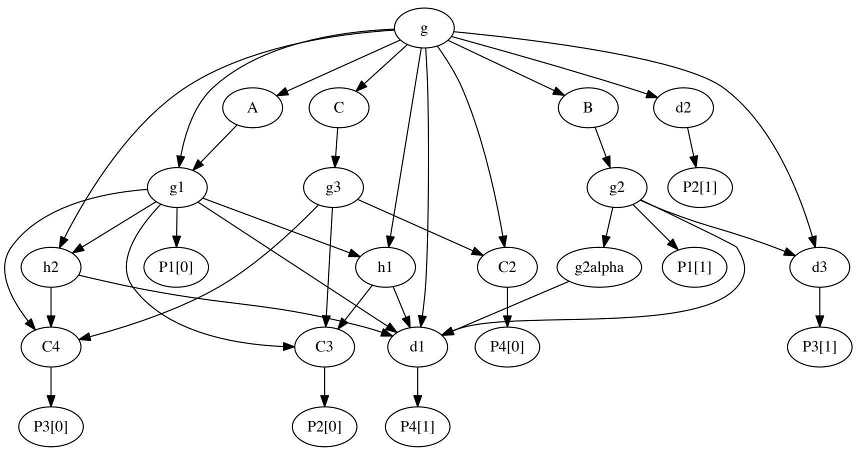
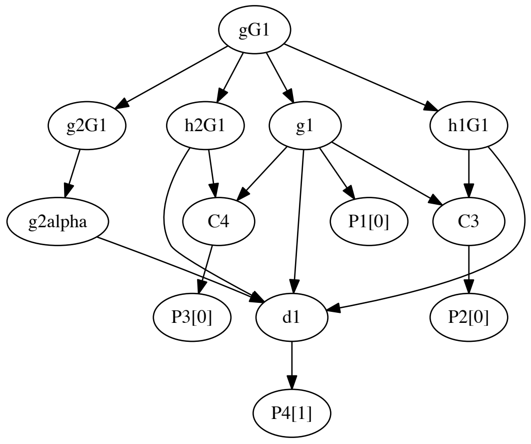


Figure 5: The merged dependency graph for the assumption, reduction to DBDH, and the BB HIBE scheme. This graph was generated by AutoGroup+.

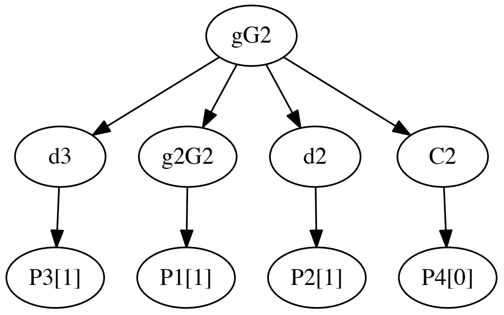
**Step 4: Assignment of Variables** With the merged graph, we simply encode the pairing equations as a logical formula as in AutoGroup but also provide the dependency graph as additional constraints to the solver (with optimization requirements):

$$P1[0] \neq P1[1] \wedge P2[0] \neq P2[1] \wedge P3[0] \neq P3[1] \wedge P4[0] \neq P4[1]$$

**Step 5: Search for an Optimal Solution** In our BB HIBE example, the goal is to minimize the number of splits in the master public parameters *params*, so this requires specifying the following parameters of the EvalGraph function. Let $B = \{g, g_1, g_2, h_1, h_2\}$ be the set of elements in the public parameters we wish to minimize and let $G$ be an encoding of the merged dependency graph shown in Figure 5. As reflected in Table 8, the solver identifies 16 possible solutions for the BB HIBE scheme and computes the following on each solution as $S_j = \mathsf{EvalGraph}(A_j, B, G)$ where $A_j$ is the $j$-th set of possible variable assignments. Recall that EvalGraph simply applies a given



(a) Showing $\mathbb{G}_1$ elements in the scheme



(b) Showing $\mathbb{G}_2$ elements in the scheme

Figure 6: The dependency graphs for the asymmetric translation of BB HIBE scheme only (with PK optimization). This graph was generated by AutoGroup+.

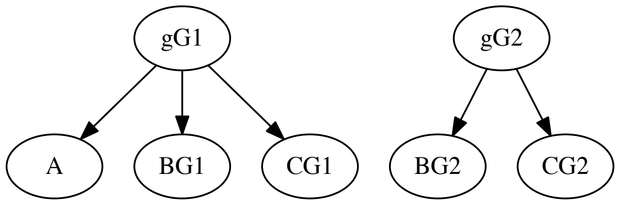


Figure 7: The dependency graph for the co-DBDH assumption and generated by AutoGroup+.

solution to $G$ and records how elements of $B$ are assigned. From the set $S$, the solver finds an assignment that has the fewest number of duplicated public key elements with the smallest overall size. Based on this criteria, the solver returned a optimal solution in the fifth step which consisted of 2 splits (i.e., two duplicated elements). The new public key elements are assigned as $B' = \{g, \tilde{g}, g_1, g_2, \tilde{g_2}, h_1, h_2\} \in \mathbb{G}_1^5 \times \mathbb{G}_2^2$. This constitutes only an addition of 2 group elements in $\mathbb{G}_2$.

**Step 6: Assignment of Variables** In the last step, AutoGroup+ splits the graph as dictated by the optimal solution found by the solver. The resulting graphs for $\mathbb{G}_1$ and $\mathbb{G}_2$ assignments for the BB HIBE scheme are shown in Figure 6. AutoGroup+ programmatically converts the split graph into an asymmetric translation for the scheme and assumption. We improve on code from AutoGroup to do the former translation and write a new module to do the latter (see Figure 7 for the graph split of co-DBDH). These resulting SDL files are provided in Appendix B.2. As mentioned before, there is a publicly-available tool (see Section 2.3) for automatically turning this SDL into

| | Conversion | Number of Group Elements | | | | Assumption | Num. |
|---|---|---|---|---|---|---|---|
| | Time | Public Key | Secret Key | Ciphertext | Assumption | | Solutions |
| *ID-Based Enc.* | | | | | | | |
| BB04 HIBE [BB04a, §4] Symmetric (l = 2) | - | $\mathbb{G}^5$ | $\mathbb{G}^3$ | $\mathbb{G}^3 \times \mathbb{G}_T$ | $\mathbb{G}^4 \times \mathbb{G}_T$ | DBDH | |
| Asymmetric [Min. PK] | 592 ms | $\mathbb{G}_1^5 \times \mathbb{G}_2^2$ | $\mathbb{G}_1 \times \mathbb{G}_2^2$ | $\mathbb{G}_1^2 \times \mathbb{G}_2 \times \mathbb{G}_T$ | $\mathbb{G}_1^4 \times \mathbb{G}_2^3 \times \mathbb{G}_T$ | | 16 |
| Asymmetric [Min. SK] | 641 ms | $\mathbb{G}_1^5 \times \mathbb{G}_2^4$ | $\mathbb{G}_1^3$ | $\mathbb{G}_2^3 \times \mathbb{G}_T$ | $\mathbb{G}_1^3 \times \mathbb{G}_2^3 \times \mathbb{G}_T$ | | 16 |
| Asymmetric [Min. CT] | 626 ms | $\mathbb{G}_1^4 \times \mathbb{G}_2^5$ | $\mathbb{G}_2^3$ | $\mathbb{G}_1^3 \times \mathbb{G}_T$ | $\mathbb{G}_1^3 \times \mathbb{G}_2^3 \times \mathbb{G}_T$ | | 16 |
| Asymmetric [Min. Assump] | 582 ms | $\mathbb{G}_1^4 \times \mathbb{G}_2^5$ | $\mathbb{G}_2^3$ | $\mathbb{G}_1^3 \times \mathbb{G}_T$ | $\mathbb{G}_1^3 \times \mathbb{G}_2^3 \times \mathbb{G}_T$ | | 16 |
| BB04 HIBE [BB04a, §4] Symmetric (l = 9) | - | $\mathbb{G}^{12}$ | $\mathbb{G}^{10}$ | $\mathbb{G}^{10} \times \mathbb{G}_T$ | $\mathbb{G}^4 \times \mathbb{G}_T$ | DBDH | |
| Asymmetric [Min. PK] | 20629 ms | $\mathbb{G}_1^{12} \times \mathbb{G}_2^2$ | $\mathbb{G}_1 \times \mathbb{G}_2^9$ | $\mathbb{G}_1^9 \times \mathbb{G}_2 \times \mathbb{G}_T$ | $\mathbb{G}_1^4 \times \mathbb{G}_2^3 \times \mathbb{G}_T$ | | 2048 |
| Asymmetric [Min. SK] | 15714 ms | $\mathbb{G}_1^{12} \times \mathbb{G}_2^{11}$ | $\mathbb{G}_1^{10}$ | $\mathbb{G}_2^{10} \times \mathbb{G}_T$ | $\mathbb{G}_1^3 \times \mathbb{G}_2^3 \times \mathbb{G}_T$ | | 2048 |
| Asymmetric [Min. CT] | 15690 ms | $\mathbb{G}_1^{11} \times \mathbb{G}_2^{12}$ | $\mathbb{G}_2^{10}$ | $\mathbb{G}_1^{10} \times \mathbb{G}_T$ | $\mathbb{G}_1^3 \times \mathbb{G}_2^3 \times \mathbb{G}_T$ | | 2048 |
| Asymmetric [Min. Assump] | 20904 ms | $\mathbb{G}_1^{11} \times \mathbb{G}_2^{12}$ | $\mathbb{G}_2^{10}$ | $\mathbb{G}_1^{10} \times \mathbb{G}_T$ | $\mathbb{G}_1^3 \times \mathbb{G}_2^3 \times \mathbb{G}_T$ | | 2048 |
| GENTRY06 [Gen06, §3.1] Symmetric | - | $\mathbb{G}^3$ | $\mathbb{Z}_p \times \mathbb{G}$ | $\mathbb{G} \times \mathbb{G}_T^2$ | $\mathbb{G}^{3+q} \times \mathbb{G}_T$ | trunc. dec. $q$-ABDHE | |
| Asymmetric [Min. PK] | 669 ms | $\mathbb{G}_1^2 \times \mathbb{G}_2^2$ | $\mathbb{Z}_p \times \mathbb{G}_2$ | $\mathbb{G}_1 \times \mathbb{G}_T^2$ | $\mathbb{G}_1^{3+q} \times \mathbb{G}_2^{2+q} \times \mathbb{G}_T$ | | 4 |
| Asymmetric [Min. SK] | 718 ms | $\mathbb{G}_1^2 \times \mathbb{G}_2^3$ | $\mathbb{Z}_p \times \mathbb{G}_1$ | $\mathbb{G}_2 \times \mathbb{G}_T^2$ | $\mathbb{G}_1^{2+q} \times \mathbb{G}_2^{3+q} \times \mathbb{G}_T$ | | 4 |
| Asymmetric [Min. CT] | 723 ms | $\mathbb{G}_1^2 \times \mathbb{G}_2^2$ | $\mathbb{Z}_p \times \mathbb{G}_2$ | $\mathbb{G}_1 \times \mathbb{G}_T^2$ | $\mathbb{G}_1^{3+q} \times \mathbb{G}_2^{2+q} \times \mathbb{G}_T$ | | 4 |
| Asymmetric [Min. Assump] | 676 ms | $\mathbb{G}_1^2 \times \mathbb{G}_2^2$ | $\mathbb{Z}_p \times \mathbb{G}_2$ | $\mathbb{G}_1 \times \mathbb{G}_T^2$ | $\mathbb{G}_1^{3+q} \times \mathbb{G}_2^{1+q} \times \mathbb{G}_T$ | | 4 |
| WATERS05 [Wat05, §4] Symmetric | - | $\mathbb{G}^{4+n}$ | $\mathbb{G}^2$ | $\mathbb{G}^2 \times \mathbb{G}_T$ | $\mathbb{G}^4 \times \mathbb{G}_T$ | DBDH | |
| Asymmetric [Min. PK] | 725 ms | $\mathbb{G}_1^{4+n} \times \mathbb{G}_2^2$ | $\mathbb{G}_1 \times \mathbb{G}_2$ | $\mathbb{G}_1 \times \mathbb{G}_2 \times \mathbb{G}_T$ | $\mathbb{G}_1^4 \times \mathbb{G}_2^3 \times \mathbb{G}_T$ | | 8 |
| Asymmetric [Min. SK] | 770 ms | $\mathbb{G}_1^{4+n} \times \mathbb{G}_2^{3+n}$ | $\mathbb{G}_1^2$ | $\mathbb{G}_2^2 \times \mathbb{G}_T$ | $\mathbb{G}_1^3 \times \mathbb{G}_2^3 \times \mathbb{G}_T$ | | 8 |
| Asymmetric [Min. CT] | 767 ms | $\mathbb{G}_1^{3+n} \times \mathbb{G}_2^{4+n}$ | $\mathbb{G}_2^2$ | $\mathbb{G}_1^2 \times \mathbb{G}_T$ | $\mathbb{G}_1^3 \times \mathbb{G}_2^3 \times \mathbb{G}_T$ | | 8 |
| Asymmetric [Min. Assump] | 716 ms | $\mathbb{G}_1^{4+n} \times \mathbb{G}_2^{3+n}$ | $\mathbb{G}_1^2$ | $\mathbb{G}_2^2 \times \mathbb{G}_T$ | $\mathbb{G}_1^3 \times \mathbb{G}_2^3 \times \mathbb{G}_T$ | | 8 |
| WATERS09 (DSE) [Wat09, §3.1] Symmetric | - | $\mathbb{G}^{13} \times \mathbb{G}_T$ | $\mathbb{G}^8 \times \mathbb{Z}_p$ | $\mathbb{Z}_p \times \mathbb{G}^9 \times \mathbb{G}_T$ | $(\mathbb{G}^4 \times \mathbb{G}_T), (\mathbb{G}^6), (\mathbb{G}^6)$ | DBDH, DLIN, DLIN | |
| Asymmetric [Min. PK] | 6217 ms | $\mathbb{G}_1^{10} \times \mathbb{G}_2^4 \times \mathbb{G}_T$ | $\mathbb{G}_1^4 \times \mathbb{G}_2^4 \times \mathbb{Z}_p$ | $\mathbb{G}_1^5 \times \mathbb{G}_2^4 \times \mathbb{G}_T$ | $(\mathbb{G}_1^4 \times \mathbb{G}_2^3 \times \mathbb{G}_T), (\mathbb{G}_1^6 \times \mathbb{G}_2^6)$ | | 256 |
| Asymmetric [Min. SK] | 5871 ms | $\mathbb{G}_1^7 \times \mathbb{G}_2^{13} \times \mathbb{G}_T$ | $\mathbb{G}_1^8 \times \mathbb{Z}_p$ | $\mathbb{G}_2^9 \times \mathbb{G}_T$ | $(\mathbb{G}_1^3 \times \mathbb{G}_2^3 \times \mathbb{G}_T), (\mathbb{G}_1^6 \times \mathbb{G}_2^6), (\mathbb{G}_1^6 \times \mathbb{G}_2^6)$ | | 256 |
| Asymmetric [Min. CT] | 5858 ms | $\mathbb{G}_1^{13} \times \mathbb{G}_2^7 \times \mathbb{G}_T$ | $\mathbb{G}_2^8 \times \mathbb{Z}_p$ | $\mathbb{G}_1^9 \times \mathbb{G}_T$ | $(\mathbb{G}_1^3 \times \mathbb{G}_2^3 \times \mathbb{G}_T), (\mathbb{G}_1^6 \times \mathbb{G}_2^6), (\mathbb{G}_1^6 \times \mathbb{G}_2^6)$ | | 256 |
| Asymmetric [Min. Assump] | 6228 ms | $\mathbb{G}_1^{12} \times \mathbb{G}_2^5 \times \mathbb{G}_T$ | $\mathbb{G}_1^3 \times \mathbb{G}_2^5 \times \mathbb{Z}_p$ | $\mathbb{G}_1^6 \times \mathbb{G}_2^3 \times \mathbb{G}_T$ | $(\mathbb{G}_1^4 \times \mathbb{G}_2^2 \times \mathbb{G}_T), (\mathbb{G}_1^6 \times \mathbb{G}_2^6), (\mathbb{G}_1^6 \times \mathbb{G}_2^6)$ | | 256 |
| *Broadcast Encryption* | | | | | | | |
| BGW05 [BGW05, §3.1] Symmetric ($n$ users) | - | $\mathbb{G}^{2n+1}$ | $\mathbb{G}$ | $\mathbb{G}^3$ | $\mathbb{G}^{2l+1} \times \mathbb{G}_T$ | decision $l$-BDHE | |
| Asymmetric [Min. PK] | 530 ms | $\mathbb{G}_1^{2n+1} \times \mathbb{G}_2^{2n}$ | $\mathbb{G}_2$ | $\mathbb{G}_1^2 \times \mathbb{G}_T$ | $\mathbb{G}_1^{2l} \times \mathbb{G}_2^{2l+1} \times \mathbb{G}_T$ | | 4 |
| Asymmetric [Min. SK] | 601 ms | $\mathbb{G}_1^{2n} \times \mathbb{G}_2^{2n+1}$ | $\mathbb{G}_1$ | $\mathbb{G}_2^2 \times G_T$ | $\mathbb{G}_1^{2l} \times \mathbb{G}_2^{2l+1} \times \mathbb{G}_T$ | | 4 |
| Asymmetric [Min. CT] | 587 ms | $\mathbb{G}_1^{2n+1} \times \mathbb{G}_2^{2n}$ | $\mathbb{G}_2$ | $\mathbb{G}_1^2 \times \mathbb{G}_T$ | $\mathbb{G}_1^{2l} \times \mathbb{G}_2^{2l+1} \times \mathbb{G}_T$ | | 4 |
| Asymmetric [Min. Assump] | 544 ms | $\mathbb{G}_1^{2n+1} \times \mathbb{G}_2^{2n}$ | $\mathbb{G}_2$ | $\mathbb{G}_1^2 \times \mathbb{G}_T$ | $\mathbb{G}_1^{2l} \times \mathbb{G}_2^{2l+1} \times \mathbb{G}_T$ | | 4 |
| *Signature* | | | | | | | |
| ACDKNO [ACD+12, §5.3] Symmetric | - | $\mathbb{G}^{15}$ | $\mathbb{G}^2$ | $\mathbb{G}^8$ | $(\mathbb{G}^4), (\mathbb{G}^6), (\mathbb{G}^6)$ | CDH, DLIN, DLIN | |
| Asymmetric [Min. PK] | 18216 ms | $\mathbb{G}_1^{14} \times \mathbb{G}_2^5$ | $\mathbb{G}_2^2$ | $\mathbb{G}_1 \times \mathbb{G}_2^7$ | $(\mathbb{G}_1^2 \times \mathbb{G}_2^4)$, $(\mathbb{G}_1^2 \times \mathbb{G}_2^6)$, $(\mathbb{G}_1^2 \times \mathbb{G}_2^6)$ | | 1024 |
| Asymmetric [Min. Sig] | 14689 ms | $\mathbb{G}_1^6 \times \mathbb{G}_2^{14}$ | $\mathbb{G}_1^2$ | $\mathbb{G}_1^8$ | $(\mathbb{G}_1^4 \times \mathbb{G}_2^2)$, $(\mathbb{G}_1^6 \times \mathbb{G}_2^2)$, $(\mathbb{G}_1^6 \times \mathbb{G}_2^2)$ | | 1024 |
| Asymmetric [Min. Assump] | 18135 ms | $\mathbb{G}_1^5 \times \mathbb{G}_2^{14}$ | $\mathbb{G}_1^2$ | $\mathbb{G}_1^7 \times \mathbb{G}_2$ | $(\mathbb{G}_1^4 \times \mathbb{G}_2^2)$, $(\mathbb{G}_1^6 \times \mathbb{G}_2^2)$, $(\mathbb{G}_1^6 \times \mathbb{G}_2^2)$ | | 1024 |
| BLS [BLS04, §2.2] Symmetric | - | $\mathbb{G}^2$ | $\mathbb{Z}_p^*$ | $\mathbb{G}$ | $\mathbb{G}^4$ | CDH | |
| Asymmetric [Min. PK] | 515 ms | $\mathbb{G}_2^2$ | $\mathbb{Z}_p^*$ | $\mathbb{G}_1$ | $(\mathbb{G}_1^4 \times \mathbb{G}_2^3)$, $(\mathbb{G}_1^3 \times \mathbb{G}_2^3)$, $(\mathbb{G}_1^3 \times \mathbb{G}_2^3)$ | | 2 |
| Asymmetric [Min. Sig] | 556 ms | $\mathbb{G}_2^2$ | $\mathbb{Z}_p^*$ | $\mathbb{G}_1$ | $(\mathbb{G}_1^4 \times \mathbb{G}_2^3)$, $(\mathbb{G}_1^3 \times \mathbb{G}_2^3)$, $(\mathbb{G}_1^3 \times \mathbb{G}_2^3)$ | | 2 |
| Asymmetric [Min. Assump] | 517 ms | $\mathbb{G}_2^2$ | $\mathbb{Z}_p^*$ | $\mathbb{G}_1$ | $(\mathbb{G}_1^4 \times \mathbb{G}_2^3)$, $(\mathbb{G}_1^3 \times \mathbb{G}_2^2)$, $(\mathbb{G}_1^3 \times \mathbb{G}_2^3)$ | | 2 |
| WATERS05 [Wat05, §7] Symmetric | - | $\mathbb{G}^{4+n}$ | $\mathbb{G}$ | $\mathbb{G}^2$ | $\mathbb{G}^4 \times \mathbb{G}_T$ | DBDH | |
| Asymmetric [Min. PK] | 724 ms | $\mathbb{G}_1^{4+n} \times \mathbb{G}_2^2$ | $\mathbb{G}_1$ | $\mathbb{G}_1 \times \mathbb{G}_2$ | $\mathbb{G}_1^4 \times \mathbb{G}_2^2 \times \mathbb{G}_T$ | | 8 |
| Asymmetric [Min. Sig] | 721 ms | $\mathbb{G}_1^{4+n} \times \mathbb{G}_2^{3+n}$ | $\mathbb{G}_1$ | $\mathbb{G}_1^2$ | $\mathbb{G}_1^3 \times \mathbb{G}_2^3 \times \mathbb{G}_T$ | | 8 |
| Asymmetric [Min. Assump] | 755 ms | $\mathbb{G}_1^{4+n} \times \mathbb{G}_2^2$ | $\mathbb{G}_1$ | $\mathbb{G}_1 \times \mathbb{G}_2$ | $\mathbb{G}_1^4 \times \mathbb{G}_2^2 \times \mathbb{G}_T$ | | 8 |

Figure 8: A summary of the experimental evaluations of AutoGroup+ on a variety of schemes and optimization options. For the symmetric baseline with curve SS1536, elements in $\mathbb{G}$ are 1536 bits and $\mathbb{G}_T$ are 3072 bits. For the asymmetric translations with BN256, elements in $\mathbb{G}_1$ are 256 bits, $\mathbb{G}_2$ are 1024 bits, and $\mathbb{G}_T$ are 3072 bits. For BGW05, the private key size is listed for a single user.

C++, Python or LaTeX.

## 5 AutoGroup+: Experimental Evaluation

We tested AutoGroup+ on 8 schemes, with 3-4 optimization options and 4 different levels of BB HIBE, for 45 total translations.[9] Figure 8 summarizes the translation times and resulting scheme sizes.[10] To demonstrate the improvement in running times due to both the asymmetric setting and AutoGroup+'s optimizations, Figure 9 includes over 130 timing experiments, showing drastic improvements. In Figure 10, we summarize the effect of scheme complexity on AutoGroup+ conversion time by varying the complexity of BB HIBE. We note that even given a more complex scheme than attempted by any other tool, AutoGroup+ still provides fast conversion times.

[9]Currently the tool does not support the assumption minimization option for schemes with more than one assumption. This is future work, although we would like to explore how valuable assumption minimization is to tool users.

[10]We only give details for two variations of BB HIBE because the results are similar for all levels.

| | Time• | | | |
|---|---|---|---|---|
| | **Setup** | **Keygen** | **Encrypt/ Sign** | **Decrypt/ Verify** |
| *ID-Based Enc.* | | | | |
| BB04 HIBE [BB04a, §4] Symmetric (SS1536) (l = 2) | 346.47 ms | 84.75 ms | 118.64 ms | 133.48 ms |
| Asymmetric (BN256) [Min. PK] | 5.09 ms | 4.79 ms | 12.92 ms | 21.36 ms |
| Asymmetric (BN256) [Min. SK] | 8.15 ms | 2.95 ms | 14.95 ms | 21.32 ms |
| Asymmetric (BN256) [Min. CT] | 9.84 ms | 6.23 ms | 12.38 ms | 21.22 ms |
| Asymmetric (BN256) [Min. Assump] | 9.08 ms | 7.30 ms | 12.27 ms | 21.64 ms |
| BB04 HIBE [BB04a, §4] Symmetric (SS1536) (l = 9) | 892.69 ms | 283.11 ms | 217.39 ms | 446.10 ms |
| Asymmetric (BN256) [Min. PK] | 9.25 ms | 17.64 ms | 17.10 ms | 70.84 ms |
| Asymmetric (BN256) [Min. SK] | 20.53 ms | 11.14 ms | 24.36 ms | 71.45 ms |
| Asymmetric (BN256) [Min. CT] | 21.60 ms | 27.02 ms | 16.48 ms | 72.03 ms |
| Asymmetric (BN256) [Min. Assump] | 21.68 ms | 31.96 ms | 16.77 ms | 70.48 ms |
| GENTRY06 [Gen06, §3.1] Symmetric (SS1536) | 172.30 ms | 28.23 ms | 137.79 ms | 48.42 ms |
| Asymmetric (BN256) [Min. PK] | 2.88 ms | 2.47 ms | 21.08 ms | 10.01 ms |
| Asymmetric (BN256) [Min. SK] | 4.22 ms | 1.18 ms | 22.46 ms | 9.96 ms |
| Asymmetric (BN256) [Min. CT] | 2.93 ms | 2.53 ms | 21.02 ms | 10.02 ms |
| Asymmetric (BN256) [Min. Assump] | 2.88 ms | 2.53 ms | 21.10 ms | 10.09 ms |
| WATERS05 [Wat05, §4] Symmetric (SS1536) | 908.94 ms | 29.78 ms | 78.08 ms | 111.76 ms |
| Asymmetric (BN256) [Min. PK] | 10.31 ms | 2.04 ms | 11.98 ms | 14.23 ms |
| Asymmetric (BN256) [Min. SK] | 24.11 ms | 1.37 ms | 13.68 ms | 14.11 ms |
| Asymmetric (BN256) [Min. CT] | 25.39 ms | 3.67 ms | 11.25 ms | 14.23 ms |
| Asymmetric (BN256) [Min. Assump] | 23.81 ms | 1.36 ms | 13.71 ms | 14.38 ms |
| WATERS09 (DSE) [Wat09, §3.1] Symmetric (SS1536) | 755.50 ms | 195.27 ms | 212.88 ms | 414.79 ms |
| Asymmetric (BN256) [Min. PK] | 23.13 ms | 9.71 ms | 13.70 ms | 66.45 ms |
| Asymmetric (BN256) [Min. SK] | 36.83 ms | 7.07 ms | 20.08 ms | 66.42 ms |
| Asymmetric (BN256) [Min. CT] | 34.41 ms | 14.82 ms | 11.08 ms | 66.92 ms |
| Asymmetric (BN256) [Min. Assump] | 29.90 ms | 11.09 ms | 13.03 ms | 66.92 ms |
| *Broadcast Encryption* | | | | |
| BGW05 [BGW05, §3.1] Symmetric (SS1536) ($n = 10$) | 376.84 ms | 140.27 ms | 86.96 ms | 68.65 ms |
| Asymmetric (BN256) [Min. PK] | 55.29 ms | 13.98 ms | 11.457 ms | 6.13 ms |
| Asymmetric (BN256) [Min. SK] | 38.45 ms | 5.82 ms | 12.49 ms | 8.122 ms |
| Asymmetric (BN256) [Min. CT] | 37.75 ms | 12.32 ms | 11.18 ms | 6.27 ms |
| Asymmetric (BN256) [Min. Assump] | 37.74 ms | 12.31 ms | 11.186 ms | 6.12 ms |
| *Signature* | | | | |
| ACDKNO [ACD+12, §5.3] Symmetric (SS1536) | 395.23 ms | 497.04 ms | 275.99 ms | 937.14 ms |
| Asymmetric (BN256) [Min. PK] | 9.05 ms | 17.19 ms | 15.27 ms | 147.62 ms |
| Asymmetric (BN256) [Min. Sig] | 8.31 ms | 22.65 ms | 14.33 ms | 152.60 ms |
| Asymmetric (BN256) [Min. Assump] | 8.43 ms | 22.23 ms | 13.94 ms | 147.77 ms |
| BLS [BLS04, §] Symmetric (SS1536) | - | 93.20 ms | 92.61 ms | 167.73 ms |
| Asymmetric (BN256) [Min. PK] | - | 2.99 ms | 0.74 ms | 14.20 ms |
| Asymmetric (BN256) [Min. Sig] | - | 3.00 ms | 0.75 ms | 14.20 ms |
| Asymmetric (BN256) [Min. Assump] | - | 3.03 ms | 0.69 ms | 14.18 ms |
| WATERS05 [Wat05, §7] (SS1536) | - | 720.75 ms | 29.72 ms | 135.00 ms |
| Asymmetric (BN256) [Min. PK] | - | 10.42 ms | 2.02 ms | 21.44 ms |
| Asymmetric (BN256) [Min. Sig] | - | 25.60 ms | 1.43 ms | 23.13 ms |
| Asymmetric (BN256) [Min. Assump] | - | 10.18 ms | 2.01 ms | 21.42 ms |

•Average time measured over 100 test runs and the standard deviation in all test runs were within ±1% of the average.

Figure 9: A summary of the running times of the AutoGroup+ translations using curve BN256 as compared to the running times using the roughly security-equivalent symmetric curve SS1536 in MIRACL. The asymmetric setting plus AutoGroup+'s optimizations cut the running times by one or two orders of magnitude.

*System Configuration.* All of our benchmarks were executed on a standard workstation that has a 2.20GHz quad-core Intel Core i7-2720QM processor with 8GB RAM running Ubuntu 11.04 LTS, Linux Kernel version 2.6.38-16-generic (x86-64-bit architecture). Our measurements only use a single core of the Intel processor for consistency. The AutoGroup+ implementation utilizes the same building blocks as AutoGroup which include the MIRACL library (v5.5.4) and/or RELIC cryptographic toolkit [AG], Charm v0.43 [AGM+13] in C++ or Python code, and the Z3 SMT solver (v4.3.2).

*Limitations.* In Section 3.1.1, we provide an example of a scheme which falls into a category of things that Abe et al. warned about and on which AutoGroup gets confused. AutoGroup tries to power

|  | Conversion Time | Num. Solutions |
|---|---|---|
| BB04 HIBE [BB04a, §4] (l = 2) | - | - |
| Asymmetric [Min. PK] | 592 ms | 16 |
| Asymmetric [Min. SK] | 641 ms | 16 |
| Asymmetric [Min. CT] | 626 ms | 16 |
| Asymmetric [Min. Assump] | 582 ms | 16 |
| BB04 HIBE [BB04a, §4] (l = 6) | - | - |
| Asymmetric [Min. PK] | 2361 ms | 256 |
| Asymmetric [Min. SK] | 2019 ms | 256 |
| Asymmetric [Min. CT] | 2023 ms | 256 |
| Asymmetric [Min. Assump] | 2375 ms | 256 |
| BB04 HIBE [BB04a, §4] (l = 7) | - | - |
| Asymmetric [Min. PK] | 4555 ms | 512 |
| Asymmetric [Min. SK] | 3644 ms | 512 |
| Asymmetric [Min. CT] | 3662 ms | 512 |
| Asymmetric [Min. Assump] | 4519 ms | 512 |
| BB04 HIBE [BB04a, §4] (l = 8) | - | - |
| Asymmetric [Min. PK] | 9344 ms | 1024 |
| Asymmetric [Min. SK] | 7148 ms | 1024 |
| Asymmetric [Min. CT] | 7194 ms | 1024 |
| Asymmetric [Min. Assump] | 9299 ms | 1024 |
| BB04 HIBE [BB04a, §4] (l = 9) | - | - |
| Asymmetric [Min. PK] | 20629 ms | 2048 |
| Asymmetric [Min. SK] | 15714 ms | 2048 |
| Asymmetric [Min. CT] | 15690 ms | 2048 |
| Asymmetric [Min. Assump] | 20904 ms | 2048 |

Figure 10: A summary of the conversion times of AutoGroup+ for various levels/degrees of complexity of BB04 HIBE [BB04a, §4] and a variety of optimization options.

through and split the hash output (which it cannot really do because the discrete log is unknown), so while it eventually outputs some SDL, this SDL is not a proper translation. Unlike AutoGroup, AutoGroup+ includes logic to output a warning when processing such inputs and continues trying to translate the scheme. If the verification check of a valid split fails (*e.g.*, due to hash split), then AutoGroup+ identifies the split as invalid and attempts checking the next best solution. If there are no such solutions, AutoGroup+ outputs no solution.

## 5.1 Comparison with ACSC/Charm

Our experiments have four schemes in common with public implementations in the Advanced Crypto Software Collection [Con] and Charm [AGM+13]. Where we have matches, our new results confirm the security and optimality of those (unproven) implemented translations.

For Waters 2009 [Wat09], we compare with the Charm implementation by Fan Zhang. For our PK-size optimization, our translation is 3 elements shorter (we split only $g$, whereas they split $g, w, u, h$.) For our ciphertext-size optimization, it looks the closest to theirs, but they do not match. Both translations have short ciphertexts leaving all base elements in $\mathbb{G}_1$. However, the Charm translation appears to have shifted some elements from the public key to the secret key and dropped some elements from the master secret key (e.g., we split $v$ and include both in the MSK, because that is the naive way to do it, but they use the $v$ split for $\mathbb{G}_1$ only in the Setup and then drop it from the MSK.) While we cannot confirm the security of this implementation using our tool (so we believe this is left as an open question), the tool did produce a translation with the same ciphertext-size that is secure.

For BGW 2005 [BGW05], we compared with the C implementation on the ACSC website by Matt Steiner and Ben Lynn. Indeed, our translations that minimize the public parameters or ciphertext size are the same, and the same as their manual translation. We confirm security and PP/ciphertext-size optimality.

For BB HIBE [BB04a], Charm has a full HIBE implementation. We tested it for a minimum of 2 levels, but their implementation matches ours for ciphertext minimization, except that they add a precomputed pairing (element in $\mathbb{G}_T$) to the public key so that it does not have to be done per encryption. This impacts only efficiency. We confirm security and ciphertext-size optimality.

For BLS [BLS04], our translations also match. This is a simple case with only two translation options.

Charm [AGM+13] also includes variants of the Waters encryption and signature schemes [Wat05] from 2005, but we translated the original schemes (as did [AGH13, AGOT14]), so our translations are not directly comparable to these Charm variants.

## 5.2 Comparison with Abe et al.

Abe et al. [AGOT14] tested their method on two encryption schemes: Waters 2005 [Wat05] and Waters 2009 (Dual System Encryption) [Wat09]. They looked at minimizing the size of the public key and the Type-III assumption. We conjecture that practitioners would be more interested in minimizing ciphertext or private key size, so our summary also includes those optimizations.

For Waters 2005, AutoGroup+ found the same construction as their semi-automated method. As remarked in Section 3.1.1, their dependency graph for this scheme included some unnecessary dependencies. Waters [Wat05] clearly states to choose $g_2, u', u_i$ as fresh random generators, but Abe et al. explicitly "assume" that they are generated from a common generator $g$. From a functionality and security standpoint *of the Type-I scheme*, this distinction certainly does not matter. However, it does change the intermediate dependency graphs, which could in some cases affect the output (though it does not in this situation). Both their partial automation and our full automation of Waters 2005 took under one second.

For Waters 2009, AutoGroup+ found a PK-optimized construction with one less group element than the PK-optimized construction of Abe et al. (They published an efficiency summary, but not the scheme itself.) The way we count public parameters/keys appears to differ from Abe et al., so a direct comparison to their tables can be misleading. We count all elements that must appear in the public key, including generators, whereas they appear to count only elements that must be calculated (or were perhaps misled by an implicit inclusion in [Wat09].) Waters09 [Wat09] must have 13 elements in $\mathbb{G}$ for the PK in his symmetric construction (because $g$ is needed for encryption, as we write), but Abe et al. report it as 12 elements in $\mathbb{G}$ (we assume they omitted generator $g$). We end up with 14 total elements in the asymmetric translation, whereas they report 13, but it seems they are omitting the splitting of $g$ into generators for $\mathbb{G}_1$ and $\mathbb{G}_2$, which would make it 15.

A good take-away from this is not whether we found a slightly better construction for one case or not, but simply that these translations are difficult and complex, and will very much benefit from a new automation tool. Drawing and merging the dependency graphs for Waters09 by hand is a nightmare and becomes infeasible for a complex scheme like [ACD+12]. In addition, their graph splitting program took 1.75 hours for Waters09, whereas our tool handled everything in 6.5 seconds. In fairness, Abe et al. was primarily focused on theoretical results which helped make this more systems-focused work possible.

### 5.3 Comparison with AutoGroup

The publicly-available AutoGroup software tool [AGH13] was used as the starting point for our implementation, hence the name of AutoGroup+. Our 45 translation experiments overlap with AutoGroup in 12 points (six schemes in common and they do fewer optimizations). For these 12, the tools found the same constructions. However, a major difference is that with AutoGroup+, we have security guarantees. This required us to write new SDL descriptions for all the assumptions and proofs involved.

Indeed, one crucial question was how the security logic would increase translation times. We focused our effort on leveraging an SMT Solver to help handle this security logic, which kept the running times of AutoGroup+ within a few seconds of AutoGroup.

In addition to the security logic we added, we also found that the public key optimization flag for encryption was not implemented. Because we wanted to compare our results with [AGOT14], we implemented it.

AutoGroup was tested on two signature schemes omitted here. Camenisch-Lysyanskaya [CL04] is not valid for AutoGroup+, because it uses an interactive complexity assumption (also not handled by [AGOT14].) Boneh-Boyen [BB04c] has a nested proof structure that makes its reduction very complex to translate to SDL.

### 5.4 Comparison with manual translations

The Dual System Encryption scheme of Waters [Wat09] has a few manual translations with a security analysis. Ramanna, Chatterjee and Sarkar [RCS12] provide a variety of translations, one with the smallest public parameter/key size, at the cost of introducing some mild complexity assumptions. Similarly, Chen, Lim, Ling, Wang and Wee [CLL$^+$13] presented a translation introducing the SXDH assumption, which achieved the shortest ciphertext size. These results are superior to those derived by AutoGroup+ and [AGH13, AGOT14], but it is not yet clear how to generalize and systematize the human creativity used.

## 6 Conclusions

Automation is the future for many cryptographic design tasks. This work successfully demonstrates automating a complex translation of a scheme from one algebraic setting to another. There was a demonstrated need for such a compiler both for pairing designers and implementors. Its realization combined and improved on contributions from the systems [AGH13] and theory [AGOT14] communities. The result is a practical software tool, AutoGroup+, that enables secure pairing translations for everyone.

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## A Current Efficiency Numbers for Type-I and Type-III Pairings

| | **Size (in bits)** | | | **Exp. Time (in milliseconds)** | | | |
|---|---|---|---|---|---|---|---|
| Sym. vs. Asym. Setting | $\mathbb{G}_1$ | $\mathbb{G}_2$ | $\mathbb{G}_T$ | $\mathbb{G}_1$ | $\mathbb{G}_2$ | $\mathbb{G}_T$ | **Pairing Time** |
| SS1536 (or Type-I) | 1536 | 1536 | 3072 | 5.3 ms | 5.3 ms | 1.0 ms | 14.9 ms |
| BN256 (or Type-III) | 256 | 1024 | 3072 | 0.2 ms | 1.2 ms | 2.1 ms | 2.2 ms |

Table 1: Comparing Size and Efficiency of Pairing-based Curves.

We include current efficiency numbers for Type-I and Type-III groups as implemented in the highly efficient RELIC cryptographic toolkit version 0.4 [AG] (using the GMP library [Gt12] for big number operations and the default configuration options for prime field arithmetic) measured on a standard workstation.[11] In Table 1, we show the differences between Type-I and Type-III pairings at the same security level in terms of group representation and efficiency. [12] A typical candidate for Type-I are supersingular elliptic curves (or SS) [Gal01, PSV06] in which the embedding degree is typically small (*i.e.*, $k \leq 6$). One such example is a supersingular curve at the 128-bit security level where the prime order of the group is large, $|p| = 1536$-bits, and the embedding degree is $k = 2$. Conversely, one common Type-III candidate at the same security level are Barreto-Naehrig (BN) [BN06] curves in which the embedding degree is much larger (*e.g.*, $k = 12$) and the prime order can be as small as $|p| = 256$-bits. As reflected in Table 1, group operations and pairing times in the Type-III setting can be drastically more efficient and have shorter representations than the Type-I setting.

We remark on hashing into Type-I and Type-III pairing groups. In the Type-I setting, it is feasible to hash arbitrary strings into $\mathbb{G}$, e.g., for the SS curve, hashing arbitrary strings to $\mathbb{G}$ takes on average 36.8 ms. In the Type-III setting (*e.g.*, over ordinary elliptic curves), it is feasible to hash arbitrary strings into both $\mathbb{G}_1$ and $\mathbb{G}_2$ independently with different costs, e.g., for the BN curve, hashing to $\mathbb{G}_1$ takes 0.04 ms and to $\mathbb{G}_2$ takes 0.37 ms on average (a ratio of roughly 9 to 1 from $\mathbb{G}_2$ to $\mathbb{G}_1$). See [GPS06] for more details.

[11] 2.4 GHz Intel Core i5 processor and 8GB of RAM (1067 MHz DDR3) running Mac OS X Lion version 10.7.5

[12] A careful reader may observe that the exponentiation time for $\mathbb{G}_T$ in SS1536 appears surprisingly small. We reassure the reader that this is not a typo. With the SS1536, $\mathbb{G}_T = {F_p}^2$ is a lower extension of a larger field, whereas with BN256, $\mathbb{G}_T = {F_p}^{12}$, which is a higher extension of a smaller field. Thus, even though the elliptic curve points are larger with SS1536, the field multiplication operation in $\mathbb{G}_T$ is quite efficient. This does not apply to $\mathbb{G}_1, \mathbb{G}_2$ as those are doing scalar multiplication.

## B SDL Descriptions for Section 4

We now provide examples of the input and output Scheme Description Language (SDL) for AutoGroup+.

### B.1 SDL as Input

First we will show our SDL transcription of the DBDH assumption:

```
name := DBDH
setting := symmetric

BEGIN :: types
a := ZR
b := ZR
c := ZR
z := ZR
END :: types


BEGIN :: func:setup
input := None
 a := random(ZR)
 b := random(ZR)
 c := random(ZR)
 z := random(ZR)
 g := random(G1)

 assumpKey := list{g, a, b, c, z}

output := assumpKey
END :: func:setup

BEGIN :: func:assump
input := assumpKey
 assumpKey := expand{g, a, b, c, z}
 A := g ^ a
 B := g ^ b
 C := g ^ c

 coinflip := random(bin)
 BEGIN :: if
 if { coinflip == 0 }
   Z := e(g, g) ^ (a * b * c)
 else
   Z := e(g, g) ^ z
 END :: if

 assumpVar := list{g, A, B, C, Z}
output := assumpVar
END :: func:assump
```

Then, the full SDL transcription for the symmetric BB HIBE scheme [BB04b]:

```
name := BB04HIBE
setting := symmetric

BEGIN :: types
M := GT
ID1 := ZR
ID2 := ZR
END :: types


BEGIN :: func:setup
input := None

g := random(G1)
alpha := random(ZR)
g1 := g ^ alpha
h1 := random(G1)
h2 := random(G1)
g2 := random(G1)
g2alpha := g2 ^ alpha

msk := list{g2alpha}
pk := list{g, g1, g2, h1, h2}
output := list{msk, pk}
END :: func:setup


BEGIN :: func:keygen
input := list{pk, msk, ID1, ID2}
pk := expand{g, g1, g2, h1, h2}
msk := expand{g2alpha}

r1 := random(ZR)
r2 := random(ZR)
d1 := g2alpha * \
   (((g1^ID1)*h1)^r1)* \
       (((g1^ID2)*h2)^r2)
d2 := g ^ r1
```

```
d3 := g ^ r2

sk := list{d1, d2, d3}
output := sk
END :: func:keygen


BEGIN :: func:encrypt
input := list{pk, M, ID1, ID2}
pk := expand{g, g1, g2, h1, h2}

s := random(ZR)
C1 := (e(g1,g2)^s) * M
C2 := g ^ s
C3 := ((g1^ID1) * h1)^s
C4 := ((g1^ID2) * h2)^s

ct := list{C1, C2, C3, C4}
output := ct
END :: func:encrypt


BEGIN :: func:decrypt
input := list{pk, sk, ct}
pk := expand{g, g1, g2, h1, h2}
ct := expand{C1, C2, C3, C4}
sk := expand{d1, d2, d3}

M := C1*((e(C3,d2) * \
     e(C4,d3))/(e(C2,d1)))

output := M
END :: func:decrypt
```

Finally, the reduction from [BB04b] for the BB HIBE scheme:

```
name := BB04
setting := symmetric
l := 2
k := 2

BEGIN :: types
l := Int
j := Int
k := Int
M := list{GT}
ID := list{ZR}
```

```
IDstar := list{ZR}
alphai := list{ZR}
h := list{G1}
r := list{ZR}
di := list{G1}
Ci := list{G1}
msk := G1
END :: types


BEGIN :: func:setup
input := list{IDstar}

 a := random(ZR)
 b := random(ZR)
 c := random(ZR)
 z := random(ZR)
 g := random(G1)
 A := g^a
 B := g^b
 C := g^c

 coinflip := random(bin)
BEGIN :: if
 if { coinflip == 0 }
    Z := e(g, g)^(a * b * c)
 else
    Z := e(g, g)^z
END :: if

 g1 := A
 g2 := B
 g3 := C

BEGIN :: for
 for{i := 1, l}
   alphai#i := random(ZR)
   h#i := (g1^-IDstar#i) * (g^alphai#i)
END :: for

 pk := list{g, g1, g2, h}
 assumpVar := list{A, B, C, Z}
 reductionParams := \
     list{g3, alphai, IDstar}
output := list{msk, pk, \
      reductionParams, assumpVar}
END :: func:setup


BEGIN :: func:queries
input := list{j, pk, ID, reductionParams}
pk := expand{g, g1, g2, h}
reductionParams := \
   expand{g3, alphai, IDstar}

BEGIN :: for
 for{i := 1, j}
    r#i := random(ZR)
END :: for

dotProd1 := init(G1)
BEGIN :: for
 for{v := 1, j}
    dotProd1 := dotProd1 * \
       (((g1^(ID#v - IDstar#v)) * \
          (g^alphai#v))^r#v)
END :: for

 d1 := (g2^((-alphai#j)/ \
           (ID#j - IDstar#j)))*dotProd1

BEGIN :: for
 for{i := 1, j}
 BEGIN :: if
   if {i == j }
      di#j := \
      (g2^(-1/(ID#j - IDstar#j))) * \
      (g^r#j)
   else
      di#i := g^r#i
 END :: if
END :: for

sk := list{d1, di}
output := sk
END :: func:queries


BEGIN :: func:challenge
input := list{M, ID, \
      reductionParams, assumpVar}
pk := expand{g, g1, g2, h}
assumpVar := expand{A, B, C, Z}
reductionParams := \
   expand{g3, alphai, IDstar}

 b := random(bin)

 C1 := M#b * Z
 C2 := g3

BEGIN :: for
 for{i := 1, k}
    Ci#k := g3 ^ alphai#i
END :: for

 ct := list{C1, C2, Ci}
output := ct
END :: func:challenge
```

We provide the configuration file that embeds the metadata required by AutoGroup+ to perform the translation:

```
schemeType = "PKENC"
assumption = ["DBDH"]
reduction = ["reductionBB04HIBE"]
short = "public-keys"

masterPubVars = ["pk"]
masterSecVars = ["msk"]

keygenPubVar = "pk"
keygenSecVar = "sk"
ciphertextVar = "ct"

reducCiphertextVar = "ct"
reducQueriesSecVar = "d"
```

## B.2 Translated Scheme and Assumption SDL Descriptions

We now show the SDL outputs of AutoGroup+. The first is the SDL output of the co-DBDH assumption:

```
name := DBDH
setting := asymmetric
```

```
BEGIN :: types
a := ZR
b := ZR
c := ZR
z := ZR
END :: types

BEGIN :: func:setup
input := None
a := random(ZR)
b := random(ZR)
c := random(ZR)
z := random(ZR)
gG1 := random(G1)
gG2 := random(G2)
assumpKey := \
   list{gG1, gG2, a, b, c, z}
output := assumpKey
END :: func:setup

BEGIN :: func:assump
input := assumpKey
assumpKey := \
   expand{gG1, gG2, a, b, c, z}
A := (gG1^a)
BG1 := (gG1^b)
BG2 := (gG2^b)
CG1 := (gG1^c)
CG2 := (gG2^c)
coinflip := random(bin)
BEGIN :: if
if {coinflip == 0}
Z := (e(gG1,gG2)^((a * b) * c))
else
Z := (e(gG1,gG2)^z)
END :: if
assumpVar := list{gG1, gG2, A,\
       BG1, BG2, CG1, CG2, Z}
output := assumpVar
END :: func:assump
```

The second SDL output is the asymmetric BB HIBE scheme [BB04b] that optimally minimizes the public key parameters:

```
name := BB04HIBE
setting := asymmetric

BEGIN :: types
M := GT
ID1 := ZR
ID2 := ZR
END :: types

BEGIN :: func:setup
input := None

gG1 := random(G1)
gG2 := random(G2)
alpha := random(ZR)
g1 := (gG1^alpha)
h1 := random(ZR)
h1G1 := (gG1^h1)
h2 := random(ZR)
h2G1 := (gG1^h2)
g2 := random(ZR)
g2G1 := (gG1^g2)
g2G2 := (gG2^g2)
g2alpha := (g2G1^alpha)
msk := list{g2alpha}
pk := list{gG1, gG2, g1, \
      g2G1, g2G2, h1G1, h2G1}

output := list{msk, pk}
END :: func:setup

BEGIN :: func:keygen
input := list{pk, msk, ID1, ID2}
pk := expand{gG1, gG2, g1, \
      g2G1, g2G2, h1G1, h2G1}
msk := expand{g2alpha}

r1 := random(ZR)
r2 := random(ZR)
d1 := ((g2alpha * \
      (((g1^ID1) * h1G1)^r1)) * \
      (((g1^ID2) * h2G1)^r2))
d2 := (gG2^r1)
d3 := (gG2^r2)
sk := list{d1, d2, d3}
```

```
output := sk
END :: func:keygen

BEGIN :: func:encrypt
input := list{pk, M, ID1, ID2}
pk := expand{gG1, gG2, g1, \
      g2G1, g2G2, h1G1, h2G1}

s := random(ZR)
C1 := ((e(g1,g2G2)^s) * M)
C2 := (gG2^s)
C3 := (((g1^ID1) * h1G1)^s)
C4 := (((g1^ID2) * h2G1)^s)
ct := list{C1, C2, C3, C4}

output := ct
END :: func:encrypt


BEGIN :: func:decrypt
input := list{pk, sk, ct}
pk := expand{gG1, gG2, g1, \
      g2G1, g2G2, h1G1, h2G1}
ct := expand{C1, C2, C3, C4}
sk := expand{d1, d2, d3}

M := (C1*((e(C3,d2) * \
     e(C4,d3))/e(d1,C2)))

output := M
END :: func:decrypt
```